

Energy transport through coordinated wake control in offshore wind farm clusters

An alternative to cable-based transmission

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Energy transport through coordinated wake control in offshore wind farm clusters

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Cover: Wind farm and transmission cable made by A. Bongers and
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Preface

This thesis is part of the requirements for my Master's degree in Applied Physics, within the Energy Technology track.

Energy systems used to be straightforward. Power plants were built close to where energy was needed, and their output could be adjusted to match demand at any moment. Renewable energy resources are different. Now, where and how much energy is generated follows the weather and the landscape: hydropower in the mountains, wind turbines offshore and on open plains. Renewable energy is often generated far from where it's needed most. This connection between the energy system and nature is what drew me to this thesis.

I am proud to have designed a model that uses the connection between atmospheric conditions and the energy system, making smart use of the wind to improve the system as a whole.

My sincere thanks go to my supervisors for their support. It was very valuable to have two supervisors specialised in the two fields this thesis tries to combine: Remco Verzijlbergh, with his extensive knowledge of energy systems, and Marcus Becker, with his expertise in wind farm dynamics.

Finally, I want to thank my friends and family for their support throughout this thesis, and for the fun discussions about the subject. Special thanks go to Miekje Schwippert, who carefully read this thesis and helped me improve my writing, and to Annemiek Bongers, who revised the figures and layout.

In memory of Peter Bongers, who worked on the wind turbine control long before I did.¹

Dennis Hoeflaak
Delft, September 2026

¹Peter .M.M. Bongers, *Modeling and Identification of Flexible Wind Turbines and A Factorizational Approach to Robust Control Design*, PhD thesis, TU Delft, 1994.

Summary

Offshore wind farms in the North Sea are built in proximity of each other. Close enough that one can block and reduce each other's incoming wind. This thesis asks whether that coupling can be used on purpose. Curtailing an upwind farm weakens its wake, increasing the incoming wind speed at the downwind farm. The upwind farm gives up energy, and the downwind farm partly recovers it. That is what a transmission line does, but less efficiently.

Whether the trade-off is worth making depends on the wake physics and the electricity market. Efficiency and direction of the trade-off depends on wake and atmospheric physics. Whether the energy is more needed downwind depends on the energy-market conditions.

Combining them in one optimisation problem is difficult, because the wake is non-linear in exactly the quantity being optimised. The Jensen wake model is therefore linearised in two steps. First, the power curve, the power coefficient curve and the mapping to the axial induction factor are replaced with piecewise linear fits. Second, the remaining dependence on curtailment is removed with a Taylor expansion around two reference points. The result maps a single scalar to the output of every turbine in a two-farm cluster and remains tractable for clusters of 374 turbines.

The cluster is placed in two market settings. In the first, day-ahead prices are fixed, and the operator maximises cluster revenue. Applied to the Belgian–Dutch cluster on 6 August 2021, curtailing the Belgian farm raises revenue from 81.8 to 96.7 kEUR, an increase of 18 %. Kenis et al. report 21 % for the same hour with a Gaussian wake model and multiplicative superposition. The two studies agree on direction and order of magnitude, and the difference follows from the wake model that the linearisation permits. Over 2021 as a whole, the control strategy increases revenue in 310 of 8688 hours and raises annual cluster revenue by 1.1 %.

In the second setting, the price is no longer an input. The cluster is embedded in a nodal model of the Dutch and Belgian electricity systems, which minimises total system cost. The question becomes whether curtailment serves the system rather than the wind farm operator. Four cases are examined; IJmuiden Ver with Hollandse Kust West shows no benefit: a detour with spare capacity always remains available, and the system prefers the cable. Borssele with the Belgian zone in the 2020 system shows none either, because neither country holds a surplus of cheap generation. Adding 10 GW of planned Dutch capacity changes that. At windy hours, the Netherlands generates more cheap wind energy than the demand and interconnectors can handle. Curtailing the upwind farm increases the downwind farm's energy generation, which can displace energy generation from fossil gas. Over a 24 h window 2.2 GW h of gas generation is avoided. Reducing the Belgian nuclear fleet to its 2026 capacity increases the windows when curtailment is useful, because a flexible unit is left on the margin more often. Measured as power gained downwind over power given up upwind, the cluster transports at 29 to 63 % efficiency.

Cluster control is therefore not a substitute for transmission, and the conditions under which it pays are narrow. A congested border, a surplus of cheap generation in the upwind zone, a flexible generator still on the margin downwind, and a wind direction running from one to the other must all hold at the same time. Wherever a route with spare capacity exists, the system takes it instead. In the cases studied here, those conditions were met in a small share of the year, and only after 10 GW of planned capacity was added to the system. But where they do hold, the effect is not marginal: a single eight-hour period displaced 3.1 GW h of gas generation, and the periods over which the wind allows it can last for days rather than hours.

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Nomenclature

Abbreviations

<i>Abbr.</i>	<i>Description</i>	<i>Location</i>
BE	Belgium	p.10
BZ	Bidding zone	p.10
CFD	Computational Fluid Dynamics	p.18
DA	Day-ahead	p.48
ENTSO E	European Network of TSOs for Electricity	p.10
EWM	Engineering Wake Model	p.18
HKW	Wind farm site Hollandse Kust West	p.15
LES	Large-Eddy Simulation	p.18
LP	Linear Programming	p.16
MIQP	Mixed-Integer Quadratic Programming	p.16
NL	The Netherlands	p.10
NLP	Non-Linear Programming	p.16
NWP	Numerical Weather Prediction	p.18
QP	Quadratic Programming	p.16
RANS	Reynolds-Averaged Navier–Stokes	p.18
SOS2	Special Ordered Set of type 2	p.17
TSO	Transmission system operator	p.6

Mathematical Symbols

<i>Symbol</i>	<i>Description</i>	<i>Unit</i>	<i>Location</i>
α_N	Share of installed capacity at which nuclear units are fixed	[–]	p.38
α_Z	Minimum dispatch share of available solar output	[–]	p.38
Δ	Step size of the forward difference in the Taylor expansion	[–]	p.31
η	Curtailment coefficient	[–]	p.22
η^*	Reference optimum curtailment, curtailment found by the non-linear wake model	[–]	p.45
η_0	Taylor expansion reference point	[–]	p.31
$\hat{\eta}$	Optimal curtailment	[–]	p.44
κ	Scaling of the ideal actuator disk relation	[–]	p.23
λ^{DA}	Day-ahead electricity price	EUR/MWh	p.36
ρ	Air density	kg/m ³	p.20
ζ_l	Transmission efficiency of line l	[–]	p.38
A	Rotor area	m ²	p.20
a	Axial induction factor	[–]	p.23
C_T	Thrust coefficient	[–]	p.23
CF_s	Capacity factor of renewable unit s	[–]	p.38
D	Rotor diameter	m	p.24
D_h	Electricity demand at hub h	MW	p.38
F_h	Net power flow into hub h	MW	p.38

F_l	Power flow on transmission line l	MW	p.38
F_l^{max}	Transmission capacity of line l	MW	p.38
K_g	Installed capacity of generating unit g	MW	p.38
k_w	Wake expansion rate	[-]	p.24
P_g	Aggregated power output of generating unit g	MW	p.38
U_∞	Free-stream wind speed	m/s	p.24
W	Wake coefficient matrix	[-]	p.26
X_h	Net exchange at hub h with countries outside the system	MW	p.38
x_{nj}	Streamwise distance between turbines n and j	m	p.25
y_{nj}	Lateral distance between turbines n and j	m	p.25
p	Power output of a single turbine	W	p.20
p^{eff}	Effective power output under curtailment	W	p.22
$p_{\text{up/down}}$	power outputs of the upwind or downwind farm, $p_{\text{up/down}} = (p_n)_{n \in \mathcal{J}_{\text{up/down}}}$	W	p.29
C_P	Power coefficient	[-]	p.20
C_P^{eff}	Effective power coefficient under curtailment	[-]	p.22
$C_{P,\text{up/down}}$	Power coefficients of the upwind or downwind farm, $C_{P,\text{up/down}} = (C_{P,n})_{n \in \mathcal{J}_{\text{up/down}}}$	[-]	p.29

Sets

<i>Symbol</i>	<i>Description</i>	<i>Location</i>
$\text{from}(l), \text{to}(l)$	Hubs at either end of transmission line l	p.38
G	Set of all generating units in the market	p.38
H	Set of all hubs, one per province	p.38
$h(g)$	Hub to which generating unit g is connected	p.38
$\mathcal{J}$	Set of turbines in the controlled two-farm cluster	p.38
$\mathcal{J}_{\text{down}}$	Set of turbines in the downwind farm	p.38
$\mathcal{J}_{\text{up}}$	Set of turbines in the upwind farm	p.38
L	Set of all transmission lines connecting the hubs	p.38
N	Set of all nuclear generating units	p.38
S	Set of renewable generating units outside the controlled cluster	p.38
$\text{up}(n)$	Set of turbines whose wake reaches turbine n	p.38
V	Set of all non-renewable generating units	p.38
Z	Set of all solar generating units	p.38

1. Introduction

Europe's energy landscape is undergoing a fundamental transformation. Initially, this change was driven solely by the need to reduce greenhouse gas emissions. The European Commission prioritised decarbonisation by phasing out coal-fired generation, with natural gas serving as an interim bridge fuel. Meanwhile, renewable energy capacity was scaled up toward the 2050 climate-neutrality target set under the European Green Deal [1]. However, Russia's invasion of Ukraine in 2022 introduced a second, equally urgent driver: strategic energy security. The resulting political and ethical constraints on Russian gas imports exposed the vulnerability of relying on a single dominant supplier [2]. Taken together, these developments reinforce the case for renewable energy sources, which reduce greenhouse gas emissions while diminishing dependence on unreliable third parties.

The core sustainable energy sources are biomass, solar, nuclear, hydropower, and wind. These sources are constrained by site-specific geographical and climatological conditions, making their applicability region-dependent. In the Netherlands, three limitations are particularly prominent. First, the country's flat topography eliminates viable sites for conventional hydropower generation. Second, the Netherlands receives a Global Horizontal Irradiance of approximately 1.000 to 1.050 kWh/m²/y [3], roughly 60% of the irradiance available in Southern European countries such as Spain (~ 1.700 kWh/m²/y) [3]. The third is the constraint of land scarcity. As one of the most densely populated countries, the Netherlands faces intense competition for available land. Competition exists between agriculture, housing, infrastructure, and nature conservation. This renders land-intensive energy technologies such as large-scale solar farms or biomass at a disadvantage. However, the Netherlands benefits from consistently high wind resources [4] and access to the shallow North Sea, where spatial competition is substantially lower than on land. This makes the North Sea particularly well-suited for harvesting wind energy.

As renewables grow, site-specific geographical and climatological dependencies increase, making their applicability region- and time-dependent. Wind and solar produce when the weather allows, not when electricity is needed, and a country the size of the Netherlands sits under a single weather system. On its own, it can have too much power when the wind blows in the North Sea and too little when it does not. Europe as a whole is large enough to span several weather systems at once, so a calm North Sea can coincide with wind elsewhere. Interconnections between these regions can therefore extend the periods in which both have access to cheap, clean electricity.

This thesis introduces a new way of transporting energy through wind instead of transmission cables. This transport is possible when two wind farms lie in proximity. The thesis investigates whether it can be advantageous to curtail an upwind wind farm, so that the downwind farm experiences less wake and therefore produces more energy. In other words, energy is reduced upwind and increased downwind. This is similar to what a transmission line does, though with lower efficiency.

The remainder of this thesis is structured as follows. Chapter 2 reviews the literature on wake modelling, curtailment and energy markets and closes with the research gap and the questions that follow from it. Chapter 3 sets out the research approach and builds the wind farm cluster model: it defines the curtailment coefficient, applies the Jensen wake model to a two-farm cluster, and linearises the result so that it can be embedded in an optimisation problem. The cluster model is first placed under fixed energy prices and then in a nodal representation of the Dutch and Belgian electricity system. Chapter 4 applies the coupled model to four cases and discusses what the results do and do not support. Chapter 5 presents the conclusions and answers the research questions.

2. Literature review

As discussed in Chapter 1, offshore wind plays a crucial role in achieving the Netherlands' renewable targets. The rapid expansion of offshore wind capacity in the North Sea introduces a problem that does not exist at smaller scales: clusters of wind farms become large enough to influence the electricity market while also being physically coupled to one another through atmospheric wake effects. These two coupling mechanisms, one aerodynamic, one economic, have each been studied extensively, but rarely together. This thesis examines the consequences of treating them jointly.

This chapter establishes the context required to identify the gap that follows. Section 2.1 introduces wake interactions and the role of atmospheric stability in their persistence across cluster scales. Section 2.2 outlines the layered structure of European electricity markets and the constraints they impose on wind farm operation. Section 2.3 brings these two perspectives together, showing where they interact and demarcating the present work against the closest existing study. Section 2.3.1 then asks why the literature has not yet comprehensively addressed this integrated perspective, and Section 2.4 formulates the resulting research gap.

2.1. Wake losses persist over tens of kilometres and grow under stable stratification

To reduce emissions and increase energy security, plans are being underway to rapidly expand offshore wind capacity in the North Sea. The European Commission has set targets of at least 60 GW of installed offshore wind capacity by 2030, and 300 GW by 2050 [5]. Figure 2.1 illustrates how the North Sea is expected to transform as a result, showing both current and planned wind farm locations. Offshore wind is therefore set to become a cornerstone of the Dutch energy mix. As offshore wind capacity expands at scale, two principal challenges emerge: inter-farm wake interactions and energy system integration.

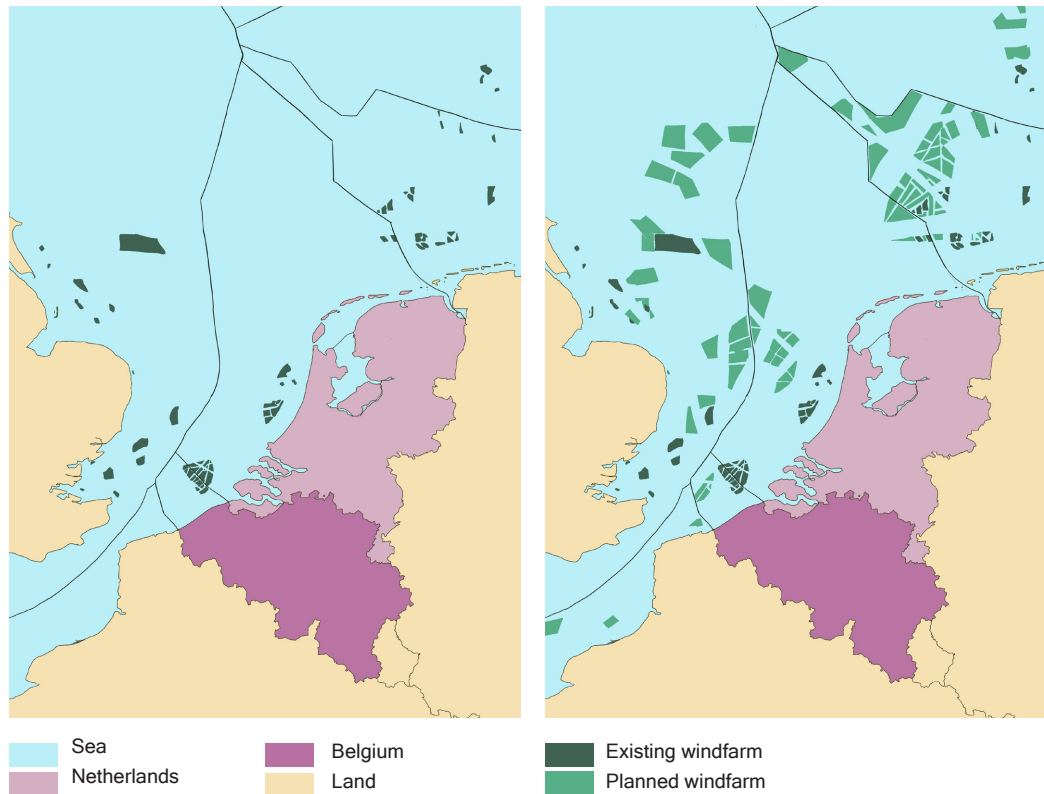


Figure 2.1: Current and future wind farm locations in the North Sea [6].

Wake recovery

Wind farms in close proximity can significantly reduce each other's power output through wake effects. As wind passes through a wind farm, a momentum deficit region emerges (the wind farm wake). This region is characterised by reduced wind speed and increased turbulence intensity. Recent studies have demonstrated that such wakes can propagate downwind over distances up to 55 km [7, 8, 9]. Stable atmospheric stratification can increase the wake distance (discussed below). The magnitude of the resulting production losses on downstream farms depends strongly on inflow conditions. This effect raises new challenges for wind farm planning and siting. Beyond the technical implications, these interactions carry legal and financial dimensions. Operators of neighbouring facilities (potentially across national boundaries) may face conflicting interests regarding wake-induced power and revenue losses. Research from Finserås in 2024 [10] shows how the United Nations Law of the Sea Convention 1982 leaves a regulatory gap regarding cross-border wind wake effects.

The impact of wind farm cluster interactions in the Dutch North Sea has been investigated extensively as part of the WIN50 program [11]. One representative example is the analysis of wake-induced production losses at the planned 4 GW IJmuiden Ver wind farm, resulting from the presence of surrounding offshore wind farms (see wind farm A in Figure 2.2). The study found that neighbouring wind farms re-

duce the energy yield of IJmuiden Ver by approximately 4 %. However, this production deficit is strongly dependent on wind direction and wind speed. At wind speeds exceeding 14 m/s (7 Bft), wake-induced losses are negligible. In contrast, for wind speeds between 6 and 12 m/s (4 and 6 Bft) average production losses of approximately 10 % were observed. Under stable atmospheric stratification production losses rose up to 20 %. Stable atmospheric stratification hinders wake recovery and allows momentum deficits to persist over greater distances [12].

Multiple studies have drawn similar conclusions about slow recovery under stable atmospheric conditions [9, 13, 14].

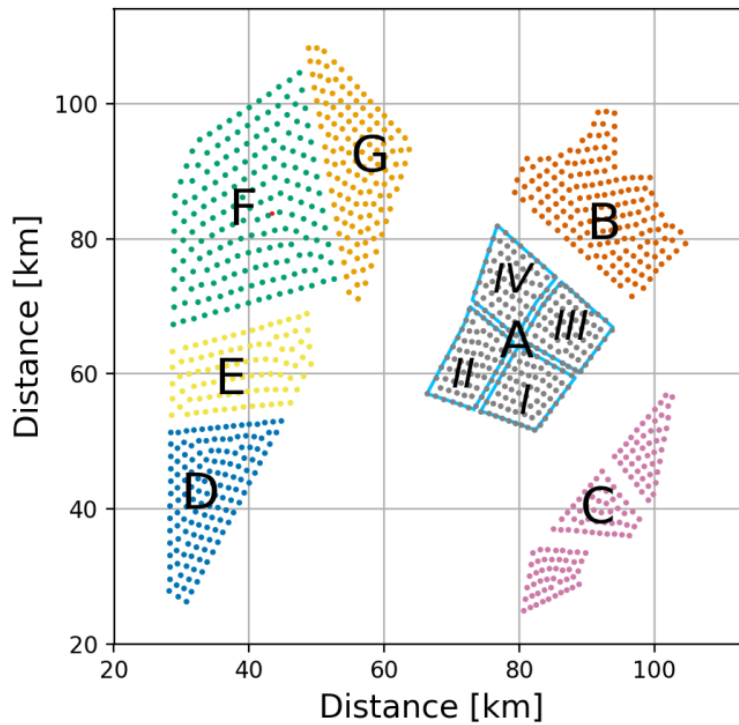


Figure 2.2: Overview of simulated wind farms around IJmuiden Ver [12]. The four separate sites of the IJmuiden Ver wind farm are indicated with numbers I to IV.

As mentioned above, wakes recover more slowly under stable atmospheric conditions. A recent study offered new insights into the wake behaviour of large-scale wind farms by examining the impact of the atmospheric boundary layer on wake recovery, particularly the height of the capping inversion [15]. The capping inversion is a layer of warm air situated above the cooler air of the atmospheric boundary layer. In the study, two distinct recovery regimes can be distinguished: (i) a high inversion layer, allowing for a deep boundary layer, and (ii) a low inversion layer, resulting in a shallow boundary layer.

First, in deep boundary layers, the wake is primarily replenished from above through vertical turbulent entrainment. Faster-moving air from higher altitudes mixes down into the slower-moving air in the wake region. This mechanism is relatively efficient, resulting in comparatively rapid wind speed recovery downstream of the farm.

Secondly, in shallow boundary layers, the capping inversion acts as a lid that blocks vertical mixing. Under these conditions, the wake no longer replenishes from above, and instead receives momentum primarily through horizontal mixing from the sides. This horizontal mixing is considerably less efficient than vertical mixing, resulting in a slow recovery. As a consequence, wakes in shallow boundary layers persist over much greater distances, remaining measurable up to 70 km downstream of the wind farm. These two recovery regimes are clearly visible in Figure 2.3, which shows vertical cross-sections of normalised wind speed downstream of a wind farm for four capping inversion heights.

These studies show that wake length depends strongly on atmospheric conditions, with losses reaching up to 20 % under stable stratification and wakes persisting well beyond 70 km downstream. These findings highlight that wake effects play an important role for wind farms in proximity.

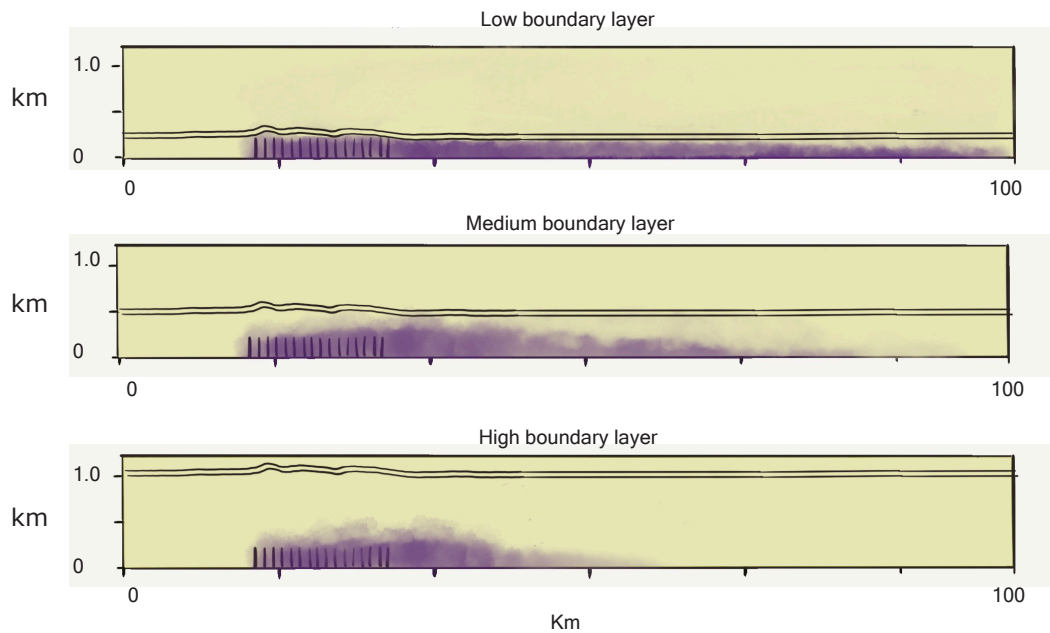


Figure 2.3: Effect of capping inversion height on the far wake. Schematic representation of the effect of the capping inversion height H on the wake behind a wind farm, for four inversion heights. A lower inversion confines the flow to a shallower layer and slows the recovery of the velocity deficit, so the wake persists further downstream. The figure is illustrative and carries no quantitative scale. Drawn after the simulations of Lanzilao et al. [15].

2.2. Market structure and the constraints it imposes on wind farm operation

The wake losses of the previous section are a physical constraint on how much energy a cluster can produce. The surrounding electricity system sets the value of that energy and whether it can be delivered. Energy system integration poses two interconnected challenges for offshore wind.

First, wind energy is variable, creating a time-varying mismatch between supply and demand. During periods of excess generation, the lack of large-scale storage means wind installations must be curtailed. Vice versa, during low-generation periods, expensive conventional generators must run to ensure sufficient supply. This variability results in volatile electricity prices.

Second, Renewable energy generation is location-dependent, introducing a spatial mismatch between supply and demand. Offshore wind capacity is concentrated in the North Sea, yet the electricity generated must be distributed across the entire national grid. This places considerable strain on transmission infrastructure, requiring considerable transmission capacity to transport power from generation sites to consumption centres.

The electricity market mediates these two challenges: temporal mismatch and spatial concentration. The economic mechanisms that govern this market, and the constraints they impose on wind farm operation, are the subject of the next section.

Energy system

To manage the structural tensions between varying supply and demand, electricity systems rely on a layered system of trading markets. Each system operates at a different time frame. In the Netherlands, five distinct market mechanisms facilitate this balancing function, as summarised in Figure 2.4 and described below [16].

- **Futures and forward market:** electricity can be traded up to four years in advance of delivery. This allows producers to hedge against price volatility by locking in an agreed price and volume in advance.
- **Day-ahead market:** Prices are established one day before delivery, based on expected supply and demand. This market is predominantly used by renewable energy sources such as wind and solar. Because weather forecasts lose reliability beyond roughly one week, the futures and forward markets are of limited use for committing wind generation far in advance.
- **Intraday market:** Trading continues after the day-ahead market closes until five minutes before delivery. This lets market participants adjust positions in response to updated forecasts and changing conditions.
- **Imbalance market:** Once the intraday market closes, real-time trading takes place in the balancing market. Operated by the transmission system operator (TSO), it maintains grid stability by activating flexible reserves to correct supply-demand imbalances. Beyond national balancing, the TSO also uses this market to address regional congestion. When transmission infrastructure cannot transport electricity from surplus areas to areas of deficit, the TSO may curtail generation in the congested region while activating additional generation elsewhere. This process, known as redispatching, ensures regional supply-demand balance without overloading the transmission network.
- **Imbalance settlement:** After delivery, deviations between each market participant's scheduled and actual production or consumption are settled financially against the imbalance price. This mechanism incentivises accurate scheduling and supports overall system security.

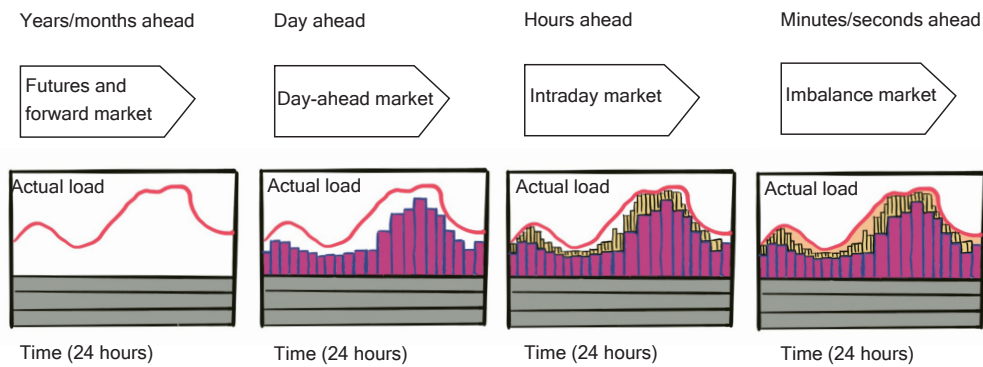


Figure 2.4: The four principal electricity trading markets arranged by time horizon, from years ahead (futures and forward market) to minutes ahead (balancing market) [17].

Price determination and merit order

Electricity prices are set at fifteen-minute intervals, driven by the supply-demand balance. On the demand side, day-and-night patterns and seasonal variations play a major role. Electricity peaks, for example, during the summer when air conditioners are widely used. Fuel costs and the availability of renewable energy resources determine electricity prices on the supply side.

The underlying pricing mechanism is the merit order: generators are ranked by marginal cost of production, from cheapest to most expensive. The market price is set by the most expensive generator required to meet demand. Renewable sources such as wind and solar have near-zero marginal costs because they require no fuel. When dispatched, they displace more expensive conventional generators, lowering the clearing price accordingly.

Market constraints for offshore wind

These markets collectively function as a price-based coordination mechanism. The electricity market affects offshore wind farms primarily through two types of constraints. The first are economic constraints. During periods of peak renewable generation, characterised by high wind speeds and intense solar irradiance, an oversupply of low-marginal-cost electricity drives prices down, incentivising producers to reduce output. When generation persistently exceeds demand, prices can turn negative, at which point operators voluntarily curtail generation to avoid selling electricity at a loss [18].

The second type involves physical constraints, which arise when transmission infrastructure cannot accommodate the generated electricity. Maintaining grid stability requires a continuous balance between electricity supply and demand. When generation exceeds the grid's capacity to absorb or transmit available power, system operators may enforce curtailment [19]. Unlike economic curtailment, which is a market-driven response to adverse price conditions, physical curtailment is imposed by system operators.

Wind farms as market participants

Conventionally, individual wind farms acted as price-takers, as their output was too small to measurably influence the clearing price. However, as installed capacity has grown, wind generation increasingly sets prices during high-wind periods, directly shaping the merit order. As capacity expands further to the scale planned for the North Sea, this effect is expected to become structural rather than incidental.

Wind farms participate in electricity markets as individual entities, bidding into the market much like conventional power plants. System operators can curtail wind farm output like any other generator, and wind farm operators independently optimise output. However, a key distinction separates wind farms from conventional generators. As discussed in Section 2.1, wind farms are not independent. Wake effects mean that the operation of one farm can directly influence the power output of neighbouring farms up to 55 km downstream.

2.3. Wake coupling and market coupling act on the same cluster

The challenges outlined in the preceding sections do not operate in isolation. Wind farms are simultaneously subject to physical wake interactions and electricity market dynamics. These relationships are summarised in Figure 2.5. First (a), wind farm operators use numerical weather prediction models to forecast their expected power output. Based on these forecasts (b), operators submit bids into the electricity market, influencing both the clearing price and the spatial distribution of generation. In response to market or grid conditions (c), system operators may instruct wind farms to curtail output when generation exceeds demand or transmission capacity. Finally, (d) neighbouring wind farms are physically coupled through wake effects, whereby the operation of an upstream farm reduces wind speed and power availability for downstream installations.

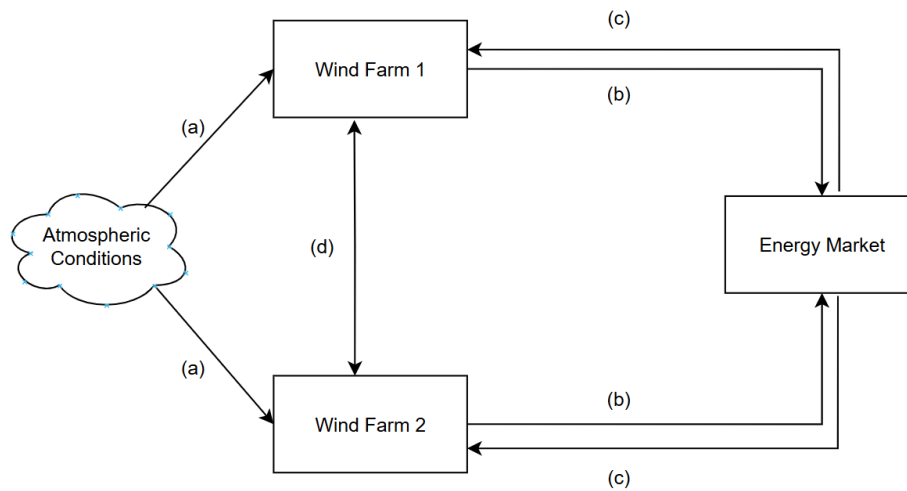


Figure 2.5: Research done in wind energy systems. (a) The Power forecasting, (b) The market participation, (c) The grid constraints and (d) The wake interactions between wind farms.

Two developments have fundamentally altered this picture. First, as mentioned in section 2.2, planned offshore wind deployments have reached a scale where individual clusters exert a measurable influence on the energy market. Planned wind farm clusters in the Netherlands in the order of 4 GW [20] represent a significant share of total generation capacity, and can substantially affect market prices, congestion patterns, and system balancing requirements. Operational decisions such as curtailment not only affect the targeted farm but can also propagate through the cluster, altering inflow conditions in ways current control frameworks do not account for.

Second, the spatial distribution of generation becomes relevant when considering multiple wind farms rather than a single facility. Whereas the output of a single wind farm is injected at a single grid node, a cluster of wind farms spans a much larger geographic area [21], with each farm connected to a different node in the transmission network, as illustrated in Figure 2.6. As shown in the figure, neighbouring wind farms are often closer to one another than to their respective onshore connection points. This amplifies the spatial distribution effect, as aerodynamically coupled farms may inject power into electrically distant nodes. This spatial distribution means a wind farm cluster's power can affect network loading on transmission cables connecting the distant nodes.

TenneT's most recent investment plan projects a further increase in both the number and severity of congestion points across the 380 kV and 220 kV high-voltage network. The planned expansion of offshore wind capacity are identified as a primary driver of this congestion growth.

Where planned grid reinforcements prove insufficient, TenneT must resort to additional measures. However, integrating new 380 kV connections into the existing infrastructure is both spatially complex and technically challenging. This is illustrated by the proposed Randstad reinforcement, a new 380 kV link

between Noord- and Zuid-Holland, which was removed after analysis concluded that it was neither spatially nor technically feasible [22].

