Theoretical study of long distance measurement using frequency comb laser

M. Zeitouny[a], N. Bhattacharya[a], H.P. Urbach[a], and S.A. van den Berg[b]

[a]Optics Research Group, Department of Applied Sciences, Technical University Delft, Lorentzweg 1, 2628 CJ Delft, The Netherlands

[b]NMi Van Swinden Laboratory, Thijsseweg 11, 2629 JA Delft, The Netherlands

ABSTRACT

Frequency and distance metrology have been revolutionized with the arrival of stabilized frequency comb lasers. We discuss several aspects of distance metrology specially the contribution of the dispersion in by air.

1. INTRODUCTION

In the recent years, frequency and distance metrology have undergone a revolution with the progress of stabilized frequency combs lasers. Here we discuss a theoretical study of a long distance measurement using a stabilized femtosecond frequency comb. This technique is promising for measuring distances in space between satellites or in air. We study this technique when applied in air. The maximum distance we can measure is mainly determined by the dispersion of air and the noises introduced by the laser source. An experimental demonstration of this technique has been validated [1] for a distance of $\sim$ 15 cm. In this measurement the pulse train was sent into a Michelson interferometer consisting of a measurement arm and reference arm. The detection of the interference takes place by measuring a cross-correlation function. The cross correlation function is obtained by scanning the delay line with a piezo-element [see figure 1]. A coherence maximum is obtained once the path length difference between arms is a multiple of the cavity length (l_{pp}). $\Delta L = m \times l_{pp}$.

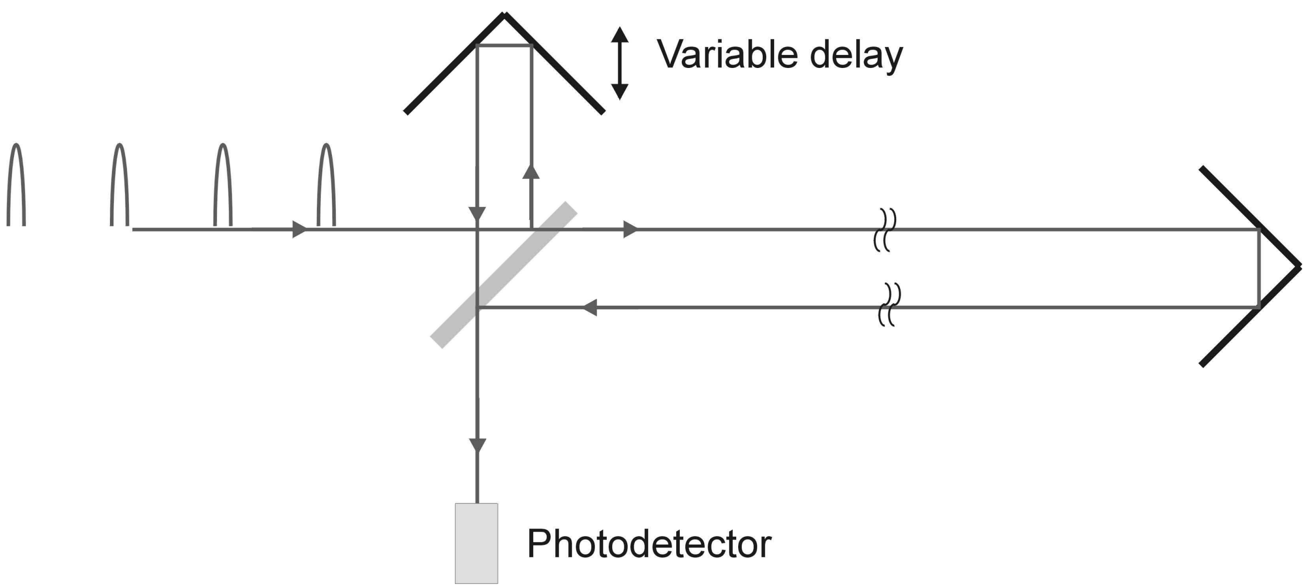

Figure 1. Schematic of the experimental setup

Advanced Optical and Mechanical Technologies in Telescopes and Instrumentation, edited by Eli Atad-Ettedgui, Dietrich Lemke, Proc. of SPIE Vol. 7018, 70183R, (2008) · 0277-786X/08/$18 · doi: 10.1117/12.789098

Downloaded from SPIE Digital Library on 19 Mar 2010 to 131.180.130.114. Terms of Use: http://spiedl.org/terms

2. THEORETICAL ASPECT OF THE AUTO-CORRELATION PATTERN

In this section we shall introduce a mathematical decription of the auto-correlation pattern. E_1 and E_2 represent the electric field in the reference and the measurement arm respectively. Since the interference pattern occurs between two trains of pulses where both are considered infinitely long, therefore E_1 and E_2 will be treated as a sum of fourier expansion in frequency domain.

