The two-dimensional Godunov scheme and what it means for continuum pedestrian flow models

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An efficient simulation method for two-dimensional continuum pedestrian flow models is introduced. It is a two-dimensional and multi-class extension of the Godunov scheme for one-dimensional road traffic flow models introduced in the mid 1990's. The method can be applied to continuum pedestrian flow models in a wide range of applications from the design of train stations and other travel hubs to the study of crowd behaviour and safety at religious and cultural events. The combination of the efficient simulation method with continuum models enables the user to get simulation results much quicker than before. This opens doors to real time crowd control and to more advanced optimisation of planning and control.

1 Simulation method

We extend the one-dimensional Godunov scheme for road traffic flow [1, 4] and its multi-class adaptations [5] to two dimensions for applications in crowd simulation. To illustrate the simulation method, we apply a simple yet generic continuum crowd flow model [2, 3].

Given an initial state, the simulation predicts future states, including dynamical changes over time. Therefore, the model equations are solved numerically. The computational domain is divided into cells sized Δx times Δy, the number of pedestrians in each of these cells is computed and updated each time step Δt depending on inflow into the cell and outflow out of the cell. Therefore, one time step consists of the following substeps:

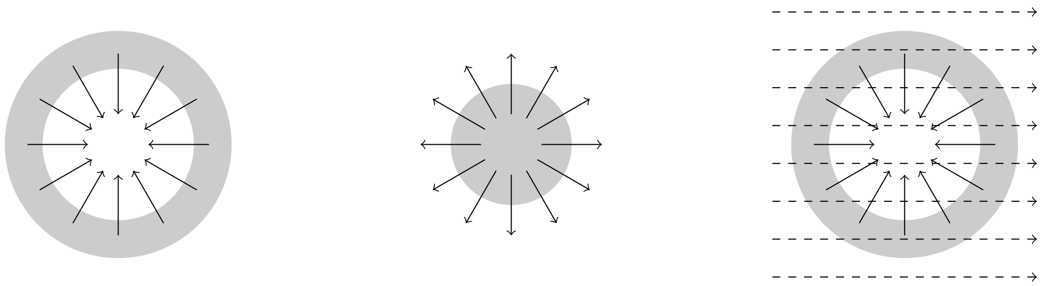
1. Determine class specific speed v_u and walking direction $\vec{e}_u$ in each cell. This is, apart from location and time, fully determined by class specific densities (number of pedestrians of each class per space unit) and their gradients.
2. Determine class specific demand and supply per edge of the cell:
   (a) determine class specific demand per cell over each edge,
   (b) determine total supply per cell over all edges,
   (c) distribute supply per cell over cell edges and classes using class specific demand per edge.
3. Compute class specific flow over each edge and update class specific density in each cell accordingly.

Each step will be discussed in more detail in the full paper. While the first and last step are very similar to their equivalents in a one-dimensional model, the main challenges lie in step 2. This is because walking direction is dynamic and depends on class. Therefore, pedestrians of one class may cross a cell's edge in one direction, while those of an other class may cross it in an other direction. Furthermore, most cells (except those on the boundaries) have four edges that could act as inflow and/or outflow edges, instead of only one for inflow and one for outflow as in the one-dimensional road traffic case.

2 Tests and results

We test the new numerical method with a few simple test cases. In this extended abstract we show the results of three of them. In the full paper, there will be more examples and the results will be discussed more in-depth.

In the test cases, the pedestrians are initially located in a circular area. As Figure 1 shows, they are then attracted to the centre (case 1 and 3) or they are repelled from the centre (case 2). In case 3, there is furthermore a second class that moves from left to right, through the area with the pedestrians of the first class.



