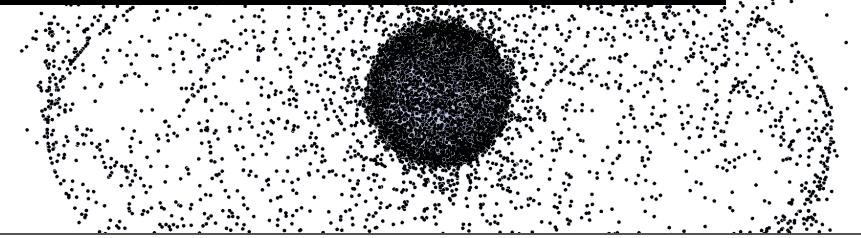
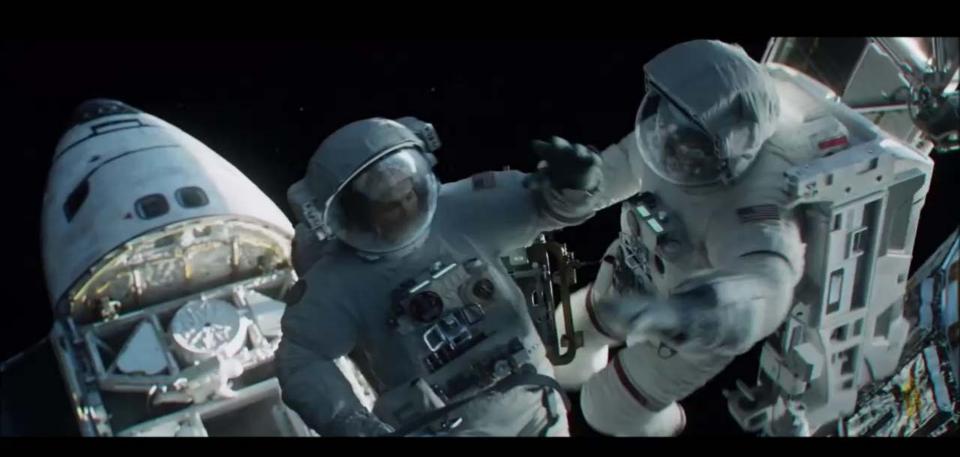
### Rigid-Body Simulation With Gaming Engines

A study on the usability of gaming industry software for simulating engineering problems



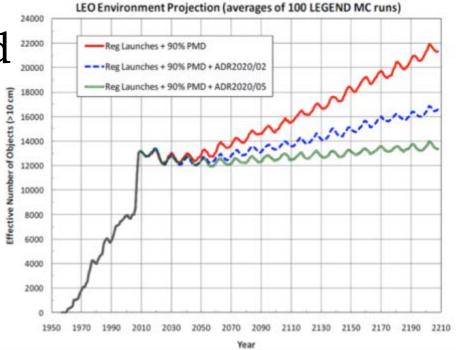




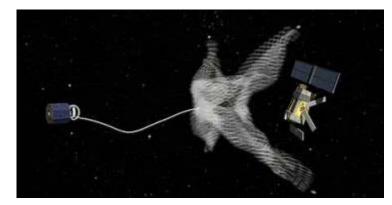


Project background

- Exponential increase in orbital debris (Kessler syndrome)
- Stabilize by de-orbiting several large objects per year
- Capture satellites with nets
- Simulations required for development
- Traditional tools expensive and not user-friendly



Source: J.C. Liou, NASA



Source: ESA





### Project background

 Video games include complex physics



- Suspected advantages
  - Flexible & free
  - Ease of use
  - Fast
- Net simulations with Bullet physics engine & 3D-modelling tool Blender











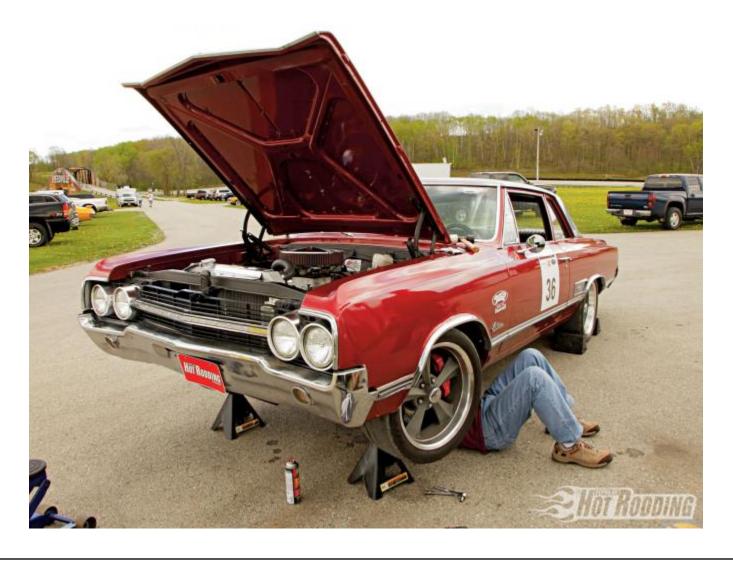
### Research objectives

- Assess gaming engine accuracy
- Verify suspected advantages
- Identify possible engineering applications
- Suggest implementation of an engineering tool





