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Optimal planning of flood defence system reinforcements using a greedy search algorithm

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ABSTRACT

Climate change and deterioration require a continuous effort to reinforce flood defences and meet reliability requirements. To efficiently upgrade flood defence systems, insight in costs and benefits of measures at a system level is required throughout the process of planning and design. Due to the size of flood defence systems the number of possible decisions is large, which hampers system optimization. We describe a greedy search algorithm that can find (near-)optimal combinations of reinforcement measures for dike segments. The algorithm has been validated by comparing results for 2800 different dike segments to an integer programming implementation. The difference in objective value (Total Cost) is only 0.04% on average, which is small compared to other uncertainties in assessment and design of dike segments. The algorithm is applied to a reinforcement project for a dike segment of 41 independent sections, and compared to the common design practice which uses reliability-based requirements on a section level. It is found that the resulting reinforced dike segment is 42% cheaper to construct than the one obtained from the common approach, based on the same input information. This illustrates the practical and societal value of the design approach using a greedy search algorithm in this context.

1. Introduction

In many delta regions around the world, systems of flood defences (e.g., dikes, dunes and hydraulic structures) have been constructed to mitigate risks from flooding [1,2]. Due to increasing economic activity [3], in combination with climate change consequences [4], reliability of these flood defence systems has to be improved continuously, often resulting in a backlog of improvement projects. In this paper we focus on design of dike segments, which are series systems of dikes (i.e., earthen flood defences) with a length of ≈ 20 km. However, the methods are also applicable for other types of flood defences.

The subsoil in delta regions is typically very heterogeneous, which means that the strength of dike segments is very non-homogeneous. This means that the system reliability is determined by that of different (partially) independent elements. This is illustrated for a dike segment in Fig. 1: the reliability of such a dike segment depends on different sections (length ≈ 1 km) that are modelled as a cross section. Typically the strength of different sections is uncorrelated, and the load is correlated. Each section is susceptible to various failure modes, most notably inner slope instability, piping erosion and overtopping. However, this is not always to the same extent, and the cost for increasing reliability is not

the same for each failure mode and can vary per section (for instance due to lack of space due to neighbouring residential areas).

To ensure acceptable flood risk for the future, in the Netherlands safety standards (i.e., reliability requirements) for dike segments have been derived based on loss-of-life and Societal Cost Benefit Analysis [5,6]. In order to meet these requirements many dike segments have to be reinforced, and flood defence managers have to translate segment reliability requirements to design alternatives for different dike sections and vice versa. Generally this is done using a reliability-based design approach where the segment requirements are translated to section requirements in a uniform way along the segment. A section is then modelled as a single representative cross section. However, it is found that this approach is often conservative, as it does not take into account the various causes for non-homogeneity of the segment, most notably: current differences in reliability between sections, and differences in costs for improving different failure modes at different sections (e.g., differences due lack of space for expansion) [7]. Overall, this approach leads to relatively high reinforcement costs.

Research on optimization of flood defence systems has generally focused on two aspects: either determining optimal safety targets for dike segments [e.g. 8–10], or optimal design alternatives for dike
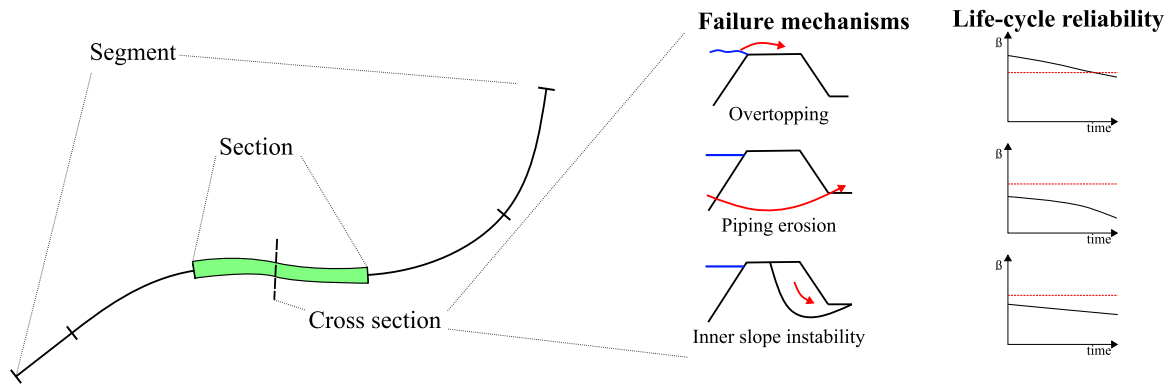
Fig. 1. Relation between a dike segment, section and representative cross section, failure modes considered for each cross section and (conceptually) resulting relations between reliability index β and time. The average length of a segment is approximately 20 km and the length of a section varies between 200 and 2000 m (average $\approx$ 1 km).

cross sections [e.g. 7,11]. van Dantzig [12] first elaborated the derivation of optimal flood defence system safety targets. Later this was advanced in the approach used to derive the new safety standards in the Netherlands [5,13]. As the Linear Programming approach that was used could not deal with non-homogeneous segments it was improved by Brekelmans et al. [14], and later extended to an integer programming approach by Zwaneveld et al. [8], and a graph-based approach [9,15]. Both can handle non-homogeneous segments and more complex systems. Nevertheless, in order to prevent state space explosion, typically one has to significantly simplify the optimization problem, which is a major issue when analysing large dike segments. Optimization of cross-sections using a target reliability typically considers multiple failure modes as well as influence of deterioration such that design alternatives can be optimized in time (e.g. [7,10] and [11]). Nevertheless, optimization of improvement planning of dike segments in connection to the optimization at a cross section level has not been addressed explicitly.

Especially for long term planning of flood protection systems decision makers also have to deal with significant uncertainty in future economic value and climate change effects on extreme flood conditions [e.g. 16]. To deal with this, various approaches have been used to incorporate this uncertainty in strategic planning decisions. For instance, real options analysis such as considered by Woodward et al. [17] explicitly accounts for the flexibility of investment decisions, in order to assure that investments are robust under a wide variety of possible future conditions. Kwakkel et al. [18] and Woodward et al. [19] use a model-driven approach using a Multi-objective evolutionary algorithm (MOEA) that can find robust optimal solutions under a wide variety of uncertainties. Such approaches can be valuable if decisions are sensitive to large (future) uncertainties, or for cases with a high degree of complexity [e.g. 20,21]. A common class of MOEA are genetic algorithms such as the NSGA-II algorithm [22]. An advantage is that these algorithms do not suffer as much from state space explosion as e.g. integer programming, although they do not provide a guaranteed optimal solution.

