

Satellite-Based Observation of North Sea Wave Dynamics

Capturing Infragravity Waves and Spatial Sea State Estimates with the Surface Water and Ocean Topography (SWOT) Mission

Master Thesis

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Satellite-Based Observation of North Sea Wave Dynamics

Capturing Infragravity Waves and Spatial Sea State Estimates with the Surface Water and Ocean Topography (SWOT) Mission

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Preface

This thesis marks the completion of my Master's in Hydraulic Engineering and the final milestone of my Civil Engineering studies at TU Delft. Having grown up on Curaçao and later lived close to the coast in the Netherlands, I have seen firsthand how vulnerable coastal communities can be and how closely daily life is tied to water.

Those experiences sparked my interest in hydraulic engineering. That interest deepened during high school, when a project on climate change confronted me with the scale of its impacts, particularly for low-lying and coastal regions. Combined with the reality of sea level rise, it made the urgency of coastal safety tangible. Since then, I have been drawn to engineering work that turns this urgency into practical, robust solutions.

During my Master's, I increasingly gravitated toward coastal engineering, a field where complexity and societal relevance come together. My interest in coastal protection and wave modelling developed further during my second-year international project in Yucatán, Mexico. In a multidisciplinary team, we designed a floating offshore barrier to mitigate coastal erosion associated with the mechanical removal of Sargassum. My contribution focused on modelling the local hydrodynamics and wave conditions along the Yucatán coast to support decisions on the barrier's placement and configuration.

Building on that experience, this thesis gave me the opportunity to explore in what extent remote sensing can support the validation of wave model outputs. Learning how satellite observations can be used to extract additional insight into coastal and oceanographic processes has been both valuable and inspiring.

I am deeply grateful to everyone who supported me throughout this journey. Jose, thank you for your constructive feedback and our open conversations. Ad, your expertise and our discussions on infra-gravity waves truly strengthened this thesis. Paco, thank you for taking the time to guide me through the complexities of SWOT processing and for reaching out to mission experts when questions arose. I also sincerely thank my supervisors at Deltares, Marieke and Joana, for their enthusiasm and encouragement throughout the research process. Your genuine passion for the subject made the project both enjoyable and motivating, and I truly appreciate it.

To my family and friends, thank you for your wholehearted support. A special thanks to Jeroen and Sjoerd, my two graduation companions, with whom I spent two unforgettable weeks in Zeeland. Having a professional workspace with the coast within walking distance was a unique experience and made those weeks remarkably productive. Finally, to my housemates, thank you for making this thesis journey enjoyable and for supporting me along the way.

*Stephan van Eps
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Abstract

The increasing frequency of extreme weather events intensifies the demand for accurate monitoring of nearshore wave dynamics, as these dynamics directly affect shoreline stability, flood risks, and coastal operations. While operational wave models provide essential forecasts, they often underestimate significant wave height (SWH) during energetic conditions and omit infragravity (IG) waves. The latter can substantially amplify coastal impacts such as flooding, dune erosion, and harbor seiching. These limitations are further exacerbated by the limited spatial coverage of in-situ buoys, making model validation in coastal waters particularly challenging.

The launch of the Surface Water and Ocean Topography (SWOT) mission presents a promising yet underutilized opportunity to improve coastal monitoring and support the validation of hydrodynamic models. Using Ka-band Radar Interferometry (KaRIn), SWOT provides high-resolution, two-dimensional measurements of sea surface height (SSH), offering spatial detail that exceeds the capabilities of buoys and traditional altimeters. While initial validations in the open ocean have confirmed SWOT's capacity to retrieve SWH and resolve long-period waves, its performance in shallow, morphologically complex coastal seas remains largely untested.

This research presents a novel approach comprising (i) a multi-pixel match-up strategy for SWH retrieval, and (ii) the first satellite-based spectral analysis for detecting IG wave energy in the Southern North Sea. Through three targeted case studies, the study investigates SWOT's two-dimensional SSH and SWH patterns, retrieval accuracy, and sensitivity to key processing parameters across varying sea states and bathymetric regimes. Validation is performed using in-situ buoy observations, platform-mounted radar measurements and numerical wave models to assess consistency, spatial performance, and retrieval accuracy.

By applying both single- and multi-pixel match-up strategies, our findings revealed the trade-off between statistical variance reduction through spatial averaging and the preservation of local wave variability. Similarly, for spectrally derived IG wave energy, different analysis box sizes were evaluated against platform-mounted radar observations in the Southern North Sea. Results show that SWOT-derived SWH exhibits strong agreement with buoy data (bias < 7 cm) and outperforms operational wave models during energetic events. IG wave height estimates also demonstrate good correspondence (MAE $\approx$ 0.9 cm), provided that residual noise amplification is carefully managed. This finding underscores the need for improved noise modelling in future spectral retrieval frameworks. A key uncertainty identified is the role of spatial heterogeneity, which induces representation errors due to the inherent mismatch in spatial and temporal scales between SWOT observations, in-situ buoys, and wave models.

Despite these uncertainties and other limitations, the results confirm SWOT's capacity to observe nearshore wave dynamics with high spatial detail. The proposed configurations and filtering strategies offer a transferable framework for future applications. These insights support SWOT's integration into coastal wave model validation, boundary condition improvement, and data assimilation schemes across coastal scales. Ultimately, this thesis advances high-resolution coastal wave monitoring by demonstrating how SWOT's two-dimensional observations can enhance our understanding of spatial sea state variability, particularly during energetic conditions in morphologically complex environments.

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Nomenclature

Abbreviations

Abbreviation	Definition
AZC	Azimuthal cutoff
BIG	Bound infragravity
CNES	Centre National d'Etudes Spatiales
CTD	Cross-track distance
DTS	Distance to shore
ECMWF	European Centre for Medium-Range Weather Forecasts
EDF	Equivalent degrees of freedom
FFT	Fast Fourier transform
FIG	Free infragravity
FWHM	Full width at half maximum
IG	Infragravity
KaRIn	Ka-band Radar Interferometry
LR	Low Resolution
MAE	Mean absolute error
MATROOS	Multifunctional Access Tool foR Operational Oceandata Services
NASA	National Aeronautics and Space Administration
NCRS	Normalized Radar Cross-Section
NVAL	Number of valid match-ups
OBP	On-Board Processor
PSD	Power spectral density
PTR	Point target response
R^2	Coefficient of determination
RB	Range bunching
RMSE	Root mean square error
SAR	Synthetic Aperture Radar
SI	Scatter index
SLC	Single-look complex
SNR	Signal-to-noise ratio
SSH	Sea surface height
SWH	Significant wave height
SWAN	Simulating WAVes Nearshore
SWOT	Surface Water and Ocean Topography
TC	Triple collocation
WGF-IG	Wave group-forced infragravity
WMCN	Dutch Water Management Centre

Symbols

Symbol	Definition	Unit
A	Amplitude	[m]
B	SWOT baseline between antennas	[m]
c	Phase speed	[m/s]
c_g	Group velocity	[m/s]
$E(f)$	Energy density spectrum (frequency space)	[m ² /Hz]
$E(k_x, k_y)$	2D energy density spectrum (wavenumber space)	[m ⁴ /cycles ²]

Symbol	Definition	Unit
f	Frequency	[Hz]
g	Gravitational acceleration	[m/s ²]
H	Satellite altitude	[m]
H_{E10}	Low-frequency SWH (0.03 Hz - 0.10 Hz)	[m]
h	Water depth	[m]
H_{IG}	IG wave height	[m]
H_{m0}	Significant wave height	[m]
H_s	Significant wave height	[m]
k	Wavenumber	[rad/m]
k_N	Nyquist wavenumber	[cycles/m]
k_x	Cross-track wavenumber	[cycles/m]
k_y	Along-track wavenumber	[cycles/m]
k_z	Geometric coefficient	[1/m]
L	Wavelength	[m]
L_{box}	Spectral box length	[m]
L_{tile}	Spectral tile length	[m]
λ	Radar wavelength	[m]
λ_c	Azimuth cutoff wavelength	[m]
r	Range of antenna	[m]
σ_0	Radar backscatter	[dB]
σ_h	Standard deviation of the surface elevation	[m]
σ_u	Root mean squared orbital velocity	[m/s]
T	Wave period	[s]
T_{m02}	Mean wave period	[s]
U_{10}	Wind velocity at 10 m above the surface	[m/s]
η	Sea surface elevation	[m]
γ	Coherence	[-]
γ_{vol}	Volumetric decorrelation	[-]
$\Delta\varphi$	Interferometric phase	[rad]
θ	Mean wave direction	[°]
θ_{look}	Look angle	[°]
ω	Angular frequency	[rad/s]

Introduction

Research context

Coastal regions face increasing challenges due to climate change, sea level rise, and the growing frequency of extreme weather events. With increased attention to flood resilience and coastal management, accurately understanding wave dynamics near the coast is more critical than ever. Many coastal systems and applications are sensitive to low-frequency waves (long-period swell and infragravity IG), which drives processes such as wave run-up, overtopping, dune erosion, and affects vessel operability [1, 2]. For the Netherlands, where large areas lie below mean sea level, reliable wave forecasts are essential for effective flood warnings and the safe operation of marine activities [3].

Traditionally, in-situ sensors such as wave buoys provide detailed time series at single locations, but offer limited spatial coverage. Numerical models, by contrast, simulate wave conditions over broader domains. However, these results are difficult to validate given the sparse buoy network and often misrepresent low-frequency wave dynamics in shallow coastal waters. Operational models used for the Dutch coast systematically underestimate low-frequency swell, and during major storms, both significant wave height (SWH) and low-frequency energy parameters (e.g., H_{E10}) are underpredicted [4, 5, 1]. Recent studies further indicate that free infragravity (FIG) waves, which are very low-frequency components that are not bound to local sea-swell groups, can dominate nearshore IG energy during storm conditions. This has implications for flooding, dune erosion, and harbour seicheing [6, 7]. Yet current wave forecasts do not incorporate IG waves, introducing a systematic underprediction risk for coastal impacts. Moreover, IG waves are not accounted for in the instrumentation used to assess the safety levels of Dutch flood defences, adding further uncertainty to flood-probability estimates [8].

Together, these inaccuracies compromise the quality of wave forecasts and coastal safety assessments. This underlines the need for improved observational techniques and validation strategies that can resolve SWH and IG wave energy over a broad spatial domain in relatively shallow waters. This is particularly important for the unstructured SWAN (Simulating WAVes Nearshore) model used to predict FIG dynamics in the North Sea, which is strongly dependent on calibration data [6].

Research problem

Remote sensing techniques, particularly satellite-based altimetry, offer the potential to extract wave properties over large spatial scales. Since the launch of the Seasat mission in 1978, satellite-based oceanographic observation has advanced significantly, enabling continuous monitoring of the sea surface along satellite ground tracks [9]. However, traditional satellite altimetry (including Synthetic Aperture Radar (SAR) modes) faces limitations in coastal zones. The quality of altimeter-derived datasets degrades notably near the coastline due to land contamination of the radar footprint and the increased complexity of nearshore physical processes, which reduce effective measurement retrieval [10].

The Surface Water Ocean Topography (SWOT) mission presents a novel opportunity through its use of wide-swath radar interferometry. Unlike conventional altimeters, SWOT provides spatially continuous two-dimensional observations of sea surface height (SSH) at high resolution [9]. Recent studies have demonstrated that SWOT's Low Resolution (LR) product can accurately resolve long-period swell waves (wavelength ≈ 500 -1,050 m; wave heights $\gtrsim 0.03$ m) in open-ocean settings [11]. In another study Bohé et al. (2025) have shown through interferometric coherence that SWOT can retrieve SWH with centimeter-level relative biases versus GPS buoys, Sentinel-3 SAR altimetry, and the SWOT nadir altimeter [12]. Extending to coastal conditions, Turki et al. (2025) reported good SWOT-buoy SWH agreement (RMSE ≈ 0.88 m, $R^2 \approx 0.95$) up to ~ 4 km from the coast using single-pixel LR data over the English Channel. Performance degraded only in very shallow, topographically complex zones [13].

Despite this promise, key limitations remain. The spatial sampling (~ 250 m) and effective resolution (~ 500 m) of the LR product impose a lower bound on resolvable wavelength ($L \gtrsim 500$ m), constraining spectral analyses to long-period swell. Moreover, current processing relies on an azimuthal cutoff parameterization that remains unvalidated for interferometric SAR systems [14]. For coherence-based SWH retrieval, challenges persist at far cross-track ranges where increasing distance amplifies SWH biases, particularly at low wave heights [12]. Crucially, applications in shallow, morphologically complex coastal environments (e.g., the Dutch continental shelf) remain largely unexplored. In these regions, variable bathymetry, coastal backscatter, and wave transformation processes may introduce additional sources of error [9].

To date, no study has systematically assessed SWOT-derived SWH in the North Sea while testing sensitivity to satellite–buoy match-up radii and cross-track position. Likewise, the feasibility of retrieving IG wave parameters in shallow to intermediate depths has not yet been evaluated. This thesis addresses these gaps by (i) evaluating SWOT-derived SWH using varying match-up radii, and (ii) extending the open-ocean spectral method of Arduin et al. (2024) to test IG-band retrievals (0.033–0.005 Hz) under North Sea conditions. The core challenge lies in the transformation of interferometric observations into meaningful wave parameters (e.g. SWH, mean wave period, propagation direction), while quantifying sensitivity to processing choices and nearshore heterogeneity.

Research objectives

The objective of this research is to assess the performance and potential of SWOT for validating North Sea wave models by evaluating SWH retrievals across offshore-to-nearshore validation sites and testing the feasibility of infragravity wave detection through spectral analysis.

Research scope

This thesis investigates the applicability of SWOT satellite data for characterising wave conditions along the Dutch coast, using a structured top-down approach. The analysis comprises three case studies: (I) visual inspection of spatial SSH and SWH fields, (II) validation and sensitivity analysis of SWOT-derived SWH, and (III) spectral analysis focused on IG wave detection. The analysis window spans early November to late December 2024, covering both calm and energetic conditions. Within this period, four Storm Surge Flashes were issued by the Dutch Water Management Centre [15, 16, 17, 18]. These events provide the basis for the selection of calm and energetic evaluations. In-situ validation is conducted using buoy locations over the Dutch continental shelf, spanning a range of water depths and coastal complexities.

The analysis encompasses both methodologies introduced by Arduin et al. (2024) and Bohé et al. (2025), which are applied selectively across the case studies. Coherence-based SWH is derived from SWOT's Expert product (2×2 km grid) and validated against buoy observations and SWAN model output at offshore-to-nearshore validation sites. For spectral analysis, the effective resolution of SWOT LR Unsmoothed product (~ 500) limits resolvable wavelengths to $L \gtrsim 500$ m. Consequently, validation of IG waves is restricted to water depths that support such wavelengths, based on the lower bound of the IG frequency range (0.033 Hz) and the linear dispersion relation. An important methodological extension to the spectral analysis is the use of an IG-specific mask, rather than the swell mask originally defined by Arduin et al. (2024). This study focuses on three key processing steps that influence spectral estimates: (i) the computation of the power spectral density (PSD), (ii) the application of transfer functions, and (iii) the formulation of a spectral mask that bounds the region from which wave parameters are derived.

Research questions

The research is guided by the following main question:

Research question

To what extent can SWOT satellite observations be used to validate numerical wave model outputs for significant wave height and infragravity wave characteristics in the North Sea under both calm and storm conditions?

To address this main question, the following sub-questions are formulated:

1. Which wave patterns can be identified in SWOT SSH, SWH and IG wave height fields during energetic conditions across the North Sea and how do these patterns relate to wind, bathymetry and coastline influences?
2. How does SWOT-derived SWH compare to in-situ buoy data and numerical wave models across the Dutch continental shelf, and how sensitive are these estimates to match-up radius, cross-track distance, sea state, and bathymetric setting?
3. To what extent can SWOT retrieve infragravity wave height (H_{IG}) in shallow to intermediate North Sea conditions, and how sensitive are these estimates to choices made in the three key spectral processing steps?
4. What configuration of processing parameters yields the most reliable SWOT-based wave estimates in the North Sea, and how do these recommendations differ from open ocean applications?

Research outline

This thesis is structured around three main components, each building upon the previous to assess SWOT's capability for characterising wave conditions in the North Sea. Chapter 2 presents the theoretical foundation. It introduces key physical parameters influencing wave conditions in the North Sea and the underlying measurement principles of the SWOT mission. This chapter develops a high-level framework of factors and processing steps that affect the performance of SWOT's LR product, providing the conceptual basis for the subsequent analysis.

Chapter 3 outlines the methodological approach, which follows a top-down strategy, progressing from visual inspection to spectral analysis. The chapter also details the sensitivity analysis performed for SWH retrieval, the adaptations made to spectral processing for IG wave energy detection, and the validation strategy involving both in-situ measurements and numerical wave model output.

Chapter 4 presents the results of the three case studies, highlighting the accuracy and consistency of SWOT estimates during calm and energetic conditions. It includes visual analysis of spatial SSH and SWH fields, validation of SWH from the Expert product against buoys/wave models across different coastal regimes, and spectral derived IG wave characteristics. Sensitivity results and configuration testing directly inform the fourth sub-question, which synthesises SWOT's potential for validating wave models in the North Sea. A schematic overview linking the four sub-questions to their respective deliverables is provided in Figure 1.1.

Chapters 5-7 provide the discussion, outline the main conclusions, and offer recommendations for future research and operational applications.

By addressing the research questions, this thesis bridges the gap between open-ocean validation studies and the application of SWOT in the North Sea by evaluating SWH retrieval sensitivity and testing the feasibility of IG wave detection.

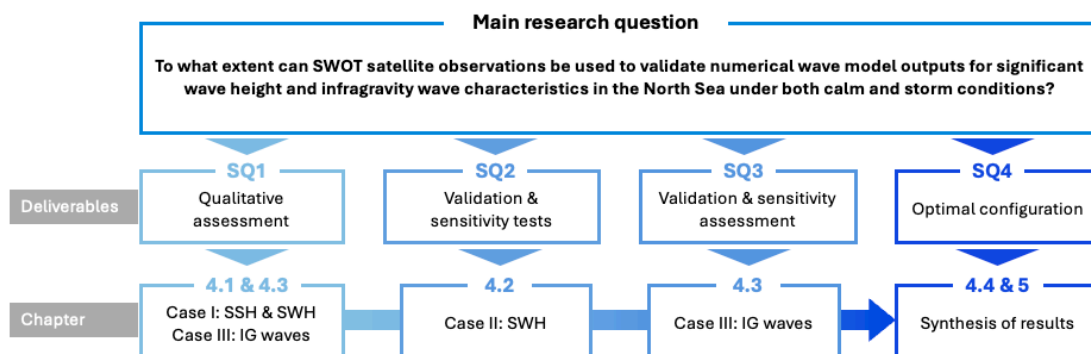
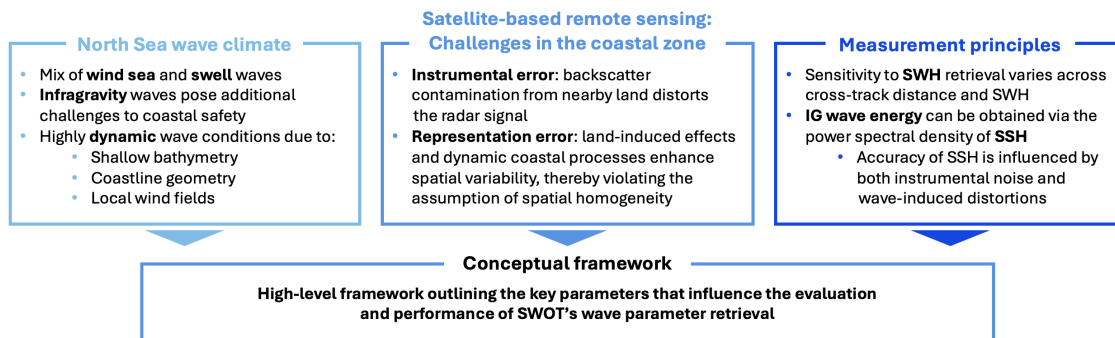


Figure 1.1: Research framework including the deliverables per sub-question (SQ). Structure follows the progression from visual to quantitative assessment of SWOT-derived wave parameters.

Theoretical background

By addressing the physical processes, remote sensing validation procedures and measurement principles of SWOT, this review lays the foundation for understanding the key parameters influencing its retrieval performance.



This chapter provides the theoretical foundation for the research, presenting key concepts, measurement principles, and known limitations of SWOT's technique for resolving sea state estimates. It begins by introducing the dominant wave systems in the North Sea and outlines the key parameters used to describe sea state. The second section introduces the challenges specific to retrieving sea state information in coastal zones using remote sensing (processing) techniques. Subsequently, the third part of this chapter focuses on the underlying measurement principles of SWOT's Ka-band Radar Interferometer (KaRIn) and addresses its constraints in resolving sea state parameters. Finally, the chapter concludes by synthesizing the key theoretical insights into a conceptual framework that supports the methodological design of this study. This framework identifies the key parameters of interest, and outlines the rationale for the processing and validation approach used in the thesis.

2.1. North Sea wave climate and sea state definitions

Coastal shelf regions exhibit complex and highly variable hydrodynamic behavior due to the interaction of multiple physical processes at varying spatial and temporal scales. These environments differ significantly from the open ocean, primarily because of the influence of shallow bathymetry and coastline geometry which induces wave transformation processes. As waves approach the coast, they interact with the seabed, undergoing shoaling, refraction, diffraction, and eventually breaking. In the surf zone strong currents are generated due to wave breaking, which exacerbate coastal erosion. During storm events, these processes further contribute to dune breaching and wave overtopping [19].

The Dutch coast, situated along the shallow and funnel-shaped North Sea basin, is especially susceptible to such dynamic wave conditions. The prevailing sea state typically consists of a mix of wind sea and swell waves. Wind sea consists of short, irregular waves generated by local winds, whereas swell waves originate from distant storm systems and travel over long distances with longer periods and more regular patterns [19]. In addition to wind sea and swell, the North Sea is also affected by infragravity (IG) waves. These low-frequency waves with periods ranging from 30 seconds to 5 minutes can grow significantly in amplitude as they reach shallow water. IG waves, often radiated from distant shorelines after wave breaking, can propagate across the entire North Sea basin and impact coastal dynamics far from their source [7]. Their presence poses challenges to coastal safety because it influences harbor resonance, contributes to dune erosion, and increases overtopping during storms [20].

To better understand these wave systems, this study first introduces the key parameters used to characterize the sea state in subsection 2.1.1. Subsequently, the generation mechanisms of wind-generated

gravity waves and infragravity waves are discussed in subsection 2.1.2 and subsection 2.1.3, respectively.

2.1.1. Sea state parameterization

Sea state refers to the properties of wind-generated surface waves which can be characterized in many ways, from summary statistics like significant wave height (SWH), to complete directional wave spectra [21]. A statistical representation of the sea state can be obtained by decomposing the surface elevation signal η at a fixed location into a series of harmonic components using Fourier analysis. This yields the energy density spectrum $E(f)$, which reveals how the total wave energy (E) is distributed across frequencies (f). When wave directionality is also considered, the two-dimensional (2D) frequency-directional spectrum $E(f, \theta)$ is obtained. A more detailed derivation of the energy density spectrum is provided in section A.1. These spectra enable the identification of distinct wave systems, such as wind sea, swell, and infragravity waves, each characterized by specific energy distributions and corresponding frequency bands [19]. Figure 2.1 illustrates the classification of wave types according to wavelength (I), wave-generating force (II) and restoring force (III). As a reference, the relative amount of energy as a function of wave period is also shown. Gravity waves, highlighted in the first panel, include both wind sea and swell waves. The large-scale input of wave energy from the wind is primarily governed by the fetch, the wind speed, and the duration [22].

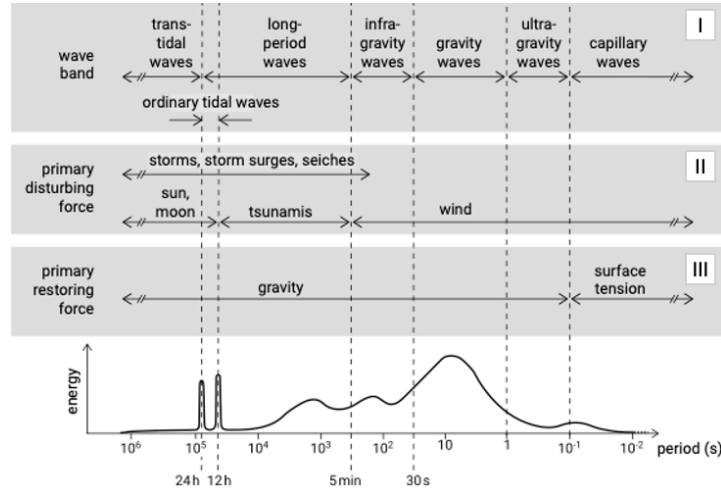


Figure 2.1: Overview of relative amounts of energy as a function of wave period. Panels I-III classify waves by wavelength, wave-generating force, and restoring force respectively [19].

Key spectral characteristics can be expressed in terms of moments (m_n) of the spectrum, which are defined as [23]:

$$m_n = \int_0^{\infty} f^n E(f) df \quad (2.1)$$

One of the most common parameters used to quantify ocean waves is the significant wave height H_s , defined as four times the standard deviation (STD) of the surface elevation. In deep water, wave heights follow a Rayleigh distribution and are solely dependent on the variance of the sea surface elevation, also referred to as the zeroth spectral moment (m_0) [23]. Since the standard deviation is given by $\sigma = \sqrt{m_0}$, the significant wave height can be expressed as:

$$H_{m0} = H_s \approx 4\sqrt{m_0} \quad (2.2)$$

The spectral moment m_0 is typically calculated over the full frequency range of the wave spectrum, yielding the SWH of all waves. However, by restricting the integration to a specific frequency band, such as the infragravity wave range (30–200 s), the SWH can be computed for individual wave classifications. In a similar way, wave periods can be derived from spectral moments. A commonly used example is the mean zero-crossing period or the mean wave period, defined as:

$$T_{m02} = \sqrt{\frac{m_0}{m_2}} \quad (2.3)$$

These parameters are based on the assumption that the sea surface is a stationary Gaussian process. However, in shallow or nonlinear multimodal conditions, this assumption breaks down. As waves become steeper or propagate into shallower water, nonlinear processes alter the wave shape. Wave crests become higher and sharper, while troughs become shallower and flatter. Consequently, the Gaussian distribution no longer holds, and spectral estimates of H_s and T become less accurate [23]. This effect is illustrated in Figure 2.2.

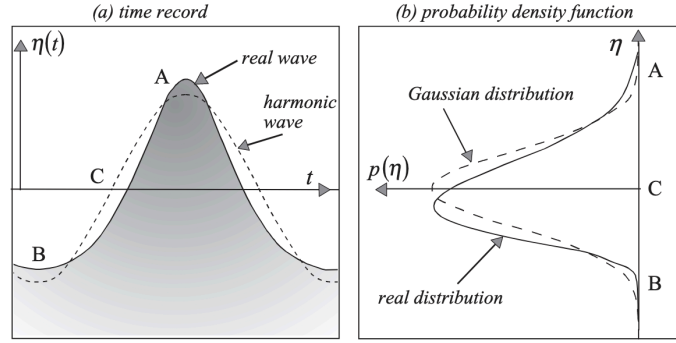


Figure 2.2: (a) Comparison of a harmonic wave (dashed) and a nonlinear wave in shallow water (solid); (b) Resulting deviation from the Gaussian wave height distribution [23].

Nonlinear processes arise when waves interact with the seabed. The influence of the seabed on wave propagation is determined by the relative water depth h/L , where h is the water depth and L the wavelength. Based on this ratio, waves are typically classified as [24]:

- Deep water waves (unaffected by the seabed): $h/L > \frac{1}{2}$
- Shallow water waves (strongly influenced by seabed): $h/L < \frac{1}{20}$
- Intermediate water depth (partially influenced): $\frac{1}{20} < h/L < \frac{1}{2}$

The linear wave dispersion relationship provides the fundamental connection between wave period, wavelength, and water depth. It describes how waves propagate in water of varying depth and is expressed as [23]:

$$\omega^2 = gk \tanh(kh) \quad (2.4)$$

where $\omega = 2\pi/T$ is the angular frequency (rad/s), $k = 2\pi/L$ is the wavenumber (rad/m), T is the wave period (s), L is the wavelength (m), h is the water depth (m), and $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration. This relationship is essential for converting between wave period and wavelength at a given depth.

2.1.2. Wind-generated gravity waves

Wind-generated waves are formed when air flows over the ocean surface, transferring energy from the atmosphere to the water. Under the influence of local winds, numerous wave components are generated with different wave periods, propagation directions and phases [24]. The superposition of these components results in a highly irregular wave pattern known as wind sea.

In contrast, swell refers to wind-generated waves that have propagated away from their area of origin. In deep water, longer-period waves travel faster than shorter ones, leading to dispersion as the wave group evolves. As a result, long-period waves separate from the wind sea and continue propagating over large distances as swell waves. These are typically more uniform and smooth in appearance, with longer periods and narrower directional spreading compared to locally generated wind sea [23]. Importantly, wind-generated waves induce low-frequency modulations of the sea surface elevation over

spatial scales much larger than individual waves (see subsection 2.1.3). These group-scale fluctuations lead to localized variations in SWH, even under otherwise homogeneous wave conditions [25].

Figure 2.3 shows a conceptual illustration of two coexisting wave systems in the North Sea. A northerly swell, generated by a storm off the Norwegian coast, meets a wind sea driven by local winds in the southern basin. The corresponding wave energy spectra are also shown: a 2D directional spectrum ($E(f, \theta)$) and a 1D frequency spectrum ($E(f)$), both illustrating the distinction between the two wave systems. Notably, the swell peak is more pronounced in the 2D spectrum due to its narrow directional spread, whereas the wind-sea appears dominant in the 1D spectrum due to its broader directional distribution [19].

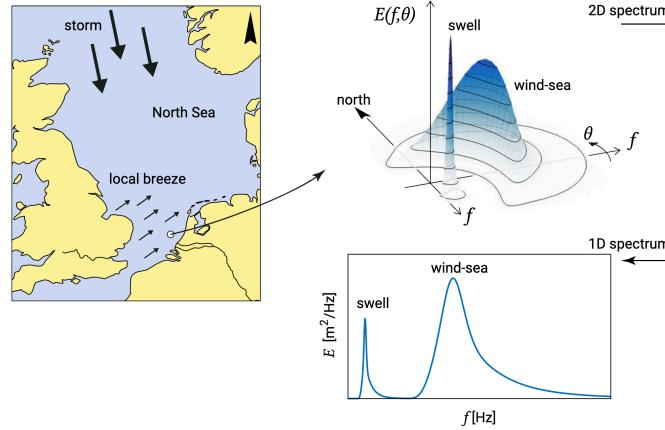


Figure 2.3: Schematic overview of two typical wave systems in the North Sea. Left: swell waves generated by a storm off the Norwegian coast and wind-sea generated by a local breeze in the Southern North Sea. Right: the corresponding 2D directional spectrum $E(f, \theta)$ and 1D frequency spectrum $E(f)$ illustrate the different characteristics of both wave systems [19].

2.1.3. Infragravity waves

Infragravity waves are long-period surface gravity waves with frequencies typically between 0.005 and 0.05 Hz [7]. These waves originate from nonlinear interactions between wind-generated waves, which convert part of their energy into longer-period subharmonics. This process is illustrated in Figure 2.4, showing how two interacting short waves generate a longer-period signal. These long oscillations, as seen in the lower panel of Figure 2.4 (red), travel bound to the short wave groups but with opposite phase [26]. These wave group-forced IG waves (WGF-IG), also called bound infragravity (BIG) waves, propagate with the group velocity (c_g). Although small in amplitude (on the order of centimetres), BIG waves are already present in deep water and can amplify significantly as they propagate toward the coast.

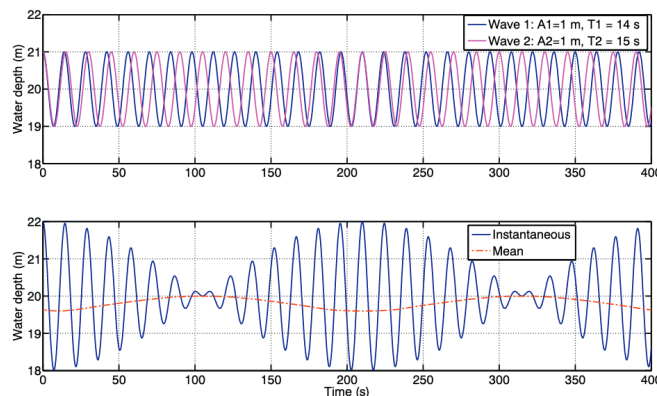


Figure 2.4: (Upper panel) Time series of two sinusoidal waves with periods of 14 s (blue) and 15 s (pink) propagating over a flat bottom at 20 m depth. (Lower panel) Resulting free surface elevation (blue) and bound wave (red) [20].

As the wave groups (lower panel (blue) in Figure 2.4), propagate into shallower water, the phase rela-

tionship between the short waves and the bound wave shifts away from 180° . This phase shift allows energy to transfer from the wave group to the bound long wave, which results in a growth of IG waves. Once the short waves break nearshore, their group structure diminishes. Here, the bound component is released and propagates further as free waves [20].

Once released, free infragravity (FIG) waves propagate independently from the wave groups. They follow the linear surface gravity waves dispersion, with larger wavelengths and phase speed (c) [26]. These are further divided into:

- Locally generated FIG waves (FIG_L): originating from nearby coasts and often reflectively trapped due to coastal geometry.
- Remotely generated FIG waves (FIG_R): radiated from distant shorelines, typically during storm-driven swell breaking [6].

In regional seas like the North Sea, FIG waves may either become trapped nearshore due to refraction or leak offshore and reach the opposite coastline. This is schematically shown in Figure 2.5. In such enclosed or semi-enclosed basins, FIG waves travel relatively short distances ($\mathcal{O}100\text{--}1000$ km) with limited energy dissipation compared to transoceanic propagation [6].

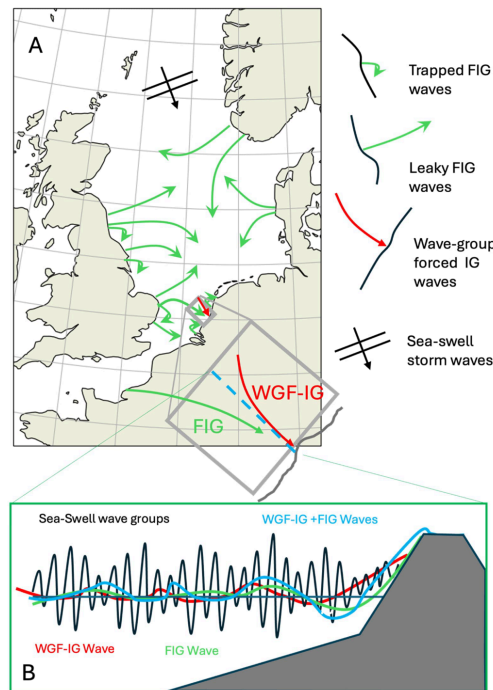


Figure 2.5: Schematic overview of IG wave generation during a storm in the southern North Sea. Free IG waves (green) may remain trapped near the coast or leak across the basin. In addition, locally generated wave group-forced IG waves (WGF-IG, red) also contribute. The total incoming IG wave height results from the superposition of FIG and WGF-IG waves, with the highest water levels occurring where both components are in phase (blue) [27].

The North Sea, with its relatively mild bathymetric gradients and enclosed geometry, is particularly susceptible to FIG wave influence. Unlike steep coasts which can trap a large part of the reflected IG energy, the North Sea's gentle slopes allow FIG waves to propagate between coasts. Reniers et al. (2021) showed that under storm conditions, the central North Sea exhibited significant IG wave heights, that could not be explained by bound IG waves alone [28, 6]. Using a spectral wave model, Rijnse et al. (2021) demonstrated that a substantial part of this energy was radiated from distant shorelines where it was generated by the incident sea-swell waves. These results emphasise the significance of remotely generated IG waves. The origin of the incoming FIG energy was found to vary across storms, largely depending on where the most energetic sea-swell waves made landfall.

Akrish et al. (2025) further quantified the relative importance of FIG versus BIG wave energy across several North Sea buoy stations during the winter of 2022 and fall/winter of 2023/24. Their analysis showed that FIG wave heights generally exceeded those of BIG waves, both offshore and nearshore,

and across a wide range of sea-swell conditions. Notably, IG wave heights increased toward the coast, with nearshore values reaching up to 0.5 m. The FIG-to-BIG energy ratio was consistently above one for nearshore stations (depths < 18.3 m) and in some cases exceeded 100, indicating a negligible contribution from bound waves. Particularly high energy ratios were observed over the more sheltered (less exposed to strong wind velocities and the associated higher sea-swell waves) measurement stations [6]. Overall, these findings underline the dominant role of FIG waves in shaping the nearshore infragravity wave field, particularly during storm conditions.

2.2. Challenges in satellite-based coastal wave retrieval

Satellite altimetry is a space-borne radar remote sensing technique that has been used for over three decades to measure SSH, SWH, and wind speed over the global ocean, including coastal seas. The underlying measurement principle is straightforward: the altimeter emits short radio-wave pulses toward the ocean surface and measures the round-trip travel time of the reflected signal. In addition to the time delay, the returned pulse's shape and amplitude provide information on sea surface roughness, allowing for the retrieval of wave-related parameters such as SWH [29].

Satellite altimetry faces considerable challenges in the coastal zone, where proximity to land and the complexity of physical processes degrade measurement reliability. Advances have been made through improved radar technology, the introduction of SAR modes with Ka-band frequency, refined geophysical corrections, and data processing workflows tailored to coastal environments [13]. Nevertheless, comparative studies reveal a decline in performance metrics for both altimeter-buoy and altimeter-model match-ups as the distance to the coast decreases [10].

A key limitation in assessing coastal altimeter performance lies in the way satellite observations are validated against in-situ data. Validation procedures rely on match-ups with in-situ wave buoys, defined by spatial and temporal windows. A match-up is typically defined when the altimeter ground track passes within 50 km of a buoy location, and the buoy has recorded data within a 30-minute window of the satellite overpass. For each match-up, altimeter-derived SWH values are averaged across the along-track measurements within a 50 km radius centered on the in situ-location. Meanwhile, buoy time series are filtered using a two-hour moving window and interpolated on the satellite overpass time [10]. This approach is statistically robust in open-ocean settings, where wave conditions are relatively homogeneous. However, in coastal zones the wave field can be highly variable over short spatial scales, making such validation procedures prone to representation errors.

For several altimetry missions, including SAR-operating configurations, a systematic increase in SWH bias $\mathcal{O}(1 \text{ dm})$ is observed for buoys located within 50 km of the coast. This degradation in performance can be attributed to four main factors:

1. Land-induced effects such as coastal shading and wave refraction alter the wave field within relatively short distances, violating assumptions of spatial homogeneity.
2. Dynamic coastal processes, including tidal currents and variable winds, increase the variability of wave conditions within the satellite footprint. This further undermines the assumption that wave parameters remain uniform across the altimeter footprint.
3. Backscatter contamination from nearby land distorts the returned radar signal, reducing the accuracy of SSH and SWH retrievals.
4. Nearshore altimeter records are more frequently rejected during data editing procedures, further biasing the dataset toward offshore conditions.

Collectively, these factors contribute to increased uncertainty and systematic bias in altimeter-derived wave products in coastal regions [10]. Specifically, points 1 and 2 introduce representation errors due to enhanced spatial heterogeneity of the wave field, while points 3 and 4 relate to instrumental signal degradation. Although SAR altimetry offers improved precision and along-track resolution, it remains fundamentally limited by its nadir-only geometry and narrow swath width. In contrast, the SWOT mission, captures SSH fields in two dimensions across a much wider swath and offers improved resolution of cross-track variability. This broader spatial coverage enables more localized and precise buoy match-ups, reducing the reliance on spatial averaging over large, heterogeneous footprints. This capability is particularly valuable in coastal zones, where fine-scale variability and proximity to land challenge traditional altimetry systems.

2.3. SWOT mission

The SWOT mission, launched in December 2022, represents a significant advancement in satellite altimetry. By employing two SAR antennas mounted on either end of a 10-meter mast, it enables interferometric measurements that provide the first two-dimensional high-resolution observations of water surface elevations from space. SWOT is a collaborative effort between NASA and CNES (the French Space Agency), with additional contributions from the Canadian Space Agency and the UK Space Agency [30]. Its primary objective is to perform a global survey of surface water, capturing both the spatial and temporal variability of fine-scale ocean surface topography.

To assess the capability of SWOT to retrieve wave characteristics in relatively shallow coastal environments, it is necessary to understand both its underlying measurement principles and its limitations for sea state analysis. This section outlines the operational concept of the KaRIn interferometer, explains how SWH is estimated from interferometric correlation, and describes the processing approach used to derive power spectral densities (PSD) and obtain frequency dependent SWH.

2.3.1. Interferometric SAR topography

Unlike the method used to measure surface height in conventional altimetry, the SWOT mission employs two Ka-band SAR antennas. These antennas operate in a bistatic configuration (also known as non ping-pong mode), measuring two swaths on either side of the satellite's nadir track, as illustrated in Figure 2.6a. Each 50 km wide swath is covered with a different polarization, transmitted alternately. In the first pulse (red), antenna 2 transmits towards the left swath using horizontal ('H') polarization. In the subsequent pulse (blue), antenna 2 transmits towards the right swath using a vertical ('V') polarization [31].

The principle is to combine two SAR images of the same ground location, captured simultaneously from the two receiving antennas (1 and 2), which are separated by a baseline distance B . These measurements are repeated at a high frequency, and by assembling the successive profiles, the processor constructs a continuous image of the surface height across the full 50 km swath widths.

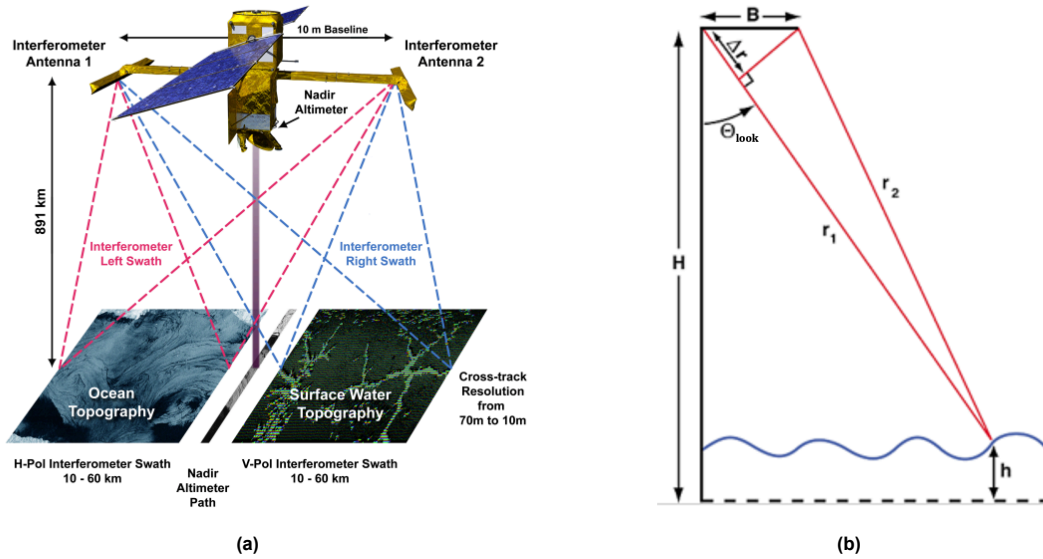


Figure 2.6: (2.6a) SWOT's bistatic mode for KaRIn swath coverage with a Jason-class altimeter in the nadir gap [32]; (2.6b) Illustration of the measurement geometry for cross-track SAR interferometry to obtain the surface height (h) [33]

Each backscattered radar pulse is received by both antennas, and the difference in path lengths between the two signals results in a phase offset $\Delta\varphi$. This interferometric phase can be expressed in terms of the range difference Δr between the two antennas as [32]:

$$\Delta\varphi = \varphi_2 - \varphi_1 = -\frac{2\pi}{\lambda} \Delta r \quad (2.5)$$

A simplified schematic of the measurement geometry is provided in Figure 2.6b. The phase difference $\Delta\varphi$ is related to the look angle θ_{look} via the following expression [31]:

$$\Delta\varphi \approx kB \sin \theta_{look} \quad (2.6)$$

where, $k = 2\pi/\lambda$ is the electromagnetic wavenumber, λ is the radar wavelength, and B is the known baseline between the antennas. Once the look angle θ_{look} is determined, the surface elevation h can be derived from the geometry as follows [31]:

$$h = H - r_1 \cos \theta_{look} \quad (2.7)$$

Here, H represents the satellite altitude and r_1 the slant range to the first antenna. The altimeter operating at nadir provides the orbital height, while the onboard microwave radiometer corrects for atmospheric water vapor. Additionally, the GPS and DORIS systems onboard allow for precise orbital tracking [34]. Together, these instruments enable accurate geolocation and elevation estimates for each radar pixel. Equation 2.7, as well as other expressions in this subsection involve approximations and simplifications. The purpose is to provide a general, introductory overview of the measurement approach, rather than a detailed account of the exact processing techniques.