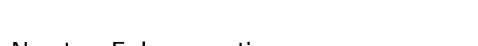
# Theoretical Analysis







# Time integration of free motion





$$\sum F = m\ddot{q}$$

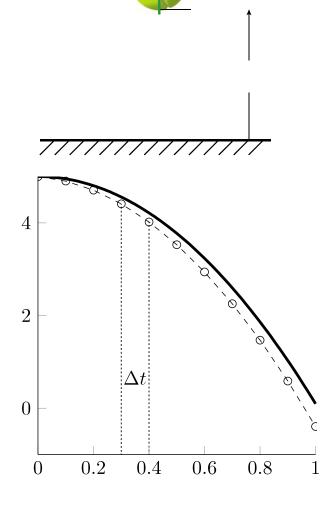
Forward Euler

$$\dot{q}^{n+1} = \dot{q}^n + \Delta t \ddot{q}^n$$
$$q^{n+1} = q^n + \Delta t \dot{q}^n$$

Semi-implicit Euler

$$\dot{q}^{n+1} = \dot{q}^n + \Delta t \ddot{q}^n$$
$$q^{n+1} = q^n + \Delta t \dot{q}^{n+1}$$

 Symplectic integrator (conserves energy)





### Contact model

- Rigid-bodies and geometry simplified to contact points
- Penalty based method (Adams)

$$F_c = f(\delta q, \delta \dot{q}) \ge 0$$

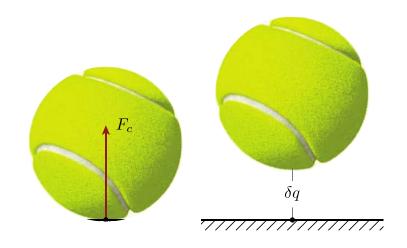
Constraint based method (Bullet)

find 
$$F_c \ge 0$$
 such that  $\delta \dot{q} \ge 0$ 

Complementarity condition

$$F_c \delta \dot{q} = 0$$







# The complementarity problem

 Including friction & joints yields linear complementarity problem (LCP)

$$\underbrace{\begin{bmatrix} \boldsymbol{M} & -\boldsymbol{J}_e^T & -\boldsymbol{J}_c^T & -\boldsymbol{D} & 0 \\ \boldsymbol{J}_e & 0 & 0 & 0 & 0 \\ \boldsymbol{J}_c & 0 & 0 & 0 & 0 \\ \boldsymbol{D}^T & 0 & 0 & 0 & \boldsymbol{E} \\ 0 & 0 & \mu & -\boldsymbol{E}^T & 0 \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} \boldsymbol{u}^{n+1} \\ \lambda_e \\ \lambda_c \\ \boldsymbol{\lambda}_f \\ \gamma \end{bmatrix}}_{\boldsymbol{x}} - \underbrace{\begin{bmatrix} \boldsymbol{M}\boldsymbol{u}^n + h\boldsymbol{f}_{\text{ext}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\boldsymbol{b}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ a \\ \boldsymbol{\sigma} \\ \zeta \end{bmatrix}}_{\boldsymbol{w}}$$

$$\begin{bmatrix} a \\ \boldsymbol{\sigma} \\ \zeta \end{bmatrix} \ge 0 \quad , \quad \begin{bmatrix} \lambda_c \\ \boldsymbol{\lambda}_f \\ \gamma \end{bmatrix} \ge 0 \quad , \quad \begin{bmatrix} a \\ \boldsymbol{\sigma} \\ \zeta \end{bmatrix}^T \begin{bmatrix} \lambda_c \\ \boldsymbol{\lambda}_f \\ \gamma \end{bmatrix} \ge 0$$

- Solved at every time-step of the simulation
- Iterative solvers





### Inaccurate behavior

#### Constraint stabilization & elastic collisions

Penetration errors

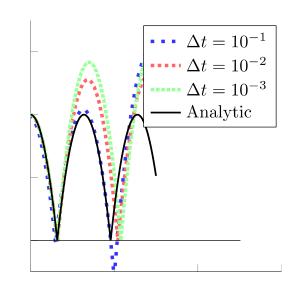


Baumgarte stabilization

$$\delta \dot{q} \ge 0$$

$$\delta \dot{q} \ge -\frac{k_{\rm erp}}{\Delta t} \delta q$$

- Pseudo random energy creation in collisions
- No convergence

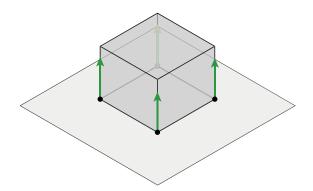


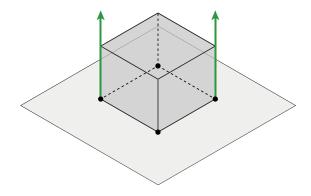


### Inaccurate behavior

#### MNCP solver convergence & uniqueness

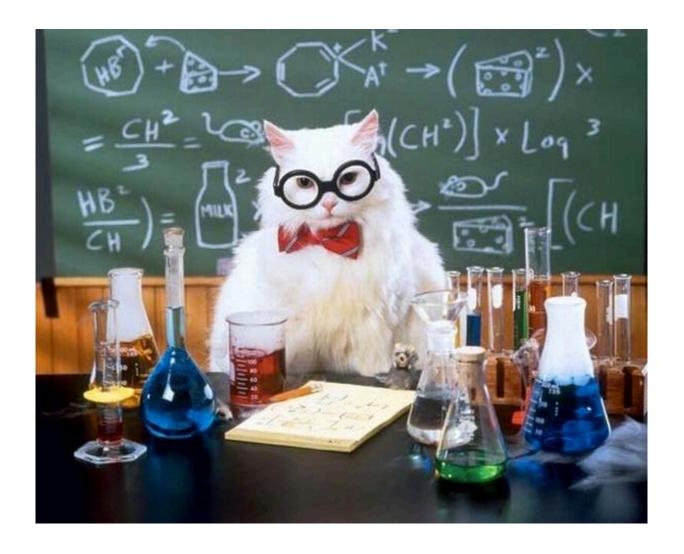
- Projected Gauss-Seidel (PGS) solver often used
  - Fast initial convergence, slow afterwards
  - No error based convergence criterion
  - Fixed number of iterations
- Uniqueness not guaranteed