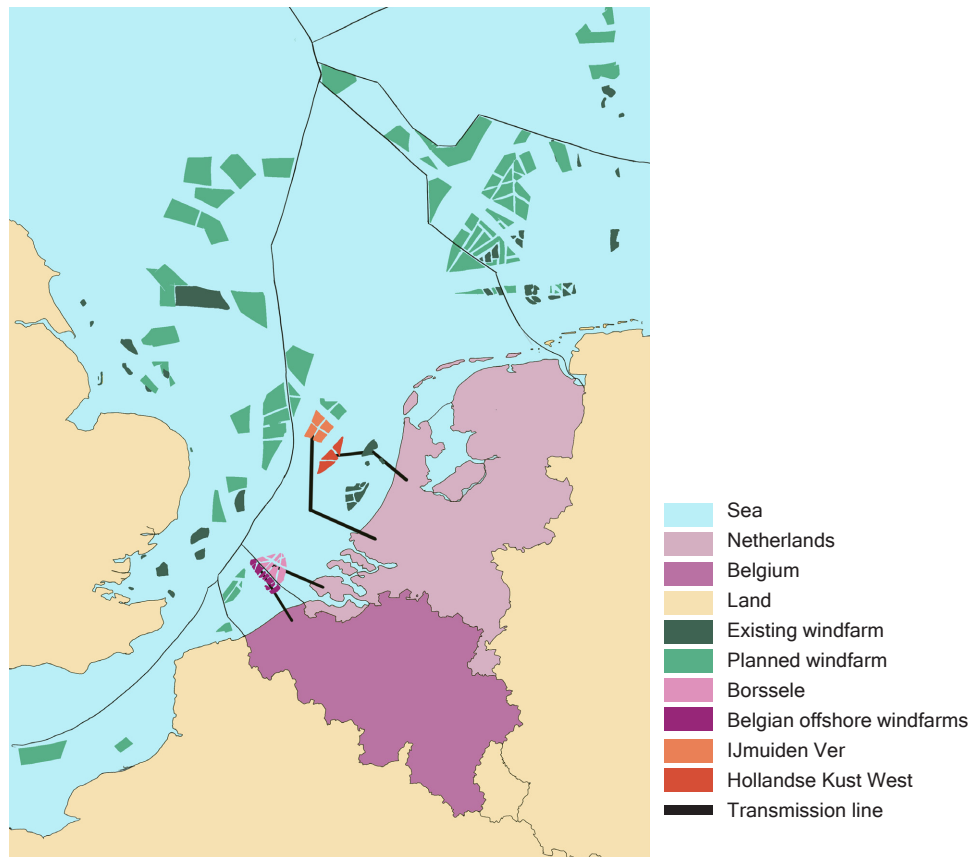


Figure 2.6: Current and planned offshore connections between the North Sea wind farms and the onshore transmission network. The distance between the landing points of two farms can exceed the distance between the farms themselves. Landing points are as planned by TenneT [23].

Wind farms influencing each other's power output creates opportunities for coordinated operation. Curtailing an upwind farm can sometimes increase the downstream farm's output, effectively redistributing power spatially. This spatial redistribution can relieve transmission congestion by shifting generation away from constrained grid nodes. When neighbouring farms operate in different pricing zones, coordinated curtailment can increase aggregate revenue by reallocating output to higher-priced zones.

Cross-border wake interactions

A concrete example of cross-border revenue optimisation is the interaction between neighbouring wind farms located on either side of the Belgian-Dutch border, as illustrated schematically in Figure 2.6. Under south-westerly wind conditions, the Belgian farm operates upstream and imposes a wake-induced power deficit on the downstream Dutch farm. As electricity prices differ between the two bidding zones, as shown in Figure 2.7, coordinated curtailment of the upstream farm can increase aggregate cluster revenue by reallocating generation to the higher-priced zone. Kenis et al. [24] show that such cross-border wake interactions can be exploited economically through tradable wind consumption rights, whereby the upstream operator voluntarily reduces output in exchange for financial compensation from the downstream operator. These opportunities can only be fully exploited through coordinated cluster-level control.

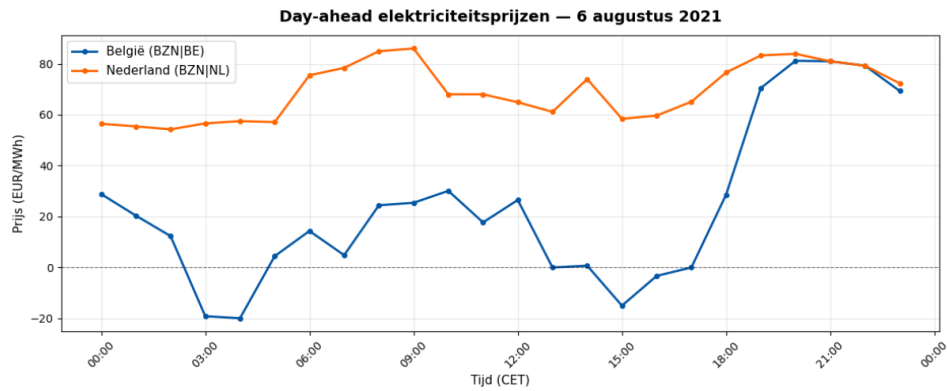


Figure 2.7: Day-ahead electricity prices for Belgium (BZN|BE) and the Netherlands (BZN|NL) on 6 August 2021, illustrating the price differential between the two bidding zones. Source [25]

To translate this theoretical opportunity into a practical assessment, two conditions must coincide: As discussed in Section 2.1, wind speeds in the range of 6 to 12 m/s, under which wake-induced losses are most impactful. Second, a sufficient price difference between the two bidding zones. Figure 2.8 illustrates day-ahead electricity prices for Belgium and the Netherlands throughout August 2025, alongside the hourly price difference between the two zones. A threshold of 10 EUR/MWh is shown as indicator of economically meaningful difference. Based on hourly day-ahead clearing prices for the bidding zone Belgium, BZ-BE and the BZ-NL from the European Network of TSOs for Electricity (ENTSO-E) [25] over the period 1 January – 31 December 2025, this threshold is exceeded approximately 20 % of the time. This suggests that cross-border arbitrage opportunities are not incidental but occur frequently.

The Belgian-Dutch border is a relatively well-interconnected bidding-zone boundary. Cross-border interactions between the Netherlands and the United Kingdom involve subsea interconnectors of considerably greater length and capacity constraints. These constraints are associated with larger and more persistent price differentials. Consequently, energy-system-aware cluster control may yield even greater economic benefits in such configurations.

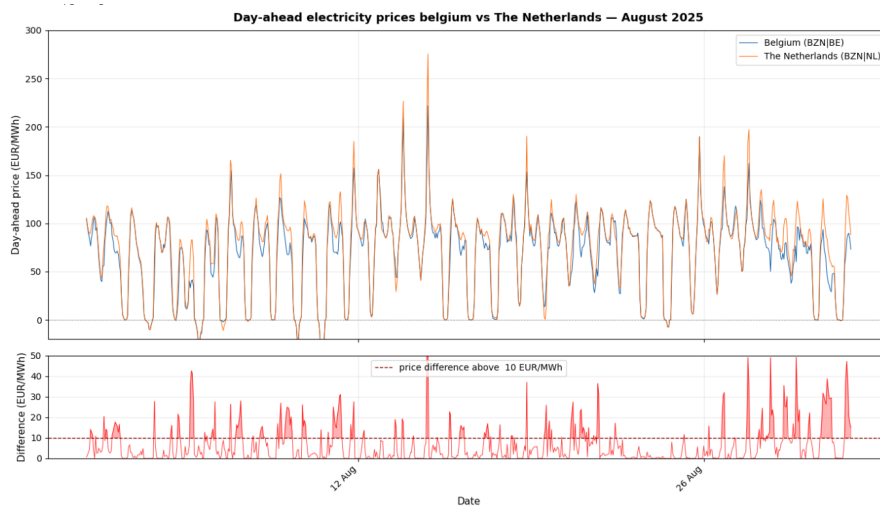


Figure 2.8: Day-ahead electricity prices for Belgium (BZN|BE) and the Netherlands (BZN|NL) throughout August 2025 (top), and the hourly price difference between the two bidding zones (bottom). Shaded regions indicate periods where the price differential exceeds 10 EUR/MWh, a threshold above which coordinated curtailment of an upstream wind farm may be economically justified [25].

Differentiation from Kenis et al.

The framework proposed by Kenis et al. [24] most closely matches the integrated perspective adopted here. Their analysis quantifies the arbitrage potential of cross-border wake interactions over a historical

reference year. The study shows that curtailing the upwind farm yields a positive aggregate revenue during specific hours. The increased revenue was found under realised market and wind conditions. This study also addresses the prospective control problem: how an operator should decide, ahead of delivery, which farms to curtail and by how much, given forecasts of the atmospheric conditions.

A further distinction concerns how market prices are treated. Kenis et al. model wind farms as price-takers, with day-ahead and intraday prices entering the model as fixed input time series. This assumption is appropriate for the present-day Belgian-Dutch configuration, in which installed offshore capacity remains modest relative to total market volume. At the capacity levels projected for the North Sea by 2030 and beyond, however, the assumption no longer holds. As discussed in Section 2.2, wind clusters of the order of 4 GW act as price-setters during high-wind periods, directly displacing marginal conventional generation and shaping the clearing price. Coordinated curtailment of such a cluster therefore alters not only the physical wake field, but also the price signal against which the curtailment decision is evaluated. This feedback loop, in which the control action itself modifies the optimisation objective, is not addressed in [24] and has, more broadly, received limited attention in the cluster-control literature.

2.3.1. Why this gap persisted

Given the relevance of energy-system-aware cluster control, one might ask why the literature has not yet comprehensively addressed this topic. Two methodological barriers stand out, alongside a more structural barrier rooted in the disciplinary boundaries between the fields involved.

Computational cost of high-fidelity wake modelling

Research into wind farm cluster interactions is itself still maturing. Wake behaviour at this scale is highly sensitive to atmospheric conditions and to farm-specific parameters such as layout, turbine type, and installed capacity. This variability means no general, universally applicable solution has emerged, and the field still relies heavily on high-fidelity simulation methods to capture the relevant physics. Such models are computationally expensive, which makes the systematic sweeps over control settings that an optimisation framework requires prohibitively costly. Engineering wake models offer a computationally tractable alternative, but their accuracy at cluster scale remains uncertain. Most engineering wake models typically do not account for complex conditions such as thermal stratification or Coriolis forcing [26].

Fischereit et al. [27] examined the wake behaviour of two wind farm clusters using three engineering models: the Jensen model [28], the Gaussian wake model [29], and the model proposed by Zong et al. [30]. All three systematically overestimate the rate of wake recovery in the far-wake region downstream of the farms, while producing acceptable predictions within the farm boundaries themselves. The result is a methodological gap: high-fidelity models capture the physics but cannot be embedded in optimisation; engineering models are fast enough but lose accuracy precisely where cluster-cluster interactions occur.

Mathematical structure of the optimisation problem

A second barrier concerns the mismatch between how the two domains are mathematically formulated. Energy systems are typically solved in optimisation frameworks. Because of the large number of linear decision variables and the market's tendency to seek the cheapest solution, optimisation is more suitable than simulation. Wind farm aerodynamics, by contrast, are inherently nonlinear: wake interactions, power curves, and atmospheric effects do not lend themselves naturally to linear approximations. Embedding aerodynamic models in energy system optimisation therefore requires either restrictive linearisations that sacrifice physical fidelity, or nonlinear formulations that sacrifice tractability. Neither option is straightforward, which has discouraged integrated treatments.

Disciplinary separation

Beyond these two methodological barriers, a structural factor shapes the literature: the problem sits at the intersection of disciplines that rarely interact. Energy system operators, best positioned to optimise power flows across the network, typically have limited expertise in wind farm aerodynamics. Wind farm researchers, who understand the wake physics, in turn have little visibility into system-level constraints, market mechanisms, and the operational concerns of transmission system operators. Bridging this divide requires interdisciplinary collaboration that has, to date, been limited. As a result, even where the methodological barriers above could in principle be overcome, the relevant expertise is rarely combined within a single research effort.

2.4. Research Gap

The barriers identified above collectively explain why researchers have not yet integrated wake interactions and energy system constraints into a unified control framework. The preceding sections have established that wind farms are interconnected through two distinct mechanisms: physical wake interactions and the shared electricity market infrastructure. Researchers have studied both mechanisms extensively in isolation, yet their combined effect at the cluster level remains largely unaddressed. This gap motivates a broader question: to what extent can active, coordinated control at the cluster level improve energy system outcomes while accounting simultaneously for inter-farm wake interactions and the operational requirements of the electricity system?

At the scale of the planned clusters such as IJmuiden Ver (4 GW), wake-induced production losses become a significant problem. Wakes propagate across farm boundaries for more than 50 km reducing power output for more than 20 %. At the same time, wind clusters are shifting from price-takers to price-setters, directly shaping market-clearing prices. The large share of renewable energy introduces transmission congestion that existing infrastructure can no longer accommodate. These two challenges do not operate independently: operational decisions such as curtailment propagate through both the wake field and the electricity market simultaneously, in ways that current control frameworks do not account for.

The knowledge gap identified in this review is be visualised by extending the framework introduced in Figure 2.5. The missing link is the active, coordinated control of wind farm clusters that simultaneously accounts for inter-farm wake interactions and energy system constraints, as highlighted in Figure 2.9 While the individual components of this framework have been studied in isolation, their integration into a unified control strategy remains largely unexplored. Consequently, the influence of such cluster-level control on both wind farm power output and energy market operation remains highly uncertain.

This uncertainty directly motivates the research question formulated in the following chapter.

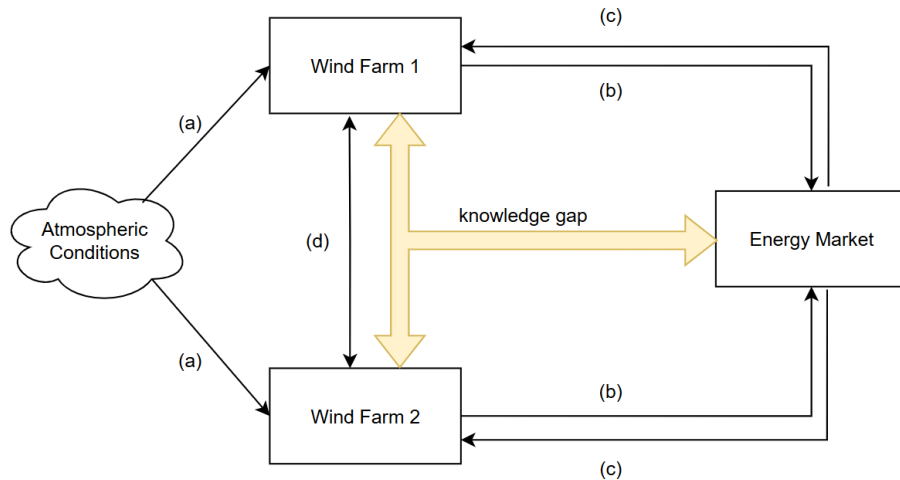


Figure 2.9: Research done in wind energy systems. (a) The Power forecasting, (b) The market participation, (c) The grid constraints and (d) The wake interactions between wind farms.

2.5. Research question

Building on the knowledge gap identified in the preceding literature review, this Section formulates the central research question and the supporting subquestions that structure this thesis. As established, active, coordinated control of offshore wind farm clusters that accounts simultaneously for interfarm wake interactions and energy system constraints remains insufficiently studied.

Based on this research gap, the following research question is defined:

"To what extent can active control of offshore wind farm clusters influence energy system outcomes, including power output and spatial power distribution?"

The main research question is divided into four subquestions. These subquestions focus on different aspects of the problem, the model accuracy, computational cost, economic feasibility, and grid interaction. Answering each individually helps clarify the overall research question. Together, they answer the main research question and help evaluating the model's performance and applicability.

- How can engineering wake models capture cluster-scale wake interactions sufficiently accurately while keeping computational cost low enough for integration into an optimisation framework?
- Under which atmospheric and market conditions does curtailment of an upstream wind farm yield a net economic benefit at cluster level?
- How can coordinated cluster-level control redistribute power output to alleviate transmission congestion, and what is the trade-off in total energy yield?

3. Methodology

This thesis's main question is whether active cluster control can influence energy system outcomes. That question sits between two fields that model their subject differently. Wind farm studies simulate a specific layout, turbine by turbine, and evaluate the resulting flow. Energy system studies formulate an optimisation problem; dispatch decisions are nearly linear, and the number of decision variables is large: every generator can produce anywhere between zero and its installed capacity.

The optimal curtailment depends on both. Atmospheric conditions and cluster layout determine how much curtailment redistributes power, while demand, transmission capacity, and price determine whether that redistribution is worth it. To combine the two, the wind farm is linearised and embedded in the energy system optimisation.

3.1. Research approach and data

The effect of cluster control is measured by comparing two curtailment strategies of the same system. In the first strategy, the curtailment coefficient of the upwind farm is fixed at $\eta = 1$: the farm extracts as much energy from the wind as its turbines allow. This is referred to as *greedy* throughout. In the second, η is a decision variable and the market clearing selects its value; this is referred to as *cluster control*. Every other part of the system is identical between the two runs, so the difference between them isolates the effect of the curtailment. How η is found is set out in Sections 3.2 and 3.3.

Note that both dispatches minimise total system cost. The two runs do not represent a selfish operator against a system planner; they represent a selfish variable only, which is free in one case and fixed in the other.

Four cases are simulated. They are ordered by how favourable the conditions are for the strategy, from a configuration where it is expected to have no effect to one where it should have the most room.

1. **IJmuiden Ver and Hollandse Kust West (HKW).** Two Dutch farms connecting to different on-shore nodes. This case tests whether cluster control can relieve domestic congestion, where the network offers alternative routes between the two connection points.
2. **The Belgian–Dutch cluster.** The same cluster studied by Kenis et al. [24]. The two farms lie on opposite sides of a bidding-zone boundary, so marginal prices differ between them. Cross-border transmission capacity is typically lower than intra-country capacity, making congestion more likely.
3. **The Belgian–Dutch cluster with additional Dutch capacity.** The same configuration, with IJmuiden Ver, Hollandse Kust West and a share of Nederwiek added as uncontrolled farms. This increases the Dutch offshore wind capacity by 10 GW. More Dutch offshore capacity pushes more energy towards the border than the interconnectors can carry, which should widen the hours in which curtailment is selected.
4. **The Belgian–Dutch cluster with reduced Belgian nuclear capacity.** The same configuration as case 3, with the Belgian nuclear capacity reduced to the units still operating in 2026. Nuclear output is fixed in this model and cannot step aside for cheaper energy. Removing must-run capacity should leave a flexible unit on the margin more often, leaving more room for redistributed wind to matter.

Cases 3 and 4 are hybrid systems: the 2020 system with one or two elements changed. They are not forecasts of a future system, but tests of which condition drives the result.

Two quantities are reported for each case. The first is the volume of fossil generation displaced, in MWh, which follows directly from the difference between the two dispatches and translates into a reduction in system cost at the marginal cost of the displaced unit. It indicates the reduction in system cost because fossil output is replaced by cheaper nuclear or renewable generation, and it indicates the reduction in emissions because the sources replacing fossil output emit little. The second is the marginal price, the cost of the most expensive generator dispatched. A single unit sets the marginal price and therefore responds discretely, making it a less robust indicator than displaced volume; it is reported as a secondary measure.

Solving a full year is not tractable at this problem size, so each case is screened for hours in which the strategy could plausibly matter, and a 24-hour window is then simulated in full. Chapter 4 presents the screening criteria and resulting windows; Section 4.4 reports the computational cost that motivates this restriction.

3.1.1. Optimisation models

This thesis relies on optimisation throughout. In the energy system model this is conventional: market clearing and unit dispatch are routinely formulated as optimisation problems. In the wake model, this is not the case, as wake effects are normally evaluated by simulation rather than optimised. Both are stated here as mathematical programmes, in which the best solution is sought within a set of equations describing the system.

This section introduces the concepts required for that formulation: the key components of a mathematical programme, the linearisation of non-linear relations, and the way both are implemented and solved.

Key components

Mathematical programming offers a systematic way of stating optimisation problems. Such a problem is composed of four components:

- **Parameters.** Quantities that are fixed before the problem is solved and enter as input, such as the day-ahead electricity prices, the free-stream wind speed and direction, and the characteristics of the transmission network.
- **Decision variables.** The quantities the model is free to choose. They may be continuous, semi-continuous, integer or binary, and together they span the space of possible solutions. The curtailment coefficient η of Section 3.2.2 is the main decision variable of this thesis.
- **Constraints.** Equalities and inequalities that restrict the solution space, separating the solutions that are physically or operationally admissible from those that are not. For example, generation and demand must balance at every node.
- **Objective function.** The quantity to be maximised or minimised, such as revenue or operating cost. A problem may also be stated without one, in which case it seeks any point satisfying the constraints.

Every optimisation problem in this thesis is stated in these terms.

Problems are further classified by the form of their objective and their constraints, which determines how hard they are to solve. In a linear programme (LP) both are linear, so the optimum lies at a vertex of the feasible region and only the vertices need to be searched. If the objective or the constraints are quadratic (QP) or non-linear (NLP), the optimum may lie anywhere in the solution space, and this guarantee is lost; quadratic problems remain tractable when the objective is convex. Integer and binary variables can be added to a linear problem without making it non-linear, giving a mixed-integer linear programme (MILP), at the cost of a search tree over the integer variables. Applying the same extension to the quadratic and non-linear cases gives MIQP and MINLP, respectively. The wind farm model of Chapter 3.2 is an NLP in its original form. It is first reduced to a MINLP through the piecewise linear constraints of Section 3.2.6, and then to an MILP through the Taylor expansion of Section 3.2.7.

Special ordered sets of type 2

A special ordered set of type 2 (SOS2) replaces the non-linear turbine curves with piecewise linear ones. The curve is sampled at several breakpoints, and the solution is written as a weighted combination of those breakpoints, subject to the restriction that at most two weights are non-zero and that these two must be adjacent. Any point on the curve is then a linear interpolation between two neighbouring breakpoints. The solver enforces the restriction with binary variables it introduces internally. The formulations in this thesis therefore contain no explicit binary variables, but are mixed-integer nonetheless. Each additional breakpoint improves the approximation and adds one weight variable, so the number of breakpoints trades accuracy against problem size.

Model implementation

All models are formulated in Pyomo, an open-source Python package for formulating and solving optimisation [31], and solved with Gurobi v13.0.1 [32]. A relative MIP gap of 10^{-5} is used throughout, and each time step is solved as a separate problem. Computations were performed on an Intel Core Ultra 7 255H with 32 GB of RAM, running Windows 11.

3.1.2. Wake models

This section discusses the computational tools available for wake modelling at cluster scale, and the two choices that follow from the requirement that the wake model be embedded in an optimisation framework: the wake model itself and the superposition method.

The review by Ouro et al. [26] compares several wake models and shows how the physical scale of interest, from a single turbine to a cluster, determines the appropriate computational framework. Figure 3.1 reproduces that classification and shows how model complexity and computational cost rise sharply from engineering models to mesoscale CFD.

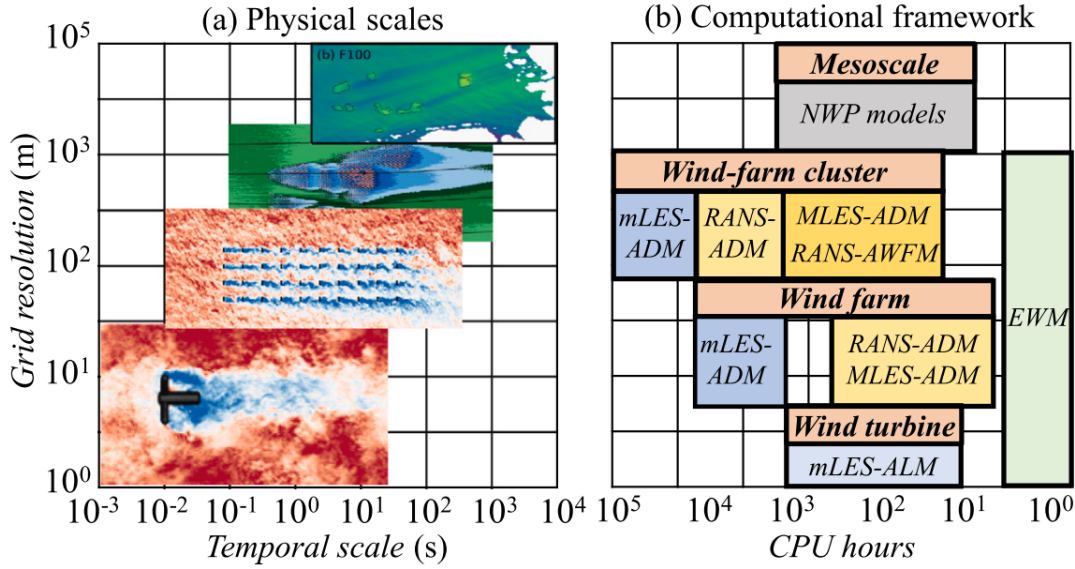


Figure 3.1: (a) Physical scales involved in flows around individual wind turbines, single wind farms, and wind farm clusters at meso- and micro-scales, and (b) classification of computational frameworks available for simulation of these flows, with indicative compute cost in CPU hours. EWM: Engineering Wake Models; NWP: Numerical Weather Prediction; mLES: microscale Large-Eddy Simulation; MLES: Mesoscale Large-Eddy Simulation; RANS: Reynolds-Averaged Navier–Stokes; ADM: Actuator Disc Model; ACM: Actuator windfarm canopy Model; ALM: Actuator Line Model. Reproduced from [26].

This thesis models interactions at the cluster scale while adjusting each farm’s thrust over time. Because many thrust configurations must be evaluated, the wake model must be cheap to evaluate. High-fidelity methods are unsuited to simultaneous energy system optimisation and wind farm control.

Engineering wake models (EWM) describe the wind speed deficit behind a single turbine with a prescribed shape. That shape and the deficit depth depend on inflow conditions, the wake recovery rate, and the turbine type. The Jensen model is one of the earliest and simplest, and Archer et al. [33] find it performs competitively against other engineering models across farms and wind directions. The Jensen model is linear in the induction factor, making it usable in an MILP. It is therefore adopted throughout this thesis; Section 3.2.3 develops it in detail.

Where the wakes of several upstream turbines overlap, their deficits are combined by superposition. The most common of these is linear superposition [34], in which the individual deficits are summed. This method is not derived from first principles and conserves neither mass, energy, nor momentum. It also allows negative wind speeds when many wakes overlap. Superposition methods based on squared deficits address part of these shortcomings [35, 36], but introduce a non-linearity that cannot be carried into the optimisation problem. Linear superposition is therefore used throughout, as discussed in Section 2.3.1.

3.1.3. Data

This section describes the model inputs. The energy system is simulated as a merit-order dispatch, in which the cheapest available source is dispatched first. Which source that is depends on the demand

at each node, the marginal cost of each generator, the capacity of the network connecting them, and the atmospheric conditions that determine how much wind and solar power is available. Table 3.1 lists these quantities and their sources. We take the time series directly from the listed sources; the remaining parameters are tabulated in Appendix A.

Table 3.1: Input data used in the energy system model.

Quantity	Source	Resolution	Year
Day-ahead electricity prices (NL, BE)	ENTSO-E [25]	hourly	2021
Electricity demand per node	ENTSO-E [25]	hourly	2020
Cross-border exchange	ENTSO-E [25]	hourly	2020
Installed capacity per technology	Thesis TU Delft [37]	–	2020
Efficiency per generating unit	Thesis TU Delft [37]	–	2020
Wind speed and direction at 100m	ERA5 [38]	hourly	2020, 2021
Solar capacity factor	Renewable Ninja project [39]	hourly	2020
onshore/offshore capacity factor	Renewable Ninja project [39]	hourly	2020
Transmission line capacity	Github Calliope-NL [40]	–	
Transmission line length	Github Calliope-NL [40]	–	
Wind farm locations and layout	Kenis et al. [24]	–	
Onshore connection points	TenneT [23]	–	
Turbine power and thrust curves	IEA [41, 42]	–	
Fuel Price	World Bank [43]	–	2020
Carbon Tax	ICE [44]	–	2020
Emission Factor	IPCC 2006 Guidelines [45]	–	2020

1 ENTSO-E reports load at the national level. Demand is regionalised proportionally to each province's GDP.

The energy system is simulated for 2020. The price-taker comparison in Section 4.2.1 uses 2021 instead, the reference year of the study by Kenis et al. [24] against which those results are compared.

The marginal cost of a conventional generator follows from its fuel price, its efficiency and the carbon tax on its emissions:

$$MC = \frac{\text{fuel price} + \text{emission factor} \times \text{carbon tax}}{\zeta}, \quad (3.1)$$

where

- MC is the marginal cost of the generator [EUR/MWh],
- ζ is the efficiency of the generator [-],
- the fuel price is the cost of the fuel per unit of thermal energy [EUR/MWh_{th}],
- the carbon tax is the price paid per unit of emitted CO₂ [EUR/tCO₂], and
- the emission factor is the CO₂ emitted per unit of thermal energy burned [tCO₂/MWh_{th}].

3.2. Wind farm cluster model

This chapter develops the wind farm model used throughout this thesis. The model aims to satisfy two requirements. It aims to accurately reproduce how curtailing the upwind wind farm changes the power output of the downwind wind farm. It must do so in a form that can be embedded in an optimisation framework and solved as a MILP. These requirements conflict: the wake physics is non-linear precisely in the curtailment, the quantity being optimised

Section 3.2.1 introduces turbine power production. Section 3.2.2 defines the curtailment coefficient η , the decision variable of this thesis, and relates it to the velocity deficit the turbine leaves behind. Section 3.2.3 scales this behaviour up to a two-farm cluster. Section 3.2.5 removes the non-linearities in two steps: piecewise linear fits for the turbine curves, and a Taylor expansion for the remaining dependence on η . Section 3.2.10 gives an overview of the resulting formulation.

3.2.1. wind turbine fundamentals

Wind farms interact through wakes; the velocity deficit one farm leaves behind changes the wind resource available to downstream farms. The wind farm wake is the combined effect of wakes from individual turbines. Understanding cluster-scale interaction therefore starts with a single turbine. This section introduces the fundamentals of turbine power production, which is scaled up to wake models in Section 3.2.3.

Wind turbines convert the kinetic energy of moving air into electricity in two steps. The wind drives the rotor blades, generating mechanical power. A generator converts this power into electrical power. The output of a single turbine is described by actuator disk theory:

$$p = \frac{1}{2} \rho A U^3 C_P, \quad (3.2)$$

where:

- p is the power output of a single turbine [W],
- ρ is the air density [kg/m^3],
- A is the rotor area [m^2],
- U is the incoming wind speed [m/s], and
- C_P is the power coefficient [-], the fraction of available wind power the turbine converts to electrical power.

As shown in the equation above, the turbine power output scales with the cube of wind speed, so small changes in U produce large changes in power. The power coefficient is bound by the Betz limit, $C_P \leq 16/27 \approx 0.593$. The Betz limit is the theoretical maximum fraction of kinetic energy a turbine can extract from the incoming wind.

Three characteristic wind speeds bound turbine operation: cut-in, rated, and cut-out. Below the cut-in speed, the wind carries not enough force to overcome drivetrain friction, so the turbine produces no power. Above the rated wind speed (u_{rated}), the rotor would otherwise extract more power than the generator and drivetrain can handle. The pitch angle is adjusted to reduce the aerodynamic force on the blades. This keeps the power output constant at p_{rated} rather than letting it continue to grow with U^3 . Beyond the cut-out speed, wind loads exceed what the turbine structure can safely withstand. The turbine stops entirely, turning its blades to minimise load until wind speed drops back below this threshold.

Figure 3.2 illustrates these regions for a reference 15 MW turbine and 10 MW turbine. Below cut-in, both power output and C_P are zero. Between cut-in and rated speed, C_P remains near its maximum as the turbine extracts power as efficiently as possible, while power output rises with the cube of wind speed. Beyond u_{rated} , power output flattens at p_{rated} while C_P declines, reflecting the deliberate loss of aerodynamic efficiency introduced by pitch control. At cut-out, both curves drop to zero as the turbine stops. Both the rated and cut-out wind speeds are control measures that limit structural loading.

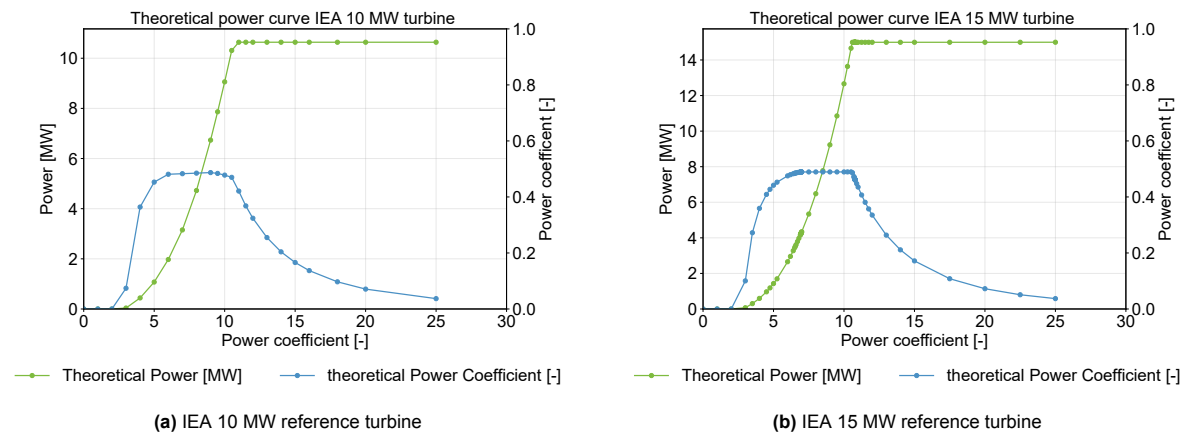


Figure 3.2: Power output (p) and power coefficient (C_P) curves for the IEA 10 MW and IEA 15 MW reference turbines.

3.2.2. Curtailment

Curtailment is the deliberate reduction of a turbine's power output below what the wind conditions would otherwise allow. Figure 3.3 illustrates a turbine operating at 20% of its uncurtailed power output. In practice, this is achieved by changing the pitch angle, which reduces the fraction of the wind's kinetic energy the blades convert. It also reduces the force the blades exert on the free stream, and therefore weakens the wake. How a turbine is curtailed falls outside the scope of this thesis; only the resulting reduction in power output and its effect on the wake matter here.

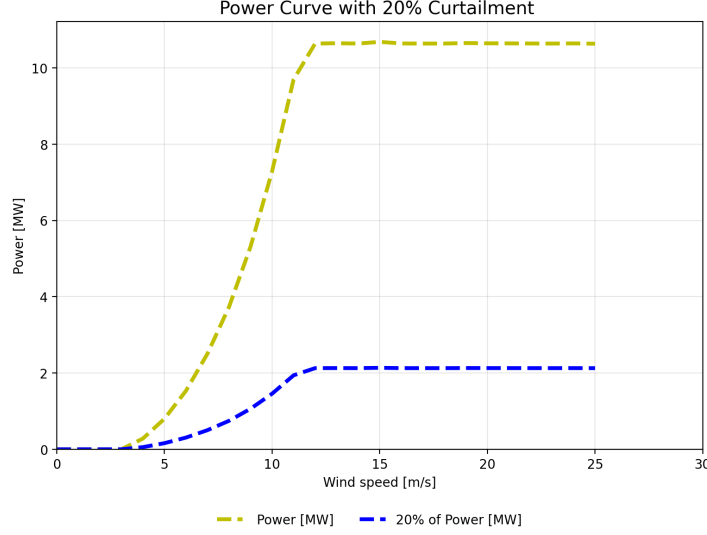


Figure 3.3: Power curve of a turbine operating at 20% of its uncurtailed power output. The blue line shows the curtailed power, and the green line shows the uncurtailed power output p_{greedy} .