A frequency comb laser is generated from a phase-stabilized mode-locked laser. The periodic train of pulses emitted from the laser has its counterpart in the frequency domain as a comb of equidistant modes with a mutual separation equal to the circular repetition frequency ω_r.

Generally, we describe our pulses as follows:

$$ReE_1(t) = Re\left\{\int_{-\infty}^{\infty} a_1(\omega)e^{i\omega t}d\omega\right\} \tag{1}$$
$$ReE_2(t) = Re\left\{\int_{-\infty}^{\infty} a_2(\omega)e^{i\omega t}d\omega\right\}$$

Where $a_1(\omega)$ ans $a_2(\omega)$ define the spectra of both pulses (or the Gaussian envelopes in this case). In frequency domain, Eq.1 is written as

$$ReE_1(t) = Re\left\{\sum_{n=1}^{N} a_{1,n}e^{i\omega_n t}\right\} \tag{2}$$
$$ReE_2(t) = Re\left\{\sum_{n=1}^{N} a_{2,n}e^{i\omega_n t}\right\}$$

Conventionally, we select $n = 1$ at the same positon on the left of both pulses. The frequency band intervals of both spectra are not necessarily equal, but we can always select N large enough to cover both spectra. After propagating in two different distances in air, labelled as d_1 and d_2 respectively, and after passing through some optical components such as beam splitters, mirrors, we have

$$ReE_1(t,d_1) = Re\left\{\sum_{n=1}^{N} a_{1,n}R_{1,n}e^{i(\omega_n t - k_{1,n}d_1 + \phi_{1,n})}\right\} \tag{3}$$
$$ReE_2(t,d_2) = Re\left\{\sum_{n=1}^{N} a_{2,n}R_{1,n}e^{i(\omega_n t - k_{2,n}d_2 + \phi_{2,n})}\right\}$$

where,

$$\omega_n = \omega_0 + n\omega_r \tag{4}$$

$k_{1,n}$ and $k_{2,n}$ are the wavenumbers of each train of pulses. We ignore the environment properties fluctuations (such as temperature, pressure and humidity) between both arm, thus $k_{1,n} = k_{2,n} = k$. The absorption of air is temporarily ignored, then

$$k = \frac{n_\omega \times \omega}{c} \tag{5}$$

Where n_ω is the refractive index of air determined by the updated Edlen's Equation [2].The effect of optical components are included in the item $R_{1,n}e^{i\phi_{1,n}}$ and $R_{2,n}e^{i\phi_{2,n}}$, $R_{1,n}$, $R_{2,n}$, $\phi_{1,n}$ and $\phi_{2,n}$ are supposed to be real functions.

The first order cross-correlation gives an intensity as follows:

$$I_\omega(t,d_1,d_2) = (ReE_1 + ReE_2)^2 = (E_1 + E_2)(E_1 + E_2)^* \tag{6}$$

Since the APD (detector)is slow compared to the pulse duration, the time averaged intensity in Eqn.8 is given by:

$$\langle I_w(d_1,d_2)\rangle = \langle (E_1 + E_2)(E_1 + E_2)^*\rangle \tag{7}$$

Downloaded from SPIE Digital Library on 19 Mar 2010 to 131.180.130.114. Terms of Use: http://spiedl.org/terms

Substitute Eqn.3 into Eqn.9, then $x = (d_2 - d_1)$ and $\Delta\phi_{1,2} = (\phi_2 - \phi_1)$, then the time averaged intensity is given by:

$$\langle I_\omega(x)\rangle = \sum_{n=1}^{N} |a_{1,n}|^2 R_{1,n}^2 + |a_{2,n}|^2 R_{2,n}^2 \tag{8}$$

$$+2\sum_{n=1}^{N} |a_{1,n}||a_{2,n}| R_{1,n} R_{2,n} cos[k \times x - \Delta\phi_{1,2}]$$