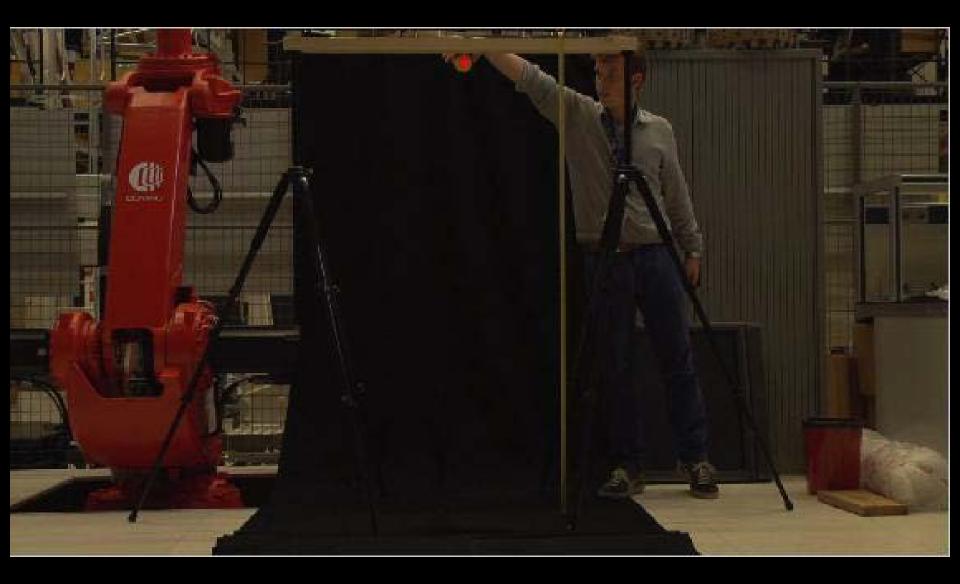




# Experimental Analysis

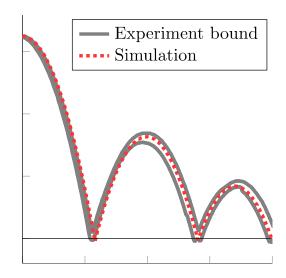


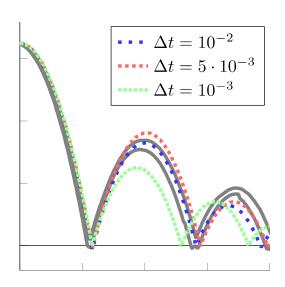




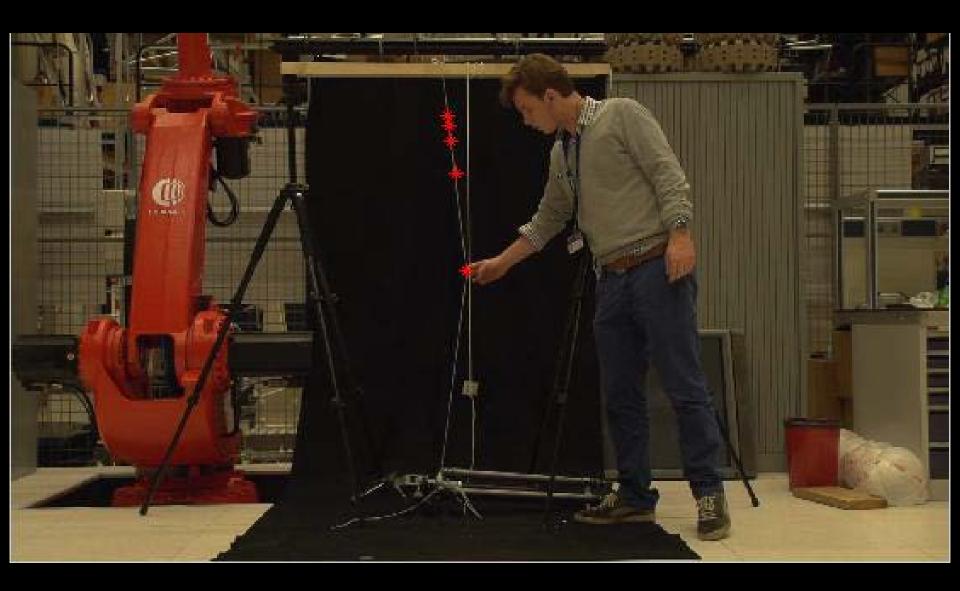
# Tennis ball experiment

- High-speed camera & video tracking
- Pseudo-random behavior too large



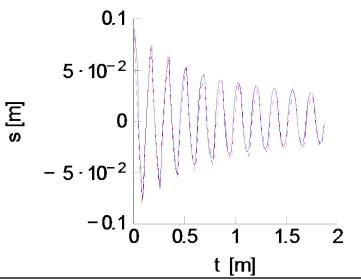


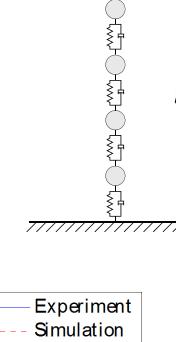


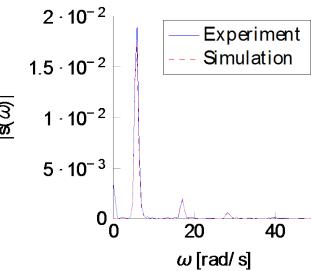


# String experiment

- Lumped mass model
  - Experimentally determined stiffness
  - Estimated aerodynamic damping
- Accurate for simple excitations