Applying curtailment changes the fraction of the available wind power the turbine can convert to electricity, and therefore reduces C_P . The curtailed power then follows from Equation 3.2 in the same form, with the effective power p^{eff} and effective power coefficient C_P^{eff} replacing their uncurtailed counterparts:

$$p^{\text{eff}} = \frac{1}{2} \rho A U^3 C_P^{\text{eff}}. \quad (3.3)$$

Since curtailment does not change the air density, rotor area, or wind speed, these terms remain identical to Equation 3.2. The power coefficient captures the entire reduction, dropping from C_P to C_P^{eff} .

Dividing the effective power by the uncurtailed power gives the efficiency of the turbine under curtailment:

$$\eta = \frac{p^{\text{eff}}}{p} = \frac{\frac{1}{2} \rho A U^3 C_P^{\text{eff}}}{\frac{1}{2} \rho A U^3 C_P} = \frac{C_P^{\text{eff}}}{C_P}. \quad (3.4)$$

Rearranging Equation 3.4 expresses the effective power coefficient and effective power directly in terms of η :

$$C_P^{\text{eff}} = \eta C_P, \quad p^{\text{eff}} = \eta p. \quad (3.5)$$

This ratio, $\eta \in [0, 1]$, is the curtailment coefficient, and the variable which will be optimised in this study. At $\eta = 1$, the farm operates greedily, and at $\eta = 0$ it is shut down, leaving the flow undisturbed; every intermediate value trades power generated upstream against wind speed downstream. This wind speed can be potential power for the wind farm downstream.

Wind speed reduction

The power coefficient describes the fraction of available wind energy the turbine extracts. Actuator disk theory relates this fraction to the velocity deficit behind the rotor. It links the power coefficient to the turbine's operating state through the axial induction factor a . The axial induction factor describes the fractional reduction in wind speed at the rotor plane relative to the incoming wind speed U . From actuator disk theory:

$$C_P^{\text{eff}} = 4a(1 - a)^2, \quad (3.6)$$

$$C_T^{\text{eff}} = 4a(1 - a), \quad (3.7)$$

where:

- a is the axial induction factor [-];
- C_T is the thrust coefficient [-], analogous to C_P but describing the fraction of available thrust force the turbine extracts from the flow.

Equation 3.6 describes an ideal actuator disk, and using it in this form raises two problems. First, the power coefficient obtained from the reference turbines is not ideal: it includes the ideal actuator disk plus efficiency losses, such as conversion from mechanical to electrical power. Second, Equation 3.6 is not invertible: every C_P below its maximum is produced by two distinct induction factors, so the inverse is not uniquely defined. Both problems are addressed below.

The Betz limit, $C_P \leq 16/27$, remains the theoretical ceiling, however the reference turbines used in this thesis peak well below it, at $C_{P,\text{max}} = 0.478$ and 0.490 for the IEA 10 MW and IEA 15 MW respectively. Their power coefficient curves peak below the ceiling because they also include inefficiencies unrelated to induction. Inverting Equation 3.6 directly would attribute those losses to a reduced induction factor, and so understate the wake the turbine leaves behind. In reality, these inefficiencies are not constant but vary with the operating conditions; this thesis assumes them to be constant.

Assumption 3.1 (Scaled actuator disk relation). The induction factor of a real turbine follows the ideal actuator disk relation up to a constant scaling (κ).

Under Assumption 3.1, the power coefficient is written as:

$$C_P = \kappa \cdot 4a(1 - a)^2, \quad \kappa = \frac{C_{P,\text{max}}}{16/27}, \quad (3.8)$$

with κ calibrated so that the turbine attains its maximum power coefficient at the induction factor of the Betz optimum, $a = 1/3$, on the assumption that the blades are designed to aim for that point. The scaling affects only the height of C_P , so the stationary point of Equation 3.8 remains at $a = 1/3$ and the relation retains two roots for any C_P below its maximum: one below and one above that value. Recovering a from a given C_P therefore requires selecting a branch.

Assumption 3.2 (Curtailment at minimum thrust). For a given reduction in power output, the upwind farm operates at the lowest induction factor that delivers it.

Assumption 3.2 selects the lower root, which restricts the domain to $a \in [0, 1/3]$. The mapping between a and C_P is one-to-one on this domain, making it possible to recover a from C_P , as required by the wake model of Section 3.2.3. The assumption reflects the operating strategy of a wind farm operator, who has no reason to load the rotor more heavily for no additional power.

Applying curtailment reduces C_P^{eff} below its uncurtailed value, which under Equation 3.8 corresponds to a lower induction factor. A curtailed turbine therefore induces a smaller velocity deficit than an uncurtailed one, weakening its wake.

3.2.3. Jensen wind farm model

Section 3.2.1 and 3.2.2 established how curtailment reduces a turbine's power coefficient C_P , and how the resulting C_P can be mapped back to an induction factor a via Equation 3.6. This induction factor, in turn, determines the wake the turbine leaves behind; the more energy a turbine extracts from the flow, the larger the downstream velocity deficit.

This thesis considers a two-farm cluster, in which turbines are curtailed depending on their farm's position relative to the incoming wind. Let n denote an individual turbine in the cluster, and let $\mathcal{J}_{\text{up}}$ and $\mathcal{J}_{\text{down}}$ denote the sets of turbines belonging to the upstream and downstream farm, respectively. For this thesis, curtailment is applied only to the upstream farm:

$$\eta_n = \begin{cases} \eta_{\text{up}}, & n \in \mathcal{J}_{\text{up}} \\ 1, & n \in \mathcal{J}_{\text{down}} \end{cases}. \quad (3.9)$$

Turbines in the downstream park therefore operate at their uncurtailed factor ($\eta = 1$), while turbines in the upstream park operate at a reduced a_n following from η_{up} via Equation 3.5 and 3.6. The curtailment factor determines the value of a_j used in the wake model below, where j indexes any upstream turbine acting on turbine n .

The Jensen wind farm model [46] describes this deficit under two assumptions: the wake expands linearly with downstream distance, and the cross-stream integral of the velocity deficit is conserved. The wind speed at turbine n caused by the wake of an upstream turbine j is then given by:

$$U_{nj} = U_{\infty} \left(1 - \frac{2a_j}{(1 + 2k_w \frac{x_{nj}}{D})^2} \right). \quad (3.10)$$

Where:

- U_{nj} is the wind speed at turbine n due to the wake of turbine j [m/s],
- U_{∞} is the free-stream wind speed [m/s],
- a_j is the axial induction factor of the upstream turbine j [-],
- x_{nj} is the downstream distance between turbines n and j aligned with the wind direction[m],
- D is the rotor diameter [m], and
- k_w is the wake expansion rate [-].

The wake expansion rate k_w depends on ambient turbulence intensity and is typically set to 0.075 onshore and 0.04 offshore [33, 47, 48]; this thesis adopts the offshore value throughout. Figure 3.4 illustrates the resulting wake geometry: the wake expands linearly from the rotor diameter D to a wake diameter $D_w = D + 2k_w x_{nj}$, within which the wind speed is reduced from U_{∞} to U .

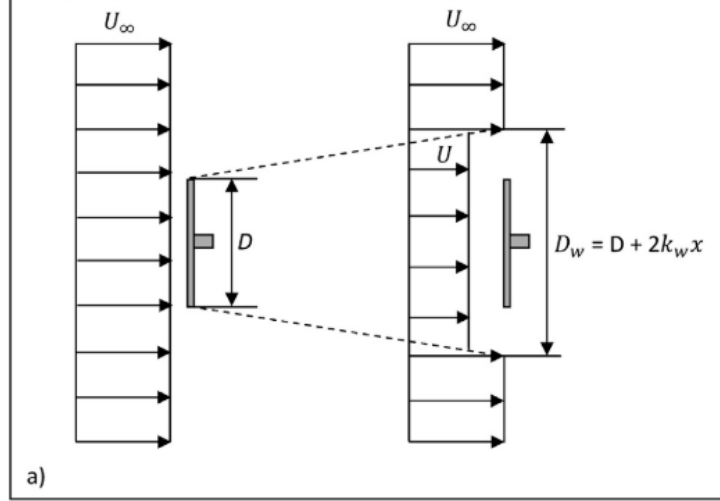


Figure 3.4: Schematic representation of the Jensen wind farm model, showing the linear expansion of the wake diameter D_w downstream of a turbine, and the resulting wind speed deficit from U_∞ to U [33].

The wake of an upwind turbine only affects turbines that lie within it. To determine whether a downwind turbine is affected, decompose the distance between the two turbines into streamwise and lateral components. The streamwise component $x_{n,j}$ is the distance appearing in Equation 3.10; the lateral component $y_{n,j}$ is the distance perpendicular to it. Both follow from the distance vector and the wind direction:

$$x_{nj} = \mathbf{r}_{nj} \cdot \hat{\mathbf{w}}, \quad y_{nj} = \sqrt{\|\mathbf{r}_{nj}\|^2 - x_{nj}^2}, \quad (3.11)$$

where:

- $\mathbf{r}_{n,j}$ is the distance vector from turbine j to turbine n [m], and
- $\hat{\mathbf{w}}$ is the unit vector in the direction of the free-stream wind [-].

The wake of turbine j expands linearly with x_{nj} , so at the location of turbine n , it has radius:

$$R_w(x_{nj}) = \frac{D}{2} + k_w x_{n,j}. \quad (3.12)$$

The downwind turbine lies within the wake when the radius of the wake is larger than the lateral component of the distance x_{nj} from the upwind turbine. The set $\text{up}(n)$ collects these turbines:

$$\text{up}(n) = \{j : x_{nj} < 0 \wedge y_{nj} \leq R_w(x_{nj})\}. \quad (3.13)$$

The condition $x_{nj} < 0$ excludes turbine n itself and every turbine downwind of it. Note that $\text{up}(n)$ is defined per turbine, not per farm: it contains every turbine whose wake reaches turbine n , regardless of whether that turbine belongs to $\mathcal{J}_{\text{up}}$ or $\mathcal{J}_{\text{down}}$. Turbines within $\mathcal{J}_{\text{up}}$ also experience wake effects.

3.2.4. Superposition

When multiple upstream turbines overlap at the turbine n , the resulting wind speed follows from the linear superposition of their individual deficits [34]:

$$U_n = U_\infty \left(1 - \sum_{j \in \text{up}(n)} \frac{2a_j}{(1 + 2k_w \frac{x_{nj}}{D})^2} \right). \quad (3.14)$$

As discussed in Section 3.1.2, this thesis adopts linear superposition because it is tractable within an optimisation framework: squared-deficit alternatives introduce non-linearities that cannot be tractably solved within the optimisation problem developed in this thesis.

Let W_{nj} collect the wake coefficient between turbine pair (n, j) :

$$W_{jn} = \begin{cases} \frac{2}{\left(1 + 2k_w \frac{x_{nj}}{D}\right)^2} & j \in \text{up}(n) \\ 0 & \text{otherwise} \end{cases} . \quad (3.15)$$

Collecting these coefficients into the matrix W , and the induction factor of every turbine in the cluster into the vector a_n , allows the wake equation of Equation 3.14 to be written for the entire cluster at once:

$$U_n = U_\infty (1 - W_{jn} a_j) . \quad (3.16)$$

Importantly, $\text{up}(n)$ is defined per turbine, not per farm: it contains every turbine physically upwind of turbine n , regardless of whether that turbine belongs to $\mathcal{J}_{\text{up}}$, $\mathcal{J}_{\text{down}}$, or n 's own farm. Turbines within $\mathcal{J}_{\text{up}}$ therefore also experience each other's wakes. Equation 3.16 is used in this form throughout the remainder of this thesis.

Equations 3.11–3.14 depend only on the cluster layout and the wind direction. For a given time step, they are evaluated once, before the optimisation is solved, and the result is stored in the wake matrix W of Equation 3.15. They add no variables to the problem.

3.2.5. Non-linearities

The wind farm model developed in the preceding sections is non-linear in several places. The power curve and power coefficient curve of the reference turbine are non-linear functions of wind speed. The mapping from C_P to a is non-linear. Finally, as shown in Section 3.2.3, the power output of a curtailed turbine depends non-linearly on the wind speed it receives, which itself depends on curtailment η . Embedding these relations directly into an optimisation framework is therefore not tractable.

This section introduces two approximation strategies used throughout the remainder of this thesis. Section 3.2.6 first approximates the power curve, the power coefficient curve, and the C_P -to- a mapping using piecewise linear fits. Building on these approximations, Section 3.2.7 then linearises the remaining non-linear dependency on curtailment η through a Taylor expansion around a reference operating point. Figure 3.5 shows where each of these non-linearities enters the pipeline.

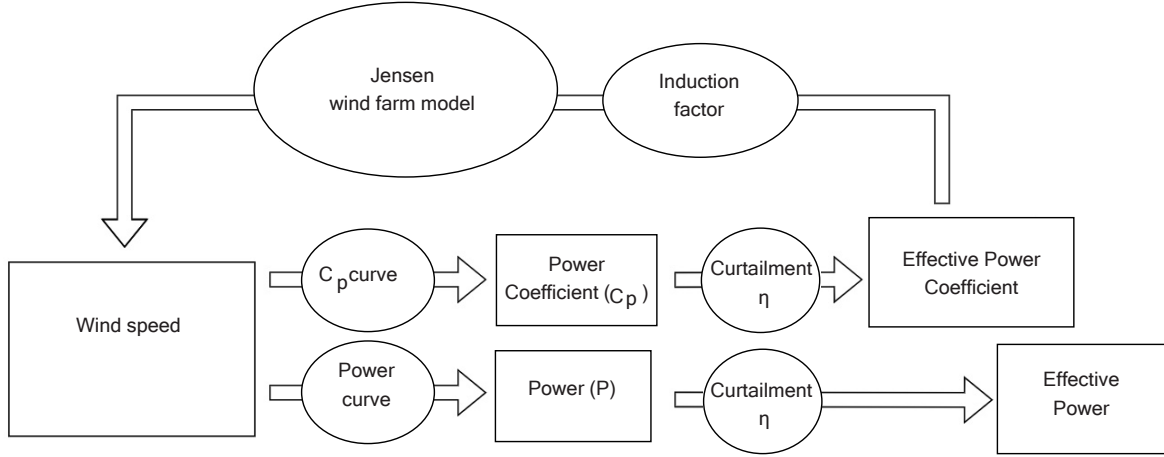


Figure 3.5: Schematic representation of the wind farm model developed in this chapter, for a single turbine. The induction factor relation of Section 3.2.1 and the wake model of this section (top) determine the wind speed, which enters the piecewise linear curves of Section 3.2.6 below. Curtailment η then scales the result to the effective power and power coefficient. η is shared by every turbine within a farm: $\eta = \eta_{up}$ for the upwind farm, $\eta = 1$ for the downwind farm.

3.2.6. Piece wise linear

As discussed in Chapter 3, a non-linear relation can be linearised by dividing it into linear segments. The relation then becomes tractable within the optimisation framework. In this thesis, each piecewise relation is enforced with a special ordered set of type 2 (SOS2) constraint. This restricts the solution to a single segment or to a breakpoint between two adjacent segments.

Figure 3.6 show the piecewise linear fits of the power curve and power coefficient curve for the IEA 10 MW and IEA 15 MW reference turbines. The power curve is fitted using 8 breakpoints and the power coefficient curve using 9 breakpoints, chosen to balance approximation accuracy against the size of the resulting optimisation problem. Figure 3.7 shows the corresponding fit of the C_P -to- a mapping, using 5 breakpoints, compared against the exact relation of Equation 3.6. Each SOS2 set introduces one weight variable per breakpoint. Applied to every turbine at every time step, this adds $N \times T \times (8+9+5) = 22NT$ weight variables and $3NT$ SOS2 constraints to the optimisation problem, where N is the number of turbines and T is the number of time steps. In all three cases, the piecewise linear approximation closely tracks the underlying curve.

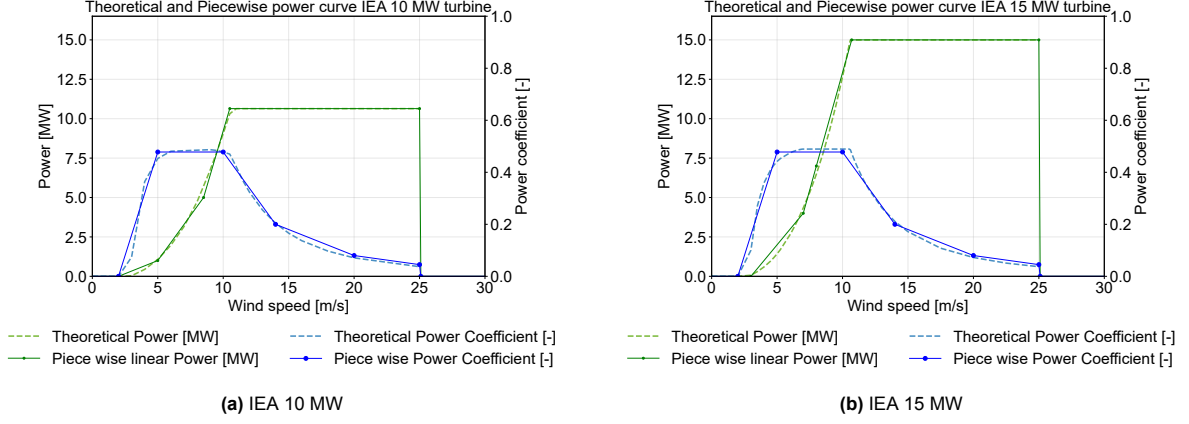


Figure 3.6: Piecewise linear approximation of the power curve and power coefficient curve for the IEA 10 MW and IEA 15 MW reference turbines.

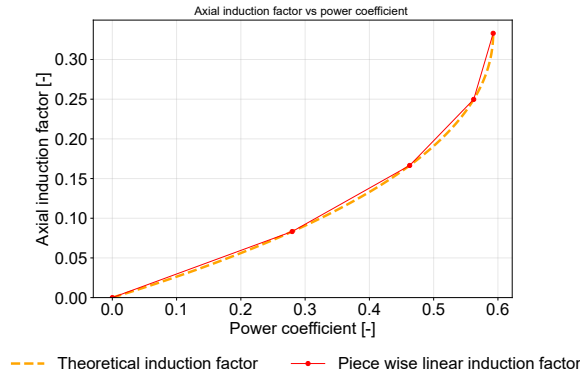


Figure 3.7: Piecewise linear approximation of the C_P -to- a mapping, compared against the exact relation of Equation 3.6.

The piecewise linear fits developed in this section resolve three of the four non-linearities identified at the start of this chapter: the power curve, the power coefficient curve, and the C_P -to- a mapping are now tractable within the optimisation framework. The remaining non-linearity, the dependence of curtailed power output on η through the wind speed U , is addressed in Section 3.2.7 using a Taylor expansion around a reference operating point.

3.2.7. Linear expanding

Section 3.2.6 linearised the mapping from C_P^{eff} to a , and the mappings from wind speed U back to C_P and p . The wake model relating a to U , introduced in Section 3.2.3 using the Jensen model and linear superposition, is linear by construction. Composed together, $C_P(U(\eta))$ and $p(U(\eta))$ are therefore themselves linear in η .

Equation 3.5 relates the curtailed quantities to η , through $C_P^{\text{eff}} = \eta C_P$ and $p^{\text{eff}} = \eta p$. When η is fixed for the downwind farm, $\eta_{\text{down}} = 1$ for an uncurtailed turbine, $C_{P,\text{down}}$ and p_{down} are only functions of η_{up} and therefore linear, giving no problem to the MILP. For the upwind farm, where η_{up} is the decision variable of the optimisation problem, C_P^{eff} is no longer linear, but a function of two quantities that are each linear in η_{up} : $C_P(U(\eta_{\text{up}}))$ and η_{up} itself. This makes $C_{P,\text{up}}^{\text{eff}}$ non-linear in η_{up} . The same holds for $p_{\text{up}}^{\text{eff}}$.

Simplified example with three turbines

To clarify, consider three turbines placed in a row. Each turbine has its own power and power coefficient, and the turbines belong to the same upstream farm, so they share a single curtailment factor η . To simplify the derivation, every turbine is assumed to operate on a single segment of the piecewise linear

approximation, so that each relation can be written as $y = \zeta x + \vartheta$. Neglecting the constant terms ϑ for now, the resulting functions are:

$$p = f(U_{\text{in}}) = x_1 U_{\text{in}} , \quad (3.17a)$$

$$C_P = f(U_{\text{in}}) = x_2 U_{\text{in}} , \quad (3.17b)$$

$$a = f(C_P^{\text{eff}}) = x_3 C_P^{\text{eff}} . \quad (3.17c)$$

The first turbine sees the free stream velocity, $U_1 = U_\infty$, so its effective power is:

$$p_1^{\text{eff}} = \eta p_1 = \eta x_1 U_\infty . \quad (3.18)$$

Curtailing the first turbine also lowers its induction factor, which in turn raises the wind speed at the second turbine. Using linear superposition of the wake deficits and no wake decay for simplification,

$$U_2 = U_\infty (1 - a_1) = U_\infty (1 - x_3 C_{P,1}^{\text{eff}}) = U_\infty (1 - x_3 \eta C_{P,1}) = U_\infty (1 - x_2 x_3 \eta U_\infty) . \quad (3.19)$$

The power of the second turbine therefore becomes:

$$p_2^{\text{eff}} = \eta x_1 U_2 = x_1 U_\infty (\eta - x_2 x_3 U_\infty \eta^2) , \quad (3.20)$$

which is quadratic in the curtailment factor. Repeating the same steps for the third turbine, with $a_2 = x_2 x_3 \eta U_2$, gives:

$$\begin{aligned} U_3 &= U_\infty (1 - a_1 - a_2) , \\ &= U_\infty (1 - x_3 C_{P,1}^{\text{eff}} - x_3 C_{P,2}^{\text{eff}}) , \\ &= U_\infty (1 - x_2 x_3 \eta U_\infty - x_2 x_3 \eta U_2) , \\ &= U_\infty (1 - 2x_2 x_3 U_\infty \eta + x_2^2 x_3^2 U_\infty^2 \eta^2) , \end{aligned} \quad (3.21)$$

so that

$$p_3^{\text{eff}} = \eta x_1 U_3 = x_1 U_\infty (\eta - 2x_2 x_3 U_\infty \eta^2 + x_2^2 x_3^2 U_\infty^2 \eta^3) . \quad (3.22)$$

Each additional turbine in the wake chain raises the order of η by one. The resulting function is a polynomial whose degree equals the number of waked turbines downstream.

Taylor expansion

Curtailment is applied uniformly across the upstream farm, so that η_{up} takes a single value shared by every turbine within the upwind farm. This section works with turbine sets grouped by farm, rather than treating each turbine individually. Let $\mathcal{J}_{\text{up}}$ and $\mathcal{J}_{\text{down}}$ denote the sets of turbines belonging to the upstream and downstream farm, respectively, as introduced in Section 3.2.3. The power output, power coefficient, and wind speed of each turbine are collected into vectors over these sets:

$$p_{\text{up}} = (p_n)_{n \in \mathcal{J}_{\text{up}}} , \quad p_{\text{down}} = (p_n)_{n \in \mathcal{J}_{\text{down}}} , \quad (3.23)$$

$$C_{P,\text{up}} = (C_{P,n})_{n \in \mathcal{J}_{\text{up}}} , \quad C_{P,\text{down}} = (C_{P,n})_{n \in \mathcal{J}_{\text{down}}} , \quad (3.24)$$

$$U_{\text{up}} = (U_n)_{n \in \mathcal{J}_{\text{up}}} , \quad U_{\text{down}} = (U_n)_{n \in \mathcal{J}_{\text{down}}} . \quad (3.25)$$

Using this notation, the curtailment relations of Equation 3.5 extend directly to the level of an entire farm. For the upstream farm, curtailment is applied uniformly through η_{up} :

$$C_{P,\text{up}}^{\text{eff}} = \eta_{\text{up}} C_{P,\text{up}}(U_{\text{up}}(\eta_{\text{up}})) , \quad (3.26)$$

$$p_{\text{up}}^{\text{eff}} = \eta_{\text{up}} p_{\text{up}}(U_{\text{up}}(\eta_{\text{up}})) . \quad (3.27)$$

Here U_{up} is itself a function of η_{up} , even though a turbine does not wake itself. The dependence runs through the other turbines in the same farm: since η_{up} is shared by all of them, curtailing the farm

weakens the wakes that its front rows leave on its own rear rows. The inflow of an upwind turbine is therefore a function of the very setting that is then applied to it.

The downstream farm is not curtailed $\eta_{\text{down}} = 1$, but the wind speed U_n it experiences is. Its power output depends on η_{up} through the wake model:

$$C_{P,\text{down}}^{\text{eff}} = C_{P,\text{down}}(U_{\text{down}}(\eta_{\text{up}})), \quad (3.28)$$

$$p_{\text{down}}^{\text{eff}} = p_{\text{down}}(U_{\text{down}}(\eta_{\text{up}})). \quad (3.29)$$

The model has two curtailment coefficients, η_{up} and η_{down} . The downwind farm is never curtailed, so $\eta_{\text{down}} = 1$ and it drops out of the equations. Only η_{up} remains as a variable, and it is written simply as η from here on.

As shown above, Equations 3.26 and 3.27 are non-linear in η_{up} , while Equations 3.28 and 3.29 remain linear. The following Taylor expansion resolves this remaining non-linear dependence.

$$C_{P,n}^{\text{eff}} = \eta C_{P,n}(U_n(\eta)) \approx \eta_0 C_{P,n}(U_n(\eta_0)) - (\eta_0 - \eta) \left(C_{P,n}(U_n(\eta_0)) + \eta_0 \frac{dC_{P,n}(U_n(\eta))}{d\eta} \Big|_{\eta_0} \right), \quad (3.30)$$

$$p_n^{\text{eff}} = \eta p_n(U_n(\eta)) \approx \eta_0 p_n(U_n(\eta_0)) - (\eta_0 - \eta) \left(p_n(U_n(\eta_0)) + \eta_0 \frac{dp_n(U_n(\eta))}{d\eta} \Big|_{\eta_0} \right). \quad (3.31)$$

Evaluating the Taylor expansion in Equations 3.30 and 3.31 at a chosen reference point η_0 requires the corresponding physical quantities $C_{P,n}(U_n(\eta_0))$, $p_n(U_n(\eta_0))$, $\frac{dC_{P,n}(U_n(\eta))}{d\eta} \Big|_{\eta_0}$ and $\frac{dp_n(U_n(\eta))}{d\eta} \Big|_{\eta_0}$.

These follow from solving the wind farm sub-model with η fixed at η_0 .

3.2.8. A Taylor-Expanded Wind Farm Cluster Model

The remainder of this thesis presents its constrained problems using standard mathematical programming notation: decision variables are stated first, followed by the constraints they are subject to. Where a problem has an objective, this is written as $\min$ or $\max$ over the decision variables, as in Chapter 3.3; where it does not, as here, it is written as Find . The system below fixes η at η_0 and has no objective. The system determines the resulting state of the wind farm sub-model at η_0 , which is used to construct the reference-point quantities required by the Taylor expansion of Equations 3.30 and 3.31:

$$\begin{aligned} &\text{Find } p_{\text{up}}, C_{P,\text{up}} \\ &\text{s.t. } U_n = U_\infty (1 - W_{jn} a_j) \end{aligned} \quad (3.32a)$$

$$a_n = f(C_{P,n}^{\text{eff}}) \quad (3.32b)$$

$$C_{P,n} = f(U_n) \quad (3.32c)$$

$$p_n = f(U_n) \quad (3.32d)$$

$$C_{P,\text{up}}^{\text{eff}} = \eta_0 \cdot C_{P,\text{up}}(U_{\text{up}}) \quad (3.32e)$$

$$p_{\text{up}}^{\text{eff}} = \eta_0 \cdot p_{\text{up}}(U_{\text{up}}) \quad (3.32f)$$

$$C_{P,\text{down}}^{\text{eff}} = C_{P,\text{down}}(U_{\text{down}}) \quad (3.32g)$$

$$p_{\text{down}}^{\text{eff}} = p_{\text{down}}(U_{\text{down}}). \quad (3.32h)$$

The same pipeline was presented schematically at the start of this section in Figure 3.5. Equation 3.32a is the wake equation of Section 3.2.3. Equations 3.32b–3.32d are the piecewise linear fits of the C_P -to- a mapping, the power coefficient curve, and the power curve, obtained from the graphs in Section 3.2.6

(Figures 3.7 and 3.6). Equations 3.32e and 3.32f are Equations 3.26 and 3.27, evaluated at $\eta = \eta_0$, which is what makes the system solvable for p_{up} and $C_{P,\text{up}}$. Equations 3.32g and 3.32h are Equations 3.28 and 3.29, likewise evaluated at $\eta = \eta_0$. The downwind farm itself remains uncurtailed, but its wind speed U_{down} , and therefore $C_{P,\text{down}}$ and p_{down} , still depends on η_{up} through the wake model.

The derivatives $\left. \frac{dC_{P,\text{up}}(U_{\text{up}}(\eta))}{d\eta} \right|_{\eta_0}$ and $\left. \frac{dp_{\text{up}}(U_{\text{up}}(\eta))}{d\eta} \right|_{\eta_0}$ in Equations 3.30 and 3.31 are approximated with a forward difference quotient. The system of Equations 3.32 is solved twice, once with η_0 , and once with $\eta_0 + \Delta$ substituted into Equations 3.32e and 3.32f, leaving all other equations unchanged. The change in $C_{P,\text{up}}$ and p_{up} between the two solves, divided by Δ , gives the local slope:

$$\left. \frac{dC_{P,\text{up}}(U_{\text{up}}(\eta))}{d\eta} \right|_{\eta_0} \approx \frac{C_{P,\text{up}}(U_{\text{up}}(\eta_0 + \Delta)) - C_{P,\text{up}}(U_{\text{up}}(\eta_0))}{\Delta}, \quad (3.33)$$

$$\left. \frac{dp_{\text{up}}(U_{\text{up}}(\eta))}{d\eta} \right|_{\eta_0} \approx \frac{p_{\text{up}}(U_{\text{up}}(\eta_0 + \Delta)) - p_{\text{up}}(U_{\text{up}}(\eta_0))}{\Delta}. \quad (3.34)$$

A forward difference is used rather than a central one. The composed mapping from η_0 to $C_{P,\text{up}}$ and p_{up} runs entirely through the piecewise linear fits of Section 3.2.6 and the linear wake model of Section 3.2.3, and is therefore linear in η_0 within one segment. The difference quotient is exact there. A central difference would also be more expensive. The solve at η_0 is needed for the expansion itself in either case; the forward difference reuses it and adds one solve at $\eta_0 + \Delta$, whereas a central difference needs another solve at $\eta_0 \pm \Delta$. That is three solves per reference point per time step instead of two.

With Equations 3.33 and 3.34, every quantity required by the Taylor expansion of Equations 3.30 and 3.31 is now available. The wind farm cluster model is therefore fully linear in η .

3.2.9. Reference point selection and Verification of the model

The previous section developed the Taylor expansion without specifying the reference point η_0 . This section presents the method used to select the reference points. Two expansions are used, one around $\eta_0 = 0.4$ and one around $\eta_0 = 0.8$, each applied on its own interval of η . Two points are used rather than one because a single expansion cannot cover the full range $\eta \in [0, 1]$ at acceptable accuracy. Each additional point, however, increases the optimisation problem. The reference-point quantities are not fixed constants. The values $C_{P,n}(U_n(\eta_0))$, $p_n(U_n(\eta_0))$ and the derivatives of Equations 3.33 and 3.34 follow from solving the system of Equation 3.32. These values depend on the free-stream wind speed U_∞ and on the wake matrix W_{nj} , and therefore on the cluster layout and the wind direction. They are recomputed for every time step of the optimisation.

The Taylor expansion is compared to the non-linear reference model. The reference model is the non-linear set of equations before applying the Taylor expansion. The reference model is given by:

$$\begin{aligned} &\text{Find } p_{\text{up}}, C_{P,\text{up}} \\ &\text{s.t. } U_n = U_\infty (1 - W_{jn} a_j) \end{aligned} \quad (3.35a)$$

$$a_n = f(C_{P,n}) \quad (3.35b)$$

$$C_{P,n} = f(U_n) \quad (3.35c)$$

$$p_n = f(U_n) \quad (3.35d)$$

$$C_{P,\text{up}}^{\text{eff}} = \eta \cdot C_{P,\text{up}}(U_{\text{up}}) \quad (3.35e)$$

$$p_{\text{up}}^{\text{eff}} = \eta \cdot p_{\text{up}}(U_{\text{up}}) \quad (3.35f)$$

$$C_{P,\text{down}}^{\text{eff}} = C_{P,\text{down}}(U_{\text{down}}) \quad (3.35g)$$

$$p_{\text{down}}^{\text{eff}} = p_{\text{down}}(U_{\text{down}}). \quad (3.35h)$$

Figure 3.9 compares both expansions against the non-linear reference model of the above Equation 3.35 at four free-stream wind speeds, 7 m/s, 10 m/s, 13 m/s and 16 m/s—the first two lie below and near the rated wind speed. The last two lie above it and differ in whether the waked turbines remain above rated. Farm power outputs are shown separately over the full range of η . The cluster consists of 200 IEA 10 MW turbines at 5 D (5×196 m) spacing, with the wind aligned along the y-axis as shown in figure 3.8; the forward difference of Equations 3.33 and 3.34 uses $\Delta = 0.05$.