2.3.2. Interferometric correlation

As described in subsection 2.3.1, SWOT is primarily a topography mission. Its main data product is surface elevation, derived from the interferometric phase difference between signals received by the two SAR antennas. This phase information, along with the amplitude and noise components, is one of the four real-valued components that can be used to construct detailed information about the observed surface [35].

As presented by Bohé at WISE 2024, each antenna receives a signal composed of amplitude, phase, and noise. When no spatial averaging is applied, these signals are referred to as single-look complex (SLC) images [32]. The complex SLC signals received by the two antennas can be expressed as:

$$s_1 = A_1 e^{i\varphi_1} + n_1 \quad (2.8)$$

$$s_2 = A_2 e^{i\varphi_2} + n_2 \quad (2.9)$$

Here, A_1 and A_2 are the amplitudes and n_1 and n_2 represent additive noise. From the single channel amplitudes $|s_1|$ and $|s_2|$, Normalized Radar Cross-Section (NRCS) images can be derived, which are useful for estimating wind speed and other surface processes [12]. A fourth independent combination of the complex received signals is the interferometric correlation, also known as coherence. The coherence quantifies the normalized covariance between the two signals:

$$\gamma = \frac{\langle s_1^* s_2 \rangle}{\sqrt{\langle |s_1|^2 \rangle \langle |s_2|^2 \rangle}} \quad (2.10)$$

The brackets (" $\langle \rangle$ ") denote ensemble averages over both instrumental noise and surface variability. The coherence ranges from 0 to 1, with lower values indicating noisier or less reliable interferometric data. Several factors contribute to decorrelation, and these can be decomposed multiplicatively as follows [12]:

$$\gamma = \gamma_{SNR} \cdot \gamma_{geo} \cdot \gamma_{ang} \cdot \gamma_{vol} \quad (2.11)$$

Here, the first three terms act as instrumental sources of decorrelation: signal-to-noise ratio (γ_{SNR}), geometric decorrelation (γ_{geo}), and angular decorrelation (γ_{ang}). Most importantly is the last term: volumetric decorrelation (γ_{vol}) which is influenced by wave-induced surface roughness. The latter is directly related to wave parameters and follows, under the assumption of a Gaussian distributed ocean surface elevation [36]:

$$\gamma_{vol} = \exp^{-\frac{1}{2} k_z^2 \sigma_h^2} \quad (2.12)$$

where σ_h is the STD of the surface elevation. Under the Gaussian distributed surface elevation assumption, the SWH equals four times the STD. Substituting this relationship into Equation 2.12 yields:

$$\gamma_{vol} = \exp^{-\frac{1}{2} k_z^2 \left(\frac{SWH}{4}\right)^2} \quad (2.13)$$

The coefficient κ_z depends on the interferometric geometry and varies across the SWOT swath. As such, the γ_{vol} exhibits strong spatial variation, with coherence generally decreasing for larger wave heights and in the near-nadir region, where instrument sensitivity is highest.

This behavior is illustrated in Figure 2.7a, which shows coherence across the swath. Coherence is maximal (≈ 1) under calm conditions and at far cross-track ranges, where sensitivity to variations in SWH diminishes. Small deviations in γ_{vol} (eg. an error of 10^{-3} on γ_{vol} in Figure 2.7b) can lead to large uncertainties up to $\mathcal{O}(\text{dm})$ in derived SWH values in the far range. These conditions represent some of the most challenging regimes for accurate SWH estimation from KaRIn observations [35].

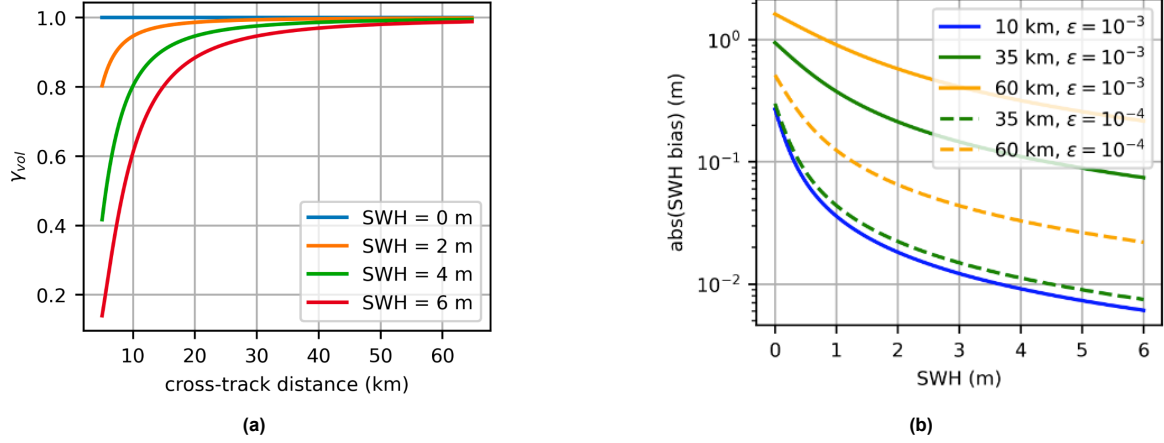


Figure 2.7: (2.7a) Volumetric decorrelation γ_{vol} as a function of cross-track distance for various SWH; (2.7b) Absolute SWH bias (log scale) introduced by two different levels of error (ϵ) on γ_{vol} illustrated for various cross-track distances [12].

2.3.3. Spectral analysis

As introduced in section 2.1, a statistical representation of the sea state can be obtained by decomposing the surface elevation signal at a fixed location into a series of harmonic components using Fourier analysis. The resulting energy density spectrum describes how total wave energy is distributed across frequencies and enables identification of distinct wave systems. When wave direction is included, a 2D frequency-directional spectrum $E(f, \theta)$ is obtained. This spectrum can also be represented in wavenumber space as $E(k_x, k_y)$, where wave energy is distributed across two orthogonal wavenumber components (k_x, k_y) [37].

From SWOT's 2D SSH observations (cross-track and along-track directions), a wavenumber power spectral density (PSD) can be computed. This PSD describes the distribution of observed SSH variance across spatial wavenumbers. From the PSD, low-frequency wave properties such as height, period, and propagation direction can be inferred. Ardhuin et al. (2024) demonstrated that SWOT's LR "Unsmoothed" product can effectively retrieve long-period (18-30 s) swell characteristics in the open ocean, providing the methodological basis used for this research [11].

The processing approach relies on the estimation of the SSH PSD, denoted $E_s(k_x, k_y)$, from large continuous square regions within the SWOT swath. To correct for distortions introduced by onboard filtering and wave-induced measurement distortions, the measured spectrum is corrected using a transfer function $G(k_x, k_y)$. This transfer function includes the point target response (PTR), which represents the spatial impulse response of the radar system (see section A.2 for more details), and enables recovery of the underlying ocean wave spectrum $E(k_x, k_y)$ via:

$$E(k_x, k_y) = E_s(k_x, k_y) / G(k_x, k_y) \quad (2.14)$$

Following Ardhuin et al. (2024), G combines multiple filtering effects. In both the cross-track and along-track directions, it includes the On-Board Processor (OBP) averaging filters (G_{OBP}). In the along-track direction, it further includes the PTR term (G_{PTR}) and the azimuthal cutoff term (G_{AZC}) related to velocity bunching. A fourth effect, range bunching (the surfboard effect), primarily influences the cross-track signal. Although the corresponding component (G_{RB}) was neglected in the study of Ardhuin et al.

(2024), its impact may become significant during storm conditions. Velocity and range bunching are further discussed in subsection 2.3.4.

To compute the SSH PSD, Ardhuin et al. (2024) apply a 2D Welch method to a large continuous square region of length L_{box} centered within the left or right swath. This region is divided into overlapping tiles of length L_{tile} , and each tile is multiplied by a window function (e.g., Hanning window) to suppress edge discontinuities. This approach mitigates spectral leakage, a phenomenon in which energy is spread across multiple wavenumber bins. A fast Fourier transform (FFT) is applied to each windowed tile, and the resulting 2D periodograms ($|FFT|^2$) are averaged to yield the final spectral estimate $E_s(k_x, k_y)$ [38].

By averaging overlapping, windowed tiles, Welch's method reduces the variance of the PSD estimate and improves the signal-to-noise ratio, but at the cost of lower spectral resolution relative to single-record estimation methods [38, 39].

Compact derivation for cross-spectra and coherence

The Welch method involves segmenting the dataset into overlapping tiles and modifying the sections using 2D data windows $w(x, y)$. For a total of N segments of length L_{tile} , the 2D FFT of the windowed SSH and radar backscatter (σ_0) data for each segment n can be expressed as:

$$E_n(k_x, k_y) = \mathcal{F}\{w SSH\}_n \quad (2.15)$$

$$S_n(k_x, k_y) = \mathcal{F}\{w \sigma_0\}_n \quad (2.16)$$

The corresponding periodograms and cross-spectral estimates per segment are:

$$P_{SSH,n} = |E_n|^2 \quad (2.17)$$

$$P_{\sigma_0,n} = |S_n|^2 \quad (2.18)$$

$$P_{SSH,\sigma_0,n} = E_n S_n^* \quad (2.19)$$

The asterisk (*) in Equation 2.19 denotes the complex conjugate. Averaging over all N segments, provides the PSD estimates: $\hat{P}_{SSH}$, $\hat{P}_{\sigma_0}$, and $\hat{P}_{SSH,\sigma_0}$. A normalization factor is applied to compensate for the power loss introduced by the data window and to properly scale the discrete FFT coefficients into PSD units. This correction is performed after the FFT and before averaging over the total number of segments.

The spectral coherence^a (or magnitude-squared coherence) C and cross-spectral phase θ_{SSH,σ_0} can be obtained by [40]:

$$C = \frac{|\hat{P}_{SSH,\sigma_0}|^2}{\hat{P}_{SSH} \hat{P}_{\sigma_0}} \quad (2.20)$$

$$\theta_{SSH,\sigma_0} = \arg(\hat{P}_{SSH,\sigma_0}) \quad (2.21)$$

By averaging overlapping, windowed tiles, the Welch method reduces the variance of the estimated PSD. The number of independent FFT computations or equivalent degrees of freedom (EDF) controls the variance and is given by [38]:

$$\text{E.D.F.}\{\hat{P}(f_j)\} = 2N, \quad f_j = \frac{j}{L_{tile}}, \quad j = 0, \dots, \frac{L_{tile}}{2} \quad (2.22)$$

While L_{box} defines the spatial extent of the analysis region, L_{tile} determines the spectral resolution (wavenumber spacing $\Delta k_{x,y}$). Larger L_{tile} results in finer wavenumber resolution (smaller Δk), allowing improved representation of spectral structure and peak localization. For open-ocean conditions, Ardhuin et al. (2024) applied this method using 40×40 km samples with 5 by 5 km tiles and 50% overlap. This configuration yields an EDF of 128, providing statistically stable spectral and cross-spectral density estimates while maintaining sufficient resolution to resolve dominant swell components. A schematic of this configuration is provided in Figure B.3. The implications of box and tile dimensions on spectral estimation are further discussed in subsection 3.6.2.

^aCoherence refers to the cross-spectral coherence between σ_0 and SSH

To compute the SWH associated with low-frequency wave energy, Ardhuin et al. (2024) first isolate swell components using the coherence between SSH and radar backscatter (σ_0), derived from their cross-spectrum. A coherence threshold is applied to define a "swell mask" within which Equation 2.14 is valid. The swell height is estimated as the sum of energy within the masked region. Additionally, the phase of the SSH- σ_0 cross-spectrum provides the swell propagation direction [11].

As part of their code release, Ardhuin et al. (2024) also included a watershed partitioning technique, which segments the spectral domain into distinct regions by detecting energy peaks and assigning individual masks. However, this method was not applied in the main analysis of their study [40].

The full methodological approach used by Ardhuin et al. (2024) is further described in section A.3. In contrast to their approach, focused on isolating individual swell systems, this study aims to retrieve the infragravity wave energy integrated across all directions. To achieve this, a different masking strategy is adopted, based on fixed wavenumber thresholds corresponding to the IG wave regime of interest. This method is explained in detail in section 3.6.

2.3.4. Phenomena influencing SWOT performance

The accuracy of SWOT's SSH retrievals is influenced by both instrumental noise and wave-induced distortions. Eva Peral et al. (2015) systematically examined these sources of error and demonstrated how ocean waves, especially in the near range of the swath, can significantly degrade measurement performance. Their analysis distinguishes between random noise (including thermal noise, volumetric decorrelation due to waves, geometric and angular decorrelation, and pointing errors) and systematic wave-induced errors such as wave aliasing, the surfboard effect, and backscatter modulation. Both types of error increase with SWH. Random noise increases most notably in the near and far edges of the swath, whereas wave-induced errors are most pronounced in the near range. Figure 2.8, based on WAVEWATCH-III simulations, illustrates how both error types vary with cross-track position and wave energy. Random noise (solid lines) remains relatively constant across different SWHs in the center of the swath, while wave-induced errors (dashed lines) increase sharply with SWH and dominate the total error budget at higher sea states. When SWH exceeds 5 m, wave-related effects become the primary contributor to height error, surpassing the random instrument noise.

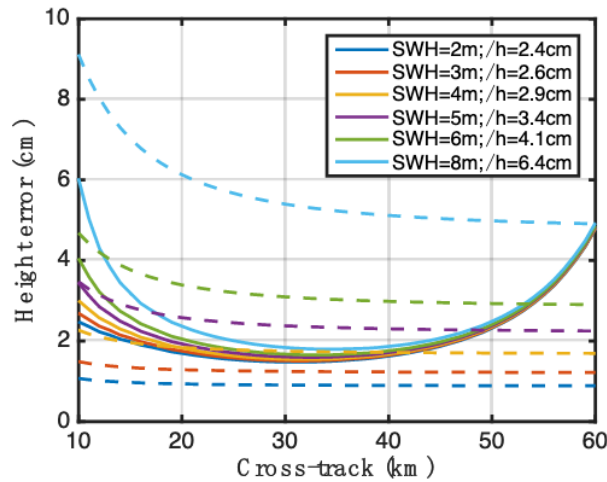


Figure 2.8: Simulated height error for 1 km $\times$ 1 km pixels due to random noise (solid) and due to the presence of waves (dashed) as a function of cross-track. Different SWH have been used to present its impact on the error distribution, with the average height error added in the legend [36].

As highlighted in subsection 2.3.3, Ardhuin et al. (2024) propose a spectral transfer function to correct for distortions introduced by onboard filtering and wave-related effects. While their formulation accounts for velocity bunching, the impacts of range bunching (the surfboard effect) and backscatter modulation are not included. However, all three phenomena can introduce energy distortions into the SSH PSD and must be carefully considered when interpreting SWOT measurements through spectral analysis. The following subsections elaborate on how each mechanism affects SWOT's interferometric SSH retrievals and the implications for wave spectral estimates.

Wave/velocity bunching

Relative motion between surface waves and the sensor of SAR introduces Doppler shifts that distort phase. At near-nadir incidence, these shifts are dominated by the vertical component of the orbital velocity. For waves propagating along-track, vertical motion is greatest between the crests and troughs of the wave, and its sign reverses on either side of the crest. Consequently, the wave in the SAR image appears sharpened or flattened (more/less “peaky”) compared to the true profile [32, 41]. This nonlinear transformation of wave patterns redistributes energy among along-track wavenumbers, making the image spectrum differ from the ocean spectrum. KaRIn’s onboard processing applies power-weighted complex averaging to reduce sensitivity to bunching (when echo power scales with contributing surface area), though it cannot remove it entirely [32]. Figure 2.9 sketches how vertical velocities shift surface points and alter the along-track image.

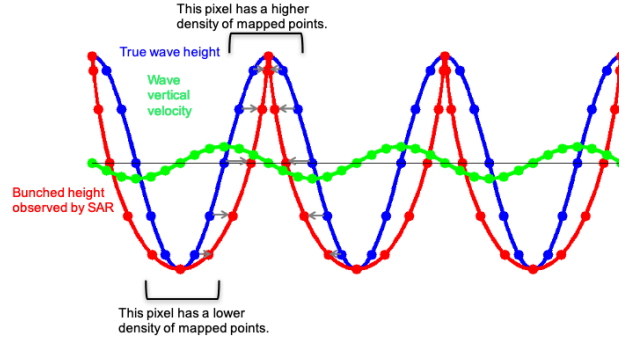


Figure 2.9: Illustration of the wave bunching effect: the spatially varying vertical velocity (green) induces shifts of the surface points captured by SAR (red) compared to the true surface (blue) [32].

Azimuth cut-off

The vertical orbital motions of waves traveling in the azimuth (along-track) direction introduce Doppler shifts that distort the returned radar signal and can create either constructive or destructive imaging effects. For waves with a strong space-time correlation (e.g. swell), velocity bunching tends to be constructive. For wind-sea, where velocities are more random, it is largely destructive. This results in an apparent blurring of the along-track wave structure. This along-track distortion acts as a low-pass filter and thus limits the shortest detectable wavelength observed by SAR [41]. The corresponding azimuthal cutoff wavelength λ_c can be expressed as [42]:

$$\lambda_c = \pi \frac{R}{V} \sigma_u \quad (2.23)$$

where R/V is the range-to-platform velocity ratio (with $R = 875$ km), and σ_u the root mean squared (rms) orbital velocity along the radar line of sight. This last term can be approximated as a function of SWH and mean zero up-crossing period T_{m02} :

$$\sigma_u = \frac{\pi H_s}{2 T_{m02}} \quad (2.24)$$

In SWOT onboard processing the azimuth cutoff λ_c , appears in the transfer function as an azimuthal attenuation factor:

$$G_{AZC}(k_y) = \exp(-k_y^2 \lambda_c^2) \quad (2.25)$$

which damps short along-track wavelengths before inverting the SSH PSD ($E_s(k_x, k_y)$). While the approximation of the azimuth cutoff is well established for SAR systems, the direct application of this equation to KaRIn interferometric phase/coherence remains under investigation [43].

Surfboard effect

KaRIn samples the surface along iso-range planes (“surfboards”) which can cause two distortions, illustrated in Figure 2.10. First, depending on sea state and incidence angle of KaRIn, a single plane can intersect the sea surface at multiple locations, creating layover points (red points in Figure 2.10a). During onboard processing, the estimated surface height is severely distorted when averaging these

layover points separated by large distances. Second, wave slopes facing the radar are sampled at lower density than slopes facing away from the radar, causing foreshortening of toward-facing slopes or stretching of away-facing slopes (see Figure 2.10b) [36]. Consequently, the interferometric averages no longer correspond to uniformly spaced ground points, biasing both apparent heights and their spatial distribution. The dominant spectral consequence, is the introduction of spectral components (wavenumbers) that do not exist on the ground (with leakage toward lower wavenumbers), thereby biasing the SSH spectrum and adding height noise [33]. The statistical behaviour of the surfboard effect depends on the amplitude, wavelength, and direction of the waves [32, 44]. The impact is strongest for cross-track propagating waves and during high-SWH (storm) conditions [11, 33].

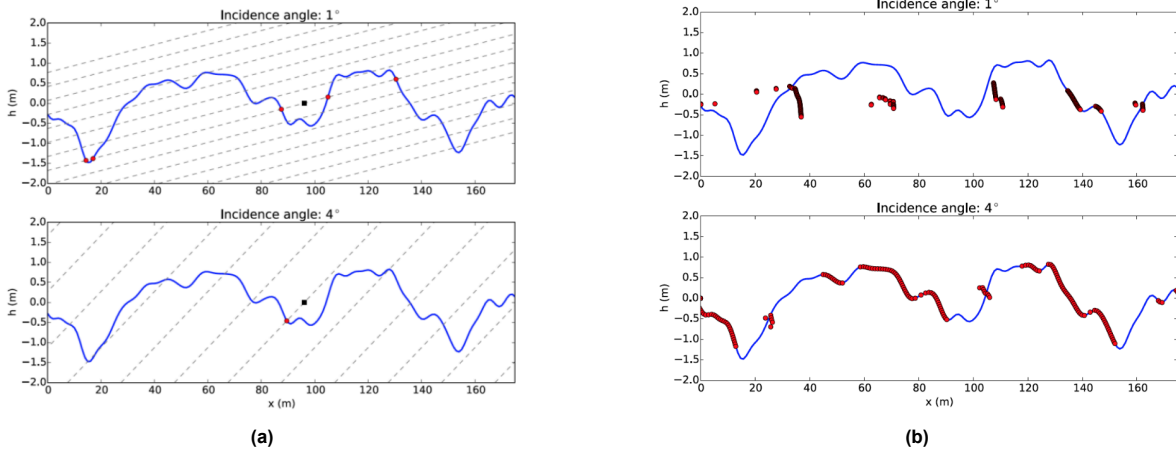


Figure 2.10: (2.10a) Ocean wave field (blue) sampled by a set of iso-range planes (dashed-gray) with incidence angle 1° (upper-panel) and 4° (lower-panel). Radar pulses propagate from left to right with sampled surface elevations shown in red and the intersection of a sample plane with the mean level in black; (2.10b) Corresponding estimated surface elevations (red) as a function of estimated cross-track, assuming a noiseless measurement [36].

Backscatter modulation

Backscatter modulation is another nonlinear source of height error, in which long ocean waves modulate the intensity of the backscattered radar signal. This modulation affects the accuracy of retrieved sea surface height, and is driven by two mechanisms. First, tilt modulation arises when long-wave slopes alter the local incidence angle of the radar pulse, thereby influencing the radar backscatter σ_0 . Second, hydrodynamic modulation occurs as the orbital motion associated with long waves modifies the short-wave slope variance, further influencing the backscatter signal [36, 45]. The backscattering modulates the radar response such that measured height is a nonlinear function of the wave height. As a result, similar to surfboard sampling effects, wave energy spreads to longer wavelengths that are not filtered by the OBP [36].

Unlike the surfboard effect, which is strongest for cross-track propagating waves, backscatter-modulation errors are largely independent of wave direction and cross-track position. This results in a more uniform increase in both the height standard deviation and systematic bias across the satellite swath [36].

2.3.5. Physical limitations

Both the sampling resolution of the LR "Unsmoothed" product and the spectral processing choices constrain which wave components can be resolved.

Shortest resolvable wavelength

With native sampling $\Delta x, \Delta y \approx 250$ m, the spatial sampling rate per axis is $f_s = 1 / \Delta x \approx 4$ cycles km^{-1} . The per axis Nyquist wavenumber k_N is therefore [46]:

$$k_N = \frac{f_s}{2} \approx 2 \text{ cycles km}^{-1}, \quad (2.26)$$

so the shortest resolvable wavelength (per axis) is:

$$\lambda_{min} = \frac{1}{k_N} \approx 500 \text{ m} \quad (2.27)$$

Waves with wavelengths smaller than 500 m, alias back into the interval $[-k_N, k_N]$. The transfer function $G(k_x, k_y)$ strongly attenuates high- k content, but does not eliminate aliasing entirely [47]. The filter applied exhibits sidelobes beyond the cut-off frequency, meaning some of the wave spectrum energy at short wavelengths will be aliased to longer wavelengths [36]. In Figure 2.11, the along-track and cross-track slices of the spectral filter G implemented by Arduin et al. (2024) are shown. The vertical dashed red line at $cpk = 2$ marks the Nyquist limit of SWOT's LR mode. The progressively steeper roll-offs (from right to left) illustrate how each component of G increasingly suppresses high-wavenumber power. The strongest attenuation is introduced by the OBP filter, which is primarily intended to reduce data volume for downlink [32, 44].

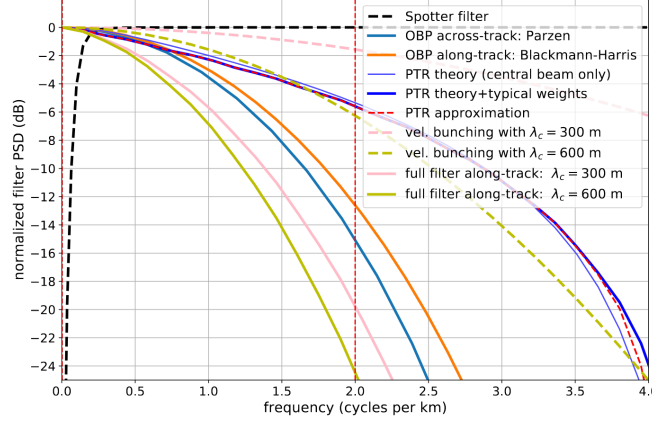


Figure 2.11: Illustration of the components of the cross-track and along-track spectral filter response G as defined in the python library `lib filters obp.py` by Arduin (2024) [47]. The Spotter filter (black dashed) represents the processing filter used for surface wave buoys. The OBP filters (blue and orange) show the cross-track Parzen and along-track Blackman-Harris onboard averaging filters. The PTR curves (violet, dark blue, red dashed) compare the theoretical PTR using only the central beam, the PTR including 9 overlapping azimuth beams with typical beam weights, and the simplified PTR approximation used in their study. The velocity-bunching filters are computed from wave orbital velocities estimated with a wave model.

Longest resolvable wavelength

As discussed in subsection 2.3.3, the SSH PSD is estimated with a 2D Welch method using overlapping tiles of size $L_{tile} \times L_{tile}$. The longest wavelength (λ_{max}) that can be resolved within each tile is approximately equal to the tile length: $\lambda_{max} \approx L_{tile}$. Wavelengths longer than the tile dimension cannot be represented in that tile's spectrum.

Each tile is transformed to the spectral domain using a FFT, resulting in a discrete grid in wavenumber space. The spacing between grid points in wavenumber, Δk , is inversely proportional to the tile size ($\Delta k = 1/L_{tile}$), meaning that larger tiles provide finer resolution in the low-wavenumber range. Increasing L_{tile} thus enhances the ability to resolve long waves (low k), but also reduces the number of independent segments within a given analysis box (L_{box}), which increases the variance of the PSD estimate. The chosen tile size therefore reflects a trade-off between spectral resolution and estimator variance.

2.4. Framework for SWOT North Sea wave assessment

This section synthesizes the theoretical insights from the previous sections to define a framework that guides the evaluation of SWOT satellite data for nearshore wave analysis. Given the absence of established processing methodologies for retrieving sea state characteristics from SWOT data in coastal and relatively shallow environments, this thesis adopts an exploratory, twofold approach. The first track focuses on evaluating the accuracy and consistency of SWOT-derived spatial estimates of SWH. The second track assesses the reliability of IG wave parameter estimates obtained via spectral decomposition. As discussed in section 2.3, both methods of Bohé et al. (2025) and Arduin et al. (2024) face known constraints and challenges. These must be considered in context of the shallow and intermediate waters analysed in this thesis. The objective of the framework presented in this section is to identify the key parameters that influence the evaluation of SWOT data. A high-level schematic of this framework is shown in Figure 2.12. Figure 2.12a highlights the parameters influencing SWH retrieval, whereas Figure 2.12b shows the parameters impacting the accuracy of retrieving IG wave energy.

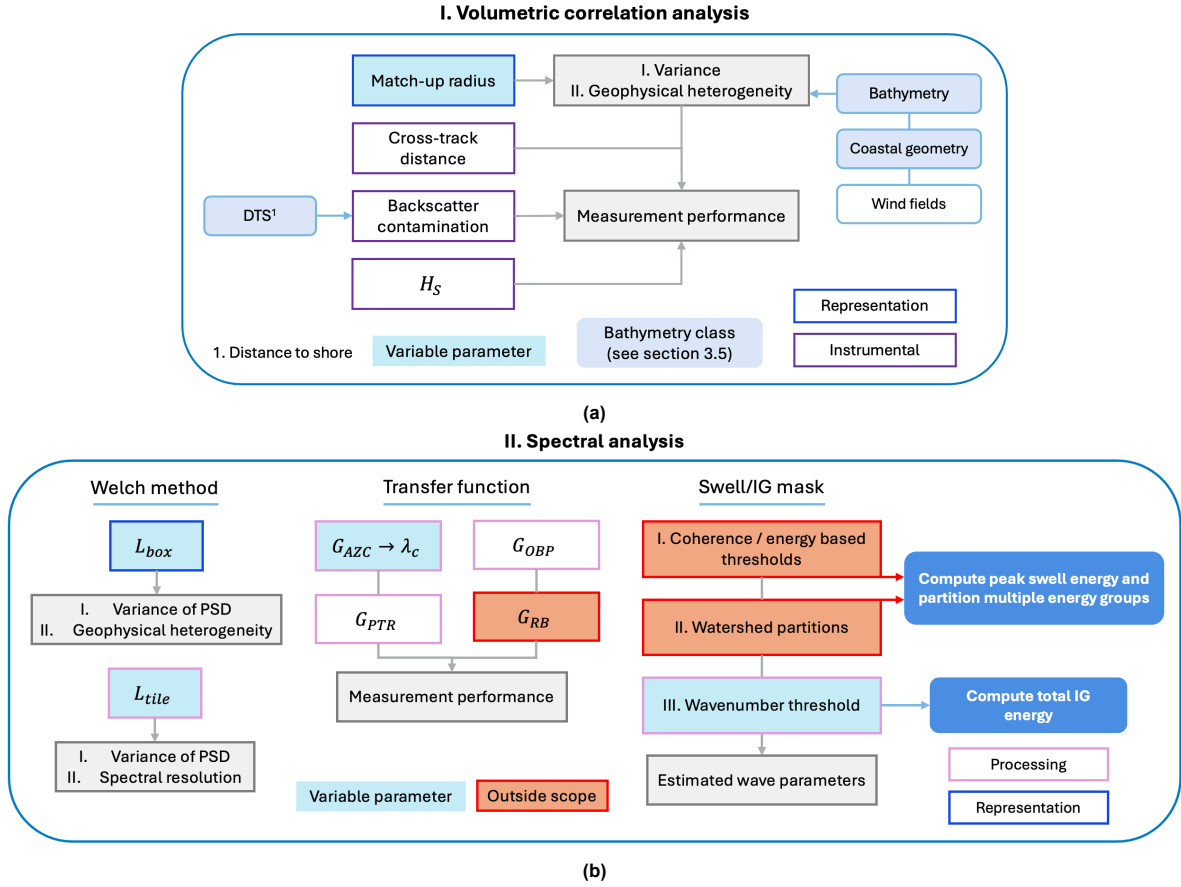


Figure 2.12: High-level framework of key parameters influencing the evaluation and outcome of SWOT's performance. The squared boxes are color-coded to indicate which parameters are influenced by representation (dark blue), instrumental (purple) and processing-related (pink) errors. This thesis focuses on analysing the sensitivity of the parameters highlighted in light blue: match-up radius, L_{box} , L_{tile} , the spectral response influenced by the azimuthal cutoff G_{AZC} and the wavenumber threshold approach for the IG mask. (2.12a) shows the variables which impact SWH retrieval; (2.12b) highlights the variables impacting the retrieval of IG wave parameters.

The framework distinguishes three main sources of error: instrumental, representation, and processing-related. Representation errors arise from the inherent temporal and spatial mismatch between SWOT observations and reference instruments (e.g. buoys or models). SWOT captures an instantaneous spatial snapshot over a two-dimensional swath, whereas wave buoys provide a temporally averaged point measurement. Differences between these spatial and temporal estimates become more pronounced when the wave field is inhomogeneous or unsteady, such as in the presence of rapidly evolving seas and atmospheric processes. Additionally, random fluctuations in wave height due to wave groups introduce further mismatch in sampling [48]. Here, the term geophysical heterogeneity refers to the spatial variability of sea surface height across the analysis domain.

Instrumental errors in the volumetric correlation analysis include parameters such as cross-track distance (CTD), backscatter contamination, and SWH. For spectral analysis, instrumental limitations are related to SWOT's radar signal and its interaction with the sea surface, as described in subsection 2.3.4. These include both random noise and systematic errors induced by wave presence.

Processing-induced errors originate from methodological choices in the retrieval workflow. These include the selection of transfer functions, IG mask definitions, and spectral resolution settings. Parameters labeled as variable in the framework are explicitly tested for sensitivity in the case-study analysis.

Overall, this framework directly informs the methodological workflow described in chapter 3, and serves as the foundation for identifying the optimal processing strategy for resolving sea state parameters in shallow to intermediate water depths.

Methodology

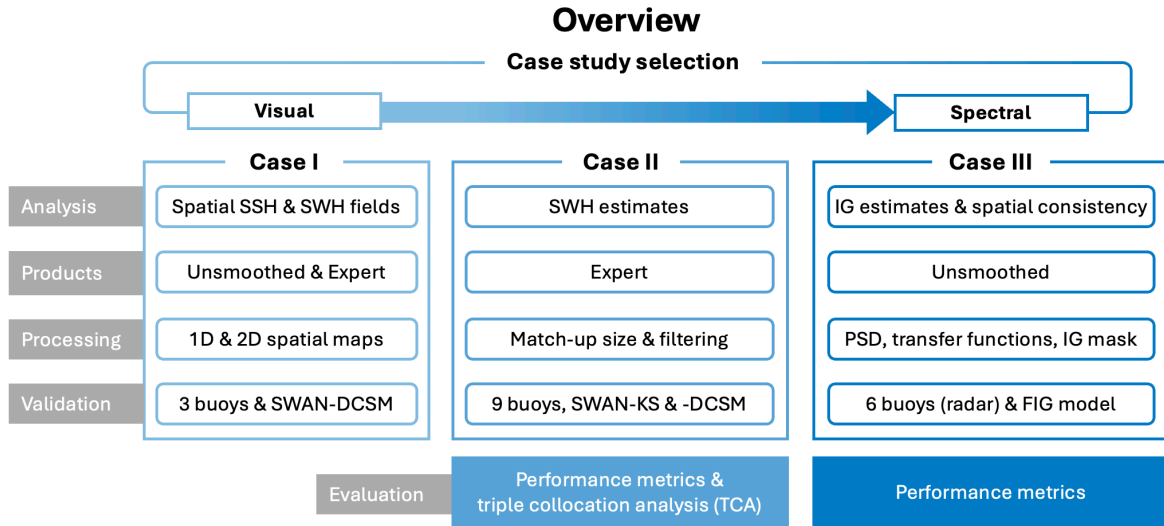


Figure 3.1: Overview of the structured approach following the top-down narrative progressing from qualitative assessment of spatial SSH and SWH fields, to the extraction of quantitative wave parameters.

This chapter outlines the structured approach used to evaluate the capability of SWOT to resolve sea state characteristics in the North Sea during the observation period from 5 November to 28 December 2024. This study is structured around three case studies, each targeting a different aspect of SWOT’s potential to observe and characterize sea state conditions in the North Sea (see Figure 3.1):

- Case I: Visual analysis of spatial SSH and SWH fields.
- Case II: SWH validation and sensitivity to match-up size and filtering choices.
- Case III: Spectral analysis of infragravity waves, including sensitivity assessment to spectral method choices.

Case I explores both 2D and 1D spatial patterns in SSH and SWH during energetic sea states, assessing SWOT’s potential to qualitatively characterize wave fields. The goal is to determine whether spatial insights into wave dynamics can be obtained from SWOT observations alone. Case II focuses on quantitative validation of SWH, delivering a recommended collocation strategy based on sensitivity to match-up size and filtering choices. Case III develops a reproducible spectral processing workflow for detecting infragravity (IG) wave energy and estimating key IG parameters: H_{IG} , L_{IG} , and θ_{IG} . Building on the framework by Ardhuin et al. (2024), methodological adaptations are introduced to enable IG wave retrievals under North Sea conditions.

Section 3.1 describes the case selection, storm classification, and analysis period. Section 3.2 introduces the SWOT data products used. Section 3.3 outlines the reference datasets (buoys and models) and the validation strategy. Sections 3.4-3.6 present the processing pipelines for each case, and section 3.7 defines the performance metrics, including triple collocation, used to evaluate accuracy and uncertainty. The codebase implementing these workflows is included in section B.5.

3.1. Case study selection and storm definition

This study focuses on the Southern North Sea, where SWOT overpasses coincide with a network of 11 in-situ measurement stations distributed across a broad range of coastal and offshore environments. These stations are located at water depths ranging from 7.2 to 42.5 m and distances to shore between 2.1 and 164 km, enabling validation across varying bathymetric and wave conditions. The case study selection leverages this spatial coverage to assess SWOT's capability in both qualitative and quantitative wave characterization. For Case III, the spectral analysis of IG waves is restricted to locations where the instrumental limitations (see subsection 2.3.5) underlying the retrieval method remain valid. Resolving wavelengths down to 500 m within the lower IG frequency range (0.033 Hz) requires a minimum water depth of approximately 29.6 m following the linear dispersion theory. Figure 3.2 displays the buoy locations used for validation (H_s and H_{IG}) and defines the spatial scope of this study.

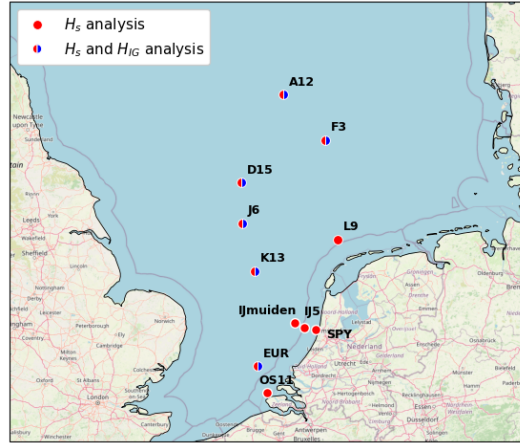


Figure 3.2: Locations of measurement stations used for H_s and H_{IG} validation.

3.1.1. Time period selection

The study focuses on SWOT's Science Orbit Phase, during which the satellite operates on a 21-day repeat cycle and provides global coverage, including overpasses of the North Sea. On average, SWOT revisits any given location in the study area approximately once per five days, depending on the exact coordinates. To ensure sufficient sampling, a two-month time window is selected, between 5 November and 28 December 2024, coinciding with multiple high-energy events along the Dutch coast. Storm identification is based on Storm Surge Flashes published by the Dutch Water Management Centre (WMCN). Within this time window, four Storm Surge Flashes were published, which provide a brief summary of the meteorological and hydrological conditions. A Storm Surge Flash is issued when at least three coastal sectors receive a pre-warning, or when a warning of code yellow or higher is provided for one or more sectors [49]. A summary of the four Storm Surge Flashes is included in section B.2.

To link SWOT observations to in-situ stations, overpasses are selected based on a proximity criterion: the center of a SWOT pixel must fall within 2 km of the buoy location. This selection ensures accurate spatial collocation between satellite and in-situ observed data. For Case II and III, the full two-month time window is utilized, covering all available SWOT overpasses that meet the proximity requirement. In Case I, the focus is restricted to the overpass occurring during the last Storm Surge Flash (22 December 2024). This event is selected to explore spatial wave patterns during energetic sea state.

Figure 3.3 shows the time series of observed SWH at buoy F3 and EUR respectively. Peaks in these figures correspond to the storm events identified in the Storm Surge Flashes issued by WMCN, with colored markers indicating selected time events used for specific in-depth case analysis. In total, four time instances are selected across different sea state conditions to test SWOT's performance under varying scenarios:

- 26 Nov 2024 ($t_{energetic,1}$): Overpass matches eight buoys. This event is not included in the official Storm Surge Flash list but exhibits high wave energy.
- 22 Dec 2024 ($t_{energetic,2}$): The final storm event of 2024, coinciding with the last Storm Surge Flash and SWOT overpass at buoy EUR.

- 7 Dec 2024 (t_{moderate}): An example of moderate conditions, one day after the 5–6 Dec Storm Surge Flash. This event is selected to assess the performance during declining sea state.
- 27 Dec 2024 (t_{calm}): A calm sea state to evaluate SWOT’s sensitivity in low-energy conditions.

The second event ($t_{\text{energetic},2}$) is used in the first Case. All four moments are used in Case III, to test sensitivity of several spectral parameters.

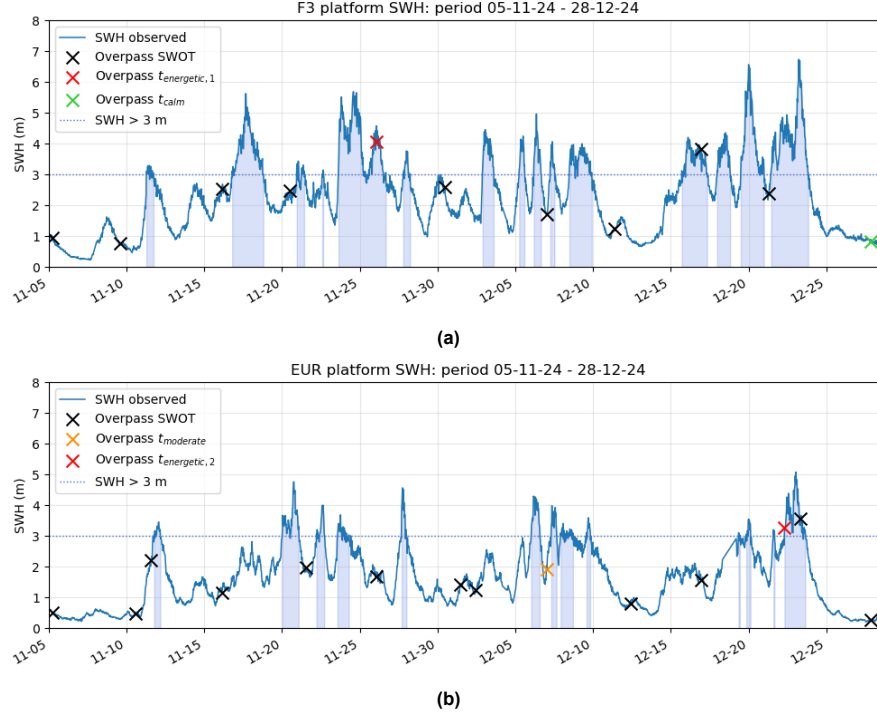


Figure 3.3: (3.3a) In-situ SWH at platform F3 during the full study time window, with the red and light orange marker indicating $t_{\text{energetic},1}$ and t_{calm} respectively; (3.3b) EUR observed SWH during the full study time window, with the red and orange marker indicating $t_{\text{energetic},2}$ and t_{moderate} respectively.

3.2. SWOT products

This thesis makes use of two primary SWOT data products:

1. The Expert product, used for SWH analysis.
2. The Unsmoothed product, used for spectral analysis.

The SWH from the Expert product is derived from the volumetric correlation at 250 m posting, 500 m resolution. This value is averaged down to fixed grid quantities of 2-km using a Hamming filter [50]. For the spectral based analysis, this research builds upon the open-source processing tool developed by Arduin et al. (2024), which enables the extraction of wave parameters from SWOT’s KaRIn interferometric measurements. For this method, the Unsmoothed L2_LR product is used which preserves native sampling at approximately 250 m, with an effective resolution of ~ 500 m. These measurements represent beam-combined averages from all nine Doppler beams and follow a swath-aligned but not geographically fixed grid [14]. An overview of the two different products including the corresponding outputs and grid sizes is provided in Table 3.1. Both products are based on the Version C PIC2 release. PIC0 refers to the original release, whereas the PIC2 version incorporates updated algorithms and resolves several previously identified issues [51].

The two-dimensional structure of the Unsmoothed and Expert datasets consists of along-track lines and cross-track pixels, with the grid dimensions defined by the number of lines and the number of pixels per line [14]. Further details on the Expert and Unsmoothed products are given in section B.1, including an illustration of the sampling grid and the geometry of the Doppler beams (see Figure B.1). In addition, a high-level schematic overview of the processing steps used within the L2 Unsmoothed and Expert products is presented in subsection B.1.1.

Table 3.1: Overview of SWOT-derived parameters and their corresponding products, processing methods, and spatial resolution.

Analysis	Product	Derivation method	Output	Grid size
SWH	Expert	Volumetric correlation	SWH	2 km
Spectral	Unsmoothed	Interferometric phase	SSH	~250 m

3.3. Validation strategy

The performance of SWOT is validated using a two-sided approach:

- In-situ validation: SWOT-derived wave parameters are compared with measurements and sea state from qualified offshore and nearshore buoys, as well as radar based spectral observations.
- Numerical validation: SWH estimates are benchmarked against high-resolution model results from the SWAN wave model.

A detailed overview of the selected stations and their application across the different case studies is provided in Table 3.2. For the validation of SWH derived from the SWOT Expert product, both in-situ and model estimates are used across the spatial extent of this study. The Multifunctional Access Tool foR Operational Oceandata Services (MATROOS) is used to obtain estimates of the sea state as time series. MATROOS offers easy access to all recent model and monitoring data and is therefore used as primary tool for the retrieval of observation data. Validation of H_{IG} estimates, obtained via spectral analysis, is performed using platform-mounted radar measurements, following the methodology of Akrish et al. (2025). Buoy locations include K13, A12, EUR, J6, D15, and F3.

The remainder of this section elaborates on the measurement principles of the different observational instruments used for validation.