In other fields the issue of optimal planning of interventions in complex systems has been addressed in the past for both single and multi-objective problems. For instance [23] addressed planning of bridge maintenance in a road network aimed at balancing cost and network performance, by using an adapted version of the aforementioned NSGA-II algorithm. Barone and Frangopol [24] evaluated optimal maintenance planning for a bridge of several components using different performance indicators, and encountered differences in both computational time as well as performance for each performance indicator. For instance, risk-based maintenance was more cost-effective than reliability-based maintenance. Cavdaroglu et al. [25] determined schedules for network restoration measures after an non-routine disruption of an interdependent network using a heuristic solution method.

The accuracy of the developed heuristic solution method was high (in comparison with some commercial solution methods), and had a much lower computational cost thus making it more accessible to decision makers. Bagloee et al. [26] solved a very practical prioritization question for road improvement projects using a combination of supervised learning and integer programming in order to optimally schedule projects within a given budget. Also in the field of project scheduling for offshore asset construction [27] and concrete bridges [28] amongst others, there are several (heuristic) approaches that deal with solving scheduling tasks with multiple objectives and uncertainty in present and future conditions. In summary, there is ample work in other fields that we can utilize for optimization of planning of dike segment reinforcements.

In general, both for flood defences and other applications, once problems become interdependent and the number of variables increases, some type of heuristic is applied in order to reach a near-optimal solution. In this paper we develop a greedy algorithm that employs heuristics based on the engineering problem of improving a dike segment consisting of many non-homogeneous sections. The advantage of this approach is that the heuristics are relatively easy to understand for dike managers, and that it can reach a (near-) optimal solution quickly on commonly available hardware. This is an advantage compared to existing solutions which are typically less transparent and require a simplification of the problem, making them less useful for application in design and planning of dike reinforcement projects.

In Section 2 we describe the principles of (improving) dike segment reliability and how this can translate to heuristics for a greedy algorithm. In Section 3 we describe a general optimization problem for planning reinforcements of a dike segment, in Section 4 we compare the results for the greedy algorithm with a Mixed Integer Programming (MIP) implementation similar to [8]. In Section 5 we demonstrate the applicability of the approach for the design and planning of an actual dike system consisting of 41 dike sections.

2. Methods

2.1. Reliability of dike segments

In safety assessments of dike segments in the Netherlands the reliability of a segment is compared to the reliability requirement, where a segment has a relatively uniform load (e.g., all sections loaded by the same river) and the consequences of flooding are relatively uniform (i.e., the same area is flooded, independent of where in the segment a failure occurs) [6].

Typically different failure modes are assessed for different independent sections, based on a representative cross section. For river dikes there are three dominant failure modes: overtopping, piping erosion and inner slope instability [29]. Overtopping failure occurs when the

hydraulic loads (water level and waves) exceed the dike crest and cause erosion of the inner slope [30]. Piping erosion is the failure mode where soil particles are eroded from a granular layer below the dike due to ground water flow [31]. Inner slope instability occurs when the inner slope of a dike fails due to saturation (and corresponding increased weight and decreased shear resistance) as a result of a lasting high water level [32]. The failure modes are schematically represented in Fig. 1 (middle).

To obtain the segment reliability, the limit state functions of the different mechanisms and different sections can be combined in order to obtain the overall reliability for the dike segment. Methods such as the Equivalent Planes method [33] can account for correlation between sections and failure modes. For piping erosion and inner slope instability the variability of the subsoil is typically dominant, and therefore different sections have hardly any correlation. For overtopping along a river the failure is mostly water level dominated, meaning that sections are strongly correlated.

In practice often a simplified approach using upper and lower bounds is used. The upper bound of the failure probability of a segment $P_{f,segment}$ of N (uncorrelated) sections at time t is then given by Vanmarcke [34]:

$$P_{f,segment}(t) = \prod_i^N 1 - P_{f,i}, \tag{1}$$

whereas the lower bound, corresponding to a system with N fully correlated sections is given by:

$$P_{f,segment}(t) = \max_{i \in N} P_{f,i}. \tag{2}$$

It was shown by Vanmarcke [34] that even systems with equally reliable components and relatively strong correlation ($\rho \approx 0.8$) are quite well approximated by Eq. (1), and even more so for components with unequal reliability (such as a typical dike segment). In our case, we can thus approximate the segment reliability for piping erosion and inner slope instability using Eq. (1), and for overtopping using Eq. (2), after which the overall segment failure probability for all M mechanisms is obtained from:

$$P_{f,segment}(t) \approx 1 - \prod_{m \in M} 1 - P_{f,m,segment}(t). \tag{3}$$

Due to temporal changes in load and strength, reliability will change over time (see Fig. 1). Deterioration of the strength can consist of, amongst others settlement of the dike crest resulting in additional overtopping, settlement of the inner toe resulting in larger hydraulic head for piping erosion and decreased slope stability. Temporal changes in loads are typically caused by climate change resulting in higher water levels and higher waves [11]. This has a direct and relatively large effect on overtopping reliability, but the influence on piping and slope instability is smaller as there is damping of these effects in the subsoil. In Section 4 we will further outline how these effects are incorporated.

2.2. Improving reliability of dike segments

Aside from the fact that dike segments consist of many non-homogeneous sections, dike asset owners in the Netherlands currently have to deal with at least the following three considerations when planning dike reinforcement projects:

- The dike segment has to meet a reliability requirement, but they typically have time to achieve this.
- The strategy to achieve this has to be explainable to other stakeholders.
- They can choose between many different types of reinforcement methods, with different costs at each dike section.

Note that in practice also considerations of other functions and regional developments are of importance, but in this study we focus on achieving the required reliability in a cost-optimal manner.

Dikes provide utility in terms of reduction of economic damage and loss-of-life due to flooding [35]. Based on analysis of costs and utility, optimal target reliability requirements have been obtained that ensure optimal risk levels [e.g. 6]. Ideally, any investment in dikes would be evaluated to yield minimal total cost (flood risk + investment). The practical approach in the Netherlands is to first define optimal reliability requirements after which investments are aimed at meeting these. As a major part of the optimization is based on optimal total cost, these reliability requirements are typically very close to what would follow from a total cost optimization.