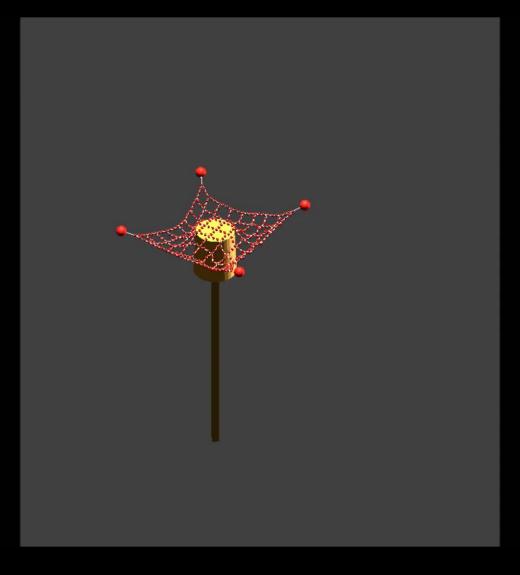




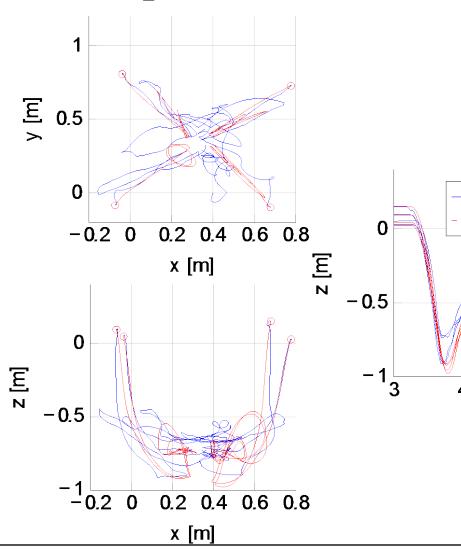


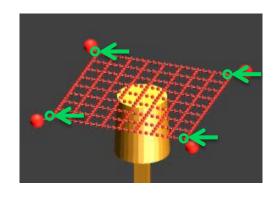






### Net experiment





Experiment Simulation

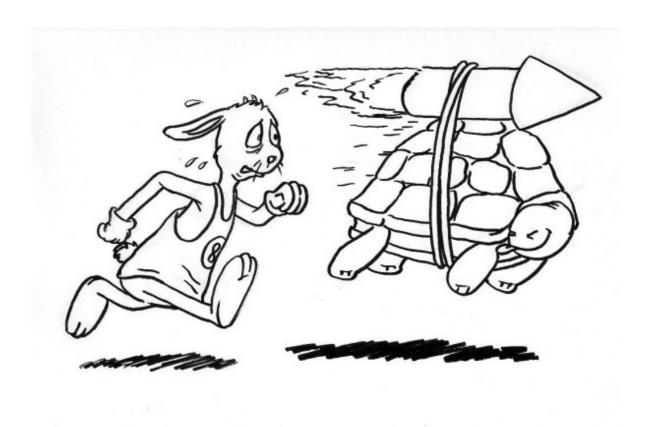
5

t [s]