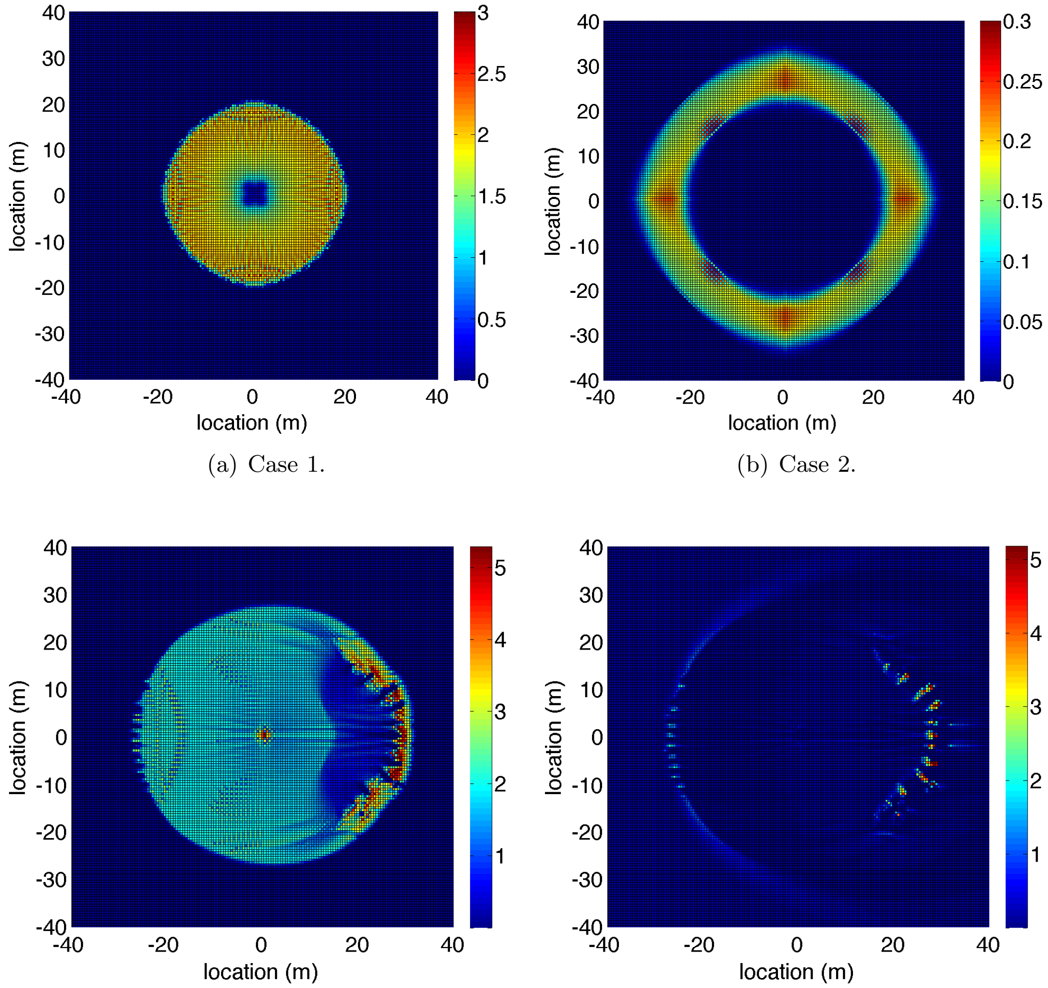
(a) Case 1

(b) Case 2

(c) Case 3. Dashed arrows: class 1, solid arrows: class 2.

Figure 1: Initial positioning (gray areas) and walking direction (arrows).

The test results (Figure 2) show that the numerical method can deal well with flows where densities strongly increase (e.g. towards the centre in case 1), where densities strongly decrease (e.g. away from the centre in case 2) and with flows of different directions (case 3). Our preliminary results for case 1 and 2 show almost perfect rotational symmetry, indicating only little influence of the walking direction and grid orientation on the computed velocity. Furthermore, preliminary results for case 3 (not shown) indicate an influence of the resolution (grid cell size) on the width of the lanes that emerge. This shows that numerical resolution has to be considered in relation to the scale at which one desires to reproduce phenomena: small scale phenomena can not be accurately reproduced on a coarse grid. The details of this finding are discussed in the full paper.



(a) Case 1.

(b) Case 2.

(c) Case 3, class 1 (towards centre).

(d) Case 3, class 2 (left to right).

Figure 2: Space density plot at time $t = 20$ s.

3 Outlook for full paper

In the full paper, we discuss the simulation method, the tests and their results in more detail. We show that self-organising phenomena may occur differently with different numerical settings, indicating that a good setting is important. Two aspects are studied in detail. Firstly, the orientation of the grid may have an influence on the occurrence of self-organisation. Secondly, the size of the grid cells may have an influence on the scale at which self-organisation occurs, e.g. the width of emerging lanes.

Future research includes more detailed comparison of the proposed numerical method with other methods. Furthermore, the two-dimensional Godunov method will and already has been applied in (future) research to study continuum pedestrian flow models [2, 3]. Since one of the objectives of these studies is to investigate self-organisation, it is important to apply an accurate numerical method such as the one proposed in this contribution.

References

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