STATISTICAL ASPECTS OF THE BEHAVIOUR OF MOORED FLOATING STRUCTURES

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A review is given of recent developments in the methods to obtain statistical data on the motions of moored vessels and on the loads in the mooring system with emphasis on low-frequency components of motions and loads. It is concluded that analytical methods which are aimed at obtaining statistical data directly from knowledge of the equation of motion of the vessel and the statistics of the low-frequency environmental forces, while giving insight in the main factors determining the behaviour and loads, are as yet inadequate for application to many practical cases. Time-domain numerical simulations which allow treatment of more degrees of freedom of the system and the inclusion of non-linearities in both the vesselmooring system and the environmental effects are finding more and more application in the design processes. However, the success of such methods depends to no small degree on the availability of empirical data on such important effects as damping of low-frequency motions on the one hand and on the other hand on efficiency of numerical methods in coping with long duration simulation computations. Model tests remain the most straightforward means to obtain statistically reliable data and to verify computer predictions but in order to be economically attractive, a fresh look is required with respect to the role of model tests in the design process, the scope of the model tests and the procedures followed in the preparation of tests.

1. INTRODUCTION

Reliable data on the behaviour of moored floating structures are required for determining the design loads, the fatigue loads and the operational aspects such as the workability under given conditions. The design loads are generally governed by a limited set of extreme sea conditions which are considered to have a low probability of occurrence at the location of the vessel. The probability of occurrence of the design sea-state can be related to the projected life of the structure under consideration. In this respect the so-called '5 times rule' can be mentioned which states that the design condition should have a probability of occurrence of 5 times the projected life of the structure. Thus a structure with a 20 year projected life should be designed to withstand a sea-condition with a probability of occurrence of once in 100 years. See reference [1].

The design loads are often specified as the 'most probable maximum value' being the load value which coincides with the peak of the distribution of the extreme loads. It is the task of the designer to determine these values under the condition that the values obtained need to 'reliable', preferably to some stated degree. When considering the fatigue life of a structure more detailed knowledge of the dynamics of the stresses set up in the structure as a result of the environmental loads is required. What is required here is an assessment of the stress history in a number of vital locations in the structure as cumulated throughout it's life. Not only is it necessary to determine the response of the system in terms of stresses to a limited selection of sea-conditions, for the fatigue analysis the stress response to the entire environmental history must be accounted for.

Workability analyses generally involve more the behaviour of the structure than the structural loads. It is related to the ability of the vessel to fullfill it's mission uninterupted by the prevailing sea-conditions for as long as possible and, when an interruption occurs due to weather conditions, that the task can be resumed as soon as conditions permit. Analysis of workability therefore not only require knowledge of the probability of occurrence of given sea-conditions and the response of the vessel to these conditions but also insight in the sequence in which these can occur. See reference [2]. The latter is the domain of the oceanographer and will not be dealt with here.

From the foregoing it will be clear that more and more, for different reasons, the designer needs to be able to assess the response of the floating structure and the loads and stresses in the structure. Knowledge of such data needs to be supplemented by indications of the reliability of the results as such additional data influences the credibility of his assessment and consequently the factors of safety which will be applied to the design.

In the following developments with respect to the methods available to the designer in order to determine the response of the system to the environment will be reviewed. Attention will be paid to analytical methods, time-domain simulation computations and to model tests. Full-scale testing, or the monitoring of the behaviour in real-life situations is left out of consideration although it should be mentioned that such tests are considered to be of great value in verifying the overall design procedures.

The methods developed are heavily influenced by the particular characteristics of the phenomena driving the process. In the past much attention has been paid to the hydrodynamic aspects of the environmental forces, in particular the low frequency drift forces.

Since the earlier efforts of Shu and Blenkarn [3] and Remery and Hermans [4], many papers have been published on this subject. Much progress has been made in identifying the characteristics of the environmental forces. Special effort has been put into developing methods to compute the mean and slowly varying drift forces and including these in time-domain simulations. Pinkster [5] developed a method to compute the quadratic transfer function for drift forces in 6 degrees of freedom based on the direct integration of pressure on the instantaneous wetted hull. Contributions due to second order potentials were approximated. In ref. [6] the pressure integration method is applied to the case of directionally spread seas.

Sclavounos [7] has developed more complete methods to determine the contribution due to the second order potential. Wichers [8] identified and investigated the wave drift damping effect which is important for vessels moored in extreme sea conditions. In recent years much effort has been devoted to developing means to compute this effect. See ref. [9], [10], [11], [12], [13]. Current and wind forces on vessels still elude evaluation by computational means. Model tests are generally required for a quantitively accurate assessment of these effects. Wind and current forces have up to now usually been assumed to be constant for a given heading of the vessel. With the increase in the knowledge concerning the dynamic effects present in wind and in current such effects in the resultant forces are also being investigated. See ref. [14] and [15]. In general it is found that, in open sea conditions, waves tend to dominate the environmental forces.