6

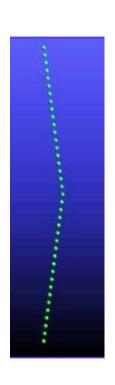


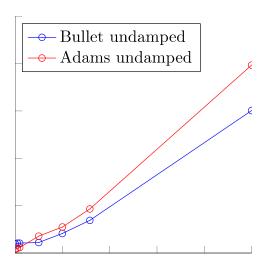


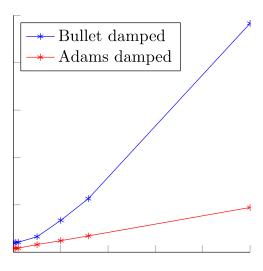




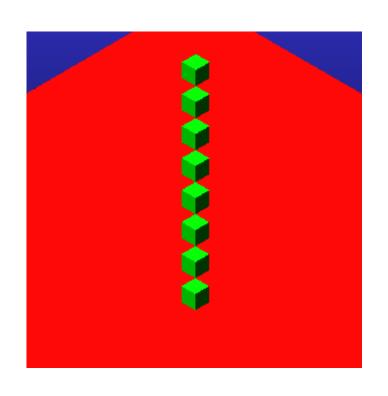


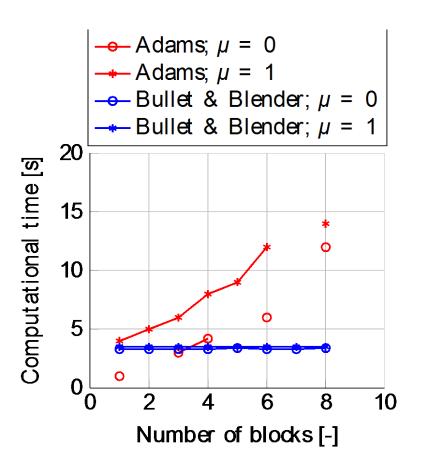




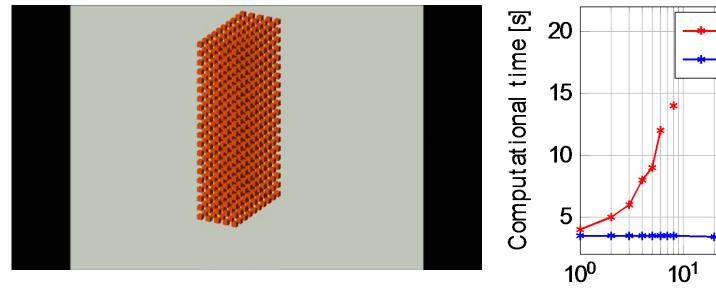


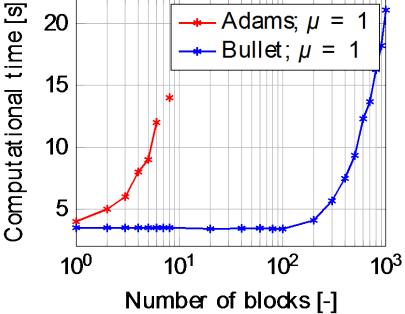














### Conclusions

- Assess gaming engine accuracy
  - Simple models, but can be accurate
  - Important to select proper combination of algorithms
- Verify suspected advantages
  - Much faster in handling contact
- Identify possible engineering applications
  - Rigid-body simulations with many contacting bodies
  - Real-time applications
- Suggest implementation of an engineering tool
  - Use Bullet & Blender as a basis

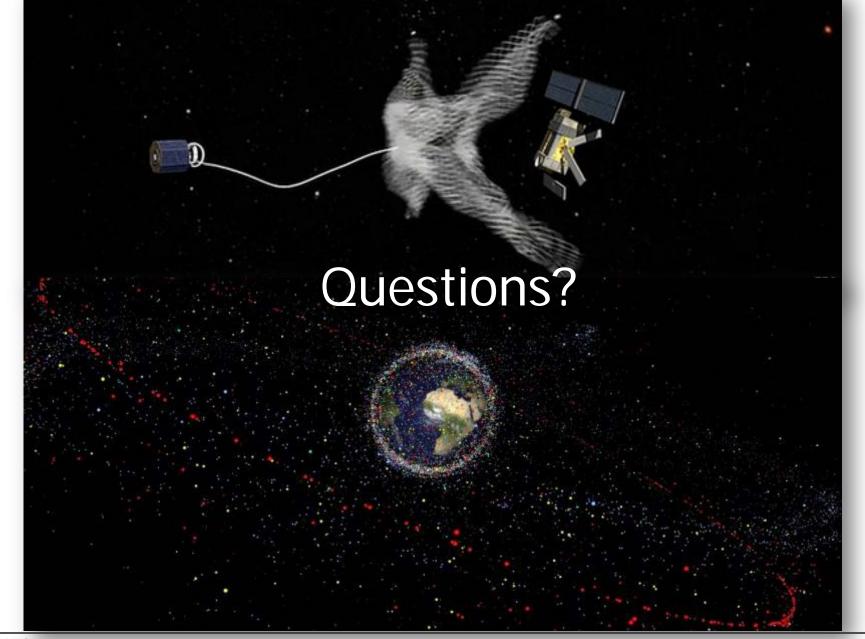


### Recommendations

- Making a useful tool from Bullet & Blender
  - More control over Bullet from Blender
  - Add additional modules for post-processing
- Future research
  - Thorough investigation on string & net dynamics
  - Adaptive time-stepping methods to improve efficiency of simulation of stiff systems

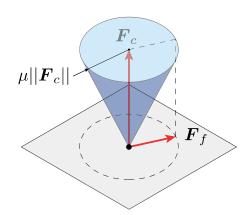








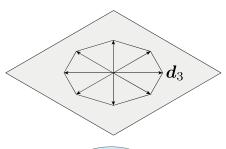
### Friction

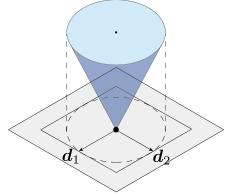


Coulomb friction model

$$||\boldsymbol{F}_f|| \leq \mu ||\boldsymbol{F}_c||$$

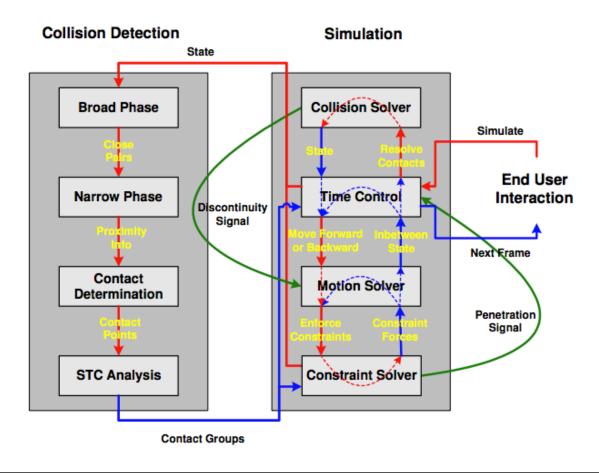
- Simplifications for simulators
  - Linearized friction model
  - Decoupled friction model
- Linearized model converges with more vectors but slower
- Static friction in decoupled model overestimated by √2







### Tennis ball experiment



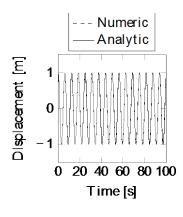


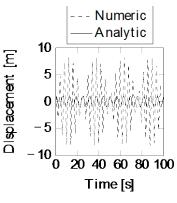
### Semi-implicit Euler integration

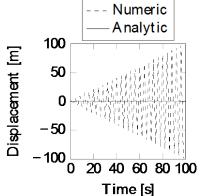
$$\Delta t \ll \frac{1}{\omega}$$

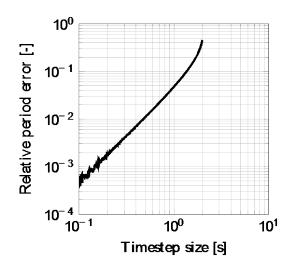
$$\frac{1}{\omega} < \Delta t < \frac{2}{\omega}$$