In the Netherlands, the overall goal is to meet the reliability requirements in 2050 for the entire country. The main reason is that it is not achievable to reinforce all dike segments to the required level before that time with the available budget and capacity. Practically this means that dike managers have to ensure two things:

- That they meet the target reliability in 2050.
- That they achieve this target in an efficient way within existing (budget) constraints.

Thus, we can distinguish two phases in the optimization problem: the period up to the year when the requirement has to be met (2050), and the period after that. In the first period it is of importance that investments are efficient in terms of total cost (investment and risk costs). It has been found that as for example loss-of-life risk is typically strongly correlated to economic damage, optimizing the total economic cost is a good approximation for other risk indicators as well [36]. Thus, we can use a total cost optimization to determine an optimal planning of reinforcement measures for the first phase. In the second phase, after 2050, a total cost optimization is still a good approach, but additionally it should be required that the reliability remains above the target level.

Secondly, aside from societally cost optimal, a planning of dike reinforcements needs to be explainable and transparent, as many stakeholders are involved. In the context of a dike reinforcement there is for instance involvement by financing organizations, inhabitants, nature preservation organizations, local farmers and local and regional governments. This means that in practice a decision on a dike reinforcement is risk-informed, rather than risk-based, and as was argued by Bohnenblust and Slovic [37] a technical analysis should aim to focus discussion between stakeholders on key issues rather than providing a clear cut solution. For our analysis this means that an approach that not only gives an optimal planning but is also explainable to non-technical stakeholders is to be preferred.

Lastly, there are many techniques available for dike reinforcement, although typically dikes are reinforced by heightening the crest (to prevent overtopping) and widening berms (to counter instability and piping erosion issues) with additional soil material. However, also structural measures such as diaphragm walls and sheetpiles [38], as well as innovative measures such as Vertical Sandtight Geotextile [39] are applied. These are specifically interesting for countering threats from instability and piping erosion in densely populated areas. The main reason is that they require less space, although some of these structural measures are much more expensive than enlarging the dike profile with additional soil.

These measures can be classified in two main dimensions, extent and type, as is illustrated in Fig. 2. For the extent we distinguish between full measures that impact all relevant failure modes, and partial measures aimed at improving reliability for only 1 failure mode. A renewal type measure alters the structural behaviour of the dike, whereas a renovation measure maintains the general structural behaviour but increases the dimensions. For instance, Fig. 2 (top left) shows a full renewal using a diaphragm wall, which affects all mechanisms and completely alters the structural behaviour. The bottom right figure shows a crest heightening: the structural behaviour remains the same
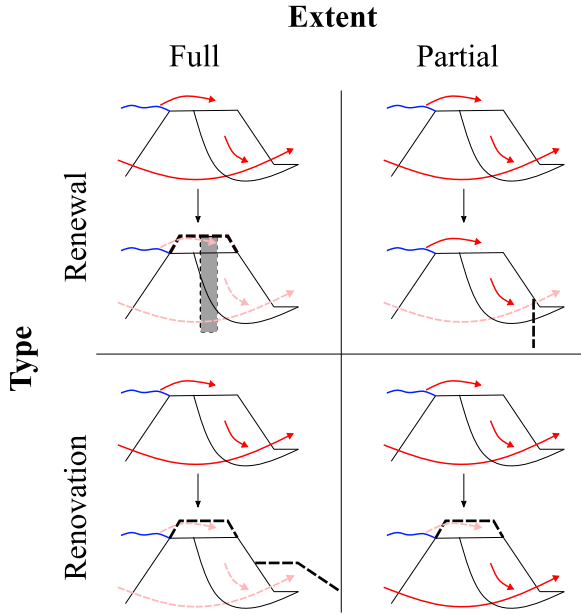
Extent

Full   **Partial**

Fig. 2. Categorization of types of reinforcement measures. Red arrows indicate mechanisms as defined in Fig. 1 with a safety deficit, pink dotted arrows indicate if a measure has resolved this deficit. For each type an illustrative reinforcement measure is shown (dashed black lines). From top left, clockwise: a full renewal using a diaphragm wall, a partial renewal using a Vertical Sandtight Geotextile, a partial renovation through crest heightening and a full renovation with combined berm widening and crest heightening.

and only the failure probability of overtopping is reduced. In terms of life-cycle performance each type of measure will have different behaviour in terms of degradation of performance, and for renewal measures the impact of different uncertainties on the performance might shift. It is important to include these effects in planning decisions, as this might affect the reliability over time significantly.

3. Approach

3.1. Definition of the problem

In this paper we consider a dike segment of N sections for a period of T years. The current safety level of the segment is significantly below the safety standard, and measures for dike reinforcement have to be determined. A variety of measures for different dike sections is available. The goal is to determine the optimal combination of measures that ensures that the reliability requirement in year t_{req} is met, in a cost optimal way and considering different failure modes. Table 1 presents the used notation. In this section we describe the general problem, in the following subsections we describe the solution methods that are used.

Our general objective may be written as follows:

$$\min TC = TR + TLCC \tag{4}$$

where TR is the total flood risk cost over the considered period T and $TLCC$ is the total life cycle cost of all measures. TR is defined as:

$$TR = \sum_{t \in T} P_{\text{f,total}}(t) \cdot D(t) \tag{5}$$

with:

$$P_{\text{f,total}}(t) = 1 - \prod_{n \in N} \left(\sum_{s_{\text{h}} \in S_{\text{h}}} \sum_{s_{\text{g}} \in S_{\text{g}}} \left(\widehat{P_{\text{f,pip}}} \cdot \widehat{P_{\text{f,inst}}} \cdot C_{\text{int}}(n, s_{\text{h}}, s_{\text{g}}) \right) \right.$$
$$\left. \cdot \left(1 - \left(P_{\text{f,over}} \cdot D_{\text{int}}(t, n, s_{\text{h}}) \right) \right) \right) \tag{6}$$
$$\text{for } t \in T,$$

where $\widehat{P_{\text{f,pip}}} = 1 - P_{\text{f,pip}}(n, s_{\text{g}}, t)$ and $\widehat{P_{\text{f,inst}}} = 1 - P_{\text{f,inst}}(n, s_{\text{g}}, t)$.