2. ANALYTICAL METHODS

For the present discussion, we consider analytical methods to be such means as are used to determine the system response to the environmental effects whereby it is attempted to achieve the results in terms of spectra and distribution functions of motions and loads based on knowledge of the equation of motion of the system and the statistics of the environmental forces obtained from frequency domain analyses. The results obtained correspond to the expected values of the response spectra and distributions and are free from finite duration effects. An advantage of analytical methods lies in the fact that reliable results are obtainable for low probability levels of the quantities. This is of importance for determining the extreme behaviour.

Roberts [16] was one of the first to adress the problem of determining the distribution function of the motions of a vessel moored in irregular waves with a mooring system with non-linear restoring characteristics. Under the assumption that the wave drift forces can be described as a normally distributed white noise process the distribution function for the low-frequency surge motions was computed based on the solution of a Fokker-Planck-Kolmogorov equation for the transition probability density function of the motions. It was concluded that non-linearities in the restoring force characteristics were a major cause of the motion distribution function deviating from predictions based on the assumption of a Rayleigh distribution.

Naess [17], [18] has studied the statistics of low-frequency motion response to wave drift forces and the combined statistics of wave-frequency and low-frequency response in irregular waves. Simplifying assumptions were made in order to render the general theory concerning the distributions of the responses tractable. Such assumptions being, among others, linearity of the system and the uncoupling of the motions. One of the results obtained by Naess [17] is the following asymptotic expression for the expected extreme of the low-frequency response based on the quadratic nature of the exciting force and under the assumption of a linear system:

$$E[X_{1f}] = \sigma_{1f} \cdot \ln\{\frac{T}{T_{1f}} \cdot [1 - r^2 (\frac{T}{T_{1f}})^{-\alpha}]\} \qquad (1)$$

in which:

σ_{1f} = R.m.s. of the response

$\alpha = \dfrac{1 - r}{1 + r}$

$r = e^{-\pi \cdot \delta}$

$\delta = \dfrac{b}{2\sqrt{cm}}$

T = duration considered

T_{1f} = natural period of the low-frequency response

b = damping coefficient

c = restoring coefficient

m = virtual mass of vessel

Evaluation of the extreme low-frequency response requires knowledge of the R.M.S. of the response. For the case of a linearly moored vessel with low damping characteristics Pinkster [19] gives the following approximation:

$$\sigma_{1f} = \sqrt{\frac{\pi}{2.b.c}} \cdot S_f \qquad (2)$$

in which:

S_f = Spectral density of the low-frequency excitation force at the natural frequency of the moored vessel. For low natural frequencies, the value at zero frequency is sufficiently accurate.

b = damping force coefficient

c = restoring force coefficient

In reference [17] Naess also gives the following expression for the extreme of the total response of a linearly moored vessel:

$$X_t = X_m + \{ X^2_{wi} + \gamma (X^2_{wf} + X^2_{1f}) \}^{½} \qquad (3)$$

in which:

X_m = mean excursion

X_{wi} = extreme motion due to wind

X_{1f} = extreme motion due to low-frequency drift forces

X_{wf} = extreme motion due to wave-frequency forces

γ = factor, in the range of 1.0-1.2

A noteworthy result in this expression is that the high- and low-frequency extremes are given equal weight in the estimation of the total extreme. This assumes that the wave frequency and low-frequency responses are independent processes. For the surge motions of a large tanker moored in irregular head seas experimental evidence suggests that this is the case. See ref. [20]. The extreme of the wave frequency components of the response can be determined in the usual way based on linear theory. The following equation applies for the expected extreme:

$$E[X_{wf}] = \sigma_{wf} \cdot \{2(\ln \frac{T}{T_{wf}} - \ln(-\ln 0.5))\}^{½} \qquad (4)$$

in which:

σ_{wf} = R.M.S of the wave frequency response components determined from the wave spectrum and the frequency transfer function of the response.

T_{wf} = mean period of the wave frequency response.

The above equations give an estimate of the extreme response for a linearly moored vessel and takes into account the non-linear nature of the low-frequency drift force excitation.
It should be remembered however, that non-linearities in the restoring characteristics of the mooring have a pronounced effect on the response statistics as pointed out Roberts [16] and confirmed by other authors. See for instance ref. [21].

Recently Johnsen and Naess [22] discussed the influence of wave drift damping (see Wichers and van Sluijs [8]) on the statistics of the extreme motions. It was concluded that the varying part of the wave drift damping has a considerable influence especially on the statistics of the extremes and therefore should be included in the analysis.