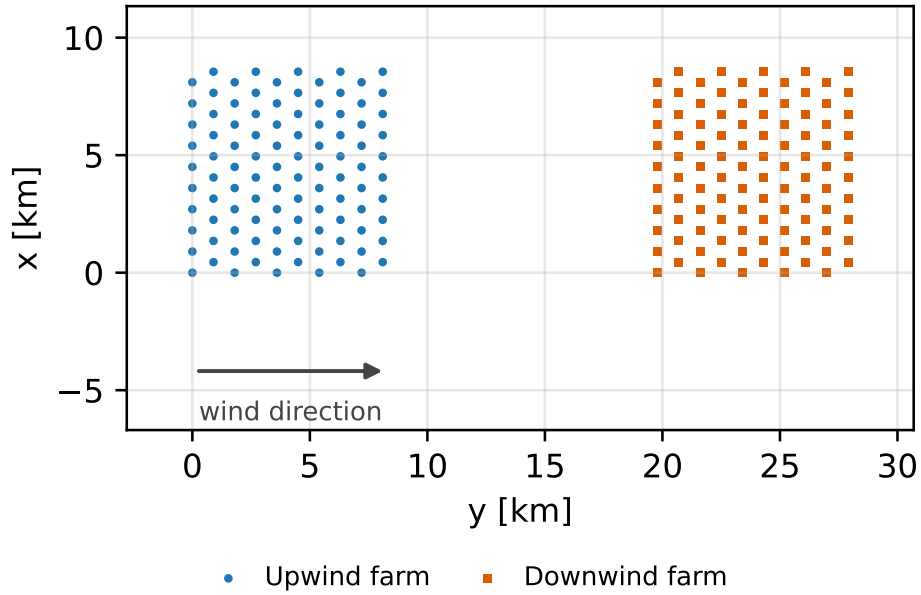


Figure 3.8: Layout of the two-farm cluster, with 100 IEA 10 MW turbines in each farm at a spacing of 5D (980 m). The wind is aligned with the y axis, so the left farm is upwind throughout.

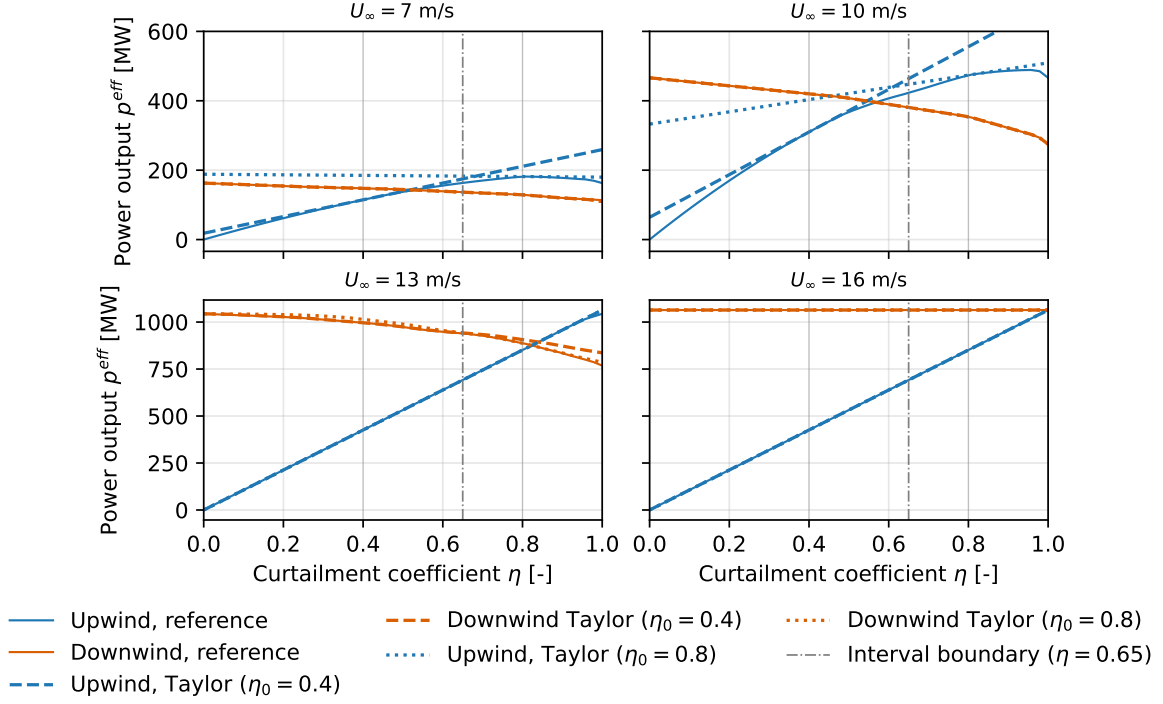


Figure 3.9: Power output of the upwind and downwind farm as a function of the curtailment coefficient η , at four free-stream wind speeds. The non-linear reference model of Equation 3.35 is shown together with two Taylor expansions, around $\eta_0 = 0.4$ and $\eta_0 = 0.8$. Each expansion is drawn over the full range of η ; the dash-dotted grey line marks the interval boundary at $\eta = 0.65$ at which the model switches between them.

The two reference points are chosen so that the Taylor expansion approximates the p_n^{eff} curve as closely as possible. Below and near the rated free-stream wind speed, the curve first rises linearly with η and then flattens as η approaches unity. One expansion is placed on each of these regions: $\eta_0 = 0.4$ on $[0, 0.65]$ and $\eta_0 = 0.8$ on $[0.65, 1]$.

Above rated wind speed $U_\infty = 16 \text{ m/s}$ the curve is linear over the full range, and both expansions reproduce the full model exactly. The turbines experiencing the wake of the upwind turbines remain above rated wind speed, where p_n is insensitive to U , so p_n^{eff} reduces to $\eta p_{n,\text{rated}}$. The wake remains, but no longer affects power output. The turbines also operate at a lower $C_{P,n}$ in this region, which further weakens the wake.

Figure 3.10 shows the resulting piecewise approximation: the expansion around $\eta_0 = 0.4$ is used for $\eta \leq 0.65$ and the one around $\eta_0 = 0.8$ above it, so that together they cover $\eta \in [0, 1]$. The grey curves, read against the right-hand axis, give the deviation from the non-linear model of Equation 3.32, as a percentage of the maximum power output of that farm at the given wind speed. The deviation grows with the distance to the expansion point and is therefore largest at $\eta = 0$ and at the transition at $\eta = 0.65$; across wind speeds, it peaks at 10 m/s , where the farms operate just below rated power, and the response to curtailment is most non-linear.

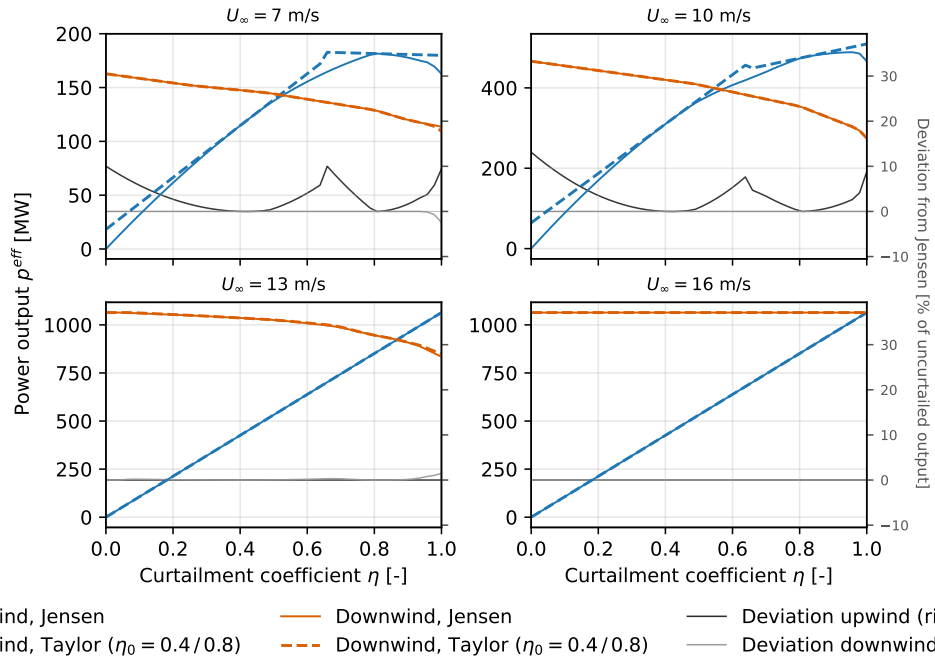


Figure 3.10: Power output of the upwind and downwind farm under the piecewise Taylor approximation, with each expansion restricted to its own interval. The dash-dotted line marks the interval boundary at $\eta = 0.65$. The grey curves give the deviation from the non-linear reference model, in MW, on the right-hand axis.

3.2.10. Resulting wind farm cluster model

The model developed in this section maps a single scalar, the curtailment coefficient of the upwind farm, onto the power output of every turbine in a two-farm cluster, using only linear relations. This section collects that formulation.

Equations 3.36 restate the wind farm cluster model in the form in which it enters Chapter 3.3. It is not solved on its own: It has no objective, and the market model determines η . The resulting wind farm model is given by:

$$U_n = U_\infty (1 - W_{jn} a_j) \quad (3.36a)$$

$$a_n = f(C_{P,n}^{\text{eff}}) \quad (3.36b)$$

$$C_{P,n} = f(U_n) \quad (3.36c)$$

$$p_n = f(U_n) \quad (3.36d)$$

$$C_{P,\text{up}}^{\text{eff}} = \eta_0 C_P(U(\eta_0)) - (\eta_0 - \eta) \left(C_P(U(\eta_0)) + \eta_0 \frac{dC_P}{d\eta} \Big|_{\eta_0} \right) \quad (3.36e)$$

$$p_{\text{up}}^{\text{eff}} \leq \eta_0 p(U(\eta_0)) - (\eta_0 - \eta) \left(p(U(\eta_0)) + \eta_0 \frac{dp}{d\eta} \Big|_{\eta_0} \right) \quad (3.36f)$$

$$C_{P,\text{down}}^{\text{eff}} = C_{P,\text{down}}(U_{\text{down}}) \quad (3.36g)$$

$$p_{\text{down}}^{\text{eff}} \leq p_{\text{down}}(U_{\text{down}}) \quad (3.36h)$$

$$\eta \in \begin{cases} [0, 0.65] & \text{for } \eta_0 = 0.4 \\ [0.65, 1] & \text{for } \eta_0 = 0.8 \end{cases} \quad (3.36i)$$

Equation 3.36a is the wake model of Section 3.2.3, with $\mathbf{W}$ given by Equation 3.15. Equations 3.36b–3.36d are the piecewise linear fits from Section 3.2.6, each enforced with an SOS2 constraint. Equations 3.36e and 3.36f are the Taylor expansions from Section 3.2.7, evaluated at the reference points selected in Section 3.2.9; the reference quantities are recomputed at every time step from the free-stream wind speed and the wake matrix. Equations 3.36g and 3.36h describe the downwind farm, which is not curtailed but whose inflow still depends on η , because they are in the wake of the upwind farm.

The power relations are stated as inequalities rather than equalities. This relaxation permits the cluster to spill available power, which is required in Chapter 3.3, where the market may clear below the available wind output.

Selecting between the two intervals of Equation 3.36i requires the system to be solved twice, on both intervals of the curtailment η . These two intervals are compared at each time step, and the best solution is chosen. The best solution depends on the full model, so the full model must be solved twice. The computational cost of adding more Taylor expansions to span the range of η depends on the size of the energy system model, since each additional interval requires one further solve of the full coupled problem.

The linearisation reproduces the spatial distribution of power output of the non-linear model in a linear model. The linear model deviates most from the non-linear model at the boundaries of the Taylor-expansion region. To reduce the error, the found curtailment η is afterwards used in the Jensen model. Equation 3.36 is used unchanged in Chapter 3.3, where the market determines the value of the power it shifts between the two farms.

3.3. Wind farm cluster optimising maximum revenue under fixed day-ahead prices

The previous sections developed a model that connects the power output of a two-farm cluster to a single scalar, the curtailment coefficient (η). Because the model uses only linear relations, it can be used in an optimisation framework. Under the rloptimisation conditions, the two farms are connected through their wakes, and power can be redistributed between them by curtailing the upwind farm. Whether that redistribution is worth anything depends on the market in which the cluster sells, and this section embeds the wind farm model in market representations.

In this section, the cluster is treated as a price taker: the two farms sell their energy at prices they cannot influence, and the operator chooses curtailment to maximise the cluster's profit. The two farms may sit in different bidding zones and face different prices, which is what makes redistributing power between them worth anything.

The price difference between the two farms is introduced through the following objective function:

$$\max_{\eta} \quad \lambda_{up}^{DA} \sum_{n \in \mathcal{J}_{up}} p_n^{\text{eff}} + \lambda_{down}^{DA} \sum_{n \in \mathcal{J}_{down}} p_n^{\text{eff}}, \quad (3.37)$$

where λ^{DA} is the day-ahead electricity price in the upwind and downwind farms. Combining this objective with the wind farm model of Section 3.2.10 gives:

$$\max_{\eta} \quad \lambda_{up}^{DA} \sum_{n \in \mathcal{J}_{up}} p_n^{\text{eff}} + \lambda_{down}^{DA} \sum_{n \in \mathcal{J}_{down}} p_n^{\text{eff}} \quad (3.38a)$$

$$\text{s.t.} \quad U_n = U_{\infty} (1 - W_{jn} a_j) \quad (3.38b)$$

$$a_n = f(C_{P,n}^{\text{eff}}) \quad (3.38c)$$

$$C_{P,n} = f(U_n) \quad (3.38d)$$

$$p_n = f(U_n) \quad (3.38e)$$

$$C_{P,up}^{\text{eff}} = \eta_0 C_{P,up}(U_{up}(\eta_0)) - (\eta_0 - \eta) \left(C_{P,up}(U_{up}(\eta_0)) + \eta_0 \frac{dC_{P,up}}{d\eta} \Big|_{\eta_0} \right) \quad (3.38f)$$

$$p_{up}^{\text{eff}} = \eta_0 p_{up}(U_{up}(\eta_0)) - (\eta_0 - \eta) \left(p_{up}(U_{up}(\eta_0)) + \eta_0 \frac{dp_{up}}{d\eta} \Big|_{\eta_0} \right) \quad (3.38g)$$

$$C_{P,down}^{\text{eff}} = C_{P,down}(U_{down}) \quad (3.38h)$$

$$p_{down}^{\text{eff}} = p_{down}(U_{down}) \quad (3.38i)$$

$$\eta \in \begin{cases} [0, 0.65] & \text{for } \eta_0 = 0.4 \\ [0.65, 1] & \text{for } \eta_0 = 0.8 \end{cases}. \quad (3.38j)$$

The wind farm sub-model enters unchanged from Section 3.2.10; only the objective is added, equation 3.37. The formulation is a mixed integer linear program and is solved per time step.

3.4. Wind farm cluster optimisation minimising system cost under Energy system

In this section, the two wind farm clusters are analysed within an energy system: the *IJmuiden Ver* and *Hollandse Kust West* cluster, and the *Borssele* and Belgian farm cluster. In the previous section, the price was an independent parameter, fixed regardless of what the cluster produced. Here it varies because the cluster's output shifts the merit order: more output pushes cheaper generation to the margin and lowers the price the cluster receives. Curtailing has the opposite effect. For a small wind farm, this effect is negligible, but for clusters of this size it can be significant. Treating the price as a variable also shows under which system conditions a cluster strategy is useful; with a fixed price, this is not possible.

Figure 3.11 shows the two offshore clusters and the nodal network connecting them to the onshore energy system.

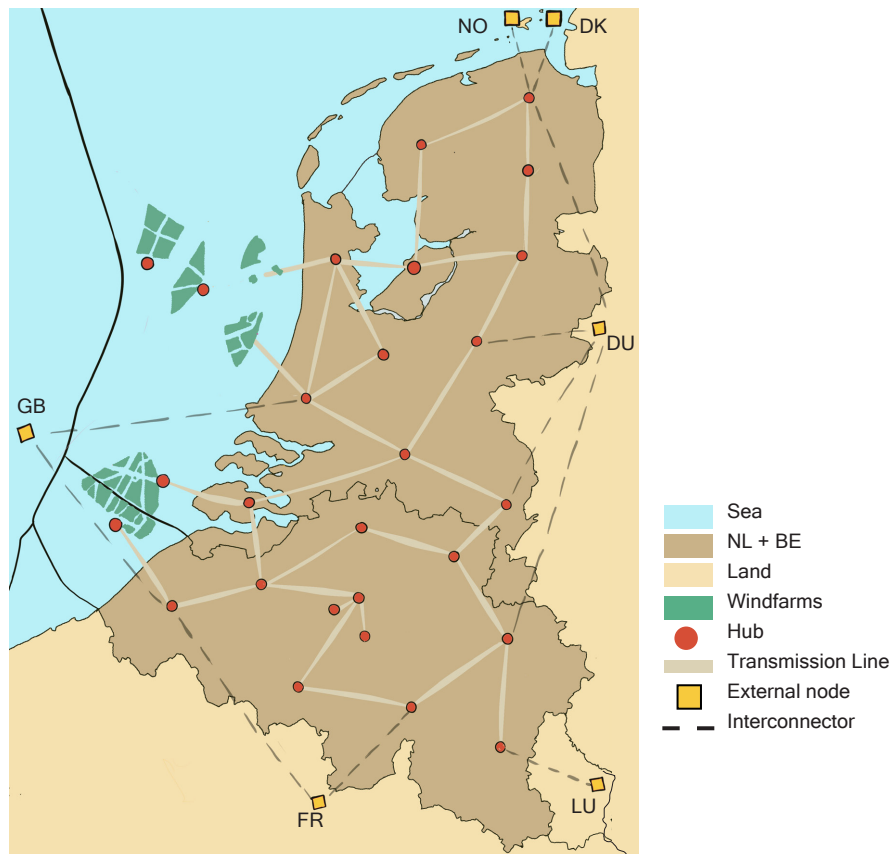


Figure 3.11: Modal representation of the Dutch and Belgian electricity system. Each hub is a province with its own demand, conventional generators and renewable capacity, connected by transmission lines of finite capacity. The four offshore zones of the two clusters studied here are shown: IJmuiden Ver and Hollandse Kust West connect to South-Holland and North-Holland, respectively; Borssele zone to Zeeland; and the Belgian zone to West-Vlaanderen.

Each node represents a province and has its own demand, conventional generators, and wind & solar capacity. Each transmission line has its own capacity. In addition, imports and exports from countries outside the system are the same as in 2020. The figure also shows that the wind farms connect to different nodes: IJmuiden Ver to Noord-Holland and Hollandse Kust West to Zuid-Holland, while in the Dutch–Belgian cluster the Borssele park connects to Zeeland and the Belgian farms to West-Vlaanderen.

To understand the energy system, we first define a few sets. These describe the generating units, hubs, and transmission lines that make up the system:

$\mathcal{J}$	Set of turbines belonging to the two-farm cluster (upwind and downwind) to which the active curtailment control strategy is applied, with $\mathcal{J} = \mathcal{J}_{\text{up}} \cup \mathcal{J}_{\text{down}}$.
G	Set of all generating units in the market. Each upwind and downwind farm enters G as a single generating unit, with $G = S \cup V \cup \{\text{up}, \text{down}\}$.
V	Set of all non-renewable generating units.
S	Set of all renewable generating units that are <i>not</i> part of the controlled cluster, including any other, uncontrolled offshore wind farms elsewhere in the system.
Z	Set of all solar generating units
N	Set of all nuclear generating units, with $N \subseteq V$.
H	Set of all hubs, one per province.
L	Set of all transmission lines connecting the hubs.
$\text{from}(l), \text{to}(l)$	The hubs at either end of transmission line $l \in L$.
$h(g)$	The hub to which generating unit $g \in G$ is connected.

The parameters appearing in the model are:

p_n	Power output of an individual turbine $n \in \mathcal{J}$, and the vector collecting the outputs of all turbines in that farm, respectively [MW].
P_g	Aggregated power output of generating unit $g \in G$, as seen by the market [MW].
MC_g	Marginal cost of generating unit g [EUR/MWh].
K_g	Installed capacity of generating unit g [MW].
CF_s	Capacity factor of renewable unit $s \in S$, the share of installed capacity available under the weather conditions at its location [-].
α_Z	Minimum dispatch share of the available solar output, set to 0.8 [-].
α_N	share of installed capacity at which nuclear units are fixed [-].
F_l	Power flow on transmission line $l \in L$, positive in the direction from $\text{from}(l)$ to $\text{to}(l)$ [MW].
F_l^+, F_l^-	The flow on line l in the forward and reverse direction respectively, with $F_l = F_l^+ - F_l^-$ and $F_l^+, F_l^- \geq 0$ [MW].
$F_l^{\max}$	Transmission capacity of line l [MW].
ζ_l	Transmission efficiency of line l , the share of the power entering the line that arrives at the other end [-].
$\mathcal{F}_h$	Net power flow into hub $h \in H$ from the lines connected to it [MW].
D_h	Electricity demand at hub h [MW].
X_h	Net exchange at hub h with countries outside the modelled system, positive for imports [MW].

In Appendix A the values of these parameters are presented.

The energy system is simulated with the objective of dispatching available energy with the lowest marginal cost first. This objective depends on constraints on load balance, location, and the capacity of both generators and transmission lines. These constraints closely match the Dutch and Belgian systems.

The objective function therefore becomes:

$$\min_{\eta, P} \sum_{g \in G} MC_g \cdot P_g, \quad (3.39)$$

where

- P_g is the aggregated power output of generating unit g [MW],
- MC_g is the marginal cost of generating unit g [EUR/MWh], and
- η is the curtailment of the upwind farm $\mathcal{J}_{\text{up}}$ [-].

The optimum is the combination of generator outputs and curtailment that meets demand at the lowest total cost. A hard coal unit has a high marginal cost, so the system dispatches any cheaper source ahead of it whenever constraints allow.

Note that this objective is that of the system, not of the wind farm operators. No part of this formulation maximises either farm's revenue, and the curtailment the market clearing selects may well reduce it. The results of this section therefore answer a different question than those of Section 4.2.1: not whether an operator would choose to curtail, but whether curtailing serves the system.

Wind has a marginal cost close to zero, so curtailment barely lowers the total cost directly. Instead, it redistributes energy generation from the upwind node to the downwind node, much like a transmission line between the two. However, this happens at a loss; part of the energy given up upstream is never recovered downstream. Whether that loss is worth taking depends on the rest of the total energy system.

The objective is dependent on several constraints, as mentioned earlier. Energy produced in southern Belgium, for example, cannot readily serve demand in the northern Netherlands. Furthermore, generators are bounded by their installed capacity. Taking these factors and the constraints discussed further below into account, the full system becomes:

$$\min_{\eta, P} \sum_{g \in G} MC_g \cdot P_g$$

subject to both

Wind farm cluster model:

$$U_n = U_\infty (1 - W a_n) \quad (3.40a)$$

$$a_n = f(C_{P,n}^{\text{eff}}) \quad (3.40b)$$

$$C_{P,n} = f(U_n) \quad (3.40c)$$

$$p_n = f(U_n) \quad (3.40d)$$

$$C_{P,\text{up}}^{\text{eff}} = \eta_0 C_{P,\text{up}}(U_{\text{up}}(\eta_0)) - (\eta_0 - \eta) \left(C_{P,\text{up}}(U_{\text{up}}(\eta_0)) + \eta_0 \frac{dC_{P,\text{up}}}{d\eta} \Big|_{\eta_0} \right) \quad (3.40e)$$

$$p_{\text{up}}^{\text{eff}} = \eta_0 p_{\text{up}}(U_{\text{up}}(\eta_0)) - (\eta_0 - \eta) \left(p_{\text{up}}(U_{\text{up}}(\eta_0)) + \eta_0 \frac{dp_{\text{up}}}{d\eta} \Big|_{\eta_0} \right) \quad (3.40f)$$

$$C_{P,\text{down}}^{\text{eff}} = C_{P,\text{down}}(U_{\text{down}}) \quad (3.40g)$$

$$p_{\text{down}}^{\text{eff}} = p_{\text{down}}(U_{\text{down}}) \quad (3.40h)$$

$$P_{\text{up/down}} \leq \sum p_{\text{up/down}}^{\text{eff}} \quad (3.40i)$$

$$\eta \in \begin{cases} [0, 0.65] & \text{for } \eta_0 = 0.4 \\ [0.65, 1] & \text{for } \eta_0 = 0.8 \end{cases} \quad (3.40j)$$

and

Energy system sub-model:

$$0 \leq P_v \leq K_v \quad \forall v \in V \quad (3.40k)$$

$$P_s \leq CF_s \cdot K_s \quad \forall s \in S \quad (3.40l)$$

$$P_z \geq \alpha_z \cdot CF_z \cdot K_z \quad \forall z \in Z \quad (3.40m)$$

$$P_k = \alpha_N \cdot K_k \quad \forall k \in N \quad (3.40n)$$

$$-F_l^{\text{max}} \leq F_l \leq F_l^{\text{max}} \quad \forall l \in L \quad (3.40o)$$

$$\mathcal{F}_h = \sum_{\substack{l \in L \\ \text{to}(l)=h}} \zeta_l F_l^+ + \sum_{\substack{l \in L \\ \text{from}(l)=h}} \zeta_l F_l^- - \sum_{\substack{l \in L \\ \text{from}(l)=h}} F_l^+ - \sum_{\substack{l \in L \\ \text{to}(l)=h}} F_l^- \quad \forall h \in H \quad (3.40p)$$

$$D_h = \sum_{\substack{g \in G \\ h(g)=h}} P_g + \mathcal{F}_h + X_h \quad \forall h \in H. \quad (3.40q)$$

Equations 3.40a–3.40j are the wind farm sub-model of Section 3.2.10, reproduced unchanged. Equation 3.40i connects the individual turbine output to the market: the two farms enter the merit order as single generating units, whose output is the sum over their turbines.

The remaining constraints (Equations 3.40k–3.40q) describe the energy system. Conventional units are bounded by their installed capacity K_v in Equation 3.40k, and may operate anywhere between zero and that bound. Renewable units outside the cluster are bounded above by their capacity factor CF_s , which follows from the weather at their location. Wind energy can be curtailed and operates anywhere between zero and that bound. Solar generation cannot. Much of the installed capacity sits on household roofs and feeds directly into the distribution network, where the market operator has no dispatch control. Equation 3.40m imposes a minimum dispatch share α_Z on the available solar output.

Nuclear units are treated differently again. Equation 3.40n fixes their output rather than bounding it above, because they cannot follow demand on the timescale considered here.

Assumption 3.3 (Nuclear as must-run). Nuclear units generate at a fixed share α_N of their installed capacity in every hour, independent of demand and price. In this thesis $\alpha_N = 1$.

Together with the lower bound on solar output in Equation 3.40m, Assumption 3.3 is what produces negative prices in the model. When demand is low and renewable output is high, neither source can step aside, and generation exceeds what the market would otherwise clear. Prices then fall below zero, since these units are willing to pay to remain dispatched.

Transmission is bounded in both directions by the line capacity F_l^{max} in Equation 3.40o. Equation 3.40p gives the net flow $\mathcal{F}_h$ into hub h : the flows arriving from connected lines, reduced by the transmission efficiency ζ_l , minus the flows leaving it. Transmission lines lose 1% efficiency every 100 km. The load balance of Equation 3.40q then requires that demand D_h at every hub is met by local generation, net inflow, and the exchange X_h with countries outside the modelled system. This constraint ties the cluster to its location: power produced offshore is only useful where the network can deliver it.

We assume the exchange X_h with countries outside the system is a set parameter.

Assumption 3.4 (Fixed external exchange). Imports from and exports to countries outside the Netherlands and Belgium follow the realised 2020 exchange. It does not respond to the dispatch computed by the model.

A disadvantage of fixed imports and exports is that they do not respond in future scenarios, so unrealistic imports and exports may occur.

The curtailment $\hat{\eta}$ obtained from 3.40 is subsequently substituted back into the non-linear formulation. With η fixed, the non-linear term vanishes and the system can be solved directly, which removes the approximation error introduced by the Taylor expansion.

The complete solution procedure is therefore:

1. the Taylor coefficients of 3.32 are computed by solving the wind farm sub-model at curtailment η_0 and at $\eta_0 + \Delta$;
2. the optimal curtailment $\hat{\eta}$ is determined for each of the two expansion regions, $\eta \in [0, 0.65]$ and $\eta \in [0.65, 1]$;
3. the optimum of the two regions is selected per time step;
4. the greedy dispatch ($\eta = 1$) is computed as a reference;
5. the controlled dispatch is computed at the selected curtailment $\hat{\eta}$.

Steps 1–5 are executed independently for every hour in the simulated window. The difference between the dispatch of steps 4 and 5 isolates the effect of the curtailment strategy, and all results reported in 4 are expressed as that difference.

4. Results

This chapter presents the results of this thesis. Section 4.1 first shows the output of the wind farm model, compares it to the non-linear reference model, and verifies that the results are physically plausible. In Section 4.2, the wind farm model is optimised for total revenue under a range of atmospheric conditions and price spreads; these results are benchmarked against the non-linear model and against the study by Kenis et al. [24]. Section 4.3 embeds the wind farm in a full nodal network, minimising system cost subject to market constraints. The final Section, Section 4.4.1, discusses the results.

4.1. Wind farm cluster model

The behaviour of the linearised wind farm model is first checked against the non-linear wake model. Figure 4.1 shows the power output of every turbine in the cluster at $U_\infty = 10 \text{ m/s}$, for four values of η , computed with the non-linear model (top) and with the Taylor approximation (middle). The bottom row gives the difference between the two.

Both models reproduce the same behaviour. At $\eta = 0$, the upwind farm extracts no power, and the entire output of the cluster is generated by the downwind farm. The downwind farm now operates in undisturbed flow. At $\eta = 1$, the situation reverses: the upwind farm operates at its uncurtailed induction factor and generates the most power. The downwind farm now receives the lowest incoming wind speed and generates less energy. Within each farm, the output decreases with distance into the array, as the front row operates in the flow that reaches it undisturbed and each subsequent row experiences the wake of the turbines ahead of it.

These features follow from the Jensen model of Section 3.2.3. The linearised formulation reproduces them, confirming that the expansion does not distort the spatial distribution of power output.

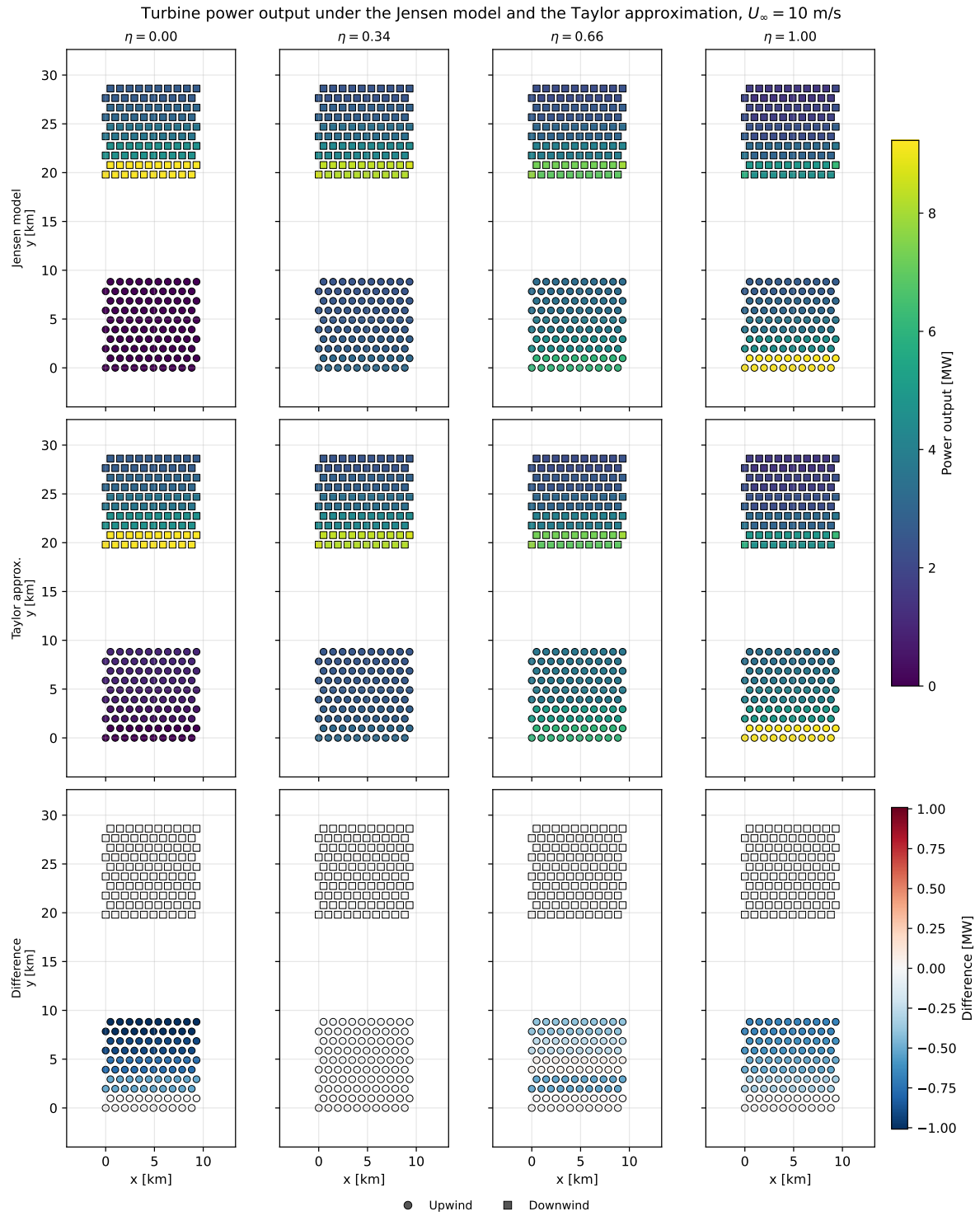


Figure 4.1: Turbine power output under the Jensen model and under the Taylor expansion. The bottom row shows the difference between the two. Values are given per individual turbine rather than as a total farm output.

4.2. Wind farm cluster optimising maximum revenue under fixed day-ahead prices

In this Section, the model is verified for a two-farm cluster in which the two farms sell their energy at different prices. This is done in two ways: First, all curtailment levels are simulated, and the resulting revenue is compared with the optimum found by the solver; Second, the simulation is run for two smaller farms, for which the optimum can be computed directly from the non-linear MINLP formulation. That result serves as the reference against which the linearised model is compared.