$TLCC$ is defined as:

$$TLCC = \sum_{n \in N} \sum_{s_{\text{h}} \in S_{\text{h}}} \sum_{s_{\text{g}} \in S_{\text{g}}} LCC(n, s_{\text{h}}, s_{\text{g}}) \cdot C_{\text{int}}(n, s_{\text{h}}, s_{\text{g}}). \tag{7}$$

This is subject to the following constraints:

$$\sum_{s_{\text{g}} \in S_{\text{g}}} \sum_{s_{\text{h}} \in S_{\text{h}}} C_{\text{int}}(n, s_{\text{h}}, s_{\text{g}}) = 1 \text{ for } n \in N \tag{8}$$

$$\sum_{n \in N} \sum_{s_{\text{h}} \in S_{\text{h}}} D_{\text{int}}(t, n, s_{\text{h}}) = 1 \text{ for } t \in T \tag{9}$$

$$\sum_{\substack{s_{\text{h}} \in S_{\text{h}} \\ P_{\text{f,over}}(n, s_{\text{h}}, t) > \\ P_{\text{f,over}}(n^*, s_{\text{h}}*, t)}} C_{\text{int}}(n, s_{\text{h}}, s_{\text{g}}) +$$

$$\sum_{n_{\text{h}}} \sum_{\substack{s_{\text{h}} \in S_{\text{h}} \\ P_{\text{f,over}}(n, s_{\text{h}}, t) \leq \\ P_{\text{f,over}}(n^*, s_{\text{h}}^*, t)}} D_{\text{int}}(t, n, s_{\text{h}}) \leq 1 \tag{10}$$

$$\text{for } t \in T, n \in N, n^* \in N, s_{\text{h}}^* \in S_{\text{h}}$$

$$P_{\text{f,total}}(t) < P_{\text{req}} \text{ for } t \in T, t_{\text{req}} \leq t \leq t_{\text{horizon}} \tag{11}$$

$$C_{\text{int}}(n, s_{\text{h}}, s_{\text{g}}) \in 0, 1 \text{ for } n \in N, s_{\text{h}} \in S_{\text{h}}, s_{\text{g}} \in S_{\text{g}} \tag{12}$$

$$D_{\text{int}}(t, n, s_{\text{h}}) \in 0, 1 \text{ for } t \in T, n \in N, s_{\text{g}} \in S_{\text{g}} \tag{13}$$

Eq. (4) describes the objective of our approach, namely to minimize the total cost consisting of flood risk and investment costs over the considered time period T, which are both written more explicitly in Eqs. (5) and (7). Eqs. (8) and (9) describe relatively simple constraints that ensure that there is only 1 combination of s_{h} and s_{g} chosen per dike section, and that for each time t only 1 section n is the weakest for overtopping. Eq. (10) is a bit more complicated, but it ensures that if there is an investment s_{h} at a section n, it cannot be the weakest section n^*, s_{h}^* at the same time, so investments in reducing $P_{\text{f,over}}(n, s_{\text{h}}, t)$ are always done at the section with the highest $P_{\text{f,over}}(n, s_{\text{h}}, t)$. Eq. (11) is a constraint that ensures that after some year t_{req} the system reliability requirement is satisfied. This is optionally limited by t_{horizon}, which is the horizon for which this is to be satisfied. Eqs. (12) and (13) ensure that the variables $C_{\text{int}}(n, s_{\text{h}}, s_{\text{g}})$ and $D_{\text{int}}(t, n, s_{\text{h}})$ are binary.

Before the optimization is started $P_{\text{f,inst}}(n, s_{\text{g}}, t)$, $P_{\text{f,over}}(n, s_{\text{h}}, t)$, $P_{\text{f,pip}}(n, s_{\text{g}}, t)$, $LCC(n, s_{\text{h}}, s_{\text{g}})$ and $D(t)$ are precalculated. These are input for both approaches used to minimize the objective function. The formulations for these precalculations will be discussed in Section 4.

3.2. Finding a solution

The problem as described above is implemented as a Mixed Integer Programming (MIP) problem in CPLEX 12.9 [40] and solved using branch-and-cut. An advantage of branch-and-cut is that it can be used to exactly solve integer programmes with optimality guarantee [41]. However, with the number of investment options that is relevant for a typical dike segment, this is only feasible for relatively small segments up to about 13 dike sections with 16 GB available RAM, which is much smaller than our real world problem. As an illustration, in the case study in Section 5 we consider $\approx 10^{100}$ possible combinations of reinforcement measures.

To overcome issues with computational speed and hardware we develop a greedy search algorithm. Greedy algorithms are a class of algorithms that use the locally optimal choice at each stage in order to obtain or approach the global optimum [42]. This means that these algorithms can handle much larger state spaces, which is useful in the context of the large dike segments that we consider. An important property of greedy algorithms is that it never reverses choices but always continues with the next optimal choice until it finds a solution or is stopped.

Table 1
Descriptions of used symbols.

Symbol	Description
N	Set of all dike sections at which investments are possible
n	A dike section index $n \in |N|$
T	Set of all years in the analysed period T
t	Considered point in time $t \in |T|$
t_{req}	Point in time $t_{\text{req}} \in |T|$ where the reliability requirement is to be met
S_{h}	Set of investment options for overtopping
s_{h}	Index of investment option for overtopping $s_{\text{h}} \in |S_{\text{h}}|$
S_{g}	Set of investment options for geotechnical failure modes
s_{g}	Index of investment option for geotechnical failure $s_{\text{g}} \in |S_{\text{g}}|$
$P_{\text{f,req}}$	Failure probability requirement for all dike sections N
$P_{\text{f,over}}(n, s_{\text{h}}, t)$	Overtopping failure probability of dike section n, given investment option s_{h}, at time t
$P_{\text{f,pip}}(n, s_{\text{g}}, t)$	Piping failure probability of dike section n, given investment option s_{g}, at time t
$P_{\text{f,inst}}(n, s_{\text{g}}, t)$	Instability failure probability of dike section n, given investment option s_{g}, at time t
$LCC(n, s_{\text{h}}, s_{\text{g}})$	Total life-cycle cost (in €) for the combination of investment options s_{h} and s_{g} at dike section n
$D(t)$	Discounted flood damage at year t in €
$C_{\text{int}}(n, s_{\text{h}}, s_{\text{g}})$	Binary value that takes value 1 or 0 and indicates whether measure s_{h} and s_{g} at section n have been taken (1) or not (0)
$D_{\text{int}}(t, n, s_{\text{h}})$	Binary value that indicates if section n with measure s_{h} implemented is the weakest section (1) for overtopping in year t or not (0)
TC	Value of the objective function of all costs over the period $|T|$
TR	Total flood risk cost over the period $|T|$
$TLCC$	Total Life Cycle Cost over the period $|T|$
$BC(n, s_{\text{h}}, s_{\text{g}}, C_{\text{int}})$	Benefit–cost ratio of investment s_{h}, s_{g} at section n with initial situation C_{int} at the beginning of a greedy search iteration

In many problems greedy algorithms have been found to achieve (near-)optimal solutions, but in general it is hard to prove that a solution is optimal. In order to show that the heuristics yield (near-)optimal solutions, we evaluate the performance of the greedy algorithm and compare it with a MIP implementation in CPLEX 12.9 [40] using branch-and-cut, for a large number of different dike segments. A potential advantage of greedy algorithms is that the heuristic rules that are used for determining the optimal steps are often easy to understand and can be adapted to the problem at hand. In our case we can use the formulations of segment reliability as well as the principle of total cost optimization as basis for the heuristics.

