Omega 143 (2026) 103546

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

Review

Best-Worst Method: A decade of evolution and future prospects ☆

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ARTICLE INFO

Keywords:
Best-Worst Method
Multi-criteria decision-making
Preference elicitation
Anchoring bias

ABSTRACT

Best-Worst Method (BWM) is a structured approach for eliciting criteria weights in multi-criteria decision-making. This review paper revisits the original BWM formulation and procedural logic, and discusses behavioral motivations for dual anchoring and structured elicitation. It then synthesizes major methodological developments, covering linear and multiplicative formulations, interaction modeling extensions (nonadditive models), Bayesian formulations, group-aggregation models, tradeoff-based elicitation, parsimonious and disaggregation-based variants, sorting extensions, and fuzzy and belief-based treatments of imprecision and epistemic uncertainty. Because the reliability of inferred weights depends on judgment quality, the paper consolidates and compares consistency checking approaches and discusses their implications for practice. Representative application domains are reviewed to illustrate how BWM is deployed in practice. Finally, future research directions are outlined, emphasizing behavioral validation, integration into complete decision pipelines, scalable elicitation support, and cautious human–AI co-production that preserves problem-specific preference meaning. These directions also include transferring BWM's anchored elicitation principle to other preference elicitation approaches, including MACBETH, UTA, and conjoint analysis.

Contents

☆ Area: Decision Analysis and Preference Modeling This manuscript was processed by Associate Editor S. Ben Amor.
E-mail address: j.rezaei@tudelft.nl.

https://doi.org/10.1016/j.omega.2026.103546
Received 29 July 2025; Accepted 26 February 2026
Available online 28 February 2026

1. Introduction

Decision-making lies at the heart of human cognition and purposeful action. It reflects our fundamental need to impose structure on complexity, to choose between competing pathways, and to justify those choices in light of values, goals, and constraints. In contexts ranging from individual reflection to organizational strategy, decision-making embodies both analytical reasoning and normative judgment. The challenge becomes particularly evident when decisions involve multiple, often conflicting objectives, requiring trade-offs, prioritization, and the synthesis of qualitative and quantitative inputs. Insights from philosophy, economics, and cognitive science have shown that such decisions are rarely guided by perfect rationality. Instead, they are shaped by limitations in knowledge, perception, and computation. In response, the field of decision analysis has developed a range of formal methods designed to support individuals and institutions in structuring problems, articulating preferences, and evaluating alternatives in a transparent and coherent manner.

One of the most influential and extensively studied frameworks in decision analysis is multi-criteria decision-making (MCDM). MCDM is used when there are several alternatives to choose from, and each alternative must be judged based on multiple criteria, such as cost, quality, or sustainability. Since the criteria often conflict with one another (e.g., low cost vs. high quality), decision-makers must make trade-offs. This is where MCDM methods provide structure and guidance. Over the past decades, MCDM has grown into a mature field, with many methods applied in business, engineering, healthcare, and public policy. Prominent families include value-based approaches such as MAUT and MAVT [1] and preference-disaggregation models such as UTA [2]. Within this value-focused tradition, elicitation procedures such as SMART and SWING [1,3,4], mid-value splitting and the Tradeoff procedure [1], Best-Worst Tradeoff [5], and MACBETH [6] are widely used to construct value functions and scaling constants. In parallel, pairwise-comparison weighting methods such as AHP/ANP [7,8] and BWM [9] derive ratio-scale weights from comparative judgments; outranking methods such as ELECTRE [10] and PROMETHEE [11] support non-compensatory modeling and are often combined with weight-elicitation tools such as the Deck-of-Cards method [12,13]; and compromise-ranking techniques such as TOPSIS and VIKOR [14,15] remain widely used, alongside robustness-oriented frameworks for uncertainty and heterogeneity (e.g., SMAA [16]).

Across these method families, two modeling components recur: (i) the representation of each alternative's performance on each criterion, and (ii) the aggregation of this information into an overall evaluation, often through criteria weights. The elicitation and interpretation of weights is therefore central: in some settings, weights reflect trade-offs between criteria, whereas in others they are better interpreted as resource shares or voting power [17,18]. This diversity of interpretations helps explain why the design of weight-elicitation procedures, including their cognitive demands, their behavioral validity, and their ability to diagnose inconsistent judgments, remains a major theme in MCDM research.

One of the most recent and widely adopted methods in MCDM is the Best-Worst Method (BWM), introduced by Rezaei [9]. Despite its relatively recent introduction, BWM has gained substantial recognition in the decision sciences literature. In a review of fifty years of MCDM research, BWM is identified as one of the most prominent contemporary methods [19]. See, also, the bibliometric analysis of publications in the journal *Omega*, the journal in which BWM was first introduced [20] for the position of the BWM in the field of OR&MS. This growing prominence underscores the need for a review that clarifies the foundations, developments, and applications of BWM within the broader MCDM literature.

BWM introduces a structured approach to eliciting preference information from decision-makers. The decision-maker first identifies the most important (Best) and least important (Worst) criteria. Subsequently, the decision-maker performs pairwise comparisons between the Best criterion and all others, and between all criteria and the Worst criterion, yielding only two vectors of judgments. These inputs form the basis of an optimization model that derives a set of criterion weights by minimizing the maximum deviation from perfectly consistent comparisons. In contrast to traditional methods based on full pairwise comparison matrices, BWM requires significantly fewer judgments, only $2n - 3$ for n criteria, while often achieving superior consistency. Its compact elicitation structure, transparent logic, and built-in consistency mechanism have contributed to its widespread adoption in both academic research and practical decision contexts. BWM has been applied across a wide range of domains, including sustainability assessment, supply chain management, healthcare planning, energy policy, and risk evaluation.

Since its introduction, BWM has been extended in many directions, such as linear and multiplicative versions, Bayesian formulations for group decisions, models under uncertainty, and methods that include interactions among criteria. However, these developments are spread across many studies, and there is currently no comprehensive review that brings together the theoretical foundations, extensions, behavioral aspects, and applications of BWM.

This paper does not aim to provide a comprehensive systematic review of all existing BWM-related publications, an endeavor made increasingly difficult by the rapid proliferation of extensions and applications across disciplines. Early and more recent review studies have already synthesized important parts of this growing literature (see, e.g., [21–23]). In addition, a bibliographical dataset, along with other methodological resources and related information, is maintained at bestworstmethod.com, providing a convenient entry point for those seeking a broader overview of the literature. Instead, the present work takes a more focused approach: it synthesizes the main methodological developments of BWM over the past decade, discusses its behavioral underpinnings, and highlights key patterns in its use.

The remainder of this paper is structured as follows: Section 2 provides an overview of the original BWM, including its behavioral and procedural foundations. Section 3 provides information on the major extensions of BWM. Section 4 discusses consistency checking of the input pairwise comparisons. Section 5 gives an overview of representative studies that have applied BWM over the last decade, with illustrative application examples. Section 6 proposes future research directions. Finally, Section 7 concludes the paper.

2. Original BWM model and its foundation

In this section, I present the original formulation of the Best-Worst Method, which laid the foundation for a growing family of methods in multi-criteria decision-making. This initial model, commonly referred to as the non-linear BWM, was developed to address the limitations of earlier pairwise comparison methods by introducing a structured and cognitively efficient preference elicitation process based on two reference points: the best and the worst criterion. The model's formulation and rationale are discussed first, followed by behavioral and procedural foundations.

2.1. Non-linear BWM

The method consists of the following five steps :

1. **Identify a set of decision criteria**
   The decision-maker first identifies a set of n criteria used to evaluate the alternatives:

   $C = \{c_1, c_2, \ldots, c_n\}$

   These criteria define distinct dimensions or attributes for evaluating alternatives, ensuring a comprehensive yet minimally redundant assessment framework.

2. **Determine the Best and Worst criteria**
The decision-maker identifies the most important criterion as well as the least important criterion from the set of criteria C. These are called Best (B) and Worst (W) respectively.
3. **Determine the preference of the Best criterion over all others**
The decision-maker conducts a pairwise comparison between the Best and all the other criteria[1] using a number from 1 to 9 (see the description in Table 1). This results in the Best-to-Others (BO) preference vector:

$$BO = (a_{B1}, a_{B2}, \dots, a_{Bn})$$

where a_{Bj} represents the preference of the Best criterion B over criterion j.
These judgments are interpreted as *ratio-scale* statements of relative importance. Accordingly, the BO vector provides anchored ratio information of the form $w_B/w_j \approx a_{Bj}$.
4. **Determine the preference of all criteria over the Worst criterion**
The decision-maker conducts a pairwise comparison between all the criteria[2] and the Worst using a number from 1 to 9 (see the description in Table 1). This results in the Others-to-Worst (OW) preference vector:

$$OW = (a_{1W}, a_{2W}, \dots, a_{nW})^T$$

where a_{jW} represents the preference of criterion j over the Worst criterion W.
Together, the BO and OW vectors express anchored ratio relations that the model seeks to represent through a coherent weight vector.
5. **Compute the weights**
The consistent weights w_j^* for the criteria are achieved when, for each pair (w_B, w_j) and (w_j, w_W), the equalities $\frac{w_B}{w_j} = a_{Bj}$, $\frac{w_j}{w_W} = a_{jW}$ hold. However, in most real-world decision-making scenarios, achieving these exact equalities is impractical due to subjective judgments and inherent inconsistencies. Therefore, the objective is to minimize the maximum absolute deviations $\left|\frac{w_B}{w_j} - a_{Bj}\right|, \left|\frac{w_j}{w_W} - a_{jW}\right|$, for all j. In other words, the objective defines a worst-case discrepancy measure over the set of anchored ratio statements. This approach ensures that the derived weights remain as consistent and close to ideal as possible. By incorporating the non-negativity and sum-to-one constraints on the weights, the resulting optimization problem is formulated as follows:

$$\begin{aligned} \min_{\mathbf{w}} \max_{j} \quad & \left\{ \left|\frac{w_B}{w_j} - a_{Bj}\right|, \left|\frac{w_j}{w_W} - a_{jW}\right| \right\} \\ \text{s.t.} \quad & \sum_{j=1}^{n} w_j = 1 \\ & w_j \geq 0, \quad \forall j \end{aligned} \tag{1}$$

where, w_B, w_W, and w_j are the weights of the Best, Worst, and other criteria, respectively.

Table 1
Fundamental scale for pairwise comparisons [8].

Intensity of importance	Definition
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong or demonstrated importance
9	Extreme importance
2, 4, 6, 8	Intermediate values between the two adjacent judgments

Model (1) is transformed into the following model.

$$\begin{aligned} \min_{\xi, \mathbf{w}} \quad & \xi \\ \text{s.t.} \quad & \left|\frac{w_B}{w_j} - a_{Bj}\right| \leq \xi, \quad \forall j \\ & \left|\frac{w_j}{w_W} - a_{jW}\right| \leq \xi, \quad \forall j \\ & \sum_{j=1}^{n} w_j = 1 \\ & w_j \geq 0, \quad \forall j \end{aligned} \tag{2}$$

The preference information represented by the non-linear BWM is therefore a set of anchored ratio constraints derived from BO and OW judgments. In the fully consistent case, there exists a strictly positive weight vector $\mathbf{w}$ (typically normalized by $\sum_{j=1}^{n} w_j = 1$) such that $w_B/w_j = a_{Bj}$ and $w_j/w_W = a_{jW}$ for all j, which induces a complete and transitive importance ordering over criteria. When judgments are not fully consistent, the solution $\mathbf{w}$ can be viewed as the weight vector that is *closest* to the elicited ratio statements under the discrepancy definition encoded by the optimization model.

In his 2015 study, Rezaei [9] conducted an experiment comparing BWM and AHP, demonstrating that BWM outperforms AHP in terms of consistency ratio, minimum violation, total deviation, and conformity across multiple evaluation criteria. Although BWM is often discussed alongside AHP due to their shared use of ratio-scale pairwise comparisons and the common 1–9 intensity scale (Table 1), the methods are structurally distinct: BWM relies on two anchored comparison vectors rather than a full reciprocal matrix and derives weights via a min–max model. Nevertheless, long-standing critiques of discretized pairwise-comparison elicitation are potentially relevant whenever BWM adopts the same scale; these issues are briefly addressed in Section 2.2. Wu et al. [24] provided an analytical approach to find the solutions of Model (2). The original BWM adopts a *min–max* objective: it chooses the weight vector that minimizes the *largest* absolute deviation between any weight ratio and its corresponding pairwise comparison. By constraining the worst-case deviation, this formulation guaranties that *every* individual pairwise comparison remains within a controlled tolerance. Alternative formulations, such as the goal-programming variant proposed by Amiri and Emamat [25], minimize the *sum* (or average) of all deviations. Although a sum-of-deviations objective might yield a lower aggregate inconsistency, it may do so at the expense of fairness across comparisons, because large deviations on a few ratios can be masked by small deviations on many others. Hence, the min–max model provides a safeguard against outlier inconsistencies, whereas a summative objective emphasizes overall fit, but may do so by allowing a few comparisons to be fit poorly if many others are fit well. More broadly, the choice of objective function is not only a mathematical design choice: it specifies the *deviation model* used to reconcile inconsistent ratio judgments and thus affects the implied preference representation in the inconsistent case. For example, additive-deviation (linearized) objectives and multiplicative (ratio-symmetric/log-ratio) objectives correspond to different notions of distance to the elicited BO and OW ratios and may therefore yield different best-fitting weights when the judgments are inconsistent, while coinciding in the fully consistent case.