3.3.1. In-situ measurements

In-situ sea state parameters used for validation in this study are obtained from two primary observational systems: the MATROOS buoy network and platform-mounted radar measurement stations. For the analysis of SWH detailed in section 3.5, buoy data is retrieved from the MATROOS server, which provides wave statistics derived from vertical displacement time series. For the spectral analysis focused on infragravity waves, radar stations providing 1D spectra from sea surface elevations, are used to estimate H_{IG} and L_{IG} .

Name	Lat [°]	Lon [°]	Depth [m]	DTS (km)	Case study	In-situ	Model
OS11	51.644	3.481	7.2	7.8	H_s	Buoy	NS & KS
SPY	52.465	4.518	15.0	2.1	H_s	Buoy	
IJ5	52.493	4.268	20.9	19.5	H_s		NS & KS
L9	53.617	4.967	24.0	29	H_s	Buoy	NS & KS
IJmuiden	52.550	4.058	24.9	35	H_s	Buoy	NS & KS
K13	53.218	3.219	28.1	100	H_s & H_{IG}	Buoy & radar	NS & KS
A12	55.399	3.810	29.7	240	H_s & H_{IG}	Buoy & radar	NS
EUR	51.999	3.275	30.1	40	H_s & H_{IG}	Buoy & radar	NS & KS
J6	53.823	2.944	35.5	142	H_s & H_{IG}	Buoy & radar	NS
D15	54.325	2.934	39.9	175	H_s & H_{IG}	Radar	NS
F3	54.851	4.717	42.5	164	H_s & H_{IG}	Buoy & radar	NS

Table 3.2: Overview of the measurement stations and their specifications. The observations were collected between 5 November and 28 December 2024. Data availability per station is indicated in the last two columns. Water depths corresponding to the Lowest Astronomical Tide (LAT) are derived by using the coordinate locations (from MATROOS) of the stations in the EMODnet bathymetry map [52]. The wave models in this study include the SWAN-North Sea (NS) and SWAN-Kustrook (KS) models.

MATROOS buoys

The MATROOS buoys estimate SWH based on vertical displacement measurements, which capture the frequency-dependent energy content of the wave field. These signals are processed through a series of steps to derive the wave energy spectrum using Fourier analysis. The resulting SWH values are obtained from 20-minute energy density spectra with a frequency resolution of 10 mHz [53]. For this study, data are sampled at 30-minute intervals, meaning that every half hour a SWH estimate is produced based on this spectral processing method.

Radar stations

For the validation of SWOT-derived IG wave parameters, platform-mounted radar measurements are used, following the approach of Akrish et al. (2025). These downward-looking radars, positioned 10–30 m above mean sea level, provide discrete surface elevation time series sampled at 2.56 Hz [6]. From these, one-dimensional frequency spectra (60-minute time window, with a frequency resolution of 1.67 mHz) are computed in the 30–200 s band to estimate H_{IG} and L_{IG} .

3.3.2. Wave models

Numerical wave model data used for validation of SWH are obtained from the MATROOS server, which provides sea state estimates from two nested SWAN models. The SWAN model, developed at Delft University of Technology, is a third-generation spectral wave model designed to simulate the generation and propagation of random, wind-generated waves in coastal and shallow water environments [3].

The SWAN Dutch Continental Shelf Model (SWAN-DCSM) also known as the North-Sea (NS) model, covers the broader North Sea and has a spatial resolution ranging between 2.5 and 4 km [5]. To accurately resolve wave dynamics in shallow nearshore areas where bathymetric gradients are more pronounced, the SWAN-Kuststrook (SWAN-KS) model is used. This high-resolution nested model features a curvilinear grid with spatial resolution varying from approximately 35 m nearshore to 2600 m at its offshore boundaries. Boundary conditions are derived from the European Centre for Medium-Range Weather Forecasts Wave Analysis Model (ECMWF-WAM) [54]. An overview of the spatial coverage and nesting of the SWAN-DCSM and SWAN-KS domains is presented in Figure 3.4.



Figure 3.4: Model domains of SWAN-DCSM (NS) and SWAN-Kuststrook [55].

It is important to note that both the SWAN-KS and SWAN-NS do not resolve IG wave energy, as they are limited to sea-swell wave processes. Consequently, no numerical model data is used for direct validation of H_{IG} in this study. However, to examine the spatial consistency and behaviour of SWOT-derived IG wave height, comparisons are made with outputs from the FIG model developed by Akrish et al. (2025), which simulates free IG wave fields in the North Sea. This comparative approach is further detailed in subsection 3.6.7.

3.4. Case I - Spatial analysis of SSH/SWH fields: processing steps

3.4.1. Data selection

For Case I, SSH patterns are analysed using the Unsmoothed product, while SWH patterns are examined using the Expert product. Both analyses cover the Southern North Sea (1°E - 6°E , 51°N - 56°N). For both products, only observations with quality flag (QF) 0 are retained to exclude degraded or low-quality data.

3.4.2. SSH analysis

SSH data is retrieved from the `ssh_karin_2` variable, which represents the fully corrected SSH measured by the KaRIn instrument, relative to the reference ellipsoid specified in the product metadata [14]. For visualization purposes, the SSH field is mean-centered by subtracting its average value.

To inspect spatial SSH patterns, a 40×40 km box is extracted around buoy K13 (53.2°N , 3.2°E). This approach follows Ardhuin et al. (2024), who applied the least-squares method for detrending. An illustrative example of this method is provided in Figure 3.5, which shows clear long-period swell propagation for SWOT cycle 005, track 328 near the Australian coast on 16-10-2023 (38.05°S , 94.56°E). The corresponding 2D PSD, corrected for filtering effects, reveals a distinct swell energy peak (red mask).

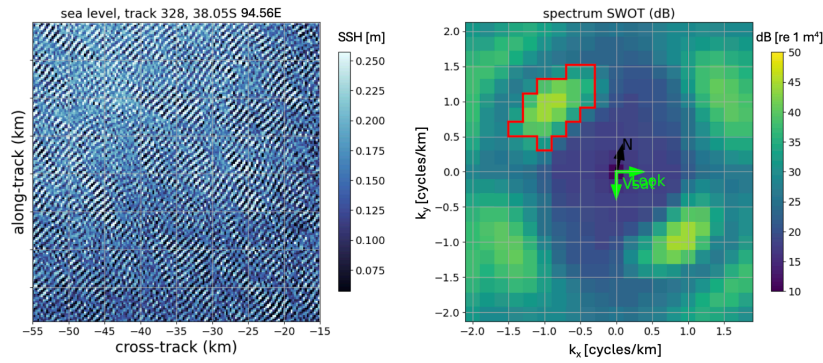


Figure 3.5: Detrended SSH derived from a 40 km box near the Australian coast on 16-10-2023 (left); corresponding PSD in the k_x, k_y domain (right). The red mask highlights the swell peak used for wave parameter estimation [40].

3.4.3. SWH analysis

SWH estimates from the Expert product are analysed over the same spatial domain, together with modelled wind directions from SWAN-DCSM at selected buoy locations. During the SWOT overpass, wind speeds ranged from 14.0 to 17.7 m/s across six buoys, with corresponding wind directions between 280° and 295° , indicating relatively uniform conditions. An overview of the wind conditions per buoy is provided in Table B.4.

To examine spatial patterns in SWOT-derived SWH and assess the potential influence of bathymetry, values are extracted along three transects aligned with pixel indices 40, 49, and 51. These pixels correspond to the satellite-buoy matches with K13, J6, and D15, respectively. The spatial extent ranges from 52 to 56°N for transects over J6 and D15, and from 51.8 to 56°N for K13, encompassing a range of water depths from offshore to shallow coastal zones. Water depths are extracted from the EMODnet bathymetry dataset using the pixel coordinates along the transects.

3.5. Case II - SWH validation: processing and sensitivity analysis

3.5.1. Data selection

To evaluate the performance of SWOT-derived SWH estimates on the North Sea, two datasets are constructed. Both include 22 SWOT overpasses where the center of at least one pixel falls within 2 km of a buoy location, ensuring spatial collocation with in-situ observations. Only data with a quality flag of zero is retained. The base dataset follows the approach of Turki et al. (2025), where single-pixel SWH values located as close as possible to buoy coordinates are extracted. No further filtering is applied, allowing direct comparison with results from the English Channel (Turki et al., 2025) and serving as a benchmark to assess the impact of additional processing.

A second, filtered dataset is used to investigate under which conditions SWOT most accurately retrieves SWH. This dataset introduces two targeted processing steps. First, all zero-valued SWH estimates are removed, as these correspond to retrieval failures in capturing small waves. This limitation was observed by Turki et al. (2025) in the earlier PIC0 version. While this issue has been partially addressed, zero values still remain present in the PIC2 version used in this study. Second, outlier values are filtered based on the range of observed and modelled SWH within the two-month analysis window. Given that the maximum recorded SWH from buoys and the wave model are 6.84 m and 4.92 m respectively, a conservative upper threshold of 8 m is applied. Only one clear outlier (at buoy J6, 2024-12-07) exceeds this value. During the same overpass, D15 also shows a high estimate of 6.6 m, while model data reports 1.96 m.

The filtered dataset is used to examine the sensitivity of SWOT performance to key parameters highlighted in Figure 2.12, including CTD, sea state, and bathymetric setting. This enables a systematic evaluation of the conditions under which SWOT can reliably and accurately estimate SWH in the North Sea.

3.5.2. Match-up criteria

An important methodological extension for the second case study is the refinement of the match-up criteria, as motivated in section 2.2. Traditional altimeter–buoy validation typically applies a fixed spatial window of 50 km and a temporal tolerance of 30 minutes. However, such thresholds become problematic in coastal regions due to the increased spatial variability of the wave field.

The high spatial resolution and wide-range swath coverage of SWOT enables a more localised assessment of match-up sensitivity. Specifically, this study investigates how varying the spatial averaging window impacts the accuracy of SWOT–buoy comparisons in nearshore wave conditions. The SWH estimated by SWOT is derived by averaging the γ_{vol} over a spatial domain of 2 by 2 km [50]. Match-up radii in this study are defined by the number of pixels used in the averaging process. The following grid sizes are tested:

- Small: 2×2 km (single pixel)
- Mid: 4×4 km (four pixels)
- Large: 8×8 km (sixteen pixels)

In addition to spatial alignment, temporal consistency between SWOT and in-situ observations is required. Each SWOT SWH estimate represents a single temporal snapshot corresponding to a 2×2 km patch. To ensure reliable comparison, in-situ or model-based SWH values, computed from the variance of sea surface elevation, are interpolated to the pixel time of the SWOT overpass. The in-situ measurement estimates are derived from a dataset with a 30 minute interval, while the SWAN model outputs are produced hourly. The model data is linearly interpolated between consecutive hourly values to obtain continuous time series. Missing in-situ observations are not extrapolated beyond the available measurement periods.

3.5.3. Cross-track distance

As discussed in subsection 2.3.2, the accuracy of SWH retrieval is dependent on the cross-track range. In particular, the combination of low SWH values and observations taken at far-range positions of the swath can introduce large uncertainties. To examine the effect of such far-range observations, a CTD threshold of 50 and 55 km is applied. Observations beyond these thresholds are excluded to assess the potential improvement in retrieval accuracy when focusing on the more central region of the SWOT swath.

3.5.4. Bin categories

Beyond the application of a fixed cross-track threshold, the sensitivity of SWOT-derived SWH estimates is further analysed using a binning approach. As described in 2.3.2, the accuracy of SWOT-derived SWH estimates depends on both CTD and sea state. To quantify these effects for North Sea conditions, several cross-track and H_s bins are applied. In addition to the SWH and cross-track position, coastal proximity and seabed characteristics are also known to affect retrieval performance.

Bathymetry class

Turki et al. (2025) showed that in the English Channel, higher SWH retrieval accuracy can still be achieved in nearshore locations, provided that the local water depth is sufficient (18 m in their study) [13]. To explore this further, a third classification, the bathymetry class, is introduced. This classification accounts for a combination of water depth, distance to shore (DTS), and bathymetric complexity to distinguish three regions:

1. Offshore (depth > 29.2 m)
2. Nearshore (DTS < 35 km)
3. Highly variable bathymetry

The nearshore threshold of 35 km is chosen arbitrarily to distinguish coastal proximity. The offshore depth threshold of 29.2 m is based on the highest in-situ observed T_{m02} within 35 km of the shore. Using the linear dispersion relationship, this wave period corresponds to a wavelength for which a water depth of 29.2 m marks the transition to deep water conditions. Figure 3.6 illustrates the spatial seabed variability of the Southern North Sea, along with the measurement stations and their assigned bathymetry classes.

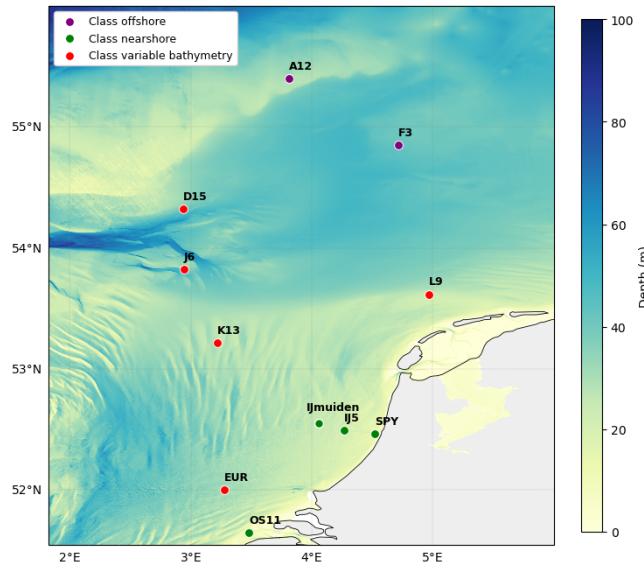


Figure 3.6: Bathymetry of the Southern North Sea with the buoy locations used for SWH validation and their corresponding bathymetry class [52].

Variability of sea surface

A fourth factor considered is the variability of the sea surface, which is particularly relevant when comparing spatially averaged SWOT retrievals with single-point buoy measurements. It is hypothesized that local variability decreases as wave fields evolve from young, wind-driven seas toward more mature swell conditions. As the wave period increases, wave groups tend to become more coherent and organized, leading to a smoother sea surface. Spatial averaging is therefore expected to perform better under longer-period wave conditions. To test this, the dataset is also binned by wave period (T_{m02}).

In addition to this wave period-based binning, an extra assessment is performed using the wave age method. This method parametrically separates wind-sea and swell conditions based on the relationship between wind speed and wave phase speed. In a 1D wave spectrum, the wave field is typically classified as wind-sea when:

$$\frac{U_{10}}{c} > \beta \quad (3.1)$$

Where U_{10} is the wind velocity at 10 m above the surface, c is the wave phase speed and β is a threshold calibration factor typically ranging from 0.50 to 0.83 [56]. While the exact value of β is not critical in this study, the ratio U_{10}/c serves as an indicator of sea state maturity. Higher values represent younger, wind-driven conditions, whereas lower values corresponding to more developed, swell-dominated wave fields. For this analysis the wind and wave related parameters are taken from the in-situ observations.

Together, these analyses characterize the key factors influencing SWOT SWH retrieval, as illustrated in Figure 2.12. An overview of the different bins per category is provided in Table 3.3. This structured categorisation enables a systematic assessment of the conditions for which SWOT most accurately and reliably estimates significant wave height in the North Sea.

Table 3.3: Overview of match-up grid sizes and bins used for sensitivity analysis of the CTD, significant wave height (H_s), wave period (T_{m02}), and bathymetry class.

Bin category				
Grid size (km)	Cross-track (km)	H_s (m)	T_{m02} (s)	Bathymetry class
2×2	0–25	0–1	3–5	Offshore
4×4	25–50	1–3	5–7	Variable bathymetry
8×8	> 50	> 3	> 7	Nearshore

3.6. Case III - Spectral analysis: processing and sensitivity analysis

The spectral analysis in this thesis builds upon the open-source processing tool developed by Arduin et al. (2024), originally designed to extract frequency-dependent sea state parameters from SWOT data in open-ocean conditions. Their method targets long-period swell (18–30 s), and adaptations are required to extend its applicability to the infragravity frequency band (30–200 s) relevant for the North Sea. The objective is to evaluate the sensitivity of key spectral processing parameters to the estimated sea state characteristics, and to identify an optimal configuration for coastal applications. This study focuses on the impact of tile length L_{tile} , box size L_{box} , the transfer functions including the azimuthal cutoff response function G_{AZC} , and the configuration of the IG mask, as introduced in section 2.4. Figure 3.7 summarizes the three main processing steps used to derive IG wave parameters, such as the infragravity wave height H_{IG} , wavelength L_{IG} and direction θ_{IG} . The remainder of this subsection further elaborates on these steps and the key parameters influencing the spectral estimates.

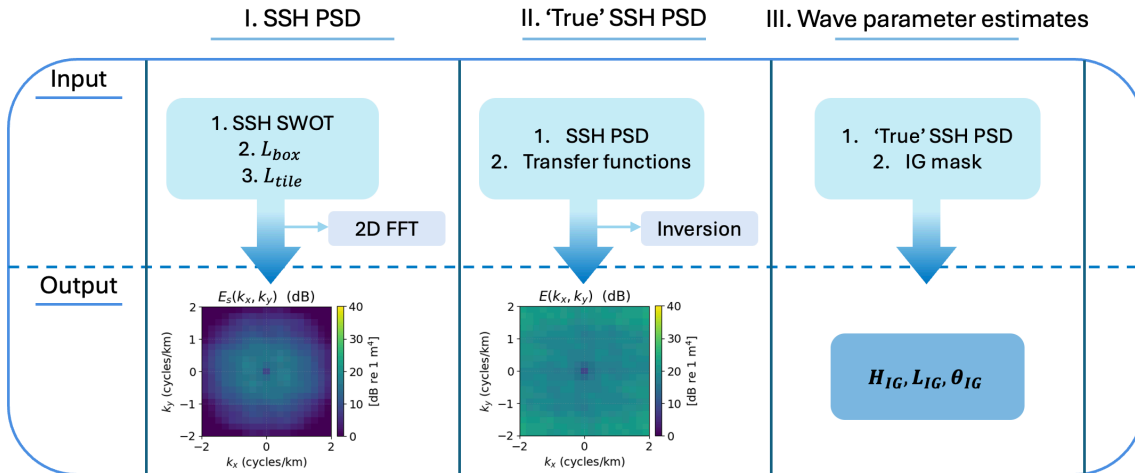


Figure 3.7: Overview of the three processing steps used in spectral analysis: (I) 2D FFT computation, (II) Inversion of the SWOT SSH PSD using transfer functions and (III) IG mask computation to isolate IG wave energy and compute final wave parameter estimates.

3.6.1. Data selection

To obtain the Unsmoothed dataset used for spectral analysis, a subset of overpasses from the SWH validation analysis is selected based on data quality. Specifically, only the overpasses are retained where all 16 pixels within the match-up area meet the quality criterion ($QF = 0$). This filtering step ensures that only high-quality observations are included. As shown in Figure 3.2, the spectral validation is conducted using six buoy locations: A12, F3, D15, J6, K13, and EUR. Applying the quality filter yields a dataset consisting of 33 SWOT-buoy matches that satisfy the pixel validity requirement based on the SWH analysis.

During the 33 SWOT-buoy match events, platform-mounted radar observations of H_{IG} ranged between 4.3 cm to 10.47 cm. To extend the validation range and assess SWOT performance under more extreme sea states, an additional event from the 2023–2024 winter season (SP23/24) is included. In the latter period, H_{IG} values are obtained from the radar measurements at buoy locations used in the study by Akrish et al. (2025). A threshold of $H_{IG} \geq 12$ cm is applied to identify periods of energetic infragravity wave activity at measurement locations K13, EUR, J6, D15, and F3. One matching SWOT overpass during such an event occurred on 21 October 2023 at F3, with an in-situ observed sea-swell SWH of 5.82 m and an H_{IG} of 13.34 cm. This case is added to the dataset to evaluate SWOT performance under higher-energy IG conditions.

3.6.2. 2D FFT approach

As detailed in subsection 2.3.3, the SWOT SSH PSD is computed using 2D windowed segments extracted along the satellite swath. As demonstrated by Morrow et al. (2019) and Peral et al. (2015), the height error associated with SWOT observations is not spatially uniform across the swath [36, 57]. Performance is best in the mid-swath region, where instrument noise is minimized. Therefore, to reduce the impact of noise and improve the reliability of spectral estimates, large square patches are extracted from the central region of the swath. To align each patch as closely as possible with the measurement station, the center of the analysis box is positioned at the same latitude.

However, this setup introduces a longitudinal offset between the box center and the buoy location. This results in a trade-off between minimizing instrumental noise and minimizing spatial mismatch with in-situ observations. To assess the sensitivity of PSD estimates to this trade-off, an additional methodological step is introduced. SWOT computes SSH estimates for both left and right swaths, covering cross-track distances from 10 to 60 km for each side. For stations located more than 20 km from the swath center, the 10 km box is repositioned as close as possible to the in-situ measurement location, within the constraints of the swath width. This allows to evaluate whether spatial alignment improves agreement with platform-mounted radar observations, and whether sea surface conditions remain sufficiently homogeneous across the swath to justify this adjustment despite increased noise exposure.

In addition to spatial placement, a second trade-off governs the choice of box and tile dimensions used in spectral estimation. Different combinations of L_{box} and tile size (L_{tile}) are tested to evaluate their effect on spectral accuracy. These configurations are summarized in Table 3.4, along with the corresponding EDF, which indicate the statistical reliability of the spectral estimates. A schematic overview of all five configurations is provided in Figure B.3. The trade-offs are outlined as follows:

- Increasing L_{box} reduces the variance of the spectral estimate, but increases the likelihood of geophysical heterogeneity within the box.
- Increasing L_{tile} improves spectral resolution and enables detection of longer wavelengths, but leads to higher estimation variance due to fewer segments.

To guide the choice of L_{tile} , a physical constraint is imposed based on the lower frequency bound of the IG wave band (200 s) at the maximum water depth considered in this study (42.5 m). According to the linear dispersion relation, a minimum tile length of 4 km is required to reliably resolve IG wave energy.

Version	L_{box} [km]	L_{tile} [km]	EDF
Alpha	40	4	200
Beta	40	5	128
Gamma	40	10	32
20 km	20	5	32
10 km	10	5	8

Table 3.4: Overview of different spatial scales used in box-tile combinations and associated equivalent degrees of freedom (EDF).

These configurations allow for systematic exploration of spectral accuracy versus spatial resolution, enabling a better understanding of how to tailor SWOT-based wave retrievals for coastal environments with higher spatial sea surface variability. As explained in subsection 2.3.3, for the computation of SSH power spectral density ($E_s(k_x, k_y)$), a two-dimensional Welch method is used. Each L_{box} is subdivided into multiple individual L_{tiles} with a 50% overlap between tile segments. The segments are modified using 2D data windows. Similar to the approach of Ardhuin et al. (2024), a 2D Hanning window is applied before performing the FFT of the SSH.

3.6.3. Transfer function components

The transfer function G is divided into along-track and cross-track filter contributions. These contributions can be expressed in cross-track (x-axis) and along-track (y-axis) as follows:

$$G_x = G_{OBP,x} \quad (3.2)$$

$$G_y = G_{OBP,y} \quad (3.3)$$

Here, the G_x accounts for the 500 m onboard averaging and the kernel shape applied by the OBP in the cross-track direction, while G_y captures the same processing effects in the along-track direction. These are combined to form the full OBP transfer function:

$$G_{OBP} = G_x G_y \quad (3.4)$$

$$G = G_{OBP} G_{PTR} G_{AZC} \quad (3.5)$$

where G_{PTR} contains the point-target response in along-track, including Doppler beam sharpening and beam weights [40]. The formulation of G_{AZC} is provided in Equation 2.25.

The spectral responses associated with the point target response and onboard processing are instrument defined and are taken as fixed in this study. In contrast, the azimuthal cutoff response (G_{AZC}), which accounts for the sea-state-dependent loss of azimuthal resolution due to velocity bunching, varies with surface conditions. For each overpass, G_{AZC} is recomputed based on wave orbital velocities estimated from local buoy observations. Because the applicability of the azimuth cut-off correction to interferometric phase is still under investigation, a sensitivity analysis is included. This involves scaling the cutoff wavelength λ_c by factors of 0.8, 1.0, and 1.2 to quantify the resulting variations in the retrieved IG wave parameters.

3.6.4. Backward modelling of SSH PSD

This study follows the inversion approach developed by Ardhuin et al. (2024), assuming that the KaRIn processing behaves linearly. That is, radar and interferometric operations are considered to affect the estimated SSH heights in the same way they affect the interferometric phase [43]. As with Ardhuin's implementation, the non-linear spectral response due to the surfboard effect is neglected.

As illustrated in Figure 2.11, the full instrumental filter (OBP + PTR + AZC) attenuates spectral power even below the per-axis Nyquist limit ($|k| < 2$ cpk). Without correcting for this attenuation, directly integrating the raw SWOT SSH PSD E_s leads to a systematic underestimation of wave energy and derived parameters such as H_{IG} .

To compensate for this spectral distortion, the inverted (or "true") ocean spectrum is obtained by dividing the observed spectrum by the transfer function G :

$$E(k_x, k_y) = \frac{E_s(k_x, k_y)}{G(k_x, k_y)} \quad (3.6)$$

Wave parameters, such as H_{IG} , are then derived by integrating $E(k_x, k_y)$ within the appropriate IG mask.

To assess the sensitivity of the results to different filtering effects, five configurations are tested:

- G_{OBP} only: isolates the impact of the OBP filters
- G_{PTR} only: isolates the effect of the PTR
- G_{AZC} only: isolates the velocity bunching filter

- No G : baseline spectrum without any correction
- Full G : combined response of G_{OBP} , G_{PTR} , and G_{AZC}

As discussed in subsection 2.3.4, instrumental and wave-induced noise, while minimized near the swath center, remains present and influences the accuracy of SSH observations. While OBP filters suppress much of this noise and mitigate aliasing effects, they do not eliminate them entirely. During spectral inversion, especially at higher wavenumbers where G strongly attenuates the signal, the inverse filter $1/G$ may amplify residual noise. Therefore, special care must be taken when interpreting high-wavenumber energy content, as these may be contaminated by enhanced noise rather than true wave signal.

3.6.5. IG mask definition

As highlighted in subsection 2.3.3, the research conducted by Ardhuin et al. (2024), focuses on extracting the wave energy associated with the dominant swell group present in the PSD. In this approach, a directional mask to the SWOT SSH spectrum is applied, followed by normalisation and thresholding to isolate the core spectral component associated with swell energy. This approach effectively isolates the dominant swell group but excludes energy from other propagation directions, making it unsuitable for estimating total wave energy.

To extract total IG wave energy for comparison with platform-mounted radar observations of H_{IG} , a direction-independent masking approach is required that captures energy from all propagation directions. The process of identifying the IG wave mask follows two steps: wavenumber magnitude filtering and energy-based thresholding. This approach comprises a binary mask that retains spectral energy within the IG wave band while suppressing noise and unrelated wave energy.

1. Wavenumber magnitude filtering

The IG mask is first defined based on the magnitude of the wavenumber vector:

$$|k| = \sqrt{k_x^2 + k_y^2} \quad (3.7)$$

This ensures that all wave energy within a specific wavenumber range is captured, regardless of propagation direction, resulting in an annular mask in 2D wavenumber space. The bounds of this mask are derived from the linear dispersion relation for IG wave periods between 30 and 200 seconds, at the local water depth:

$$k_{min} \leq |k| \leq k_{max} \quad (3.8)$$

Here, k_{min} corresponds to the wavelength associated with a 200 s period, and k_{max} to 30 s. This step filters out wave energy from swell and high-frequency noise by retaining only the wavenumber range expected for IG waves.

2. Energy thresholding

To further separate IG signal from background noise, an energy-based threshold is applied to the filtered spectrum. The maximum value of the filtered energy spectrum $E_{filtered}$ is used to normalize the data:

$$E_{max} = \frac{\alpha}{\max(E_{filtered})} \quad (3.9)$$

$$E_{norm} = E_{filtered} \times E_{max} \quad (3.10)$$

The normalized energy E_{norm} is thresholded at a fixed level of 0.5, which produces a binary mask. The relationship between the threshold ($\tau = 0.5$), normalization factor (α), and the minimum retained energy fraction is [40]:

$$E_{filtered} > \max(E_{filtered}) \frac{\tau}{\alpha} \quad (3.11)$$

The minimum energy fraction retained is τ/α , where the normalization factor α controls the sensitivity of the threshold:

- Higher α lowers the effective threshold, resulting in a more inclusive mask.

- Lower α increases the threshold, retaining only the strongest spectral peaks.

The chosen threshold balances signal retention and noise suppression, ensuring that only the most relevant spectral features within the IG band are included in further analysis. In this study, a value of $\alpha = 10$ is used, which excludes pixels below 5% of the peak. For comparison, $\alpha = 15$ lowers this threshold to 3.3%. Arduin et al. (2024) employs factors of 2-4 for swell waves. The chosen higher factor compensates for the substantially lower spectral energy density of IG waves.

An additional methodological consideration is whether to compute the IG mask from the original SWOT SSH spectrum or the inverted spectrum. This decision affects the resulting mask shape and its sensitivity to spectral energy distribution:

- Original spectrum (E_s): Represents the energy observed by SWOT, including attenuation due to instrument response. Energy in the higher wavenumber range is suppressed, resulting in fewer pixels exceeding the threshold for inclusion in the IG mask.
- Inverted spectrum (E): Reflects the estimated true sea surface spectrum, corrected for instrumental attenuation.

In this study, the IG mask is computed using the SWOT SSH spectrum. This choice avoids potential amplification of residual noise introduced during the backward modelling of the PSD.

To evaluate the sensitivity of the IG mask to the normalization factor α (10 versus 15) and the choice of spectrum, a test is performed using measurement location F3 during the overpass of $t_{energetic,1}$ and t_{calm} . The results inform the choice of α and the preferred spectrum representation for stable and reliable IG wave height estimation.

3.6.6. Computation of IG wave parameters

Following the definition of the IG mask in subsection 3.6.5, wave parameters are derived by integrating the SSH PSD values within the masked region. The IG wave height (H_{IG}), wavelength (L_{IG}) and mean wave direction (θ_{IG}) are computed as follows:

$$H_{IG} = 4 \sqrt{\sum_{k_x, k_y \in \text{IG mask}} E(k_x, k_y)} \quad (3.12)$$

The energy-weighted mean wavelength (L_{IG}) is estimated by:

$$L_{IG} = \frac{\sum_{k_x, k_y \in \text{IG mask}} E(k_x, k_y) \cdot \frac{1}{\sqrt{k_x^2 + k_y^2}}}{\sum_{k_x, k_y \in \text{IG mask}} E(k_x, k_y)} \quad (3.13)$$

Finally, the mean wave direction is computed by:

$$\theta_{IG, \text{SWOT}} = \arctan 2 \left(\sum_{k_x, k_y \in \text{IG mask}} \frac{k_x E(k_x, k_y)}{N}, \sum_{k_x, k_y \in \text{IG mask}} \frac{k_y E(k_x, k_y)}{N} \right) \quad (3.14)$$

With $E(k_x, k_y)$ the SSH PSD corrected for the different filtering effects as described in subsection 3.6.4 and N the number of pixels within the IG mask. These formulations are consistent with the approach described in the handbook of DUACS L3_LR_WIND_WAVE [43], however adapted in order to estimate the total IG wave height and corresponding wavelength and mean direction. The focus of this study is to evaluate which configuration of SWOT-derived parameters yields the best agreement with in-situ observations of H_{IG} . Since both L_{IG} and θ_{IG} are derived from the same underlying PSD $E(k_x, k_y)$, their accuracy is inherently linked to the quality of the retrieved wave energy and thus to H_{IG} performance. As such, L_{IG} and θ_{IG} are not systematically validated across the dataset. Instead, their validity is qualitatively assessed for the two energetic overpasses on 26 November and 22 December 2024. Additionally, due to the absence of directional in-situ observations, no direct validation of θ_{IG} can be performed.

3.6.7. Spatial consistency

To qualitatively assess the spatial consistency and reliability of SWOT-derived IG wave heights, the ratio between modelled FIG wave height and the total IG wave height retrieved from SWOT is evaluated. This comparison is performed for the two energetic overpasses, on 26 November 2024 and 22 December 2024, across a large spatial domain in the North Sea.

The analysis uses SWOT swath data aligned along-track, with H_{IG} and L_{IG} estimated every 10 km along the swath (approximately every 43 pixels, given the ~ 235 m pixel resolution). For the spectral analysis, the 10 km configuration is used (see Table 3.4). Corresponding water depths for each analysis box are obtained from the FIG model outputs at the exact geographic coordinates of the SWOT track. Only locations with water depths greater than 29.6 m are included, based on the minimum depth required for SWOT to fully capture the infragravity wave frequency band, as determined by the linear dispersion relation. The overpass tracks of both swaths are shown in Figure 3.8, with swath tracks indicated as 'L' (left) and 'R' (right).

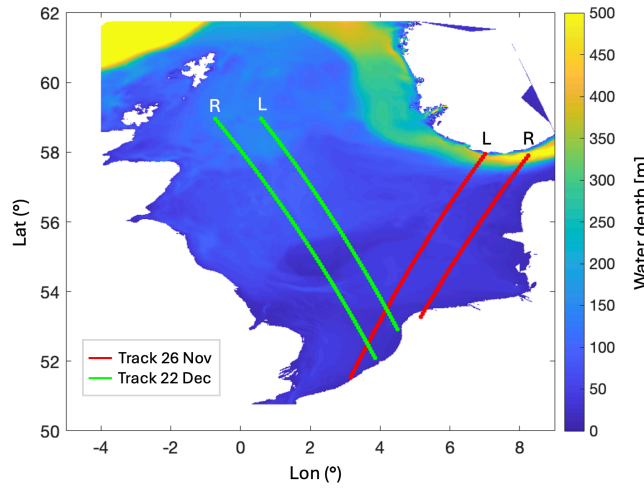


Figure 3.8: SWOT overpass tracks during $t_{energetic,1}$ (26 November 2024) and $t_{energetic,2}$ (22 December 2024), shown in red and green lines over the North Sea. Tracks are labeled as 'L' (left) and 'R' (right).

Since the FIG wave model only represents the free infragravity component, this comparison provides an upper-bound estimate of the FIG contribution relative to the total IG energy observed by SWOT. The ratio is defined as:

$$R = \frac{H_{FIG,model}}{H_{IG,SWOT}} \quad (3.15)$$

R -values of 0.7 indicate that BIG and FIG heights are similar, which follows from the quadratic relationship between wave height H and energy E ($E \propto H^2$).

Based on previous studies, the expected FIG-to-total IG ratio varies by location and sea state. Under energetic conditions on open coasts, FIG and BIG contributions are typically more balanced, leading to ratios closer to one [6]. However, as discussed in subsection 2.1.3, observations from nearshore locations in the North Sea have shown substantially higher FIG/BIG energy ratios, with FIG waves often dominating the infragravity spectrum. This contextual background is considered when interpreting the SWOT–model comparison results.

3.7. Evaluation metrics

In this study, several statistical metrics are used to evaluate the agreement between SWOT-derived wave parameters and reference data. The selected metrics follow established practices in wave data validation [13, 58, 59]:

- Bias: the mean difference between SWOT estimates and reference values. It gives an indication of a systematic over- or underestimation.
- Mean absolute error (MAE): the average of the absolute differences between each pair of SWOT and reference values (less sensitive to outliers).

- Root mean square error (RMSE): the square root of the average squared difference between SWOT and reference pairs (more sensitive to outliers).
- Scatter index (SI): measurement of the magnitude of deviation between SWOT and reference values relative to the magnitude of the buoy or model observations. The smaller the SI, the better agreement between SWOT and reference estimates.
- Coefficient of determination (R^2): measure of the association between SWOT and the observation estimates. A coefficient of one implies positive correlation, and thus a well-fitted estimation outcome.
- Standard deviation (STD): represents the variability in the residuals (SWOT–reference differences), characterizing the precision of the estimates.

The metrics can be broadly categorised as follows: bias, MAE, and RMSE measure accuracy, capturing systematic and absolute deviations. In contrast, STD, SI, and R^2 reflect precision, indicating how consistently the estimates match the reference data. The mathematical formulations of the metrics are further defined in subsection B.6.1.

3.7.1. Triple collocation analysis

A triple collocation (TC) analysis is performed to objectively quantify the instrument-specific errors and calibration coefficients associated with the three independent SWH data sources: in-situ buoys, SWOT observations, and SWAN model estimates. In addition to the validation metrics described in section 3.7 (bias, MAE, RMSE, R^2 , SI, and STD), TC analysis provides insight into the random and systematic errors inherent to each dataset. The results of the TC analysis support the selection of the final processing configuration adopted for SWOT SWH retrieval in the North Sea, including the choice of match-up size and CTD threshold. To assess the impact of data quality filters on error estimates, the datasets are filtered using a minimum SWH threshold of 1 m and a CTD limit of 55 km.

This study applies the standard TC framework, also used by Dodet et al. (2022). The method assumes that the reference system (buoy) is bias-free, that SWOT and model systems have linear calibration errors, and that random measurement errors are uncorrelated between systems and uncorrelated with the true geophysical signal [60].

However, when measurement systems operate at different spatial or temporal scales, the assumption of uncorrelated errors may be violated due to representation error. In the present study, SWOT provides instantaneous 2×2 km snapshots, buoys represent point measurements averaged over 20-minute intervals, and model outputs correspond to grid-cell averages. At the temporal scale, high-frequency variability captured by instantaneous SWOT snapshots, may be smoothed out in both buoy and model data. As a result, this unresolved variability appears as error in the coarser-resolution systems. The potential impact of representation error on the TC results is discussed further in chapter 5.

The derivation of the calibration coefficients and random error variances are detailed in subsection B.6.2. In this study two key final error estimates are derived for each measurement system:

1. Absolute error: The standard deviation of the random error

$$\sqrt{\langle \varepsilon_i^2 \rangle} \quad (3.16)$$

2. Relative error: The normalized standard deviation of the random error

$$\frac{\sqrt{\langle \varepsilon_i^2 \rangle}}{\langle H_{s,i} \rangle} \quad (3.17)$$

where $\langle \varepsilon_i \rangle$ denotes the variance of the random error, and $\langle H_{s,i} \rangle$ refers to the mean significant wave height of the considered measuring system i .

To reduce the influence of outliers, an iteratively reweighted least-squares robust regression is applied, following the approach described in [61]. Data triplets with weights below 0.1 in any of the three pairwise comparisons are excluded from the TC analysis.

Results

This chapter presents the main findings of the thesis, following a structured progression from visual inspection, to SWH validation and spectral analysis of SWOT-derived IG wave parameter estimates. The results are organized into the three case studies, each addressing a specific aspect of SWOT's ability to observe and characterize sea state conditions in the North Sea:

- Case I: Visual analysis of spatial SSH and SWH fields
- Case II: SWH validation and sensitivity
- Case III: Spectral analysis of infragravity waves

The chapter closes with a synthesis linking spatial patterns, validation results, and spectral findings to inform practical recommendations for integrating SWOT into coastal wave modelling and monitoring strategies.

4.1. Case I: Visual analysis of spatial SSH and SWH fields

This section presents the spatial fields of SSH and SWH as observed by SWOT, focusing on the energetic sea state during the overpass on 22 December 2024. The analysis provides a qualitative assessment of wave dynamics in the Southern North Sea, offering initial insights into SWOT's capability to capture spatial variability under storm conditions.

4.1.1. SSH field characteristics

The SSH observed by SWOT during the overpass on 22 December 2024 is presented in Figure 4.1 (left). The mean-centered SSH field shows elevated values (>1.5 m) near the Dutch coast and north of the Dogger Bank (west of buoy A12). Since the data are mean-centered but not detrended for tidal effects, this pattern likely reflects the spatially varying tidal signal in the North Sea. At this spatial scale, no distinct small-scale wave induced features are visible.

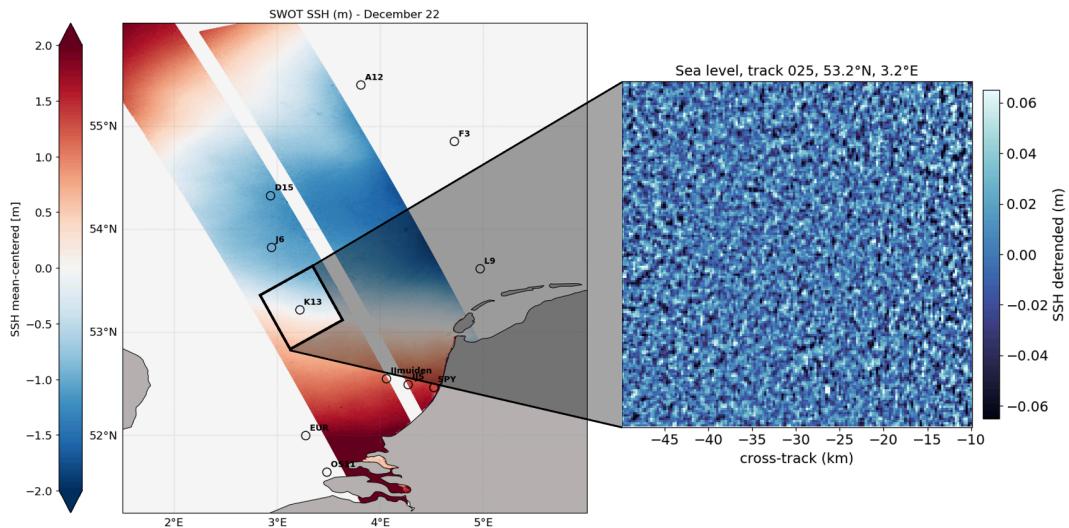


Figure 4.1: Mean-centered SSH observed by SWOT during cycle 542, track 25, over the Southern North Sea (left). Zoom-in of a 40x40 km box extracted on buoy K13, with least-squares detrending applied (right).

The right panel of Figure 4.1, illustrates a 40x40 km zoom-in centered on buoy K13. The detrended SSH field appears randomly distributed with no clear wave orientation or coherent swell signatures. This aligns with the shallow bathymetry (28.1 m at K13) and wind-sea-dominated conditions, where shorter wavelengths prevail. Furthermore, the 500 m effective resolution of the SWOT LR product limits the detection of smaller-scale features. The absence of organized wave patterns reflects the local, fetch-limited wave generation typical in the Southern North Sea. This stands in contrast to the coherent swell patterns observed in deeper ocean conditions, as illustrated in the left panel of Figure 3.5.

4.1.2. SWH field characteristics

In contrast to the SSH field, the SWH field as presented in Figure 4.2a, reveals distinct wave patterns that reflect local wind and fetch conditions. The right swath ('L' track) shows elevated SWH values, with regions exceeding 5 m around 54.2°N, 4°E. The corresponding bathymetry map, provided in Figure 4.2b, indicates no strong depth gradients in this area (south-west of buoy F3). This suggests that these elevated SWH values are primarily driven by local wind forcing or wave-current interactions rather than bathymetric effects. A notable spatial contrast emerges between the nearshore regions where the two tracks intersect the coastline. SWH values near the coastline north of buoy SPY (right swath) reach 3.45 m (SWOT SWH at SPY), while the left swath, crossing the coastline near The Hague and Hoek van Holland, shows SWH values remaining below 3 m. This pattern reflects the longer fetch available in the right swath (extending westward toward the UK), where waves can develop over an extended length under southeasterly-directed winds. In contrast, the fetch length in the left swath is more constrained due to the proximity of the UK coastline around 1.8°E, limiting wave development.

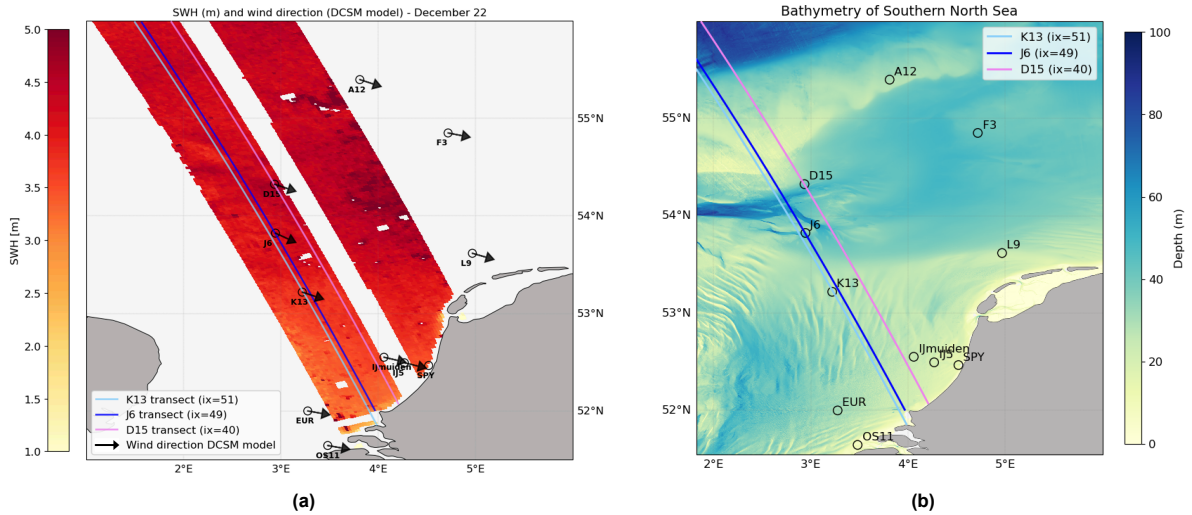


Figure 4.2: (4.2a) SWH (m) measured by SWOT, including buoy locations with modelled (SWAN-DCSM) wind directions (left); (4.2b) Corresponding map of the Southern North Sea including water depths [52]. Both figures include the three (K13, J6, and D15) transects used for SWH analysis over longitude-axis.

