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# Adiabatic Computing for Power Flow Analysis

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**Abstract**—We investigate the performance of different annealers for power flow analysis using adiabatic computing. The annealers include D-Wave’s simulated annealer Neal, D-Wave’s quantum-classical hybrid annealer, D-Wave’s Advantages system (QA), Fujitsu’s classical simulated annealer, and Fujitsu’s digital annealer V3 (DA). We implement Quadratic Unconstrained Binary Optimization (QUBO) and Ising model formulations, with the latter offering finer control over complex voltage adjustments. Different test systems are experimented with to systematically evaluate the annealers. The evaluation is based on the accuracy, the annealer’s capability to handle the decision variables, and the computational time needed. QA and DA show superior performance over classical annealers for our application. DA effectively manages larger test systems, whereas QA encounters difficulties embedding the problem graph onto the hardware graph because of the limited qubit connectivity. This constraint confines QA to the 14-bus system with the QUBO formulation and the 4-bus system with the Ising model formulation. The best performance is associated with different annealers across different test systems, which suggests that adjusting the threshold can improve precision if the compiler and annealer are capable of handling the number of variables involved.

## I. INTRODUCTION

The AC power flow (PF) equations are non-linear and non-convex, which rules out their exact analytical solution. Hence, the state-space representation of the power system is conventionally solved using iterative numerical methods, such as the Newton-Raphson (NR) methods [1]. The use of conventional PF analysis methods, however, can cause convergence problems in large-scale modern power systems and is very time-consuming [2]. Therefore, conventional PF analysis approaches are becoming ineffective at supporting modern power system operation and planning [3].

Despite the significant progress made in PF analysis algorithms (e.g., [4]), there remains a critical need for scalable, computationally efficient, and numerically stable PF analysis algorithms to meet the growing requirements of modern power systems. In particular, high levels of distributed energy resources, variable loads, and bidirectional power flows require more precise and efficient PF analysis approaches. In addition, the increasing deployment of advanced control and optimization techniques, such as real-time monitoring and control, demand response, and distributed energy resource management, necessitates the development of more sophisticated PF analysis algorithms. Therefore, although state-of-the-art methods have shown improvements, they are not enough to address the needs of modern power systems [5], [6].

The present work employs the adiabatic quantum computing model (AQC) to enhance PF analysis (e.g., [7]). We develop and validate a combinatorial PF algorithm and an adiabatic quantum PF (AQPF) algorithm. The algorithms use Quadratic Unconstrained Binary Optimization (QUBO) and the Ising model formulations for PF analysis. The algorithms iteratively update the complex voltages based on the outcome of a minimization problem that is solved by AQC. The scalability of the AQPF algorithm and of a partitioned variant is also numerically demonstrated by applying them to various power system benchmarks, i.e., 4-bus, 9-bus, and 14-bus, test systems. We use five different classical/digital/quantum annealers, i.e., D-Wave’s simulated annealer Neal (SA<sub>1</sub>), D-Wave’s quantum-classical hybrid annealer (HA), D-Wave’s Advantages system (QA), Fujitsu’s classical simulated annealer (SA<sub>2</sub>), and Fujitsu’s digital annealer V3 (DA).

## II. METHODOLOGY

For each bus  $i$  in a power system, the mismatch between the given active  $p_i^d$  and reactive power  $q_i^d$ , and their net counterparts,  $p_i$  and  $q_i$ , can be expressed as

$$p_i^d - p_i = 0 \quad (1)$$

$$q_i^d - q_i = 0 \quad (2)$$

where  $p_i$  and  $q_i$  are given by

$$p_i = \sum_{j=0}^n \mu_i g_{ij} \mu_j + \omega_i g_{ij} \omega_j + \omega_i b_{ij} \mu_j - \mu_i b_{ij} \omega_j \quad (3)$$

$$q_i = \sum_{j=0}^n \omega_i g_{ij} \mu_j - \mu_i g_{ij} \omega_j - \mu_i b_{ij} \mu_j - \omega_i b_{ij} \omega_j \quad (4)$$

Here,  $g_{ij}$  and  $b_{ij}$  are the real and imaginary parts of the bus admittance matrix  $Y$ , respectively.  $\mu_i$  and  $\omega_i$  are the real and imaginary parts of the complex voltages needed to be determined. To solve this problem using an annealer,  $\mu_i$  and  $\omega_i$  need to be discretized using *binary* variables. For the QUBO formulation,  $\mu_i$  and  $\omega_i$  are discretized as

$$\mu_i = \mu_i^0 + x_{i,0}^\mu \Delta \mu_i - x_{i,1}^\mu \Delta \mu_i \quad (5)$$

$$\omega_i = \omega_i^0 + x_{i,0}^\omega \Delta \omega_i - x_{i,1}^\omega \Delta \omega_i \quad (6)$$

where  $x_{i,\{0,1\}}^{\{\mu,\omega\}} \in \{0, 1\}$  are *binary decision variables* that determine if  $\mu_i^0$  and  $\omega_i^0$  are increased, decreased, or kept

TABLE I  
COMPARISON OF AQPF WITH THE NEWTON-RAPHSON CLASSICAL SOLVER (NR). THE AQPF ALGORITHM IS IMPLEMENTED USING THE QUBO FORMULATION AND QA SOLVER FOR THE 4-BUS TEST SYSTEM.