[1] For elicitation convenience, it can be useful to start with the Best-to-Worst comparison a_{BW} (i.e., the BO entry with $j = W$) to anchor the response scale before completing the remaining BO comparisons.

[2] Note that the Best-to-Worst judgment a_{BW} is common to both BO and OW. Hence, it does not need to be elicited again in this step; it can be taken directly from the BO vector.

2.2. Behavioral foundations of the best-worst method

The choice of Best and Worst

One of the distinctive features of BWM is its reliance on two reference criteria: the *Best* and the *Worst*. This modeling choice is not merely technical, it draws upon cognitive and philosophical foundations that support the special role of extreme points in human evaluation.

In human judgment, extreme outcomes naturally serve as cognitive anchor points, a fact established across psychology, cognitive science, and even classical philosophy. Foundational cognitive models like Parducci's range–frequency theory show that people internally calibrate evaluations according to the best and worst values in a context: "the judgment of any given stimulus… [is] determined in accordance with the two endpoints of the contextual range" [26]. Likewise, psychological research demonstrates that when people summarize experiences or options, they disproportionately focus on the most extreme moments: under the *peak–end rule*, overall retrospective evaluations depend largely on an experience's *peak* (most intense positive or negative point) and its *end*, rather than the average of every moment [27]. Empirically, decision-makers often gravitate to considering the best-case versus worst-case scenarios, using these extremes as reference points because they are more salient, unambiguous, and diagnostically informative than moderate cases [28,29]. Even philosophy has long recognized the primacy of extremes in defining values: Aristotle's doctrine of the *golden mean* explicitly frames each virtue as a mean between two vices (an excess and a deficiency), i.e., as an intermediate between extremes [30]. The use of reference points not only reflects intuitive cognitive strategies, but also has foundational support in decision science methodologies. For example, techniques like SMART (Simple Multi-Attribute Rating Technique) [3] and the Swing method [4] begin by establishing anchor levels, typically the best or most desirable outcomes, against which all other attribute levels are evaluated. Together, these converging insights indicate that eliciting preferences via best and worst anchors is not only cognitively natural and behaviorally grounded, but also resonates with a deep-seated structure of human thought: leveraging our intuitive tendency to judge relative to salient extremes confers both practical ease and theoretical soundness.

The dual-anchoring mechanism

While the use of extremes as reference points facilitates clearer and more consistent judgments, it also introduces a potential vulnerability: anchoring bias. Anchoring bias occurs when decision-makers disproportionately rely on initial information (the "anchor"), resulting in systematic judgment distortions [31]. Traditional elicitation methods, which typically use a single anchor or reference point, may inadvertently exacerbate this bias, causing systematic over- or underestimation of criteria importance [32]. BWM addresses this issue through its explicit use of dual anchors, specifically, the simultaneous designation of the most important (Best) and least important (Worst) criteria. This dual-anchor approach establishes two distinct reference points that cognitively balance each other. Initially, when the decision-maker compares the Best criterion to other criteria, the Best criterion serves as a high anchor. Due to anchoring effects, this comparison naturally biases the decision-maker toward underestimating the relative importance of the other criteria. Conversely, when the same decision-maker evaluates criteria relative to the Worst criterion, this criterion acts as a low anchor, inherently biasing the decision-maker toward overestimating the relative importance of the other criteria. Crucially, these two vectors (Best-to-Others and Others-to-Worst) are not considered separately but integrated simultaneously within a single optimization framework. This deliberate structural arrangement leverages cognitive biases strategically: the systematic underestimations caused by the high anchor (Best criterion) and the systematic overestimations caused by the low anchor (Worst criterion) effectively counterbalance each other. Consequently, the final derived weights benefit from an implicit self-correction mechanism, considerably enhancing judgment consistency and reducing the net influence of anchoring distortions. The efficacy of this dual-anchor debiasing mechanism (or consider-the-opposite strategy) aligns with empirical findings from behavioral decision-making research [33–35]. These studies have demonstrated that judgments calibrated through opposing reference points tend to be more stable and accurate compared to single-anchor judgments. Several empirical studies have shown the effectiveness of this strategy within BWM [36–38].

Preference-elicitation methods for deriving criteria weights span a broad spectrum in both the number and the nature of judgments required. At one extreme are minimal-elicitation approaches rooted in MAUT/MAVT, such as SMART, SWING, and the Tradeoff procedure [1,3,4], which can determine n weights from as few as $n-1$ judgments by anchoring all assessments to a reference criterion or consequence. These approaches rely on carefully structured elicitation protocols and internal logical checks, but they do not typically provide a standalone, quantitative index for diagnosing inconsistency in the elicited judgments. At the other extreme are full pairwise-comparison approaches such as AHP, which rely on a complete pairwise comparison matrix and therefore require $n(n-1)/2$ judgments (e.g., in AHP [7]), leading to a rapidly increasing elicitation burden as n grows. The BWM occupies an intermediate position. Its structured use of dual anchoring not only introduces a debiasing mechanism but also supports a more efficient elicitation process: in contrast to full pairwise comparison matrices, BWM requires only $2n-3$ comparisons. This reduction in elicitation burden is consistent with known limits of human information processing capacity [39].

Pairwise comparison

"Every thing in this world is judg'd of by comparison". (*David Hume, A Treatise of Human Nature*). Pairwise comparison has a long-standing foundation in decision theory, psychology, and measurement science. The seminal work of Thurstone [40] introduced the law of comparative judgment, establishing the basis for deriving preference scales from pairwise evaluations. In MCDM, Saaty [7] formalized the Analytic Hierarchy Process (AHP) and introduced a discretized 1–9 ratio scale, motivated by the idea that humans can reliably discriminate only a limited number of intensity levels [41]. Importantly, the *homogeneity axiom* [8] provides a normative scope condition for this scale: elements should not differ by more than roughly an order of magnitude, which is precisely the regime in which a bounded discrete scale is intended to be meaningful. Psychophysical research further supports the use of comparative judgments as cognitively natural elicitation primitives [42,43], and methods such as MACBETH [6] build on these foundations using qualitative pairwise assessments. At the same time, discretized ratio scales remain subject to two well-known concerns: *semantic ambiguity* and *bounded discrimination* [44–47]. First, verbal intensity labels can be interpreted heterogeneously across decision-makers (and even within the same decision-maker across contexts) [48,49], which motivates careful calibration and, in group settings, semantic harmonization. Second, because the range is bounded, extremely strong judgments may be compressed, while discretization can reduce sensitivity when preferences are close [46,47]. These critiques are not independent of the homogeneity axiom [50]: when comparisons violate homogeneity (i.e., true ratios exceed the intended range), scale compression becomes likely, and the elicitation task itself becomes less reliable. Despite proposals for alternative or non-linear scales [47], empirical and comparative evidence suggests that such transformations do not yield systematic improvements over the standard 1–9 scale in typical applications [51]. Accordingly, BWM applications should report the elicitation protocol transparently, document consistency diagnostics (Section 4), and, where appropriate, complement results with robustness checks.

Reference range

Another important behavioral advantage of BWM lies in its structured sequencing of pairwise comparisons. Traditional methods such as

AHP [7] typically begin with comparisons between randomly ordered criteria, without first establishing global reference points. This can result in a retrospective inconsistency phenomenon: a decision-maker may initially judge criterion c_a to be extremely more important than c_b and assign the maximum score (e.g., 9). However, upon subsequently comparing c_a to a third criterion c_h, the decision-maker may again assign a 9 to reflect a similar degree of superiority, only to later realize that c_h is less important than c_b. This realization often forces the decision-maker to retroactively revise earlier judgments, potentially requiring further downstream adjustments, and thus triggering a cascade of recalibration that undermines consistency and increases cognitive load. BWM circumvents this issue by anchoring the comparison process on two explicitly chosen criteria, the Best and the Worst, prior to eliciting any judgments, and first conducting a pairwise comparison between Best and Worst. Once these anchors are identified, and a_{BW}, is expressed by the decision-maker, all pairwise comparisons are structured relative to them, forming two vectors: Best-to-Others and Others-to-Worst. Because the decision-maker always compares each criterion to a fixed reference, the likelihood of retrospective contradiction is significantly reduced.

min–max objective function

A core feature of the original BWM formulation is its use of a min–max optimization model to derive the weights. Specifically, the objective is to minimize the maximum absolute deviation between the weight ratios and the corresponding pairwise comparison values provided by the decision-maker. This conservative-yet-informative choice is both theoretically grounded and behaviorally justified. From a mathematical standpoint, the min–max criterion, often referred to as Chebyshev approximation, is well-established in robust decision-making, where the aim is to minimize the worst-case error or inconsistency across constraints [52–56]. In the context of preference modeling, this approach ensures that no individual pairwise judgment is violated beyond a tolerable threshold, thereby aligning with the principle of max-regret minimization [57,58]. This is particularly relevant in decision settings where consistency is paramount but perfect accuracy cannot be guaranteed. Furthermore, psychological studies show that individuals are more likely to maintain bounded rationality, providing judgments that are locally coherent but not globally consistent [59]. The min–max model respects this cognitive constraint by seeking to find weights that are maximally consistent with the most extreme inconsistencies, rather than averaging across all inconsistencies, as in least-squares or sum-of-deviation formulations. While alternative formulations (e.g., sum-of-squared deviations) may smooth inconsistencies across all pairwise comparisons, they risk masking significant local inconsistencies, an outcome that may reduce transparency or accountability in high-stakes decision-making. The min–max criterion, in contrast, provides a clear and interpretable consistency measure, enabling decision-makers to assess whether their most extreme judgments align with a coherent weight structure.

Ordered structure

An additional cognitive advantage of selecting extreme attributes in pairwise comparisons, as practiced in BWM, is its capacity to mitigate equalizing and splitting biases. Behavioral studies have shown that decision-makers often exhibit an equalizing bias, where they allocate nearly identical weights across attributes. Splitting bias, which has been studied by Borcherding and von Winterfeldt [60], Jacobi and Hobbs [61] and Weber et al. [62], and others, refers to the tendency for a branch of a decision tree to receive a disproportionately high overall weight when an attribute within it is subdivided into relatively more sub-attributes compared to other branches, particularly when weights are elicited using non-hierarchical methods. By requiring respondents to explicitly identify the most and least important attributes, and following a hierarchical structure, BWM forces deliberate differentiation between attributes, which helps counteract the tendency toward uniform weight allocations and the inflation of weights due to attribute splitting. This targeted approach enhances discrimination among attributes, improving the accuracy and reliability of elicited weights in MCDM contexts. Rezaei et al. [63] have provided empirical evidence supporting the ability of BWM to reduce equalizing and splitting bias in the elicitation of attribute weight.

3. Notable contributions

In this section, I provide an overview of the major extensions of BWM that have been developed over the past decade. These extensions reflect the method's adaptability to various decision-making contexts, including uncertainty, group settings, and hybrid frameworks, and illustrate how the core BWM logic has been generalized to address a wide range of theoretical and practical challenges.

3.1. Linear BWM

Model (2) is non-linear, and the optimal value of its objective function serves as a measure of the inconsistency in the decision-maker's judgments. Due to its non-linearity, the model may yield multiple optimal solutions. Readers could refer to Rezaei [64] and Wu et al. [24] to see how to find the optimal solutions and how to compare the weights (as they are now in a format of interval with possible overlap). The linear version of BWM typically offers a unique set of optimal weights while producing results that remain highly comparable to those of the non-linear model.

The linear BWM [64] is different from the non-linear BWM only in step 5 (optimization problem). That is, here the objective is to find the optimal weights w_j^* for the criteria such that for each pair (w_B, w_j) and (w_j, w_W), the equalities $w_B = a_{Bj} w_j, w_j = a_{jW} w_W$ hold. With the same reasoning as in most real-world decision-making scenarios, achieving these exact equalities is impractical due to subjective judgments and inherent inconsistencies, the objective is to minimize the maximum absolute deviations $\left|w_B - a_{Bj} w_j\right|, \left|w_j - a_{jW} w_W\right|, \forall j$. By incorporating the non-negativity and sum-to-one constraints on the weights, the resulting optimization problem is formulated as follows:

$$
\begin{aligned}
&\min_{\mathbf{w}} \max_{j} \quad \left\{ \left|w_B - a_{Bj} w_j\right|, \left|w_j - a_{jW} w_W\right| \right\} \\
&\text{s.t.} \quad \sum_{j=1}^{n} w_j = 1 \\
&\qquad w_j \geq 0, \quad \forall j
\end{aligned}
\tag{3}
$$

Model (3) can be transformed to a model similar to Model (2), and solved. It yields an optimal weight vector (often unique in practice), and the optimal objective value can again be used as an indicator of the consistency of the elicited comparisons; relative to the non-linear BWM, it preserves the same anchored BO and OW preference interpretation but measures misfit through additive deviations in the anchored equalities.