In Figure 4.3a, the progression of SWH along the three transects depicted in Figure 4.2 is presented. The corresponding water depths along the longitudinal axis are provided in Figure 4.3b, where the buoy locations of D15, J6 and K13 are indicated by vertical dashed lines. In all three transects, SWH varies between 3.8 and 4.8 m up to 2.8°E. Wave transformation processes due to the strong depth decrease up to approximately 20 m water depth at the Dogger Bank (1.8°E-2.7°E) appear to be negligible. At buoy location K13, a T_{m02} of 5.98 s is observed, which results in a h/L ratio of 0.5 (deep waters) given the local water depth of 28.1 m. Around 3.0°E, passing buoy K13, a consistent decrease in SWH can be observed. Here, water depths gradually decrease until the coastline. Interestingly, around buoy locations D15 and J6, a sudden increase in SWH reaching 4.8 m can be seen along the D15 transect. At this location, water depths increase to approximately 55 m, which corresponds to deep water conditions (assuming a relatively constant wave period of 6 s). This suggests that the locally observed increase in SWH cannot be attributed solely to the water depth at the buoy location.

Considering the broader transect, it is observed that D15 is situated near the southern edge of the Dogger Bank. Along much of the preceding wave propagation path, waves encounter substantially

shallower water before entering deeper regions. Such spatial variations in bathymetry may influence wave growth and transformation prior to reaching the buoy location. In particular, refraction over the Dogger Bank may lead to a convergence of wave energy near D15. This process could contribute to the locally enhanced SWH observed at this location.

Furthermore, estimated SWH remain relatively stable up to the coastline, with small variations between transects. At transect ix=51, passing K13, one outlier can be observed around 3.9°E , where SWH increases from 3.3 to 4.9 m. A similar outlier is observed at the D15 transect at the last pixel, located 800 m from the coastline, where SWH increases from 2.9 to 4.0 m. Despite these anomalies, SWH retrieval appears to remain consistent up to 1.3, 2.8 and 2.5 km from the coastline for transects K13, D15, and J6, respectively. Initial evidence that SWOT can accurately retrieve SWH is demonstrated by the in-situ observed wave height at K13 (red marker), which shows good agreement with SWOT observations (light blue). Modelled SWH at D15 and J6 are also presented (pink and blue markers) and show an initial indication that SWAN-DCSM underestimates SWH values during energetic conditions. SWOT estimates the SWH to be 4.67 m, while SWAN DCSM estimates 3.56 m. At buoy J6 estimations are 4.36 and 3.29 m for SWOT and SWAN respectively.

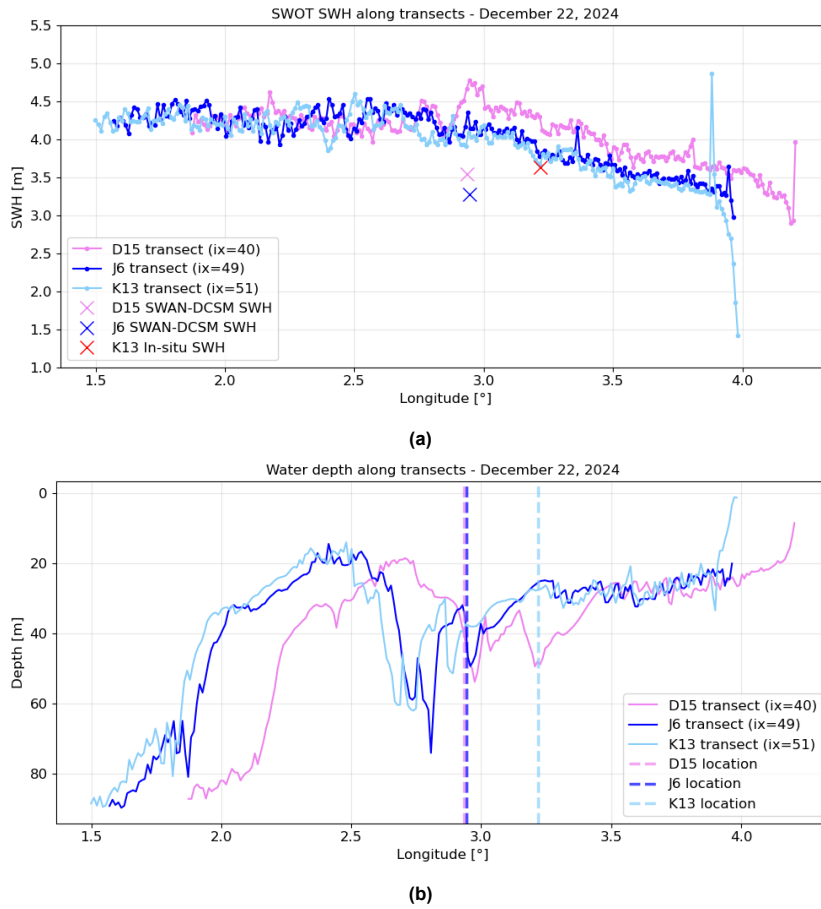


Figure 4.3: (4.3a) SWOT estimated SWH along transects ix=40 (D15), ix=49 (J6) and ix=51 (K13) as a function of longitude. Transects progress from offshore to nearshore. Corresponding colored crosses indicate the in-situ observed and SWAN-DCSM estimates of SWH at buoy locations; Corresponding water depths (y-axis) along the three transects. Dashed vertical lines represent the buoy locations (4.3b).

4.2. Case II: SWH validation and sensitivity analysis

This section presents the validation and sensitivity analysis of SWOT-derived SWH estimates. The evaluation is conducted using both the base and filtered datasets described in section 3.5. In subsection 4.2.1, the overall agreement between SWOT estimates and in-situ buoy observations is assessed, alongside comparisons with SWAN-KS and SWAN-NS model outputs. Building on this, subsection 4.2.2 presents the influence of key parameters on SWH retrieval accuracy, as highlighted in

Figure 2.12. Lastly, in subsection 4.2.3 the results of the triple collocation analysis are presented.

4.2.1. Overall validation performance

The overall performance of SWOT-derived SWH estimates is summarised in Table 4.1, comparing both the base and filtered datasets using single-pixel extraction. The base dataset is strongly affected by a single outlier observed at buoy J6 on 07-12-2024, where SWOT recorded a value of 9.4 m compared to the observed 2.0 m. Excluding this outlier in the filtered dataset leads to substantial improvements: RMSE decreases by 83% to 0.21 m, MAE by 54% to 0.17 m, and SI by 83% to 11.59%. The coefficient of determination (R^2) increases by 171% to 0.97, indicating strong agreement with observed values. With the outlier removed, SWOT demonstrates reliable SWH retrieval, with a MAE of 0.17 m.

Table 4.1: Performance metrics for SWOT-derived SWH compared to in-situ buoy observations, based on the base and filtered datasets. Sample size is given with NVAL.

Reference	Bias (m)	MAE (m)	RMSE (m)	SI	R^2	STD (m)	NVAL
SWOT base	0.33	0.37	1.25	68.52	0.36	1.22	36
SWOT filtered	0.12	0.17	0.21	11.59	0.97	0.17	35
Change (%)	-62%	-54%	-83%	-83%	171%	-86%	

Using the filtered dataset, SWOT performance is further evaluated against model estimates from SWAN-KS and SWAN-NS. Figure 4.4 shows that SWOT consistently outperforms both models, particularly under energetic conditions. Although SWAN-KS exhibits the lowest bias (-6.6 cm), SWOT outperforms both models on MAE (16.7 cm vs. 17.5 cm for SWAN-KS and 26.5 cm for SWAN-NS), RMSE (21 cm vs. 24.8 cm and 33.6 cm), SI (12% vs. 17% and 21%), and STD (17.2 cm vs. 24.1 cm and 24.9 cm). The fitted line for SWOT (slope = 0.94) closely follows the 1:1 reference line, while both models systematically underestimate SWH. SWAN-NS shows the lowest slope (0.83) and highest negative bias (-22.8 cm), indicating reduced accuracy under energetic conditions. SWOT slightly overestimates SWH by an average of 12.5 cm, but this bias remains stable across the SWH range. SWOT observations capture energetic conditions well ($\text{SWH} > 3.5$ m), with measurement errors within 5–8.5% of observed values. In contrast, errors are more pronounced for small waves ($\text{SWH} < 1$ m), where error margins range between 13–40 cm, corresponding to 1.16–1.7 times the in-situ observed wave height.

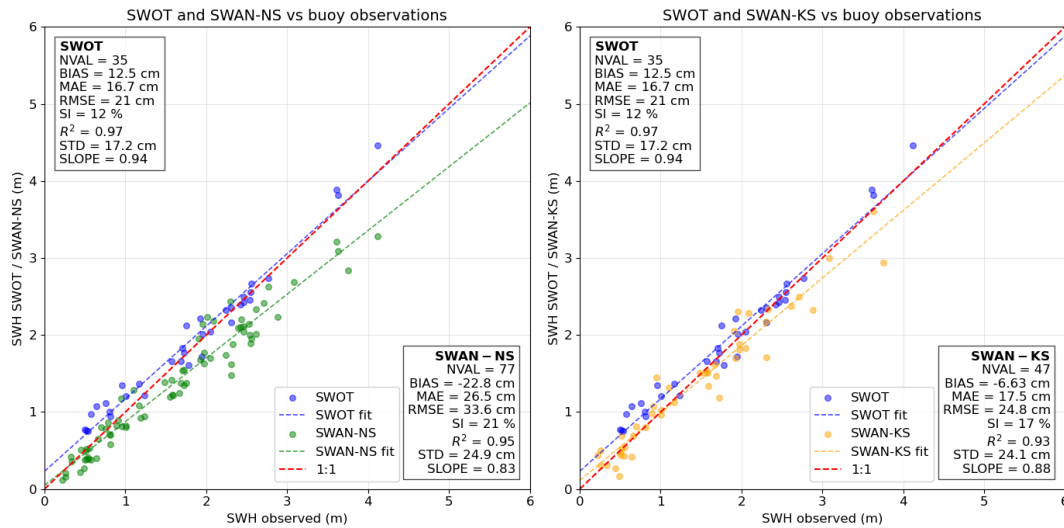


Figure 4.4: Comparison of SWOT and model-derived SWH estimates against in-situ observations. Left: SWOT vs. SWAN-NS; right: SWOT vs. SWAN-KS. The 1:1 line (red) and linear fits are shown for reference. Note, the difference in NVAL between NS (77) and KS (47), which arises because NS includes more buoy locations (10 vs. 6).

This better performance under energetic conditions is also illustrated in the time series at buoy F3 (Figure 4.5). SWOT observations align closely with the in-situ measurements during the energetic events

on 26 November and 17 December, when observed SWH exceeded 3.5 m. In contrast, both SWAN-KS and SWAN-NS models consistently underestimate the peak SWH values during these events.

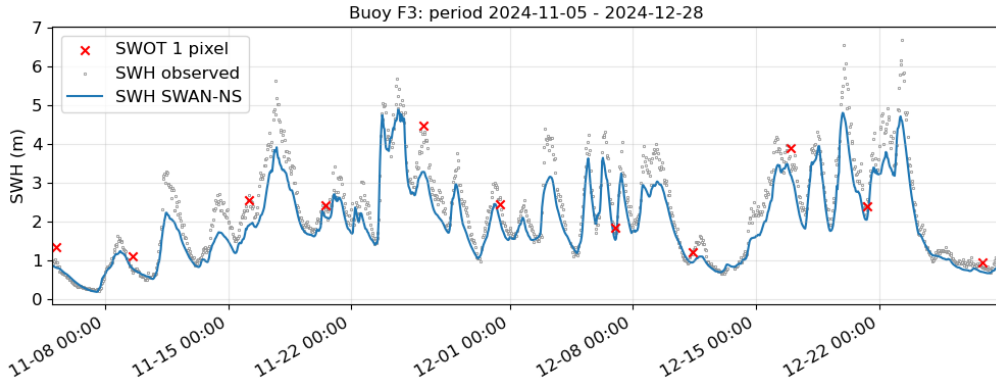


Figure 4.5: Time series of SWH at buoy F3 from SWOT (red markers), SWAN-NS (blue line), and in-situ measurements (gray dots) over the period 5 Nov–28 Dec 2024.

4.2.2. Key parameters influencing SWH accuracy

The influence of key parameters on SWOT SWH retrieval accuracy is examined through the relationship between absolute error and CTD for 1-pixel and 16-pixel observations (Figure 4.6). The fitted regression lines show a positive slope ($y = 0.219x + 8.93$ for 1-pixel and $y = 0.210x + 9.03$ for 16-pixel), indicating that absolute error increases with CTD. This trend is consistent across both pixel sizes, suggesting that measurements farther from nadir are less accurate. A similar trend can be observed for the 4-pixel configuration, having the highest slope of all three regression lines ($y = 0.513x - 1.35$, see Figure C.1). Observations with SWH below 1 m are encircled in red. A total of 8 out of 9 (1-pixel) and 13 out of 17 (16-pixel) low SWH observations fall above the respective regression lines, confirming that SWOT retrieval accuracy is reduced in calm wave conditions. The color gradient representing water depth does not reveal a consistent pattern, implying that depth may play a less dominant role than CTD or wave height. However, it is important to note that the influence of depth is modulated by factors such as local bathymetry and the wave period. These factors can complicate the detection of clear depth-dependent trends. Within these two plots, CTD and SWH demonstrate a stronger association with absolute error, whereas any potential influence of water depth remains inconclusive.

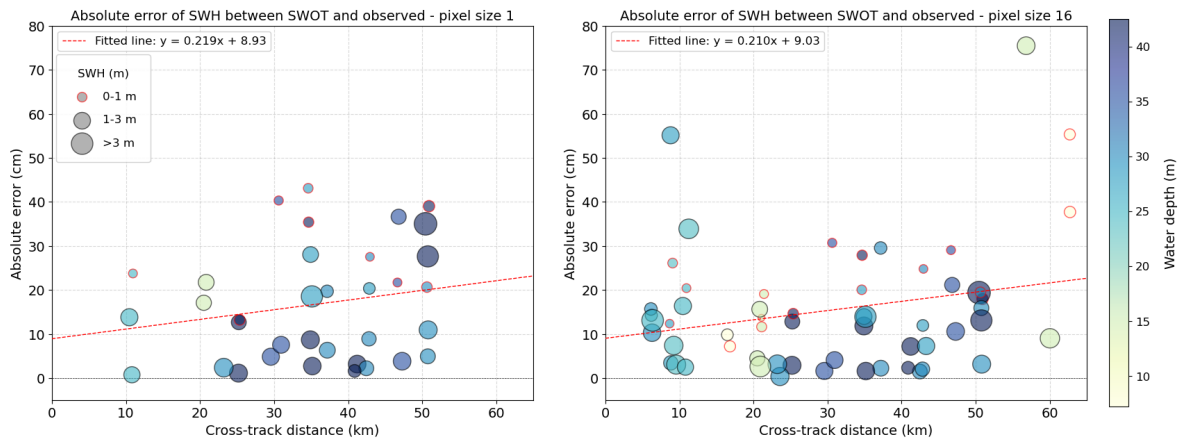


Figure 4.6: Absolute error (cm) between SWOT and observed SWH as a function of CTD. Marker size represents observed SWH. Red circles highlight cases where SWH < 1 m. Left: 1-pixel observations; right: 16-pixel observations. Linear fits (red lines) are shown for reference.

When comparing pixel sizes, the 16-pixel observations yield a systematically lower bias (7.2 cm) compared to 1-pixel observations (12.5 cm), suggesting that spatial averaging helps reduce systematic error. However, the 16-pixel dataset exhibits more outliers, particularly at CTDs > 55 km, and includes

a prominent outlier around 10 km CTD with an absolute error of 55 cm. This observation corresponds to buoy K13 during the overpass on 07-12-2024, which is the same overpass that produced the outlier > 8 m in the base dataset. While the effect of water depth remains unclear from this figure, the analysis clearly demonstrates that CTD negatively influences SWH retrieval accuracy, and that this accuracy decreases for low SWH values (< 1 m), where relative errors become proportionally larger.

To quantify the influence of CTD on retrieval accuracy, performance metrics for 1-pixel observations are binned by CTD and presented in Table 4.2. In the near-nadir range (0–25 km), SWOT shows a slight negative bias (–5.3 cm), along with the lowest MAE (13.2 cm) and RMSE (15.9 cm), indicating optimal performance. As distance increases to 25–50 km, the bias becomes positive (14.4 cm) and errors increase (MAE = 16.0 cm, RMSE = 20.7 cm). For the outer swath (> 50 km), performance further degrades: bias rises to 23.0 cm, with MAE and RMSE increasing to 23.0 cm and 26.1 cm, respectively. These results reinforce the visual trends from Figure 4.6, confirming that CTD is a primary factor influencing SWH retrieval accuracy.

Table 4.2: Performance metrics for SWOT-derived 1-pixel estimates compared to in-situ buoy observations, binned by CTD.

Cross-track	Bias (m)	MAE (m)	RMSE (m)	STD (m)	NVAL
0–25 km	–0.05	0.13	0.16	0.16	6
25–50 km	0.14	0.16	0.21	0.15	23
> 50 km	0.23	0.23	0.26	0.13	6

The influence of pixel size on SWH retrieval accuracy is summarised in Table 4.3. Overall, the performance across pixel sizes (1, 4, and 16 pixels) is relatively consistent, with slight variations in key metrics. Bias decreases with increasing pixel-sizes, from 12 cm to 7 cm (4- and 16-pixel), suggesting that spatial averaging reduces systematic overestimation. MAE similarly declines from 16.7 cm (1-pixel) to 15.6 cm (4-pixel) and 15.4 cm (16-pixel). Scatter index and R^2 values remain stable across pixel sizes, further confirming consistent performance. However, STD is lowest for 1-pixel observations (17.2 cm) and increases for 4-pixel (24.1 cm) and 16-pixel (20.4 cm) cases. This indicates that while averaging reduces bias, it may introduce greater variability in the residuals between satellite and buoy measurements in certain conditions or bathymetric regions. Note that the number of match-ups (35 for 1-pixel, 44 for 4-pixel, and 58 for 16-pixel) increases with pixel size, which may influence these statistics by including a broader range of conditions. This increase of match-ups reflects the higher likelihood that at least one pixel within the larger spatial averaging window meets the quality flag criterion. As a result, more distant but valid pixels can contribute to the estimate, improving spatial data coverage near buoy locations.

Table 4.3: Performance metrics for SWOT-derived SWH retrieval across different pixel sizes (1, 4, and 16 pixels), compared to in-situ buoy observations. Larger pixel sizes represent spatially averaged SWH estimates over wider areas.

Pixel size	Bias (m)	MAE (m)	RMSE (m)	SI	R^2	STD (m)	NVAL
1	0.12	0.17	0.21	11.59	0.97	0.17	35
4	0.07	0.16	0.25	13.29	0.93	0.24	44
16	0.07	0.15	0.21	11.63	0.96	0.20	58

Cross-track distance and SWH sensitivity

As demonstrated in Table 4.2 and Figure 4.6, the accuracy of SWH retrieval decreases with increasing CTD. To further quantify this effect, cross-track thresholds of 50 and 55 km are applied to exclude observations beyond this range. The results for the 50 km threshold are shown in Table C.2, and indicate relatively modest improvements (1–8% in MAE, 0–4% in STD) compared to the 55 km threshold dataset. In Table 4.4 the results of the 55 km threshold are presented. This threshold has no effect on the 1-pixel dataset, as all observations already fall within the 55 km range. However, for the 4-pixel and 16-pixel datasets, excluding far-range observations substantially improves performance.

For 4-pixel observations, STD decreases from 0.24 m to 0.13 m (45% reduction), RMSE from 0.25 m to 0.17 m (33%), MAE from 0.16 m to 0.13 m (16%), and R^2 increases from 0.93 to 0.98 (5%). Similarly, for 16-pixel observations, STD reduces from 0.20 m to 0.14 m (28%), RMSE from 0.21 m to 0.17 m (20%), MAE from 0.15 m to 0.13 m (14%), and R^2 improves from 0.96 to 0.98 (2%). With this additional filtering, the 4-pixel and 16-pixel configurations reach optimal performance (MAE = 0.13 m, RMSE = 0.17 m), while also achieving larger sample sizes ($N = 43$ and $N = 54$, respectively) compared to 1-pixel observations ($N = 35$). However, this improvement comes at the cost of increased bias. For 4-pixel observations, bias increases from 7.4 cm to 10 cm (35%), and for 16-pixel observations from 7.2 cm to 9 cm (25%), though these remain below the 1-pixel bias of 12.5 cm. These results demonstrate that spatial averaging (4-pixel and 16-pixel) can achieve superior accuracy and precision compared to single-pixel observations, provided that far-range measurements are excluded.

Table 4.4: Comparison of performance metrics between SWOT-derived and in-situ SWH after applying a CTD threshold of 55 km. Color tones indicate the percentage change between the filtered dataset and the thresholded dataset.

Size	Metrics - cross-track < 55 km									
	Bias (m)		MAE (m)		RMSE (m)		STD (m)		r^2	NVAL
1 pixel	0.12	0	0.17	0	0.21	0	0.17	0	0.97	35
4 pixels	0.10	41%	0.13	-16%	0.17	-33%	0.13	-45%	0.98	43
16 pixels	0.09	25%	0.13	-14%	0.17	-20%	0.14	-28%	0.98	54

The relationship between absolute error and SWH magnitude, categorised by CTD, reveals a decreasing pattern that aligns with theoretical expectations demonstrated in Figure 2.7b. This is evidenced in Figure 4.7, for all three cross-track bins (0–25 km in green, 25–50 km in blue, and >50 km in red) the fitted regression lines exhibit a decreasing trend in absolute error, with a minimum around 2.5–2.75 m SWH. The regression line of the near-range bin (0–25 km) shows the lowest absolute error overall, declining from approximately 24 cm at low SWH to near zero around 2.8 m SWH. The fitted line of the mid-range bin (25–50 km) starts with errors around 33 cm, reaching a minimum of ~8 cm around 2.5 m SWH. The far-range bin (>50 km, red line) follows a similar trend, but its regression line remains slightly above the mid-range line across the SWH range. These patterns are consistent with theoretical predictions, and show that absolute bias decreases with increasing SWH and increases with CTD. However, for SWH > 3 m, errors in the mid- and far-range bins begin to rise again. The three observations with SWH > 3.5 m suggest that accuracy is not solely driven by CTD and wave height magnitude, but may also be affected by additional factors under energetic conditions.

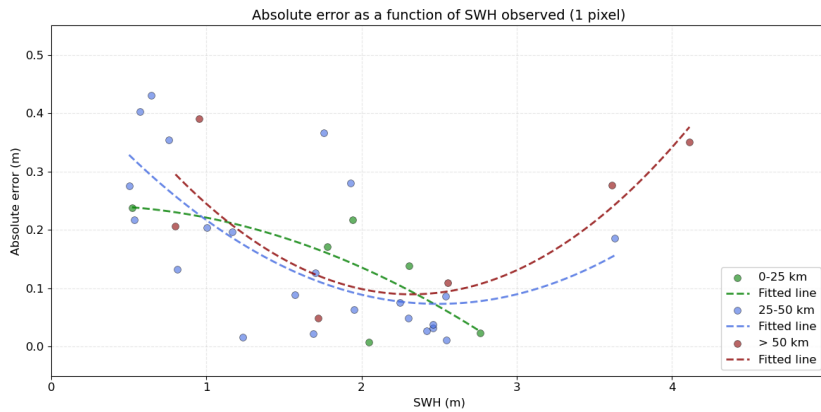


Figure 4.7: Absolute SWH error (in m) as a function of observed SWH. Fitted regression lines are shown for three CTD bins: 0–25 km (green), 25–50 km (blue), and >50 km (red).

Sensitivity to match-up size and bathymetry class

As shown in Table 4.4, the overall performance of SWOT SWH retrieval is highest for the 4- and 16-pixel match-up sizes. To further evaluate the influence of spatial averaging across locations, Figure 4.8 presents the distribution of absolute errors per bathymetry class and water depth bin, for pixel sizes 1, 4, and 16.

Even without excluding outliers beyond 55 km CTD, the median absolute error remains below 20 cm for all three match-up sizes, indicating robust performance across all configurations and geographic settings. In the nearshore class and lower water depths (0–20 m), the 4-pixel match-up size achieves the lowest median error (~ 9 –11 cm), with the 16-pixel configuration showing slightly increased errors (~ 12 –14 cm). In the variable bathymetry class and 20–40 m water depth range, the 16-pixel size yields the lowest median error (~ 12 –13 cm), though it does contain one outlier corresponding to buoy K13 during the overpass on 07-12-2024. In the offshore class and water depths > 40 m, the 4-pixel size again shows the lowest median error (~ 10 –12 cm).

While similar trends emerge across both bathymetry- and depth-based groupings, it remains difficult to disentangle the effects of water depth, bathymetric complexity, and distance to shore, as these factors are spatially correlated. For example the variable bathymetry class includes 4 out of the 6 buoys located in mid-range water depths (20–40 m). Rather than attributing error differences to a single factor, the results suggest that areas with variable bathymetry consistently show higher median errors compared to nearshore and offshore classes, regardless of pixel size. This highlights the particular challenge of retrieving accurate SWH in dynamically complex coastal regions.

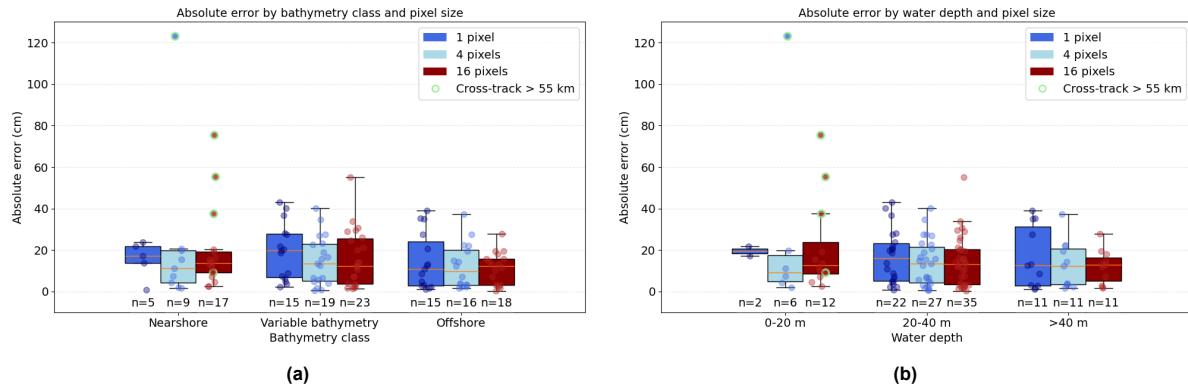


Figure 4.8: (4.8a) Absolute error distribution between SWOT and observed SWH per bathymetry class and pixel size; (4.8b) Absolute error distribution per water depth bin and pixel size. Outliers corresponding to CTDs > 55 km are highlighted in green.

Observations with CTDs exceeding 55 km occur only in the nearshore and lower water depth bins, as visualised in Figure 4.8. This co-occurrence is coincidental and not indicative of an underlying relationship between CTD and bathymetry class. To improve reliability, these outlier points are excluded from the analysis. Additionally, observed SWH values below 1 m are removed, as these cases show a higher likelihood of absolute error, as evidenced in Figure 4.6 and Figure 4.7. These filtering steps help isolate meaningful spatial performance differences across bathymetric classes. Excluding these data points considerably improves performance. Table 4.5 summarises the MAE and STD performance by bathymetry class and pixel size. In the nearshore class, the 4-pixel configuration shows a 28% lower MAE compared to 1-pixel (9.6 cm vs. 13.3 cm), while the 16-pixel configuration shows a similar improvement of 26% (9.8 cm vs. 13.3 cm). Across all bathymetry classes, larger match-up sizes (4- and 16-pixel) consistently outperform single-pixel estimates. The best overall performance is observed in offshore locations using the 4-pixel configuration, with the lowest MAE (8.2 cm) and STD (7.6 cm). Likewise, in the variable bathymetry class, the 4-pixel size yields the lowest MAE (9.3 cm) and STD (8.5 cm), matching the performance in offshore and nearshore classes. A full overview of the corresponding median absolute errors is provided in Table C.1.

Compared to the results shown in Figure 4.8, the 4-pixel configuration shows improved retrieval accuracy for the variable bathymetry class, with performance decreasing progressively from offshore to nearshore. In contrast, for both the 1- and 16-pixel grid sizes, the variable bathymetry class continues to exhibit the highest MAE values (14.2 and 13.1 cm), consistent with earlier findings in Figure 4.8. The corresponding absolute error distributions by bathymetry class and individual buoy are shown in Figure C.2.

Table 4.5: MAE (left) and STD (right) between in-situ observations and SWOT SWH estimates per bathymetry class and pixel size. Observations beyond 55 km CTD and in-situ observations with SWH < 1 m are excluded. Color tone indicates magnitude of error (light to dark blue).

MAE (cm)				STD (cm)			
Class	Pixel size			Class	Pixel size		
	1 pixel	4 pixels	16 pixels		1 pixel	4 pixels	16 pixels
Offshore	10.0	8.2	8.5	Offshore	11.7	7.6	16.0
Variable	14.2	9.3	13.1	Variable	13.8	8.5	9.2
Nearshore	13.3	9.6	9.8	Nearshore	9.0	10.5	11.7

Sea surface variability

Figure 4.9a presents the distribution of absolute error across three wave period bins, excluding cases where SWH < 1 m to remove low wave height noise. A general trend of decreasing absolute error with increasing wave period is observed. For the 3–5 s bin, median absolute errors are approximately 17 cm (1-pixel), 13 cm (4-pixel), and 12 cm (16-pixel). These decrease in the 5–7 s bin to 8 cm (1-pixel), 6 cm (4-pixel), and 9 cm (16-pixel), and further drop in the >7 s bin to 5 cm (1-pixel), 3 cm (4-pixel), and 1 cm (16-pixel).

However, a notable exception occurs in the 5–7 s range, where observations with SWH > 3 m (highlighted in orange) exhibit increased errors, particularly between 6 and 7 s wave period. As shown in Figure 4.9b, absolute error increases with wave period in this regime, suggesting that energetic sea states introduce greater local variability. This variability appears to challenge the effectiveness of spatial averaging, even at the 2×2 km scale used in single-pixel SWOT retrievals.

Despite this, the lower errors for 4- and 16-pixel configurations across most wave period bins confirm the general benefit of spatial averaging. In young seas (3–5 s), the 16-pixel configuration performs best (12 cm median error), suggesting that large-area averaging helps mitigate higher surface variability. In contrast, for the 5–7 s regime, the 4-pixel configuration yields the lowest median error (6 cm), potentially reflecting a trade-off between noise reduction and preserving local variability.

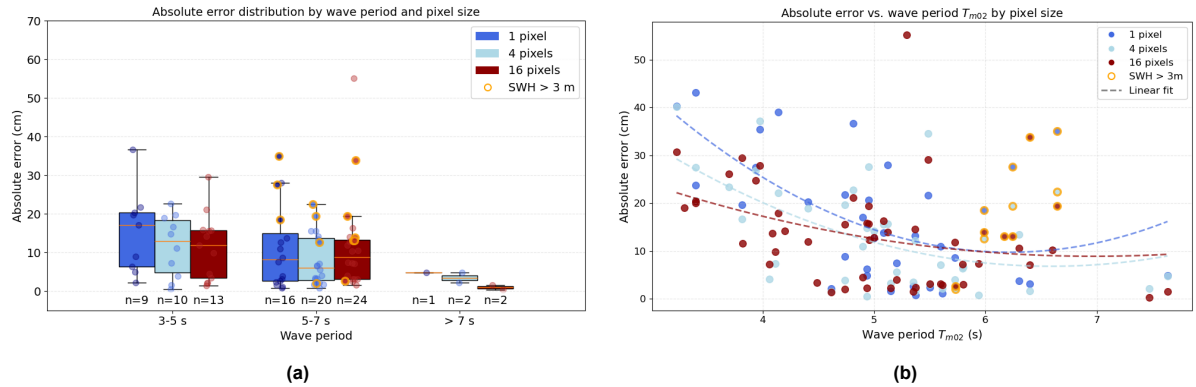


Figure 4.9: (4.9a) Distribution of absolute errors between SWOT and observed SWH per wave period bin and pixel size. Observations with CTD > 55 km and SWH < 1 m are excluded, and those with SWH > 3 m are marked in orange; (4.9b) Absolute error (cm) as a function of T_{m02} , highlighting increased error for energetic sea states in the 6–7 s regime.

The relationship between absolute error and wave age provides additional insight into how sea state characteristics affect SWOT SWH retrieval accuracy. Figure 4.10 shows absolute error as a function of wave age (values > 0.5 typically correspond to wind-sea) across all pixel sizes. For the 1- and 4-pixel configurations, regression lines remain relatively flat, suggesting no clear relationship between wave age and retrieval error. In contrast, the 16-pixel configuration exhibits a positive trend, with errors increasing under higher wave age conditions. This implies that large-scale spatial averaging (8×8 km) becomes less effective in young, wind-driven seas, where local variability is greater and less spatially coherent.

The dimensionless ratio h/L , which characterizes the influence of seabed-induced wave transformation processes, is visualized in Figure 4.10 using orange markers for observations where $h/L < 0.5$. These values correspond to shallow to intermediate depth conditions, where wave transformation processes

are expected to be more pronounced. However, no consistent increase in absolute error is observed for these conditions. For the 4-pixel configuration, 5 out of 7 observations with $h/L < 0.5$ fall below the fitted regression line, while for the 16-pixel configuration this holds for 6 out of 9 cases. In the 1-pixel configuration, 1 out of 3 observations falls below the regression line. Additional distributions of absolute error as a function of h/L are provided in Figure C.3.

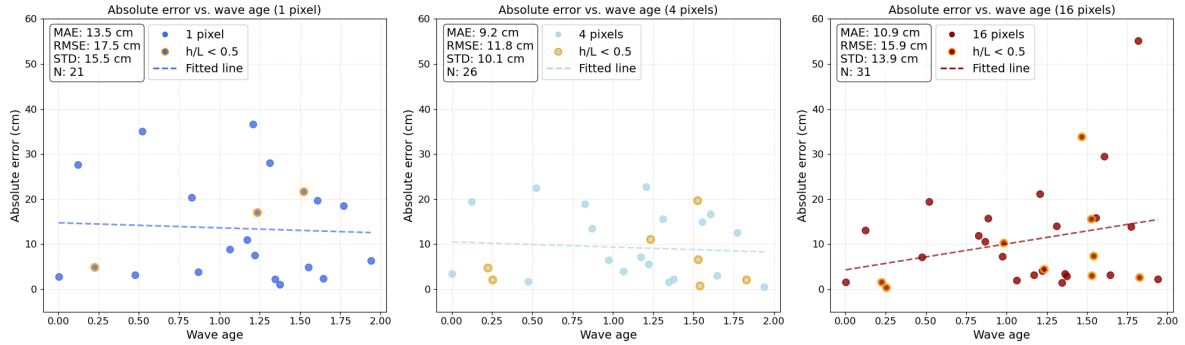


Figure 4.10: Absolute SWH error (in cm) as a function of wave age. Observations with CTD > 55 km and SWH < 1 m are excluded, and those with $h/L < 0.5$ are marked in orange. Fitted regression lines are shown as reference.

To further examine the sea surface variability and its impact on SWH estimation, two energetic overpasses ($t_{energetic,1}$ and $t_{energetic,2}$) are analysed using zoomed-in spatial maps of SWOT-derived SWH over a 45×45 km box. As reference, the pixel boxes used for 4-pixel and 16-pixel match-up sizes are illustrated with dashed lines. The pixel used for single SWH extraction is shown with a blue cross.

Figure 4.11 shows the overpass on 26 November 2024 near buoy F3, where observed SWH was 4.11 m, wave period (T_{m02}) was 6.64 s, and wind speed was 5.4 m/s. From the left panel, showcasing the overpass from 51° to 56° N, the local variability in SWH is difficult to discern. However, in the right panel showing part of the center to outer swath, this variability becomes visible. The single-pixel SWH estimate at the buoy location is 4.46 m, overestimating by 35 cm. Interestingly, the third and fourth closest pixels (1.7–2.1 km from the buoy) yield estimates of 4.25 m and 4.28 m, respectively. Both estimates are closer to the buoy-observed 4.11 m value. Averaging over 4- and 16-pixel areas lowers the estimate to 4.34 m and 4.31 m, respectively. The variability within the 16-pixel box is modest (4.23–4.46 m), and all values overestimate the observed SWH, indicating a slight regional overestimation by SWOT under these conditions.

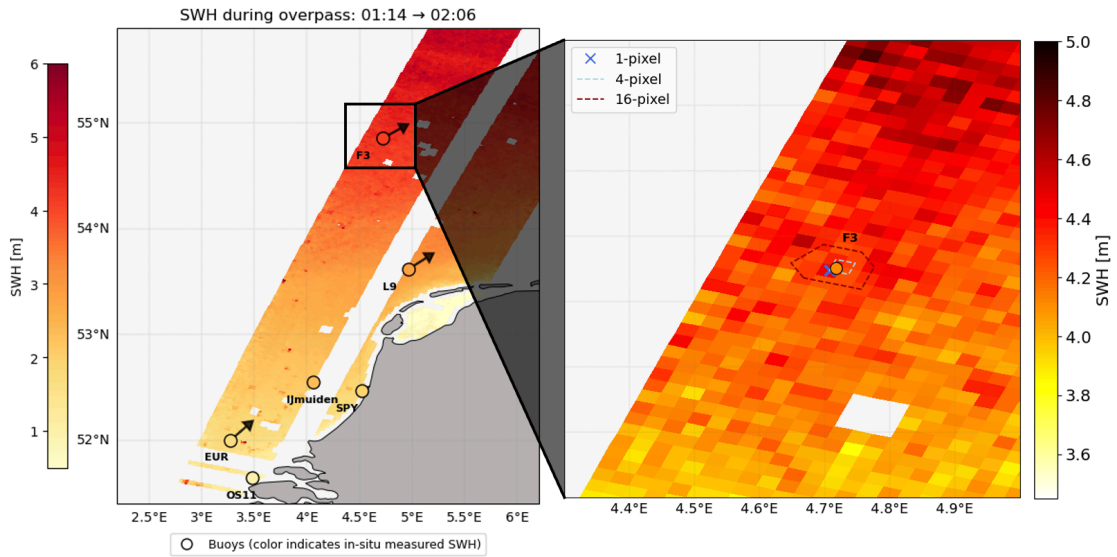


Figure 4.11: Zoom-in on SWOT-derived SWH values during the 26 November 2024 overpass near buoy F3. Dashed lines show the 4- and 16-pixel match-up areas, the blue cross indicates the 1-pixel retrieval. SWH ranges from 4.23–4.46 m within the 16-pixel box.

Figure 4.12 shows the overpass on 22 December 2024 near buoy K13, where observed SWH was 3.63 m, wave period was 5.98 s, and wind speed was 16.5 m/s. The single-pixel error is lower here (19 cm), reduced further to 13–14 cm with spatial averaging. The 16-pixel box exhibits limited variability (STD = 4.8 cm), with SWH values ranging from 3.66 m to 3.84 m. However, zooming out of the 16-pixel box reveals a strong northward SWH gradient. Just 20 km north, SWH rises to 4.2 m. This case underscores how SWH can vary by up to 50 cm over small spatial scales, reinforcing the need for spatial averaging to obtain representative estimates.

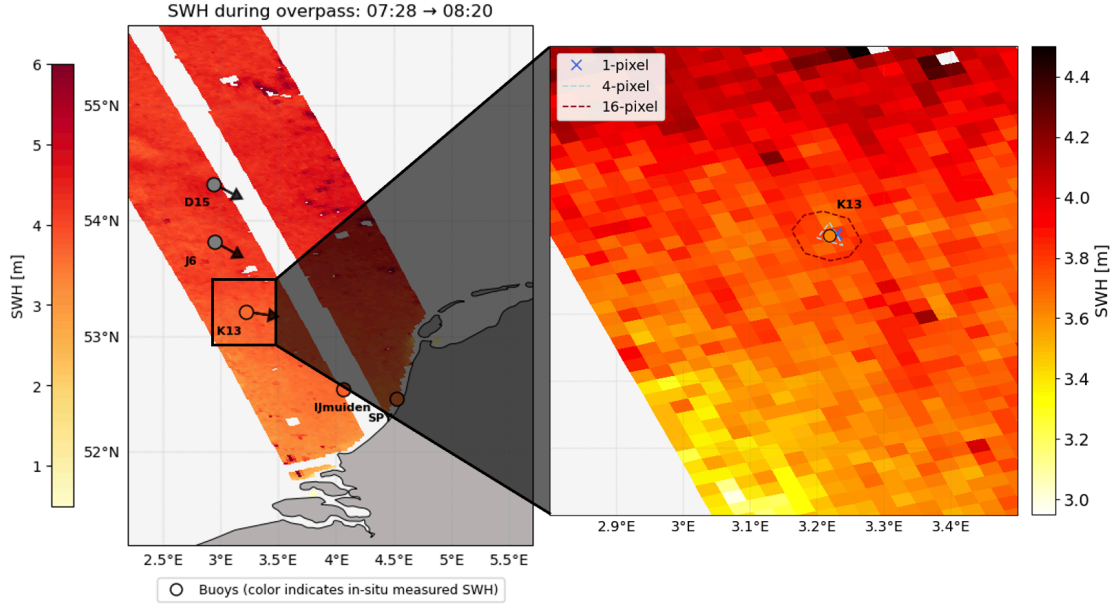


Figure 4.12: Zoom-in on SWOT-derived SWH values during the 22 December 2024 overpass near buoy K13. Dashed lines show the 4- and 16-pixel match-up areas, the blue cross indicates the 1-pixel retrieval. Within the 16-pixel box, SWH ranges from 3.66–3.84 m.

4.2.3. Triple collocation analysis

Table 4.6 presents the error statistics from the triple collocation analysis using the three different pixel configurations (1, 4, and 16 pixels) and two SWAN wave models (NS and KS) as the third measurement system. For all three match-up size configurations using the NS model dataset, one outlier is removed, corresponding to the previously discussed SWOT observation at buoy J6 on 2024-12-07.

Absolute and normalized errors

For both error estimates and SWAN models, the numerical model shows the highest errors, with absolute errors ranging from 0.16 to 0.24 m and normalized errors from 10.7 to 15.1%. SWOT error statistics rank second, with absolute errors ranging from 0.08 to 0.19 m depending on the configuration and wave model system used. The in-situ buoy data shows the lowest errors, with normalized errors ranging from 0.1 to 5.8%.

Table 4.6: Absolute (left) and normalized error (right) statistics of the three independent measurement instruments. Six different datasets are used, with each dataset using either 1-, 4- or 16 pixels for SWH extraction and the NS or KS SWAN wave model as third reference system.

Absolute error (m)					Normalized error (%)				
Configuration	System				Configuration	System			
	Buoy	SWOT	Model	NVAL		Buoy	SWOT	Model	NVAL
NS - 1 pixel	0.04	0.14	0.17	33	NS - 1 pixel	2.0	7.2	11.3	33
NS - 4 pixel	0.03	0.10	0.16	40	NS - 4 pixel	1.8	5.2	10.7	40
NS - 16 pixel	0.09	0.13	0.17	50	NS - 16 pixel	5.0	6.7	11.3	50
KS - 1 pixel	0.09	0.10	0.19	12	KS - 1 pixel	5.8	5.9	13.0	12
KS - 4 pixel	0.05	0.08	0.20	18	KS - 4 pixel	2.9	4.6	13.2	18
KS - 16 pixel	0.00	0.19	0.24	26	KS - 16 pixel	0.1	10.7	15.1	26

Overall, the 4-pixel configuration shows the lowest SWOT errors across most configurations, consistent with the MAE results from the bathymetry class analysis when CTDs < 55 km are excluded. The normalized errors for SWOT are relatively low (4–7%) across all datasets, except the KS - 16 pixel configuration which results in an error of 10.7%.

Impact of pixel configuration

Interestingly, the highest SWOT errors are present in the 16-pixel dataset with the KS wave model (absolute error: 0.19 m, normalized error: 10.7%). The KS model, includes 5 out of 6 buoys which are within 40 km of the shoreline. While the 16-pixel nearshore bathymetry class shows similar accuracy to the 4-pixel configuration in the standard validation metrics, the triple collocation analysis reveals errors more than double the size of the 4-pixel configuration. This discrepancy suggests that in nearshore regions with high spatial heterogeneity, the larger 16-pixel spatial averaging window and the model grid-cell averaging are too coarse to capture the small-scale variability that is resolved by the point buoy measurements. The buoy's point measurement captures local wave conditions that differ from the spatially-averaged SWOT and model values, creating a scale mismatch that results as increased representation errors in the triple collocation framework.