The optimiser searches for the curtailment (η) that maximises the cluster revenue. The cluster is the same as in Section 3.2.7, with 100 turbines in each farm; the layout is shown in Figure 3.8. The optimal point depends on the price difference and on the wind speed. Figure 4.2 shows the curtailment found by the linearised optimiser, together with the revenue over the full range of η according to the non-linear reference model. The upwind and downwind farms receive 10 EUR and 50 EUR for every Megawatt-hour they respectively generate.

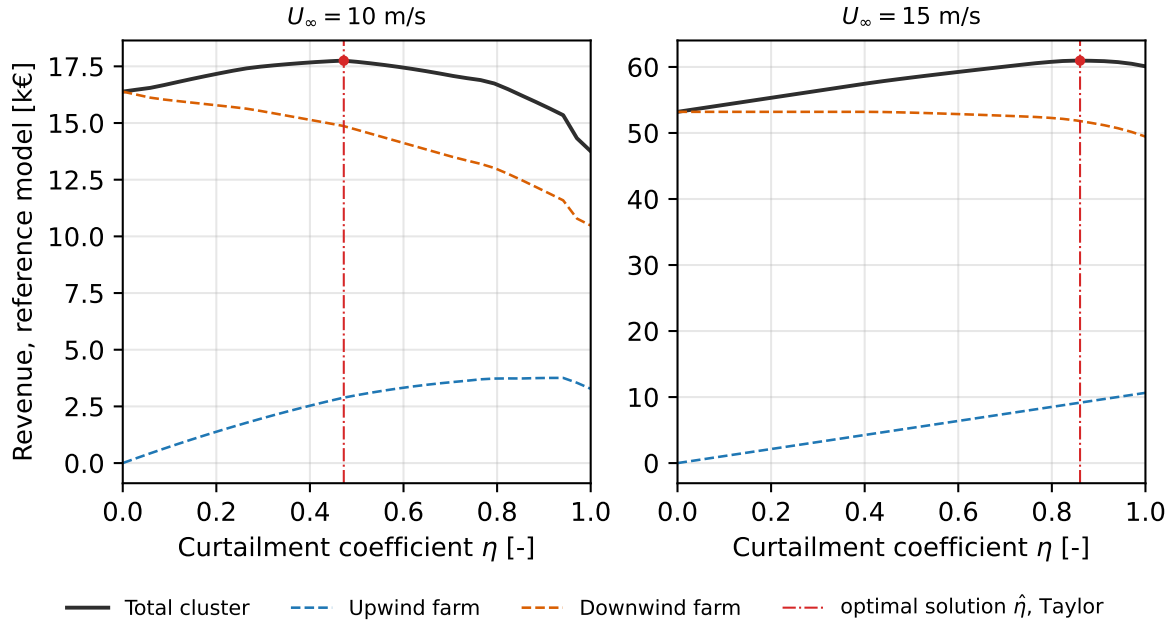


Figure 4.2: Cluster revenue as a function of the curtailment coefficient η , evaluated in the non-linear reference model, at two free-stream wind speeds. The upwind farm receives 10 EUR/MWh and the downwind farm 50 EUR/MWh. Revenue is shown for each farm separately, and for the cluster as a whole. The dash-dotted red line marks the optimum $\hat{\eta}$ returned by the linearised optimiser. The cluster layout is that of Figure 3.8.

The figure shows that under these conditions the optimiser finds the optimal curtailment $\hat{\eta}$. This optimum is lower for lower wind speeds, which follows from the power coefficient. Below rated wind speed, the turbines operate near their maximum power coefficient, C_P . Therefore, they induce the strongest velocity deficit in this region, and curtailing reduces this deficit the most. This increases the impact on the downwind farm, and distributing energy to the downwind farm is more efficient. So when the downwind day-ahead price is higher, curtailing the upwind farm has more effect below rated wind speed than above it.

The same comparison is repeated for the Belgian-Dutch cluster used in the upcoming Chapter 3.3, at 10 EUR/MWh for the upwind and 50 EUR/MWh for the downwind farm. This cluster has 374 turbines and a different spacing. This tests whether the agreement above also applies to the layout used in later Sections of the thesis.

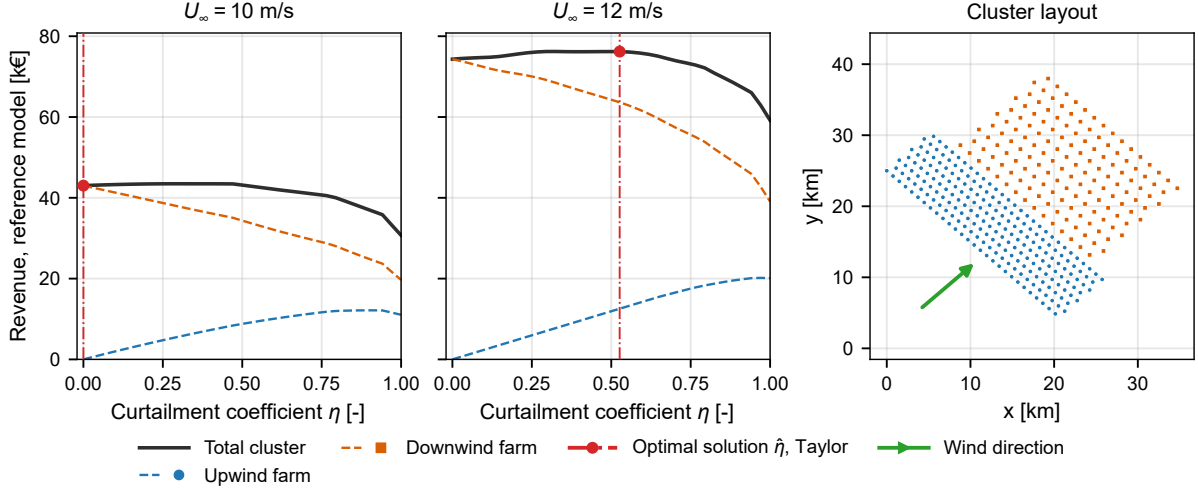


Figure 4.3: As in Figure 4.2, but for the Belgian-Dutch cluster of 374 turbines, whose layout is shown in the right-hand panel. The upwind farm receives 10 EUR/MWh and the downwind farm 50 EUR/MWh. At 12 m/s the linearised optimiser recovers the optimum of the reference model; at 10 m/s it returns $\hat{\eta} = 0$ while the reference optimum lies at $\eta^* = 0.294$.

At $U_\infty = 12$ m/s, the optimiser again finds the optimum of the reference model. At windspeeds of 10 m/s, the optimiser returns $\hat{\eta} = 0$, which shuts the upwind farm down entirely. The non-linear reference optimum lies at $\eta^* = 0.294$, an significant difference. The aggregate revenue curve explains this difference. Near its optimum, the revenue curve is almost flat. Hence, the revenue lost by moving away from η^* is small. The Taylor expansion, meanwhile, overestimates p^{eff} most strongly at the interval boundaries. If the overestimation exceeds the curvature of the revenue curve, the boundary becomes the reported optimum. This overestimation influences the selected point and yields a lower curtailment than the reference model.

The consequence is limited. Evaluating both $\hat{\eta}$ and η^* in the reference model gives 43.02 and 43.47 kEUR respectively, a loss of 1 %. The curtailment error is large, but the flat revenue curve that causes it also bounds the revenue error.

The Taylor expansion overestimates the power output; therefore, the revenue it reports at $\hat{\eta}$ is not the revenue the cluster would actually earn. To evaluate the curtailment on equal terms, $\hat{\eta}$ is substituted into the reference model, and the power output and revenue are computed. Since $\hat{\eta}$ is then fixed, the non-linear terms vanish and the system reduces to:

$$\begin{aligned} &\text{Find } p_{\text{up}}, C_{P,\text{up}} \\ &\text{s.t. } U_n = U_\infty (1 - W_{jn} a_j) \end{aligned} \quad (4.1a)$$

$$a_n = f(C_{P,n}^{\text{eff}}) \quad (4.1b)$$

$$C_{P,n} = f(U_n) \quad (4.1c)$$

$$p_n = f(U_n) \quad (4.1d)$$

$$C_{P,\text{up}}^{\text{eff}} = \hat{\eta} \cdot C_{P,\text{up}}(U_{\text{up}}) \quad (4.1e)$$

$$p_{\text{up}}^{\text{eff}} = \hat{\eta} \cdot p_{\text{up}}(U_{\text{up}}) \quad (4.1f)$$

$$C_{P,\text{down}}^{\text{eff}} = C_{P,\text{down}}(U_{\text{down}}) \quad (4.1g)$$

$$p_{\text{down}}^{\text{eff}} = p_{\text{down}}(U_{\text{down}}) \quad (4.1h)$$

This reduced system evaluates the curtailment found by the linearised model. The resulting output is compared to the reference model. The model is also compared to the reference model for a range of price ratios between the upwind and downwind farms. The reference model is an MINLP and is not tractable for clusters of more than roughly 100 turbines, so a smaller cluster of 49 turbines per farm is used; see Figure 4.4 for the layout.

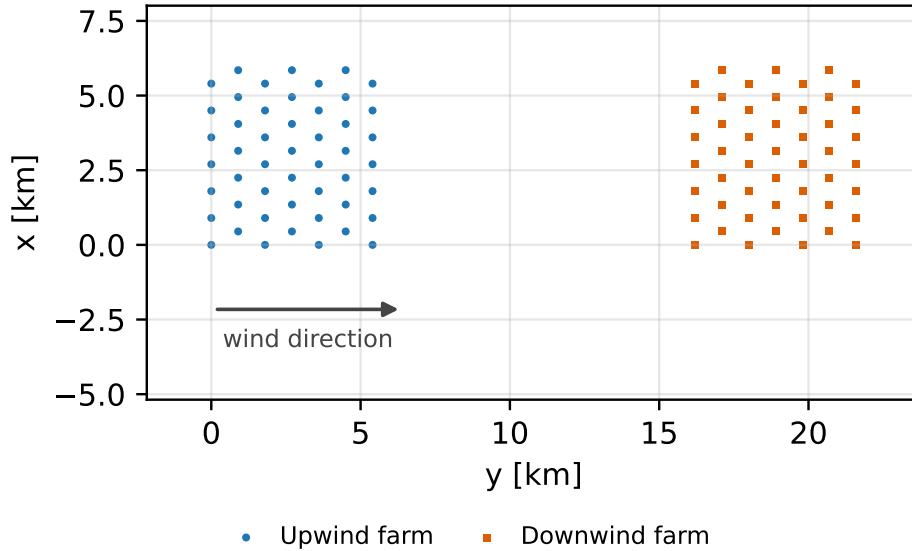


Figure 4.4: Layout of the reduced two-farm cluster, with 49 IEA 10 MW turbines in each farm at a spacing of $5D$ (980 m). The wind is aligned with the y axis, so the left farm is upwind throughout. This is the largest cluster for which the quadratic reference model of Equation 3.35 remains tractable.

Figure 4.5 on the next page shows the optimal curtailment, $\hat{\eta}$ and η^* for the Wind farm model and the Reference model respectively, for different price ratios. These price ratios range from 1:1 to 1:10, where the total price is fixed. For example, 30 EUR/MWh for both farms or 10 and 50 EUR/MWh for the upwind and downwind farms, respectively. Also, the revenue of the cluster is plotted for the reference and Taylor models.

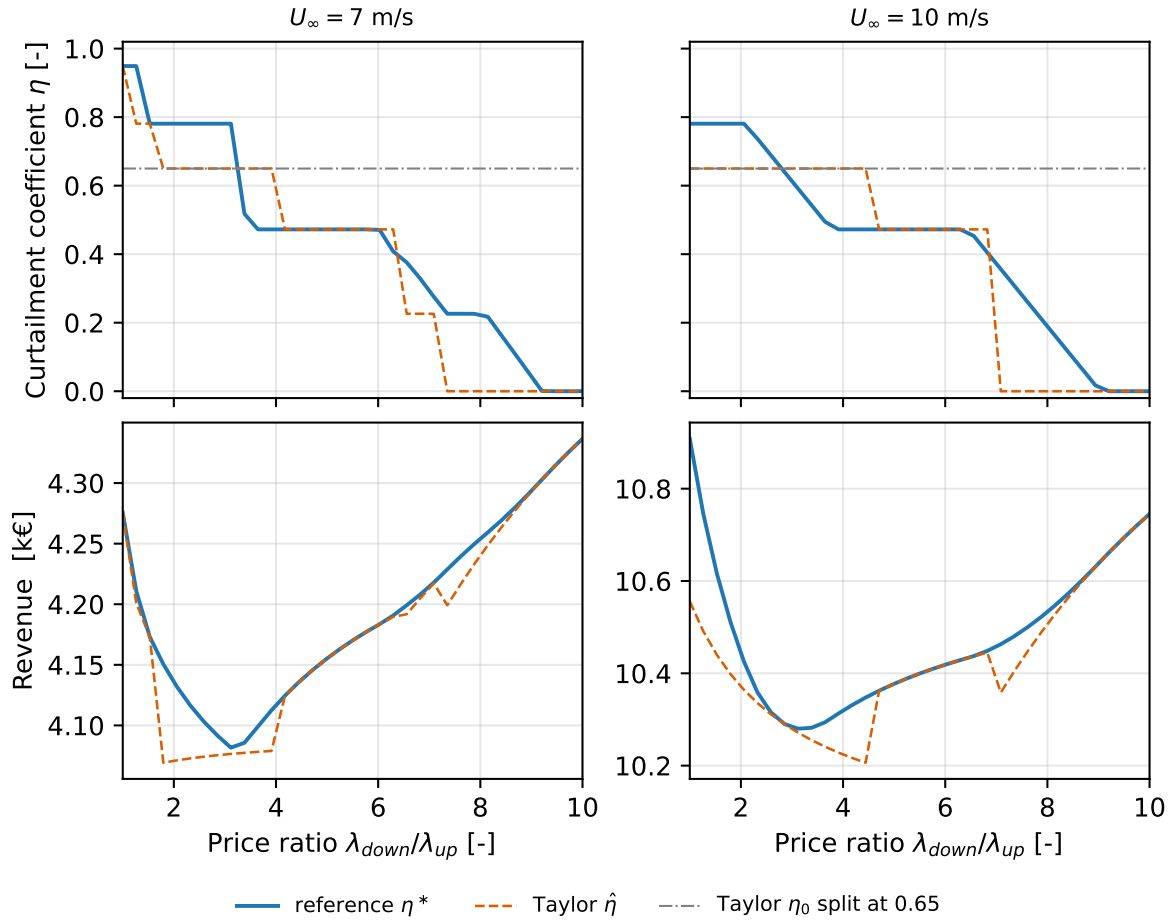


Figure 4.5: Optimal curtailment (top) and cluster revenue (bottom) as a function of the price ratio between the downwind and upwind farm, at free-stream wind speeds of 7 m/s and 10 m/s. The linearised model ($\hat{\eta}$) is compared against the non-linear reference model (η^*) for the reduced cluster of 49 turbines per farm shown in Figure 4.4, the largest cluster for which the non-linear reference model remains tractable. The total price is fixed, so a ratio of 1:5 corresponds to 10 and 50 EUR/MWh. The curtailment found by the linearised model deviates by more than 20 % at some ratios, but the resulting revenue loss stays below 2.5 %.

The curtailment found by the linearised model follows the same trend as the reference curtailment η^* . As expected, it is drawn toward the boundaries of the two Taylor regions, as discussed in Section 3.2.9. The expansion overestimates power output the most at these boundaries.

At some price ratios the difference between $\hat{\eta}$ and η^* exceeds 20 %. Both are evaluated in the reference model, where η^* is optimal by construction, so revenue at $\hat{\eta}$ can only be lower. Over the price ratios considered here, the largest loss is 2.5 %. The revenue curve is flat near its optimum, as Figure 4.2 shows, so a substantial error in curtailment translates into a small loss in revenue.

4.2.1. Belgian-Dutch cluster

In this Section, the wind farm cluster on the Belgian-Dutch border is modelled; see Figure 4.6. This is the same cluster studied by Kenis et al. [24], allowing comparison of the results from both models.

The Dutch farm sells its electricity on the Dutch day-ahead market and the Belgian farm on the Belgian market. The combined revenue of the two farms is maximised through the following objective function:

$$\max_{\eta} \quad \lambda_{BE}^{DA} \sum_{n \in \mathcal{J}_{BE}} \mathbf{p}^{\text{eff}}[n] + \lambda_{NL}^{DA} \sum_{n \in \mathcal{J}_{NL}} \mathbf{p}^{\text{eff}}[n]. \quad (4.2)$$

Where λ^{DA} is the day-ahead electricity price in Belgium and the Netherlands, respectively. The wind direction determines which of the two farms is upwind, so curtailment is not tied to a fixed farm: η applies to whichever farm lies upwind at that moment. This results in the following system:

$$\max_{\eta} \quad \lambda_{BE}^{DA} \sum_{n \in \mathcal{J}_{BE}} p_n^{\text{eff}} + \lambda_{NL}^{DA} \sum_{n \in \mathcal{J}_{NL}} p_n^{\text{eff}} \quad (4.3a)$$

$$\text{s.t.} \quad U_n = U_{\infty} (1 - W a_n) \quad (4.3a)$$

$$a_n = f(C_{P,n}^{\text{eff}}) \quad (4.3b)$$

$$C_{P,n} = f(U_n) \quad (4.3c)$$

$$p_n = f(U_n) \quad (4.3d)$$

$$C_{P,\text{up}}^{\text{eff}} = \eta_0 C_{P,n}(U_n(\eta_0)) - (\eta_0 - \eta) \left(C_{P,n}(U_n(\eta_0)) + \eta_0 \frac{dC_{P,n}}{d\eta} \Big|_{\eta_0} \right) \quad (4.3e)$$

$$p_{\text{up}}^{\text{eff}} = \eta_0 p_n(U_n(\eta_0)) - (\eta_0 - \eta) \left(p_n(U_n(\eta_0)) + \eta_0 \frac{dp_n}{d\eta} \Big|_{\eta_0} \right) \quad (4.3f)$$

$$C_{P,\text{down}}^{\text{eff}} = C_{P,\text{down}}(U_{\text{down}}) \quad (4.3g)$$

$$p_{\text{down}}^{\text{eff}} = p_{\text{down}}(U_{\text{down}}) \quad (4.3h)$$

$$\eta \in \begin{cases} [0, 0.65] & \text{for } \eta_0 = 0.4 \\ [0.65, 1] & \text{for } \eta_0 = 0.8 \end{cases} \quad (4.3i)$$

Following Kenis et al. [24], the offshore clusters are approximated by two rectangular farms. The Belgian farm measures $8.5 \text{ km} \times 29.5 \text{ km}$ and the Dutch farm $15.6 \text{ km} \times 23.6 \text{ km}$. The Belgian farm consists of 224 IEA 10 MW turbines in 8 rows and 28 columns, and the Dutch farm consists of 150 10 MW turbines in 10 rows and 15 columns. In this study, the resulting power densities are 9.53 MW/km^2 and 4.35 MW/km^2 . In the study by Kenis et al., the resulting power densities are 9.48 MW/km^2 and 4.35 MW/km^2 . The real layout has power densities of 9.49 MW/km^2 and 4.37 MW/km^2 . Figure 4.6 shows the layout used in this thesis.

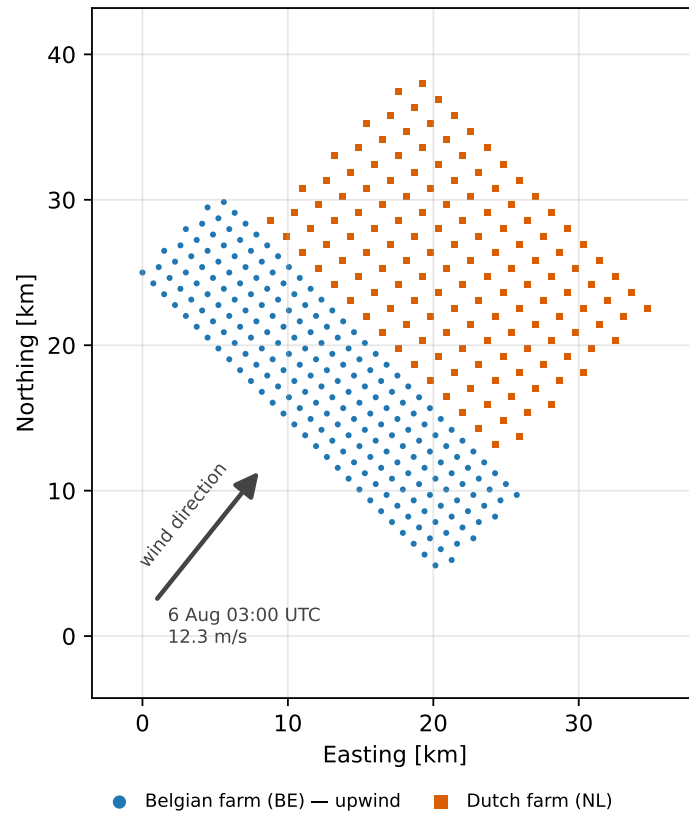


Figure 4.6: Layout of the Belgian and Dutch offshore wind farm zones, consisting of 224 IEA 10 MW turbines in the Belgian zone and 150 in the Dutch zone. The arrow indicates the wind direction on 6 August 2021 at 03:00 UTC.

Figure 4.6 shows the wind direction and wind speed on 6 August 2021. The free-stream wind speed was 12.3 m/s, and the day-ahead prices were 4.45 and 57.09 EUR/MWh for the upwind Belgian and the downwind Dutch farm, respectively.

In Figure 4.7 the revenue obtained under two control strategies is compared. Under cluster control, the upwind farm is curtailed to the model-optimal level. Under greedy control, it disregards the downwind farm and operates at $\eta = 1$. The hour highlighted in the figure, 6 August 2021 at 03:00 UTC, is the same as reported by Kenis et al. [24].

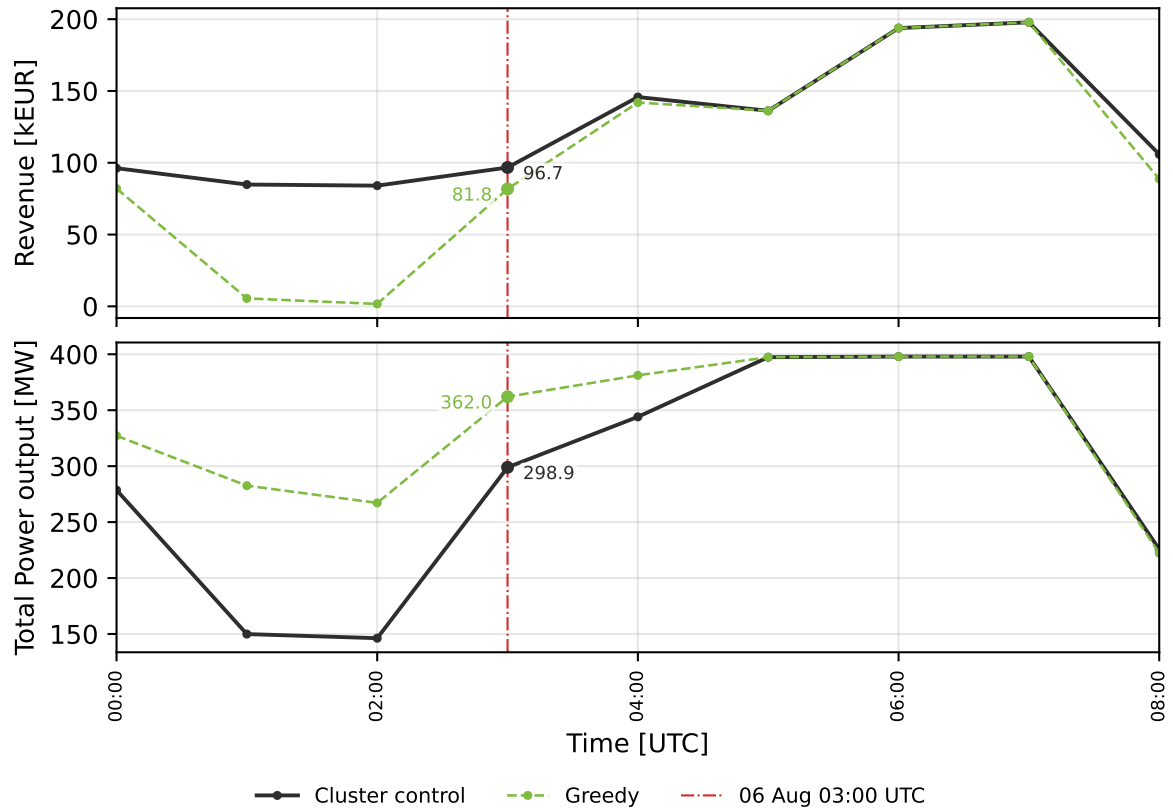


Figure 4.7: Revenue and power output of the total wind farm cluster over a day in 2021. The day analysed in detail is highlighted in red.

The figure shows an advantage to curtailing the upwind farm: the cluster's total power output decreases while its revenue increases. At 01:00 and 02:00 UTC, the Belgian day-ahead price is negative, so curtailing is trivially profitable, and these hours carry little information.

At 03:00 UTC the model finds a revenue increase from 81.8 to 96.7 kEUR, which is an increase of 18.23 %. Kenis et al. [24] report an increase from 70.1 to 84.9 kEUR for the same hour, showing an increase of 21.03 %. The two studies agree on the direction and order of magnitude of the effect, but not on its size.

The difference is explained by the wake-simulation model used. Kenis et al. use the Gaussian wake model of Lanzilao and Meyers with a multiplicative superposition model. At the same time, the linearisation required here restricts this thesis to the Jensen model with linear superposition.

4.2.2. Results maximising revenue over 2021

The profit between cluster control and greedy is now analysed over the whole year of 2021. To reduce the calculation time, only hours for which a control strategy would work are included. These are the hours where the day-ahead price at the downwind farm is higher than at the upwind farm. The wind speed is between the cut-in and cut-out velocity. The ratio between the downwind/upwind is at least 1.1. The hours when the day-ahead price at the upwind farm is negative or zero are neglected since it is trivial that the upwind farm should be curtailed.

The figure below shows the results from a full year, comparing the greedy situation with the control situation.

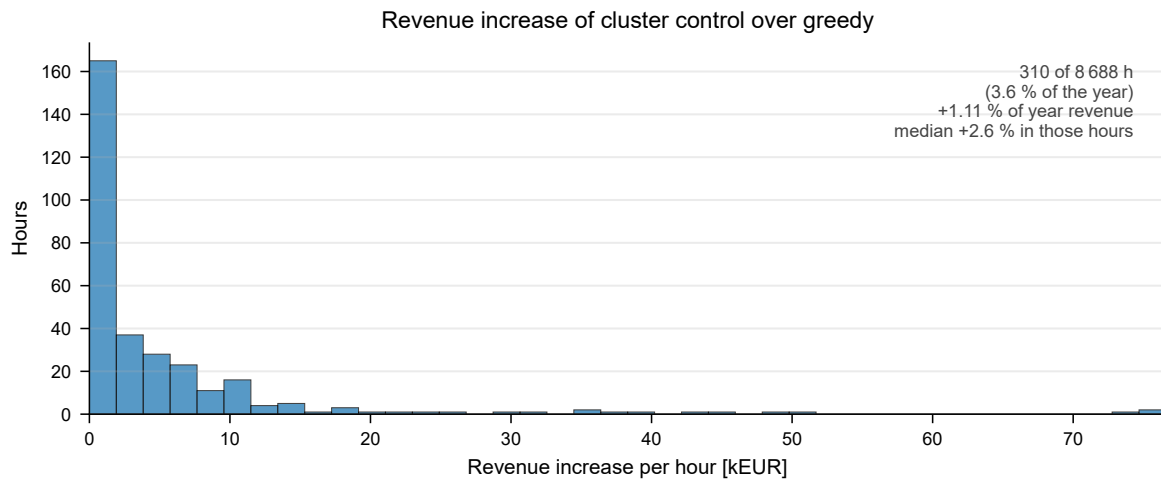


Figure 4.8: Distribution of the hourly revenue increase of cluster control over greedy operation ($\eta = 1$) for the Belgian–Dutch cluster in 2021. Only hours passing the screening criteria of Section 4.2.2 are shown: 310 hours in total, 3.6 % of the year. Together, they raise annual cluster revenue by 1.11 %, with a median increase of 2.6 % during those hours.

Figure 4.8 shows the distribution of that increase over the 310 hours in which cluster control raises cluster revenue. Those hours make up 3.6 % of the year, so the effect on annual revenue is small: 1.11 % over 2021 as a whole. Within those hours the median increase is 2.6 % of cluster revenue.

The distribution is strongly skewed. In most of the 310 hours, the gain is below 2 kEUR, but the tail reaches almost 70 kEUR in a single hour.

4.3. Wind farm cluster optimisation minimising system cost under Energy system

This Section analyses the four cases introduced in Section 3: the IJmuiden Ver – Hollandse Kust West cluster, and the Borssele – Belgian offshore zone in three configurations.

Curtailment acts as an inefficient transmission line. Part of the energy given up upstream is never recovered downstream. The system therefore uses this route only when no more efficient route is available.

Each case is screened in three steps First, the year is searched for hours in which the critical transmission lines are at their limit These are the lines connecting the two wind farm zones Second, these hours are checked for wind direction: the wind must blow along the congested flow Third, the free-stream wind speed must lie at or below rated, because only then does curtailment significantly change the wake output An example is wind from Borssele towards the Belgian zone at a moment when the Netherlands exports to Belgium at full interconnector capacity Hours that fail any of the three criteria are discarded From the hours that remain, a window is selected and simulated in full.

For each window, the results report the optimal curtailment factor η , the dispatch under greedy and under cluster control, and the output of both farms. The benefit is quantified as the reduction in fossil generation, the corresponding reduction in system cost, and the change in marginal price.

4.3.1. IJmuiden Ver and Hollandse Kust West

The first cluster is IJmuiden Ver, together with Hollandse Kust West, analysed for 2020. Neither farm existed in that year, so both are added to the 2020 system. The two farms connect to shore at different locations, so their power enters the system at different hubs. The strategy therefore depends on the lines between Zuid-Holland, Noord-Holland, and Utrecht. Figure 4.9 shows the configuration.

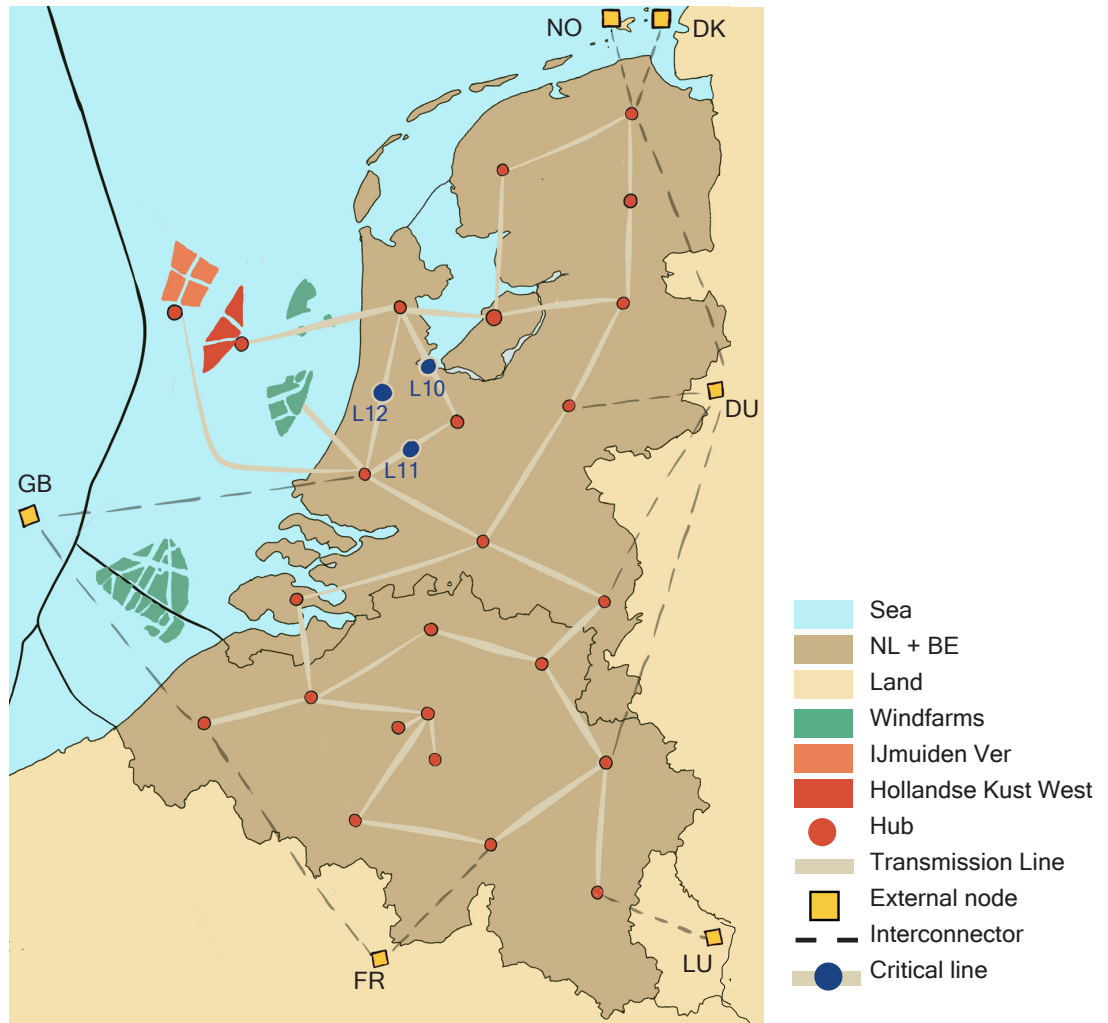


Figure 4.9: Schematic overview of the nodal network. Two wind farm zones are highlighted: IJmuiden Ver and Hollandse Kust West.

The two farms connect onshore at different locations, so the power they generate enters the system at different hubs. In this configuration, Borssele and the Belgian farms are uncontrolled and are simulated as any other uncontrolled wind farm, delivering to the hub at their capacity factor CF_s .

The strategy depends on the lines between Zuid-Holland, Noord-Holland, and Utrecht, highlighted in Figure 4.9, which carry flow between the two hubs. Figure 4.10 shows their loading over 2020.