3.3. A greedy algorithm for planning of flood defence improvements

In this section we introduce the greedy search algorithm. In the implementation of heuristics, we need to ensure two main points: Firstly, that the search method is in line with the objective of finding minimal Total Cost, and secondly that the relation between element and system reliability is properly dealt with for all failure modes.

For the search method the work by Špačková and Straub [43] can be used, who demonstrated that for a case without budget limitation the optimal solution is found if $-\delta C < \alpha \cdot \delta R$, with δR the risk reduction and δC the cost increment. α is an arbitrary factor indicating risk averseness, so how much risk has to be reduced for a cost increment δC. This criterion guarantees that Pareto optimal solutions are found in all cases (although not all Pareto optimal solutions are found). For cases such as ours where deterministic costs are assumed this is equal to the benefit–cost ratio $BC = E[(-\Delta R)/\Delta C]$. Špačková and Straub [43] considered single investment strategies to achieve a certain protection level. In our case we aim to determine the combination of measures that is on (or very close) to the Pareto optimal solution for $\alpha = 1$ (we assume risk neutrality). If the utility of our investment is the risk reduction, we would expect that for each subsequent measure that we take, our marginal risk reduction will reduce in accordance with the law of diminishing marginal returns. Thus if we find a path where we continuously maximize the BC until $E[(-\Delta R)/\Delta C] = 1$ we should obtain at least a local optimal solution. Thus the main parameter used for evaluating the steps in the greedy algorithm is the benefit–cost ratio, which is defined as:

$$BC(n^*, s_{\text{h}}^*, s_{\text{g}}^*, C_{\text{int}}(N, S_{\text{h}}, S_{\text{g}})) = \frac{TR - TR^*}{\sum_{n^* \in N} \sum_{s_{\text{h}}^* \in S_{\text{h}}} \sum_{s_{\text{g}}^* \in S_{\text{g}}} LCC(n^*, s_{\text{h}}^*, s_{\text{g}}^*)}$$

(14)

where TR denotes the Total Risk of flooding over a period T for a set of measures defined by $C_{\text{int}}(n, s_{\text{h}}, s_{\text{g}})$, and TR^* denotes the same for a case where measures $s_{\text{h}}^*, s_{\text{g}}^*$ have been taken at sections n^*. $LCC(n^*, s_{\text{h}}^*, s_{\text{g}}^*)$ denotes the cost of these measures. If $BC(n^*, s_{\text{h}}^*, s_{\text{g}}^*, C_{\text{int}}(N, S_{\text{h}}, S_{\text{g}})) < 1$ for all combinations of $n^*, s_{\text{h}}^*, s_{\text{g}}^*$ the optimum with minimal total cost TC has been reached, as the marginal total costs are smaller than 0. This can be either a local or global optimum, depending on the performance of the search routine. In our case this search routine is based on the definitions of system reliability in Eqs. (1) and (2).

As was outlined in Section 2.1 the relation between element and system reliability for overtopping and geotechnical failure modes (piping erosion and slope instability) differs. In order to prevent missteps in deriving the local optimal solution in the greedy search algorithm, this has to be dealt with properly in the implemented heuristics. Therefore we need to implement different rules for deciding on investments to improve geotechnical reliability and investments to improve overtopping reliability. The main steps of the algorithm are listed below and displayed in the flowchart in Fig. 3.

Input for the search algorithm are precalculated arrays of failure probabilities and $LCC(n, s_{\text{h}}, s_{\text{g}})$ for all measures $n \in N$, $s_{\text{h}} \in S_{\text{h}}$ and $s_{\text{g}} \in S_{\text{g}}$. Initial failure probabilities are given per dike section.
Step 0: at the beginning of each iteration the set of existing measures $C_{\text{int}}(n, s_{\text{h}}, s_{\text{g}})$ is updated with all measures that have been prioritized in previous iterations.
Step 1a: In the first part of the second step $BC(n^*, s_{\text{h}}^*, s_{\text{g}}^*, C_{\text{int}}(N, S_{\text{h}}, S_{\text{g}}))$ is computed for all individual measures, based on the existing situation $C_{\text{int}}(n, s_{\text{h}}, s_{\text{g}})$. Due to the formulation of system reliability for geotechnical failure modes (see Eq. (2)) this approach works well for determining optimal priority orders for segments dominated by these failure modes. The reason is that each dike section contributes to the overall risk, so improving any individual dike section will have a direct influence on the total risk.
Step 1b: While considering individual sections is adequate for geotechnical measures, overtopping system reliability is governed by the weakest section (see Eq. (1)). Therefore we introduce a second heuristic where we compute the BC-ratio for a bundle of measures aimed at reducing $P_{\text{f,over}}(n, s_{\text{h}}, t)$. By only considering individual dike sections as in Step 1a, situations can occur where improving overtopping reliability at a single section has a low BC-ratio. Considering a bundle of different improvements at different sections can have a much higher BC-ratio as all weak sections are improved simultaneously. This can result in a much
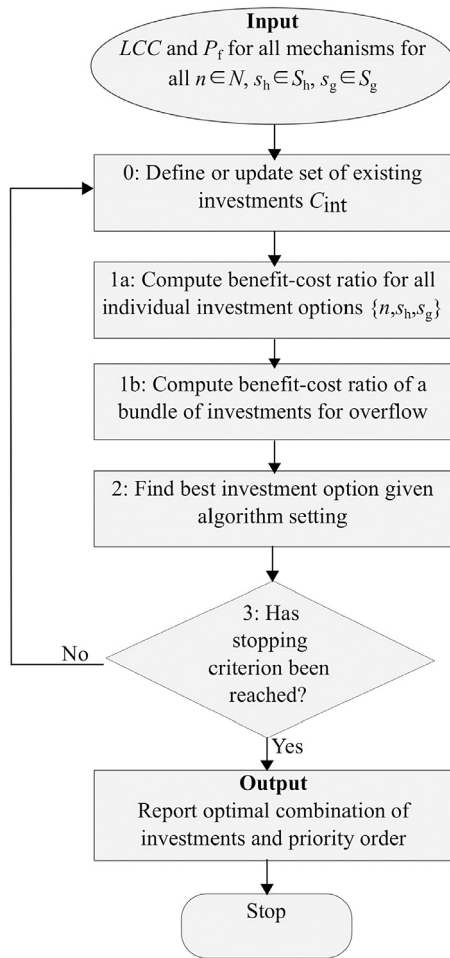
Fig. 3. Outline of the steps taken in the greedy search algorithm.