Stansberg [23] assumed that the drift forces are proportional to the square of the wave envelope and as such exponentially distributed and that the system characteristics were linear. A simple and robust procedure was developed on the basis of which the extreme values can be predicted. Governing parameter in the results is the ratio between the bandwidths of the motion response and the wave group spectrum. A very narrow response spectrum (low damping) leads to extremes which are Rayleigh distributed. Broader spectra lead to exponentially distributed response. For non-dimensional damping values greater than 20 % the exponential distribution is considered to be more appropriate.

At the present time it must be concluded that analytical methods are necessary to provide the basic insight into the major factors influencing the processes. Practical cases, however, require quantitive data on complex systems which are influenced substantially by such effects as non-linearities in the mooring systems, large heading changes of the vessel relative to the wave direction etc.. Analytical methods are not able to supply such data as of yet.

3. TIME-DOMAIN SIMULATION METHODS

Time domain simulation methods allow more complete descriptions of the system characteristics and the environmental effects than analytical methods and are being applied more frequently for the analysis of vessel motions. See, for instance, references [24] through [30]. Time-domain simulation computations can range from simple one-degree-of-freedom cases limited to low frequency behaviour to very complex systems involving two of more bodies, each with 6 degrees of freedom and including both low frequency and wave frequency forces and motions and loads due to impacts. See Van de Boom [31]. Time-domain simulations are not always suitable to determine the statistics of design loads. Simulations of the more complex systems involving multiple degrees of freedom rapidly become computationally a heavy burden. It is therefore of importance, also from this point of view, to develop computationally efficient codes which allow long duration simulations aimed at producing statistically reliable data at reasonable costs.

One of the most important and complex items from the point of view of computations is the generation of wave drift force time records. For the case of long-crested irregular waves the complete expression for the drift forces in the time domain involves a double summation of the following type:

$$F(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i \zeta_j P_{ij} \cos\{(\omega_i - \omega_j)t + (\varepsilon_i - \varepsilon_j)\} + \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i \zeta_j Q_{ij} \sin\{(\omega_i - \omega_j)t + (\varepsilon_i - \varepsilon_j)\} \quad (5)$$

in which:

ζ_i, ζ_j = amplitude of wave component with frequency ω_i and ω_j

$\varepsilon_i, \varepsilon_j$ = random phase angles

P_{ij}, Q_{ij} = in- and out-of-phase component of the wave drift force quadratic transfer function.

N = Number of frequencies used to describe the wave spectrum.

This double summation expression may be also be expressed as a single summation as pointed out in ref.[32] by dividing the wave spectrum in discrete, equidistant frequency steps. The following expression is then found which is an order faster than the above expression:

$$F(t) = \sum_{k=0}^{M} \{A_k \operatorname{Cos} \omega_k t + B_k \operatorname{Sin} \omega_k t\} \tag{6}$$

in which:

$\omega_k = k \cdot \Delta\omega$

$\Delta\omega$ = frequency step used to discretize the wave spectrum

$$A_k = \delta_k \sum_{j=1}^{N-k} \zeta_{j+k} \cdot \zeta_j \{P_{j+k,j} \cdot \operatorname{Cos}(\underline{\varepsilon}_{j+k} - \underline{\varepsilon}_j)$$

$$+ Q_{j+k,j} \cdot \operatorname{Sin}(\underline{\varepsilon}_{j+k} - \underline{\varepsilon}_j)\}$$

$$B_k = \delta_k \sum_{j=1}^{N-K} \zeta_{j+k} \cdot \zeta_j \{Q_{j+k,j} \cdot \operatorname{Cos}(\underline{\varepsilon}_{j+k} - \underline{\varepsilon}_j)$$

$$+ P_{j+k,j} \cdot \operatorname{Sin}(\underline{\varepsilon}_{j+k} - \underline{\varepsilon}_j)\} \tag{7}$$

in which:

$\delta_k = 1$ for $k = 0$

$\delta_k = 2$ for $k \neq 0$

N = number of frequency used to describe the wave spectrum.

$M = N - 1$

The use of fast fourier transformation techniques further serve to reduce considerably the computer time for these evaluations. See ref.[33]. In irregular directionally spread seas the drift forces can be expressed as a quadruple summation which involve summation of components arising from the interactions of regular wave components with different frequencies and directions. See ref.[6]. These can also be simplified in a similar manner.

The above expressions can be used in time-domain simulations with or without wave frequency components. If only low-frequency behaviour is being studied, or wave frequency components are added as an independent process, the wave drift force record may be generated based on an exponentially distributed white noise process which has the required mean and spectral density values. The required spectral density of the force in this case is taken equal to the 'true' density at the natural frequency of the moored vessel, or, as is usually sufficient, the spectral density for zero frequency. The selection of the exponential distribution is related to the fact that the envelope square process of the irregular waves, to which the drift forces are directly related through the quadratic transfer function, is exponentially distributed. See ref.[5]. The following expression is obtained:

$$F_e(t) = \sigma_f \cdot (1 + \ln(\text{Rand})) + F_m \tag{8}$$

in which:

Fm = mean force

Rand = random number, homogeneously distributed between 0 and 1.