While the non-linear BWM model is the most cited in the BWM literature, the linear variant is arguably the most widely applied, likely due to its ease of implementation, and ability to provide a unique optimal solution.

3.2. Multiplicative BWM

The Multiplicative Best-Worst Method, proposed by Brunelli and Rezaei [65], modifies the classical BWM by incorporating a multiplicative consistency framework, ensuring ratio-scale properties in weight derivation. This model is also only different in step 5 (the optimization problem). In other words, we argue that the distance should be defined based on nature of pairwise comparison (ratio). So, for this model, the objective is to find the optimal weights w_j^* for the criteria such that for each pair (w_B, w_j) and (w_j, w_W), the equalities $a_{Bj}/(w_B/w_j) = 1, a_{jW}/(w_j/w_W) = 1$ hold. With the same reasoning as in most real-world decision-making scenarios, achieving these exact equalities is impractical due to subjective judgments and inherent inconsistencies, the objective is to minimize deviation. As deviation means that these ratios could become smaller or bigger than 1, we opt for minimizing the maximum of each ratio and its reciprocal across all criteria. This choice preserves the anchored BO and OW ratio interpretation and treats inconsistency symmetrically in multiplicative terms, thereby defining a ratio-consistent notion of closeness to the elicited judgments. By incorporating the non-negativity and sum-to-one constraints on the weights, the resulting optimization problem is formulated as follows:

$$\begin{aligned} \min_{\mathbf{w}} \max_{j} \quad & \left\{ \max\left\{ \frac{a_{Bj}}{w_B/w_j}, \frac{w_B/w_j}{a_{Bj}} \right\}, \max\left\{ \frac{a_{jW}}{w_j/w_W}, \frac{w_j/w_W}{a_{jW}} \right\} \right\} \\ \text{s.t.} \quad & \sum_{j=1}^{n} w_j = 1 \\ & w_j > 0, \quad \forall j \end{aligned} \tag{4}$$

The problem is first transformed into its logarithmic form, converting the multiplicative constraints into a linear structure in the log-space. This transformation simplifies the optimization process, allowing the problem to be solved using convex optimization techniques. By doing so, computational efficiency is significantly improved while ensuring that the multiplicative consistency of the original model is preserved.

Similar to Wu et al. [24] and Ratandhara and Kumar [66] provided an analytical approach to solve multiplicative BWM.

3.3. Nonadditive BWM

The BWM models discussed in this paper implicitly assume no interaction among criteria. In many real-world decision-making settings, this assumption does not hold: criteria can be interdependent, redundant, or complementary, which cannot be captured by standard additive weighting. To address this limitation, the nonadditive Best-Worst Method (nonadditive BWM) extends BWM by incorporating interaction effects through the Choquet integral [67]. The key motivation is to account for synergistic (positive) and redundant (negative) relationships among criteria, providing a more realistic representation of preferences.

This extension affects both the optimization model and the elicitation process. In addition to the Best-to-Others and Others-to-Worst comparisons, the decision-maker provides qualitative judgments about pairwise criteria interactions.

Criteria interaction matrix. Let $C = \{c_1, c_2, \dots, c_n\}$ be the set of criteria, and let $\Gamma = (t_{ij})$ denote the criteria interaction matrix. For $i \neq j$,

- $t_{ij} = +$ if c_i and c_j have a synergistic (positive) interaction,
- $t_{ij} = -$ if c_i and c_j exhibit redundancy (negative interaction),
- $t_{ij} = \Delta$ if there is no interaction,
- $t_{ij} = u$ if the interaction is unknown,

and Γ is symmetric, i.e., $t_{ij} = t_{ji}$.

2-Additive fuzzy measure, interaction degree, and importance. Using a 2-additive fuzzy measure, the pairwise interaction degree I_{ij} (with $I_{ij} = I_{ji}$) satisfies, for $i < j$,

$$\begin{aligned} & I_{ij} > 0, && \text{if } t_{ij} = +, \\ & I_{ij} < 0, && \text{if } t_{ij} = -, \\ & I_{ij} = 0, && \text{if } t_{ij} = \Delta, \\ & I_{ij} \in [-1, 1], && \text{if } t_{ij} = u. \end{aligned} \tag{5}$$

Let μ be a 2-additive fuzzy measure with Möbius representation $\{\mu_j, \mu_{ij}\}$. The interaction degree is set equal to the Möbius term:

$$I_{ij} = \mu_{ij}, \qquad i \neq j \tag{6}$$

The relative importance of criterion c_j is defined through the Shapley value (under the 2-additive representation):

$$I_j = \mu_j + \frac{1}{2} \sum_{c_i \in C \setminus \{c_j\}} \mu_{ij}, \qquad \forall j \tag{7}$$

Non-linear nonadditive bwm. Let c_B and c_W denote the best and worst criteria, and let a_{Bj} and a_{jW} be the Best-to-Others and Others-to-Worst comparisons, respectively. The non-linear nonadditive BWM minimizes the maximum absolute deviation between the implied ratios and the elicited comparisons:

$$\begin{aligned} \min \max_{j} \quad & \left\{ \left| \frac{I_B}{I_j} - a_{Bj} \right|, \left| \frac{I_j}{I_W} - a_{jW} \right| \right\} \\ \text{s.t.} \quad & \mu(\emptyset) = 0 \\ & \sum_{j=1}^{n} \mu_j + \sum_{1 \le i < j \le n} \mu_{ij} = 1 \\ & \mu_j + \sum_{c_i \in T} \mu_{ij} \ge 0, \quad \forall c_j \in C,\ \forall T \subseteq C \setminus \{c_j\},\ T \neq \emptyset \\ & I_{ij} = \mu_{ij}, \quad \forall i \neq j \\ & I_j = \mu_j + \frac{1}{2} \sum_{c_i \in C \setminus \{c_j\}} \mu_{ij}, \quad \forall j \\ & I_{ij} > 0 \quad \text{if } t_{ij} = +, \quad \forall i < j \\ & I_{ij} < 0 \quad \text{if } t_{ij} = -, \quad \forall i < j \\ & I_{ij} = 0, \quad \text{if } t_{ij} = \Delta, \quad \forall i < j \\ & I_{ij} \in [-1, 1], \quad \text{if } t_{ij} = u, \quad \forall i < j \end{aligned} \tag{8}$$

As in the previous BWM variants, the min–max objective in (8) can be reformulated into an equivalent constrained optimization model by explicitly bounding the maximum deviation, which can then be solved with standard optimization solvers.

Linear nonadditive bwm. Analogous to the linear BWM, a linear nonadditive variant minimizes the maximum absolute deviations in the linear equalities:

$$\begin{aligned} \min \max_{j} \quad & \left\{ \left| I_B - a_{Bj} I_j \right|, \left| I_j - a_{jW} I_W \right| \right\} \\ \text{s.t.} \quad & \mu(\emptyset) = 0 \\ & \sum_{j=1}^{n} \mu_j + \sum_{1 \le i < j \le n} \mu_{ij} = 1 \\ & \mu_j + \sum_{c_i \in T} \mu_{ij} \ge 0, \quad \forall c_j \in C,\ \forall T \subseteq C \setminus \{c_j\},\ T \neq \emptyset \\ & I_{ij} = \mu_{ij}, \quad \forall i \neq j \\ & I_j = \mu_j + \frac{1}{2} \sum_{c_i \in C \setminus \{c_j\}} \mu_{ij}, \quad \forall j \\ & I_{ij} > 0, \quad \text{if } t_{ij} = +,\ \forall i < j \\ & I_{ij} < 0, \quad \text{if } t_{ij} = -,\ \forall i < j \\ & I_{ij} = 0, \quad \text{if } t_{ij} = \Delta,\ \forall i < j \\ & I_{ij} \in [-1, 1], \quad \text{if } t_{ij} = u,\ \forall i < j \end{aligned} \tag{9}$$

Aggregation via the choquet integral. Given an alternative a_p and normalized criterion values $v(g_j(a_p))$, the 2-additive Choquet integral (under the Möbius representation) is:

$$C_\mu(a_p) = \sum_{j=1}^{n} \mu_j\, v(g_j(a_p)) + \sum_{1 \le i < j \le n} \mu_{ij} \min\{ v(g_i(a_p)), v(g_j(a_p)) \} \tag{10}$$

The weights I_j and interaction degrees I_{ij} obtained from the above models are then used in (10) to evaluate and rank alternatives.

Eliciting interactions adds interpretive complexity. That is, experts may construe *interaction* differently (e.g., complementarity, redundancy, causal influence). This parallels the broader ambiguity of *importance* in preference elicitation [45]. Hence, applications should state an operational definition of interaction and document the elicitation protocol.

Another study addresses interdependencies between criteria from a different perspective [68]. The approach begins with a standard BWM solution and then adjusts the initial weights using an influence–intensity matrix that captures the extent to which each criterion affects the others. A fully consistent adjusted weight vector is obtained by solving a constrained optimization problem that integrates both the original preferences and the interdependencies

3.4. Bayesian BWM

Bayesian BWM is designed for group decision-making under probabilistic reasoning [69]. Let $k \in \{1, \ldots, K\}$ index the decision-makers. The method estimates the aggregated (group) weights $\mathbf{w}^* = (w_1^*, \ldots, w_n^*)$ together with the individual weights $\mathbf{w}^k = (w_1^k, \ldots, w_n^k)$ by means of a Bayesian hierarchical model. The observed inputs for each decision-maker are the Best-to-Others vector $A_B^k = (a_{B1}^k, \ldots, a_{Bn}^k)$ and the Others-to-Worst vector $A_W^k = (a_{1W}^k, \ldots, a_{nW}^k)$.

Following the probabilistic interpretation of the original Bayesian BWM [69], the OW vector A_W^k is modeled by a multinomial distribution with probability vector $\mathbf{w}^k$, whereas the BO vector A_B^k is modeled by a multinomial distribution with probability vector proportional to the element-wise inverse of $\mathbf{w}^k$. To keep the multinomial parameters well-defined (i.e., summing to one), define the normalized inverse-weight vector

$$\hat{\mathbf{w}}^k = \left(\hat{w}_1^k, \ldots, \hat{w}_n^k\right), \qquad \hat{w}_j^k = \frac{1/w_j^k}{\sum_{r=1}^n 1/w_r^k}, \quad \forall j \in \{1, \ldots, n\} \tag{11}$$

Moreover, let $N_B^k = \sum_{j=1}^n a_{Bj}^k$ and $N_W^k = \sum_{j=1}^n a_{jW}^k$ denote the totals in the two comparison vectors. The Bayesian hierarchical model is then:

$$\begin{aligned} &A_B^k \mid \mathbf{w}^k \sim \text{Multinomial}\left(N_B^k, \hat{\mathbf{w}}^k\right), \quad \forall k \\ &A_W^k \mid \mathbf{w}^k \sim \text{Multinomial}\left(N_W^k, \mathbf{w}^k\right), \quad \forall k \\ &\mathbf{w}^k \mid \mathbf{w}^*, \gamma \sim \text{Dir}(\gamma\, \mathbf{w}^*), \quad \forall k \\ &\gamma \sim \text{Gamma}(a, b) \\ &\mathbf{w}^* \sim \text{Dir}(\mathbf{1}) \end{aligned} \tag{12}$$

Here, Dir($\mathbf{1}$) denotes a Dirichlet distribution with an all-ones parameter vector. As in Mohammadi and Rezaei [69], we assume $\gamma \sim \text{Gamma}(a, b)$ with hyperparameters a and b. The Dirichlet distribution $\text{Dir}(\gamma\, \mathbf{w}^*)$ has mean $\mathbf{w}^*$, while the concentration parameter γ controls the dispersion of the individual weights around the group weights (larger γ implies tighter concentration).

This model does not admit a closed-form posterior; therefore, Markov Chain Monte Carlo (MCMC) methods are typically used to sample from the posterior distribution of $\mathbf{w}^*$ and $\{\mathbf{w}^k\}_{k=1}^K$ (e.g., using probabilistic programming environments such as JAGS or Stan). By drawing S posterior samples of the aggregated weights, denoted by $\{\mathbf{w}^{*(s)}\}_{s=1}^S$, one can characterize group preferences and uncertainty.