For simplicity we will take $\Delta\phi_{1,2} = 0$, $R_{1,n} = R_{2,n} = 1$. Typical values are:

$$\begin{array}{ll} \lambda_0 = 0.85\,\mu\text{m}, & \nu_0 = c_0/\lambda_0 = 3.53 \times 10^{14}\,\text{Hz}, \\ T_0 = 2.83 \times 10^{-15}\,\text{s}, & \omega_0 = 2.22 \times 10^{15}\,\text{Hz}. \end{array} \tag{9}$$

Furthermore, the half the duration of the pulse, τ, (2τ is FWHM) has typical value:

$$\tau = 2 \times 10^{-14}\,\text{s} = 7.06\,T_0, \quad \sigma = \frac{1}{\tau} = 5 \times 10^{13}\,\text{Hz} = 0.141\,\nu_0 = 2.254 \times 10^{-2}\,\omega_0. \tag{10}$$

3. PROPAGATION IN AIR

We have done a series of simulations for various distances (30, 60 and 120 m) to see the dispersion effects of air. The temperature and pressure have been set to $20\,C^\circ$ and $1013.25\,hPa$ respectively. We assume that the piezo-element is scanning the measurement arm within $80\,\mu m$, which means that the propagation distance in air $(d_{\,air})$ is varing as: '$d_{air} - 40(\mu m) \leq d_{air} \leq d_{air} + 40(\mu m)$' with a step of $50\,nm$. Let us mention here that the propagated distance is the path length difference between two arms (i.e back and forth). Thus, the measured distance is $d_{\,air}/2$ [see reference 1]. We can see clearly from Fig.2 that after propagation, the auto-correlation patterns become wider.

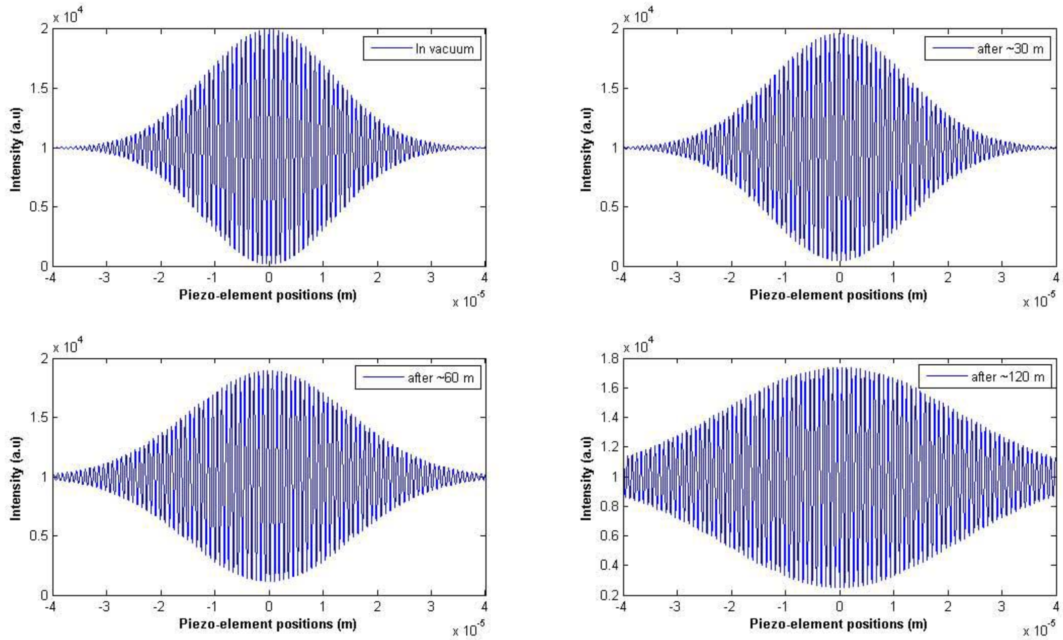

Figure 2. illustration of the auto-correlation patterns for different propagation distances in air

3.1 Effects of pressure and temperature

Downloaded from SPIE Digital Library on 19 Mar 2010 to 131.180.130.114. Terms of Use: http://spiedl.org/terms