$\sigma_f = \sqrt{S_f \cdot \dfrac{\pi}{\Delta t}}$

Δt = time step of the simulation

S_f = spectral density of drift force at the natural frequency of the moored vessel or, if the frequency is very low, at zero frequency

This expression was used as the basis for long duration simulations of the low frequency surge motions of a moored tanker in head seas presented in reference [21].

The success of the above formulation relies on the response of the vessel to act as a filter which is highly tuned about its natural frequency. In such cases even the distribution of the drift force is not a matter of great importance as results presented in ref. [21] have shown that the distribution of the low frequency surge motion is almost Gaussian with the extremes following the Rayleigh distribution.

In cases where the system damping is larger and the vessel also reacts appreciably to frequencies beside the natural frequency, a more realistic record of the wave drift force may need to be generated. This can be achieved, for instance, by passing the aforegoing exponentially distributed white noise force record through a simple filter such that the spectral form of the simulated record resembles more closely that of a record based on the full expression for the drift force. For our example we have chosen a simple first order system in order to reach the required result. The final expression is as follows:

$$F(t+\Delta t) = (F(t) - F_e(t))\, e^{-\frac{\Delta t}{a}} + F_e(t) \qquad (9)$$

in which $F_e(t)$ is obtained from equation (8) and 'a' is a coeffiecient dependent on the shape of the 'true' drift force spectrum.

The frequency transfer function of the filter is:

$$\frac{F_a}{Fe_a}(\omega) = \frac{1}{\sqrt{1 + a^2\omega^2}} \qquad (10)$$

The variance of the drift force is found by integration the spectral density of the force:

$$\sigma_F{}^2 = \int_0^\infty \left| \frac{F_a}{Fe_a}(\omega) \right|^2 \cdot S_f \; d\omega$$

$$= \frac{\pi}{2a} \cdot S_f \qquad (11)$$

The unknown parameter 'a' may be estimated by comparison with the variance of the wave envelop squared process which, being the driving process behind the drift force may also be described in the same way. In this case however, the variance of the envelop squared process can also be derived directly from the wave spectrum so that the value of 'a' can be derived. The following result is found:

$$a = \frac{\pi \cdot S_{A^2}(0)}{8\, m_0{}^2} \qquad (12)$$

in which:

$$S_{A^2}(0) = 8 \int_0^\infty S^2{}_\zeta(\omega)\, d\omega$$

$$m_0 = \int_0^\infty S_\zeta(\omega)\, d\omega$$

S_ζ = wave spectral density

In figure 1 the spectra of the drift force, on a fully loaded 200 kdwt tanker in irregular head seas, based on the exponentially distributed white noise and the filtered white noise are shown compared with the spectrum of the force obtained on the basis of the full expression given in equation (5) using the quadratic transfer function given by Wichers [34]. The wave spectrum is shown in figure 2. The distributions of the force, which were obtained based on time-domain records generated based on equations (5), (8) and (9) respectively, are shown in figure 3. An example of the time record of the force obtained using the unfiltered noise and the filtered noise is given in figure 4. The following data have been used in the calculations:

Significant wave height : 12.33 m
Mean period : 14.00 s

Spectral density $S_f(0)$: 115000 tf².s
Coefficient 'a' : 13.95 s
Time step dt : 10.0 s

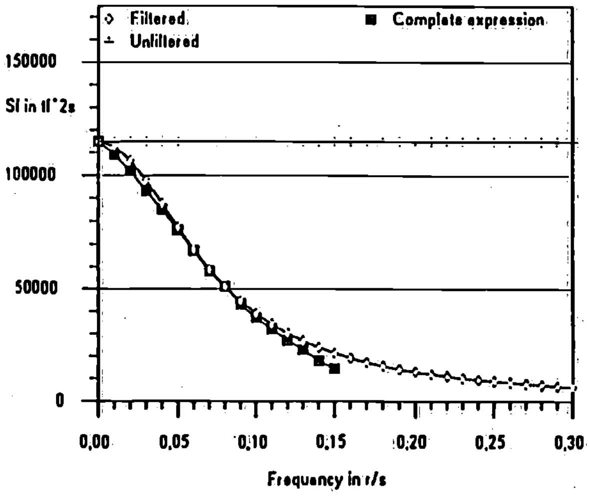

Figure 1: Spectral density of low-frequency surge forces.