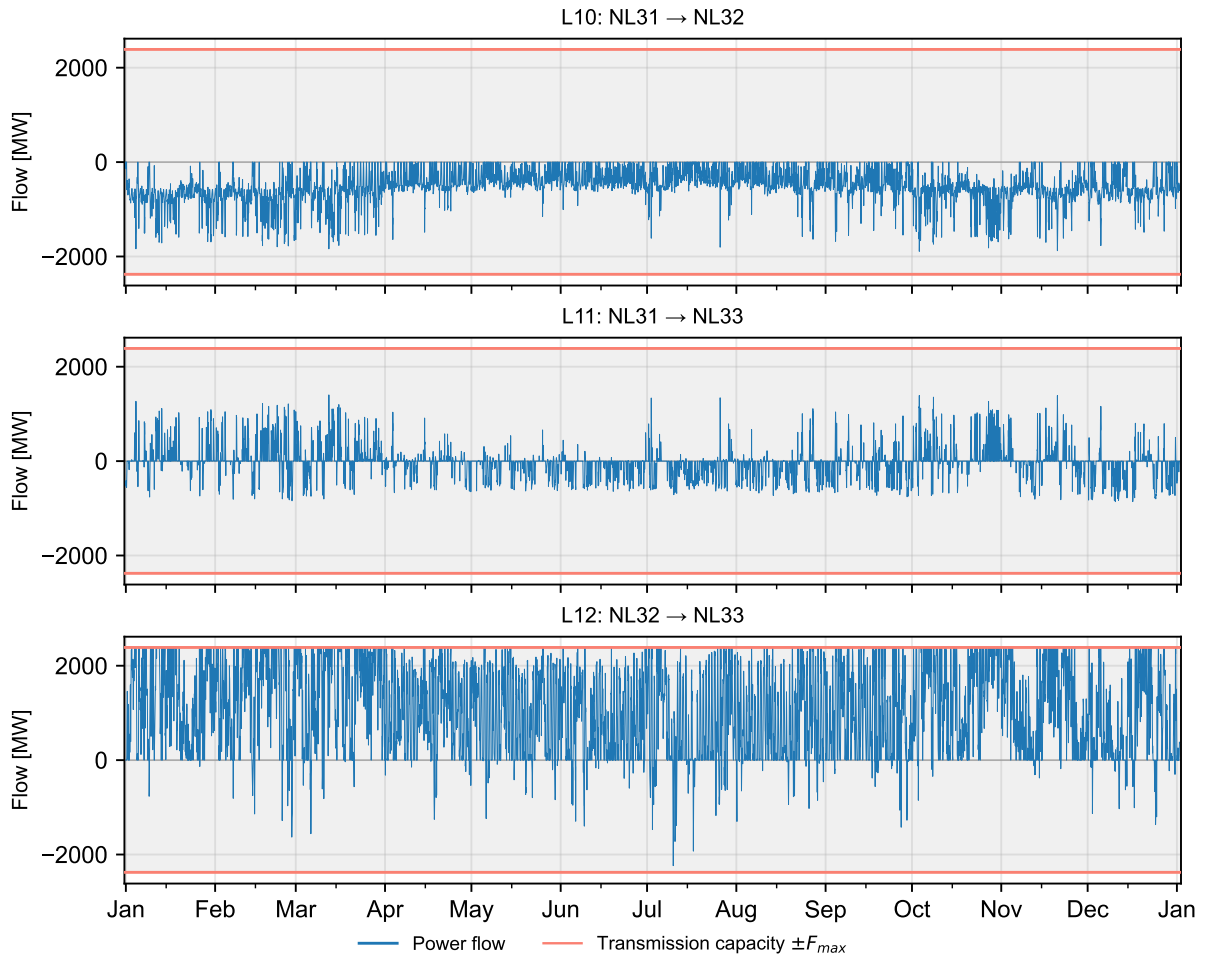


Figure 4.10: Power flow on the critical lines L10, L11 and L12 over 2020, against their transmission capacity $\pm F_l^{max}$.

Of the three, only L12 becomes congested: it reaches its capacity in 13 % of the hours, at an average utilisation of 47 %. In those hours, the flow can be rerouted through L10 and L11, which average 20 percent and 6 percent utilisation, respectively, and never reach their limit. A detour therefore remains available whenever the direct line is full.

The screening returns no candidate hours for this cluster Rerouting over a line with spare capacity costs 1 % per 100 km, whereas shifting the same energy through the wake means giving it up upstream and recovering only part of it downstream. As long as a detour exists, the system prefers the network, and curtailment is never selected. The result is insensitive to the cluster control strategy: curtailment only lowers total output, so lines with spare capacity under greedy control retain it under cluster control.

4.3.2. Borssele and Belgian offshore farms

The second cluster is the Borssele wind farm zone together with the Belgian offshore wind farm zone, again for the year 2020. Both were operational in that year, so they are removed from the uncontrolled wind farm model by setting their capacity K_s to zero and are instead modelled as the controlled cluster. IJmuiden Ver and Hollandse Kust West did not exist in 2020 and are not modelled here. Figure 4.11 shows the resulting configuration.

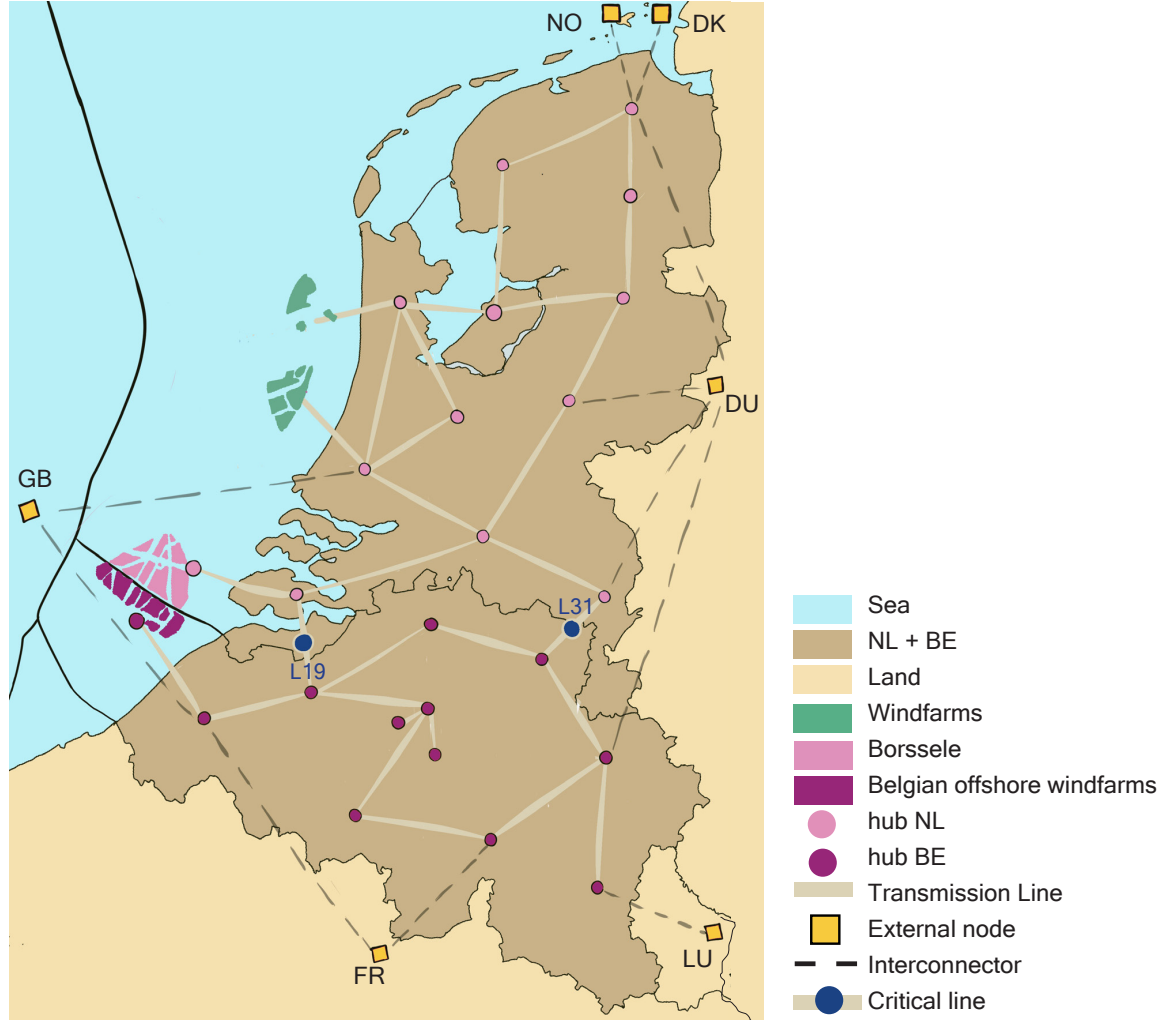


Figure 4.11: The Dutch and Belgian system with the Borssele Windfarm zone and the Belgian offshore windfarm zone as the controlled cluster, with the interconnectors L19 and L31 highlighted.

The two farms connect onshore at different hubs, Borssele to Zeeland and the Belgian farms to West-Vlaanderen, so the strategy here depends on the interconnectors between the two countries. Both highlighted lines have a capacity F_l^{max} of 2377 MW and a length of 100 km. The layout of the cluster is the same as in Section 4.2.1 and is shown in Figure 4.12. The figure also shows the wind conditions for one hour in the analysed window.

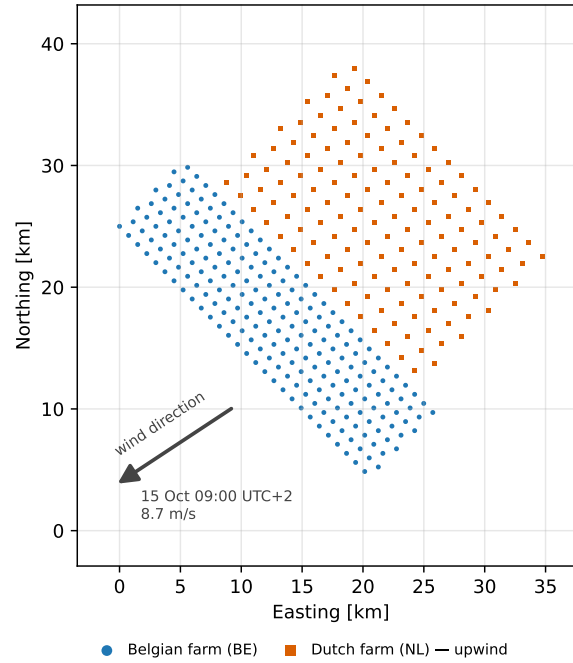


Figure 4.12: Layout of the Belgian and Dutch offshore wind farm zones, consisting of 224 IEA 10 MW turbines in the Belgian zone and 150 in the Dutch zone. The arrow indicates the wind direction on 15 October 2020 at 09:00 UTC + 2.

This cluster differs from the previous one in the location of both wind farm zones. This is a cross-border cluster, and that has some interesting implications. The system is solved with a single objective function covering both countries, so any price difference between them follows from the model rather than being an input: the price at a hub is the marginal cost of the most expensive generator dispatched to serve it. Cross-border capacity is typically lower than intra-country capacity, since these lines involve more parties and can be politically sensitive.

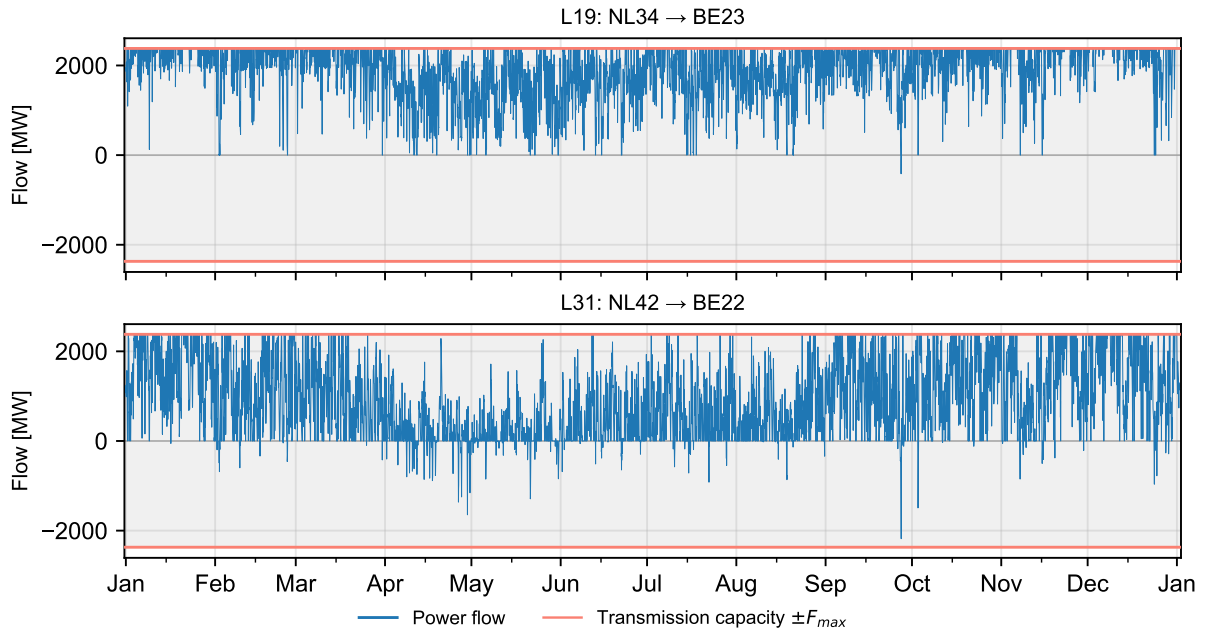


Figure 4.13: Power flow on the critical lines L19 and L31 over 2020, against their transmission capacity $\pm F_l^{\max}$.

Figure 4.13 shows the loading of the two interconnectors over 2020. Both lines are congested for a significant share of the year. L19 reaches its capacity in 34 % of the hours at an average utilisation of 78 %, and L31 in 14 % of the hours at an average utilisation of 40 %. L19 is more relevant: L31 lies farther from the cluster, so congestion on it may originate elsewhere in the network. Unlike the previous case, no detour with spare capacity is available when L19 and L31 are full, and the screening returns candidate hours. These are hours when the wind blows from the Dutch farm toward the Belgian one.

A congested window: 14-15 October

One such window runs from 14 October 12:00 to 15 October 12:00. Over these 24 hours L19 is at its capacity in 18 hours and L31 in 2. The wind is north-easterly throughout, averaging 51° with a range of 36° to 62° , so the Dutch farm is upwind for the whole window. The free-stream wind speed averages 9.89 m/s, between 7.91 and 12.27 m/s, and therefore stays at or below rated.

All three criteria are met, and the window is simulated with the full system of Equations 3.40. Figure 4.14 shows the resulting optimal curtailment.

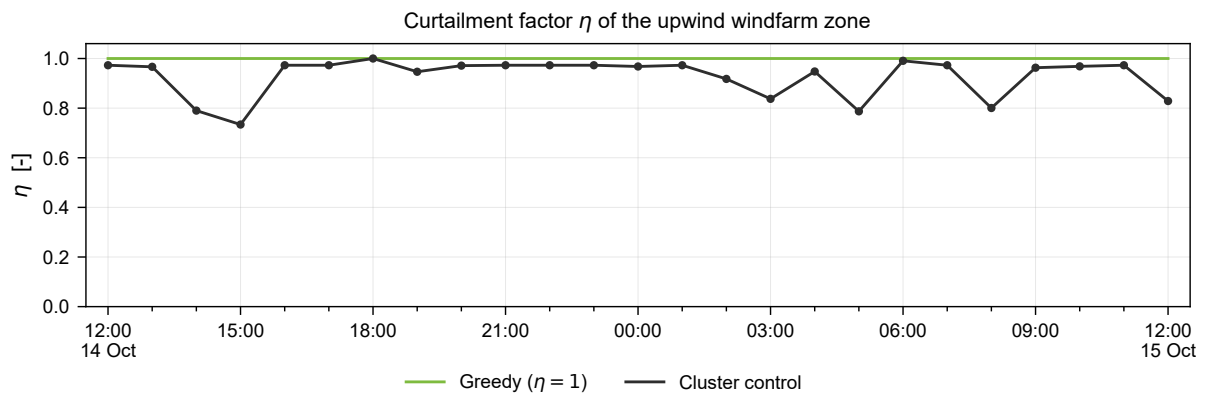


Figure 4.14: Optimal curtailment factor η of the upwind (Dutch) farm over the window of 14-15 October, against the greedy baseline $\eta = 1$.

Figure 4.14 shows the resulting optimal curtailment. The system curtails the upwind farm in almost every hour of the window, but only slightly. Figure 4.15 shows the corresponding output of the two farms and of the cluster as a whole. The total cluster output barely changes, and neither farm shifts significantly.

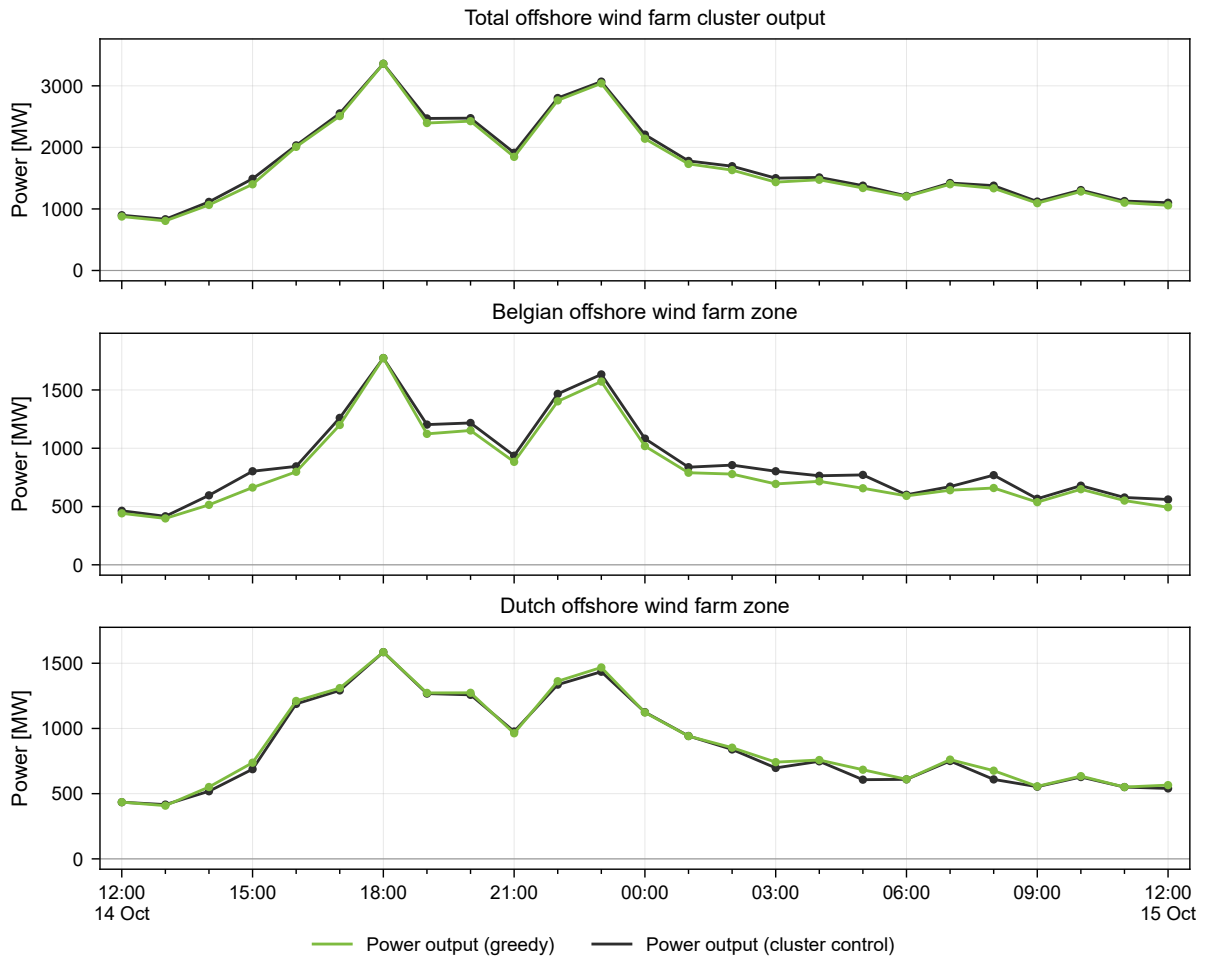


Figure 4.15: Power output of the Belgian and Dutch farms and of the cluster as a whole, under greedy and under cluster control. The Dutch farm is upwind throughout.

Figure 4.16 shows why neither the Netherlands nor Belgium has enough cheap renewable or nuclear generation to cover its demand in this window, and both dispatch gas. Curtailing the upwind farm therefore raises costs on the Dutch side and lowers them on the Belgian side, but by less. The total system cost increases, and curtailment is not profitable.

This adds a further criterion for cluster control to be useful: the upwind zone must have a surplus of cheap energy while the downwind zone still dispatches expensive fossil generation. The next section tests that criterion by adding IJmuiden Ver, Hollandse Kust West and a share of Nederwiek to the Dutch system.

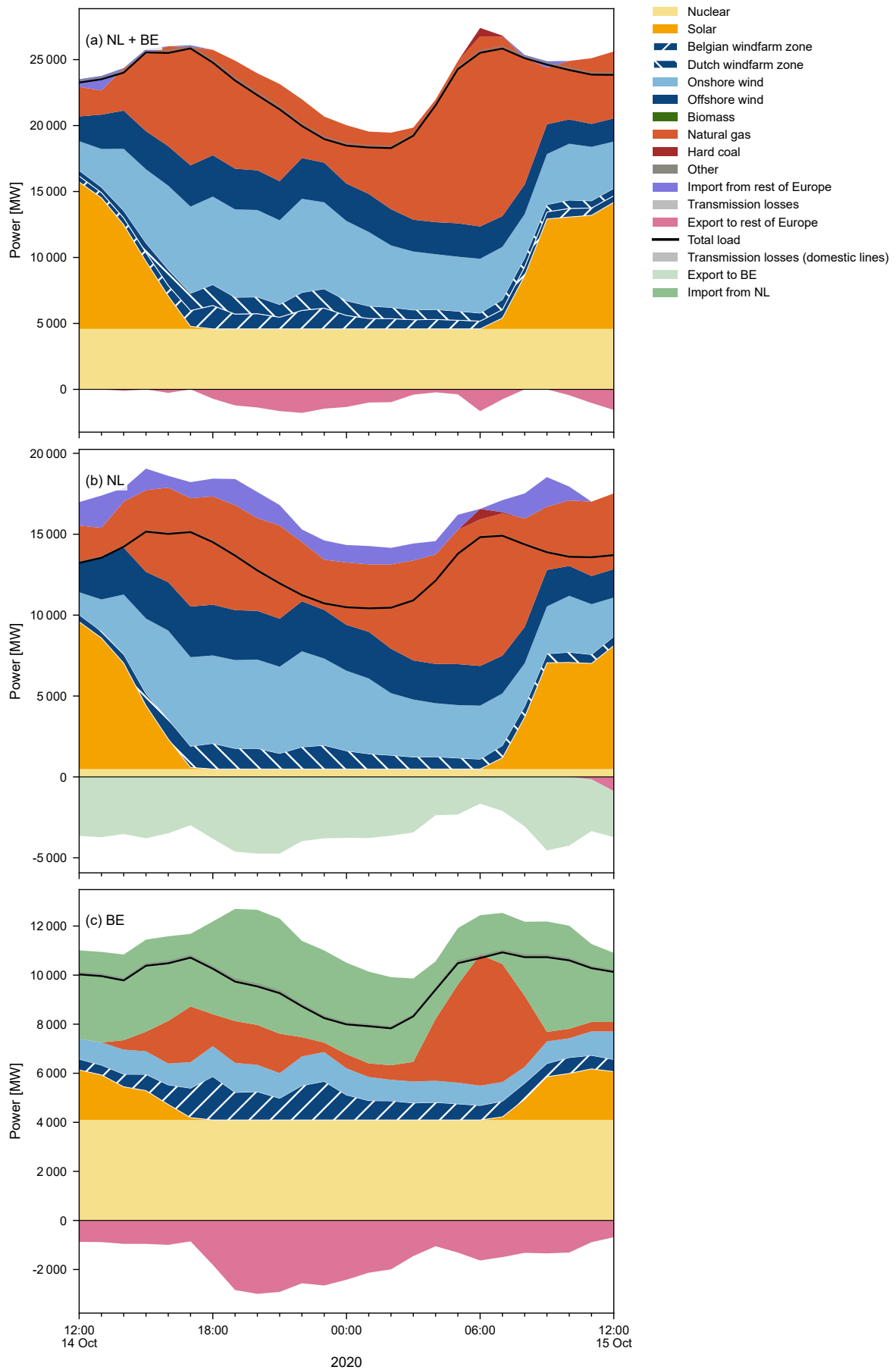


Figure 4.16: Dispatch over the congested window of 14–15 October 2020 under greedy operation ($\eta = 1$). Panel (a) shows the combined system, (b) the Netherlands and (c) Belgium. The Belgian and Dutch offshore zones of the controlled cluster are shown separately from the remaining offshore wind. Neither country covers its demand with cheap generation in this window, and both dispatch gas throughout.

4.3.3. Borssele and the Belgian offshore farms with the planned Dutch farms

The third case is the same Borssele – Belgian cluster as in Section 4.3.2, but with IJmuiden Ver, Hollandse Kust West and a share of the Nederwiek zone added to the system. Together these add 10 GW of installed capacity to the Dutch system. None of the three existed in 2020. All three are added as uncontrolled wind farms and deliver to the hub they connect to at their capacity factor CF_s . Section 4.3.2 closed on the observation that the strategy requires a surplus of cheap energy in the upwind zone. This case tests whether additional Dutch offshore capacity creates that surplus and whether a coordinated curtailment strategy is profitable. Figure 4.17 shows the resulting configuration. In the figure, the extra wind farms are shown:

- A Windfarm zone Nederwiek 2 GW
- B windfarm zone IJmuiden Ver 6 GW
- C Windfarm zone Hollandse Kust west 4 GW

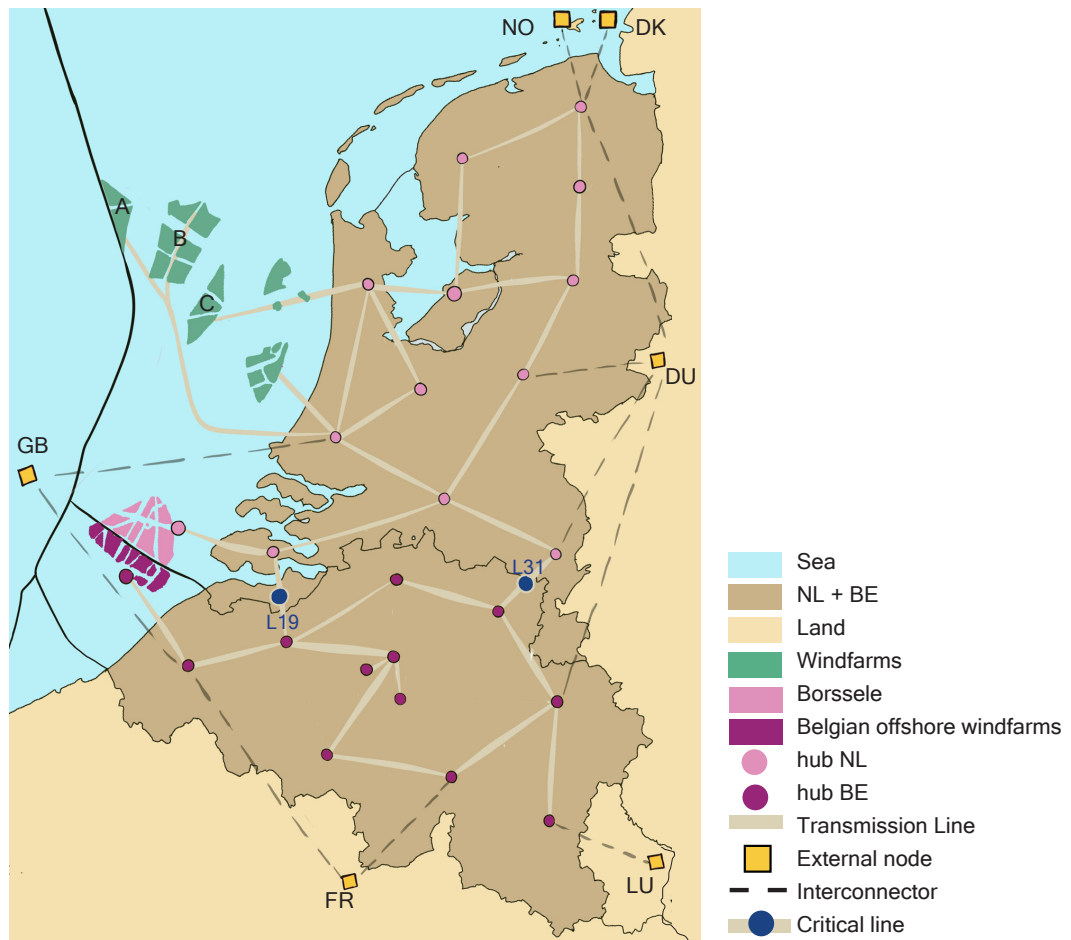


Figure 4.17: The Dutch and Belgian system with IJmuiden Ver and Hollandse Kust West added as uncontrolled farms, with the controlled cluster and the interconnectors L19 and L31 highlighted.

Apart from the three added farms, the model, cluster layout, and year analysed remain unchanged. Figure 4.18 shows the loading of the two interconnectors over 2020.

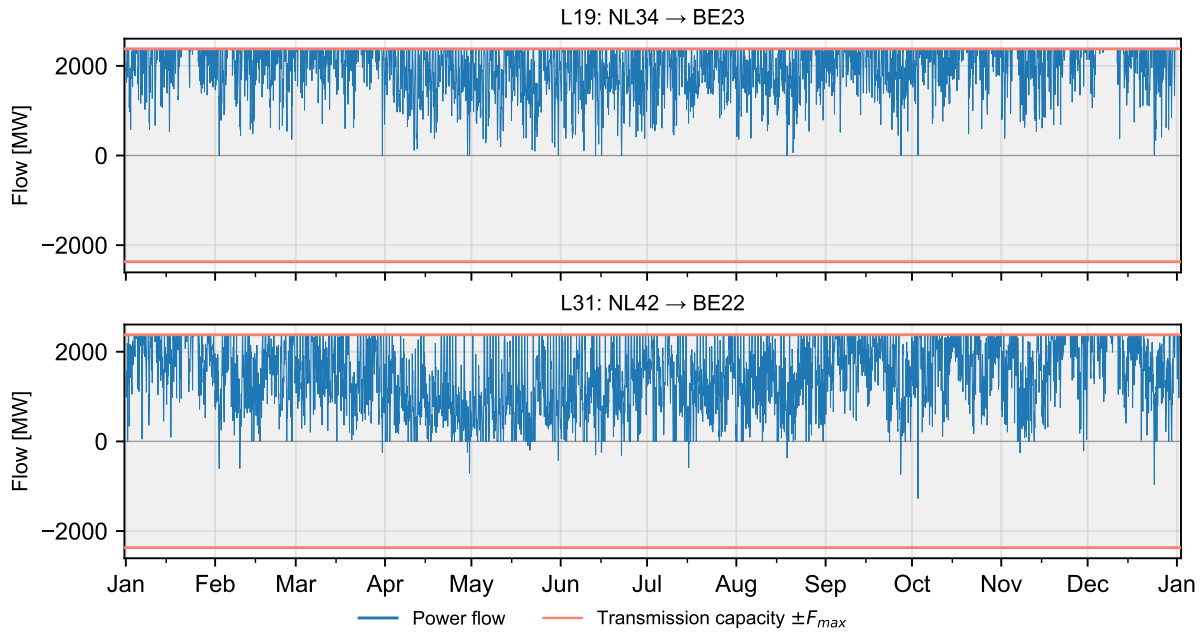


Figure 4.18: Power flow on the interconnectors L19 and L31 over 2020 with the planned Dutch farms added, against their transmission capacity $\pm F_l^{max}$.

Both lines are more heavily loaded than in Section 4.3.2; L19 now reaches its capacity in 61 % of the hours at an average utilisation of 88 %, against 34 % and 78 % before. L31 rises further, from 14 % to 20 % of the hours at its max capacity, and from 40 % to 46 % average utilisation. The additional Dutch capacity therefore pushes more energy towards the border than the interconnectors can carry. That is the condition under which curtailment can substitute for transmission.

A congested window: 14-15 October

The same window is analysed as in Section 4.3.2. Both lines average above 92 % utilisation, the wind blows from the Dutch zone towards the Belgian one throughout, and the free-stream wind speed averages 9.89 m/s. Figure 4.19 shows the optimal curtailment found under this configuration.

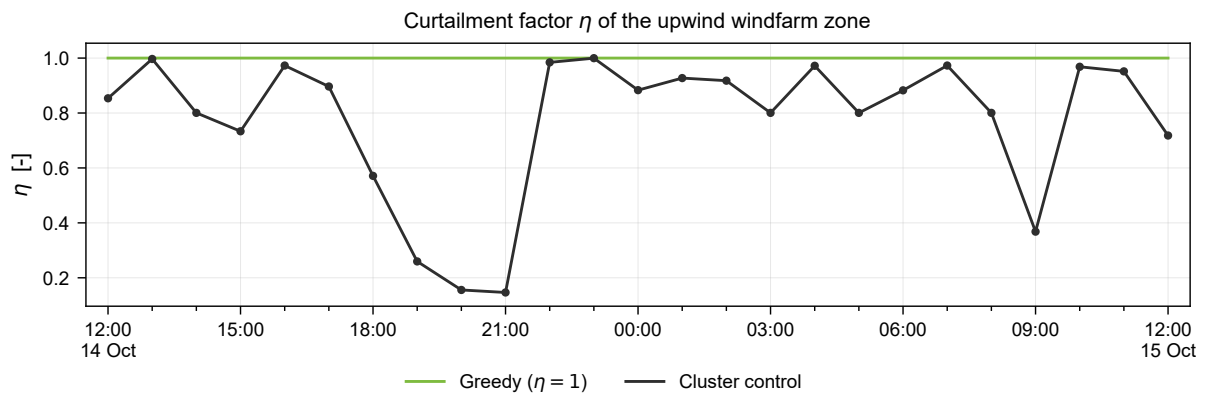


Figure 4.19: Optimal curtailment factor η of the upwind (Dutch) farm over the window of 14-15 October, against the greedy baseline $\eta = 1$.