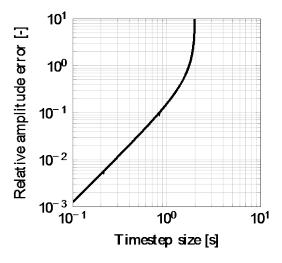
$$\Delta t \geq \frac{2}{\omega}$$











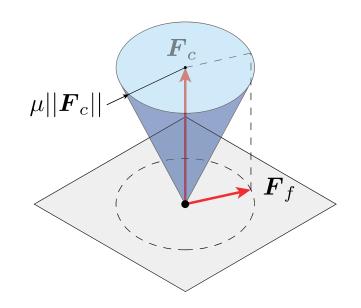
#### Friction models

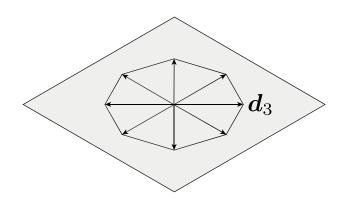
Coulomb friction model

$$||\boldsymbol{F}_f|| \le \mu ||\boldsymbol{F}_c||$$

Linearized friction model

$$m{F}_f = m{D} m{\lambda}$$
 where  $m{D} = [m{d}_1, \dots, m{d}_{\eta}]$  and  $\sum_i \lambda_i \leq \mu ||m{F}_c||$ 







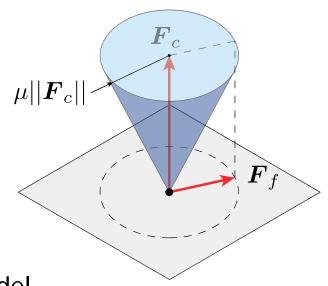
#### Friction models

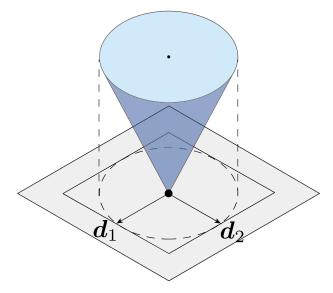
Coulomb friction model

$$||\boldsymbol{F}_f|| \le \mu ||\boldsymbol{F}_c||$$

Linearized & decoupled friction model

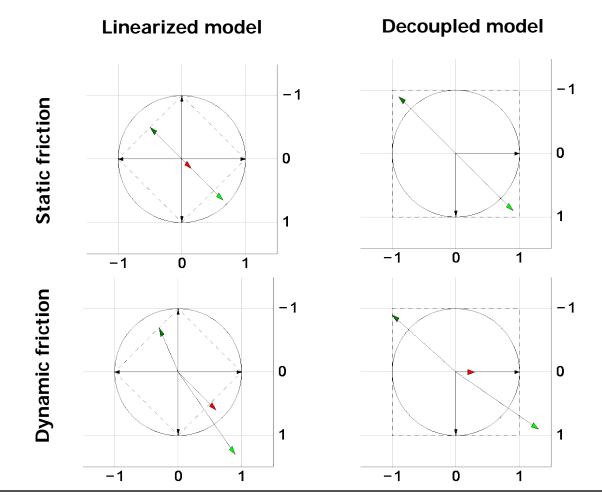
$$egin{aligned} m{F}_f &= m{D} m{\lambda} \ & ext{where} \quad m{D} = [m{d}_1, m{d}_2] \ & ext{and} \quad -\mu || m{F}_c || \leq \lambda_i \leq \mu || m{F}_c || \end{aligned}$$