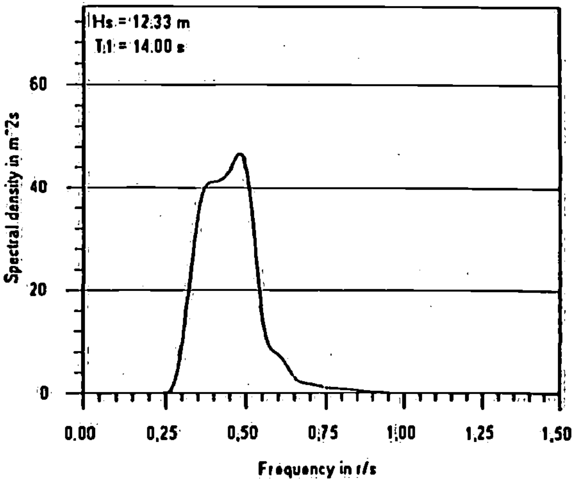

Figure 2: Wave spectral density.

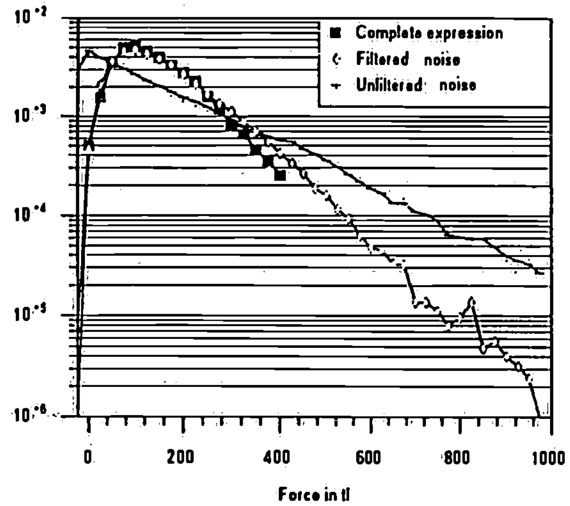

Figure 3: Distribution function of drift force records.

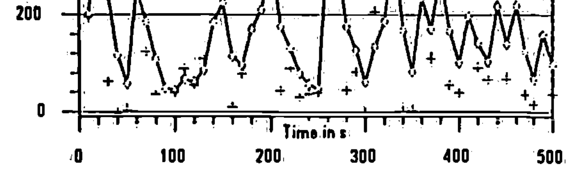

Figure 4: Time records of surge drift forces.

As can be seen, the filtered white noise resembles the 'true' force record reasonable well both with respect to the distribution and the spectrum. Using the filtered white noise costs only a small fraction of the computer time required for the full expression. It is easily adapted to take into account such effects as changes in heading of the vessel which requires only an adaptation of the input spectral density Sf(0) of the drift force at zero frequency.

In order to investigate the influence of the use of a filtered noise instead of straightforward, unfiltered noise some calculations were carried out in the frequency domain of the R.M.S. values of the low-frequency surge motions of the same tanker moored in head seas. The mooring system was assumed to be linear. The same sea-condition was applied as given in the aforegoing. The following additional data were used:

virtual mass of the vessel in surge

$m = 38940 \text{ tf.sec}^2/\text{m}$

surge damping

$b = 50\text{-}500 \text{ tf.sec/m}$

restoring coefficient of mooring

$c = 10/100 \text{ tf/m}$

The following three cases were investigated with respect to the surge drift force excitation:

- Noise 1: Exponentially distributed noise with $S_f = S_f(0)$
- Noise 2: Exponentially distributed noise with $S_f = S_f(\omega_e)$ $\omega_e = \sqrt{c/m}$
- Filtered noise : Filtered exp. distr. noise with $S_f = S_f(0)$

The results of the computations are shown in figure 5 in the form of the R.M.S. of the low-frequency motions to a base of damping coefficient for the different mooring stiffnesses and excitation models. It is seen that for c=10 all three models give virtually the same result even for relatively high damping values. For the value of c=100, differences occur which can be mainly ascribed to the difference in the level of Sf chosen for the computations. If the value of Sf is selected at the frequency corresponding to the natural surge frequency, the result is again virtually the same as that obtained using the filtered noise case. It can be concluded that in this particular case, the use of the filtered noise model, even though this leads to a more realistic record for the wave drift force, the effect in the end result is not very large. Additional time domain simulations which allowed comparison of such quantities as the most probable maximum restoring force values for the different excitation models showed a difference of at most 10% in the results.

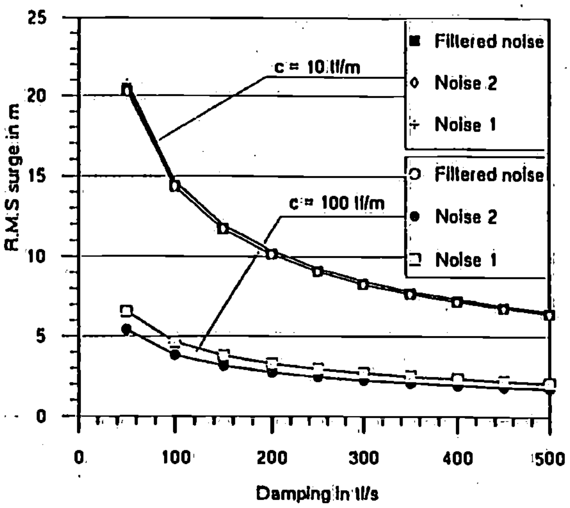

Figure 5: Surge motions in head seas.