The system now curtails the upwind farm in clearly identifiable hours. Between 17:00 and 21:00 on 14 October, the Dutch farm is curtailed. A second, sharper peak continuously occurs around 15 October 09:00. Figure 4.20 shows the corresponding output of the two farms and of the cluster as a whole.

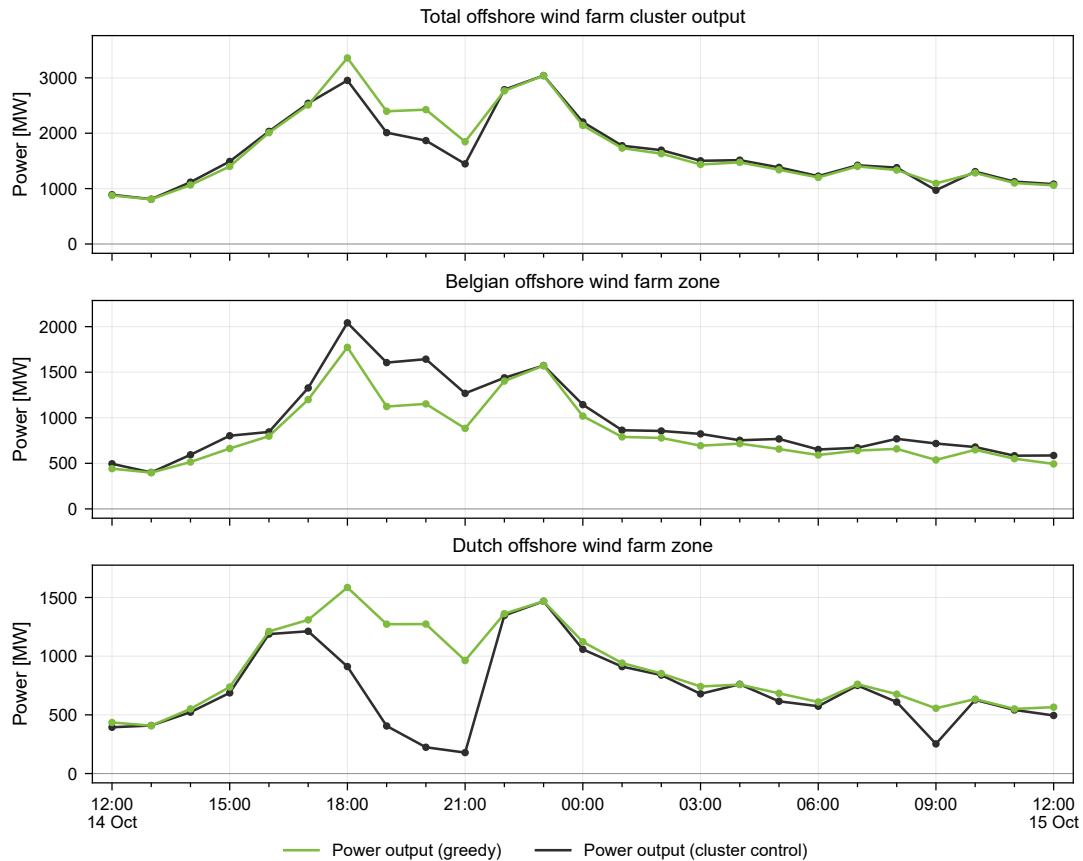


Figure 4.20: Power output of the Belgian and Dutch farms and of the cluster as a whole, under greedy and under cluster control. The Dutch farm is upwind throughout.

The output follows expectations when the upwind farm is curtailed. The Dutch farm output drops, and the Belgian farm output rises. The peak at 15 October 09:00 shows the same behaviour.

Under cluster control, the cluster itself acts as a transmission line. The energy gained at the downwind farm comes from the energy given up at the upwind farm so that the redistribution can be measured. Effective efficiency is the power gained downwind divided by the power given up upwind.

That efficiency depends strongly on wind speed and wind direction. It also depends on the energy system. If the upwind zone has no use for its wind energy, only the energy gained downwind matters, even at a low efficiency. The optimiser therefore does not select the curtailment at which transport is most efficient, but the one that minimises system cost. The efficiencies reported here are not the highest reachable. Table 4.1 reports the efficiency at three moments of the window.

Table 4.1: Effective transmission efficiency of the cluster at three moments of the 14–15 October window, with extra wind capacity added to the system. The efficiency is the power gained at the downwind farm divided by the power given up at the upwind farm.

Date	Given up upwind [MW]	Gained downwind [MW]	Efficiency [%]
14 Oct 18:00	674	268	39.8
14 Oct 20:00	1050	491	46.8
15 Oct 09:00	303	180	59.3

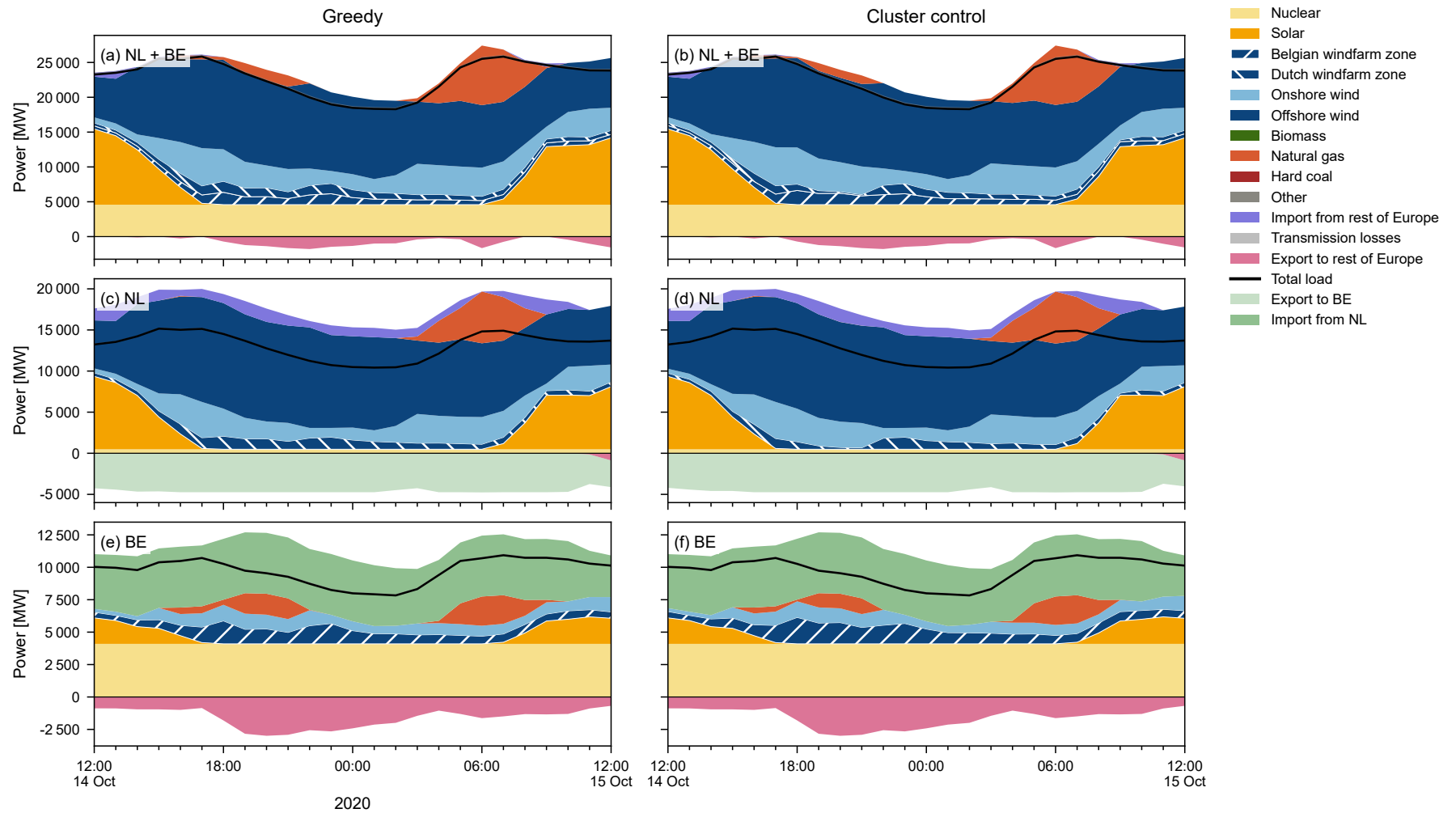


Figure 4.21: Dispatch over the congested window of 14–15 October 2020 under greedy operation (left) and under cluster control (right). Panels (a) and (b) show the combined system, (c) and (d) the Netherlands, and (e) and (f) Belgium. The Belgian and Dutch offshore zones of the controlled cluster are shown separately from the remaining offshore wind. The effect is clearest between 19:00 and 21:00 on 14 October: the Netherlands holds a surplus of wind it can neither place domestically nor export; output from the Dutch zone falls, output from the Belgian zone rises, and Belgium dispatches less gas.

Figure 4.21 shows the dispatch over the window. The Netherlands' energy mix changes considerably. Offshore wind now dominates, and some onshore wind is shut down, which would not have happened without the added farms. The Netherlands still imports energy while holding a surplus. This follows from the fixed 2020 import and export profiles, which are kept as a parameter even though a system with this much wind capacity would not export in these hours.

The effect of curtailment is hard to see in the dispatch curves, because the redistributed power is at most 0.5 GW against a system of about 16 GW. The hours between 17:00 and 21:00 on 14 October show the result. The Dutch dispatch curve shows a drop in output from the Dutch wind farm zone. In the same hours, the Belgian curve shows an increase in output from the Belgian wind farm zone, and Belgium dispatches less gas.

Two system conditions produce this outcome, and both are visible in Figures 4.20 and 4.21. The first is a large Dutch wind share between 17:00 and 21:00. The Netherlands then generates more wind than it can place domestically and more than the interconnectors can carry. Because Dutch installed wind capacity is substantially larger than Belgian capacity, this condition is asymmetric, and the strategy works only from the Netherlands to Belgium. It ends after 21:00, when Belgium stops dispatching expensive fossil units.

The second condition is an imbalance in solar output, as at 15 October 09:00, when one country has enough solar to displace its gas and the other does not. This occurs in the morning and in the late afternoon rather than at midday, when solar alone is generally abundant enough that the wind contribution no longer sets the margin. The hour identified in Section 4.2.1 also falls in the morning, which is consistent with this. Because either country can hold a solar surplus, this second condition can work in both directions, but it lasts only a few hours. The first condition can last as long as the wind blows from the Netherlands toward Belgium.

Displaced fossil generation

The strategy's benefit is quantified in two ways: the reduction in fossil generation, which translates directly into lower system cost, and the change in the marginal price. Figure 4.22 shows the dispatched fossil generation over the window.

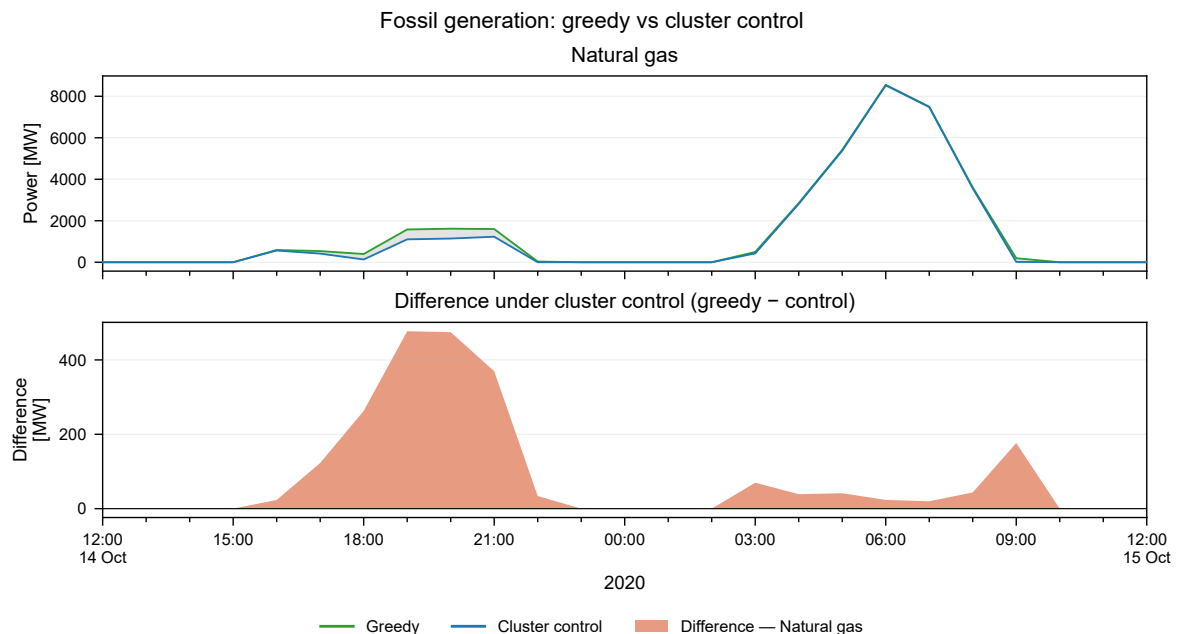


Figure 4.22: Natural gas generation under greedy and under cluster control over 14-15 October, and the difference between the two.

Gas generation decreases clearly, most of all in the hours in which the upwind farm is curtailed by more than 50 %. Over the whole window 2.2 GWh of gas generation is avoided. The period from 17:00 to 22:00 on 14 October is the most relevant. Here, curtailment is beneficial over several consecutive hours rather than in a single isolated hour, which matters because a wind farm needs time to adjust its curtailment and the wake needs time to reach the downwind zone. Over this period 1.76 GWh of gas generation is avoided. At a marginal cost of 30 €/MWh for natural gas, this lowers the system cost by 52.8 k€.

The displaced gas generation also reduces emissions. Life-cycle emissions of natural gas range from 290 to 930 kg CO₂-eq/MWh, whereas nuclear and wind emit around 11–12 kg CO₂-eq/MWh [49]. Replacing the 2.2 GWh of avoided gas generation with nuclear and wind therefore reduces emissions by roughly 0.6 to 2.0 kt CO₂-e over the full window.

The fuel prices used throughout are those of 2020, when European gas was historically cheap. Physical energy redistribution does not depend on fuel price, because gas remains far more expensive than renewables or nuclear in either case. The strategy's value does depend on it. In 2025, European gas reached about 102 €/MWh [50], at which the same displaced volume would be worth about 179.5 k€. Whether gas would still be the displaced fuel is a separate question. At that price level, hard coal, at about 96 €/MWh, moves ahead of gas in the merit order. The emissions avoided would be even more.

Marginal price

The marginal prices in Figure 4.23 should be read with care. The curtailment is selected in the linearised model, which overestimates p^{eff} , and is then evaluated in the reference model of Equation 3.35. A curtailment that clears the last gas unit in the linearised model can therefore fall just short of clearing it in the reference model, leaving the marginal price unchanged or higher. Because a single unit sets the marginal price, this discretisation makes it a far less robust indicator than the fossil volume of Figure 4.22.

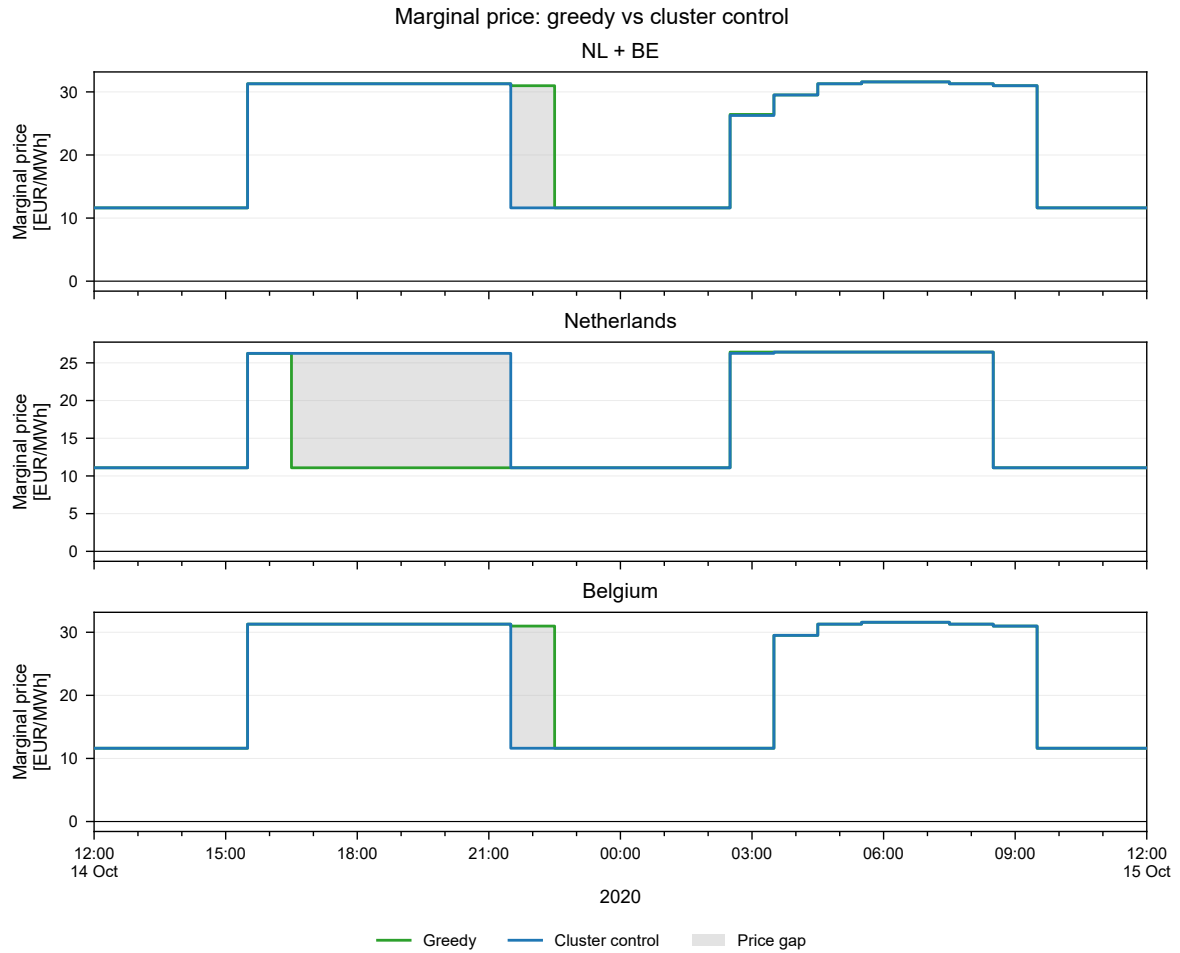


Figure 4.23: Marginal price under greedy operation and under cluster control over 14–15 October 2020, for the combined system (top), the Netherlands (middle) and Belgium (bottom). The marginal price is the marginal cost of the most expensive generator dispatched. During the curtailed hours, the price rises in the Netherlands and falls in Belgium, but in most hours the redistributed power is too small to change it.

Figure 4.23 shows this. During the curtailed period between 17:00 and 21:00, the marginal price in the Netherlands (the upwind zone) increases. This is partly an artefact of the overestimated wind output in the linearised model. It is also physical: cheap wind energy is given upwind, raising the price. The figure further shows that in most hours the additional power in the downwind farm is too small to change the price at all.

4.3.4. Borssele and the Belgian offshore farms with 2026 installed nuclear capacity

The fourth case repeats the configuration of the previous Section 4.3.3, with the Belgian nuclear fleet reduced to the units still in operation in 2026, with increased wind capacity. Doel 1, Doel 2 and Tihange 1 were shut down in 2025 [51]. Together these units carried about 1.7 GW, roughly half of the Belgian nuclear capacity in the 2020 system. Only Doel 4 and Tihange 3 remain.

That variable matters for the strategy. Nuclear output is fixed at its installed capacity in the model, so it cannot step aside for cheaper energy. In the previous sections, there were hours when nuclear, wind, and imports together already covered Belgian demand. Redistributed wind is worth nothing in those hours, because no flexible unit is left on the margin to be displaced. Removing 1.7 GW of must-run capacity puts gas back on the margin, and widens the set of hours in which curtailment pays off.

Sections 4.3.2 and 4.3.3 analysed a window where Belgium exports electricity as well. That window was chosen because Belgian gas was already dispatched and could be displaced. This Section analyses a window in which the 2020 nuclear configuration consumes no Belgian gas at night, so no curtailment is selected, and asks whether the reduced nuclear capacity changes that outcome.

A window with less Belgian nuclear in 2020: 27-28 March

The window runs from 27 March 15:00 to 28 March 09:00. Over these hours, L19 is at its capacity in 88 % of the hours and L31 in 80 %, both at an average utilisation above 90 %. The Dutch farm is upwind throughout the whole window. Figure 4.24 shows the resulting optimal curtailment.

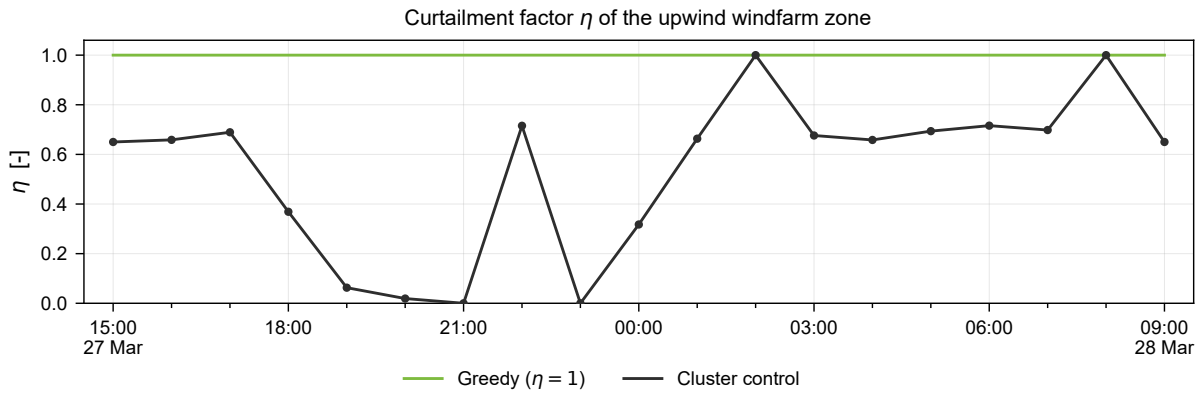


Figure 4.24: Optimal curtailment factor η of the upwind (Dutch) farm over the window of 27–28 March, with the Belgian nuclear fleet reduced to its 2026 capacity, against the greedy baseline $\eta = 1$.

The system curtails the upwind farm in most hours of the window. Between 27 March 18:00 and 28 March 01:00, the curtailment factor lies mostly below 0.5, so the Dutch farm runs at less than half its greedy output. Over most of the remaining hours, the factor is around 0.65. Figure 4.25 shows the corresponding output of the two farms and of the cluster as a whole.

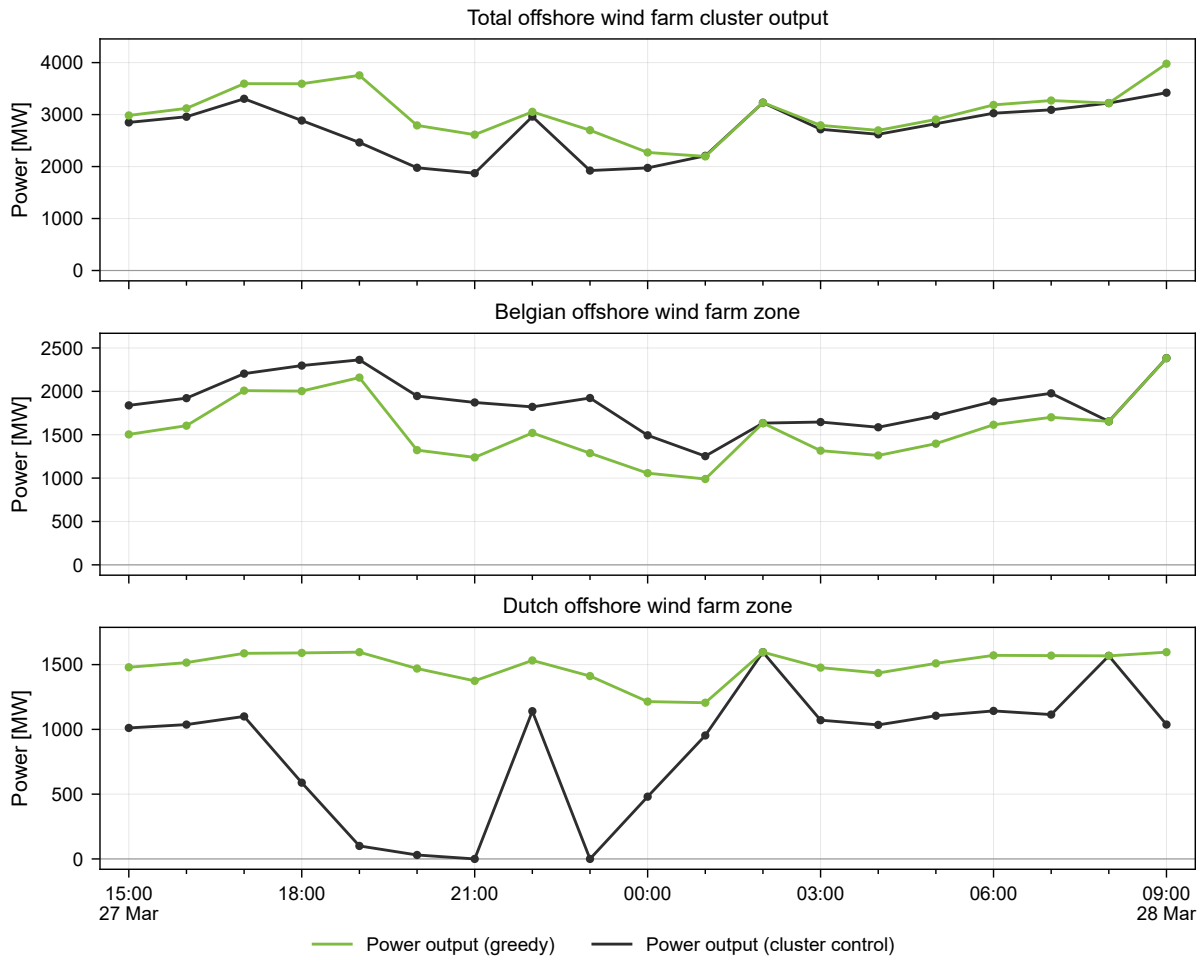


Figure 4.25: Power output of the Belgian and Dutch farms and of the cluster as a whole, under greedy and under cluster control, over 27–28 March with the 2026 Belgian nuclear fleet. The Dutch farm is upwind throughout.

In the evening hours, the total cluster output drops clearly. The Belgian farm gains roughly 100 to 300 MW in each of those hours, while the Dutch farm loses more than 500 MW. In the morning, total output drops only slightly. However, the power is still shifted from the Dutch farm to the Belgian one. Note that the two panels use different vertical scales, which makes the upwind loss look larger relative to the downwind gain than it is.

Table 4.2 reports the effective transmission efficiency at three moments of the window. Efficiency depends on atmospheric conditions, but also on the value of the energy given up. The optimiser minimises system cost rather than transport efficiency, so it accepts a poor efficiency whenever the upwind energy has no value at home.

Table 4.2: Effective transmission efficiency of the cluster at three moments of the 27–28 March window, with the Belgian nuclear fleet reduced to its 2026 capacity and extra Dutch wind capacity. The efficiency is the power gained at the downwind farm divided by the power given up at the upwind farm.

Moment	Curtailment η [-]	Given up upwind [MW]	Gained downwind [MW]	Efficiency [%]
27 Mar 18:00	0.40	1000	294	29.4
27 Mar 21:00	0.00	1374	633	46.1
28 Mar 06:00	0.65	428	269	62.9

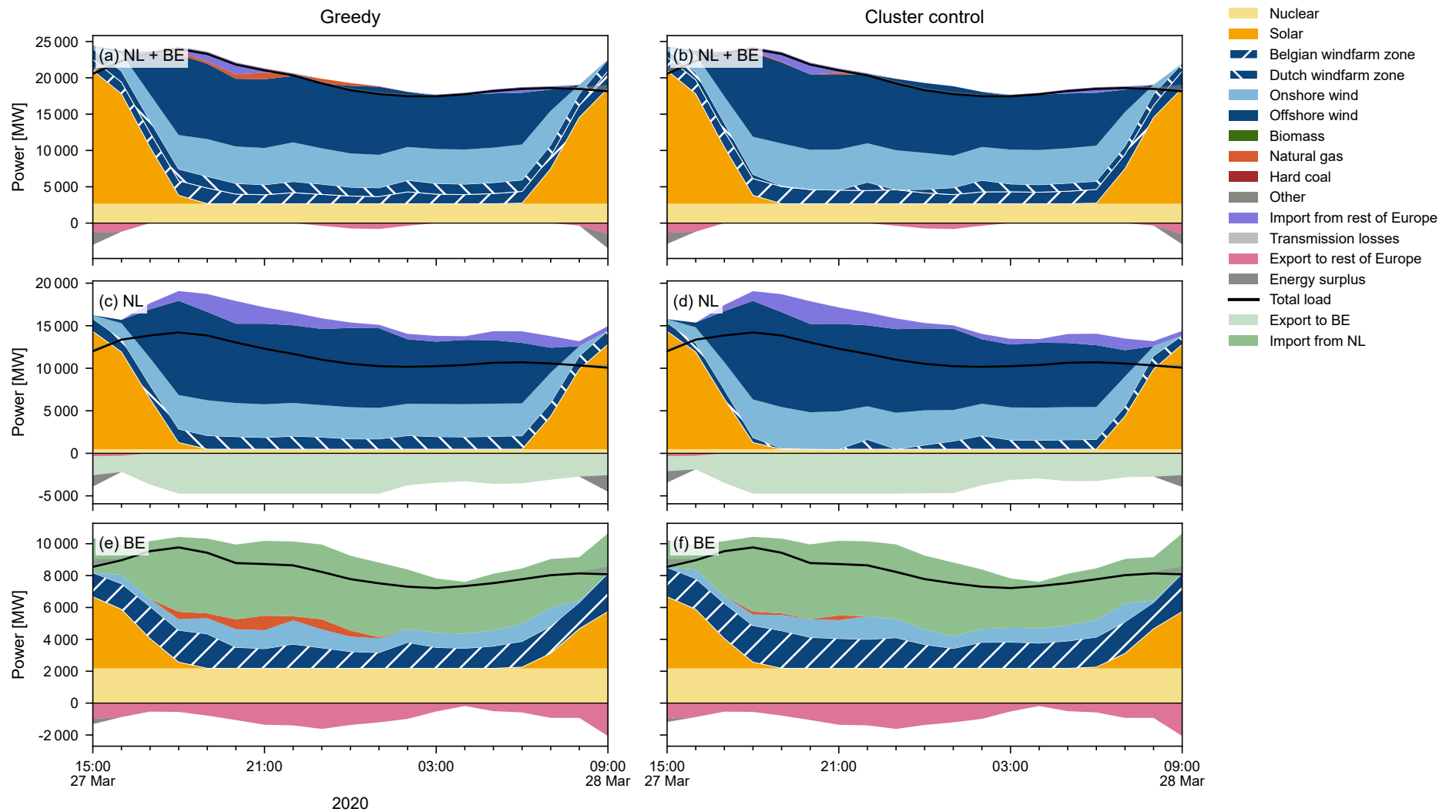


Figure 4.26: Dispatch over 27–28 March under greedy control (left) and under cluster control (right), with the Belgian nuclear fleet reduced to its 2026 capacity. Panels (a) and (b) show the combined system, (c) and (d) the Netherlands, and (e) and (f) Belgium.

Figure 4.26 shows why the night hours are curtailed. With Doel 1, Doel 2 and Tihange 1 removed, nuclear, wind and imports no longer cover Belgian demand, and gas units are dispatched between about 18:00 and 08:00. Under the 2020 nuclear capacity, the same hours were served without gas, and no curtailment was selected. At the same time, Dutch demand at night is low enough that the Netherlands holds a surplus of cheap wind. Wind that cannot be placed in the Netherlands is worth nothing at home and displaces gas in Belgium, so the system moves it across the border through the wake. These two conditions, a Dutch surplus and a Belgian unit on the margin, are the same pair identified in Section 4.3.3. The reduced nuclear capacity does not create a new mechanism; it only makes the conditions easier to meet.

Displaced fossile generation

The benefit is quantified as in the previous sections, as the reduction in dispatched fossil generation. Figure 4.27 shows the dispatched gas generation over the window. Cluster control reduces gas generation for eight hours in the window, from 27 March 15:00 to 23:00. Over the window, 3.1 GWh of gas generation is avoided. At a marginal cost of 30 €/MWh this lowers the system cost by 93 k€, and at the 2025 price of 102 €/MWh by about 316 k€.

The displaced gas generation also reduces emissions. Replacing the 3.1 GWh of avoided gas generation with nuclear and wind therefore reduces emissions by roughly 0.8 to 2.8 kt CO₂-e over the full window.