The Bayesian BWM framework also enables a credal ranking of criteria based on posterior samples.

Credal ordering: A pairwise ordering of two criteria c_i and c_j is defined as

$$O = (c_i, c_j, Rel, d) \tag{13}$$

where $Rel \in \{<, >, =\}$ denotes the asserted relation between c_i and c_j, and $d \in [0, 1]$ is the confidence level associated with that relation.

Credal ranking: For a given set of criteria $C = \{c_1, \ldots, c_n\}$, the credal ranking is the set of credal orderings that includes all pairs (c_i, c_j) for $c_i, c_j \in C$. Using posterior samples, the degree of preference for a given pair (c_i, c_j) can be estimated as

$$P(c_i > c_j) = \frac{1}{S} \sum_{s=1}^S \mathbb{I}\left(\mathbf{w}_i^{*(s)} > \mathbf{w}_j^{*(s)}\right) \tag{14}$$

where $\mathbb{I}(\cdot)$ is the indicator function that equals 1 if its argument is true and 0 otherwise. Analogously, $P(c_j > c_i)$ is obtained by swapping i and j, and a conventional (crisp) ranking can be recovered by applying a threshold such as 0.5 to these pairwise probabilities.

There are also other studies that have extended BWM to group decision-making; we refer to Dehshiri et al. [70], Emamat et al. [71], Qin et al. [72] and Liang et al. [73].

3.5. Best-worst tradeoff

The Best-Worst Tradeoff Method (BWT) developed by Liang et al. [5] integrates BWM and the Tradeoff procedure, which is a procedure to elicit the criteria weights (scaling constants) within the Multi-Attribute Value Theory (MAVT) [1]. While BWM is widely recognized for its structured pairwise comparison approach and consistency measurement, it does not explicitly account for attribute ranges. Conversely, the Tradeoff procedure explicitly considers the range of attributes, but it lacks a consistency checking mechanism. BWT combines the strengths of both methods, ensuring both consistency validation and range sensitivity. This range sensitivity is not merely a technical refinement but addresses a well-documented issue in preference elicitation. That is, importance judgments may depend on the criterion ranges and performance levels implicitly considered by the decision-maker [74]. By grounding comparisons in explicit hypothetical consequences, BWT mitigates such range effects while retaining the structured elicitation and consistency checking of BWM.

Liang et al. [5] follows a structured approach similar to the Tradeoff procedure, incorporating preference ordering, indifference relations, and deriving the rates of substitutions. First, the decision-maker establishes an order of importance among the criteria using pairwise comparisons of hypothetical consequences, where only two criteria vary while the others remain at their worst levels. This allows for a preference ranking of the criteria, similar to the Tradeoff procedure [1]. Next, the decision-maker determines indifference relations, but unlike the traditional Tradeoff approach, BWT introduces two sets of tradeoff-based indifference comparisons:

- *Best-to-Others (BO) Indifference*: The decision-maker compares the most important (Best) criterion to each other criterion, determining how much of the Best criterion c_B is needed to make the decision-maker indifferent to receiving the best level of another criterion c_j. This results in a value $x_B^{B,j}$, representing the required amount of c_B.
- *Others-to-Worst (OW) Indifference*: The decision-maker compares each criterion c_j to the least important (Worst) criterion c_W, determining how much of c_j is needed to make the decision-maker indifferent to receiving the best level of c_W. This results in a value $x_j^{j,W}$.

Using elicited value functions (e.g., midvalue splitting technique, Keeney and Raiffa [1]) and linear interpolation, the values of $x_B^{B,j}$ and $x_j^{j,W}$ are transformed into tradeoff ratios:

$$a_{jB} = v_B(x_B^{B,j}), \quad a_{Wj} = v_j(x_j^{j,W}) \tag{15}$$

Applying the reciprocity property, we obtain:

$$a_{Bj} = \frac{1}{a_{jB}}, \quad a_{jW} = \frac{1}{a_{Wj}} \tag{16}$$

These values form the Best-to-Others (BO) vector and Others-to-Worst (OW) vector, analogous to those in the BWM. Finally, using the same definition of consistency as in BWM, an optimization problem is formulated to derive the final weights (scaling constants). The resulting model takes a form identical to Model (1) (if adopting the distance definition from non-linear BWM) or Model (3) (if adopting an alternative linear definition). This ensures that BWT maintains consistency validation while incorporating tradeoff-based weight derivation.

A probabilistic version of BWT has been developed by Mohammadi et al. [75].

3.6. Parsimonious BWM

The Parsimonious Best-Worst Method (PBWM) was introduced to address the challenge of applying BWM in decision problems with a large number of alternatives [76,77]. While BWM efficiently determines criteria weights, its direct use for evaluating many alternatives may be impractical due to the cognitive burden of eliciting pairwise comparisons. PBWM mitigates this issue by employing a *reference set* of alternatives, reducing the number of pairwise comparisons required while maintaining decision quality.

The key idea of PBWM is parsimonious elicitation: the decision-maker first assigns a rating to all alternatives, and then performs traditional BWM pairwise comparisons only for a carefully selected subset of reference alternatives. The priorities of these reference alternatives are subsequently used to infer the priorities of all alternatives via linear interpolation.

Let $\mathcal{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_m\}$ denote the set of alternatives. PBWM proceeds as follows:

1. **Alternative rating.** The decision-maker assigns a rating $r(\mathbf{x}_i)$ to each alternative $\mathbf{x}_i \in \mathcal{X}$, reflecting its relative standing on the considered evaluation dimension (or an aggregate rating used for screening).
2. **Selection of reference alternatives.** Select a subset of alternatives,

   $$\mathcal{R} = \{\mathbf{x}_{\gamma_1}, \mathbf{x}_{\gamma_2}, \dots, \mathbf{x}_{\gamma_t}\} \subseteq \mathcal{X}, \qquad t \ll m,$$

   such that the ratings $\{r(\mathbf{x}_{\gamma_s})\}_{s=1}^{t}$ cover the full rating range and are well distributed. This creates multiple reference points and reduces susceptibility to anchoring effects.
3. **Apply BWM to the reference alternatives.** The decision-maker identifies the Best and Worst alternatives within $\mathcal{R}$. Pairwise comparisons are elicited between the Best and all other reference alternatives and between all reference alternatives and the Worst. Solving a BWM model (e.g., analogous to Models (1) and (3)) yields the priorities $\{v(\mathbf{x}_{\gamma_s})\}_{s=1}^{t}$ of the reference alternatives.
4. **Interpolate priorities for all alternatives.** For any alternative $\mathbf{x}_i \in \mathcal{X} \setminus \mathcal{R}$, identify two adjacent reference alternatives $\mathbf{x}_{\gamma_s}$ and $\mathbf{x}_{\gamma_{s+1}}$ such that

   $$r(\mathbf{x}_{\gamma_s}) \le r(\mathbf{x}_i) \le r(\mathbf{x}_{\gamma_{s+1}}), \qquad s \in \{1, \dots, t-1\},$$

   and estimate its priority by linear interpolation:

   $$v(\mathbf{x}_i) = v(\mathbf{x}_{\gamma_s}) + \frac{r(\mathbf{x}_i) - r(\mathbf{x}_{\gamma_s})}{r(\mathbf{x}_{\gamma_{s+1}}) - r(\mathbf{x}_{\gamma_s})}\left(v(\mathbf{x}_{\gamma_{s+1}}) - v(\mathbf{x}_{\gamma_s})\right) \tag{17}$$

   If $r(\mathbf{x}_i) = r(\mathbf{x}_{\gamma_s})$ for some s, set $v(\mathbf{x}_i) = v(\mathbf{x}_{\gamma_s})$.

PBWM substantially reduces the cognitive load associated with pairwise comparisons in large decision problems. Instead of requiring $2m-3$ comparisons to compare all m alternatives in a classical BWM-style elicitation, PBWM requires only $2t-3$ comparisons for the reference set, where $t \ll m$. Moreover, by introducing multiple reference points (rather than relying solely on extreme comparisons with one Best and one Worst), PBWM can mitigate anchoring effects in the elicitation process.

3.7. Best-worst disaggregation method

Whereas the previously discussed versions of BWM follow an aggregation approach, the Best-Worst Disaggregation (BWD) method, developed by Brunelli et al. [78], is based on disaggregation principles similar to those used in the UTA family. The fundamental idea behind BWD is to infer a value function from holistic preference information.

Suppose we have a set of m alternatives $\mathcal{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_m\}$ evaluated with respect to n criteria. Following the BWD approach, a reference set of alternatives $\mathcal{R} \subseteq \mathcal{X}$ is identified. Within this reference set, the Best and Worst *alternatives* are determined and labeled as $\mathbf{x}_B$ and $\mathbf{x}_W$, respectively.

The preference elicitation process involves holistic comparisons between the Best alternative and all other alternatives, as well as comparisons between all alternatives and the Worst alternative. These preferences are structured into two sets:

$$A^{BO} = \left\{a_{Bi} \mid i \in \mathcal{R}\right\} \tag{18}$$

$$A^{OW} = \left\{a_{iW} \mid i \in \mathcal{R}\right\} \tag{19}$$

The Best-to-Others set A^{BO} contains values a_{Bi}, which represent the decision-maker's preference for the best alternative $\mathbf{x}_B$ over each alternative $\mathbf{x}_i$. Similarly, the Others-to-Worst set A^{OW} consists of elements a_{iW}, indicating the preference of each alternative $\mathbf{x}_i$ over the worst alternative $\mathbf{x}_W$. These preferences are typically expressed on a numerical scale (e.g., 1 to 9 or other predefined scales).

Under full rationality, the following consistency conditions hold:

$$V(\mathbf{x}_B) = a_{Bi} V(\mathbf{x}_i), \quad V(\mathbf{x}_i) = a_{iW} V(\mathbf{x}_W) \quad \forall i \tag{20}$$

where $V(\mathbf{x}_B)$, $V(\mathbf{x}_W)$, and $V(\mathbf{x}_i)$ represent the value function of the Best, Worst and ith alternative respectively. This implies that the ideal conditions should satisfy:

$$\begin{cases} V_L(\mathbf{x}_B) - a_{Bi} V_L(\mathbf{x}_i) = 0 \\ V_L(\mathbf{x}_i) - a_{iW} V_L(\mathbf{x}_W) = 0 \end{cases} \quad \forall i \tag{21}$$

where $V_L(\mathbf{x}_B)$, $V_L(\mathbf{x}_W)$, and $V_L(\mathbf{x}_i)$ represent the piecewise linear value function of the Best, Worst and ith alternative respectively.

To operationalize the preference model, BWD assumes an additive structure for the overall value function. That is, for any alternative $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{in})$, the overall value is computed as:

$$V_L(\mathbf{x}_i) = \sum_{j=1}^{n} v_j^L(x_{ij}) \tag{22}$$

where $v_j^L(x_{ij})$ denotes the marginal value function for criterion j evaluated at performance level x_{ij}. This additive formulation allows BWD to decompose overall preferences into interpretable, criterion-wise components.

Since achieving perfect consistency is often unrealistic in practical decision-making, the objective is to minimize the maximum deviation from this ideal condition. This leads to the following linear optimization model:

$$\begin{aligned} \min \max_i \quad & \left\{\left|V_L(\mathbf{x}_B) - a_{Bi} V_L(\mathbf{x}_i)\right|, \left|V_L(\mathbf{x}_i) - a_{iW} V_L(\mathbf{x}_W)\right|\right\} \\ \text{s.t.} \quad & v_j^L(x_j^k) \le v_j^L(x_j^{k+1}), \qquad \forall j, k \\ & \sum_{j=1}^{n} v_j(\overline{x}_j) = 1 \\ & v_j(\underline{x}_j) = 0, \qquad \forall j \end{aligned} \tag{23}$$

Solving (23) provides piecewise linear approximations of the criterion-specific value functions, while the optimal value ξ^* serves as an indicator of compatibility between the decision-maker's preferences and the inferred value function V_L. If the obtained results are deemed reliable, the derived value functions can be used to evaluate and rank all alternatives within the given set $\{\mathbf{x}_1, \dots, \mathbf{x}_m\}$, ensuring an effective preference modeling process within a disaggregation-based decision-support framework.

3.8. BWM-FlowSort

While BWM is primarily used for weight elicitation and (full) ranking, many applications require *sorting* (classification) of alternatives into a set of predefined and ordered classes. Let $\mathcal{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_m\}$ be the set of alternatives, $C = \{c_1, \dots, c_n\}$ the criteria, and $\mathscr{C} = \{C_1, \dots, C_H\}$ the ordered classes (best to worst). Classes are delimited by $H+1$

limiting profiles $\mathcal{R} = \{r_1, \dots, r_{H+1}\}$, where each $r_h = (c_1(r_h), \dots, c_n(r_h))$ and, for each (benefit-type) criterion c_j,

$$c_j(r_1) \ge c_j(r_2) \ge \cdots \ge c_j(r_{H+1}) \tag{24}$$

We assume that alternatives lie within the extreme profiles for each criterion, i.e., $c_j(r_{H+1}) \le c_j(\mathbf{x}_i) \le c_j(r_1)$ for all i and j.