In this particular case then, the choice of the excitation model is not of crucial importance. However, in general, it is of importance to have at hand simple models which, in cases where this is necessary, make it possible to simulate more realistically the low-frequency drift force characteristics at low computational costs. The first order filter model demonstrated here is one example of such a model.

As a second example of the application of time domain simulation using the experimentally distributed white noise model we have carried out a series of calculations to verify Naess' prediction of the expected extreme of low-frequency output as given by equation 1. Simulation computation were carried out for the low-frequency surge motion of a fully loaded 350 Kdwt tanker moored in irregular head seas.

The following data was used (see also Ref. [21]):

Virtual mass vessel in surge:

38543 tf sec^2/m

Surge restoring force coefficient c:

15.5 tf/m

Surge damping b:

80.6 tfs/m and 241.6 tfs/m

Mean surge drift force F_m:

-175.6 tf

Spectral density l.f. surge drift force

S_f: 206073 tf^{2}s

Significant wave height: 12.0 m

Mean period: 14.0 s

Simulation were carried out for durations corresponding to 3, 6, 12, 18 and 24 hours full scale. For each duration 10 independent simulation were carried out and the expected minimum surge motion found by averaging the maxima found from each set of 10 simulations. The results of the simulations are given in Figure 6 and Figure 7.

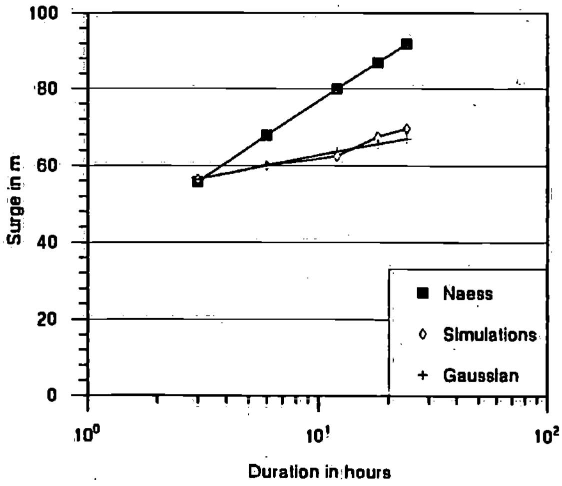

Figure 6: Expected Maximum Low Frequency Surge Motions
relative damping 0.0519.

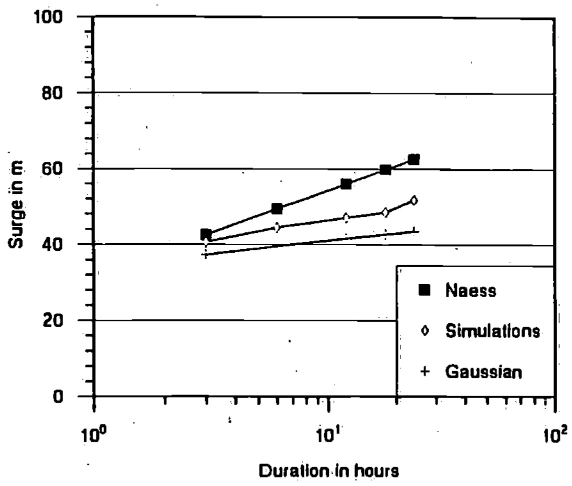

Figure 7: Expected Maximum Low-Frequency Surge Motions relative damping 0.156.

Each figure shows the expected maximum value of the surge motions, including mean offset, as predicted by Naess according to equation 1, the results of time domain simulation and as predicted by equation 4 which assume a Gaussian distribution for the low-frequency surge motions.
In order to evaluate equation 1 and equation 4, equation 2 was used to determine the value of σ_{lf}.
The results of figure 6 and figure 7 show that for the lower damping value (fig. 6) the simulated data correspond well with the assumption of a Gaussian distribution for the low-frequency surge motions. Naess' results appear to be too conservation.
For the higher damping value (fig. 7), the assumption of a Gaussian distribution results in an unconservative estimate of the extreme while Naess'results are still somewhat conservative.
This result suggests that equation 1 is applicable for high values of the damping. It should be mentioned, however, that the surge damping for this vessel as obtained from model tests (including wave drift damping) corresponded with the lowest value of 80.6 tfs/m. The high damping value of 241.6 tfs/m is simply three times the measured value and therefore already unrealistically high for such a case.

Finally, with respect to simulations of the behaviour of moored vessels it should be mentioned that at the present time, the major inaccuracies are not caused by such items as discussed above, but more by the lack of accurate data on physically relevant effects such as the damping of low-frequency motions.