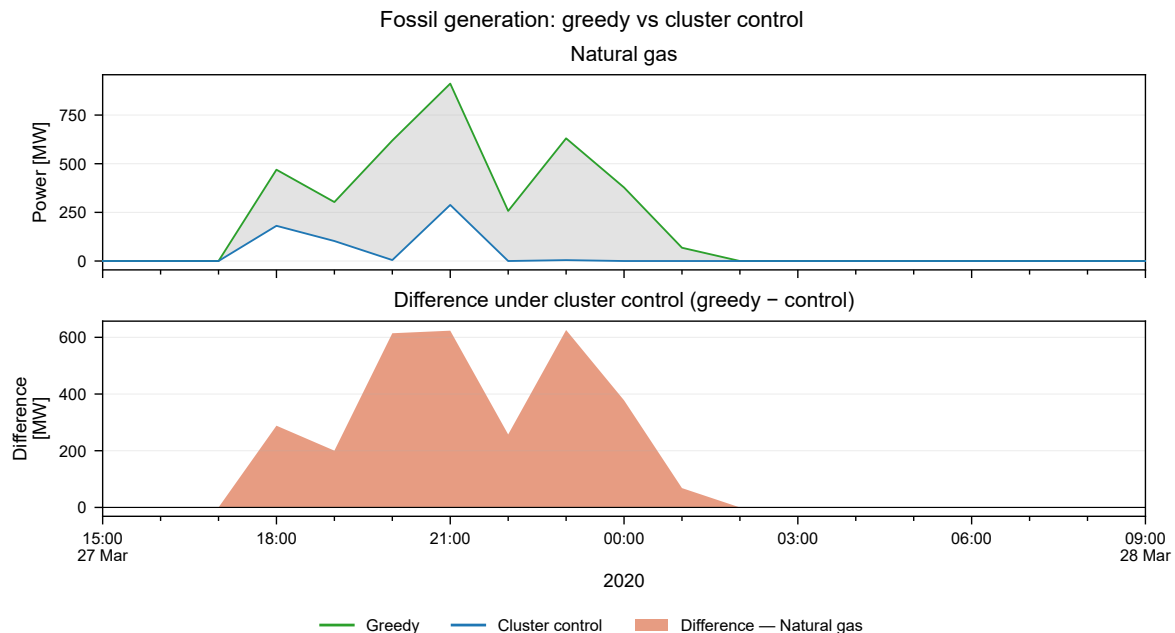


Figure 4.27: Natural gas generation under greedy and under cluster control over 27–28 March with the 2026 Belgian nuclear generators, and the difference between the two.

Gas dispatch in Belgium limits the benefit here. If Belgian nuclear capacity were reduced further, gas would be on the margin throughout the night, and curtailment would be expected to displace fossil generation over the full window rather than part of it. That case was not simulated.

4.3.5. Sensitivity study

To investigate when curtailing is useful, the same window as in Section 4.3.3 is used: 14 to 15 October. The reference case is adjusted relative to the sections above. It is set at a point where curtailing the upwind farm is close to being useful, so that the effect of each factor is visible in both directions. The Dutch offshore wind is set to 75 % of the third case, Section 4.3.3, and Belgian nuclear capacity is set to the 2026 level, Section 4.3.4.

Three factors are varied, one at a time. These are the transmission capacity between Belgium and the Netherlands, the installed Dutch offshore wind capacity, and the nuclear share.

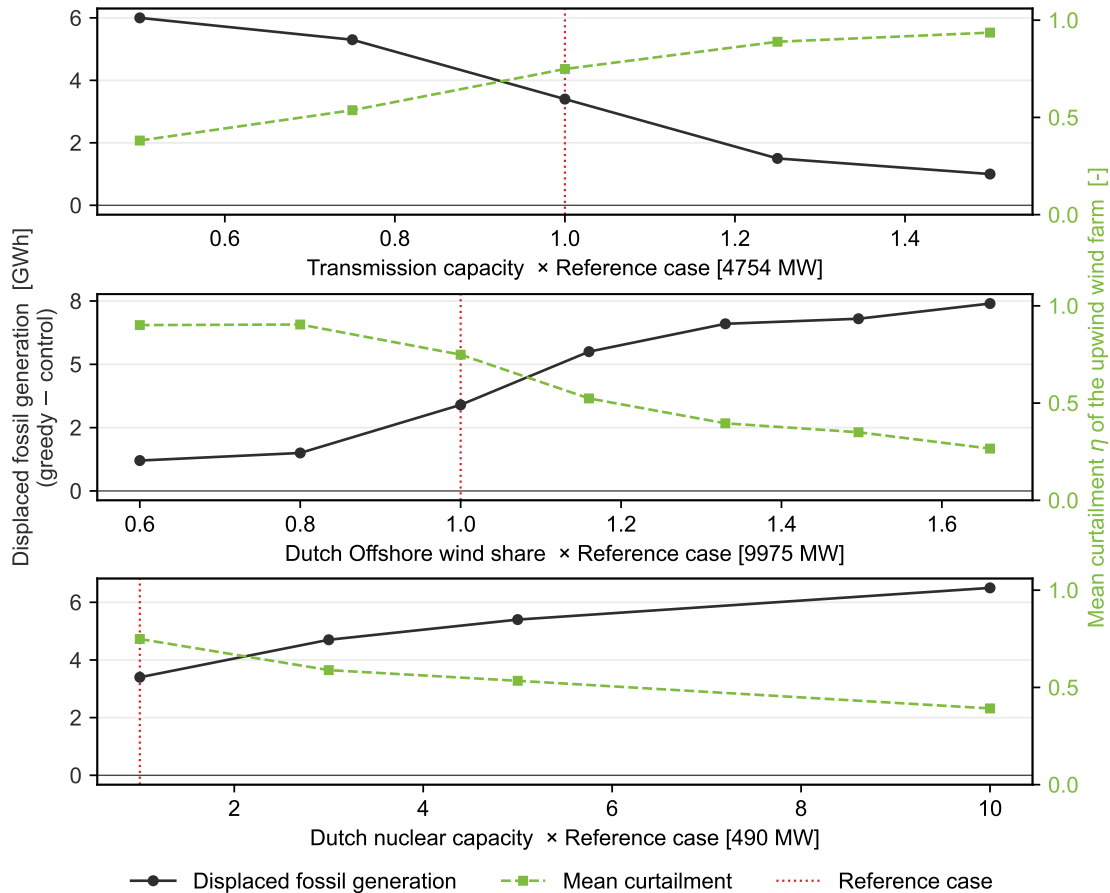


Figure 4.28: Impact of transmission capacity, installed Dutch offshore wind and Dutch nuclear capacity on the effectiveness of cluster control, over 14–15 October. Positive values denote fossil generation displaced by the control strategy (greedy – control). The reference case, marked by the dotted line, holds about 10 GW of offshore wind, 0.5 GW of nuclear and 4.7 GW of transmission between Belgium and the Netherlands.

The figures report displaced fossil generation, as in the previous sections. Displaced fossil generation stands for two things at once. It indicates the reduction in system cost because fossil output is replaced by cheaper nuclear or renewable generation, and it indicates the reduction in emissions because the sources it displaces emit little. Secondly, the figures show the mean curtailment factor over the window. A lower mean curtailment η means the upwind farm is curtailed more often and more deeply, and therefore the control strategy is used more.

The transmission sweep shows that the system prefers the cable over the control strategy. When transmission capacity is Sufficient, the mean curtailment is 0.89, against 0.38 when the line is constrained. This shows that the strategy is used more when no cheaper route exists.

Installed wind capacity works in the same direction. With more wind, the Netherlands holds more energy than it can consume or export, and curtailing the upwind farm becomes worthwhile. With less

wind, the surplus is absorbed conventionally, and the strategy has no role.

Nuclear capacity has the same effect; it adds cheap generation. Because nuclear runs as must-run, it increases generation during windy hours, and more transmission capacity is needed to consume the extra energy. This increases the effectiveness of cluster control.

These three factors only show an effect because the reference case is chosen to make them visible. With wind and nuclear output below Dutch demand, transmission capacity would make no difference, since there is no surplus to move. With Sufficient transmission capacity, the cables would absorb all of the increased wind energy generated, and curtailment wouldn't be useful. In addition, the strategy also depends on the Belgian energy system and the wind. The results show that the strategy depends on several conditions holding at once, and that all these factors significantly affect the strategy.

4.3.6. The year 2020

The windows analysed above are single occurrences. Whether the strategy is worth implementing depends on how often such conditions return and on how long they last. Duration matters in itself. A wind farm needs time to adjust its curtailment and the wake needs time to travel to the downwind zone, so isolated favourable hours are worth less than sustained periods.

The persistent condition is the wind. The imbalance in solar output identified in Section 4.3.2 lasts a few hours at most, because it depends on the position of the sun. A north-easterly wind at a suitable speed can hold for days. Table 4.3 therefore lists the longest periods of 2020 during which the wind blew from the Dutch zone towards the Belgian one at a free-stream speed of $11 \pm 4 \text{ m s}^{-1}$.

Table 4.3: Periods in 2020 during which the wind at the Borssele–Belgian cluster blew from the north-east towards the south-west, at a free-stream speed of $11 \pm 4 \text{ m s}^{-1}$. The ten longest periods are listed.

Duration [h]	Start	End
102	16 Sep 02:00	20 Sep 07:00
70	25 Mar 14:00	28 Mar 11:00
61	19 Mar 12:00	22 Mar 00:00
50	20 Apr 21:00	22 Apr 22:00
45	13 Oct 23:00	15 Oct 19:00
44	13 May 06:00	15 May 01:00
41	19 Apr 03:00	20 Apr 19:00
40	04 May 12:00	06 May 03:00
38	29 May 10:00	30 May 23:00
37	27 May 16:00	29 May 04:00
Total	528	

The ten longest periods together cover 528 hours, or 6.0% of the year. The longest runs last for more than four days. The window analysed in Sections 4.3.2 and 4.3.3 falls inside the fifth entry. The periods cluster in March to May. In September and October, and none occurs in winter or in midsummer.

The direction that makes the strategy possible is not the prevailing one. Under the same conditions as Table 4.3, with a free-stream speed of $11 \pm 4 \text{ m s}^{-1}$, the wind at the cluster blew from the south-west towards the north-east for 2364 hours of 2020, and in the opposite direction for 1027 hours. The Belgian farm is upwind 2.3 times as often as the Dutch farm.

At this location, that works against the strategy. The Netherlands is often exporting energy to Belgium, so the curtailment strategy is only useful while the Dutch farm is upwind. During the other 2364 hours, the wake would move energy from Belgium toward the Netherlands, in a direction where the border is rarely congested.

4.4. Computational time

As described in Section 3.2.10, several choices were made to keep the model tractable: piecewise linear constraints for the power curve, the power coefficient curve and the C_P -to- α mapping, and a Taylor expansion to linearise the remaining dependence on η . Both trade accuracy against computational cost. The number of breakpoints and reference points is close to the minimum. Removing a single breakpoint or a single expansion region already makes the approximation deviate visibly from the underlying curve.

The size of the resulting problem depends on the number of turbines and on the size of the nodal network, but also strongly on the atmospheric conditions. The number of wake constraints follows from the number of interacting turbine pairs. A wind direction in which few turbines lie in each other's wake therefore yields a substantially smaller problem than one in which the wind is aligned with the rows of the array and the maximum number of wake interactions is active.

Because the formulation contains no coupling between time steps, each hour can be solved independently. This keeps the memory footprint bounded and limits the number of piecewise constraints present in any single solve, at the cost of rebuilding the model T times.

Table 4.4 lists the five steps of Section 3.4 with their problem size and their minimum and maximum solution time. The average is approximately 67 s per hour. These figures apply to atmospheric conditions under which curtailment can be beneficial, at a free-stream wind speed of roughly 11 m/s. Hours below cut-in or above cut-out are solved in approximately 13 s. Most of that time is spent building the model rather than solving it; the energy system model alone solves in less than a second.

The table below shows the steps from Section 3.4 and gives their minimum and maximum computational time and number of constraints.

Table 4.4: Problem size and solution time per step of the procedure of 4.3.3, for the Borssele–Belgian cluster over the 24-hour window of 14–15 October 2020. Values are per hourly solve; ranges give the average over the window.

Step	Rows	Nonzeros	SOS2	Solves	Time [s]
1. Taylor coefficients	4488	29971	1122	4	7 - 21
2. Optimal $\hat{\eta}$	4852	30790	1122	2	2 - 334
3. Region selection	973	2622	0	1	0.5 - 1
4. Greedy dispatch	4852	30790	1122	1	2 - 5
5. Controlled dispatch	4852	30790	1122	1	2 - 5
Total (per hour)				9	14 - 366

Restricting the analysis to hours when the strategy can be used leaves 3391 hours in 2020. Solving those hours at the average rate above would require $67 \times 3391 \approx 2.3 \times 10^5$ s, or roughly 63 hours of computation. Simulating the full year for every configuration analysed in this chapter is therefore expensive, so the analysis is restricted to selected windows.

4.4.1. Discussion

This chapter has shown that curtailing the upwind farm of a cross-border cluster can displace fossil energy generation. In this Section, the limitations of the research are stated.

This Section first treats the wind farm cluster model, then the energy system model, and finally the assumptions that must hold for the strategy to be applied in practice.

The screening covered a full year. Still, only two windows were simulated with the full system. The results therefore support a statement about the mechanism, not its annual value. The windows in October and March were selected because they show visible results.

4.4.2. The wind farm cluster model

The cluster is modelled with the Jensen model and linear superposition. The Jensen model reports strong performance, according to Archer et al. [33]. Still, their evaluation concerns turbines within a single farm. For Borssele and the Belgian zone, this is not a limitation because these two farms are adjacent. The closest rows of these farms are separated by less than $10 D$. The transition between the farms is therefore no different from a transition between two rows of the same array, and the argument that the cluster may be treated as a single farm is a fair one.

For the IJmuiden Ver and Hollandse Kust west, the two farms lie farther apart from each other. There is still a lot unknown about the accuracy of engineering models for these distances. Studies such as [12] and [15] showed that the length of the wake depends a lot on atmospheric conditions; these conditions are not taken into account in the Jensen model. The IJmuiden Ver and Hollandse Kust West case return no curtailment at all, because the network always offered a detour with spare capacity. The cluster-control conclusion there rests on transmission capacity rather than wake physics. It does not depend on the wake model. The two cases that do produce a result are the ones for which the EWMS are best suited.

The curtailment assumptions

The power curve and the power coefficient curve of the reference turbines were determined for greedy operation at $\eta = 1$. Every curtailed state in this thesis is obtained by scaling those curves: $p^{\text{eff}} = \eta p(U)$ and $C_P^{\text{eff}} = \eta C_P(U)$. The turbine is assumed to reach any intermediate power level by scaling the measured curve linearly, and to do so identically at every wind speed. That behaviour has not yet been measured.

Neither the scaled curves nor the resulting wake has been validated against more sophisticated models or field measurements. The most direct way to remove this uncertainty is to establish how a real turbine moves through the (C_P, C_T) plane under curtailment. More research is needed to determine whether this curtailment approach is accurate.

The linearisation: underestimation

The Taylor expansion of the wind farm model overestimates p^{eff} ; this overestimation is strongest at the boundaries of the two expansion intervals. The optimiser is therefore drawn towards those boundaries and returns a curtailment $\hat{\eta}$ that can differ from the true optimum η^* . Since η^* is optimal by construction, the objective attained at $\hat{\eta}$ can only be worse. The linearisation underestimates the strategy's performance.

Steady state within the hour

The model is solved per hour and treats the flow as stationary within it. The wake needs time to travel: at the free-stream speed of the analysed window, roughly 10 m s^{-1} , energy released upstream reaches the downwind farm after about 30 min, depending on the location in the farm. Curtailment and its effect are therefore not simultaneous, and an hourly resolution cannot represent the delay. This matters most for single isolated hours, such as 15 October 09:00, and least for sustained periods with strong wind availability, where curtailment can persist over several consecutive hours.

4.4.3. The energy system model

The system is run with 2020 fuel prices, when European gas averaged about 30 EUR/MWh and was cheaper than coal. Gas is therefore the marginal unit in almost every hour of the window, and the

strategy's benefit is measured as gas displaced. The physical redistribution of energy does not depend on the fuel price; its value does. At the 2025 price of about 102 EUR/MWh, the same displaced volume is worth roughly three times as much.

Dutch offshore capacity is planned to grow substantially faster than Belgian capacity in 2050, 21 GW compared to 8 GW, respectively [5]. This is because the Dutch exclusive economic zone offers far more space in the North Sea. The asymmetry between the two zones widens: Dutch wind peaks grow larger, and so does demand for export capacity across the border during these peak moments. Section 4.3.3 already shows this effect, where adding IJmuiden Ver, HKW and Nederwiek raised the utilisation of both interconnectors, and active cluster control was beneficial.

Transmission capacity is also planned to grow, and it works in the opposite direction. Every megawatt of additional interconnection is a detour that the system prefers over the wake, as the IJmuiden Ver case demonstrates. Whether cluster control becomes more or less relevant depends on which grows faster.

Hydropower

The system excludes hydropower and energy storage. These generators depend on the state of charge and successive hours, which a single window cannot represent. A storage unit would therefore not give meaningful information. The impact is relatively small since Belgium and the Netherlands have a very small share of storage generators.

Favourable control conditions

The conditions under which the strategy pays off follow directly from the receiving system's composition. Belgian nuclear output is fixed at its installed capacity, so a large share of Belgian supply cannot step aside. Redistributed wind is only valuable when a flexible unit remains on the margin to be displaced. At night, when load is low, nuclear output together with imports over the interconnectors is already enough to serve Belgian demand, and curtailment is not profitable.

Three properties of a border therefore determine whether a cluster can use the strategy. The downwind zone should carry a small must-run share, so that a flexible unit remains on the margin. The interconnection should be narrow, because the narrower the link, the sooner cluster control becomes the only transmission option. Third, the two systems' renewable output should not be strongly correlated. This is a condition on the systems, not on the cluster itself: the two farms of the cluster must stand close together and see the same wind, since that creates the wake the strategy uses. But if the national portfolios behind them peak at the same hour, both systems have a surplus at the same moment, and the downwind system has no fossil generation left to displace. The value of the redistributed energy then drops to zero exactly when the cluster has most to give away. Two systems that build the bulk of their wind capacity in the same sea are therefore poor candidates.

The Netherlands and the United Kingdom are an example. Both are built in the same sea, and some planned farms lie as close together as the Borssele and Belgian zones do. That proximity is what makes the case worth considering: the wake-coupling this strategy depends on is strongest where two farms almost touch. The interconnection between the two countries is narrow, at 1 GW today, so the second condition is met. The third is not. If both countries build the bulk of their offshore capacity in the same sea, their national wind output peaks at the same hours.

However, if capacity inside the Netherlands is sufficient, the Netherlands could work as a transmission country between the United Kingdom and Germany. This transmission is currently limited by the connection between the United Kingdom and the Netherlands, but could increase with active cluster control in the future.

No cluster spans a border other than the Dutch–Belgian one in the North Sea today, so this example is about future systems rather than 2020. Read it as such throughout, and apply the same horizon to every quantity used to support it: quoting the present capacity of a single interconnector against a cluster that does not yet exist compares two different points in time.

4.4.4. Implications for further work

The linearised wind farm model makes the coupled problem solvable, but it also limits the energy system it can support. The piecewise linear constraints and the two Taylor regions slow the problem. Complex

systems such as storage had to be dropped. An alternative is to simulate the cluster separately, with software built for wind farms, and to enter the result into the energy system as a lookup table with a single piecewise linear constraint. This would allow a more detailed wake model on one side and a more detailed energy system on the other.

This route also removes a second restriction. The linearisation forces the wake to be driven by the power coefficient, which under Assumption 3.1 carries turbine inefficiencies that have nothing to do with induction.

A second route is using the thrust coefficient directly, and Assumption 3.2 would no longer be needed. The thrust coefficient can also be used within a linearised formulation: the mapping from C_T to a is linear, which removes one piecewise linear constraint. The resulting power coefficient curve, C_P^{eff} , is then not a polynomial and a less regular non-linear function. This would probably require more Taylor expansion points.

Future research should either further develop these computational implementations or apply the approach to a different wind farm cluster, such as the planned wind farm cluster between the Netherlands and the United Kingdom, Nederwiek–Doggerbank.

Given the assumptions above, the quantities reported in this chapter should be read as order-of-magnitude indications. The contribution of this thesis is not a figure for the value of curtailing an upwind farm. It demonstrates that a congested border and an inflexible downwind system can make coordinated cluster control worthwhile, and identifies the system conditions under which that happens.

Control strategies transmit energy with efficiency between 29 to 63 %; however, they cannot compete with conventional transport. During peak wind hours in the Netherlands, when the cable is full, storage is an alternative to transmission. Round-trip efficiencies of hydrogen storage fall below 50 % [52], so in those hours curtailing the upwind farm may be the better use of the surplus energy. The comparison is not clean; however, storage moves energy to another hour, which curtailment cannot do, and that flexibility may be worth more than the efficiency difference. Establishing which is preferable requires further work.

5. Conclusion

This thesis investigated whether coordinated control of an offshore wind farm cluster can influence energy system outcomes, both total power output and its spatial distribution were of interest. To make this question tractable, a wake model is developed. This is a linearised version of the Jensen wind farm model. The turbine power and power coefficient curves are replaced with piecewise linear fits. The remaining non-linear dependence on curtailment is linearised through a Taylor expansion around two reference points. The result is a set of linear relations that redistributes power between two farms as a function of a single scalar; the curtailment coefficient η . The linearised model was then embedded in an optimisation framework.

The wind farm cluster behaves as an inefficient transmission line: part of the energy disregarded upstream is recovered downstream. The efficiency of energy transport depends on atmospheric and market conditions. The model was embedded in a nodal representation of the Dutch and Belgian electricity systems and applied to four cases:

1. IJmuiden Ver and Hollandse Kust West
2. Belgian-Dutch Cluster:
 - (a) Borssele and Belgian offshore farms
 - (b) Borssele and the Belgian offshore farms with the planned Dutch farms
 - (c) Borssele and the Belgian offshore farms with 2026 installed nuclear capacity

Case results

1) The IJmuiden Ver and Hollandse Kust West cluster showed no benefit from cluster control. The direct line between the two connection points never reached its capacity; a detour with spare capacity remained available. Re-routing costs 1 % per 100 km, compared to shifting the same energy through the wake, means disregarding it upstream and recovering only part of it downstream. The system consistently preferred the transmission network. This result rests on transmission capacity rather than on wake physics.

2a) The cross-border Borssele-Belgian cluster showed no benefit either. Three conditions had to coincide: the interconnectors at their capacity, wind blowing from the Dutch zone towards the Belgian one, and a surplus of Dutch wind or solar relative to the Belgian. In the 2020 system, the Dutch renewable share was not large enough to cover domestic demand and leave a surplus to export.

2b) Adding IJmuiden Ver, Hollandse Kust West and a share of Nederwiek show considerable results. Sustained periods became profitable, the longest of which ran from 17:00 to 22:00 on 14 October. Over the full window 2.2 GW_h of gas generation was avoided, and 1.76 GW_h over that period alone. At the 2020 marginal cost of gas this lowers system cost by 52.8 kEUR, and at the 2025 price by roughly 179 kEUR.

2c) The last case reduced Belgian nuclear capacity to its 2026 level, keeping the added Dutch wind capacity. This widened the hours in which cluster control was useful. At night, when demand is low, nuclear output together with imports was already enough to serve Belgian demand under the 2020 nuclear fleet, leaving no gas units on the margin to displace. With the reduced capacity, gas returns to the margin, and the moments in which it can be displaced become more frequent. Over an eight-hour

window on 27 March, 3.1 GW h of gas generation was avoided, lowering system cost by 93 kEUR at the 2020 gas price and by roughly 316 kEUR at the 2025 price.

Sub-questions

The first sub-question asked how engineering wake models can capture cluster-scale interactions accurately enough while remaining cheap enough for optimisation. In this thesis, the Jensen model served as the ground truth, and the linearised formulation reproduces its trends. It is accurate enough to identify when curtailment is worthwhile and to establish the order of magnitude of the resulting saving. It is less reliable for the marginal price and for the curtailment level itself, where the deviation from the reference optimum can exceed 20 %. Whether the Jensen model itself is accurate at cluster scale was not established here, and remains an open question.

The second sub-question asked under which atmospheric and market conditions curtailment of an upstream farm yields a net benefit. Those conditions are tightly coupled. Curtailment becomes useful when one node wants to export to another. When the connecting lines are at capacity, a curtailment strategy can become useful. This happens when the exporting node has a surplus of cheap generation while the importing node still runs more expensive units that can be displaced. On top of that, the two farms must lie close enough for their wakes to interact, and the wind must blow from the exporting node towards the importing one. That is a demanding set of conditions, but wind and solar produce exactly the pattern that creates them: large, localised peaks. A larger renewable share sharpens both the peaks and the spatial differences, and with that the demand for transmission. The third case demonstrates this directly.

The third sub-question asked how coordinated control redistributes power and what it costs in total yield. In the cross-border case with additional Dutch capacity, the cluster transported energy at an effective efficiency of 29 to 63 % during curtailment. The transport efficiency depends strongly on wind speed, wind direction, farm layout and the state of the energy system. Curtailment can alleviate congestion, but is almost always inferior to transmission through cables. Cluster control works only when congestion occurs, and no cheaper route exists.

Main question

The main research question for this thesis was formulated as:

"To what extent can active control of offshore wind farm clusters influence energy system outcomes, including power output and spatial power distribution?"

Active cluster control can shift power between two grid nodes and displace fossil generation under a narrow set of conditions. It is not a substitute for transmission but an addition. Whenever a transmission route with spare capacity exists, the system prefers it. The wake energy transport route is taken only when the network offers no alternative. Where those conditions do hold, the effect is not marginal. Under the right conditions 3.1 GW h of fossil gas generation can be avoided in a short window of time of eight-hours.

Increasing the share of renewables also increases the generation peaks. Effectively flattening these peaks by spreading the energy becomes a central problem. Cables are the obvious answer and very effective. This thesis shows there is a second route. Under the right conditions, a wind farm cluster can act as a transmission line. Curtailing the upwind farm gives up power upstream and recovers it downstream, displacing fossil energy on the other side of the congested border.

Declaration on the use of generative AI

The author used generative AI, such as Claude, during this thesis. This section declares its use in creating the model.

Wind farm model. Generative AI was of limited use in developing the wind farm cluster model and was used only to resolve syntax errors and to assist in debugging.

Energy system model. Generative AI was more useful here, mainly still for debugging. The author shared code in small portions rather than in full, and checked and tested every suggested change before adopting it.

Writing. Generative AI was used for language editing: grammar, spelling and readability of text the author had already written. It was also used to align the phrasing and section structure with the conventions described by Whitesides [53]. Every section was drafted by the author first. Suggested edits were then thoroughly reviewed by the author. No argument, result, interpretation or reference was produced by a generative AI tool.

Data. No input data was shared; where the structure of a dataset was needed, a simplified example reproducing only its layout was used instead.

The author takes full responsibility for the content of this thesis, including any part on which a generative AI tool was consulted.

Reflection

While using AI, two things stood out. During the development of the wind farm model, the tools repeatedly proposed a different approach and insisted that the Taylor expansion was incorrect. When I asked it to work through it step by step, it withdrew the objection.

For the energy system model, the suggestions were clearer and more compact. The difference has two plausible causes: the energy model is simpler in structure, and energy system optimisation is far better documented than linearising wind farm wake models. That imbalance is why every suggestion on the wind farm side was read with extra care.

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A. Energy System Data

Table A.1: Installed Capacity and Peak demand per Hub (MW)

		Installed capacity (MW)				Dispatchable units	
Node	(province)	Peak demand (MW)	Onshore wind	Offshore wind	Solar	Units (#)	Capacity (MW)
The Netherlands							
NL11	(Groningen)	532	1 414	600	4 842	10	6 765
NL12	(Friesland)	488	880	0	1 453	0	0
NL13	(Drenthe)	367	267	0	267	0	0
NL21	(Overijssel)	1 047	113	0	2 181	0	0
NL22	(Gelderland)	1 865	255	0	3 638	0	0
NL23	(Flevoland)	353	1 466	0	2 861	2	880
NL31	(Utrecht)	1 725	58	0	671	5	1 214
NL32	(North Holland)	525	802	987 + A	1 600	5	3 279
NL33	(South Holland)	3 754	621	1658 + B + C	1 465	5	2 995
NL34	(Zeeland)	334	525	D	1 465	4	1 734
NL41	(North Brabant)	2 785	462	0	3 688	2	1 059
NL42	(Limburg)	946	93	0	1 244	2	1 584
Belgium							
BE10	(Brussels)	2 357	5	0	270	0	0
BE21	(Antwerp)	2 527	438	0	279	6	936
BE22	(Limburg)	807	337	0	1 767	1	422
BE23	(East Flanders)	1 600	509	0	834	6	2 857
BE24	(Flemish Brabant)	1 400	53	0	526	1	460
BE25	(West Flanders)	1 337	241	0	1 039	1	465
BE31	(Walloon Brabant)	630	94	0	348	0	0
BE32	(Hainaut)	949	557	0	1 789	2	763
BE33	(Liège)	920	300	0	417	4	3 616
BE34	(Luxembourg)	203	147	0	723	0	0
BE35	(Namur)	380	374	0	798	1	106
BE_OFFSHORE		–	E	–	–	–	–

The letters A to E denote offshore wind farm zones that are not represented by a single capacity value in the table. They are either modelled as individual turbines, or added to the system only in specific cases.

- A Hollandse Kust West. Installed capacity 2000 MW.
- B IJmuiden Ver, sites alpha, beta and gamma.. Installed capacity 6000 MW.
- C Nederwiek, site I only. Installed capacity 2000 MW.
- D Borssele, the existing Dutch offshore zone. Installed capacity 1590 MW. Modelled either in detail or through a capacity factor, depending on the case.
- E The existing Belgian offshore zone. Installed capacity 2374 MW. Modelled either in detail or through a capacity factor, depending on the case.

Table A.2: Transmission lines in the nodal model, with their end points, capacity, efficiency and implied length. The efficiency follows from the length at a loss of 1 % per 100 km.

Line	From	To	$F_l^{\max}$ (MW)	η (-)
Within the Netherlands				
L1	NL11	NL12	1 376	0.99
L2	NL11	NL13	2 377	1.00
L3	NL13	NL21	2 377	0.99
L4	NL12	NL23	688	0.99
L5	NL21	NL22	2 377	0.99
L6	NL21	NL23	3 066	0.99
L7	NL22	NL41	2 377	0.99
L8	NL23	NL32	2 377	0.99
L9	NL33	NL41	2 377	0.99
L10	NL31	NL32	2 377	0.99
L11	NL31	NL33	2 377	1.00
L12	NL32	NL33	2 377	0.99
L13	NL34	NL41	2 377	0.99
L14	NL41	NL42	4 755	1.00
L15	NL32	HKW	2 500	1.00
L16	NL33	IJmuiden Ver	6 000	1.00
L17	NL34	Borssele	2 500	1.00
Cross-border				
L19	NL34	BE23	2 377	0.99
L31	NL42	BE22	2 377	0.99
Within Belgium				
L20	BE25	BE23	1 204	0.99
L21	BE23	BE21	4 755	0.99
L22	BE21	BE22	2 943	0.99
L23	BE23	BE24	3 171	0.99
L24	BE31	BE24	3 323	0.99
L25	BE10	BE24	3 000	1.00
L26	BE24	BE32	3 323	0.99
L27	BE22	BE33	2 849	0.99
L28	BE33	BE34	451	0.99
L29	BE33	BE35	2 517	0.99
L30	BE32	BE35	3 566	0.99
L18	BE_OFFSHORE	BE23	2 500	0.99

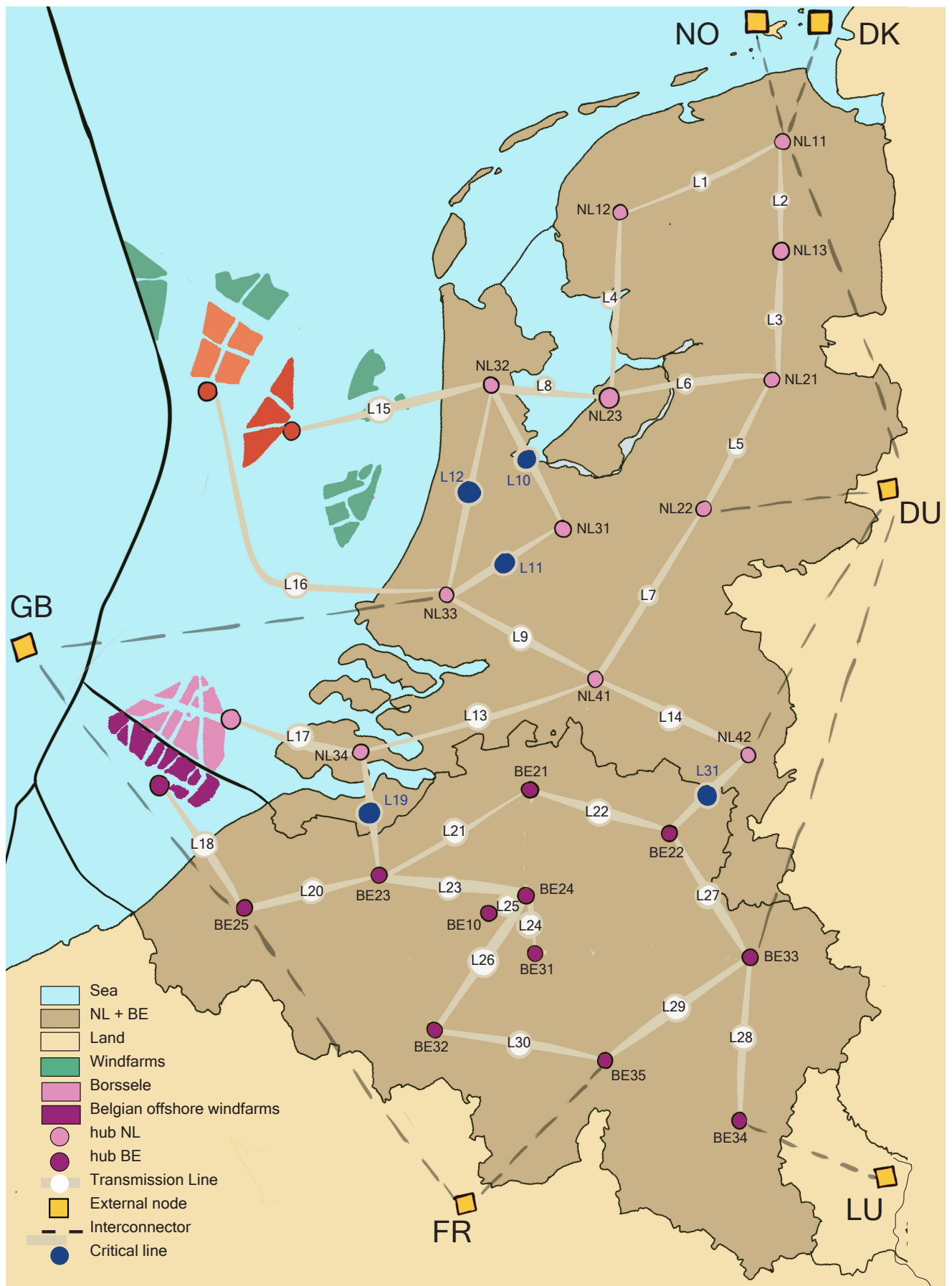


Figure A.1: Model Network Belgium and The Netherlands