However, when applying both the CTD threshold of 55 km and the SWH threshold of 1 m, the TC results show substantial improvements in SWOT error statistics (see Table 4.7). For the 16-pixel configuration, SWOT absolute errors decrease from 0.13 m to 0.01 m (92% reduction) for NS and to 0.10 m for KS (48% reduction). The 1- and 4-pixel configurations also improve, with filtered SWOT errors of 0.11 m and 0.07 m for NS, and 0.02 m and 0.03 m for KS, respectively. Model errors remain relatively stable (0.13–0.22 m), while buoy errors are highest in the 16-pixel configurations (0.13 m for both NS and KS), consistent with increased representation errors in nearshore regions. These results demonstrate the critical importance of applying quality filters, particularly for larger pixel configurations, and quantify the contribution of low wave conditions and far-range observations to the total error budget.

Table 4.7: Error statistics when the CTD threshold of 55 km is applied, and SWH below 1 m are excluded. Six datasets are used, with each dataset using either 1-, 4- or 16 pixels for SWH extraction and the NS or KS SWAN wave model as third reference system.

System	Absolute error (m)					
	Configuration					
	NS - 1 pixel	NS - 4 pixel	NS - 16 pixel	KS - 1 pixel	KS - 4 pixel	KS - 16 pixel
Buoy	0.06	0.05	0.13	0.13	0.08	0.13
SWOT	0.11	0.07	0.01	0.02	0.03	0.10
Model	0.19	0.18	0.18	0.13	0.22	0.20
NVAL	24	29	36	8	13	18

Calibration coefficients

The calibration coefficients obtained from the TC analysis are consistent with the fitted regression relationships shown in Figure 4.4. For SWOT, the estimated slopes cluster around 0.93–0.96 across all configurations, in close agreement with the regression slope of 0.94 obtained from the 1-pixel comparison. The corresponding offsets indicate a positive bias, which decreases with increasing pixel size, suggesting that spatial averaging reduces systematic bias.

For the SWAN model, the estimated slopes (approximately 0.82–0.83 for the NS dataset) closely match the regression slope shown in Figure 4.4, confirming a systematic underestimation that increases with SWH. Calibration results for the KS dataset show larger variability, particularly for the 1-pixel configuration, reflecting the smaller sample size ($n=12$ for the 1-pixel KS configuration). The full set of calibration coefficients for all configurations is provided in Table C.4.

4.3. Case III: Spectral analysis of infragravity waves

This section presents the results of the spectral analysis used to retrieve IG wave characteristics from SWOT observations. As outlined in section 3.6, the analysis is structured in three main steps: (1) testing box and tile configurations, (2) evaluating the impact of the transfer functions, and (3) applying the IG wave mask. These steps aim to assess the sensitivity of estimated H_{IG} to key methodological choices. During the 33 SWOT-buoy match events, platform-mounted radar observations of H_{IG} ranged between 4.3 cm to 10.47 cm.

4.3.1. Box and tile configurations

Figure 4.13a shows the MAE of each version per measurement station, with H_{IG} estimated by directly computing the wave energy belonging to the IG mask of the SWOT SSH PSD. As mentioned in subsection 3.6.2, applying the transfer function can amplify errors significantly. To avoid these errors, in this first step the inversion through the use of transfer function is neglected.

The results in Figure 4.13a demonstrate that box size has a greater impact on the MAE than the chosen tile size. This effect is particularly pronounced for station 'EUR', located roughly 40 km from the Dutch coast. For this site, increasing the box size from 10 km to 40 km results in an MAE increase of up to 3.5 cm, representing a sevenfold increase in error. This degradation suggests that the 40 km box captures significant coastal contamination, including land-sea interactions and shallow water transformation effects. Nevertheless, the optimal box size varies by location. The smallest box (10 km) yields superior performance for locations F3, EUR, and D15, whereas the 40 km box achieves better accuracy at locations K13, J6, and A12. When averaging over all measurement locations, the total MAE is 1.3 cm for the Alpha version and 1.4 cm for the other versions. This indicates that while location-specific effects matter, overall performance is similar across the different configurations.

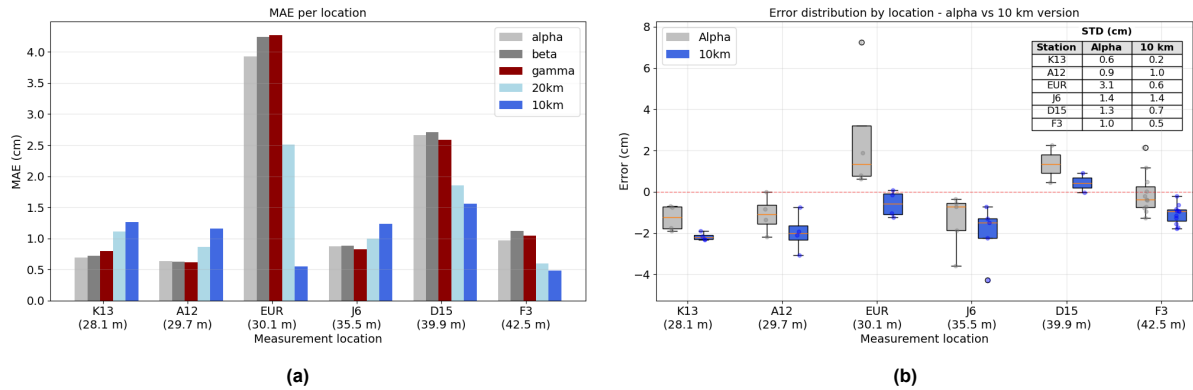


Figure 4.13: (4.13a) MAE per version per measurement location ($n=30$); (4.13b) Error distribution per measurement location for version Alpha (grey) and the 10 km (blue) version. Water depths of the stations are provided between brackets.

Differences in results between the choice of using a configuration with many tiles (version Alpha, high EDF) versus fine resolution (Gamma, lower EDF) appear insignificant. Comparing the MAE of the two versions, no considerable differences are observed, with either version performing slightly better depending on measurement location. This suggests that Alpha's advantage of lower variance of the PSD (from high EDF) is balanced by Gamma's advantage of finer spectral resolution, which can better capture the signal structure and improve the accuracy of the energy integral.

Figure 4.13b compares the error distributions for the Alpha and 10 km versions across all measurement locations, revealing systematic differences in both bias and precision. The 10 km version consistently shows lower STD values at most locations, despite having a much lower EDF (8 vs. 200). The most dramatic improvement is observed at EUR, where STD is reduced from 3.1 cm to 0.6 cm (an 81% reduction). Similar improvements are seen at D15 (1.3 cm to 0.7 cm, 46% reduction), F3 (1.0 cm to 0.5 cm, 50% reduction), and K13 (0.6 cm to 0.2 cm, 67% reduction). The exception is at A12, where both versions result in similar STD values (0.9 cm and 1.0 cm), suggesting relatively homogeneous wave conditions at this location.

These results demonstrate that precision in H_{IG} estimation depends not only on PSD variance (con-

trolled by EDF), but also on the spatial homogeneity of the analysis box. Smaller boxes (10 km version) reduce the inclusion of spatially heterogeneous conditions, such as bathymetric variations, nearshore dynamics, or coastal influences, which can distort the spectral estimate. Using a smaller spatial area for spectral analysis appears to outweigh the statistical variance reduction provided by higher EDF in larger boxes, particularly at locations with complex bathymetry or nearshore dynamics.

Both versions show one significant outlier at J6 (16-11-2024), which pushes the STD above 1 cm for both versions (1.4 cm). This outlier may be due to unusual wave conditions or data quality issues at this specific time and location instance.

SSH error versus spatial offset

Figure 4.14 reveals a systematic negative bias (median error of -1.5 cm) for buoy–swath center distances up to 20 km. Contrastingly, beyond 20 km it overestimates the H_{IG} with approximately +0.5 cm. Without applying a transfer function, the estimates computed for stations located further than 20 km from the center of swath show a 2 cm difference compared to the estimates with measurement observations located closer to the center of swath. This indicates that increasing separation between the measurement location and the spatial footprint over which the SWOT spectrum is computed introduces representation errors, which become particularly pronounced once this distance exceeds ~ 20 km.

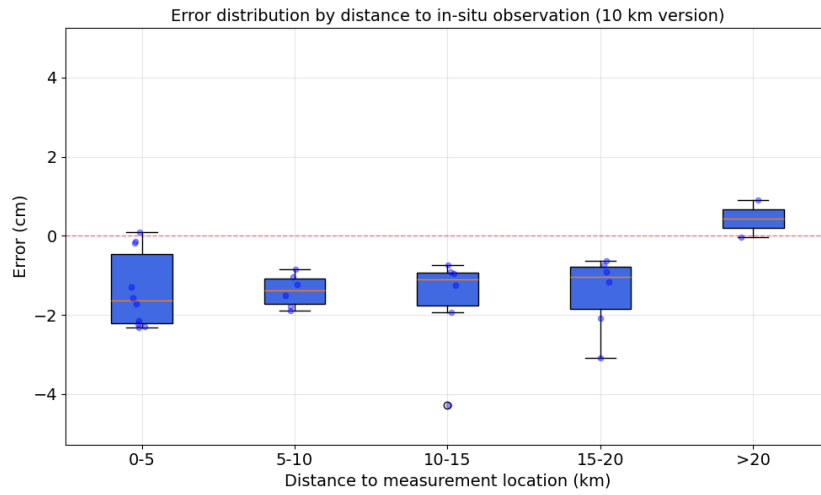


Figure 4.14: Error distribution of H_{IG} (y-axis) binned by euclidean distance (x-axis) between buoy location and center of the analysis box for the 10 km version (no transfer function applied).

To assess the impact of spatial offset between the in-situ location and analysis box, the 10 km version was recomputed with the box repositioned directly over location D15. Table 4.8 summarises the results for two overpasses where D15 is located more than 20 km from the original box center. In both cases, the buoy lies approximately 12 km cross-track from the nadir. While this repositioning minimizes spatial mismatch, it results in a notable degradation of H_{IG} retrieval due to increased exposure to instrumental and wave-induced noise near the swath edges.

Table 4.8: Overview of in-situ measurements and SWOT estimates of H_{IG} during two overpasses at D15, including spectral center and nearest-point values with respective distances.

10 km version without G				
Overpass	SWH (m)	Observed H_{IG} (cm)	Center of swath H_{IG} (cm)	Near station H_{IG} (cm)
Dec 1, 2024	2.05	5.88	5.88 (23.5 km)	8.96 (3.4 km)
Dec 22, 2024	4.22	9.46	10.37 (23.3 km)	17.93 (3.2 km)

The overpass on December 22, which occurred under higher wave conditions (SWH = 4.22 m vs 2.05 m), shows that these errors become more pronounced with increasing sea state. The results in Figure 4.14 show how larger spatial differences between buoy and spectral analysis can bias results of the estimated H_{IG} . However, results of Table 4.8 demonstrate that due to increasing SSH error in the near-range of the cross-track, SWOT accuracy degrades significantly when centering the analysis box directly on the in-situ location.

4.3.2. Transfer function analysis

After computing the 2D FFT on each sample, the SWOT SSH power spectral density (E_s) is inverted using the full spectral response function G to estimate the true wave spectrum $E = E_s/G$. This subsection evaluates the effect of each component of the transfer function across different box–tile configurations.

Table 4.9a shows the MAE of H_{IG} estimates per version, comparing results when no filter, PTR-only, AZC-only, OBP-only and the full transfer function are applied. Notably, the full inversion (OBP + PTR + AZC) leads to the highest MAE across all configurations, while applying only the OBP correction accounts for approximately 60% of the total error increase observed with the full correction. Similarly, Table 4.9b highlights a systematic positive bias in H_{IG} estimates when inversion is applied, indicating consistent overestimation of energy in the IG regime.

These results suggest that the error increase may not stem from the physical filters themselves, but from noise amplification during inversion. If the OBP and PTR are correctly modelled, and the AZC parameterization from SAR is approximately valid for KaRIn, then the systematic excess of energy likely result from one or more of the following:

- Residual noise being over-amplified during inversion (especially where OBP attenuation is strongest)
- Spectral leakage or aliasing contaminating the IG wavenumber band
- Mask definition error resulting in the IG mask capturing non-IG energy

Table 4.9: (4.9a) MAE and (4.9b) bias between in-situ observations and SWOT estimates of H_{IG} per version and transfer function (n=30). Tone of color visualizes the size of error (light to dark blue).

MAE (cm)						Bias (cm)					
Version	Transfer function					Version	Transfer function				
	No G	PTR	AZC	OBP	Full G		No G	PTR	AZC	OBP	Full G
Alpha	1.3	1.4	1.3	4.8	7.7	Alpha	-0.14	0.92	-0.04	4.84	7.74
Beta	1.4	1.5	1.4	4.8	7.5	Beta	-0.03	1.01	0.06	4.77	7.48
Gamma	1.4	1.5	1.4	4.5	7.2	Gamma	-0.12	0.91	-0.03	4.46	7.16
20 km	1.4	1.1	1.3	3.3	5.7	20 km	-0.88	0.04	-0.80	3.29	5.69
10 km	1.4	0.9	1.3	2.4	4.4	10 km	-1.33	-0.50	-1.20	2.35	4.41

(a)

(b)

To better understand the magnitude of this amplification, Figure 4.15 shows the attenuation of each filter in the 1D wavenumber domain. In this analysis, the Beta version is used to compute the transfer functions. The box and tile lengths change the number of wavenumber points, however impact on the shape and strength of each filter is negligible (see Figure C.4). The OBP filter in both cross- and along-track directions exhibits the most significant suppression of spectral energy. At 2 cycles per km (Nyquist), attenuation by the OBP reaches approximately -15 dB in both directions, equating to a power factor of ~ 0.032 (similar to the OBP filter used by Arduin et al (2024), see Figure 2.11). The PTR filter adds -5.6 dB attenuation, leading to an overall suppression of approximately -20 dB, or an amplification of a factor 100 when inverting the full G . This makes IG wave energy retrievals highly sensitive to background noise and masking near high-k.

The AZC component, based on the velocity bunching effect, is comparatively weak under North Sea conditions. Two versions of the AZC filter are plotted, representing moderate ($\lambda_c = 154$ m) and high-energy ($\lambda_c = 392$ m) sea states. In both cases, the attenuation remains under 8 dB and is negligible relative to OBP. This is supported by the MAE and bias results, which show that adding AZC has a minimal effect on H_{IG} estimation accuracy compared to OBP and PTR.

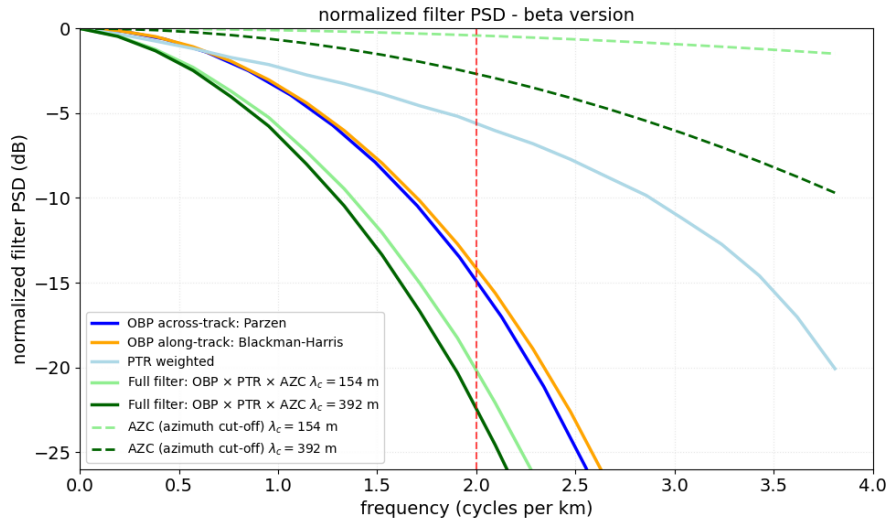


Figure 4.15: SWOT OBP cross-track filter and components of the along-track spectral filter G. PTR filter uses weights of each beam using averaged beam weights as provided by A. Bohé. Two versions of the AZC filter are demonstrated: λ_c 154 m (H_s 0.97 m, $T_{m,02}$ 3.72 s) and λ_c 392 m (H_s 4.33 m, $T_{m,02}$ 6.53 s).

Importantly, Table 4.9a shows that the 10 km box version achieves the lowest MAE (0.9 cm) when only the PTR correction is applied. This suggests that the smaller box size reduces error induced by spatial heterogeneity that could be amplified by the OBP filter. However, when applying the full transfer function, all versions show substantial degradation, indicating that the noise amplification issue is fundamental and not strongly dependent on spatial configuration.

Azimuthal cutoff sensitivity

To assess the sensitivity of the azimuthal cutoff filter G_{AZC} , two scaling factors (0.8 and 1.2) were applied to the λ_c used in the spectral response formulation. The resulting transfer functions shows negligible deviation from the reference filter shown in Figure 4.15, indicating minimal spectral impact. This is further confirmed by the resulting H_{IG} errors. For example, across all overpasses matching location F3, applying a 0.8 factor leads to a 3% increase in MAE (0.03 cm), while a 1.2 factor yields a 4% reduction in MAE (−0.04 cm) when testing the 10 km version. Similarly, the percentage change for the Alpha version is minimal (0–1%). These small variations suggest that the velocity bunching effect, as modelled through G_{AZC} , plays a minor role in the total H_{IG} estimation error under the tested North Sea conditions. The percentage change in MAE across all five versions is provided in Figure C.5.

4.3.3. Infragravity mask

Water depth sensitivity

As highlighted in section 3.6, the IG mask boundaries depend on the local water depth through the dispersion relationship. This study uses constant water depths derived from EMODnet bathymetry, but in practice, sea levels vary due to tides, storm surges, and other oceanographic processes. To evaluate how such fluctuations might affect IG wave estimates, depth variations of $\pm 10\%$ are applied for station F3 during both $t_{energetic,1}$ and t_{calm} .

The results in Table 4.10 show that water depth variations have asymmetric effects: decreasing depth (90%) has a larger impact than increasing depth (110%). This asymmetry occurs because decreasing depth shifts the upper wavenumber limit to higher values, expanding the mask into regions with higher spectral energy. Under energetic conditions with the full transfer function, a 10% depth decrease increases H_{IG} by 8.8%, while a 10% increase decreases it by only 2.7%. This asymmetry is more pronounced when the full transfer function is applied because the amplified energy near the Nyquist wavenumber (where most energy is concentrated) is more sensitive to mask boundary changes. The smaller effects observed without the transfer function (2.5% vs 0.7%) confirm that the amplification by $1/G$ enhances the sensitivity to mask definition errors.

The objective of this sensitivity analysis is not to optimize the error margin compared to the radar estimate, but to quantify the degree of variability in the H_{IG} estimate itself due to water depth uncertainty.

Table 4.10: Impact of water depth on H_{IG} with and without applying the transfer function under energetic and calm conditions at location F3. Values in parentheses show the percentage difference relative to the reference (ref) baseline.

Water depth [m]	$t_{energetic}$		t_{calm}	
	H_{IG} full G	H_{IG} no G	H_{IG} full G	H_{IG} no G
38.25 (90%)	(+8.8%)	(+2.5%)	(+12.3%)	(+2.4%)
42.5 (ref)	18.62 cm	8.77 cm	8.02 cm	4.09 cm
46.75 (110%)	(−2.7%)	(−0.7%)	(−3.5%)	(−0.7%)

The findings suggest that for calm North Sea conditions, depth fluctuations of $\pm 10\%$ introduce errors of 1–2% (no G) and 3–12% (full G). During energetic conditions these errors vary less with errors decreasing/increasing 1–3% (no G) and 3–9% (full G) when applying 10% depth changes. For high-precision applications, time-varying water depths (accounting for environmental changes) may be necessary, particularly when using the full transfer function inversion.

IG mask sensitivity to normalization and PSD representation

To assess the robustness of the IG mask definition, the impact of the normalization factor α and the choice of PSD (original E_s vs. inverted E) is evaluated for both $t_{energetic,1}$ and t_{calm} . Quantitative results in Table 4.11 show that variations in H_{IG} due to these choices are minimal. Differences remain ≤ 0.2 mm across all cases, with the exception of a 2.5 mm increase when the full transfer function is applied in the 10 km configuration under energetic conditions. Visual analysis of the PSD's presented in Figure C.6, also reveal the limited influence on the shape of the mask when applying either the original E_s or the inverted E when shaping the IG mask. Figure C.7 presents the limited influence on the shape of the mask when applying a normalization factor of 15.

These results suggest that the mask shape is primarily governed by the wavenumber bounds derived from the linear dispersion relation, and that both the energy normalization factor and the choice of PSD have limited influence on the final H_{IG} estimates.

Table 4.11: Overview of estimated H_{IG} values under energetic and calm conditions at measurement location F3 for Beta and the 10 km version. Normalization factor of 10 is applied for the rows without color, while a factor of 15 is applied for the light blue rows.

Version	$t_{energetic,1}$		t_{calm}	
	G H_{IG} (cm)	No G H_{IG} (cm)	G H_{IG} (cm)	No G H_{IG} (cm)
Beta E_s	21.04	10.32	9.33	4.75
Beta E	21.04	10.32	9.33	4.75
10 km E_s	18.62	8.77	8.02	4.09
10 km E	18.87	8.79	8.03	4.09
10 km E_s	18.87	8.79	8.02	4.09
10 km E	18.87	8.79	8.03	4.09

4.3.4. Consistency

The consistency of SWOT IG wave height estimates is evaluated by examining its accuracy across different wave height conditions and by comparing its performance to model estimates of the FIG wave height over a large spatial domain on the North Sea.

Wave conditions

Figure 4.16 demonstrates how SWOT H_{IG} estimates compare to in-situ observations across increasing IG wave heights. The left panel shows the 1:1 relationship without transfer function correction, while the right panel shows the improved performance when applying G_{PTR} inversion. Without transfer function correction, SWOT exhibits a systematic underestimation (bias = -1.33 cm) with a slope of 0.76,

indicating that the underestimation becomes more pronounced at higher wave heights. The application of G_{PTR} greatly improves the performance: bias is reduced to -0.50 cm (a 62% reduction), RMSE decreases from 1.7 cm to 1.1 cm (a 35% reduction), and the slope improves to 0.87, indicating better linearity across the full range of observed wave heights. The scatter index (SI) also improves from 25% to 17%, demonstrating reduced variability in the estimates.

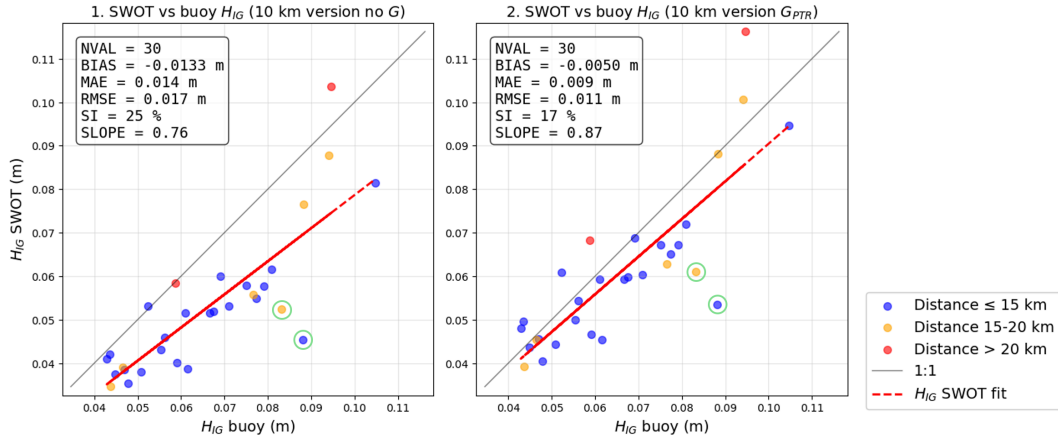


Figure 4.16: SWOT H_{IG} versus buoy estimates: 1. Without transfer function G , 2. With G_{PTR} inversion. The two outliers (green circle) belong to buoys A12 (left circle) and J6 (right circle) during the overpass on November 16, 2024.

Overall, SWOT demonstrates consistent performance for H_{IG} values below 8 cm, with all estimates falling within ± 2 cm of the observed values when using G_{PTR} . However, as IG wave height increases above 8 cm, the errors become more pronounced, with several points showing larger deviations from the 1:1 relationship and the fraction within ± 2 cm decreasing to 57%.

Table 4.12 quantifies this degradation by comparing performance metrics for $H_{IG} < 8$ cm versus $H_{IG} \geq 8$ cm. For the G_{PTR} version, the MAE increases by 112% (from 0.7 cm to 1.49 cm) and the STD increases by 160% when comparing the lower and higher wave height ranges. The average SWH during $H_{IG} < 8$ cm conditions is 1.5 m, while for $H_{IG} \geq 8$ cm conditions it is 3.2 m, suggesting that higher sea states are generally associated with higher IG wave heights and increased estimation errors. A full overview of all individual buoys and the relationship between H_{IG} observed versus SWOT (10 km version G_{PTR}) is provided in Figure C.8.

Table 4.12: Statistical comparison between No G and G_{PTR} conditions, split by low and high H_{IG} thresholds.

No G				
Condition	N	Bias (cm)	MAE (cm)	STD (cm)
$H_{IG} < 8$ cm	23	-1.19	1.20	0.71
$H_{IG} \geq 8$ cm	7	-1.79	2.04	1.69
G_{PTR}				
$H_{IG} < 8$ cm	23	-0.45	0.70	0.71
$H_{IG} \geq 8$ cm	7	-0.68	1.49	1.85

The degradation in SWOT accuracy under increased sea state conditions is further confirmed by examining the specific case during storm Babet (21 October 2023) in the SP23/24 dataset, where in-situ observed SWH was 5.8 m. Figure 4.17 shows the time series of radar-observed H_{IG} at F3, with SWOT estimates (both without and with G_{PTR} correction) indicated at the time of the satellite overpass. During this period, the radar observations show elevated H_{IG} values, with a peak of approximately 18.7 cm around 20 October 21:00, followed by a gradual decrease. At the time of the SWOT overpass (21

October 02:13), the interpolated radar H_{IG} value was 13.3 cm, while SWOT estimates were 17 cm (without G) and 19 cm (with G_{PTR}), resulting in errors of 3.6 cm and 5.7 cm, respectively.

These errors are substantially larger than the typical errors observed under moderate conditions (MAE = 0.70 cm for $H_{IG} < 8$ cm with G_{PTR} , as shown in Table 4.12), confirming that increased sea state leads to degraded accuracy. Interestingly, while G_{PTR} correction generally improves performance (as demonstrated in Figure 4.16), during this high sea state case the G_{PTR} version shows a larger error compared to the uncorrected version. This suggests that the transfer function correction may over-amplify noise or residual errors under extreme conditions.

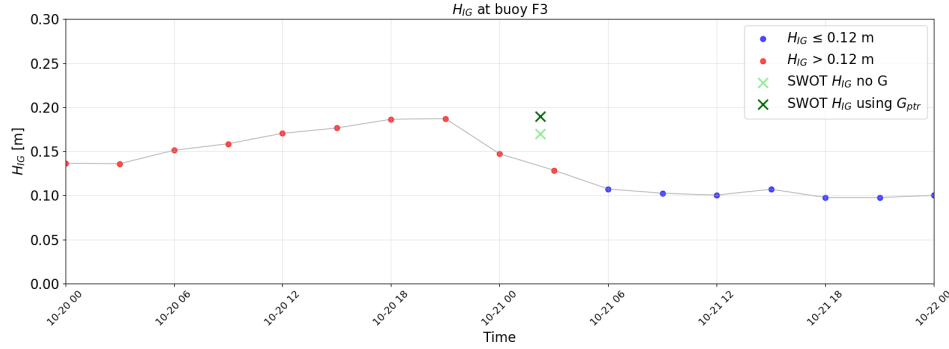


Figure 4.17: Time series of radar-observed H_{IG} at location F3 during storm Babet (21 October 2023), with SWOT estimates (without G : light green marker, with G_{PTR} : dark green marker) indicated at the satellite overpass time. The color of radar data points indicates whether H_{IG} exceeds 12 cm (red) or is below 12 cm (blue).

Two notable outliers are highlighted in green in Figure 4.16 both corresponding to track 24, cycle 113 on 16 November 2024. During this overpass, four measurement locations matched with the satellite track, as summarised in Table 4.13. For locations A12 and J6, where observed H_{IG} exceeds 8 cm (8.3 cm and 8.8 cm, respectively), the differences between SWOT and observed values are substantial (3.1 cm and 4.3 cm, respectively). In contrast, for F3 and EUR, where observed H_{IG} is below 7 cm (6.9 cm and 4.4 cm, respectively), the differences are significantly smaller (0.9 cm and 0.2 cm, respectively), representing a factor of 3-4 reduction in error. A notable contradiction emerges when examining these results more closely. F3, with the highest SWH (2.46 m) among all four locations, shows good agreement (error = 0.9 cm) despite having a similar distance to swath center (11.3 km) as J6 (12.8 km), which shows poor agreement (error = 4.3 cm). Similarly, A12 and F3 have comparable SWH values (2.44 m and 2.46 m, respectively), yet A12 shows significantly larger errors (3.1 cm) compared to F3 (0.9 cm). This suggests that increased wave-induced distortions at higher sea states and spatial offset effects alone cannot fully explain the degradation in accuracy observed at A12 and J6.

Table 4.13: Comparison of observed and SWOT-derived H_{IG} (10 km version E_s) values at different measurement locations, including in-situ observed SWH and the distance between spectral center and the measurement station.

Station	SWH (m)	10 km version without G		
		Observed H_{IG} (cm)	SWOT H_{IG} (cm)	Distance center-station
J6	1.87	8.8	4.5	12.8 km
A12	2.44	8.3	5.2	16.5 km
F3	2.46	6.9	6.0	11.3 km
EUR	1.13	4.4	4.2	3.0 km

Wavelength and wave direction

Computed IG wavelengths (L_{IG}) using the G_{PTR} transfer function and directly from SWOT's observed SSH PSD (No G) yield very similar results, as shown in Table 4.14. The largest deviation occurs at D15 on 22 December, where the PTR-based estimate is 12 m longer than the No G estimate. This

indicates that the transfer function has minimal influence on the wavelength calculation itself. However, when comparing SWOT-derived L_{IG} (using PTR) to in-situ radar observations, larger differences are observed. SWOT underestimates the IG wavelength by approximately 25, 12, and 6% for locations F3, D15, and K13, respectively. On 26 November, the best agreement is found at EUR, where SWOT slightly overestimates L_{IG} by 4%. Interestingly, these wavelength discrepancies are not directly mirrored in H_{IG} . During 22 December, SWOT overestimates IG wave height at D15 and F3 by 23% and 7%, respectively, while underestimating H_{IG} at EUR and K13 by 10%. The computation of H_{IG} depends on the total integrated energy, whereas L_{IG} depends on the distribution of that energy across wavenumbers. This suggests that the mismatch in wavelength is not simply due to overall energy scaling, but are driven by variations in spectral shape.

This is illustrated in Figure C.9, which compares the frequency distribution of IG energy from SWOT and the platform-mounted radar at F3. The peak energy lies at 0.0267 Hz for SWOT and at a significantly lower frequency (0.0084 Hz) for radar, consistent with longer observed wavelengths in the radar record.

Table 4.14: Comparison of L_{IG} and mean wave direction (θ) estimates from SWOT (using PTR and No G) with in-situ radar observations during the 26 November and 22 December 2024 overpasses. All estimates are computed using the 10 km box-tile configuration. In-situ L_{IG} is based on the observed wave period and computed using the dispersion relationship. Wave directions are given in nautical directions. In-situ observed sea-swell mean wave direction by buoys are added as reference.

26-Nov, 2024 L_{IG} (m)				26-Nov, 2024 θ ($^{\circ}$)		
Station	PTR	No G	In-situ	PTR	No G	In-situ sea-swell
F3	854	852	1145	51.1	10.9	241
EUR	900	900	865	17.4	287.4	229
22-Dec, 2024 L_{IG} (m)				22-Dec, 2024 θ ($^{\circ}$)		
Station	PTR	No G	In-situ	PTR	No G	In-situ sea-swell
D15	897	885	1024	335.6	70.3	297
K13	861	863	913	338.6	71.0	278

Table 4.14 also reveals the mean wave direction (θ) estimates derived from SWOT, alongside buoy-measured sea-swell directions for comparison. On 26 November, PTR-based directions cluster between 17.4° and 51.1° (NE), while the No G estimates range from 10.9° to 287.4° (N–NW). On 22 December, this pattern reverses. The estimated IG wave directions generally diverge from the sea-swell wave directions observed by the buoys, which predominantly originate from the west. This divergence suggests the presence of FIG waves, which propagate independently and originate from various coastlines. In contrast, BIG waves typically travel in the same direction as the primary wave groups. Further evidence for directionally distributed energy is shown in Figure C.10, which displays the PSD at F3 across different box-tile configurations. The lack of a clear spectral peak reflects a broad directional spread of energy, consistent with the presence of FIG waves rather than a single dominant source.

Numerical wave model

Figure 4.18 presents the spatial fields of SWH and IG wave height along both SWOT tracks during the overpass on 26 November 2024. Panel I displays the full-swath SWH field, illustrating a clear increase in wave height along the wind direction (northeast), reaching a maximum of 8 m near the Norwegian coastline. Panel II shows the corresponding IG wave height (H_{IG}) derived from SWOT. The spatial pattern of increasing H_{IG} from 52°N to 58°N mirrors that of the total SWH. Panel III shows the FIG wave height (H_{FIG}) estimated by the numerical wave model. Overall, SWOT-derived H_{IG} values appear consistent with the model, with a comparable longitudinal trend and magnitude, although local differences occur. In-situ radar observations from locations EUR and F3 show good agreement with SWOT-derived H_{IG} (see Table C.5). Absolute differences are within 1 cm, confirming the reliability of SWOT's IG wave detection under these conditions. One exception is a local outlier south of EUR (52°N), visible in both the map and in the longitudinal profile (Figure 4.19) around 3.3°E .

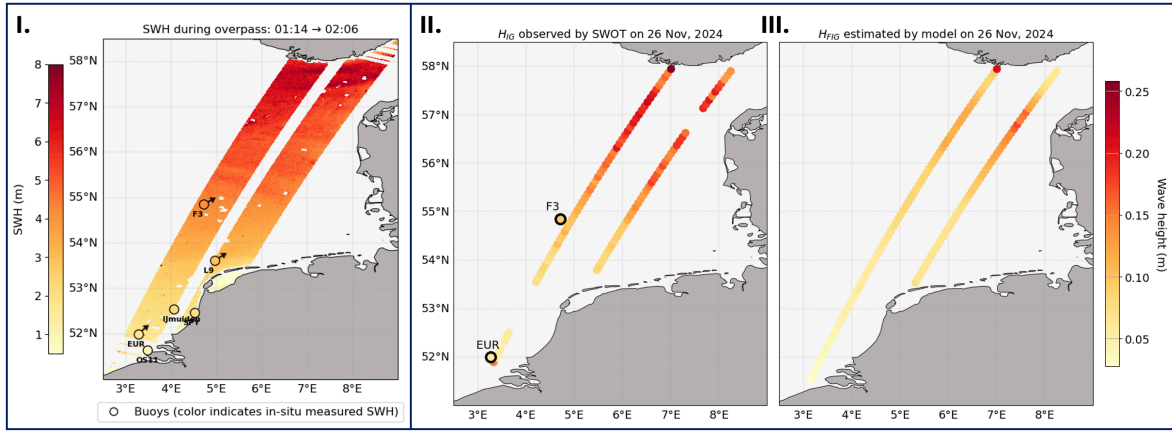


Figure 4.18: Spatial fields observed during the SWOT overpass on 26 November 2024. (I) SWH measured by SWOT. (II) IG wave height (H_{IG}) derived from SWOT. (III) Free IG wave height (H_{FIG}) computed from the numerical wave model. Markers of the measurement locations are colored with the in-situ observed wave heights (SWH or H_{IG}).

In Figure 4.19, SWOT-derived H_{IG} shows a steeper increase with longitude than the modelled H_{FIG} , especially along the left swath. Both datasets reveal a distinct drop in wave height around 6.7°E, coinciding with the abrupt depth change at the Norwegian Trench (to depths of ≈ 400 m, see Figure C.12). A similar drop occurs around 7.8°E along the right track, modulating both the total IG wave height and the FIG wave height.

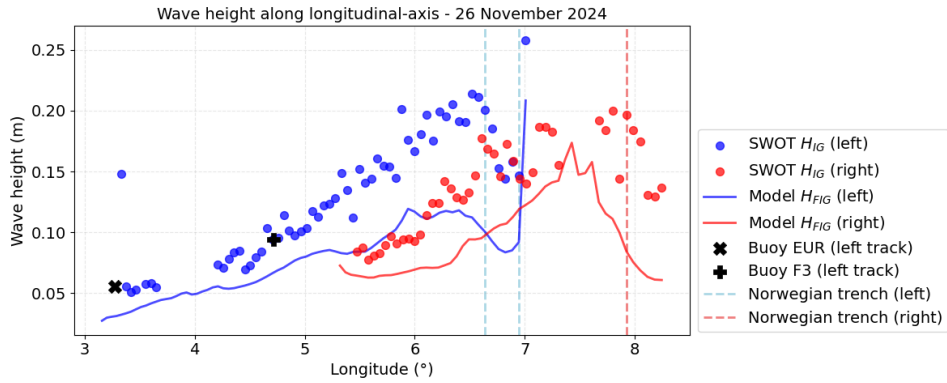


Figure 4.19: Infragravity wave height along the longitudinal-axis of SWOT tracks on 26 November 2024. SWOT-derived H_{IG} and modelled H_{FIG} are shown for both the left and right swaths. Dashed lines indicate the location of the Norwegian Trench along each track. In-situ observations of H_{IG} at buoys EUR and F3 (crosses) are shown as reference.

The R -value distribution along the longitudinal transect (Figure 4.20) remains relatively stable (0.5–0.8) up to 6.7°E. Along the left swath, a gradual decline in R is observed, from around 0.8 to 0.5. Since R expresses the relative contribution of FIG energy to the total IG wave energy, a decreasing trend suggests a growing presence of BIG waves. This pattern aligns with the consistent increase in SWH, which enhances local bound wave generation through wave group forcing. In contrast, along the right swath beyond 7°E, R -values exceed 0.8, indicating a stronger dominance of the FIG component. This shift occurs approximately 55 km offshore from the Danish coastline (around 7.5°E in Figure 4.19), just prior to the sharp bathymetric drop at the Norwegian Trench. The location of this increase suggests the presence of reflected FIG energy originating from the nearby Danish coastline. The enhanced H_{FIG} in this region may thus result from incoming free IG waves being reflected seaward, amplifying local FIG energy before the depth transition. This finding is supported by the 2D FIG model output, which shows elevated H_{FIG} values along the Danish coast (see left panel in Figure C.11). The full two-dimensional distribution of R across the SWOT swaths is shown in Figure C.13.

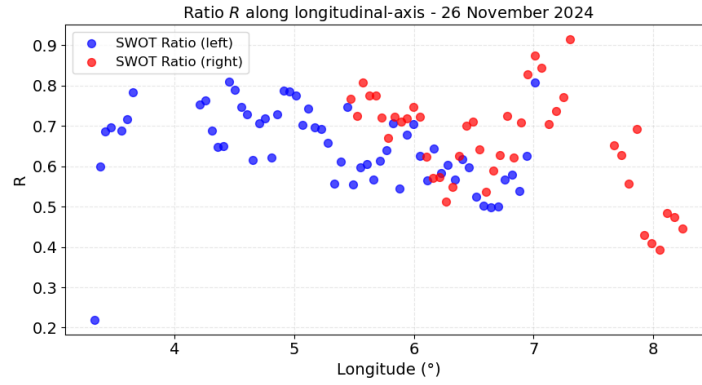


Figure 4.20: Ratio R between H_{FIG} (model) and H_{IG} (SWOT) along the SWOT tracks on 26 November 2024.

The wave directions derived from SWOT for the 26 November event indicate mean propagation angles of 46° and 32° for the left and right tracks, respectively. This suggests that the total IG wave energy predominantly propagates from the northeast. Such a direction aligns with the sea-swell wave groups approaching the Norwegian and Danish coasts, where breaking processes generate FIG waves. These free waves then reflect seaward, resulting in a southwestward propagation pattern, consistent with the directions inferred from SWOT.

In contrast, the wave climate during the 22 December overpass was dominated by SE-E directed sea-swell (see Figure 4.21 panel I). SWOT-derived IG mean wave directions for this event are 290° and 292° for the left and right tracks, respectively. These are in line with in-situ sea-swell buoy observations in the Southern North Sea, suggesting that free IG waves in this case propagate in the same direction as the bound wave groups. The spatial field of H_{IG} observed by SWOT during this second case is presented in the second panel. In both the left and right tracks, patterns of increasing and decreasing IG wave heights correspond closely to those seen in the SWH field. For example, enhanced H_{IG} values are observed between $57^\circ\text{--}59^\circ\text{N}$ in the right swath, mirroring the elevated SWH in that region. A similar co-variation is apparent around 54°N , east of measurement locations D15 and J6, where both SWH and H_{IG} increase simultaneously. These patterns indicate that during this event, the infragravity wave signal is likely governed by sea-swell transformation processes, rather than by coastal reflections.

The results of the R -value distribution during the 22 December event are not shown. Along multiple regions of both tracks the R -value exceeds 1, with maximum values of 1.5. Since $R > 1$ implies that the modelled free IG wave height exceeds the total IG wave height estimated by SWOT, these values indicate either an underestimation of H_{IG} by SWOT or an overestimation of H_{FIG} by the numerical model. In this study, it is difficult to attribute these patterns to one of these factors. The full spatial distribution of R and the corresponding longitudinal profiles are therefore provided in subsection C.2.6 for completeness rather than quantitative interpretation.

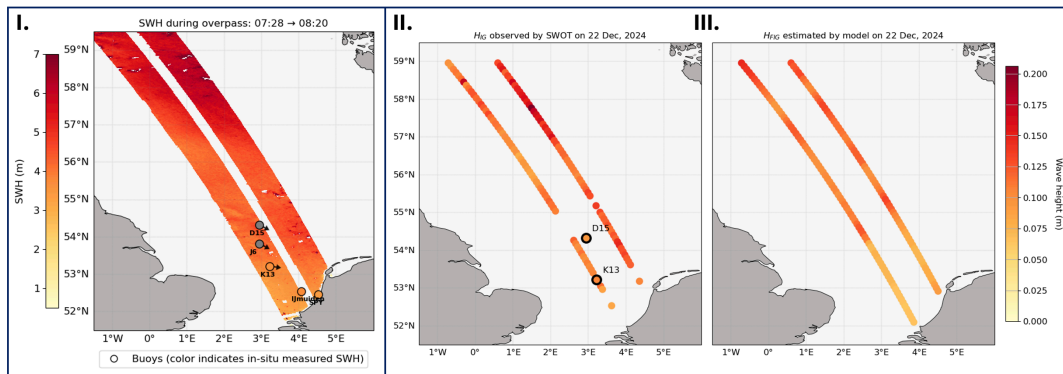


Figure 4.21: Panel I: SWH measured by SWOT during the overpass on 22 December 2024. Panel II: Total infragravity wave height (H_{IG}) estimated by SWOT. Panel III: Free infragravity wave height (H_{FIG}) estimated by the numerical wave model. Note the spatial coherence between the SWH and H_{IG} fields, particularly in the northern and central parts of the domain.

4.4. Cross-case synthesis

This section synthesizes key findings from the three case studies, focusing on how instrumental, geo-physical, and processing parameters influence SWOT-based wave retrievals. Particular attention is given to the role of spatial heterogeneity and its implications for satellite–buoy comparisons in both the SWH and IG wave regimes.

Case I

Qualitative assessment of the SSH fields (Figure 4.1 right) reveals a randomly distributed wave field in the North Sea, with no clear directional wave patterns at the $\sim 250 \times 250$ m pixel scale. In contrast, spatial maps and transects of the SWH fields (Figure 4.2) show recognizable wave patterns, with SWH increasing along longer fetch distances. While modelled wind directions and observed wind speeds (Table B.4) suggest relatively homogeneous forcing, localized SWH peaks (such as 4°E , 54.2°N) occur without strong depth gradients, possibly indicating small-scale wind variability or enhanced wave-current interaction. These observed patterns underscore the influence of spatial heterogeneity on local sea state dynamics. Notably, 1D transects (Figure 4.3a) demonstrate that SWOT can retrieve SWH values consistently down to a DTS of 1.3–2.5 km, with only one outlier (transect ix=51) showing an abrupt increase in SWH from 3.3 to 4.9 m.