The BWM-FlowSort method integrates BWM-based value elicitation with FlowSort-style use of limiting profiles to support scalable multi-criteria sorting [79].

Step 1: Criteria weights. Elicit criteria weights $w = (w_1, \dots, w_n)$ using a standard BWM model (as reviewed earlier), with $\sum_{j=1}^{n} w_j = 1$.

Step 2: Priorities of limiting profiles. For each criterion c_j, the decision-maker provides the BO and OW comparison vectors among the $H+1$ limiting profiles (with r_1 as best and r_{H+1} as worst), requiring $2H-1$ judgments. Let a^j_{Bh} denote the preference of r_1 over r_h under c_j, and a^j_{hW} the preference of r_h over r_{H+1}. The priorities $v(c_j(r_h))$ are obtained by the BWM min–max model:

$$\begin{aligned}
\min \quad & \xi_j \\
\text{s.t.} \quad & \left| \frac{v(c_j(r_1))}{v(c_j(r_h))} - a^j_{Bh} \right| \le \xi_j, \qquad h = 1, \dots, H+1 \\
& \left| \frac{v(c_j(r_h))}{v(c_j(r_{H+1}))} - a^j_{hW} \right| \le \xi_j, \qquad h = 1, \dots, H+1 \\
& \sum_{h=1}^{H+1} v(c_j(r_h)) = 1 \\
& v(c_j(r_h)) \ge 0, \qquad h = 1, \dots, H+1
\end{aligned} \tag{25}$$

If multiple optima occur, lower/upper bounds $v^{\min}(c_j(r_h)), v^{\max}(c_j(r_h))$ can be computed by solving two auxiliary programs at the optimal ξ_j^* (interval representation), and the midpoint

$$v_{\text{mid}}(c_j(r_h)) = \tfrac{1}{2}\left(v^{\min}(c_j(r_h)) + v^{\max}(c_j(r_h))\right)$$

can be used as a central representation.

Step 3: Criterion-specific value functions. *Central (midpoint) case.* Normalize the central profile priorities to $[0,1]$:

$$v^N_{\text{mid}}\big(c_j(r_h)\big) = \frac{v_{\text{mid}}\big(c_j(r_h)\big) - v_{\text{mid}}\big(c_j(r_{H+1})\big)}{v_{\text{mid}}\big(c_j(r_1)\big) - v_{\text{mid}}\big(c_j(r_{H+1})\big)},$$
$$v^N_{\text{mid}}\big(c_j(r_{H+1})\big) = 0,\ v^N_{\text{mid}}\big(c_j(r_1)\big) = 1 \tag{26}$$

For an alternative $\mathbf{x}_i$, if $c_j(r_h) \ge c_j(\mathbf{x}_i) \ge c_j(r_{h+1})$, infer its normalized value by piecewise-linear interpolation:

$$v^N_{\text{mid}}\big(c_j(\mathbf{x}_i)\big) = v^N_{\text{mid}}\big(c_j(r_{h+1})\big) + \frac{c_j(\mathbf{x}_i) - c_j(r_{h+1})}{c_j(r_h) - c_j(r_{h+1})} \times \Big(v^N_{\text{mid}}\big(c_j(r_h)\big) - v^N_{\text{mid}}\big(c_j(r_{h+1})\big)\Big) \tag{27}$$

Interval case. First normalize the bounds separately:

$$v^{N,\min}\big(c_j(r_h)\big) = \frac{v^{\min}\big(c_j(r_h)\big) - v^{\min}\big(c_j(r_{H+1})\big)}{v^{\min}\big(c_j(r_1)\big) - v^{\min}\big(c_j(r_{H+1})\big)} \tag{28}$$

$$v^{N,\max}\big(c_j(r_h)\big) = \frac{v^{\max}\big(c_j(r_h)\big) - v^{\max}\big(c_j(r_{H+1})\big)}{v^{\max}\big(c_j(r_1)\big) - v^{\max}\big(c_j(r_{H+1})\big)} \tag{29}$$

Because these two normalizations may lead to ill-formed intervals, enforce valid normalized bounds by

$$v^{L,N}\big(c_j(r_h)\big) = \min\big\{v^{N,\min}\big(c_j(r_h)\big),\ v^{N,\max}\big(c_j(r_h)\big)\big\} \tag{30}$$

$$v^{U,N}\big(c_j(r_h)\big) = \max\big\{v^{N,\min}\big(c_j(r_h)\big),\ v^{N,\max}\big(c_j(r_h)\big)\big\} \tag{31}$$

Then, if $c_j(r_h) \ge c_j(\mathbf{x}_i) \ge c_j(r_{h+1})$, define normalized interval values of $\mathbf{x}_i$ by:

$$v^{\min}\big(c_j(\mathbf{x}_i)\big) = v^{L,N}\big(c_j(r_{h+1})\big) + \frac{c_j(\mathbf{x}_i) - c_j(r_{h+1})}{c_j(r_h) - c_j(r_{h+1})} \times \Big(v^{L,N}\big(c_j(r_h)\big) - v^{L,N}\big(c_j(r_{h+1})\big)\Big) \tag{32}$$

$$v^{\max}\big(c_j(\mathbf{x}_i)\big) = v^{U,N}\big(c_j(r_{h+1})\big) + \frac{c_j(\mathbf{x}_i) - c_j(r_{h+1})}{c_j(r_h) - c_j(r_{h+1})} \times \Big(v^{U,N}\big(c_j(r_h)\big) - v^{U,N}\big(c_j(r_{h+1})\big)\Big) \tag{33}$$

(Thus, in the interval case, $v^{\min}(\cdot), v^{\max}(\cdot)$ are understood as normalized to $[0,1]$.)

Step 4: Outranking degrees. Let $\Upsilon = \mathcal{X} \cup \mathcal{R} = \{v_1, \dots, v_{m+H+1}\}$ be the set consisting of all alternatives and limiting profiles. Compute outranking degrees between r_1 and each $v_s \in \Upsilon$, and between each v_s and r_{H+1}. Define the (central) difference on the normalized value scale as:

$$d_j(r_1, v_s) = v^N_{\text{mid}}\big(c_j(r_1)\big) - v^N_{\text{mid}}\big(c_j(v_s)\big) \tag{34}$$

$$d_j(v_s, r_{H+1}) = v^N_{\text{mid}}\big(c_j(v_s)\big) - v^N_{\text{mid}}\big(c_j(r_{H+1})\big) \tag{35}$$

(Under the standing assumption $c_j(r_{H+1}) \le c_j(\mathbf{x}_i) \le c_j(r_1)$, these differences are nonnegative.)

To obtain BWM-compatible preference intensities in $[1,9]$, use the modified V-shape function (with threshold $p_j > 0$):

$$P_j(r_1, v_s) = \begin{cases} 1, & d_j(r_1, v_s) \le 0, \\ 1 + 8\,\dfrac{d_j(r_1, v_s)}{p_j}, & 0 < d_j(r_1, v_s) \le p_j, \\ 9, & d_j(r_1, v_s) > p_j, \end{cases} \qquad P_j(v_s, r_{H+1}) \text{ analogously.} \tag{36}$$

Then aggregate:

$$\pi(r_1, v_s) = \sum_{j=1}^{n} w_j\, P_j(r_1, v_s), \qquad \pi(v_s, r_{H+1}) = \sum_{j=1}^{n} w_j\, P_j(v_s, r_{H+1}) \tag{37}$$

In the interval case, define (one possible) interval-based difference as:

$$d_j(r_1, v_s) = \frac{v^{\max}(c_j(r_1)) - v^{\min}(c_j(v_s))}{\big(v^{\max}(c_j(r_1)) - v^{\min}(c_j(r_1))\big) + \big(v^{\max}(c_j(v_s)) - v^{\min}(c_j(v_s))\big)} \tag{38}$$

and then apply (36)–(37) using this $d_j(\cdot,\cdot)$.

Step 5: Classification indices. Let $IC(v_s)$ denote the (unknown) classification index of element $v_s \in \Upsilon$. Using $\pi(r_1, v_s)$ and $\pi(v_s, r_{H+1})$ as BWM-like intensity data, infer $IC(\cdot)$ by:

$$\begin{aligned}
\min \quad & \xi \\
\text{s.t.} \quad & \left| \frac{IC(r_1)}{IC(v_s)} - \pi(r_1, v_s) \right| \le \xi, \qquad s = 1, \dots, m+H+1 \\
& \left| \frac{IC(v_s)}{IC(r_{H+1})} - \pi(v_s, r_{H+1}) \right| \le \xi, \qquad s = 1, \dots, m+H+1 \\
& \sum_{s=1}^{m+H+1} IC(v_s) = 1 \\
& IC(v_s) \ge 0, \qquad s = 1, \dots, m+H+1
\end{aligned} \tag{39}$$

If multiple optimal solutions exist, compute bounds $IC^{\min}(v_s), IC^{\max}(v_s)$ by minimizing/maximizing $IC(v_s)$ over the optimal set (interval indices), and use midpoints

$$IC^{\text{mid}}(v_s) = \tfrac{1}{2}\left(IC^{\min}(v_s) + IC^{\max}(v_s)\right)$$

as central indices.

Step 6: Assignment rules. *Central indices.* With the convention that class C_h is delimited by profiles (r_h, r_{h+1}), assign alternative $\mathbf{x}_i$ by comparing its index to adjacent limiting profiles:

$$\text{if} \quad IC^{\text{mid}}(r_{h+1}) \le IC^{\text{mid}}(\mathbf{x}_i) \le IC^{\text{mid}}(r_h) \quad \text{then} \quad \mathbf{x}_i \in C_h, \qquad h = 1, \dots, H \tag{40}$$

Interval indices and belief degrees. Let $I(\mathbf{x}_i) = [IC^{\min}(\mathbf{x}_i), IC^{\max}(\mathbf{x}_i)]$ and define class intervals

$$I_h^{\min} = [IC^{\min}(r_{h+1}), IC^{\min}(r_h)], \qquad I_h^{\max} = [IC^{\max}(r_{h+1}), IC^{\max}(r_h)]$$

Compute belief degrees as normalized overlap lengths:

$$\beta_h^{\min}(\mathbf{x}_i) = \frac{\text{Len}\left(I(\mathbf{x}_i) \cap I_h^{\min}\right)}{\text{Len}\left(I(\mathbf{x}_i)\right)}, \qquad \beta_h^{\max}(\mathbf{x}_i) = \frac{\text{Len}\left(I(\mathbf{x}_i) \cap I_h^{\max}\right)}{\text{Len}\left(I(\mathbf{x}_i)\right)} \tag{41}$$

$$\beta_h(\mathbf{x}_i) = \frac{\beta_h^{\min}(\mathbf{x}_i) + \beta_h^{\max}(\mathbf{x}_i)}{2}, \qquad \Omega(\mathbf{x}_i) = \{(C_1, \beta_1(\mathbf{x}_i)), \dots, (C_H, \beta_H(\mathbf{x}_i))\} \tag{42}$$

where $\text{Len}([a, b]) = b - a$.

3.9. Fuzzy BWM

A line of research extends BWM to settings where preference judgments are assumed to be imprecise or linguistically expressed, and therefore are modeled using fuzzy sets. In this stream, the decision-maker's judgments are not translated to crisp numerical intensities, but rather linguistic assessments (e.g., "equally important", "moderately more important", etc.) are represented by fuzzy numbers. Fuzzy BWM aims to propagate this imprecision through the weight elicitation model and then (typically) return a crisp weight vector via defuzzification.

One of the earliest fuzzy extensions is due to Guo and Zhao [80]. In that approach, the BO and OW comparisons are modeled by triangular fuzzy numbers (TFNs). In contrast to the classical BWM that uses the 9-point scale, Guo and Zhao employ a five-level linguistic scale in which each linguistic term is associated with a TFN. Denoting fuzzy (BO, OW) comparisons by $\tilde{a}_{Bj}$ and $\tilde{a}_{jW}$, one writes

$$\tilde{a}_{Bj} = (l_{Bj}, m_{Bj}, u_{Bj}), \qquad \tilde{a}_{jW} = (l_{jW}, m_{jW}, u_{jW}) \tag{43}$$

and the (unknown) fuzzy weights by

$$\tilde{w}_B = (l_B^w, m_B^w, u_B^w), \qquad \tilde{w}_j = (l_j^w, m_j^w, u_j^w), \qquad \tilde{w}_W = (l_W^w, m_W^w, u_W^w) \tag{44}$$

The optimization model retains the structure of the non-linear BWM, but it is reformulated to accommodate fuzzy arithmetic (e.g., fuzzy ratios and fuzzy deviations). The solution yields fuzzy weights $\tilde{w}_j$, which are subsequently defuzzified to obtain crisp weights for downstream decision analysis.