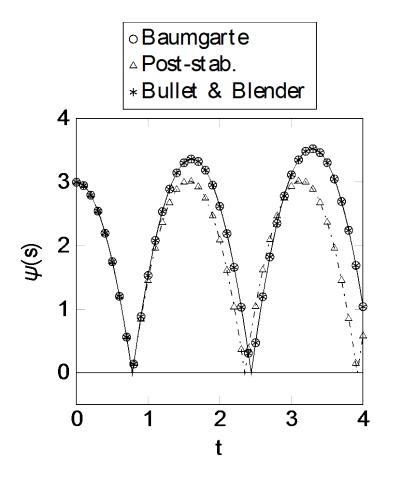
#### Friction models

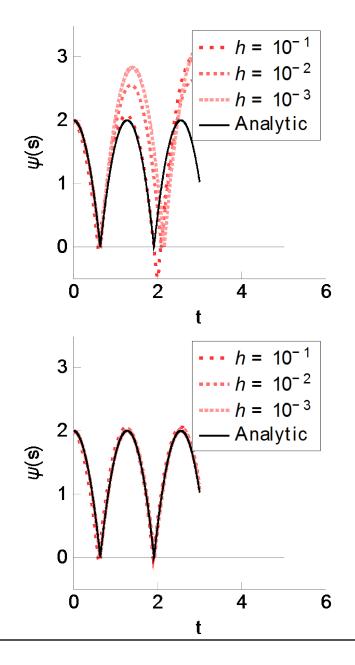






#### Error redution



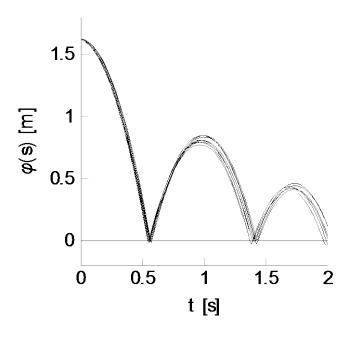


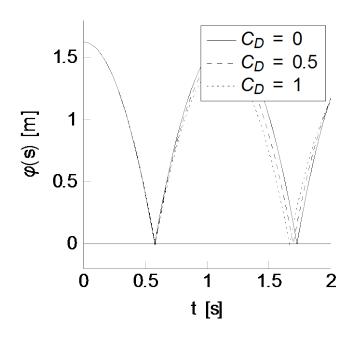




### Tennis ball experiment

$$m{F}_d = -rac{1}{2}AC_D
ho(m{u}\cdotm{u})\left(rac{m{u}}{||m{u}||}
ight)$$

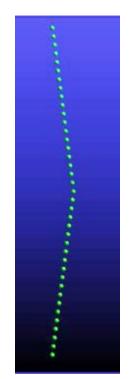


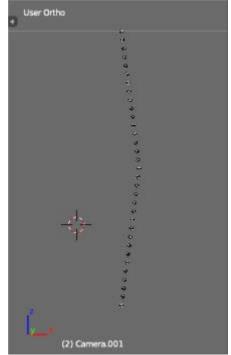


$$\mathbf{F}_{s,ij} = k_e(||\delta_{ij}\mathbf{r}|| - l_e)\delta_{ij}\hat{\mathbf{r}}$$
$$\mathbf{F}_{d,ij} = c_e(\delta_{ij}\mathbf{v} \cdot \delta_{ij}\hat{\mathbf{r}})\delta_{ij}\hat{\mathbf{r}}$$

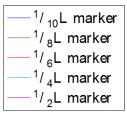
$$\boldsymbol{F}_{ij} = \begin{cases} \boldsymbol{F}_{s,ij} + \boldsymbol{F}_{d,ij} & \text{if } ||\delta_{ij}\boldsymbol{r}|| - l_e \ge 0 \\ 0 & \text{if } ||\delta_{ij}\boldsymbol{r}|| - l_e < 0 \end{cases}.$$

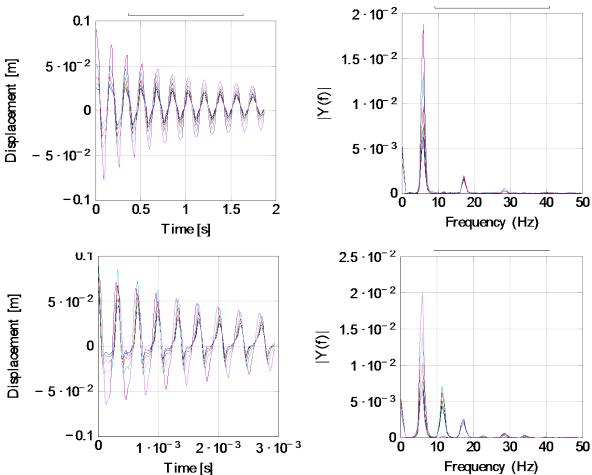
$$m{F}_{D,i} = rac{1}{2}
ho_{
m air} egin{bmatrix} 1 & 0 \ 0 & 0 \end{bmatrix} m{v}_i C_D A_h,$$