Case II

Case II further validates SWOT-derived SWH through quantitative comparison with buoy data. Filtering by CTD (<55 km) and excluding low SWH values (<1 m) improves validation metrics substantially, reducing MAE by a factor of three relative to the base dataset (see Table 4.15). The 4-pixel configuration consistently outperforms 1- and 16-pixel setups across all bathymetry classes, improving from the nearshore to the offshore class. This supports the trade-off between spatial representativeness and noise reduction, with the 4-pixel configuration offering the best balance in terms of error magnitude and representation across varying coastal regimes. A full overview showing all performance metrics while applying the full filter is provided in Table C.3.

Table 4.15: Improvement in MAE (cm) across filtering steps, from the base dataset to the fully filtered dataset (CTD < 55 km, SWH > 1 m). Color tone indicates magnitude of error (light to dark blue).

Configuration	MAE (cm)				
	Base dataset	Filtered dataset	Cross-track filter	SWH filter	Full filter
1 pixel	36.7	16.7	16.7	12.4	12.4
4 pixel	29.8	15.6	13.1	12.4	8.9
16 pixel	27.9	15.4	13.3	12.3	10.8

Case III

Case III extends the analysis into the IG wave regime. Here, the sensitivity to processing choices is most pronounced in the use of the transfer function G . Across the full dataset, the 10 km version combined with the G_{PTR} transfer function, delivers the lowest MAE (0.9 cm). In contrast, the full G inversion results in systematic overestimation, with errors up to five times higher. Other processing factors, such as the azimuthal cutoff function, normalization factor α , and PSD representation, shows negligible effects by comparison.

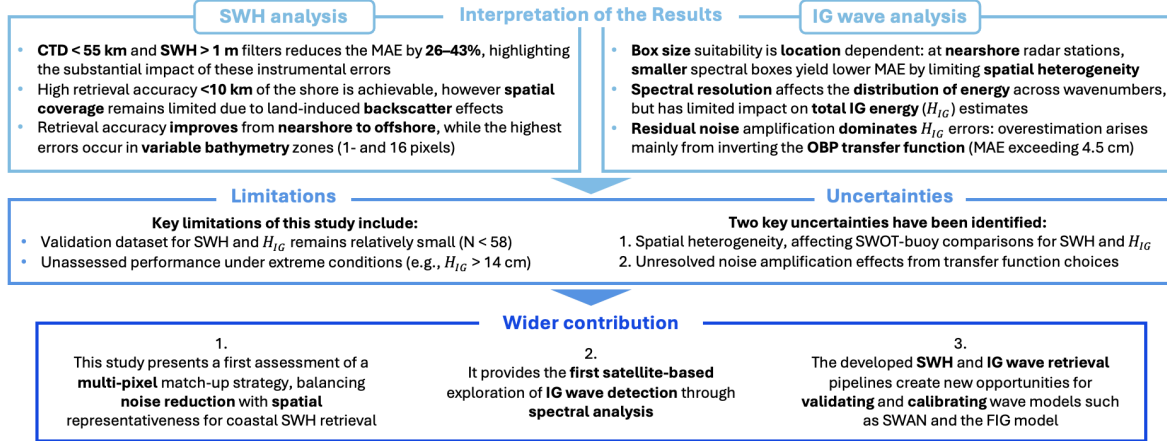
SWOT-derived H_{IG} remains consistent for values below 8 cm, typically within 2 cm of platform radar-based observations (10 km G_{PTR} configuration). A slight decrease in performance occurs for $H_{IG} > 8$ cm, which is further exemplified during the Case of storm Babet (Figure 4.17). Spatial representativeness again appears critical. The smallest box size (10 km) achieves strong agreement across all locations, while performance degrades with increasing box size, particularly nearshore (e.g., EUR located 40 km from shore, using the 40 km analysis box).

Furthermore, the consistency of both SWH and H_{IG} retrieval is illustrated during the two energetic sea states on 26 November and 22 December 2024. A clear spatial correlation between SWH and H_{IG} is observed, supporting the physical coupling between wave groups and IG energy. This is reflected in consistent increases in H_{IG} alongside SWH across both SWOT swaths.

Together, these findings inform the broader interpretation of SWOT's potential and limitations, which are further explored in chapter 5.

Discussion

By interpreting the results and discussing their limitations and uncertainties, this chapter defines the validity, scope, and appropriate application of the study, concluding with its broader scientific contribution and practical implications for operational wave modelling



This chapter reflects on the main findings of the study by interpreting the performance of SWOT-derived wave estimates across the Southern North Sea. It discusses how instrumental effects, spatial heterogeneity, and processing choices influence the retrieval of SWH and IG wave characteristics, and places the results in the context of existing validation and spectral analysis studies. The chapter further critically evaluates methodological choices and sources of uncertainty, including representation errors inherent to satellite–buoy comparisons. Finally, it considers the broader contribution of this work to the application of SWOT for coastal wave validation and analysis.

5.1. Interpretation of the results

This section provides an interpretation of the key findings presented in chapter 4. It begins with a discussion of the SWH analysis, including qualitative spatial patterns (Case I), validation results, and sensitivity to CTD, sea state, and match-up size (Case II). Subsection 5.1.2 then evaluates the feasibility of IG wave detection from SWOT SSH spectra, focusing on spatial heterogeneity and the sensitivity to spectral processing choices. The section concludes by reflecting on the suitability of SWOT to retrieve wave parameters in the North Sea.

5.1.1. SWH analysis

Instrumental error: cross-track distance sensitivity

SWOT-derived SWH results demonstrate that absolute retrieval errors tend to increase with CTD. As shown in Figure 4.6, the fitted regression lines for both the 1- and 16-pixel configurations exhibit a positive trend, indicating growing errors toward the swath edges. This is further confirmed by the performance metrics across CTD bins, with the highest MAE of 0.23 m occurring in the >50 km bin.

As illustrated in Figure 2.7b, the absolute SWH bias depends not only on CTD but also on SWH magnitude. A closer inspection reveals that the observations in the 16-pixel configuration beyond 55 km occur in lower water depths (<20 m) or coincide with low wave conditions ($SWH < 1$ m). Both of these factors are known to degrade SWH retrieval accuracy. This co-occurrence complicates attribution, making it difficult to isolate the effect of cross-track position alone.

Nevertheless, across all pixel configurations, the trend of increasing error with CTD is consistent, with the 4-pixel setup showing the steepest slope. These findings align with those of Bohé et al. (2025), who report that geolocation uncertainties, larger radar footprints, and longer atmospheric path lengths degrade retrieval performance from nadir toward the swath edges [12, 13]. Their global SWOT validation showed that KaRIn–nadir biases remain under 5 cm for SWH > 1 m but can rise to ~10 cm for SWH < 1 m. This is particularly evident in the 4-pixel configuration, where red markers (SWH < 1 m) exhibit increased errors at larger CTDs (Figure C.1).

Importantly, these results indicate that the optimal cross-track threshold may vary depending on local sea state and bathymetric conditions.

Instrumental error: backscatter contamination

One of the key instrumental limitations affecting SWOT SWH retrieval in coastal regions is backscatter contamination caused by land proximity. As the KaRIn instrument approaches the shoreline, signal returns from land surfaces can interfere with radar processing, reducing measurement quality and increasing uncertainty.

Results from the spatial transects along pixel indices 40, 49, and 51 (intersecting buoys D15, J6, and K13, see Figure 4.3a) suggest that SWOT retrieves consistent SWH values down to distances of approximately 1.3–2.5 km from shore. An exception occurs along the transect crossing buoy K13, where a sharp increase in SWH from 3.3 to 4.9 m is observed within a single pixel approximately 9.5 km offshore. This anomaly illustrates the potential for localized signal distortions when approaching the coast. Nevertheless, nearshore validation confirms that SWOT can still retrieve accurate SWH under favorable conditions. Using the filtered dataset (CTD < 55 km, SWH > 1 m), buoy observations at OS11 (7.8 km from shore) and SPY (2.1 km) show excellent agreement with SWOT-derived SWH for both the 4- and 16-pixel configurations, yielding absolute errors of 10 cm and 8 cm, respectively. These results suggest that SWOT can, under optimal circumstances, retrieve accurate SWH within the final pixel before land interference begins.

However, this performance should be considered in the context of substantial data loss due to quality issues. Of the 19 SWOT overpasses across OS11 and SPY during the 2-month analysis period, only four yielded valid (QF = 0) observations. This aligns with findings from Ho et al. (2025), who showed that nearly all KaRIn observations within 2 km of shore were flagged as degraded, suspect, or bad. Their analysis indicates that for distances within 5 km of land, roughly 80% of observations are rejected due to quality concerns [48]. This severely reduces the usable sample size for nearshore validation and introduces spatial bias toward offshore conditions.

Nonetheless, the valid observations obtained close to shore, especially at 2.1 km, demonstrate that accurate retrieval is still possible within the coastal zone. This is consistent with Turki et al. (2025), who reported robust SWOT–buoy agreement out to 4.4 km from the shore in the English Channel. Collectively, these findings highlight both the potential and the limitations of SWOT in nearshore regions. High retrieval accuracy is achievable, but spatial coverage near land remains limited due to land-induced backscatter effects.

Instrumental error: rain contamination

Beyond CTD and coastal proximity, rain contamination presents an additional instrumental challenge for SWOT's Ka-band radar. Theoretically, Ka-band pulses are sensitive to rain, especially under storm conditions when both rainfall intensity and sea state are elevated [48, 62]. During such events, rain attenuates the radar signal, degrading the reliability of KaRIn-derived SWH estimates and often resulting in suspect data classifications [14].

This effect is evident in the second case study, where a significant outlier was observed at buoy J6 on 07-12-2024. Here, SWOT recorded an SWH of 9.4 m, whereas the buoy measured only 2.0 m. A second large error occurred at buoy K13 during the same overpass, with an absolute error of 55 cm. These anomalous values coincide with regions of low radio backscatter (σ_0), as shown in Figure 5.1. σ_0 values near both J6 and K13 fall below 5 dB, indicative of significant signal attenuation likely caused by rain. While the ocean surface itself is not inherently less reflective during rain, the radar signal is weakened by rain in the atmosphere. Moreover, the KaRIn processing algorithm does not have sufficient information to fully correct for this attenuation [14]. Although the L2_LR_SSH product includes a rain flag, its limited spatial and temporal resolution makes it an unreliable sole indicator for detecting rain-affected data. In

this analysis, even observations flagged as valid ($QF = 0$) exhibit strong attenuation signatures. Thus, these low σ_0 values may help explain the elevated SWH retrieval errors near J6 and K13 during the 7 December 2024 overpass.

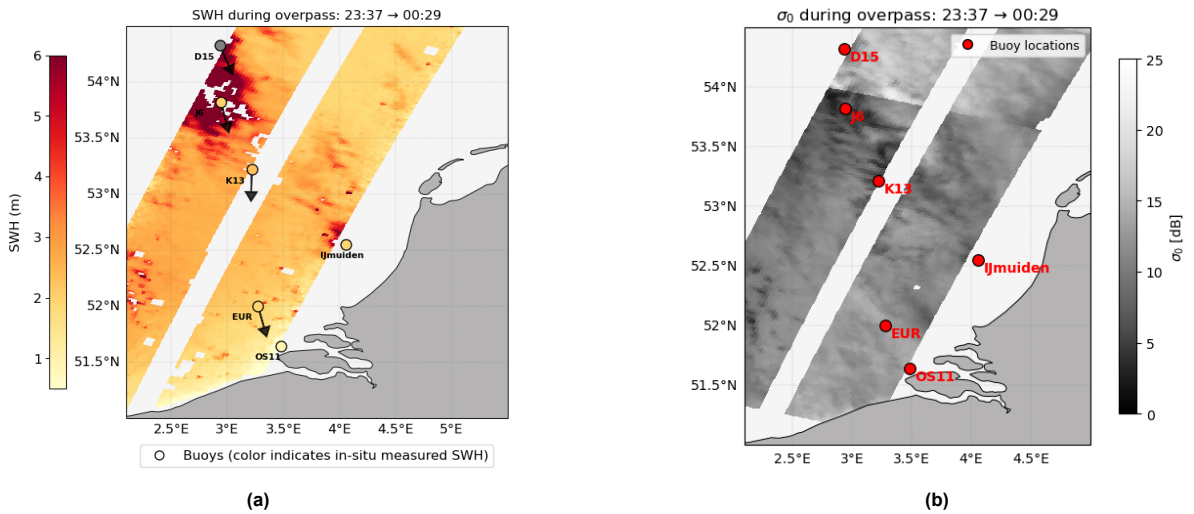


Figure 5.1: (5.1a) SWH (m) measured by SWOT, including buoy locations with observed wave directions during the overpass on 07-12-2024; (5.1b) Corresponding radio backscatter σ_0 observed by SWOT.

Representation effect: sea state sensitivity

SWOT-derived SWH shows strong agreement with in-situ observations across a range of sea state conditions, particularly for $SWH > 1$ m. As expected, the highest absolute errors occur in the 0–1 m SWH range. This is likely due to KaRIn’s reduced sensitivity under low-energy conditions, amplifying relative error contributions [12, 48].

A general trend of decreasing error with increasing wave period is observed across all configurations. This reflects the smoother, more coherent sea surfaces associated with longer-period wave conditions. However, an unexpected pattern emerges in the high-energy regime. For $SWH > 3$ m and wave periods between 6–7 s, errors increase again (Figure 4.7). This inconsistent behavior suggests that wave energy, not just wave period, influences retrieval accuracy. These conditions may reflect a physical transition from chaotic, wind-driven seas to more regular dominated wave fields. These wave fields introduce greater spatial heterogeneity, potentially amplifying representation error. Interestingly, within this challenging regime, absolute errors decrease with increasing pixel size (Table 5.1). This implies that spatial averaging helps to smooth out localized gradients in the sea surface, reducing the mismatch between SWOT observations and point-based buoy measurements.

Table 5.1: Absolute error of SWH for different pixel sizes and SWOT overpass dates at buoys F3 and K13. Observed SWH exceed 3 m, with wave periods ranging between 5 - 7 s.

Pixel size	Absolute error (m)		
	26-11-2024 (F3)	16-12-2024 (F3)	22-12-2024 (K13)
1	0.35	0.28	0.19
4	0.22	0.19	0.13
16	0.19	0.13	0.14

Notably, the wave age analysis does not exhibit a clear or robust relationship with absolute error. Although higher wave age values are generally associated with young, wind-driven seas, the 1- and 4-pixel sizes show no systematic increase in error. This contrasts with the clearer dependence observed for wave period. Within the scope of the available data, wave age alone does not emerge as a reliable indicator of spatial variability at SWOT’s resolution. This discrepancy reflects how wave period

and wave age capture different physical aspects of the sea state. Wave period captures the maturity and spatial coherence of wave groups, while wave age characterizes the relative influence of local wind forcing. A positive trend with wave age only emerges for the 16-pixel configuration, possibly due to increased spatial averaging smoothing out finer-scale variability present during active wind forcing.

Similarly, analysis of the h/L ratio shows no consistent increase in error for shallow or intermediate water depth conditions (see Figure C.3). In fact, regression lines for the 4- and 16-pixel sizes show a weak positive trend, indicating slightly higher errors under deeper-water conditions. This contradicts the hypothesis that deeper waters, free from bed-induced wave transformations, yield more reliable satellite estimates. This suggests that other processes, such as small-scale wind variability or wave-current interactions, may introduce heterogeneity even in deep water, increasing representation error.

An important consideration in interpreting both the wave age and h/L analyses is that these parameters are derived from single-point buoy observations. Sea state indicators like wave age and h/L do not inherently predict retrieval accuracy, rather, their impact depends on the spatial coherence of the wave field relative to SWOT's resolution. When wave conditions remain uniform across a 2×2 km pixel or 4 to 16 pixel averaging window, SWOT can capture sea state estimates that closely match point-based buoy data. However, when wave age or h/L vary over short spatial scales, such as during rapidly evolving wind-sea conditions or in areas with strong bathymetric gradients, spatial averaging may obscure local wave dynamics, increasing mismatch.

This relationship is further supported by the results in Table 4.5, where MAE increases from offshore to nearshore regions for the 4-pixel configuration. The variable bathymetry class shows the highest error for the 1- and 16-pixel setups, emphasizing that combining water depth, local depth gradients, and DTS provides a more complete representation of spatial sea state variability.

Representation effect: TC analysis and sensitivity to match-up size

The triple collocation analysis highlights a fundamental trade-off in selecting the optimal match-up size for SWOT SWH retrieval: statistical variance reduction through spatial averaging versus spatial representativeness relative to in-situ point measurements. The outcome of this trade-off varies depending on the sea state and spatial heterogeneity of the study region.

Larger pixel configurations (e.g. 16-pixel) reduce random instrument noise through spatial averaging, as evidenced by the comparison between pixel sizes within the TC filtered dataset (e.g., 16-pixel SWAN-NS filtered error of 0.01 m compared to 1-pixel filtered error of 0.11 m). The improvement in error statistics with increasing pixel size (filtered dataset) aligns with findings from [61], who reported that expanding the altimeter sampling size lowers random instrument error. However, this benefit must be weighed against the increased representation errors that arise when spatially-averaged measurements are compared to point-based buoy observations in regions with high spatial heterogeneity. For instance, the 4-pixel configuration consistently performs best in the variable and nearshore bathymetry classes. In the KS dataset, even the 1-pixel configuration shows the lowest error, underscoring that in such regions, finer spatial sampling is necessary to retain representativeness and minimize spatial mismatch with buoy observations.

The application of quality filters ($\text{SWH} \geq 1$ m and $\text{CTD} < 55$ km) yields substantial error reductions across all configurations and datasets. For the 16-pixel NS dataset, the absolute error improves from 0.19 m to 0.01 m. The reduction in SWOT absolute error across all configurations, from 0.08–0.19 m (unfiltered) to 0.01–0.11 m (filtered), quantifies the contribution of low wave conditions and far-range observations to the total error budget. The remaining error after filtering (0.01–0.11 m) likely reflects a combination of residual instrumental limitations and representation errors arising from scale mismatches. These include:

- Spatial scale mismatch: SWOT pixels (2×2 km) and SWAN grid cells are substantially larger than buoy footprints, potentially smoothing out fine-scale sea surface variability that buoys do capture.
- Temporal scale mismatch: SWOT provides instantaneous snapshots, while buoy and model estimates are time-averaged, smoothing out high-frequency temporal variability.

As noted by [60], when representation error plays a role, error variances are estimated with respect to the system with coarsest resolution. In this case, the unresolved variability observed by buoys is interpreted as error in the coarser SWOT or SWAN estimates. Moreover, if SWOT and SWAN

both fail to resolve this signal, their shared bias can introduce correlation between their errors. This violates the TC assumption of uncorrelated errors. Despite this limitation, the TC results still offer meaningful insight into overall system uncertainty, including both measurement and representation error components. However, sample sizes per configuration remain limited ($N_{VAL} = 8$ to 36 valid triplets), warranting caution. The 2×2 km SWOT pixels, point buoy measurements, and SWAN grid cells do not sample the same spatial footprint, and with small sample sizes, footprint mismatch can dominate the triple collocation “noise”. Thus, while TC provides valuable benchmarks for evaluating SWOT’s performance, results should be interpreted in the context of these inherent constraints.

Comparison to other missions and studies

Proximity to shore and bathymetric influence The median absolute errors across bathymetry classes reveal a consistent trend: SWH retrieval accuracy decreases when moving from offshore to nearshore regions (see Table C.1). This finding aligns with Turki et al. (2025), who reported that SWH accuracy is strongly influenced by distance to shore and local bathymetry. Their study in the English Channel showed SWOT reproduces wave characteristics with notable skill up to ~ 24 km from the coast. Highest accuracy was achieved in deeper waters, even when close to shore, such as at reflective beaches characterized by steeper slopes [13].

However, the h/L analysis in this study suggests that water depth alone does not guarantee low error. Even in deep-water conditions (where $h/L > 0.5$), elevated errors may occur, likely driven by spatial variability from wind field fluctuations or wave-current interactions. The latter phenomenon has been shown to be a major source of spatial SWH variability on scales of 1–100 km [12, 25]. These findings emphasize that bathymetric setting is only one component of the broader representation error problem.

Comparison to Ho et al. (2025) When filtering the base dataset, specifically by removing the outlier at J6 on 7 December 2024, SWOT performance in the North Sea closely aligns with results from Ho et al. (2025). In their U.S. coastal validation study using the Expert product (Version C), they reported an RMSE of 0.19 m and R^2 of 0.98. Similarly, this study finds $RMSE = 0.21$ m and $R^2 = 0.97$ in the filtered dataset (Table 4.1). One notable contrast lies in the bias. While Ho et al. observed a consistent underestimation of SWH by SWOT (-0.22 m), this study reports a general overestimation relative to in-situ buoys. Ho et al., applied an empirical correction based on Version D of the product, improving RMSE to 0.13 m, bias to 0.04 m and R^2 to 0.99. Although no empirical correction is applied here, filtering by CTD and SWH yields comparable improvements. For instance, the 1-pixel configuration achieves $RMSE = 0.16$ m, bias = 0.07 m, and $R^2 = 0.97$ (see Table C.3).

Both studies also identify significant spatial variability in SWH over short distances under energetic wave conditions. Ho et al. observed buoy–SWOT SWH differences exceeding 0.5 m over ~ 30 km, while the ECMWF wave model failed to capture this spatial variability (predicting differences of < 0.05 m) [48]. A similar pattern is visible in this study during the energetic events of 26 November and 22 December 2024. Zoomed-in SWH maps over F3 and K13 reveal gradients of ~ 0.5 m over just 20 km (Figure 4.11, Figure 4.12). These observations underscore SWOT’s capability to resolve short-scale spatial heterogeneity, that is often smoothed out in coarser-resolution wave models.

Comparison to other satellite missions SWOT also shows improved performance compared to previous altimeter missions. For example, the SARAL mission (also operating in Ka-band) reported $RMSE = 0.40$ m and $R^2 = 0.96$ across 10,800 observations covering a wide range of conditions and distances to shore [10]. Similarly, the denoised Sea State CC1 product, based on multi-mission data from 1991 to 2018, reported $RMSE = 0.44$ m and $R^2 = 0.89$ for buoys within 50 km of the coast. This dataset uses a collocation radius of 50 km and time difference threshold between buoy-satellite of 30 minutes. In contrast, this study uses a collocation radius of just 2 km, providing much closer spatial and temporal matching. The resulting RMSE values (down to 0.16 m for 1-pixel configurations) demonstrate SWOT’s improved SWH retrieval performance under more spatially precise validation. It should be noted, however, that the present validation is based on a smaller and more temporally constrained dataset, which could influence the stability of the reported error statistics.

5.1.2. Spectral analysis of IG waves

Spatial heterogeneity

The influence of spatial heterogeneity is particularly evident in the choice of the analysis box size (L_{box}). A key trade-off emerges: increasing L_{box} increases the EDF (reducing statistical variance in the PSD es-

time), but simultaneously increases spatial heterogeneity within the analysis box. The results demonstrate that this trade-off plays out differently across locations.

For instance, station A12, located in the deepest and most offshore region of the dataset, shows better agreement with in-situ H_{IG} estimates when using the largest box size (40 km). This implies that, in relatively homogeneous offshore areas, the statistical benefit of variance reduction outweighs the spatial mismatch penalty. In contrast, for stations near coastal boundaries (EUR, DTS = 40 km) or in regions with strong spatial depth gradients (especially D15), the heterogeneity penalty exceeds the statistical benefit, confirmed by the better performance using the 10 km box version. Despite the lower EDF, the 10 km configuration shows the best performance across measurement stations when applying the transfer functions. This outcome underscores that error reduction through better spatial representativeness can be more important than variance reduction in heterogeneous environments.

Further evidence comes from the analysis of the distance between the in-situ observation and the center of the SWOT swath. As shown in Figure 4.14, errors in H_{IG} shift notably (-1.5 cm to +0.5 cm) when this distance exceeds 20 km, highlighting the risk of spatial mismatch when the analysis box covers a region not fully representative of the conditions at the in-situ observation.

The per-station scatter plots of the 10 km version (Figure C.8) further highlight spatial heterogeneity effects. For example, F3 shows strong agreement (SI = 9%, bias = -0.27 cm) across a wide range of H_{IG} , including during high SWH (>4 m) and a station-to-center distance of 15–20 km. In contrast, J6 exhibits the highest scatter (SI = 26%, bias = -1.2 cm), despite similar distance-to-station characteristics. A similar underestimation trend is observed at A12 under higher H_{IG} values.

These findings suggest that location-specific factors, beyond sea state intensity, influence the agreement between SWOT and in-situ observations. The bathymetry map (Figure 3.6) reveals that A12 and J6 are located near complex features, such as the Dogger Bank or narrow channels. These regions exhibit significant depth variability across the 10 km analysis window. Two key mechanisms may explain the underestimation observed in areas with spatially varying bathymetry. First, local bed-induced wave transformations, which affect IG wave dynamics, may differ between the point-based station and the spatially averaged SWOT footprint. Since IG waves propagate in shallow to intermediate waters ($h/L \approx 0.04$ – 0.07 across the dataset), local differences in depth or slope alter the observed wave energy. Second, representation errors can arise from constructing the IG spectral mask using a single reference depth. The cutoff wavenumber used to define the mask is derived from the linear dispersion relation, assuming a uniform depth across the analysis box. However, if the analysis box spans shallower regions, valid IG energy may occur at wavenumbers higher than the buoy-defined threshold. This leads to the unintended exclusion of physically meaningful energy, resulting in systematic underestimation of H_{IG} . The sensitivity of H_{IG} to depth assumptions is further explored in subsection 5.2.2.

Spectral resolution

This study focuses primarily on validating SWOT-derived IG wave heights (H_{IG}) using platform-mounted radar observations. Since the wave height is based on the total integrated energy, the effect of spectral resolution on H_{IG} is minimal. This is supported by the negligible differences in MAE observed across the Alpha, Beta, and Gamma versions, regardless of the applied transfer function. While the spectral resolution has little effect on total energy capture, it significantly affects how energy is distributed across frequency or wavenumber bins.

The Gamma version (based on 10 km tiles) has a finer spectral resolution (0.1 cpk), allowing it to resolve more detailed spectral structure than the Alpha version (4 km tiles, 0.25 cpk). As a result, the spectral mask applied to the Gamma version is narrower and more precise, while the Alpha version includes a broader wavenumber range, slightly increasing total H_{IG} due to the inclusion of marginally more energy.

However, for the wave parameters L_{IG} and θ_{IG} , the spectral shape, not just the integrated energy, is critical. These parameters are derived from the distribution of energy over wavenumber bins. Therefore, changes in spectral resolution can shift energy distributions and alter the inferred wave characteristics. This sensitivity is illustrated in the wavenumber distributions for F3 on 26 November 2024 (see Figure 5.2). When using G_{PTR} for inversion, the total IG energy is nearly identical across the Alpha and Gamma versions (0.00089 and 0.00083 m^2 , respectively), resulting in $H_{IG} = 0.120$ m (Alpha) and $H_{IG} = 0.116$ m (Gamma). However, the peak wavenumber shifts from 1.135 cycles/km (Alpha) to 0.966 cycles/km (Gamma), and the Gamma version distributes energy more evenly across the IG wavenumber

band. This more uniform spread leads to a slightly lower estimated wavelength (909 m versus 965 m of the Alpha version) and alters directional estimates.

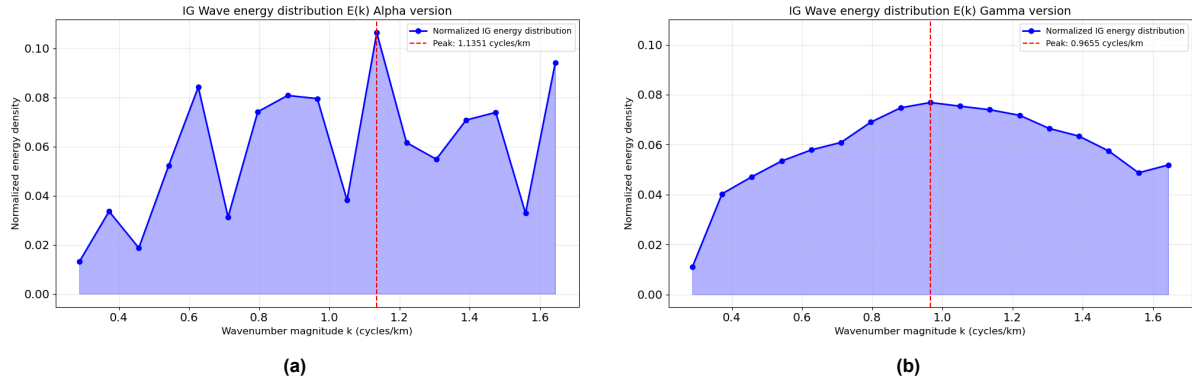


Figure 5.2: Normalized IG wave energy distributions as a function of wavenumber magnitude $|k|$ at location F3 during the SWOT overpass on 26 November 2024. (5.2a) Alpha version (4 km tiles); (5.2b) Gamma version (10 km tiles). Energy is distributed across discrete wavenumber bins using linear interpolation between adjacent bins. Although total energy is similar, spectral resolution shifts the peak (red dashed line) and redistributes energy.

These results emphasize that while H_{IG} is robust to spectral resolution, the estimated L_{IG} and θ_{IG} are more sensitive to how spectral energy is resolved and distributed. Higher spectral resolution can improve the physical interpretability of spatial IG wave characteristics without compromising energy-based accuracy.

Transfer function inversion and amplification of residual noise

The transfer function G , applied during spectral inversion of SWOT SSH PSD, accounts for two main system responses: the On-Board Processor (OBP) filtering and the azimuthal point target response (PTR) [43]. Its purpose is to correct the observed SSH PSD by compensating for signal attenuation introduced during onboard processing.

However, the analysis reveals that applying the full transfer function, especially the OBP component, leads to systematic overestimation of infragravity wave energy. Across all configurations, inverting the PSD using the OBP filter alone results in mean absolute errors exceeding 4.5 cm and positive biases of similar magnitude. The PTR-only configuration, by contrast, shows substantially lower errors (e.g., 0.9 cm MAE for the 10 km version) and smaller bias (−0.5 cm), confirming that the OBP correction dominates the error budget.

The OBP filter is designed to reduce the sampling rate to 250 m and strongly attenuate high-wavenumber content. While it effectively suppresses energy above the Nyquist frequency (2 cycles/km), it does not fully eliminate it. Residual spectral energy near the cutoff persists and is severely amplified during inversion. The OBP filter response drops to approximately −15 dB in this region, attenuating energy by a factor ~ 0.032 . As a result, dividing the observed spectrum by this small value disproportionately amplifies residual noise rather than restoring signal.

This overestimation is not primarily due to aliasing. The OBP filter attenuates most aliased energy above 2 cpk. Instead, the key issue is the amplification of residual noise that survives the FFT-based processing steps. This residual noise includes:

- Instrumental interferometric noise (thermal, phase errors, atmospheric effects) [12, 32]
- Spectral noise introduced during FFT computation
- Minor aliasing leakage near the Nyquist limit

These effects persist despite onboard filtering, as expressed in the system response:

$$\eta_{SWOT} = (\eta_{observed} + \eta_{instrumental\ noise})G_{OBP}G_{PTR} + \eta_{aliased} \quad (5.1)$$

$$\eta_{true} = \frac{\eta_{SWOT} + \eta_{processing\ noise}}{G} \quad (5.2)$$

Here η refers to the SSH signal in the spectral domain (k_x, k_y). The inversion step aims to recover the original signal but also reintroduces and amplifies residual noise. This occurs especially in the high- k range (2 cpk) where G is small. This could explain the consistently elevated H_{IG} values seen in all OBP-corrected configurations.

Additionally, the use of a broad directional IG mask (to capture energy from all directions) increases the likelihood that amplified residual noise is included in the final estimate. This is particularly problematic near the OBP attenuation edge. This further explains why the PTR-only correction performs significantly better than the full G inversion.

Finally, the dominance of OBP-induced errors is evident when compared to other sensitivity tests in this study. The following factors have negligible impact on the final H_{IG} estimates:

- Azimuth cutoff filter: 3–4% effect
- Water depth misestimation: 1–3%
- Normalization factor α and PSD representation: <0.2%

In contrast, G_{OBP} inversion contributes $\sim 60\%$ of the total MAE (see Table 4.9a), confirming that residual noise amplification via spectral inversion is the primary source of overestimation in H_{IG} retrieval.

Wave-induced errors and range bunching susceptibility

Wave-induced errors In addition to instrumental noise, wave-induced SSH errors contribute to inaccuracies in SWOT-derived H_{IG} estimates. As discussed in subsection 2.3.4, surface waves can distort the SSH PSD, by redistributing energy across wavenumbers. These effects intensify with increasing SWH, as demonstrated in Figure 2.8, where SSH height errors increase systematically with sea state.

This relationship is further explored in Figure 5.3, which shows absolute H_{IG} error against SWH for the 10 km configuration under two inversion strategies: PTR-only (5.3a) and full transfer function G (5.3b). In both cases, errors tend to increase with SWH, but the slope is significantly steeper for the full G inversion. At higher SWH, increased surface motion likely leads to higher residual noise in the SSH PSD. When the PSD is inverted using the OBP component of G , this residual noise is reintroduced. This effect is evident for locations such as K13 and F3, where absolute errors rise rapidly with SWH under the full G inversion. In contrast, when using G_{PTR} alone, the dependence on SWH is much weaker (regression slope = 0.001), and most absolute errors remain below 1.5 cm across the full SWH range (0.5–4.5 m). For F3, errors remain consistently low (<0.8 cm) even for SWH exceeding 3.5 m, including cases where the station-to-swath-center distance exceeds 15 km. A similar pattern is observed for K13.

These results indicate that the strong sea-state dependence observed with the full transfer function primarily arises from amplification of wave-induced SSH errors by the transfer function inversion, rather than from wave-induced variability alone.

The relatively flat error distribution obtained with G_{PTR} suggests that:

- The PTR correction (approximately -5.6 dB attenuation, power factor ~ 0.28) provides sufficient correction for point target response effects without introducing excessive noise amplification.
- Wave-induced SSH errors, while present, are not strongly amplified when only the PTR component is inverted, resulting in more stable H_{IG} estimates across varying sea states.

Nevertheless, increased sea state can also enhance representation error through greater spatial heterogeneity of the IG wave field. Results from Case II show that for SWH exceeding 3 m, absolute errors of SWH increase between SWOT and in-situ observations. Spatial transects during the energetic overpasses of 26 November and 22 December 2024 (Figure 4.18, Figure 4.21) further reveal a clear coupling between SWH and H_{IG} , indicating that energetic conditions can lead to stronger spatial variability in the IG wave field. This effect is visible when examining the absolute errors in 5.3a. For A12, three observations with station-to-swath-center distances greater than 15 km show increasing absolute errors with SWH, ranging from approximately 0.1 cm at $H_s = 0.8$ m to 2.2 cm at $H_s = 2.5$ m. Since amplification effects are minimal under G_{PTR} , these errors are most plausibly attributed to spatial heterogeneity in the IG wave field. As sea state increases, the mismatch between point-based measurements and spatially averaged SWOT estimates becomes more pronounced, leading to increased representation error.

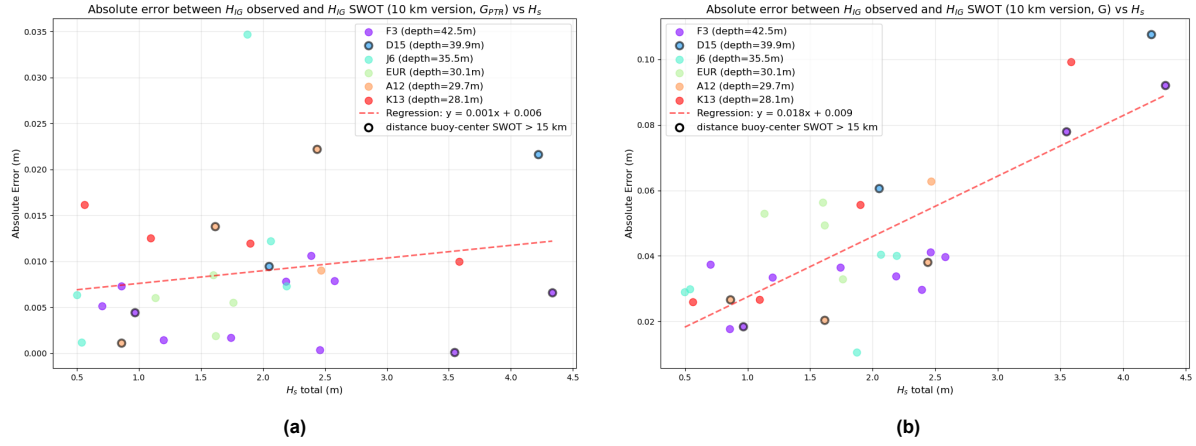


Figure 5.3: (5.3a) Absolute error between in-situ observed and SWOT H_{IG} as a function of SWH (H_s) using the 10 km version with G_{PTR} inversion, and while using the 10 km version with the full G inversion (5.3b). Fitted regression lines are added as reference. Observations with distance station to center of swath larger than 15 km are marked in black. Note, D15 is always located at a distance > 15 km from the center of swath.

Overall, these results indicate that while wave-induced SSH errors contribute to increased uncertainty under energetic conditions, their impact on H_{IG} retrieval is strongly modulated by the inversion strategy. The full G inversion amplifies wave-induced residual noise, whereas the PTR-only approach reveals more clearly the growing role of representation error with increasing sea state.

Range bunching susceptibility Part of wave-induced errors, includes the phenomena of range bunching (also known as the surfboard effect). As discussed in subsection 2.3.4, the surfboard effect can redistribute energy across cross-track wavenumbers (k_x), particularly under steep and high-amplitude wave conditions. Theoretical work by [36] indicates that the effect is strongest for waves aligned in the cross-track direction and vanishes for along-track waves.

In the storm case analysed by [47] (S5, $H_s = 7.67$ m), range bunching and potential unaccounted filtering caused a twofold underestimation of SSH variance. There, two mechanisms contributed: (1) underestimation of the true along-track filtering by the PTR, and (2) energy redistribution by range bunching not captured in the transfer function correction. The full radial mask approach used in this study, which integrates energy over all directions within a specified wavenumber range, is less prone to range bunching effects than directional analysis because energy that redistributes within the mask is still captured. However, it can still underestimate H_{IG} if energy is pushed outside the mask boundaries, particularly in energetic sea states.

For the North Sea, where average SWH and wave steepness are substantially lower than in the S5 case, strong range bunching effects are not expected. Nonetheless, the overpass on 16 November 2024 presents an interesting case. Here, SWOT significantly underestimates H_{IG} for A12 and J6 (by 3–4 cm), despite moderate SWH (2.44 and 1.87 m) and cross-track aligned waves (W–SW direction against a S–SE heading). By contrast, station F3 under similar wave conditions (SWH 2.46 m) shows good agreement (0.9 cm overestimation), suggesting that range bunching alone is insufficient to explain the underestimation at A12 and J6.

As previously discussed, both A12 and J6 are located in areas with complex bathymetry, where strong depth gradients may induce wave transformations not captured by point-based buoy observations. A combined effect of mild range bunching and representation error could plausibly explain the anomaly, but further research is needed to isolate the contributions of each mechanism and assess range bunching effects under moderate sea states.

5.1.3. Suitability of SWOT to retrieve wave parameters in the North Sea

SWH observations

SWOT-derived SWH shows strong agreement with in-situ observations across a range of sea state conditions, particularly for SWH > 1 m. The 4-pixel configuration achieves a MAE of 8.9 cm and a STD of 10 cm in the full filtered dataset, highlighting its potential as a robust method for SWH retrieval in the Southern North Sea.

SWOT retains reliable performance up to 1.3–2.5 km offshore, as demonstrated by buoys OS11 (MAE = 6 cm) and SPY (MAE = 11 cm), located 7.8 km and 2.1 km from the coastline, respectively. This nearshore capability is particularly promising, as conventional nadir altimeters typically exhibit degraded accuracy in coastal zones. SWOT's ability to maintain accuracy closer to shore represents a meaningful advance over traditional missions [13]. However, this should be considered in light of the substantial data reduction when only observations with quality flag zero are retained.

A few notable outliers remain. For instance, transect ix=51 shows a sharp local increase in SWH (from 3.3 to 4.9 m), while at buoys J6 and K13 large discrepancies were recorded on 07-12-2024 (e.g., 9.4 m SWOT vs. 2.0 m in-situ at J6). These anomalies likely result from contamination due to rain or enhanced backscatter. While such local errors warrant caution when interpreting spatial SWH fields, they do not undermine the overall reliability of SWOT-derived SWH at the regional scale. Future processing methods could benefit from implementing de-spiking techniques to flag and remove erroneous “hot pixels” (for e.g. individual pixels with values twice as high as surrounding median) that escape the current quality flags [48].

In summary, SWOT's wide-swath observations offer significant potential for monitoring wave conditions in the North Sea, particularly in coastal regions where conventional altimetry faces limitations.

IG waves

In contrast to SWH retrieval, which benefits from spatial filtering, the estimation of IG wave energy from SWOT is more sensitive to spectral processing choices and remains constrained by residual spectral noise and spatial representation errors. The results show that the observed SSH signal is strongly influenced by residual noise which is reintroduced and amplified when inverting the OBP filter. This effect becomes particularly problematic under high SWH conditions, likely due to increased wave-induced noise. Additionally, the choice of spectral box size emerges as a key methodological factor. Smaller analysis boxes result in lower MAE and STD, especially at buoys closer to shore. Smaller footprints reduce the likelihood of capturing the spatially heterogeneous regions that dominate the shallow to intermediate water depth ranges studied here.

Despite some degradation at higher wave heights and location-specific variability, SWOT shows consistent performance within the analysed dataset. When the J6 outlier (16-11-2024) is excluded and the analysis is restricted to observations within 15 km of the swath center, the G_{PTR} -corrected estimates remain within ± 2 cm of the in-situ measurements. Moreover, applying the PTR transfer function yields a 62% reduction in bias and a 36% reduction in MAE compared to the raw SWOT observed SSH PSD (no G inversion). The transfer function correction provides substantial improvements across all wave height ranges, making it an essential component for accurate IG wave retrieval from SWOT data.

While the combination of PTR and AZC correction shows potential for further improvement, the AZC response depends on external wave condition parameters (SWH and mean zero-crossing period T_{m02}), which may not be available in all applications. In this context, the PTR transfer function is recommended for estimating total IG wave energy in the North Sea, as it provides the best balance between accuracy and bias reduction while being applicable without requiring additional wave model data.

Based on these findings, SWOT can be considered a reliable instrument for retrieving IG wave height parameters in the North Sea under moderate IG wave energy conditions ($H_{IG} < 10.5$ cm). However, retrieval accuracy remains sensitive to noise amplification during transfer function inversion. Further research into noise scaling effects is needed to fully optimize the methodology.

5.2. Methodological limitations and uncertainty

This section critically evaluates key methodological decisions made in this study and the associated uncertainties they introduce. In addition, the main data limitations are presented that negatively influence the validation outcome of this research.

5.2.1. Data limitations

Temporal and spatial coverage

The ~5-day revisit time of SWOT limits the temporal resolution of the dataset, restricting the ability to capture the full range of sea state conditions. Although a two-month analysis window was selected to maximize data availability, it still omits a significant portion of energetic sea states. For instance, the

maximum H_{IG} observed during the 2024 dataset was 10.47 cm, while only one matching event from the 2023–2024 storm season captured a value exceeding 12 cm. As a result, the retrieval performance under extreme conditions remains uncertain.

In terms of spatial coverage, the strict application of the quality flag ($QF = 0$) substantially reduces nearshore data availability, skewing the validation sample toward offshore buoys. This is especially evident in the 1-pixel nearshore bathymetry class, which includes only 5 samples. This is three times smaller than the variable and offshore datasets, therefore undermining the statistical robustness of performance metrics in shallow coastal areas.

Furthermore, SWOT's orbit and track geometry do not cover all regions of the North Sea, inherently limiting the spatial scope of the analysis. While the study incorporates data from 9 buoys for SWH validation and 6 measurement locations for IG wave analysis, the uneven spatial distribution constrains regional representativeness. Nevertheless, the selected sites span a wide range of water depths, distances to shore, and local bathymetric conditions, thereby strengthening the diversification of the analysis.

β -factor for SWH retrieval

This study uses the Version C Expert product of SWOT, which does not incorporate the β -factor correction. The β angle, defined as the angle between the Earth–Sun vector and the satellite's orbital plane, influences the thermal environment of the KaRIn instrument and, consequently, the accuracy of SWH retrieval. This has especially influence on the retrieval of low wave heights and large CTDs [12].

Recent work by Ho et al. (2025) demonstrates that the updated Version D product, which includes β -factor corrections, improves SWH performance across all CTDs and for wave heights including the 1–3.5 m range [48]. Although the filtered dataset used here excludes cases with SWH < 1 m and distances > 55 km, the inclusion of the β -factor may still enhance accuracy, particularly in higher wave height regimes. Future studies using Version D products are likely to benefit from these refinements.