Temperature and pressure play an important role for precise length interferometry. In this study, the air has been considered as an homogenous medium at a given temperature and pressure (t, P). We can show that a small variation of temperature and pressure will make an important shift for the auto-correlation pattern (figures 3). Figure 3.a shows the auto correlation pattern after propagating a train of pulses in the measurement arm for a distance of $\sim$ 120 m at $20\,C^\circ$ and $1013.25\,hPa$. The peak of the auto-correlation pattern is centered on zero. We can see that for a temperature variation of $0.2\,C^\circ$ and $0.5\,C^\circ$, the position of the peak is shifted 24 and 57 microns to the forth respectively (figures 3.b, 3.c). Same for the pressure, a varition of $20\,Pa$ and $50\,Pa$ will be able to shift the peak 6 and 16 microns to the back respectively (figures 3.d and 3.e).

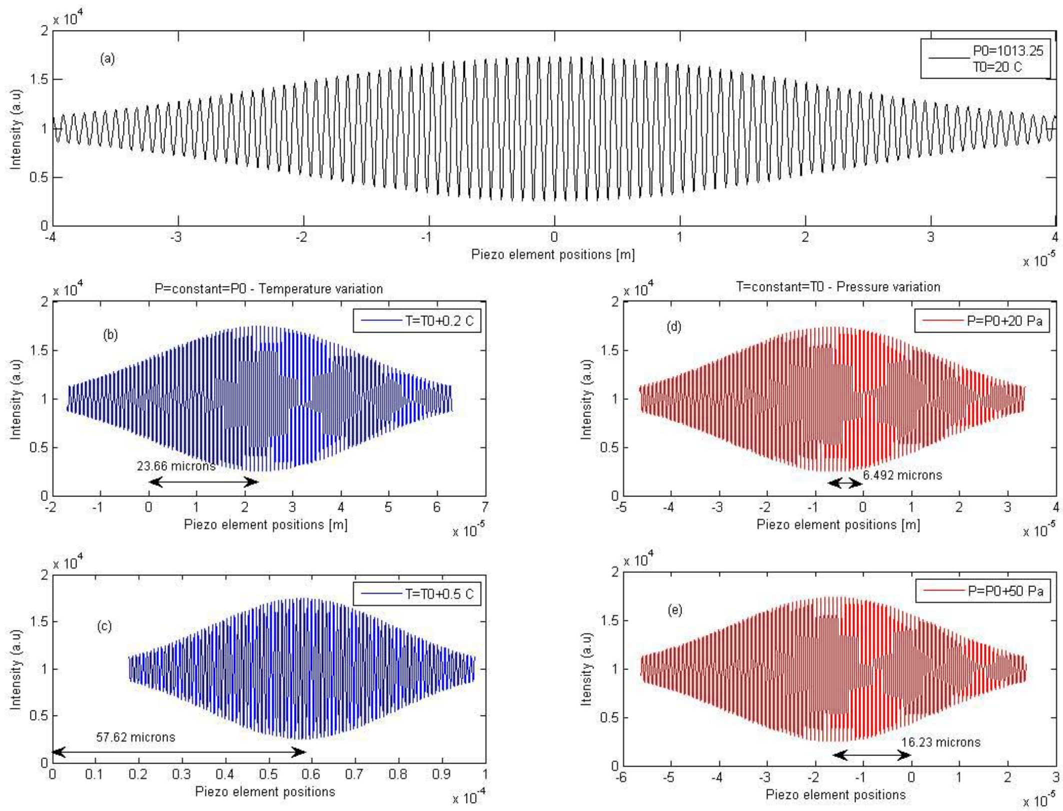

Figure 3. Temperature and pressure effects on the auto-correlation pattern - The study has been done on $\sim$ 120 m of propagation in air

4. CONCLUSION

We have presented a theoretical model of the auto-correlation function generated by a frequency comb laser. Through simulations we have seen that the auto-correlation became wider after long propagation in air. Finally, a small temperature and/or pressure variation is able to shift the pattern (back and forth) up to few microns.

5. REFERENCES

[1] M. Cui, " Experimental demonstration of distance measurement with a femto second frequency comb laser ", JEOS. **3**, 08003 (2008)

[2] K.P.Birch and M.J.Downs, An Updated Edlen Equation for the Refractive Index of Air, Metrologia **30**, 155 (1993).

Downloaded from SPIE Digital Library on 19 Mar 2010 to 131.180.130.114. Terms of Use: http://spiedl.org/terms