larger marginal increase in system reliability, and thus a larger *BC*-ratio. This step consists of the following substeps:

1. Sort the investment options based on $LCC(n, s_h, s_g)$.
2. Filter the options such that only options $s_h \in S_h$ combined with the existing investment option s_g at section *n* are considered.
3. Determine a priority order of measures, where each time the weakest section is improved with the smallest investment step available from $s_h \in S_h$.
4. Take the set of investment options with the highest *BC*-ratio.

Step 2 is to find the optimal investment option based on the *BC*-ratios obtained from steps 1a and 1b. If the *BC*-ratio from step 1b is higher than the maximum *BC*-ratio from step 1a, the bundle of investments obtained from step 1b is implemented. Otherwise the algorithm determines the next investment step based on a greediness factor f_c, where it holds that the investment step has to have a *BC*-ratio that is f_c larger than the best measure at all other dike sections. This is a factor to set the *greediness* of the algorithm, with a larger factor implying larger steps, but also a less cautious and more error-prone search routine. Additionally, multiple runs with different settings for f_c might yield different solutions, meaning that the overall accuracy of the approach increases as the best of those different solutions can be chosen.
Step 3: if the *BC*-ratio of the best available investment option is smaller than the stopping criterion (default setting: *BC*-ratio <

0.1) the optimization is stopped. Note that in most cases a stopping criterion *BC*-ratio < 1 should yield the optimal solution, but due to the properties of a greedy search in some cases a measure with *BC*-ratio < 1 is followed by one with a *BC*-ratio > 1 (due to the dependence of *BC*-ratio on preceding measures). After reaching the stopping criterion the optimal solution is obtained based on the minimum total cost of all steps in the search path, or the point where the solution meets the target reliability requirement in the years for which this is required. Note that the latter could also be used as a stopping criterion.
Output: the output of the greedy search is a sequence of arrays $C_{int}(n, s_h, s_g)$ which describe the obtained optimal solution as well as the priority order of investments leading to that solution. This is an advantage compared to for instance a MIP solution, where only the array $C_{int}(n, s_h, s_g)$ for the optimum is obtained.

4. Performance of the greedy search algorithm

The aim of this section is to validate the greedy search algorithm introduced in Section 3.3. We compare the obtained results with those of a method guaranteed to find the global optimum (in this case the Mixed Integer Programming implementation of the problem described in Section 3.1). Section 4.1 presents a general discussion on input data and the approach taken for verifying the algorithm performance. Section 4.2 presents the results of the validation. More detailed information on input data can be found in Appendix A.

4.1. Input data and approach

As test data we use data from 73 dike sections at the river Lek in the Netherlands (including the sections from the case study in Section 5). By randomly selecting subsets of dike sections we can generate many different realistic segment configurations with different numbers of sections. For each section we have information on current reliability, the reliability after taking different measures, the cost of measures and the damage in case of flooding. Note that for the validation of the search routine we do not include Eq. (11) as a constraint, as this is merely an optional additional stopping criterion.

We consider the reliability for overtopping, piping erosion and inner slope instability failures. Reliability estimates were obtained by back calculating implicated reliability indices using the semi-probabilistic assessment rules in the applicable statutory safety assessment tools [32]. It has to be noted that the approach can as easily be used with any failure model as long as it provides a probability of failure for a dike section. Note that it holds that a reliability index $\beta \approx -\Phi^{-1}(P_f)$. To properly assess reliability over time we include relevant temporal changes that impact different failure mechanisms. Higher outside water levels reduce reliability for all failure modes; increases in wave run-up due to higher wind speeds, as well as settlement of the crest reduce overtopping reliability; and settlement of the hinterland results in increased hydraulic heads which reduces piping erosion reliability. Reliability in time (see right pane of Fig. 1) was derived for each of the 73 sections based on local data (see Appendix A for formulations).

In the validation we want to consider the influence that including different types of measures has on the performance of the algorithm. Therefore we consider different sets of available measures as shown in Table 2. Set 1 is a set of all available options for investment years 2025 and 2045. In sets 2 through 5 different measures are excluded. Set 6 only considers investments in 2025. The costs are obtained from standard cost functions [44], except for soil based reinforcement. For soil based reinforcement we consider starting costs, variable costs based on the volume of added soil, and costs dependent on the number of adjacent properties to be removed for each section, which has a large impact on reinforcement costs. For all computations the economic consequences of flooding are assumed to be 5 billion €, the annual

Table 2
Different combinations of sets of measures considered. x indicates the measure is included. Set 6 considers all measures at t = 0, other sets also consider measures at t = 20. Extent and type refer to the classification in Fig. 2.

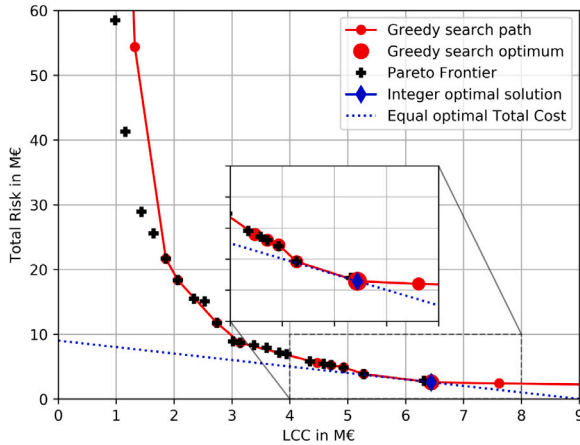| Measure	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Extent	Type	Mechanisms impacted
Diaphragm Wall	x	x	x			2025	Full	Renewal	All
Soil based reinforcement	x	x	x	x	x	2025	Full/Partial	Renovation	All
Stability screen	x		x	x		2025	Partial	Renewal	Inner slope instability
Vertical Sandtight Geotextile	x	x		x		2025	Partial	Renewal	Piping erosion

Fig. 4. Example result of a system with 5 dike sections. Red line shows the path of the greedy search, where the large dot denotes the optimal solution. Blue diamond indicates the optimum found using branch-and-cut in CPLEX 12.9. The black pluses denote the Pareto Frontier derived from several branch-and-cut computations with a budget limit. Note that here the TR and TLCC are displayed as the conflicting objectives, whereas in the optimization routine these are summed and considered as a single objective.

discount rate is assumed to be 3%. More detailed formulations for cost and reliability computations can be found in Appendix A.