Radar uncertainty and limitations

In addition to SWOT-related uncertainties, the in-situ radar measurements used for validation may introduce minor observational errors. Akrish et al. (2025) note that radar systems can exhibit deviations in capturing low-frequency wave heights, potentially affecting estimates of H_{BIG} and by extension, the total H_{IG} [6]. However, simulations by Jangir et al. (2022) demonstrate that radar instruments perform reliably up to 0.5 Hz, well above the 0.005–0.033 Hz frequency band used in this study [63]. Therefore, any impact of radar-related measurement uncertainty on the IG wave analysis is expected to be minimal.

The validation of SWOT-derived IG wave parameters in this study was limited to wave height and wavelength, as only 1D in-situ radar spectra were available. As a result, the directional estimates of IG wave energy from SWOT could not be quantitatively assessed and were therefore interpreted qualitatively. Future studies incorporating 2D spectral measurements would enable more robust validation of directional outputs and strengthen confidence in SWOT's capacity to resolve IG wave directionality.

5.2.2. Methodological limitations and uncertainties

The key methodological limitations of this study are outlined in the following subsections. For SWH retrieval, the focus is on representation errors between SWOT and in-situ observations. For spectral analysis, limitations related to the transfer function, IG mask assumptions and spectral-domain differences are discussed.

SWH analysis: representation mismatch between SWOT and in-situ observations

Wave group effects In open-ocean validation studies, wave group modulations have been shown to be a dominant source of sampling mismatch. Bohé et al. (2025) demonstrated that under extreme sea states (e.g., SWH > 9 m), wave-group sampling mismatch can explain a substantial fraction of the large (> 0.5 m) differences observed between 2 km SWOT pixels and buoy SWH estimates [12]. Ho et al. (2025) further quantified this effect by applying 2D Gaussian filters to SWOT SWH fields. When compared to hourly averaged buoy SWH, applying a 10 km spatial filter reduced RMSE by approximately 20% relative to unfiltered 2 km SWOT data [48]. Notably, when a shorter 20-minute averaging window was used for buoy data, the RMSE minimum occurred for a filter size around 4.5 km, which closely matches the optimal pixel configuration used in this study.

Both these studies apply continuous spatial smoothing to mitigate wave group modulation effects. However, compared to the Southern North Sea, a key difference lies in the spatial scale of wave groups. The analyses by Bohé et al. and Ho et al. are based in deep ocean environments, where wave groups can extend over several kilometers. In contrast, the maximum observed IG wavelength in this study is approximately 1,150 m, which also defines the upper bound for wave group size (wave group length $\approx L_{BIG}$). This implies that even within a single 2×2 km SWOT pixel, at least two wave groups are sampled, inherently averaging out group-scale variability. As a result, the sampling mismatch observed in open-ocean studies is expected to be less severe in the coastal settings of this study.

Despite the reduced influence of wave group sampling, representation errors still persist. These are primarily attributed to spatial variability of the wave field introduced by complex bathymetry and stronger wind forcing effects. This spatial heterogeneity complicates direct comparison between point-based buoy data and spatially averaged SWOT observations, especially in shallow and transition-depth regions. Even with group-scale variability largely accounted for, these dynamic local effects remain a key source of uncertainty in SWH validation.

Spatial interpolation In this study, the spatial match between SWOT pixels and in-situ buoy observations was simplified by selecting the pixel closest to the buoy within a 2 km radius. However, even for these selected pixels, a spatial offset remains between the pixel center and the actual buoy location. In the 26 November case, a detailed inspection near buoy F3 reveals that the third and fourth closest pixels yielded SWH estimates closer to the buoy measurement than the nearest one. This suggests that spatial interpolation could improve agreement with in-situ data.

Such interpolation could also be extended to multi-pixel configurations by assigning distance-based weights to each valid pixel. In that case, SWH would be estimated as a weighted average, where closer pixels contribute more strongly. A comparable approach is applied by Ho et al. (2025), who use linear interpolation between the nearest KaRIn pixels to align with buoy locations. While this method may reduce spatial mismatch and improve validation accuracy, its effectiveness in the coastal context of this study remains to be assessed in future research.

Temporal interpolation In addition to the simplification in spatial interpolation, this study applies linear interpolation of in-situ and model time series data to estimate SWH at the exact pixel time of the SWOT overpass. However, during energetic sea states, buoy data show that SWH can vary significantly, up to 0.5 m within just 30 minutes (Table 5.2). This highlights the dynamic nature of the sea surface under such conditions and may partially explain the reduced agreement between SWOT-derived and buoy-observed SWH in Case II, especially for $H_s > 2.75$ m.

Table 5.2: Time series of SWH at buoy locations F3 and K13 during selected SWOT overpasses.

Time	H_s at F3	Time	H_s at F3	Time	H_s at K13
11/26/24 01:30	4.57 m	12/16/24 22:00	3.67 m	12/22/24 07:00	3.82 m
11/26/24 02:00	4.06 m	12/16/24 22:30	3.81 m	12/22/24 07:30	3.51 m
11/26/24 02:30	4.42 m	12/16/24 23:00	3.31 m	12/22/24 08:00	3.93 m

Linear interpolation assumes a smooth transition between measurement points and may introduce timing errors if sea state conditions evolve non-linearly. This can particularly occur during storms or fast-changing wind-sea conditions. Nevertheless, it is important to recognize that the in-situ SWH estimates used here are already based on 20-minute time series windows, which inherently smooth out high-frequency variability. Similarly, SWOT pixels spatially average SWH over 2 km boxes, which suppress local wave fluctuations. Therefore, while linear interpolation introduces some uncertainty, both datasets represent smoothed estimates of the wave field, limiting the impact of this temporal mismatch on overall SWH comparisons.

Spectral analysis: transfer function assumptions

The transfer functions used to invert the SWOT SSH PSD rely on several assumptions and simplifications that may introduce uncertainties in the retrieved wave energy. Three key limitations are identified:

PTR parameterization The PTR filter G_{PTR} uses a theoretical sinc-function approximation for each of the nine Doppler beams, combined with fixed beam weights representing orbit-averaged values [47]. This simplified representation does not account for along-track variations in beam weights or wave-steepness-dependent effects, which may result in an underestimation of along-track filtering.

Neglected range bunching effects Range bunching can cause additional deviations of observed wave energy, particularly for waves propagating at oblique angles to the satellite track. Neglecting this effect may result in a systematic under- or overestimation of wave energy, especially for waves with significant cross-track components.

AZC filter assumption The azimuth cut-off filter G_{AZC} is parameterized using velocity bunching theory from traditional SAR imaging, an assumption not yet validated for interferometric measurements [43]. The direct application of this equation for the coherence phase of an interferometric system is still under investigation.

Despite these assumptions and potential limitations, the results presented in this study demonstrate that residual noise dominate the error budget when inverting the full transfer function G . The strong amplification of noise by the OBP filter (factor of ~ 32 near Nyquist) means that uncertainties in PTR parameterization, neglected range bunching, or AZC assumptions become secondary compared to the noise amplification problem. This is evidenced by the dramatic error reduction when using G_{PTR} instead of full G , where weaker amplification allows more reliable signal retrieval despite the same assumptions. The dominance of noise amplification suggests that improvements in noise modelling are of higher priority than refining the PTR or AZC parameterizations for IG wave retrieval. As noise models improve and residual errors decrease, these assumptions may become more critical, warranting further investigation into their validity for interferometric measurements.

Spectral analysis: IG mask assumptions

The IG mask applied to SWOT data uses a fixed wavenumber threshold derived from the linear dispersion relation, assuming constant water depths taken from the EMODnet bathymetry database. These depths are referenced to Lowest Astronomical Tide (LAT), whereas in reality, actual water levels vary due to tides, storm surges, and other atmospheric influences. As a result, the true water depth at the time of each overpass may differ from the assumed value, which directly impacts the upper wavenumber cutoff ($k_{\max}$) used in the IG mask, and thus the estimated IG wave energy.

Sensitivity tests at location F3 (depth = 42.5 m) show that a $\pm 10\%$ change in depth results in only a 0.7–2.5% change in H_{IG} when no transfer functions are applied. For the full G inversion, this leads to a 2.7–12.3% change under calm and energetic conditions, respectively. During the overpass of 16-11-2024, two significant underestimations at stations J6 and A12 were observed (~ 3 –4 cm, corresponding to 37% and 49% difference, respectively). While inaccuracies in assumed water depth may contribute to these underestimations, they are unlikely to fully explain the discrepancies. Nevertheless, for measurement stations located in shallower waters, variations in depth may have a larger impact. If water depths are lower, the upper wavenumber limit shifts closer to the Nyquist drop-off, where residual noise dominates the SSH PSD.

Spectral analysis: mismatch between SWOT and radar measurement domains

Another limitation arises from the fundamental difference in how IG energy is captured: SWOT analyses SSH variability in wavenumber space, while radars measure energy in frequency space. This distinction becomes critical for BIG waves, which do not propagate freely but are forced by the group velocity of wind waves [26]. These waves share the same frequency as their forcing wave groups, but propagate with a lower celerity. As a result BIG waves, while having the same frequencies, have shorter wavelengths (higher k).

Consequently, part of the IG energy observed by radar within the 0.005–0.033 Hz band may correspond to wavenumbers beyond the $k_{\max}$ threshold used in the SWOT mask and therefore be excluded from the SWOT IG energy estimate. The significance of this mismatch depends on the relative contributions of freely propagating and bound waves. When BIG is dominant, the mismatch becomes bigger, with energy in the 1D SWOT energy spectrum shifting toward higher frequencies relative to radar. This behaviour is illustrated in Panels I and II of Figure 5.4. Here, the in-situ radar spectrum is represented in wavenumber space by distributing the BIG contribution according to its bound-wave (c_g -controlled) wavenumber, which shifts energy toward higher k . A direct mapping of the full $E(f)$ to $E(k)$ using linear dispersion would yield a more similar spectral shape.

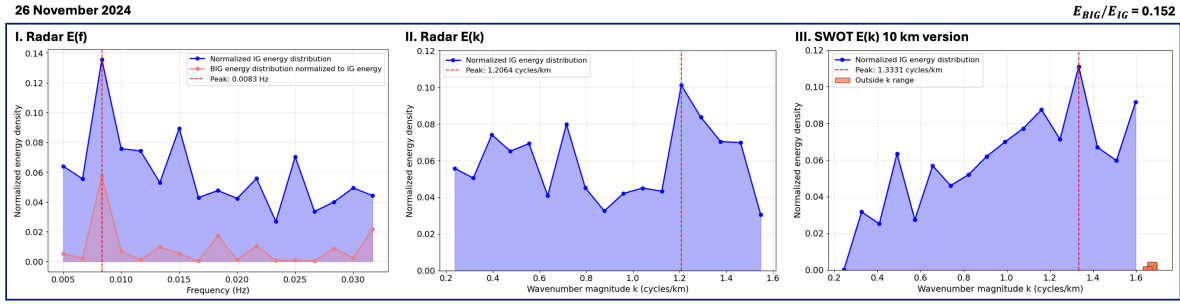


Figure 5.4: Normalized IG wave energy distributions from platform-mounted radar measurement and SWOT at F3 for 26 November 2024. Panel I: Radar $E(f)$ with IG energy (blue) and BIG energy normalized to IG (orange). Ratio between BIG/IG energy results in 0.152. Panel II: Radar $E(k)$ in wavenumber space. Panel III: SWOT $E(k)$ in 1D wavenumber space (10 km version with PTR inversion). Energy is distributed across the same wavenumber bins as the radar-measurement distribution using linear interpolation between adjacent bins. Peak frequencies/wavenumbers are marked by red dashed lines.

To ensure a consistent comparison between instruments, the most appropriate reference is therefore the wavenumber domain. Comparing Panels II and III shows that radar and SWOT spectra exhibit similar peak wavenumbers (approximately 1.2–1.3 cycles/km). However, despite these comparable peak locations, the normalized energy distributions differ in overall shape. These shape differences arise from three main factors:

1. Representation errors associated with spatial heterogeneity of the sea surface,
2. Spectral noise in SWOT SSH PSD due to instrumental and wave-induced errors,
3. Spectral-processing differences between both instruments.

The first two factors have been discussed in detail in subsection 5.1.2. The third source of mismatch originates from the fundamentally different sampling domains and statistical averaging strategies of both instruments. Radar spectra in this study are derived from SSH time-series using 600 s blocks within a 60-minute record (17 FFT blocks when using 75% overlap). In contrast, SWOT estimates spectra from spatial tiles using cross- and along-track pixels. Depending on the box-tile configuration, the number of spatial blocks used for averaging differs substantially, resulting in varying EDF (200, 128, 32, 32 and 8 for the Alpha, Beta, Gamma, 20 km and 10 km versions, respectively).

The influence of EDF on spectral shape is evident in Figure 5.2, where the Gamma configuration (200 blocks) produces the smoothest normalized distribution despite similar total masked energy. This demonstrates that spectral smoothness and variance are strongly controlled by the number of statistically independent blocks. In comparison, the in-situ radar spectrum in this study is based on only 17 temporal blocks. While this yields fewer EDF than the higher-resolution SWOT configurations (Alpha, Beta, Gamma, and 20 km), the 10 km version used in Figure 5.4 has an EDF of only 8, implying reduced statistical stability relative to the platform-mounted radar spectrum.

Additional minor discrepancies arise from differences in spectral discretization. Platform-mounted radar spectra are constructed on fixed frequency bins (with $\Delta f = 1/600$ Hz), which map to discrete wavenumber points through the dispersion relation. In contrast, SWOT directly estimates energy in discrete spatial k -space pixels. Converting these pixels to 1D wavenumber magnitudes and interpolating them onto the radar wavenumber grid may introduce small redistribution effects.

Nevertheless, the overall overestimation of H_{IG} observed when applying the full transfer function G suggests that noise amplification during spectral inversion remains the dominant source of error. Both the frequency–wavenumber misclassification associated with BIG energy and the sampling-domain and spectral-averaging differences discussed above appear to be of secondary importance. Still, these spectral mismatches can meaningfully impact retrieval of IG wavelength and mean direction, and should therefore be considered in future efforts to accurately characterize IG wave fields from SWOT.

5.2.3. Key points

The limitations outlined above do not undermine the validity of this study but help define its scope and appropriate application. Among the various uncertainties, two stand out as particularly consequential:

- Spatial heterogeneity, which affects the comparison between SWOT and in-situ measurements for both SWH and H_{IG}
- Transfer function choice, especially noise amplification effects in the spectral analysis

These two factors most strongly influence retrieval accuracy and should be a central focus of future research. For spectral analysis, improving noise modelling and evaluating the robustness of transfer functions under varying sea states is key. For SWH, spatial variability, driven by bathymetry, and potentially by wind forcing or wave-current interactions, remains a dominant source of representation error. The term “potentially” is used here because this study did not include spatial wind or current data to directly attribute observed wave height variations to these mechanisms. Nevertheless, the mismatch can be partially mitigated by using a 4-pixel averaging strategy, which smooths small-scale variability and reduces noise.

Temporal limitations are harder to address given SWOT’s 5-day revisit time, but the results indicate that meaningful insights can still be obtained within this constraint. Ultimately, the challenge of defining an observational ground truth remains central. As high-resolution satellite products continue to evolve, understanding the spatial and temporal scales of wave variability becomes increasingly important for refining validation methods [48].

5.3. Wider contribution

By extending validation into the infragravity wave regime and testing retrieval sensitivity under complex coastal conditions, this study contributes to a broader understanding of SWOT’s potential to observe nearshore wave dynamics. It addresses a key research gap by evaluating SWH and H_{IG} retrieval in shallow-to-intermediate water depths. Such environments were largely absent from previous open-ocean studies. Moreover, it presents the first satellite-based exploration of IG wave detection using spectral analysis.

The primary innovation lies in bridging SWOT’s theoretical capabilities with practical retrieval strategies suitable for coastal seas. Methodologically, the study introduces several advances:

- A single- and multi-pixel match-up strategy that reduces noise variance while maintaining spatial representativeness.
- Tailored box-tile configurations that improve agreement with in-situ IG wave energy observations across variable bathymetry.
- An adapted IG mask framework enabling estimation of H_{IG} , L_{IG} and θ_{IG} .

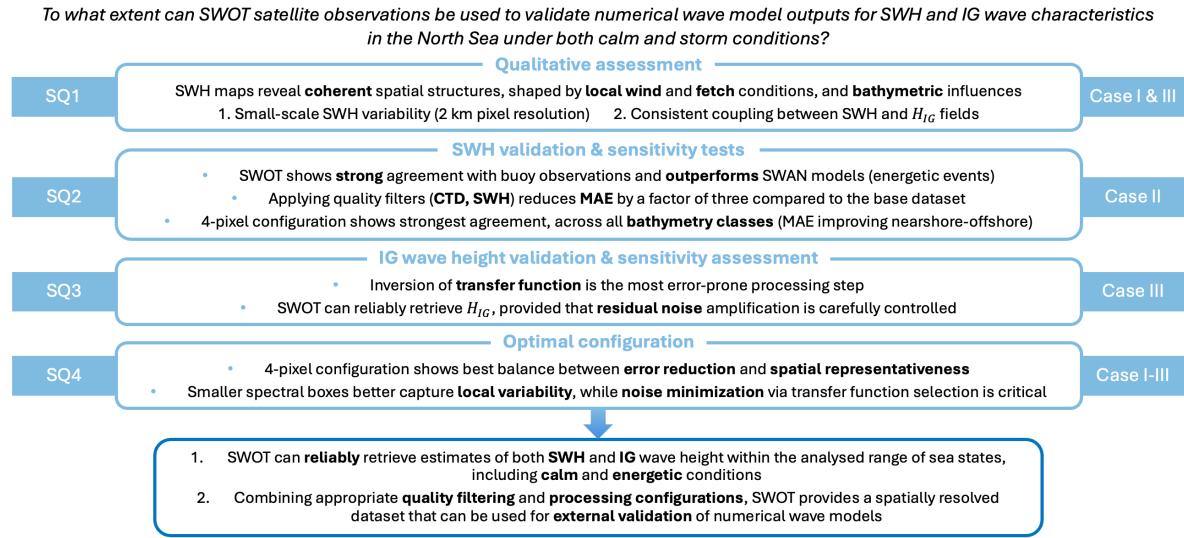
These developments enhance the reliability of SWOT-derived SWH in coastal zones and lay the foundation for consistent IG wave parameter retrieval. While focused on the Southern North Sea, the proposed configurations and filtering strategies are conceptually transferable to other regions with complex bathymetry and wind-sea dominated wave conditions.

Beyond academic insight, this study also has practical implications for coastal oceanography and operational wave modelling:

- **Boundary condition improvement:** SWOT’s high-resolution SWH observations can be assimilated into coastal models to better constrain boundary inputs, particularly during energetic events.
- **Model calibration and validation:** The IG wave retrieval pipeline offers new validation opportunities for models such as the IG wave model developed by Akrish et al. (2025). Additionally, the accurate spatial SWH estimates enable a robust validation tool for the SWAN-NS and KS models, which are known to underestimate wave energy under storm conditions.
- **Capturing small-scale variability:** SWOT’s 2×2 km resolution allows detection of sub-grid SWH variability often missed or smoothed out in models like SWAN-NS, which operates at 2.5–4 km resolution.

In the broader context of future satellite-based coastal monitoring, this study supports SWOT’s integration into multi-mission altimetry frameworks, expanding the potential of wave data assimilation. As satellite resolution improves, the retrieval strategies and IG processing tools presented here can inform next-generation validation methods and assimilation schemes to enhance the reliability of operational numerical wave models.

Conclusion



This study was structured around three cases, each targeting a specific aspect of SWOT's potential and addressing the study's four subquestions.

Case I explored spatial patterns in SSH and SWH during energetic sea states. SWOT's SSH fields revealed small-scale random variability at 250 m pixel resolution, while the SWH maps showed coherent spatial patterns shaped by local wind and fetch effects. These patterns, including isolated SWH peaks, highlight the influence of spatial heterogeneity on local sea state dynamics. This case demonstrated that SWOT can detect localized wave features and revealed its unique ability to observe 2D wave variability in coastal seas, with consistent SWH outputs up to a distance to shore of 1.3-2.5 km.

Case II focused on quantitative SWH validation and sensitivity to match-up strategies. SWOT-derived SWH shows strong agreement with buoy observations and outperforms the SWAN-NS and KS models, particularly during energetic events. Applying quality filters (e.g., cross-track distance < 55 km, SWH > 1 m) significantly improves accuracy, reducing MAE by a factor of three compared to the base dataset. The 4-pixel configuration consistently yielded the best balance between variance suppression and spatial representativeness, across varying coastal regimes. The results highlight the influence of complex bathymetries on SWH retrieval, with MAE highest at buoy locations close to shore. Nevertheless, the 4-pixel configuration shows strong agreement with buoy observations, resulting in an overall MAE of 8.9 cm across all bathymetry classes. These results support a robust, region-specific framework for coastal SWH retrieval that maintains accuracy across both nearshore and offshore conditions.

Case III extended the analysis into the low-frequency wave regime, developing a reproducible processing workflow for detecting IG wave energy and retrieving parameters such as H_{IG} , L_{IG} , and θ_{IG} . The analysis identified the inversion of the transfer function as the most error-prone processing step due to noise amplification. Other processing factors, such as the azimuthal cutoff function, normalization factor α , and PSD representation, shows negligible effects by comparison. The best-performing configuration, using a 10 km spectral box and the G_{PTR} transfer function, achieved a low MAE (0.9 cm) and minimal bias (-0.5 cm). While this indicates robust retrieval of H_{IG} , the estimation of L_{IG} and θ_{IG} remains more sensitive to spectral resolution and to the frequency–wavenumber mismatch

between radar and SWOT spectra. The enhanced performance of smaller box size, illustrates that spatial representation is key in heterogeneous coastal settings. The results confirm that SWOT can reliably retrieve H_{IG} in shallow-to-intermediate water depths, provided that residual noise amplification is carefully controlled.

A broader spatial analysis during energetic overpasses revealed consistent coupling between SWH and H_{IG} fields. Notably, enhanced IG energy was observed in front of the Danish coast, corresponding to the locally reflected FIG wave energy. These findings reinforce SWOT's capability to resolve coastal wave dynamics and spatial variability linked to bathymetry, wind, and coastline influences.

In summary, the most reliable SWH retrieval in coastal regions is achieved using a 4-pixel averaging strategy, balancing error reduction and spatial representation. Unlike open-ocean settings, where wave group effects dominate representation errors, coastal retrievals are more affected by spatial heterogeneity induced by wind forcing, and bed-induced wave transformations. For IG analysis, smaller spectral boxes better capture local variability, while noise minimization via transfer function selection (especially G_{PTR}) is critical. The use of a wavenumber-based IG mask introduces minor uncertainties, but proves to be effective with appropriate transfer function selection.

Overall, this study demonstrates that SWOT can reliably retrieve spatially resolved estimates of SWH and IG wave height within the analysed range of sea states, including both calm and energetic conditions. When combined with appropriate quality filtering and spectral processing, these observations provide a spatially distributed dataset for external validation of numerical wave models in the North Sea. This spatial coverage extends validation beyond conventional point-based measurements. The results further highlight SWOT's potential to support wave model improvement, refine boundary conditions, and enrich data assimilation efforts in coastal environments.

Recommendations

This research explored the potential of SWOT to retrieve two-dimensional estimates of SWH and IG wave parameters in the North Sea. While the results demonstrate clear promise, several opportunities remain to further refine the methodology and to advance the practical integration of SWOT observations into numerical wave modelling frameworks.

The recommendations are grouped into three main areas: (i) methodological improvements to reduce uncertainty and strengthen the technical basis of wave parameter retrieval; (ii) expansion of SWOT data usage to better understand wind-wave, wave-current and wave-bottom interactions; and (iii) exploration of higher-resolution SWOT products to extend IG analysis toward the nearshore. Together, these steps are essential for progressing from validation studies toward operational model integration.

7.1. Methodological improvements

This study combined established and novel approaches, including discrete spatial averaging for SWH validation and a spectral processing approach for IG wave parameter retrieval. Future work could strengthen this framework in several ways:

- **Expand temporal and spatial coverage**

The proposed SWH and IG retrieval configurations should be tested over longer time periods and a wider range of sea states, particularly including more extreme events. Extending the analysis to other coastal regions with complex bathymetry would further test the robustness and transferability of the approach.

- **Advance noise modelling in spectral analysis**

This study relied on backward spectral inversion to retrieve IG wave energy. Complementary forward-modelling approaches, as proposed by Ardhuin et al. (2024), could provide valuable cross-validation. In this framework, reference buoy or model spectra are converted into two-dimensional wavenumber space and filtered using the same transfer function applied to SWOT:

$$E_s^{ref}(k_x, k_y) = G(k_x, k_y) E^{ref}(k_x, k_y) \quad (7.1)$$

Comparing forward- and backward-modelled results helps validate the spectral correction approach. Agreement between the two increases confidence in the robustness of the derived wave parameters, while discrepancies may signal either noise amplification (backward-modelling) or inaccuracies in the applied transfer function G (forward-modelling).

- **Integrate SWOT water depths into IG mask**

The IG mask relies on water depth to define the upper and lower wavenumber thresholds. In this study, depths were taken from buoy data and the bathymetry map of the FIG model. The SWOT Expert product provides depth information that could potentially be used directly, but the accuracy and suitability of these estimates for spectral masking should first be validated.

Once these methodological elements are more robustly validated, both frameworks become suitable for integration with numerical wave models, where SWOT's spatial detail could provide added value.

7.2. Broadening SWOT's potential for coastal wave monitoring

While this study focused on SWH and IG wave retrieval, future research could extend the framework toward a more integrated understanding of coastal dynamics. SWOT offers opportunities to study coupled coastal dynamics by combining wave, wind, and current information. Potential directions include:

- **Wind-wave interactions**

Wind speed estimates derived from SWOT's wide-swath backscatter observations could help explain small-scale spatial variability in wave fields. Integrating these wind estimates may improve interpretation of localized SWH and IG wave patterns, particularly under rapidly evolving atmospheric conditions.

- **Wave-current interactions**

A recent study by Bôas et al. (2025) demonstrates that geostrophic currents derived from SWOT SSH gradients reveal mesoscale and sub-mesoscale features not fully resolved in operational ocean models. In open-ocean conditions, inaccuracies in current forcing have been shown to significantly impact wave model performance [25]. Although this study did not explicitly include wave-current interactions, similar mechanisms may influence wave dynamics in the North Sea. Exploring SWOT-derived current fields alongside wave observations could therefore improve understanding of observed spatial variability.

Together, these recommendations aim to support the continued refinement of SWOT-based wave retrieval methods and their integration into practical coastal wave monitoring and modeling applications. The colocated, high-resolution observations of waves, winds, and currents provided by SWOT represent a unique opportunity for advancing coupled wave-atmosphere-ocean modelling. As such, SWOT has the potential to play a central role in improving our understanding of coastal wave dynamics and the processes that govern them.

7.3. Exploration of higher-resolution products

The main limitation of the LR data product used in this study is its 250 m pixel size spacing. This sets a constraint on the minimum resolvable wavelength to the order of 500 m. For IG retrieval in the 0.005–0.033 Hz band, this wavelength limits the analysis to resolving the full IG band at water depths greater than approximately 29.6 m. The High Resolution (HR) SWOT products offer a promising pathway to overcome this limitation. With pixel resolutions down to 100 m and reduced onboard averaging, the HR data stream may enable retrieval of shorter wavelengths and thereby extend IG wave analysis closer to the coast within the same frequency band (0.005–0.033 Hz) [32].

However, several key challenges remain before HR data can be used for spectral analysis. The HR OBP differs from the LR processing chain and retains higher spatial detail with minimal averaging. While this increases spatial resolution, it also implies that instrumental filtering characteristics and associated transfer functions (e.g., OBP, PTR, and azimuthal cutoff effects) are not yet fully characterized for spectral applications. Moreover, the HR product has not been systematically evaluated for wave spectral analysis, and additional noise sources may become significant at higher wavenumbers. Future research should therefore investigate the feasibility of adapting the spectral correction framework developed for LR products to the HR data stream. If instrumental effects can be adequately characterized, the HR product may substantially broaden SWOT's capability to resolve nearshore IG wave energy and expand validation opportunities for the FIG wave model.

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Supplementary literature review

A.1. Energy density spectrum

The energy or variance density spectrum gives a complete statistical description of the ocean surface elevation, provided that the surface elevation can be seen as a stationary, Gaussian process. The sea surface can be modeled using a random-phase/amplitude approach, in which the surface elevation η is the sum of a large number of harmonic wave components. For a single harmonic wave with amplitude a , the variance is given by:

$$\eta^2 = \frac{1}{2}a^2 \quad (\text{A.1})$$

To obtain the energy density spectrum $E(f)$, the variance is computed over all frequencies:

$$E(f) = \lim_{\Delta f \rightarrow 0} \frac{1}{\Delta f} E \left\{ \frac{1}{2}a^2 \right\} \quad (\text{A.2})$$

This represents the one-dimensional (frequency-only) wave spectrum. However, ocean waves propagate in multiple directions. Figure A.1 illustrates how the sea surface results from the superposition of many harmonic components, each with different amplitudes, frequencies, directions, and phases [23].

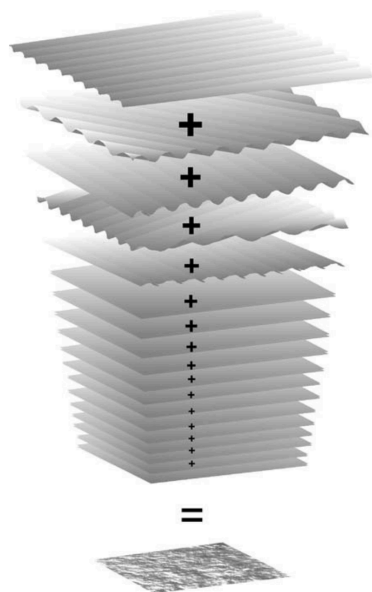


Figure A.1: Illustration showing the ocean surface as a superposition of many harmonic wave components traveling with different directions, amplitudes, and phases [23].

To account for wave directionality, the two-dimensional frequency-directional spectrum $E(f, \theta)$ is introduced. It represents the variance density of wave components propagating in direction θ relative to the x -axis. This formulation assumes a stationary and homogeneous Gaussian process in both time and x, y space:

$$E(f, \theta) = \lim_{\Delta f \rightarrow 0} \lim_{\Delta \theta \rightarrow 0} \frac{1}{\Delta f \Delta \theta} E \left\{ \frac{1}{2}a^2 \right\} \quad (\text{A.3})$$

The one-dimensional frequency spectrum $E(f)$, which does not contain any directional information, can be obtained from the frequency-direction spectrum $E(f, \theta)$ by removing all directional information by integration over all directions [23]:

$$E(f) = \int_0^{2\pi} E(f, \theta) d\theta \quad (\text{A.4})$$

A.2. Point target response of KaRIn

The Point Target Response (PTR) describes the response of the KaRIn radar system to an idealized, infinitesimally small point target on the Earth's surface. In practice, such a perfect point does not exist; the PTR is therefore a theoretical construct used to characterize the intrinsic response of the measurement system.

The PTR consists of a main lobe and weaker side lobes, whose shape and width are determined by the radar system design and signal processing. Physically, this implies that a single point on the surface does not appear as a single point in the measured signal, but rather as a spatially distributed response. The observed SSH field can therefore be modelled as the superposition of many such point responses with additive measurement noise.

The width of the PTR determines the spatial resolution of the KaRIn measurement. Within the SWOT documentation, resolution is typically defined as the full width at half maximum (FWHM), or 3 dB width, of the power of the PTR in a given direction (as illustrated in Figure A.2). This definition is independent of additive noise and may differ between the along-track and cross-track directions. Importantly, resolution is distinct from sampling: reducing pixel spacing does not improve the intrinsic resolution if the PTR width remains unchanged [32].

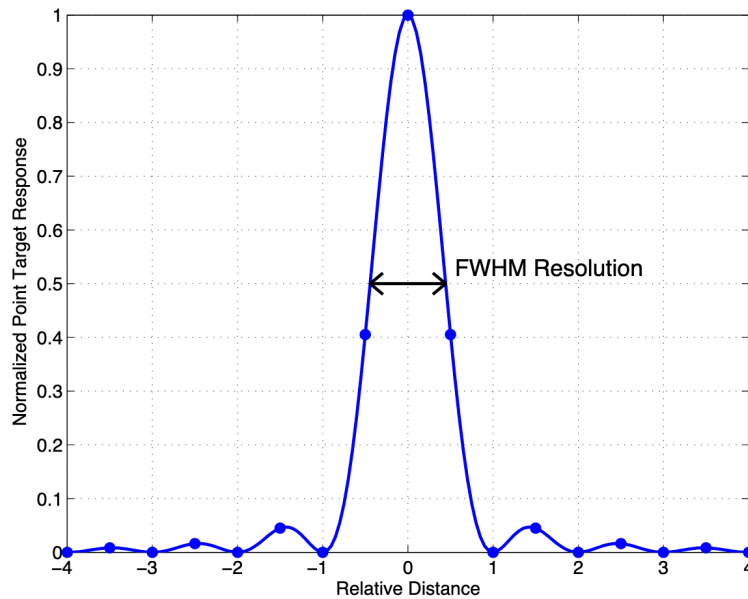


Figure A.2: Conceptual illustration of the PTR power profile and the definition of spatial resolution. The continuous curve represents an idealized sinc^2 -type PTR in the absence of noise. The FWHM of the main lobe defines the intrinsic spatial resolution of the measurement system. The blue markers represent discrete sampling of the signal, demonstrating that resolution (set by the PTR width) is distinct from sampling interval.

In spectral space, the PTR acts as a direction-dependent spatial low-pass filter. High-wavenumber components of the true sea surface are attenuated in the measured spectrum, particularly in the along-track direction. As a result, the measured SSH PSD is related to the true ocean spectrum through inversion of the PTR transfer function. This filtering effect must be accounted for when attempting to recover the underlying wave spectrum.

A.3. Spectral analysis of swell components

In the study of Ardhuin et al. (2024), swell components are isolated using the coherence between SSH and radar backscatter σ_0 , derived from their cross-spectrum. A coherence threshold of 0.05 is used, reflecting the physical correlation between surface slopes (affecting σ_0) and surface elevations (SSH) induced by long-period swell [11].

When the coherence condition is met, an energy-based mask is applied. First, spectral pixels whose energy exceeds one quarter of the spectral maximum are selected. This is achieved by normalizing the SSH spectrum to a fixed range and applying a threshold corresponding to approximately 25% of the peak energy.

This "swell mask" region is extended with one pixel to facilitate comparison with model output that may have slightly shifted peaks. This adds a one-pixel buffer around the detected swell energy which reduces sensitivity to these shifted peaks [11]. Within the masked region, the corrected SSH spectrum (Equation 2.14) is assumed to represent swell-dominated wave energy.

Directional consistency is enforced using phase information from the SSH- σ_0 cross-spectrum. The phase determines whether waves propagate in the forward or backward direction relative to the satellite motion. Combined with the sign of the along-track wavenumber component, this yields a directional product that suppresses spectral energy traveling opposite to the physically consistent swell propagation direction. As a result, the mask is restricted to a single half-plane in wavenumber space corresponding to the true swell propagation direction.

Subsequently a radial filtering step is applied to retain only spectral components within a physically consistent band around the dominant peak wavenumber k_{max} . Pixels with wavenumbers exceeding $2 k_{max}$ or below $0.6 k_{max}$ are discarded. This restriction confines the mask to a limited wavenumber range centered on the dominant swell peak and suppresses spurious energy outside the expected spectral bandwidth [43].

This procedure combines physical constraints (coherence and direction) with heuristic energy-based criteria to robustly isolate swell-dominated spectral lobes. The swell height (H_{18}) is computed as the sum of energy in the masked region, denoted by E_{mask} :

$$H_{18} = 4\sqrt{2E_{mask}}, \quad (\text{A.5})$$

where the factor two accounts for the double-sided nature of the spectrum [64].

Supplementary methodological details

B.1. SWOT products

The L2_LR_SSH products provide data derived from the LR data stream of the SWOT KaRIn instrument. KaRIn LR data are available continuously and globally, and are primarily designed for observations over ocean surfaces. The L2_LR_SSH products contain the variables used in this study, including:

- Sea surface height (SSH)
- Estimated significant wave height (SWH)
- Normalized radar cross section (NRCS), also referred to as radar backscatter σ_o
- Quality flags indicating the data quality of the variables listed above

The L2_LR_SSH is organized into multiple files, each corresponding to specific measurement variables and spatial resolutions. For the purpose of this study, two L2 products are used:

- Expert SSH with Wind and Wave ('Expert') product
- Unsmoothed SSH ('Unsmoothed') product

The Expert product contains all variables included in the Basic and WindWave files, supplemented with additional parameters intended for advanced analysis and data processing. The Unsmoothed product is organized into two groups, left and right, each corresponding to one half of the KaRIn swath on either side of the nadir. The terms 'left' and 'right' are defined from the perspective of an observer standing on the Earth's surface at the spacecraft nadir point and facing in the direction of the spacecraft velocity vector [14]. An overview of the two different products is presented in Table B.1.

Table B.1: Overview of key differences between the SWOT Expert and Unsmoothed L2 products [14].

Product	Description
Expert product	Provides measured SWH, σ_0 , wind speed derived from σ_0 and SWH, model-based wind and wave information, and quality flags on a 2 km geographically fixed grid.
Unsmoothed product	Provides SSH, sea surface height anomaly (SSHA), and σ_0 without additional smoothing relative to the native KaRIn downlink resolution on a ~ 250 m native (center-beam) grid.

The Unsmoothed product contains SSH data that have not undergone significant additional smoothing beyond that applied within the KaRIn OBP. As a result, SSH is provided on a relatively fine spatial sampling grid with a nominal posting of approximately 250 m. This sampling grid is tied to the native sampling of the center Doppler beam formed by the OBP during KaRIn processing.

The LR interferogram data downlinked from KaRIn comprise nine Doppler beams on each side of nadir (see Figure B.1a). The nine beams on a given side are sampled contemporaneously, while the beams from the two sides are slightly offset in time. The beams are numbered from 1 to 9, with Beam 5 representing the center beam, which is nominally aligned with the peak of the KaRIn antenna pattern in

the azimuthal direction. During ground processing, after several intermediate processing steps, measurements from beams other than the center beam are resampled (i.e., interpolated) onto the native sampling grid of Beam 5.

Once the measurements from the different beams on each side are mapped onto a common sampling grid, they are combined through weighted averaging across the beams at each sample location. The resulting beam-combined measurements have a spatial posting of approximately 250 m and an effective spatial resolution of approximately 500 m in both the cross-track and along-track directions [14].

To reduce noise and facilitate interpretation, the Unsmoothed measurements are subsequently resampled and spatially smoothed onto a 2 km geographically fixed grid, as illustrated in Figure B.1b. Data from the left and right sides of the swath are both mapped onto this same fixed grid. The resulting gridded data are provided in the Basic, WindWave, and Expert files, which together cover the full KaRIn swath [14].

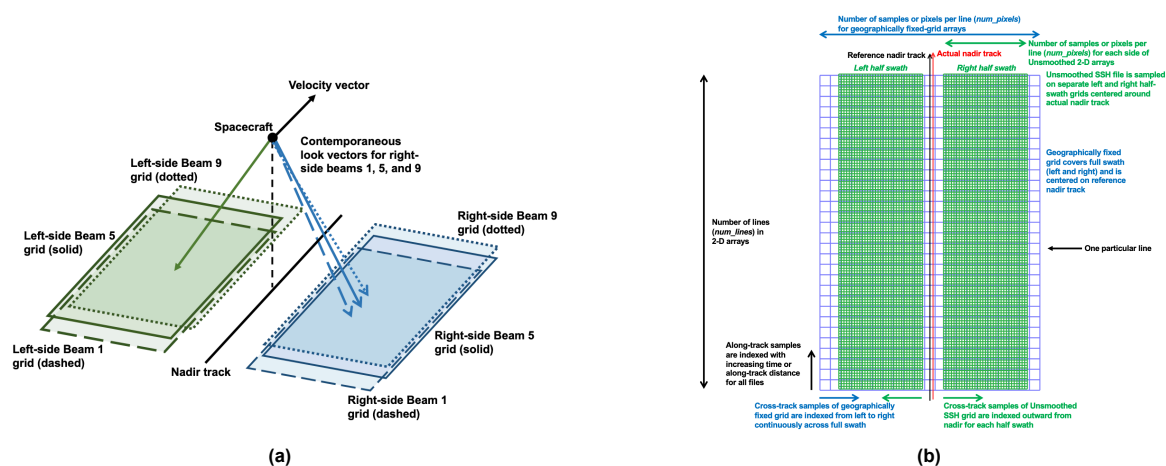


Figure B.1: (B.1a) Doppler beam geometry for each side of the KaRIn swath; (B.1b) Sampling grid used for the L2 LR SSH product [14].

B.1.1. L2_LR_SSH processing algorithm

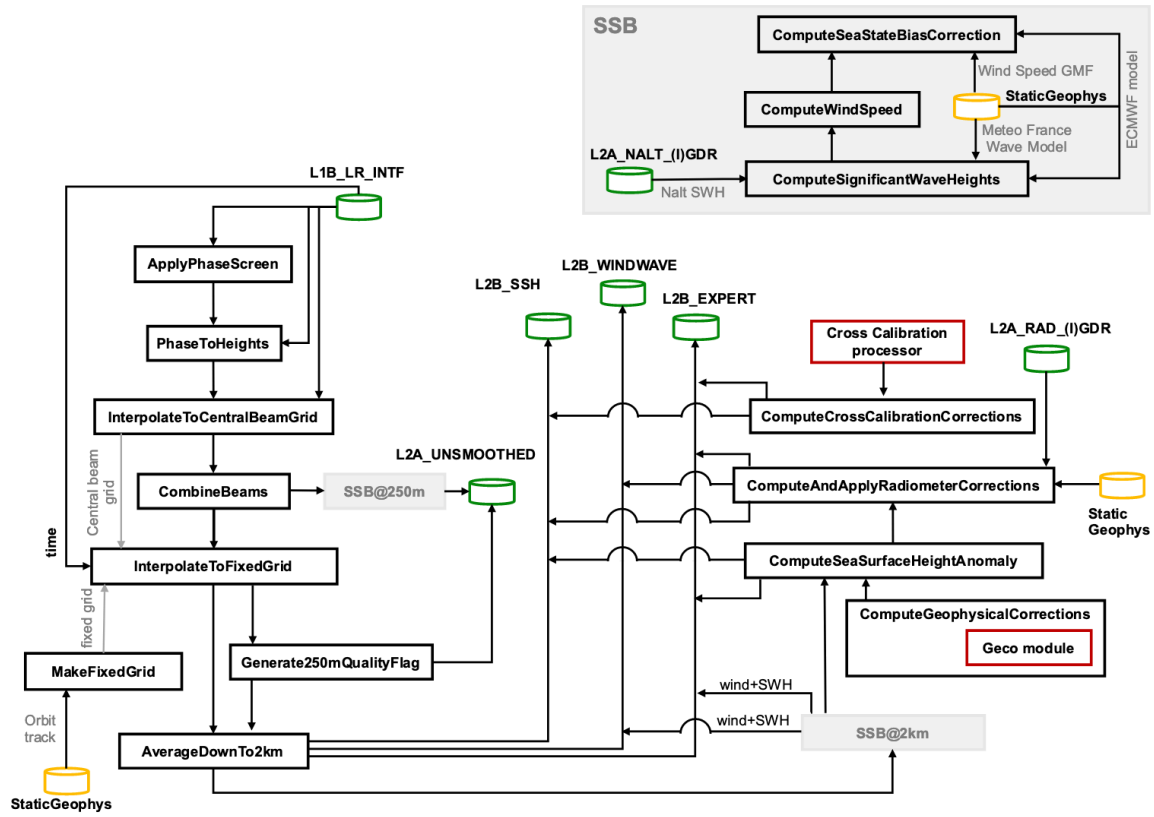


Figure B.2: Processing chain for the generation of the L2 LR SSH products. The diagram illustrates the main algorithmic steps used to derive the Unsmoothed and Expert products from the KaRIn L1B LR interferogram data. Black-edged boxes denote processing modules, whose high-level descriptions are provided in Table B.2. The Unsmoothed product is obtained after beam resampling and combination onto the native center-beam grid (≈ 250 m). Subsequent resampling and averaging to a 2 km geographically fixed grid form the basis for the 2 km SSH products, which are further corrected with geophysical, radiometric, wind, and wave information to produce the Expert product. For clarity, the complete sea state bias (SSB) correction workflow is shown separately in the grey box and referenced at the relevant points in the main processing flow [50].

Table B.2: High-level description of the processing functions used to generate the L2_LR_SSH product [50].