Beyond this early formulation, many fuzzy variants of BWM have been proposed, differing in (i) the linguistic scale and the associated fuzzy membership functions (e.g., triangular and trapezoidal), (ii) the way deviations and constraints are defined under fuzzy arithmetic, and (iii) the defuzzification rule used to obtain final crisp weights; see, for example, Dong et al. [81], Wan et al. [82], Hafezalkotob and Hafezalkotob [83], Wan and Dong [84], Dong and Wan [85] and Ratandhara and Kumar [86].

Fuzzy BWM is often proposed as a way to represent imprecise judgments, but two properties of the original BWM should be kept in mind when motivating fuzzification. First, the integers $1, \dots, 9$ are already codings of linguistic intensity statements (Table 1); in this sense, the inputs are not "precise" measurements but discretized ratio-scale judgments intended to approximate relations such as $w_B/w_j \approx a_{Bj}$ and $w_j/w_W \approx a_{jW}$. Second, the BWM weight-derivation models are formulated with a min–max inconsistency criterion, which explicitly admits deviations from these ratio relations and typically yields implied ratios that fall between adjacent integers. Consequently, the added value of fuzzification depends on (i) whether the membership functions attached to $\tilde{a}_{Bj}$ and $\tilde{a}_{jW}$ are elicited with a clear semantic protocol (e.g., whether they represent a range of plausible ratios, degree of confidence, or another interpretation) and (ii) whether the subsequent decision model can meaningfully propagate fuzzy weights rather than immediately defuzzifying them. Absent such justification, fuzzy modeling may add parameters and computational complexity without proportionate gains in interpretability, transparency, or decision quality.

3.10. Belief-based BWM

The belief-based BWM extends the original BWM by incorporating uncertainty in the form of belief structures based on Dempster–Shafer Theory. It enables decision-makers to express their preferences not as fixed numbers but as belief distributions over possible preference grades, thus capturing both *discord* and *nonspecificity*.

Let $\Theta = \{\vartheta_1, \vartheta_2, \dots, \vartheta_L\}$ be the frame of discernment with linguistic or numeric grades (e.g., $L = 9$ for grades 1–9). A belief structure is defined by assigning basic belief masses $b(\Theta_r)$ to focal elements $\Theta_r \subseteq \Theta$, such that:

$$\sum_{\Theta_r \subseteq \Theta} b(\Theta_r) = 1, \quad b(\emptyset) = 0 \tag{45}$$

The pignistic probability δ_ℓ for each grade $\vartheta_\ell \in \Theta$ is then calculated as:

$$\delta_\ell = \sum_{\Theta_r : \vartheta_\ell \in \Theta_r} \frac{b(\Theta_r)}{|\Theta_r|} \tag{46}$$

For each BO comparison (B, j) and OW comparison (j, W), let $\delta_{\ell,Bj}$ and $\delta_{\ell,jW}$ denote the corresponding pignistic probabilities of grade ϑ_ℓ. Let $\vartheta_{\ell,Bj}$ and $\vartheta_{\ell,jW}$ denote the grade values associated with these BO and OW belief structures. The belief-based BWM model then determines the weights by minimizing the maximum belief-weighted deviation:

$$\begin{aligned} \min \quad & \xi \\ \text{s.t.} \quad & \left| \frac{w_B}{w_j} - \vartheta_{\ell,Bj} \right| \cdot \delta_{\ell,Bj} \le \xi, \qquad \forall j, \ell \\ & \left| \frac{w_j}{w_W} - \vartheta_{\ell,jW} \right| \cdot \delta_{\ell,jW} \le \xi, \qquad \forall j, \ell \\ & \sum_{j=1}^{n} w_j = 1, \quad w_j \ge 0, \qquad \forall j \end{aligned} \tag{47}$$

To assess the reliability of a decision-maker, two measures are introduced: (i) an inconsistency ratio IR and (ii) an uncertainty measure AU from evidence theory. The uncertainty measure is normalized as

$$\widetilde{AU} = \frac{AU}{\log_2 L} \in [0, 1] \tag{48}$$

Here AU denotes the aggregated uncertainty of a belief structure (capturing both discord and nonspecificity) computed from the basic belief masses; we follow the definition used by Liang et al. [87]. The reliability degree (RD) is then computed as:

$$RD = 1 - \sqrt{\frac{IR^2 + \widetilde{AU}^2}{2}} \tag{49}$$

In group settings, individual weights w^g are aggregated by reliability-weighted averaging:

$$w_j^{grp} = \sum_{g=1}^{G} \lambda^g w_j^g \qquad \text{where} \quad \lambda^g = \frac{RD^g}{\sum_{g=1}^{G} RD^g} \tag{50}$$

This model allows integration of uncertain and inconsistent assessments from multiple decision-makers. There are a few other studies that have incorporated belief structures into BWM; see, for example, Fei et al. [88] and Zhang et al. [89].

Table 2
Consistency Index (CI) table [9].

a_{BW}	1	2	3	4	5	6	7	8	9
Consistency index (max ξ)	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

4. Consistency checking

Consistency checking is a crucial aspect of BWM to ensure that the decision-maker's pairwise comparisons are logically coherent. Inconsistencies in preference elicitation can lead to unreliable weight estimations, which may compromise decision-making quality. Two primary approaches to measuring consistency in BWM are the output-based consistency ratio [9] and the input-based consistency ratio [87], each evaluating consistency from a different perspective.

Output-based consistency ratio evaluates how much the optimal solution obtained from the BWM deviates from a perfectly consistent pairwise comparison system. It is defined based on the optimal objective function value of the non-linear BWM (Model (2)):

$$CR^O = \frac{\xi^*}{CI} \tag{51}$$

where ξ^* represents the optimal objective value of Model (2), and CI or $\xi_{\max}$ denotes the maximum possible inconsistency, which depends on the value of a_{BW} (see Table 2). The range of CR^O is $[0,1]$, with values closer to zero indicating a higher level of consistency. To assess whether the inconsistency is acceptable, predefined threshold values are applied (see Table 3).

Input-based consistency ratio, in contrast to the output-based CR, provides immediate feedback during the elicitation process because it is computed directly from the elicited BO and OW judgments, without solving any optimization model. Following Liang et al. [87], the global input-based consistency ratio is defined as the maximum normalized deviation between the direct best-worst comparison a_{BW} and the implied indirect comparison $a_{Bj}a_{jW}$:

$$CR^I = \max_j CR^I_j, \qquad CR^I_j = \begin{cases} \dfrac{\left|a_{Bj}a_{jW} - a_{BW}\right|}{a_{BW}(a_{BW}-1)}, & a_{BW} > 1 \\ 0, & a_{BW} = 1 \end{cases} \tag{52}$$

where CR^I_j is the *local* input-based consistency ratio associated with criterion j. The statistic CR^I lies in $[0,1]$ and can be interpreted as the maximum normalized discrepancy between a_{BW} and its indirect estimate $a_{Bj}a_{jW}$ [87]. Beyond offering immediate feedback, this formulation is easy to interpret, identifies the most problematic judgments through $j' \in \arg\max_j CR^I_j$, and is model-free in the sense that it depends only on the input comparisons and therefore applies irrespective of whether weights are computed by a non-linear or linear BWM model [87]. Empirical evidence further shows that CR^I has a very high correlation with the output-based ratio across a wide range of problem sizes and scales [87].

The local indices CR^I_j provide actionable guidance for improving consistency. In particular, the maximizer $j' \in \arg\max_j CR^I_j$ pinpoints the criterion for which the implied product $a_{Bj'}a_{j'W}$ deviates most from a_{BW}; revising $a_{Bj'}$ and/or $a_{j'W}$ is therefore a natural first step when CR^I exceeds the relevant threshold (see Table 4).

The input-based ratio CR^I satisfies several desirable axiomatic properties. In particular, Liang et al. [87, Proposition 1] show that: (i) $CR^I = 0$ if and only if the preferences are cardinal-consistent; (ii) CR^I is invariant under permutation (relabeling) of criteria; (iii) CR^I is normalized, i.e., $0 \le CR^I \le 1$; (iv) starting from a fully consistent system, moving any single judgment a_{Bj} or a_{jW} away from its consistent value (within the admissible range) increases CR^I; (v) for $a_{BW} > 1$, CR^I is continuous in the inputs; and (vi) removing a criterion that is neither Best nor Worst cannot increase CR^I. The formal proofs are provided in Liang et al. [87].

Table 3
Output-based CR thresholds [87].

	Number of criteria						
Scales	3	4	5	6	7	8	9
3	0.2087	0.2087	0.2087	0.2087	0.2087	0.2087	0.2087
4	0.1581	0.2352	0.2738	0.2928	0.3102	0.3154	0.3273
5	0.2111	0.2848	0.3019	0.3309	0.3479	0.3611	0.3741
6	0.2164	0.2922	0.3565	0.3924	0.4061	0.4168	0.4225
7	0.2090	0.3313	0.3734	0.3931	0.4035	0.4108	0.4298
8	0.2267	0.3409	0.4029	0.4230	0.4379	0.4543	0.4599
9	0.2122	0.3653	0.4055	0.4225	0.4445	0.4587	0.4747

Table 4
Input-based CR thresholds [87].

	Number of criteria						
Scales	3	4	5	6	7	8	9
3	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667
4	0.1121	0.1529	0.1898	0.2206	0.2527	0.2577	0.2683
5	0.1354	0.1994	0.2306	0.2546	0.2716	0.2844	0.2960
6	0.1330	0.1990	0.2643	0.3044	0.3144	0.3221	0.3262
7	0.1294	0.2457	0.2819	0.3029	0.3144	0.3251	0.3403
8	0.1309	0.2521	0.2958	0.3154	0.3408	0.3620	0.3657
9	0.1359	0.2681	0.3062	0.3337	0.3517	0.3620	0.3662

For transparency and reproducibility, applied studies using BWM should report: (i) the number of criteria n and the comparison scale; (ii) the identified Best and Worst criteria and the value of a_{BW}; (iii) CR^O and CR^I; (iv) the relevant threshold values (as a function of n and the scale) and whether they are satisfied; and (v) the largest local inconsistency $CR^I_{j'}$ (and the associated criterion) when CR^I exceeds the threshold. This information enables readers to assess the reliability of the elicitation and the robustness of the resulting weights. We note that some studies apply a fixed threshold of 0.1 irrespective of n and the comparison scale, effectively importing the AHP rule-of-thumb; such a practice is not appropriate for BWM, because acceptable consistency thresholds in BWM depend on the elicitation setting (including n and the scale) and the corresponding reference values.

Table 5 outlines the most prominent BWM variants developed to date, summarizing their key features, inputs and outputs, consistency measures, and appropriate use cases. This table provides a valuable reference point for researchers and practitioners seeking to select or adapt the most appropriate BWM variant for a given decision context. Importantly, the variants in Table 5 do not all pursue the same modeling goal: some retain the original weight-derivation purpose (differing mainly in the discrepancy model or constraints), whereas others repurpose the best-worst elicitation logic for distinct preference-theoretic settings or decision tasks (e.g., trade-off consistent value/utility modeling in BWT or sorting variants).

As the range of extensions summarized in Table 5 illustrates, BWM has been adapted to accommodate different modeling requirements; accordingly, no single BWM variant is universally appropriate. The standard BWM elicits relative *importance weights* and is most straightforward to use in decision models that employ weighted aggregation with at least some degree of compensation (e.g., weighted scoring or distance-to-ideal methods), where weights function as importance coefficients. When criteria exhibit substantive *interactions* (complementarity or redundancy), an interaction extension (e.g., nonadditive BWM) is required, since additive weights alone may misrepresent joint effects. When weights are intended to represent explicit *trade-offs* in an additive value or utility model, where scaling depends on attribute ranges and performance levels, BWT provides a more appropriate MAVT/MAUT-grounded elicitation basis. By contrast, in strongly *non-compensatory* settings (vetoes, strict requirements, lexicographic priorities) or when preferences are *non-monotonic* (ideal-point) or dominated

Table 5
Comparison of major extensions of the Best-Worst Method (BWM).