The results of a comparative study reported by Herfjord and Nielsen [35], indicated a large scattering in the low-frequency motions independently predicted by some 23 institutes for a deep-draft floater and a ship-shaped vessel. See figure 8.

The lack of accurate data on the motion damping is considered to be a major factor causing the large scatter. Considerable effort should be put into obtaining such data if time-domain simulations are to play a major part in determining design loads in the systems. More complex conditions including such effects as directional spreading of the irregular waves, current and large changes in heading still present formidable problems with respect to long-duration simulations aimed at generating accurate and statistically reliable design data.

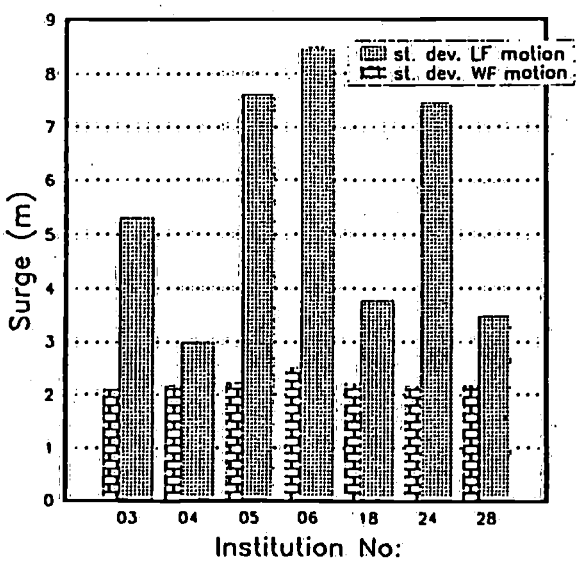

Figure 8: Comparisons of predicted Wave-frequency and Low-frequency motins of TPS-ship.
From Herfjord and Nielsen [35]

4. MODEL TESTS

Model tests have the advantage that complicated structures and conditions can be modelled limited only by the available equipment such as measuring devices and suitable model basins.

In the past model tests of moored structures were often incorporated in the design process, i.e., a preliminary design of a system was made and during the model tests, changes were made to the system until satisfactory results in terms of mooring loads etc. were found. The test duration was necessarily short due to the desire to obtain the final configuration with as little costs as possible. It was realised, however, that the statistical reliability of the results left something to be desired.

Nowadays, with the increase in the insight in the physical processes involved and the increasing confidence in mathematical models as a basis for analysing the design, model tests of moored structures carried out specifically with the aim of generating accurate and statistically reliable data on design and fatigue loads are finding their place in the arsenal of tools available to the designer. Such model tests are used less and less as a design tool but rather more as an independent and dependable means of verifying the design. As a result, in order to generate statistically reliable data, the duration of model tests has increased considerably. Whereas previously model test durations corresponded to 30 minutes reality, nowadays it is not unusual to carry out model tests for durations of 12 hours reality. In a particular case, model tests were carried out at MARIN for a duration corresponding to 48 hours full scale. Similar cases are reported in literature. Such an increase in the test duration for tests in waves places severe demands on , among others, the quality of the facility in which the tests are carried out.

A major factor inhibiting long duration testing in waves in some facilities is the reflection of the waves from the beaches and basin sides. It is our experience that, in this respect, conventional towing tanks are less suitable for carrying out such tests than are the large rectangular basins which are fitted with wave damping beaches on all sides not occupied by wave makers. A method to overcome this obstacle in conventional towing tanks is to carry out several shorter duration tests in the same wave conditions and to combine the data from these tests in order to obtain the required statistical data. The quality of a basin with respect to the reflections set up in the basin can be assessed by continually measuring the irregular waves during a test and observing the progression of the wave characteristics with time. In figure 9, results the progression of the R.M.S. of the irregular waves generated during of a long duration test reported by Pinkster and Wichers [21] are shown. The R.M.S. values were determined for successive period of 30 minutes full scale. The variations seen in the R.M.S. values are fully in the range of variations expected from such a gaussian process when taking into account the sample duration of 30 minutes and do not reveal undue effects as a result of a build-up of reflections.
Low-frequency wave activity in the form of seiches set up in the model tank can be detected by continuing to measure the wave elevation after the wave maker has been turned off. Due to the lower damping of the seiches these will continue to travel back and forth in the basin long after the short waves have damped out.
With regard to the model test duration required for a given statistical reliability of the output, no direct indication can be given for the more complex cases.