Function Name	Description
MakeFixedGrid	Generate the L2B fixed grid from reference orbit track.
ApplyPhaseScreen	Apply the phase screen.
PhaseToHeights	Compute SSH from interferometric phase for all 9 beams.
InterpolateToCentral BeamGrid	Interpolate beams to 250-m central reference grid.
CombineBeams	Combine beams on central reference grid.
InterpolateToFixedGrid	Interpolate data from Native grid to Fixed grid at 250 m posting.
Generate250mQualityFlag	Generate quality flag for unsmoothed 250 m data.
AverageDownTo2km	Average down fixed-grid quantities to 2 km and decimate to 2km posting. Generate the 2 km quality flags.
ComputeSignificant WaveHeights	Compute significant wave heights (SWH) from volumetric correlation (see Equation B.1)
ComputeWindSpeed	Retrieve wind speeds from σ_0 values, instrument geometry, and ECMWF model wind directions.
ComputeSeaStateBias Correction	Compute and apply sea state bias correction from wind speed and SWH.
ComputeGeophysical Corrections	Compute tidal, model media delays, attenuations and other geophysical corrections from models.
ComputeSeaSurfaceHeight Anomaly	Apply corrections to SSH and compute SSH anomaly.
ComputeAndApply RadiometerCorrections	Compute additional version of SSH and SSH anomaly where media delays correction from weather model applied during L1B processing are replaced with radiometer media delays.
ApplyCrossCalibration Correction	Convert XOver info into the proper format and representation.

Computation of SWH from volumetric correlation

The γ_{vol} as discussed in 2.3.2, can be inverted to obtain the SWH for each pixel of the Expert grid. To do so, the volumetric correlation is first corrected and empirically calibrated for the instrumental parameters [12]. Result is the volumetric correlation at 250 m posting, 500 m resolution. This value is averaged down to fixed grid quantities of 2-km using a Hamming filter [50]. The next step is to calculate per pixel in cross-track the geometric coefficient k_z , after which the SWH on a 2 km $\times$ 2 km grid can be estimated by:

$$SWH = \frac{\sqrt{-2\ln\gamma_{vol}}}{k_z} \quad (\text{B.1})$$

B.2. Storm Surge Flash overview

Table B.3: Overview of the last four Storm Surge Flashes in 2024, summarizing storm development, wind, affected regions, and hardest-hit Dutch areas [15] [16] [17] [18].

Characteristic	Nov 19-20, 2024	Dec 5-6, 2024	Dec 19-20, 2024	Dec 21-23, 2024
Start & development	Low pressure from South England towards the Baltic.	Disturbance crossed North Sea, rapidly deepened on Dec 5.	Depression over Norwegian Sea, affecting NL on Dec 19-20.	Low-pressure area from Scotland to Norway and Denmark.
Peak wind	W to NW, 6-8 Bft.	S-SW turning W-NW, 7-9 Bft.	NW, 6-9 Bft.	W 6-7 Bft, turning NW 6-8 Bft late Dec 22.
Decrease	After midday Nov 20.	From late morning Dec 6.	After morning Dec 20.	From morning Dec 23.
Main region of storm	Western Europe / North Sea / Baltic.	North Sea / German Bight.	Norwegian Sea.	North Sea / Norwegian coast.
Hardest-hit areas (NL)	West-Holland (Hoek van Holland, Rotterdam).	Den Helder, West-Holland, Harlingen, Delfzijl.	Delfzijl (highest surge), West-Holland, Den Helder and Harlingen.	Den Helder, Delfzijl, West-Holland, Benedenrivieren.

B.3. Case I: supplementary information

Table B.4: Overview of in-situ observed and modelled wind conditions at buoy locations during SWOT overpass on 22 Dec, 2024.

Buoy	In-situ observed	SWAN-DCSM
	Wind speed u (m/s)	Wind direction ($^{\circ}$)
D15	14.9	288
J6		295
K13	16.5	288
IJmuiden		283
IJ5		284
SPY	15.6	
EUR	17.7	280
OS11		280
L9	14.0	286
F3		281
A12	14.4	288

B.4. Case III: supplementary information

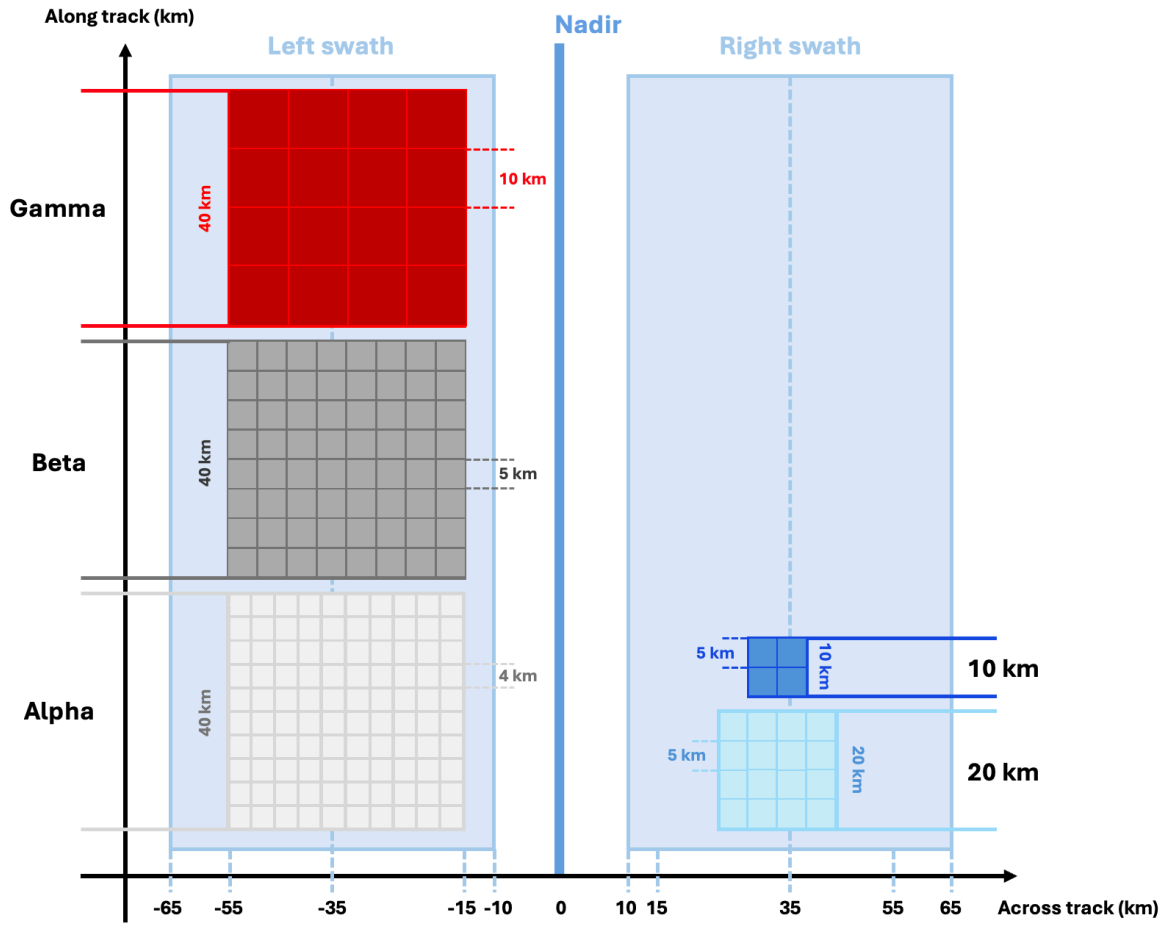


Figure B.3: Schematic representation of the spectral box-tile configurations used within the SWOT swath. The 'Beta' configuration corresponds to the setup used in Arduin et al. (2024). For all five configurations, the tile sizes (L_{tile}) are indicated by the smaller segments within the large spectral boxes. Note that this schematic illustrates only the layout of box and tile dimensions; overlapping of windowed tiles is not shown. In this study, a 50% overlap is applied in both the cross-track and along-track directions.

B.5. Data and code availability

The full codebase used for data processing, spectral analysis, and figure generation is publicly available on GitHub: <https://github.com/svaneps/swot-wave-validation-north-sea.git>. The repository contains the scripts, functions, and workflow documentation required to reproduce the results presented in this thesis.

B.6. Evaluation metrics: supplementary information

B.6.1. Performance metrics

In this study the following statistical metrics are used to evaluate the agreement between SWOT-derived wave parameters and reference data. The mathematical formulations of the metrics are:

$$\text{Bias} = \frac{1}{N} \sum X_{\text{SWOT}} - X_{\text{ref}} \quad (\text{B.2})$$

$$\text{MAE} = \frac{1}{N} \sum |X_{\text{SWOT}} - X_{\text{ref}}| \quad (\text{B.3})$$

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum (X_{\text{SWOT}} - X_{\text{ref}})^2} \quad (\text{B.4})$$

$$\text{SI} = \frac{\text{RMSE}}{\bar{X}_{\text{ref}}}, \quad (\text{B.5})$$

$$R = \frac{\sum (X_{\text{SWOT}} - \bar{X}_{\text{SWOT}})(X_{\text{ref}} - \bar{X}_{\text{ref}})}{\sqrt{\sum (X_{\text{SWOT}} - \bar{X}_{\text{SWOT}})^2} \sqrt{\sum (X_{\text{ref}} - \bar{X}_{\text{ref}})^2}}, \quad (\text{B.6})$$

$$\text{STD} = \sqrt{\frac{1}{N-1} \sum (d - \bar{d})^2} \quad (\text{B.7})$$

Here, X_{SWOT} are estimates recorded by SWOT and X_{ref} the corresponding reference values, obtained from either in situ observations or numerical model outputs. The residuals are defined as $d = X_{\text{SWOT}} - X_{\text{ref}}$, and their mean as $\bar{d}$. The total number of validation points is given by N .

B.6.2. Triple collocation analysis

Given the assumptions given in subsection 3.7.1, and with the true signal being the SWH (H_s), the three measurement systems can be expressed as:

$$x = H_s + \varepsilon_x \quad (\text{B.8})$$

$$y = a_y H_s + b_y + \varepsilon_y \quad (\text{B.9})$$

$$z = a_z H_s + b_z + \varepsilon_z \quad (\text{B.10})$$

where x , y , and z represent the buoy, SWOT, and model estimates, respectively. The coefficients a_i and b_i ($i = y, z$) denote the linear calibration slope and offset, and ε_i are the random measurement errors. Under the standard TC assumptions, the random error variances $\langle \varepsilon_i^2 \rangle$ can be estimated using the variances C_{ii} and covariances C_{ij} of the observed signals as follows:

$$\langle \varepsilon_x^2 \rangle = C_{xx} - \frac{C_{xy}C_{xz}}{C_{yz}} \quad (\text{B.11})$$

$$\langle \varepsilon_y^2 \rangle = C_{yy} - \frac{C_{xy}C_{yz}}{C_{xz}} \quad (\text{B.12})$$

$$\langle \varepsilon_z^2 \rangle = C_{zz} - \frac{C_{xz}C_{yz}}{C_{xy}} \quad (\text{B.13})$$

The calibration coefficients are obtained as:

$$a_y = \frac{C_{yz}}{C_{xz}} \quad (\text{B.14})$$

$$a_z = \frac{C_{yz}}{C_{xy}} \quad (\text{B.15})$$

$$b_y = \langle y \rangle - a_y \langle x \rangle \quad (\text{B.16})$$

$$b_z = \langle z \rangle - a_z \langle x \rangle \quad (\text{B.17})$$

Supplementary results

C.1. Case II

C.1.1. Additional absolute error results

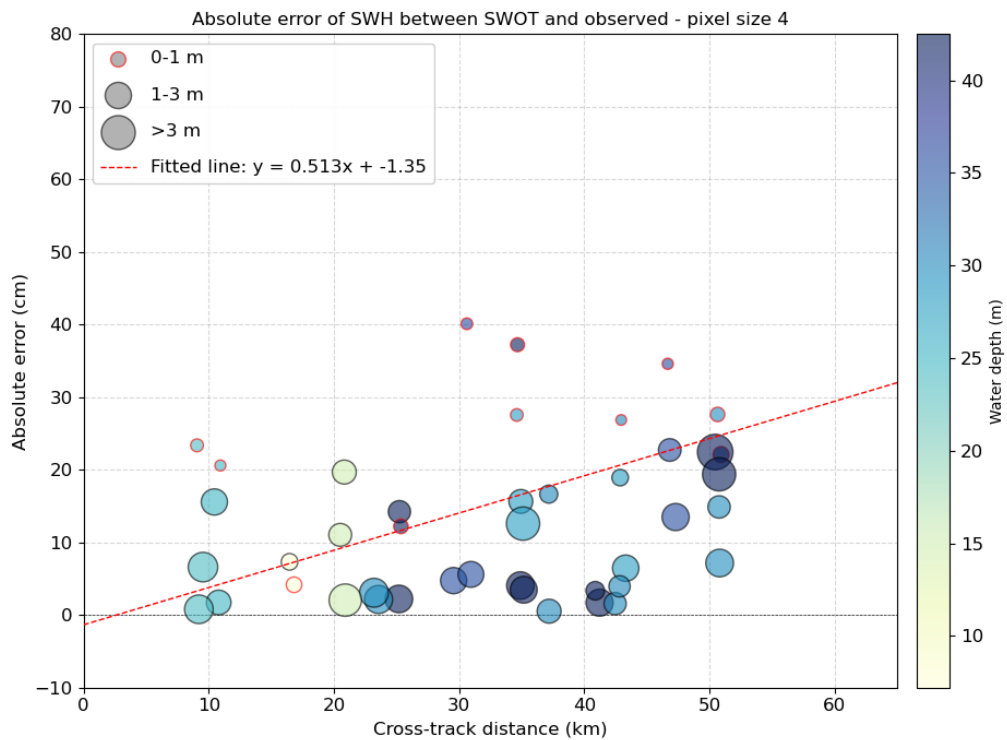


Figure C.1: Absolute error (cm) between SWOT using 4 pixels and observed SWH as a function of cross-track distance. Marker size represents observed SWH. Red circles highlight cases where SWH < 1 m. Linear fit (red line) is shown for reference.

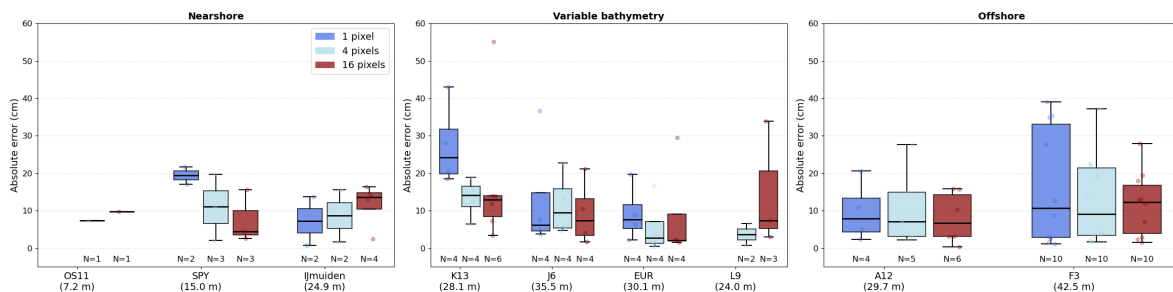


Figure C.2: Absolute error distribution between SWOT and observed SWH per bathymetry class, pixel size and buoys. Observations beyond 55 km cross-track distance and in-situ observations with SWH < 1 m are excluded.

Table C.1: Median absolute errors (cm) per bathymetry class and pixel size. This dataset encompasses both the cross-track < 55 km and SWH > 1 m filters. Sample size for each bin is indicated in parentheses.

Match-up size	Nearshore	Variable bathymetry	Offshore
1 pixel	15.45 (n=4)	8.90 (n=11)	4.93 (n=11)
4 pixels	9.18 (n=6)	6.52 (n=14)	3.77 (n=12)
16 pixels	11.46 (n=8)	7.38 (n=17)	8.73 (n=14)

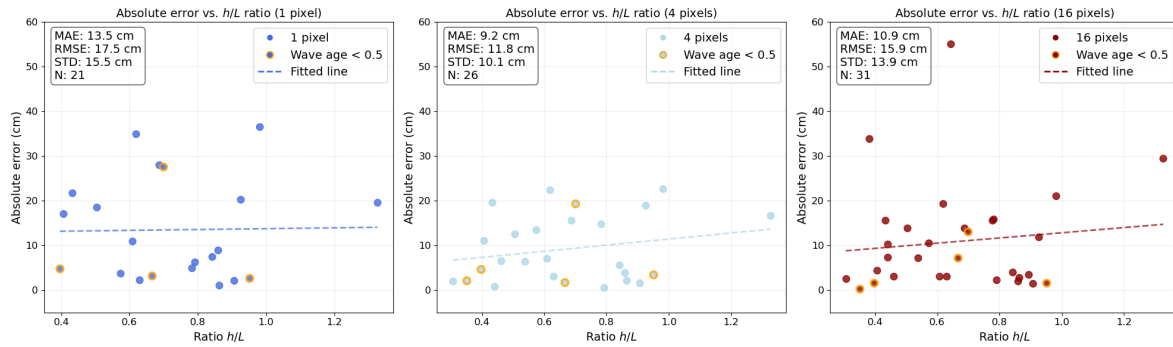


Figure C.3: Absolute SWH error (in cm) as a function of h/L . Observations with cross-track distance > 55 km and SWH < 1 m are excluded, and those with wave age < 0.5 are marked in orange. $h/L > 0.5$ marks the transition from intermediate to deep water conditions. Fitted regression lines are shown as reference.

C.1.2. Additional cross-track sensitivity analysis

Table C.2: Comparison of performance metrics between SWOT-derived and in-situ SWH after applying a cross-track distance threshold of 50 km. Color tones indicate the percentage change compared to the 55 km threshold dataset.

	Metrics - cross-track < 50 km										
Size	Bias (m)		MAE (m)		RMSE (m)		STD (m)		r ²		NVAL
1 pixel	0.10	-17%	0.15	-8%	0.20	-6%	0.17	0%	0.96	0%	29
4 pixels	0.09	-13%	0.12	-7%	0.16	-3%	0.14	2%	0.98	0%	37
16 pixels	0.08	-8%	0.13	-1%	0.17	1%	0.15	4%	0.97	0%	48

C.1.3. Final performance metrics

Table C.3: Statistical metrics of SWH estimates per pixel size. The dataset encompasses the full filter including cross-track distance < 55 km and SWH > 1 m.

Pixel size	Bias (m)	MAE (m)	RMSE (m)	SI	r^2	STD (m)	NVAL
1	0.07	0.12	0.16	7.34	0.97	0.15	26
4	0.05	0.09	0.11	5.02	0.98	0.10	32
16	0.06	0.11	0.15	6.69	0.97	0.14	39

C.1.4. TC analysis: calibration coefficients

Table C.4: Linear calibration parameters (slope a , offset b) comparing SWOT and model results across configurations.

Configuration	SWOT		Model	
	Slope a	Offset b	Slope a	Offset b
1 pixel – NS	0.94	0.24	0.83	0.01
4 pixel – NS	0.94	0.23	0.82	0.03
16 pixel – NS	0.96	0.18	0.82	0.05
1 pixel – KS	0.93	0.27	0.98	-0.08
4 pixel – KS	0.93	0.21	0.91	0.04
16 pixel – KS	0.96	0.18	0.86	0.14

C.2. Case III

C.2.1. Sensitivity of transfer function components

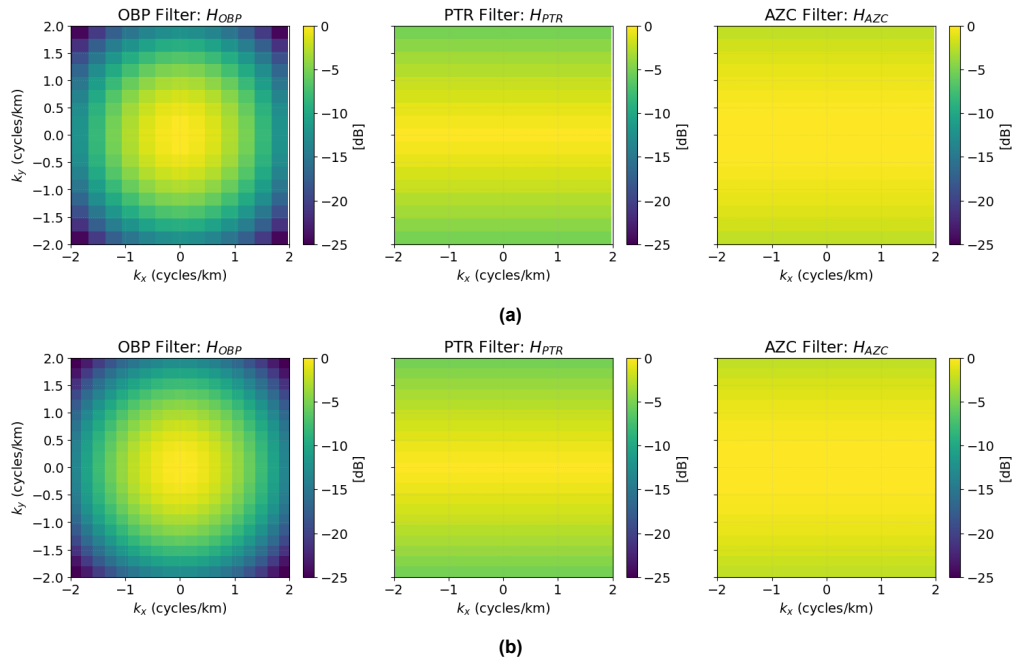


Figure C.4: (C.4a) Two-dimensional wavenumber domain responses showing the strength (dB) of transfer function components OBP, PTR, and AZC using the alpha version box-tile configuration (40 km box, 4 km tile). (C.4b) Same transfer function components using the 10 km version box-tile configuration (10 km box, 5 km tile). Both plots show filter responses as a function of wavenumber components k_x and k_y (cycles/km), with color scale ranging from 0 dB (bright yellow) to -25 dB (dark blue).

C.2.2. Sensitivity of the azimuthal cutoff wavelength

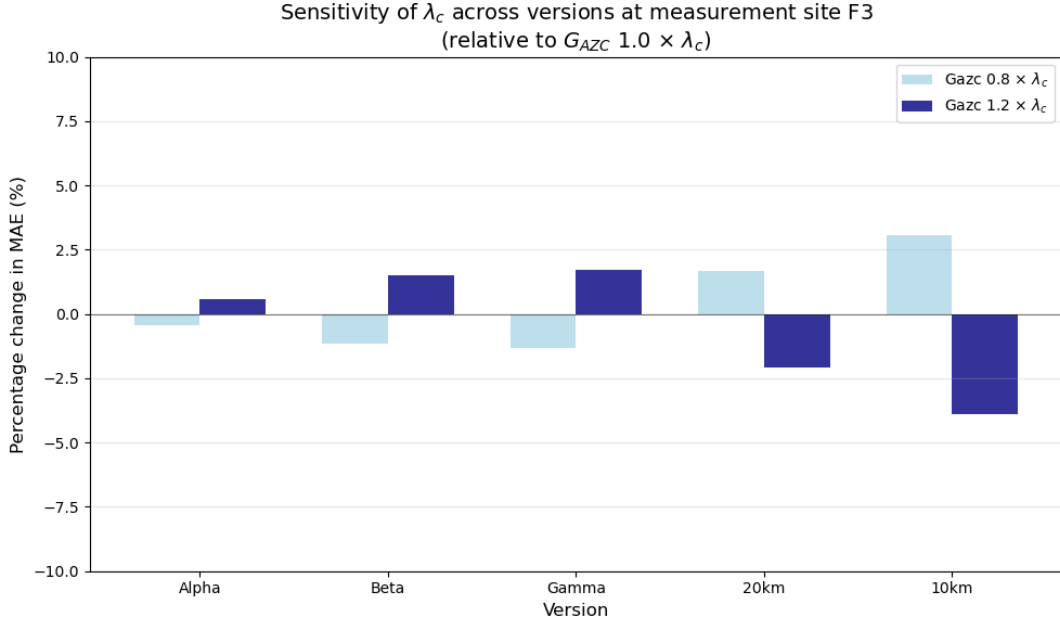


Figure C.5: Sensitivity of MAE to the azimuthal cutoff wavelength λ_c at site F3. Percentage changes in MAE are relative to the baseline ($1.0 \times \lambda_c$) for two parameter variations ($0.8 \times \lambda_c$ and $1.2 \times \lambda_c$) across different box-tile configurations.

C.2.3. IG mask sensitivity to PSD representation

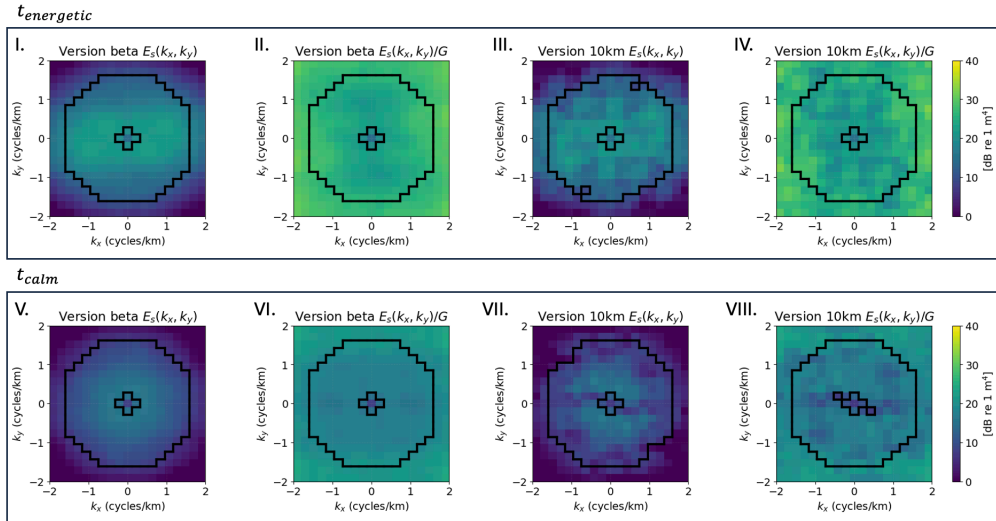


Figure C.6: SSH PSD of beta version and 10 km version at buoy F3. Top panels I-IV show PSD during energetic conditions ($H_s = 4.3$ m, $T_{m02} = 6.53$ s), while panels below V-VIII show PSD under calm conditions ($H_s = 0.7$ m, $T_{m02} = 3.36$ s). Odd panels show the PSD from SWOT SSH, and even panels the inverted version when applying the full transfer function G . Here, a normalization factor of 10 is applied. Since the 'true' SSH PSD E contains higher spectral energy, it more easily meets the energy threshold, leading to a marginally wider mask, as seen in Panels III vs. IV and VII vs. VIII. This effect is not apparent for the beta configuration but slightly more pronounced for the 10 km version, where the higher normalization factor extends the mask.

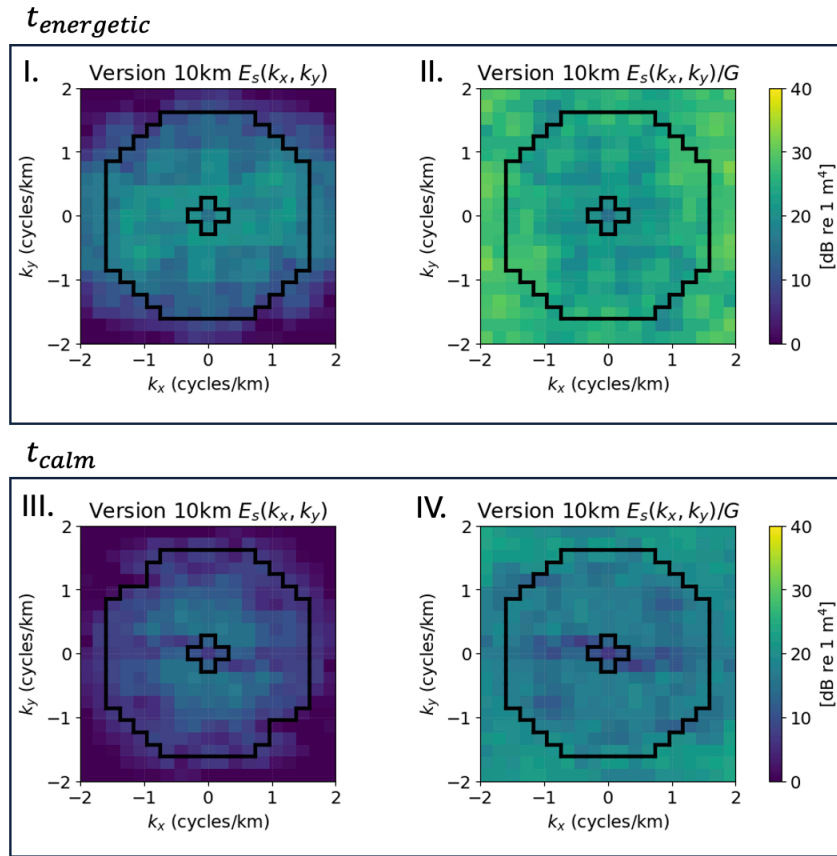


Figure C.7: SSH PSD of beta version and 10 km version at buoy F3. Here, a normalization factor of 15 is applied. Top panels I and II show PSD during energetic conditions, while panels below III and IV show PSD under calm conditions. Odd panels show the PSD from SWOT SSH, and even panels the inverted version when applying the full transfer function G .

C.2.4. Additional SWOT versus platform-mounted radar H_{IG} comparisons

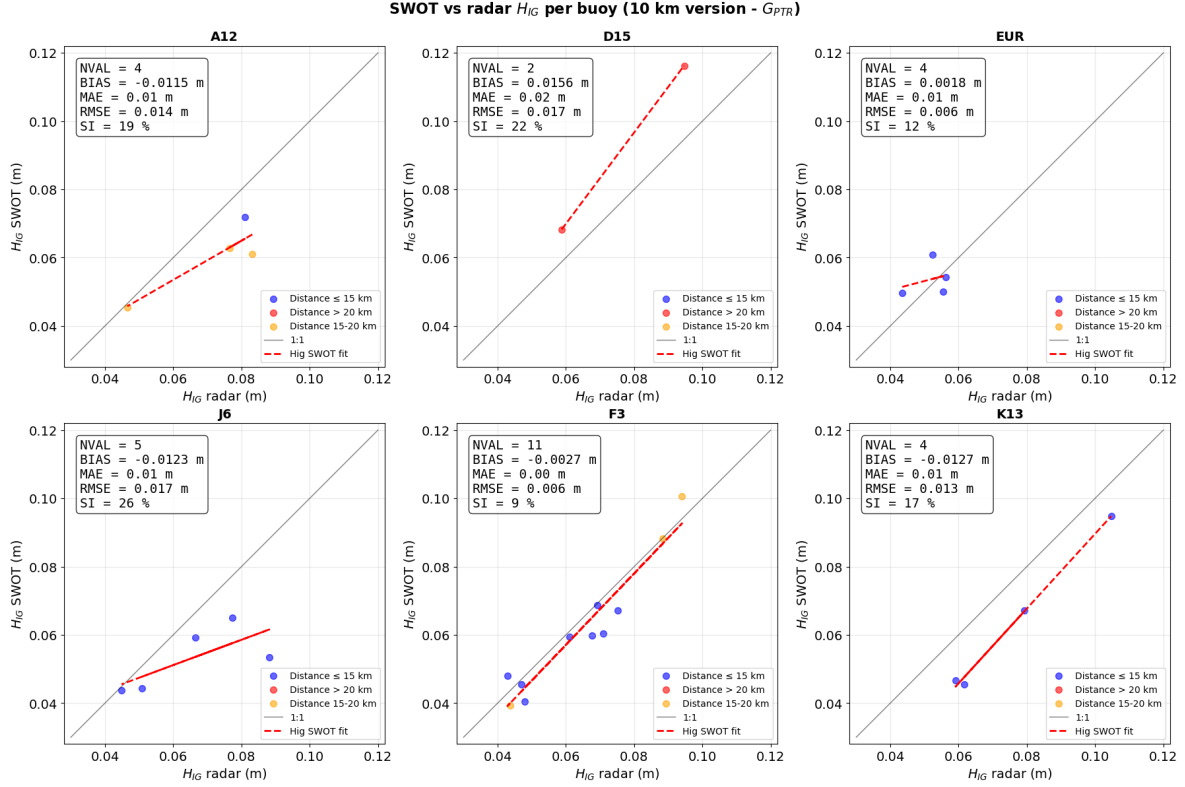


Figure C.8: SWOT H_{IG} versus platform-mounted radar estimates using the 10 km version with G_{PTR} for inversion.

C.2.5. IG wavelength and direction

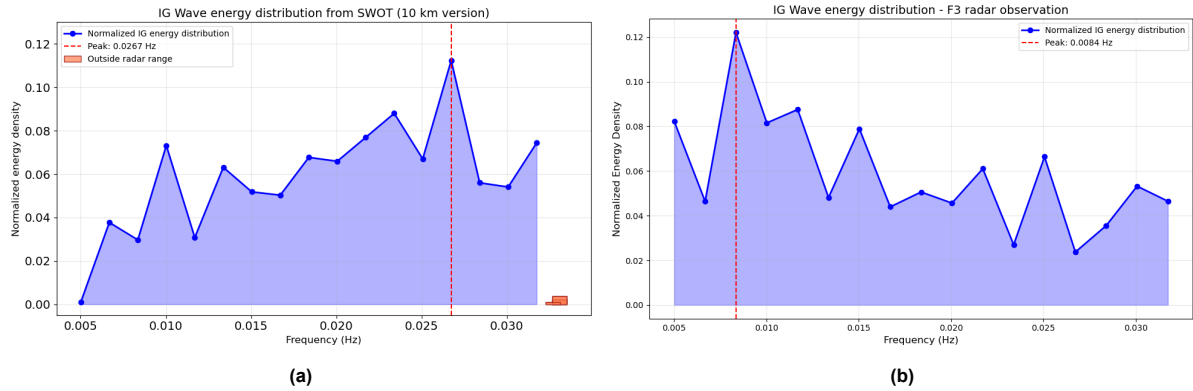


Figure C.9: Normalized IG wave energy distributions at buoy location F3 during the SWOT overpass on 26 November 2024.

(C.9a) SWOT-derived 2D PSD in wavenumber space converted to a 1D frequency distribution using the linear dispersion relationship. Energy is distributed across the same frequency bins as the radar-measurement distribution using linear interpolation between adjacent bins. The energy peak occurs at 0.0267 Hz (red dashed line). Note that SWOT's wavenumber range includes frequencies in the range 0.032703 to 0.033090 Hz (30.2 to 30.6 s) which fall just outside the radar's IG frequency band (orange bars). The excluded energy amounts to 0.0000067 m^2 , which is negligible compared to the total masked energy (1.4%); (C.9b) Radar-derived IG wave energy distribution within the 0.005–0.032 Hz frequency band, with peak energy observed near 0.0084 Hz (red dashed line). The total IG energy in the SWOT-derived mask is 0.00048 m^2 , compared to 0.00052 m^2 for the radar estimate. The corresponding IG wave heights are $H_{IG} = 0.088 \text{ m}$ for SWOT (no transfer functions) and $H_{IG} = 0.094 \text{ m}$ for the radar observation. SWOT travels in the South West direction (-161°), while sea-swell propagates in the North-East direction.

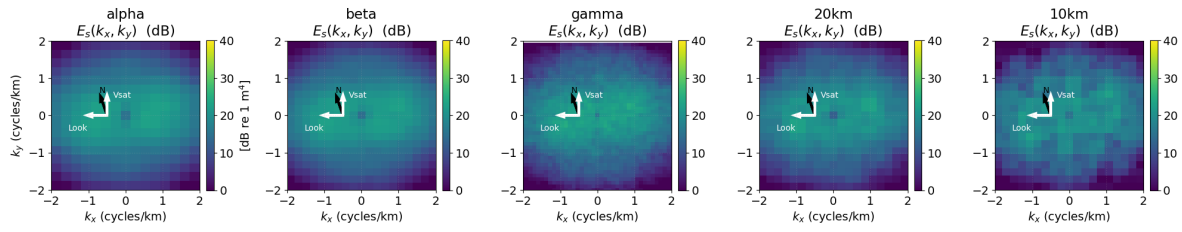


Figure C.10: PSD E_s observed by SWOT at buoy location F3 during the overpass on 26 November, 2024. The PSD's are computed using the five different box-tile configuration versions.

C.2.6. SWOT IG versus FIG model comparison

Table C.5: Comparison of in-situ and SWOT-derived values at selected stations during the overpass of 26 November and 22 December 2024.

26-Nov, 2024 H_{IG} (cm)			22-Dec, 2024 H_{IG} (cm)		
Station	In-situ	SWOT	Station	In-situ	SWOT
EUR	5.54	4.99	K13	10.47	9.47
F3	9.40	10.06	D15	9.46	11.63

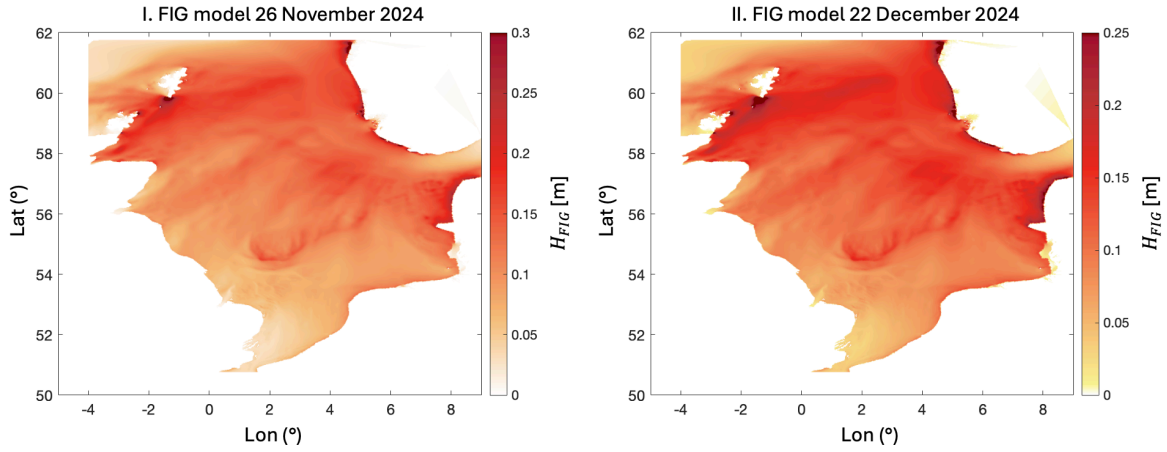


Figure C.11: Spatial FIG wave height outputs derived from the FIG model developed by Akrish et al. (2025) during the overpass on 26 November 2024 (left) and 22 December 2024 (right).

Case: 26 November 2024

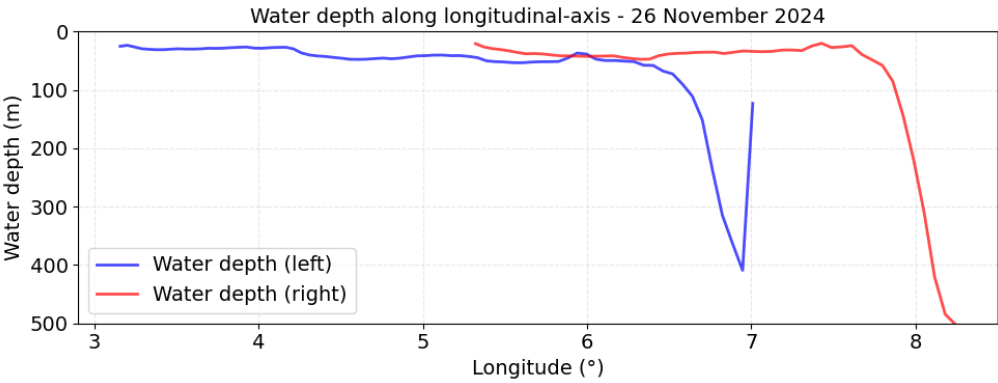


Figure C.12: Water depth along longitudinal-axis. Left swath is shown in blue and right swath in red.

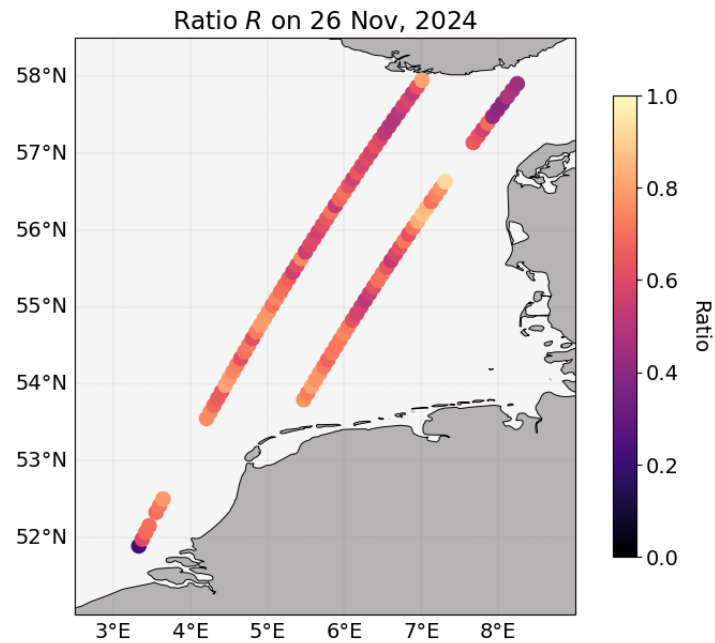


Figure C.13: Ratio R along left and right swath tracks on 26 November 2024. Only values at water depths > 29.6 m are shown.

Case: 22 December 2024

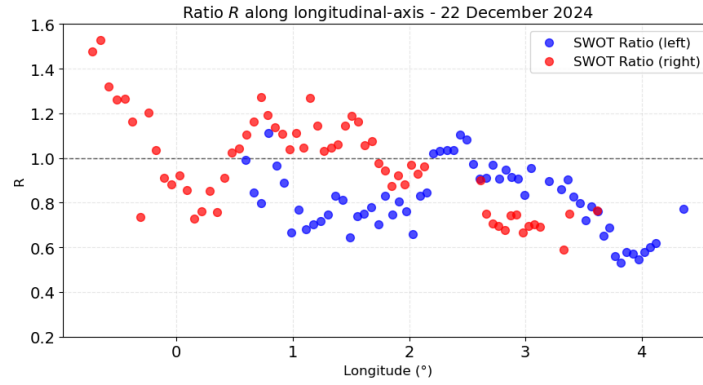


Figure C.14: Ratio R along the longitudinal axis. Colors indicate swath side relative to the satellite heading (blue: left; red: right). For southward satellite motion, the left swath appears east of nadir on a north-up map.

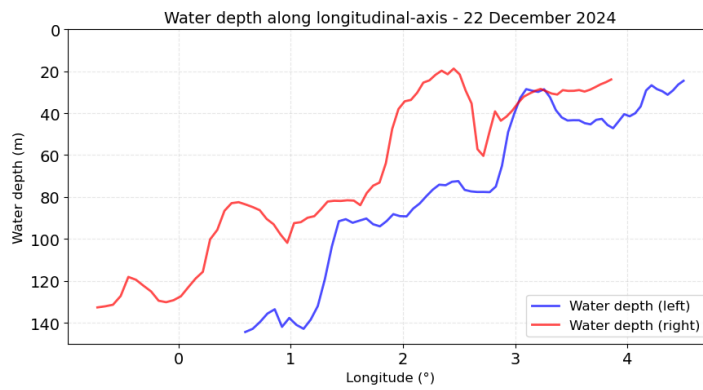


Figure C.15: Water depth along longitudinal-axis. Colors indicate swath side relative to the satellite heading (blue: left; red: right). For southward satellite motion, the left swath appears east of nadir on a north-up map.

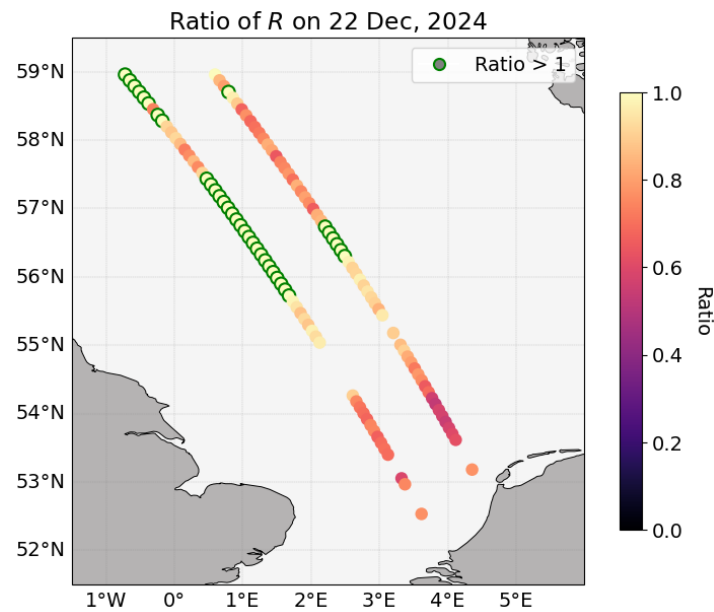


Figure C.16: Ratio R along left and right swath tracks on 26 November 2024. Only values at water depths > 29.6 m are shown. Green markers indicate locations where ratio $R > 1$.