BWM variant	Formulation and focus	Input and Output	Consistency measurement	Recommended use
Non-linear BWM	Non-linear optimization to minimize the maximum deviation of weight ratios from pairwise comparisons	Inputs: BO and OW pairwise comparisons; Outputs: Typically unique (for $n \leq 3$, and for $n > 3$ & fully consistent; multiple optimal weights (if $n > 3$ and inconsistent)	Output-based CR; Input-based CR	When interpretability is important and no significant interdependencies among criteria exist
Linear BWM	Linear approximation of the non-linear BWM	Inputs: BO and OW vectors; Outputs: Typically unique weight vector	Input-based CR	When uniqueness and simplicity of optimization are desired
Multiplicative BWM	Log-transformed formulation using multiplicative ratios	Inputs: BO and OW vectors; Outputs: Unique set of weights	Input-based CR	When ratio-scale consistency is emphasized
Nonadditive BWM	Choquet integral model with interaction terms for criteria	Inputs: BO, OW, and interaction matrix Γ; Outputs: Non-additive capacity weights (typically unique for linear model and multiple for non-linear if $n > 3$ and inconsistent)	Input-based CR	When interactions among criteria (synergy/redundancy) are relevant/significant
Bayesian BWM	Probabilistic framework for group aggregation via hierarchical modeling	Inputs: BO and OW vectors from multiple decision-makers; Outputs: Posterior distribution of weights and credal rankings	Input-based CR	When aggregating group judgments under uncertainty
Best-Worst Tradeoff (BWT)	Indifference-based tradeoff model using value functions	Inputs: Attribute value functions, tradeoff comparisons w.r.t. Best and Worst; Outputs: Typically, unique (linear) or multiple (for non-linear if $n > 3$ and inconsistent) weights (scaling constants)	Input-based CR	When weights reflect tradeoff
Parsimonious BWM	Reduced elicitation based on reference alternatives	Inputs: Ratings for all alternatives, and BO and OW comparisons on a small reference set; Outputs: Interpolated priorities for full set	Input-based CR	When evaluating many alternatives with limited decision-maker effort
Best-Worst Disaggregation (BWD)	Inverse modeling from holistic preferences using disaggregation logic	Inputs: BO and OW comparisons among alternatives; Outputs: Value functions and inferred weights per criterion	Input-based CR	When inferring preferences from choice data and rankings
BWM-FlowSort	Sorting/classification	Inputs: BO and OW comparisons (for criteria weights) and class limiting profiles (reference profiles); Outputs: Assignment of alternatives to ordered classes	Input-based CR	When the decision goal is to sort many alternatives into predefined classes rather than produce a full ranking
Fuzzy BWM	Extends classical BWM using fuzzy arithmetic	Inputs: Linguistic BO and OW judgments mapped to TFNs or other forms of fuzzy numbers; Outputs: Fuzzy weights defuzzified into crisp scores	Fuzzy consistency index or adapted input-based CR	Handling linguistic pairwise comparisons through fuzzy numbers
Belief-based BWM	Incorporates Dempster–Shafer theory to model epistemic uncertainty	Inputs: Belief structures over BO and OW comparisons; Outputs: Belief distributions or expected weights	Belief-based consistency index or plausibility measures	When uncertainty stems from incomplete evidence or imprecise beliefs

by non-linear value functions, BWM-type weights should be complemented with an appropriate preference model (e.g., outranking or explicit value-function modeling) rather than used as a stand-alone basis for ranking.

5. Application domains

Since its introduction in 2015, the BWM has been adopted in a wide range of MCDM problems in business, economics, energy, engineering, finance, management, medicine, social sciences, and technology. This broad uptake reflects the method's applicability to problems that require criteria weighting and alternative evaluation for selection, ranking, or sorting under conflicting objectives. The aim here is not to provide a comprehensive review of all published applications, for which several dedicated reviews are available (see, e.g., [21–23]), but rather to highlight a set of representative domains that illustrate the breadth and diversity of BWM's use.

To contextualize this overview, a structured bibliographic search was conducted in Scopus on 19 December 2025 using the keywords "best-worst" and "BWM". Records referring to methods other than the BWM (e.g., "best-worst scaling" [90]) were excluded. In addition, papers that merely cited BWM without applying it as a decision-making method were removed. After title- and abstract-level screening to eliminate non-relevant studies, a total of 3157 publications were retained for analysis (see Fig. 1). Approximately 88% of these records are classified as "article", with the remainder consisting of "conference paper", "book chapter", and "review". China, Iran, India, Turkey, the United States, and the United Kingdom account for the largest shares of

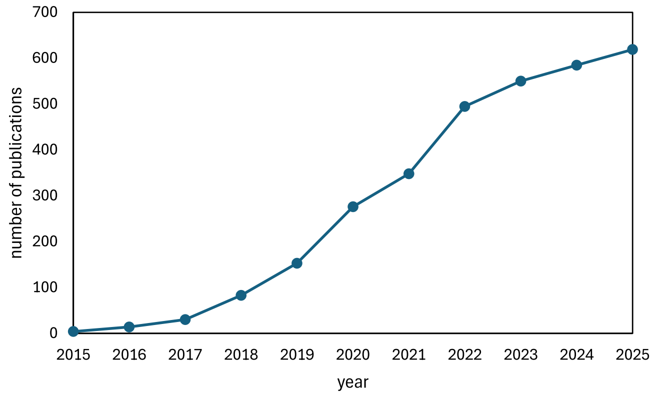

Fig. 1. Number of papers per year.

the resulting publication set. The three most cited articles within this set are Rezaei [9] with 3537 citations, Rezaei [64] with 1488 citations, and Guo and Zhao [80] with 801 citations.

Against this background, the following subsections review representative application domains in which BWM has been employed, illustrating how the method has been adapted to support decision-making in diverse problem settings.

Supply chain management and logistics. BWM has been extensively applied in supply chain management and logistics, where complex trade-offs arise between cost, risk, sustainability, technological innovation and social considerations. Reported applications include the assessment of social sustainability, analysis of technology-implementation risks, identification of procurement and circular-economy enablers, design of Logistics 4.0 capabilities, and development of inclusive workforce strategies. For instance, in Ahmadi et al. [91], BWM is used to prioritize social sustainability indicators (such as health and safety, equity and stakeholder influence) and to show that social criteria, often neglected in practice, can be decisive for sustainable supply-chain design. In Kusi-Sarpong et al. [92], BWM is employed to rank technological, organizational and environmental risks linked to big data analytics, highlighting the most critical risk factors that managers must address when adopting analytics-based solutions. Kannan [93] uses BWM to weight drivers of sustainable procurement across an extended multi-tier network, identifying top-management commitment, regulatory pressure and buyer requirements as the key levers for implementation. In Lanzini et al. [94], BWM is applied to prioritize technological, organizational and regulatory factors that influence blockchain adoption and circular supply-chain initiatives, thus providing structured guidance for SMEs and larger firms considering blockchain investments. At the level of logistics services and circular operations, Jamkhaneh et al. [95] and Tseng et al. [96] use BWM to rank service-quality dimensions and circular-economy enablers, respectively, showing which technological and organizational capabilities are most critical for sustainable, data-driven logistics and circular supply chains. On the human side, Agarwal et al. [97] applies BWM to prioritize enablers of gender-inclusive logistics workplaces, identifying inclusive policies, flexible work arrangements and skill-development initiatives as the most important factors. Finally, Modares et al. [98] shows how Bayesian BWM can be embedded in a blockchain-enabled supplier selection system to derive consensus weights for supplier–performance criteria and support transparent, data-driven supplier decisions. Taken together, these studies demonstrate that BWM has become a foundational tool in SCM and logistics, supporting decisions across strategic, technological, operational and social dimensions of modern supply-chain systems.

Sustainability, environmental management, and energy. BWM has also been widely applied in environmental management and energy-system analysis, where decisions must balance environmental impacts, technical feasibility, economic performance, and social acceptability. Reported applications span renewable-energy site selection, formulation of national renewable-energy strategies, assessment of grid-integration projects, and technology-dominance studies in emerging energy carriers. In Wang et al. [99], BWM is used to weight technical, environmental, and socio-economic criteria and to prioritize efficient wave energy sites along the Chilean coastline; the authors show that locations combining high wave power with limited socio-environmental conflicts emerge as very high-priority sites for investment. Pan and Hashemizadeh [100] proposes a circular-economy and life-cycle-based assessment of renewable energy options, using BWM to elicit expert weights for economic, environmental, and circularity indicators before applying a compromise-ranking procedure, and finds that life-cycle environmental performance and circular resource-use criteria dominate conventional cost measures in the final rankings. In Jiang et al. [101], BWM is used to derive the relative importance of several social, technical, economic, and policy challenges to sustainable biomass deployment and, combined with DEMATEL, identifies fear of public health and safety hazards, lack of professional training, infrastructural gaps, and insufficient investor interest as the most critical barriers. Zhang et al. [102] applies BWM to evaluate the economic, social, and environmental benefits of power-grid investment projects for renewable integration, showing that internal rate of return, unit investment income, and carbon-emissions reduction are the most influential sub-criteria and that some projects deliver markedly superior sustainability value. In the corporate environmental domain, Rodríguez-Gutiérrez et al. [103] use BWM to support sustainability-reporting decisions in SMEs, structuring several cost- and benefit-related sub-criteria into six categories and deriving criteria weights that highlight reputation, legitimacy and synergistic value creation as particularly influential, with the Global Reporting Initiative standard emerging as the preferred reporting option for most firms. At the level of urban environmental services, Moreno-Solaz et al. [104] apply BWM to the sustainable selection of engine technologies for municipal solid waste collection trucks, weighting several economic, environmental and social criteria under multiple feasible future scenarios and showing that, once regulatory and cost uncertainties are taken into account, electric trucks (followed by advanced diesel and CNG options) form the most robust choice, whereas hydrogen-powered trucks remain the least attractive alternative under current expectations. Finally, De Graaf et al. [105] applies BWM to elicit expert judgments on the factors driving dominant designs in maritime hydrogen fuel-cell technologies and to weight criteria such as fuel-cell cost, safety, emissions, and efficiency, concluding that cost is the most important factor and that solid oxide fuel cells currently have a slight edge over proton-exchange membrane fuel cells in their likelihood of becoming the dominant maritime solution. Together, these studies illustrate how BWM can structure complex environmental and energy decisions, from strategic planning and barrier analysis to project appraisal and technology selection, often in combination with other MCDM tools.

Food and agriculture. BWM has also been widely applied in food and agriculture, where agronomic, economic, environmental and social criteria must be balanced across farms, value chains and regional systems. Reported applications cover land evaluation, crop-system sustainability, sector-level indicator prioritization and opportunity identification for sustainable agri-food business models. For instance, Everest et al. [106] use BWM for land–suitability evaluation for paddy cultivation in northwest Turkey, eliciting weights for several soil and land characteristics (e.g., texture, drainage, depth and fertility indices) and showing that the resulting suitability classes closely match those obtained by AHP while differing markedly from the traditional Storie Index, thereby demonstrating BWM's practicality and reduced comparison burden. Alijani et al. [107] integrate eco-efficiency analysis with BWM to compare the sustainability of paddy cultivation systems, using BWM to prioritize energy and exergy efficiency, benefit–cost ratio and an environmental index. At a broader sectoral level, Streimikis et al. [108] apply a Delphi–BWM procedure to assess the economic sustainability of agriculture in the Baltic states, reducing an initial large set of indicators to nine and then ranking them with BWM, with investment intensity, income diversification, agricultural labor productivity and market access emerging as the most critical levers for long-term sectoral viability. Along agri-food value chains, Salimi [109] apply BWM within a business-model-for-sustainability framework for the Dutch dairy sector, deriving weights for technological, social and organizational dimensions and showing that social aspects are both highly important and underperforming, thus offering the strongest opportunities for sustainable entrepreneurship. Taken together, these studies demonstrate BWM's versatility in supporting decisions from plot-level land evaluation to sectoral sustainability assessment and supply-chain design in agriculture and food systems.

Healthcare and medicine. BWM has been applied in health-related decision-making, where multiple clinical, organizational and societal criteria must be balanced. Reported applications include the evaluation of healthcare system performance, hospital service quality, diagnosis and treatment selection, healthcare supply-chain management, and occupational health and safety programs. For instance, Abadi et al. [110] develop a SWOT-based strategic planning framework for medical tourism and then use BWM to weight and rank development strategies, identifying the creation of medical tourism marketing centers, the establishment of a strategic council for medical tourism, and the enactment of consistent laws to attract medical tourists as the most critical levers for policy action. Ahmadinejad et al. [111] propose a "Clean Hospital" strategy during COVID-19 and apply BWM, after a Delphi-based identification of several infection-prevention criteria, to prioritize personal protection, screening checklists, and body-temperature checks as the key factors to which scarce financial and human resources should be allocated first. At a more systemic level, Goyat and Singh [112] use BWM to rank barriers to Industry 4.0 adoption in Indian healthcare, showing that poor technological infrastructure, high development and maintenance costs, and low top-management support are the most critical obstacles, whereas data transformation and acquisition is comparatively less important, thus providing a clear agenda for policy-makers and hospital managers. Goldani and Ishizaka [113] used BWM to support the selection of healthcare waste treatment technologies, deriving weights for cost, environmental, technical and operational criteria in a group setting and showing that BWM can handle complex multi-stakeholder healthcare decisions in a transparent and structured way. These individual contributions sit within a broader pattern synthesized by Gulum Tas [22], who reviews sixty BWM applications in health, groups them into six categories (system evaluation, hospital performance and service quality, diagnosis and treatment, healthcare supply chains, support systems, and occupational health and safety), and concludes that BWM is a particularly favorable method for health-related decision-making due to its practicality, integration capability, and reliable results.