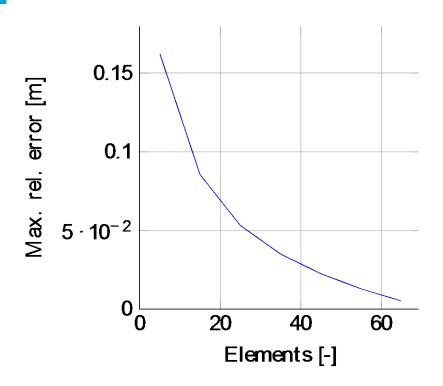


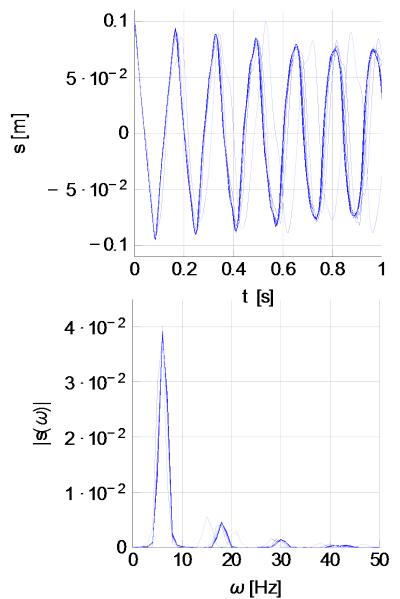




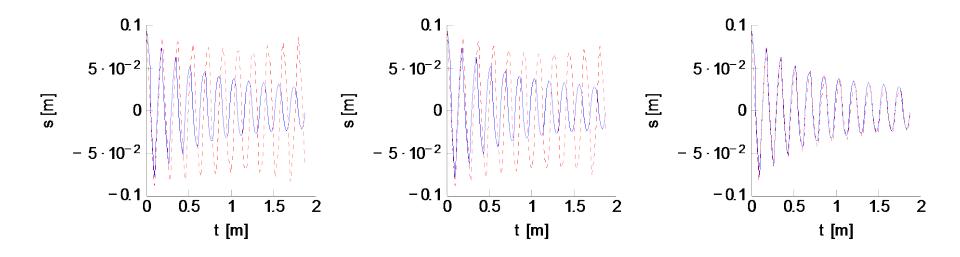






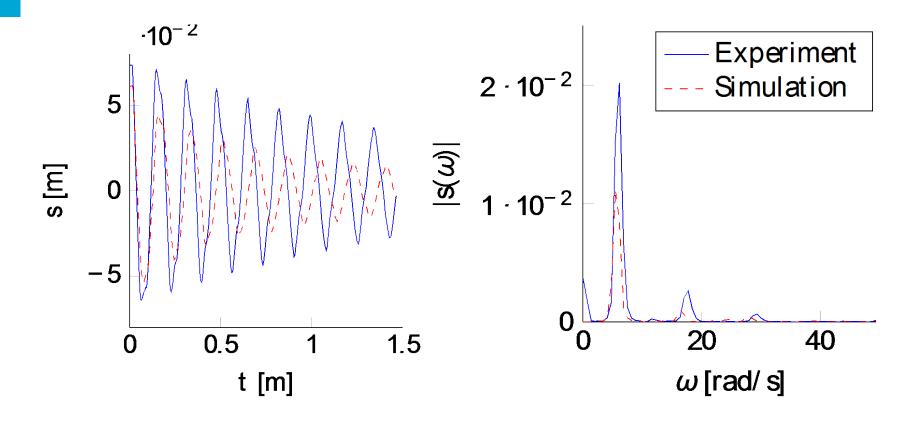








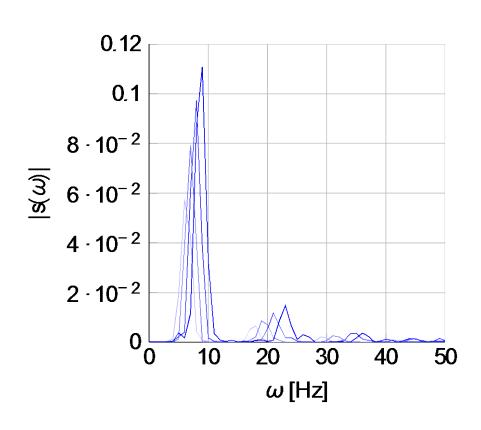


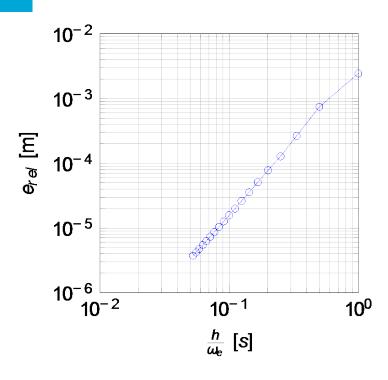


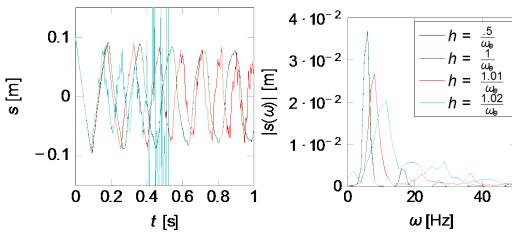




$$\mathbf{F}_{s,ij} = k_e(||\delta_{ij}\mathbf{r}|| - l_e)\delta_{ij}\hat{\mathbf{r}}$$
$$\mathbf{F}_{d,ij} = c_e(\delta_{ij}\mathbf{v} \cdot \delta_{ij}\hat{\mathbf{r}})\delta_{ij}\hat{\mathbf{r}}$$

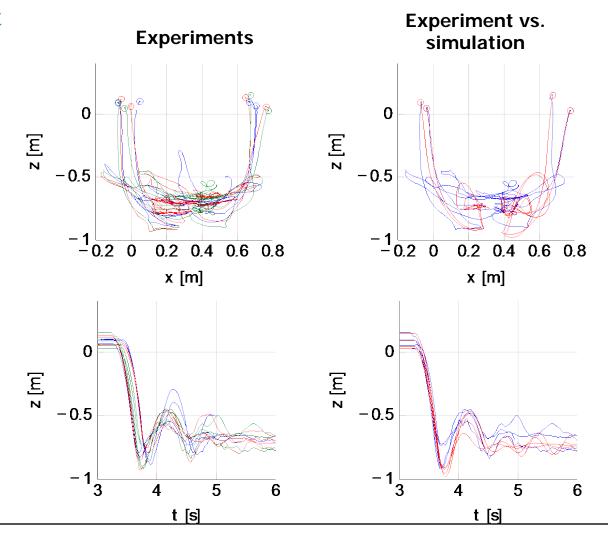








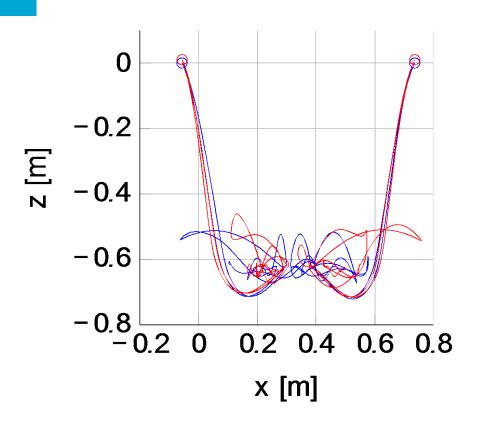
Net experiment

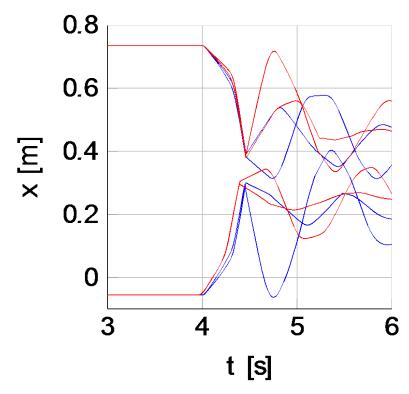






Net experiment



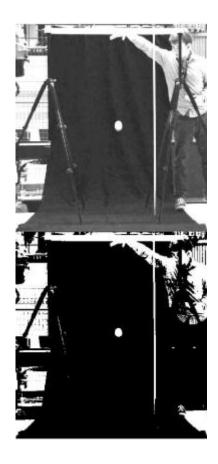






#### Image processing











Bullet algorithm overview