The combination of different system configurations and sets of measures gives us a large variety of realistic cases for which we can assess the performance of the greedy search algorithm by comparing with a MIP implementation in CPLEX 12.9.

4.2. Validation of the greedy algorithm

A typical run of the greedy search algorithm yields a stepwise prioritization of dike reinforcement measures that eventually ends at or very close to the global optimal solution, which consists of the minimum sum of $TLCC$ and TR. Fig. 4 shows results for a system with 5 dike sections. We see that the greedy search (red) reaches the global optimum, and follows the Pareto front $TLCC$ and TR (black) computed using the MIP implementation with variable budget limits (i.e., where $TLCC$ is constrained). This shows that, especially closer to the optimal solution (blue diamond) the investment path of the greedy search not only finds the optimal solution, but the intermediate steps are also (near-)optimal for that budget.

Next we consider a large set of system configurations from our dataset and combine these with different sets of measures as defined in Table 2. We randomly sample configurations of N sections. The largest system size considered is 11 sections, which is smaller than typically encountered in practice but the largest practically feasible with the available 16 GB RAM. We consider regular cases (i.e. directly sampled from our dataset) that are typically dominated by failures due to inner slope instability and piping erosion. We also consider system configurations that are dominated by overflow failures. Here crest levels of sections were modified such that the initial overflow reliability index ranges between 2.8 and 3.5, making it the dominant failure mechanism.

Results are shown in Table 3, where f_c of the greedy search is set to 1.5. Overall we see that in about 93% of all cases the greedy search

finds the global optimum. If the outcome differs from the global optimal solution, only in 1% of the cases the difference in TC is higher than 1%, and only in 5% of the cases the difference is larger than 0.18%. On average the difference is 0.04% which is negligible compared to the often large uncertainties in dike reinforcement projects. The performance for regular cases and cases dominated by overflow failures is very similar.

It is found that differences in Life Cycle Cost are larger. This is explained by the fact that many cases with large differences in LCC, are cases that are often very close to the global optimum. Overall the differences are small: only 5% of the cases has a difference larger than 0.71%. On average differences are only 0.29%. In most cases the differences arise from cases with different investment cost but close to optimal values for the objective of minimizing TC. Practically this means that even though the solution is not exactly optimal, there are many different combinations of investments that are close to optimal, even though the investment costs are different. This has the practical advantage that it allows policy makers to choose from various near optimal solutions. Also it has to be noted that small choices in for instance the schematization of different failure modes, or uncertainty in for instance sea level rise rates have a potentially much larger effect on TC [45].

Another observation from Table 3 is that the percentage of runs where the global optimum is found decreases slightly for larger systems, which is sensible from the perspective that there are many more different investment paths that can be taken. However, when looking at the investment costs, the deviation does not increase significantly, which is confirmed by the results in Fig. 5. The left pane shows that for both TC and LCC there is a clear decreasing trend of the average relative error for larger values of TC and LCC. This is also represented in a slightly different way in the scatter plots in the right panes. Thus we can conclude that for larger dike segments consisting of many sections that are relatively expensive to improve, the algorithm can be expected to provide accurate results, even though the exact global optimal solution might not be found.

As was explained in Section 3.3 the factor f_c can be used to vary the step size taken by the algorithm. The same cases have been evaluated using f_c equal to 1.0, 1.5 and 3 respectively. Table 4 shows the results for TC for all cases with different settings. Aside from the individual evaluations we also use a combination of the three settings where each time the greedy solution with the lowest TC is used. It turns out that the different settings all perform quite well, but $f_c = 1.5$ results in the highest accuracy, even to the extent that the performance metrics for the combined case are the same as for $f_c = 1.5$. It turns out that in some cases $f = 1.0$ or $f = 3.0$ yield a better result than $f = 1.5$, but these have no bearing on the overall performance (typically because the inaccuracy in these cases is already very small).

One of the advantages of the greedy search algorithm is that it does not suffer as much from state space explosion as the MIP approach. Typically for the cases considered we observe that the runtime doubles for each two sections added. At the hardware that was used (16 GB of RAM), the largest system that could be solved with MIP contained 15 dike sections (with the most extensive measure set). Furthermore it should be noted that aside from the time required to solve the MIP problem, the initialization of the problem (i.e., defining al the constraints, especially Eq. (10)) also costs significantly more time using MIP than with the greedy algorithm. Overall, initializing and solving a

Table 3

Results for 2800 different system configurations with different measure sets and system sizes. Systems derived from the case study in Section 5 ('Regular cases') are distinguished from systems that are dominated by failures due to overflow. Δ denotes the relative difference between the two methods. Δ^* denotes a threshold value exceeded by only 5% of all cases. For both Total Cost and LCC three indicators are used: the % of cases with a Δ larger than 1%, the Δ^* only exceeded by 5% of the cases and the average difference Δ.