Other application areas. Beyond the major domains discussed above, BWM has also been adopted in a variety of more specialized contexts, illustrating its flexibility in structuring complex qualitative judgments. In education, Castro et al. [114] employ BWM and TOPSIS to priorities design factors for educational technologies for children with intellectual disabilities and to rank alternative assistive solutions; their analysis shows that user perception and engagement receive the highest weights, and that adaptive educational platforms outperform sensor-based AI systems and assistive robots for supporting children with autism and related conditions. In the maintenance domain, Ghaleb and Taghipour [115] integrate BWM with correlation-based analysis in a sustainability dashboard that quantifies the contribution of different maintenance practices to an asset's overall sustainability score; BWM-derived weights for environmental, social, and economic indicators allow managers to compute an aggregate sustainability index and to identify which maintenance actions offer the largest sustainability gains. In finance, Abdel-Basset et al. [116] develop a hybrid framework including BWM for sustainable supply chain finance in the gas industry, using BWM to elicit expert weights for technological, operational, environmental, and financial attributes and showing that financial attributes and product/service management are the most critical levers for strengthening sustainable supply chain finance. In entrepreneurship studies, Salimi and Vrauwdeunt [117] apply non-additive BWM to the triple-bottom-line dimensions and sub-criteria relevant for the survival of alternative-protein ventures, eliciting expert weights that reveal the relative importance of environmental impact, consumer health and nutrition, employment, and profitability for long-term survival. Finally, in transport and infrastructure-related decision-making, Tusher et al. [118] use the Bayesian BWM to rank cyber-security risks for autonomous ships, finding that navigational systems and their subsystems are perceived as substantially more vulnerable than propulsion systems or port operations, while van de Kaa et al. [119] use BWM to weigh firm-level success factors in the standards battle among biomass thermochemical conversion technologies and show that biomass gasification currently has the highest chance of achieving standard dominance. Taken together with the sectoral applications reviewed earlier, these examples confirm that BWM has evolved into a versatile, general-purpose weighting and prioritization tool.

In many studies, BWM has been solely used as a weight elicitation method (e.g., Mondal et al. [120], Sharma et al. [121] and Salimi [109]), and in a few studies as a sole method for weight elicitation as well as driving the priority of the alternatives (e.g., Kalpoe [122]). In majority of studies, BWM has been applied with other methods. For these application, BWM has been often used as a method to elicit weights for other methods. While there are many combinations, here I refer to a few examples: with TOPSIS [123–125], with VIKOR [126–128], with ELECTRE [129–131], with PROMETHEE [132–134], with DEA (data envelopment analysis) [135–137], with DEMATEL [138–140].

6. Future directions

Despite the rapid proliferation of BWM applications, several conceptually rich and practically relevant questions remain open. Below I articulate a few research avenues that follow directly from the strengths and gaps identified in the present review.

Group-level dual anchoring

BWM's *dual-anchor* structure, Best-to-Others (BO) and Others-to-Worst (OW), is typically elicited from the same decision-maker. In group contexts, however, it is possible to gather BO judgments from one subset of stakeholders and OW judgments from another. This *split-anchor* protocol may preserve BWM's bias-mitigating property (counter-anchoring) at the collective level while reducing individual workload. Systematic laboratory and field experiments are required to test whether (i) aggregate consistency improves, and (ii) disagreement patterns differ from those produced by conventional aggregation rules.

Hierarchical modeling

Most empirical studies employ BWM only to derive criterion weights, even though the original method was conceived to support a full hierarchical evaluation of both criteria and alternatives. A promising research direction is therefore to operationalize BWM across an entire multi-level structure, first eliciting weights for top-level criteria, then repeating the procedure for each cluster of sub-criteria, and finally comparing alternatives within every terminal node. Propagating the resulting local weights upward yields global priorities that remain behaviorally coherent, because every node follows the same $2n-3$ comparison protocol and uniform consistency checks. Future work can formalize aggregation rules that transmit consistency ratios throughout the hierarchy, design elicitation sequences that minimize the total number of BO and OW vectors for large alternative sets, and empirically test, via simulations and laboratory experiments, whether a fully homogeneous BWM hierarchy delivers more stable and transparent rankings than mixed-method frameworks. In this way, BWM can fulfill its original promise as an end-to-end decision tool, preserving cognitive economy and traceability at every level of abstraction.

Human-AI co-production and large-scale data

The rapid advancement of machine learning (ML) and artificial intelligence (AI) presents compelling opportunities to extend the scope and usability of the BWM. At the same time, a central methodological point is that BWM weights are intended to represent *problem-specific* value judgments; therefore, AI should primarily be viewed as a *co-production* partner that supports structuring, elicitation, and quality control, rather than as a substitute for value elicitation. Below, three focused research directions are particularly promising at the intersection of BWM and ML and AI:

AI-assisted adaptive elicitation and consistency-driven interaction: A natural integration is to embed BWM queries in an interactive decision-support loop where the system adaptively selects the next comparison to minimize uncertainty while controlling cognitive load. This aligns with preference-elicitation research that emphasizes real-time interaction, robustness to noise, and scalable query selection based on (Bayesian) belief updates [141]. In a BWM setting, the anchored structure can be exploited to (i) propose the most informative BO or OW query, (ii) provide immediate, localized feedback using input-based consistency diagnostics (Section 4), and (iii) recommend targeted revisions by identifying the judgments that contribute most to inconsistency. This human–AI loop would operationalize *consistency checking* as an interactive coaching mechanism, rather than a purely ex-post statistic.

Criteria management at scale and cognitive support: In problems involving a large number of criteria, unsupervised ML methods (e.g., clustering or manifold learning) can pre-process and organize the criteria space. A more AI-native implementation is to combine representation learning (e.g., document embeddings) with interactive clustering to (i) deduplicate near-synonymous criteria, (ii) propose hierarchical groupings, and (iii) surface potentially missing criteria from large document collections (policies, technical standards, incident reports). Importantly, such tools should preserve traceability by presenting human-auditable rationales (e.g., representative keywords/documents) for each cluster, so that the decision-maker retains ownership of the final criteria set. This can substantially reduce elicitation fatigue while improving problem structuring, which is often a dominant source of error in applied MCDM.

Data-driven priors, bias detection, and hybrid alignment: ML techniques can in principle infer regularities from historical choices, behavioral traces, or organizational data and use them to propose candidate weights or informative priors [141–143]. However, the interpretation of such outputs as *weights for a new decision problem* is delicate: weights often depend on the criteria ranges and performance levels under consideration [74,144], and transferring weights across settings with different scales, ranges, or objectives may yield numbers with limited meaning for the decision at hand ([145], see Mistake 7). A defensible strategy is therefore to treat algorithmically derived weights as *inputs to be audited and calibrated*, rather than as final answers. In particular, BWM can serve as a transparent bias-detection layer by checking whether the ratios implied by the algorithmic weights are compatible with the decision-maker's anchored BO and OW judgments, and by pinpointing which criteria generate the largest disagreements. This makes potential domain shift, scale/range mismatch, or systematic bias *measurable and localizable* in terms that are meaningful to the decision-maker. Building on this diagnostic step, a concrete hybrid inference scheme is to compute final weights by balancing fidelity to the elicited BWM judgments with proximity to the algorithmic reference (e.g., through a regularized BWM formulation), and to iteratively align the two sources: the system highlights the largest conflicts, the decision-maker revises or confirms only the corresponding judgments, and the weights are re-estimated until consistency indices and disagreement indicators fall below agreed thresholds. This approach turns hybrid optimization into an explicit, auditable workflow that improves transparency and trustworthiness.

From choices to preferences: Incorporating beliefs

Disaggregation-based extensions of BWM (e.g., BWD) reverse the usual flow of information: instead of eliciting pairwise comparisons and producing a ranking, they infer criteria weights from an *ex ante* ranking of alternatives. Yet, observed choice alone may be insufficient to recover underlying preferences because those choices are filtered through the decision-maker's beliefs about the state of the world [146]. Ignoring such beliefs risks confounding *value* with *expectation*. A fruitful line of inquiry is therefore to augment BWD with an explicit belief-elicitation layer, using, for instance, probabilistic assessments of scenario likelihoods or Bayesian priors jointly estimated with weights. Methodological work could formulate a two-stage optimization in which (i) beliefs over uncertain attribute realizations are captured via a parsimonious scoring rule, and (ii) weights are inferred conditional on those beliefs. Experimental studies could then test whether incorporating belief structures improves out-of-sample predictive accuracy and reduces the dispersion of recovered weights across respondents faced with identical rankings. Such integration would move BWD beyond mere choice fitting toward a more complete behavioral representation of decision-making under uncertainty.

Intrinsic robustness vs. fuzzy reformulations

A growing body of work proposes fuzzy, interval, or gray BWM variants to capture vagueness in judgments. Yet the original BWM already tolerates imprecision through its optimization-based consistency minimization: small deviations from crisp ratios are endogenously absorbed. Comparative simulations and analytical proofs are needed to determine when fuzzy generalizations truly add explanatory or predictive power, and when the classic consistency ratio suffices. Such studies would clarify trade-offs between model parsimony, computational effort, and behavioral realism across the broader family of preference-elicitation methods (e.g., AHP vs. fuzzy AHP).

Leveraging the dual-anchor logic beyond BWM

A fruitful avenue for future work is to export BWM's *dual-anchor* principle, eliciting judgments against both an explicitly defined best and worst reference point, into other weighting and preference-learning procedures. In our experimental study on attribute-specific value function elicitation within MAVT, we examined the effects of anchoring by comparing single-anchor (low or high) and dual-anchor conditions. The results showed that value functions elicited under the dual-anchor condition, where respondents were exposed to both a low and a high anchor before providing midpoint judgments, were significantly less susceptible to distortion [147]. The same idea that we have used in Tradeoff procedure (BWT) can enhance swing weighting (SMART/SWING) by collecting two assessments per attribute, the value of the swing from the worst level to the midpoint and the value of the swing from the midpoint to the best level, thus tempering single-anchor bias [31,148]. MACBETH can be extended analogously by requesting each verbal difference in both upward and downward directions, tightening the internal consistency checks that underpin the approach [6]. Additive-value disaggregation models such as UTA and Robust Ordinal Regression could fix utilities for two extreme *calibration* alternatives before optimization, shrinking the feasible weight polytope and improving predictive robustness [149]. A closely related direction is to export the dual-anchor idea to non-compensatory ordinal elicitation, for example the Deck-of-Cards method [13,150]. In the standard protocol, the decision-maker orders criteria from least to most important and inserts blank cards to express strength of differences along this single direction. A natural dual-anchor extension is to elicit the spacing twice, once in the forward direction (worst to best) and once in the reverse direction (best to worst), and then compare the implied spacings. Discrepancies between the two passes can be used as a debiasing trigger and as a consistency diagnostic, which is currently largely absent from Deck-of-Cards style elicitation. Finally, discrete-choice and conjoint designs might embed extreme benchmark profiles at the start and end of a survey to stabilize part-worth estimates against context effects [151]. Systematic experimental and field testing of these dual-anchor variants could reveal whether the best–worst calibration can serve as a general debiasing layer across the broader decision-science toolkit.

7. Conclusions

Over the past decade, the Best-Worst Method (BWM) has evolved from a novel weight elicitation technique into a framework that has been widely applied and has stimulated a substantial methodological literature for multi-criteria decision-making (MCDM). Its appeal is often attributed to its anchored elicitation structure, relatively low elicitation burden, and explicit consistency diagnostics. This review has synthesized the foundational principles of BWM, its behavioral justifications, and the substantial methodological innovations that followed, including its extensions into Bayesian, fuzzy, belief-based, disaggregation, and trade-off-based variants. Rather than merely cataloging applications, this paper has provided a conceptual overview of BWM's evolution. I have highlighted how its structured comparison mechanism can support consistency assessment and, in some settings, may help mitigate anchoring-related effects by balancing high and low reference points. I also examined the method's integration into broader decision-making systems, and its adaptability to various real-world domains such as sustainability, healthcare, supply chain, and risk assessment.

The continued interest in BWM is not solely a function of domain proliferation; it also reflects the practical usefulness of a structured elicitation protocol that can be implemented with relatively limited judgmental input while retaining consistency assessment. This review not only consolidates the state-of-the-art, but also identifies promising avenues for future research. These include deeper exploration of BWM's behavioral mechanisms, formal integration into complete decision pipelines, experimental validations, and synergies with artificial intelligence to enhance scalability and personalization.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Declaration of competing interest

The author has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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