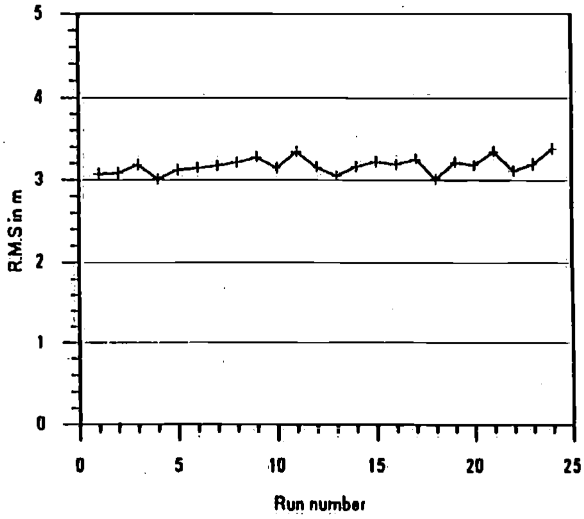

Figure 9: Time-variation of wave R.M.S. in a model basin.

Time-domain simulation computations can be usefull in this respect.
The previously referred to case with a test duration of 48 hours was selected on basis of results of a series of long duration simulations. An indication can be given if it is known that the main governing factor are the low-frequency components in the output and that the behaviour is dominated by one degree of freedom. This is certainly the case of, for instance, a permanently moored storage/production vessel in survival, head sea condition. The following expression, derived in ref.[21], has been used succesfully on a number of occasions:

$$\sigma^2{}_{\sigma^2} = \frac{1}{T\,\omega_e\,\delta} \qquad (13)$$

in which:

T = duration of model test

ω_e = natural period of the considered low-frequency motion

δ = non-dimensional damping

The above equation gives the non-dimensional Variance of Variance of the low-frequency motion components. The derivation is based on the application of a general expression previously given by Tucker [36], applied to the linear mass-spring-damper system used to describe the low-frequency motions.
The non-dimensional V.o.V given in the aforegoing equation is made non-dimensional by dividing by the square of the Variance give in equation (2).
It is generally assumed that the number of oscillation of the output determines the statistical reliability . Based on equation 13 it can be shown that this is not the case however. In equation 13, the product $\omega_e\delta$ is proportional the bandwidth $\Delta\omega$ of the spectrum of the response. The bandwidth is directly related to the frequency of the <u>envelope</u> of the output. Equation 13 therefore expresses the dependence of the V.o.V. on the number of oscillations of the <u>envelope</u> of the output and not of the output record directly.
This result is more readily understood from figure 10. In this Figure 10b a record is shown with a wide bandwidth (high value of δ) and in Figure 10a a record with a narrow bandwidth (low value of δ).

The mean frequency ω_e is the same in both cases. It will be clear that the value of the V.o.V. will be highest for the case of record a.

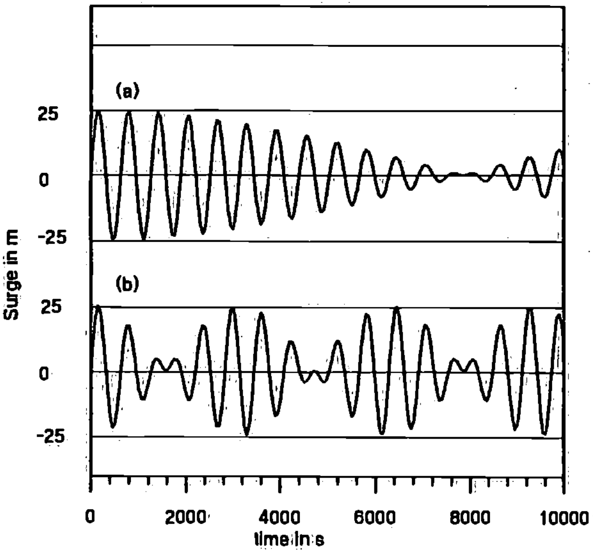

Figure 10. Narrow - and wide - band width slow motions.

5. FINAL REMARKS

In this paper we have reviewed some of the developments with respect to the methods to obtain data on the statistical properties of the response of a moored vessel in irregular waves, wind and current environment. The current state of the art does not allow straightforward determination of the statistics of extremes for many of the practical cases involving combined non-linearities of the environmental effects and the system properties or more degrees of freedom. Only for the case of one degree of freedom have results been given on the combined statistics of wave- and low-frequency responses. Progress has been reported with respect to the statistics of the low-frequency response due to a linearly moored vessel under the influence of low-frequency second order wave drift forces. These results need to be verified on the basis of extensive comparisons with the results of specific model tests and simulation computations.

Time domain simulations are becoming the standard way of analysing the behaviour of moored vessels under arbitrary conditions. Many aspects of the environmental effects are however still too complex to be applied routinely in mooring analyses. For instance, drift forces in directionally spread seas are extremely computer intensive both from the point of view of the generation of quadratic transfer functions and with respect to the effort required to generate time histories for time domain simulations.

Model tests provide a practical, straightforward means to obtain statistical data on the behaviour of moored vessels provided the model basin is suitable for long-duration model tests and is able to generate the required environment. The increase in the test duration over the past years requires a reconsideration of the procedures followed during testing in order for these to remain economically attractive.

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