	$\mu_1$	$\mu_2$	$\mu_3$	$\omega_1$	$\omega_2$	$\omega_3$
NR	0.902	0.916	0.890	-0.092	-0.080	-0.104
AQPF	0.902	0.916	0.891	-0.092	-0.081	-0.104

unchanged. For the Ising model formulation,  $\mu_i$  and  $\omega_i$  are represented as

$$\mu_i = \mu_i^0 + s_{i,0}^\mu \Delta \mu_i + s_{i,1}^\mu 2 \Delta \mu_i + s_{i,2}^\mu 3 \Delta \mu_i \quad (7)$$

$$\omega_i = \omega_i^0 + s_{i,0}^\omega \Delta \omega_i + s_{i,1}^\omega 2 \Delta \omega_i + s_{i,2}^\omega 3 \Delta \omega_i \quad (8)$$

where  $s_{i,\{0,1,2\}}^{\{\omega, \mu\}} \in \{\pm 1\}$  are *spin variables*. This allows for finer control of the increments/decrements at the cost of additional variables per bus  $i$ .

### III. RESULTS

Solving equations (1)–(2) using an annealer involves minimizing the sum of the squared terms, represented by the Hamiltonian  $H(\vec{x})$  and  $H(\vec{s})$

$$H(\cdot) = \sum_{i=0}^n (p_i^d - p_i)^2 + (q_i^d - q_i)^2. \quad (9)$$

In this context,  $\vec{x}$  and  $\vec{s}$  are vectors of binary and spin variables, respectively. We use the Python packages PyQUBO and DADK to develop the QUBO and Ising models based on the Hamiltonian (9). These packages can effectively reduce higher-order terms. Once the minimization problem is solved, the real and imaginary parts of the voltages are updated until a convergence is obtained.

Table I shows the results obtained for the 4-bus test system for the Ising model formulation using QA. It is observed that the Ising model formulation with QA achieves results closest to those derived from NR, followed by the QUBO formulation with QA, for  $\epsilon = 1 \times 10^{-3}$ . The reference bus  $i = 0$  with known values of  $\mu_0 = 1$  and  $\omega_0 = 0$  is not shown in the table.

The scalability of the AQPF algorithm is assessed using a 9-bus test system. The AQPF algorithm demonstrated scalability, with the Ising model resulted in managing more variables compared to the QUBO formulation. This poses challenges for embedding into quantum annealers due to the limited qubit connectivity in QA systems. Furthermore, SA<sub>1</sub> did not converge with the Ising model formulation. In contrast, HA, SA<sub>2</sub>, and DA effectively handled the increased number of spin variables for the 9-bus test system. Notably, the computational time per iteration on quantum hardware, excluding communication overhead, is significantly below the microsecond scale.

A partitioned AQPF algorithm provides a notable advantage by allowing power flow (PF) analysis even when data is incomplete, inaccurate, or computational resources are limited. This approach has been tested and validated on a 14-bus system, where it demonstrated a reduction of 102 binary variables, achieving up to a 20% decrease compared to the

standard AQPF algorithm. The average deviation between the active power ( $p_i$ ) calculated by the partitioned AQPF algorithm and the desired values ( $p_i^d$ ) from the Newton-Raphson (NR) method is  $3.64 \times 10^{-3} [W]$ , with a standard deviation of  $1.57 \times 10^{-2} [W]$ . For reactive power ( $q_i$ ), the mean deviation is  $1.67 \times 10^{-3} [Var]$ , with a standard deviation of  $1.42 \times 10^{-2} [Var]$ .

### IV. CONCLUSIONS

This work introduces the adiabatic quantum power flow (AQPF) algorithm, leveraging quantum annealers for power flow (PF) analysis. With Fujitsu's Digital Annealer providing up to 100,000 qubits, the AQPF can handle larger power systems compared to current gate-based quantum algorithms, which struggle with systems larger than five buses. A partitioned AQPF algorithm is also proposed to reduce the number of required qubits. This partitioned approach demonstrates significant capabilities in solving PF problems, even with incomplete or inaccurate data or limited computational resources, although it increases the number of iterations needed to reach a defined threshold.

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### REFERENCES

- [1] J. Arrillaga and B. Smith, *AC-DC power system analysis*. London: Institution of Electrical Engineers, 1998.
- [2] G. Mokryani, A. Majumdar, and B. C. Pal, "Probabilistic method for the operation of three-phase unbalanced active distribution networks," *IET Renewable Power Generation*, vol. 10, no. 7, pp. 944–954, 8 2016.
- [3] J. Sexauer, P. Javanbakht, and S. Mohagheghi, "Phasor measurement units for the distribution grid: Necessity and benefits," in *2013 IEEE PES Innovative Smart Grid Technologies Conference (ISGT)*. IEEE, 2 2013, pp. 1–6.
- [4] M. Tostado-Vélez, S. Kamel, and F. Jurado, "Power flow solution of Ill-conditioned systems using current injection formulation: Analysis and a novel method," *International Journal of Electrical Power & Energy Systems*, vol. 127, p. 106669, 5 2021.
- [5] K. Baker, "Solutions of DC OPF are Never AC Feasible," *arXiv*, 11 2019.
- [6] J. S. Giraldo, O. D. Montoya, P. P. Vergara, and F. Milano, "A fixed-point current injection power flow for electric distribution systems using Laurent series," *Electric Power Systems Research*, vol. 211, p. 108326, 10 2022.
- [7] T. Morstyn, "Annealing-Based Quantum Computing for Combinatorial Optimal Power Flow," *IEEE Transactions on Smart Grid*, vol. 14, 2023.