|	Case properties				Total cost			LCC		
	Measure set	No. of sections	Total runs	Runs where global optimum was found	$\Delta > 1\%$	Δ^* for $P(\Delta > \Delta^*) < 0.05$	Average Δ	$\Delta > 1\%$	Δ^* for $P(\Delta > \Delta^*) < 0.05$	Average Δ
Regular cases	Set 1	mixed	400	91.0%	2.25%	0.37%	0.07%	6.0%	2.2%	0.37%
	Set 2	mixed	400	93.3%	1.50%	0.17%	0.05%	6.0%	2.2%	0.35%
	Set 3	mixed	400	92.0%	0.50%	0.15%	0.03%	3.8%	0.4%	0.22%
	Set 4	mixed	400	93.0%	0.75%	0.16%	0.03%	5.5%	2.2%	0.60%
	Set 5	mixed	400	96.5%	0%	0%	0.01%	2.5%	0%	0.06%
	Set 6	mixed	400	93.3%	1.00%	0.37%	0.04%	3.0%	0.6%	0.23%
	Size 5	5	600	97.8%	0.17%	0%	0.02%	1.7%	0%	0.21%
	Size 7	7	600	95.8%	1.00%	0%	0.03%	3.3%	0%	0.22%
	Size 9	9	600	91.0%	1.33%	0.26%	0.05%	6.3%	2.2%	0.46%
	Size 11	11	600	88.0%	1.50%	0.37%	0.05%	6.5%	2.1%	0.33%
	All	mixed	2400	93.2%	1.00%	0.18%	0.04%	4.5%	0.7%	0.30%
Overflow dominant cases	Set2	5	100	99.0%	0%	0%	0.01%	1.0%	0%	0.06%
	Set2	7	100	98.0%	0%	0%	0.01%	1.0%	0%	0.05%
	Set2	9	100	94.0%	2.00%	0.13%	0.07%	4.0%	0.40%	0.34%
	Set2	11	100	86.0%	3.00%	0.39%	0.11%	11.0%	4.66%	0.49%
	All	mixed	400	94.3%	1.25%	0.19%	0.05%	4.3%	0.55%	0.24%
Total	All	mixed	2800	93.3%	1.04%	0.18%	0.04%	4.4%	0.71%	0.29%
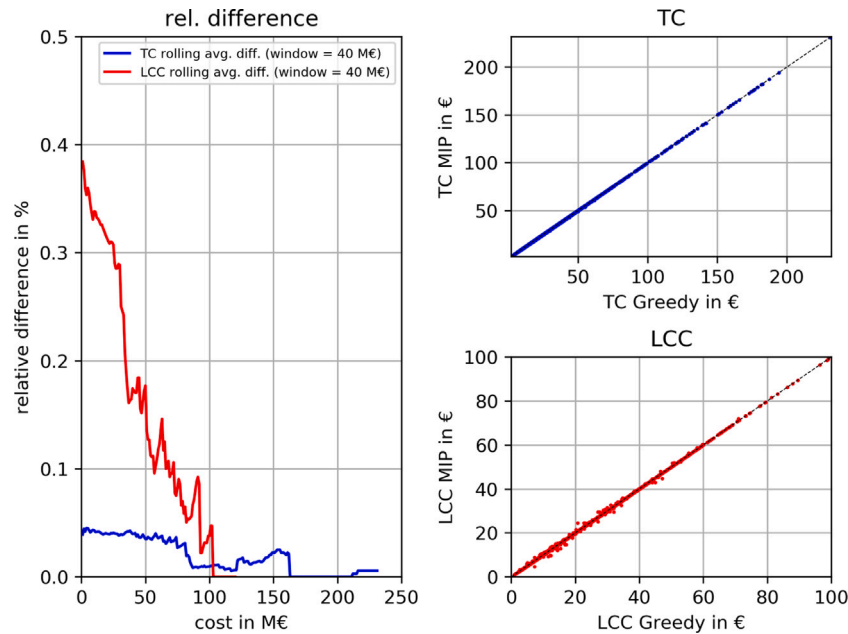
Fig. 5. Left: Relative error for Total Cost and Life Cycle Cost compared to MIP optimum. Red and blue lines denote a moving average relative difference for all evaluated cases based on a window for absolute TC or LCC of 40 M€. right: comparison of TC and LCC for MIP and Greedy computations. It can be observed that for the LCC values the scatter is slightly larger, especially for small absolute values. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

system of 15 dike sections using MIP took approximately 600 s, whereas the greedy search routine can find a solution in approximately 6 s. With more extensive hardware the computable system size could be extended, but not easily to the size of practical problems that often consist of over 30 sections. As will be shown in the next section solving such a system with the greedy search approach is easily achievable.

5. Case study application

This section presents an application of the greedy search algorithm to the planning of a reinforcement project for a dike segment along the Lek river in the Netherlands. The main aim of this section is to illustrate the practical applicability of the developed approach, and show the main advantages of using an optimization algorithm in the process of planning and design compared to the commonly used design approach based on cross-sectional target reliability.

5.1. Case description

We consider dike segment 16-4 along the Lek river in the Netherlands, located between the towns of Everdingen and Ameide. The length of this segment is approximately 20 km, and it consists of 45 sections of which 41 are considered in the calculations. The three main failure modes are overtopping, piping erosion and inner slope instability, for which calculations have been made using the statutory safety assessment tools [32,46]. If a flood occurs along this segment the estimated damage is 23 billion € [47]. Since the introduction of the new safety standards in 2017 the segment failure probability has to be less than 1/10,000 per year ($\beta \approx 3.72$). Due to the new standards (amongst other reasons) many flood defence reinforcement projects are initiated, and the improvement of dike segment 16–4 is one of them. The goal is to meet the safety standards nationwide by 2050. In this

Table 4

Comparison of results for three different settings of f_c as well as the best combination of the three. Indicators are the same as in Table 3.

	Case type			
	Setting	Normal	Overflow dominant	All cases
Runs where global optimum was found	*f = 1.0*	90.63%	92.50%	90.89%
	f = 1.5	93.17%	94.25%	93.32%
	f = 3.0	90.04%	92.75%	90.93%
	combined	93.17%	94.25%	93.32%
Cases with $\Delta > 1\%$	*f = 1.0*	2.75%	3.00%	2.79%
	f = 1.5	1.00%	1.25%	1.04%
	f = 3.0	3.13%	2.75%	2.79%
	combined	1.00%	1.25%	1.04%
Δ^* for $P(\Delta > \Delta^*) < 0.05$	*f = 1.0*	11.76%	7.40%	11.76%
	f = 1.5	5.53%	3.62%	5.53%
	f = 3.0	11.76%	7.40%	11.76%
	combined	5.53%	3.62%	5.53%
Avg. difference	*f = 1.0*	0.10%	0.12%	0.10%
	f = 1.5	0.04%	0.05%	0.04%
	f = 3.0	0.11%	0.11%	0.10%
	combined	0.04%	0.05%	0.04%
Total runs		2400	400	2800

Table 5

Values used for determining the cross-sectional target reliability.

Failure mode	a_m	b_m	ω_m
Overflow	1	1	0.24
Inner slope stability	0.033	50	0.04
Piping erosion	0.9	300	0.24

study we aim to find an optimal strategy to achieve this for segment 16–4, such that the safety standard is met in 2050 and until at least 2075. This means that Eq. (11) is now also used, with $P_{req} = 1/10{,}000$, $t_{req} = 2050$ and $t_{horizon} = 2075$. We assume that the reinforcement can start in 2025.

The commonly used design approach is a reliability-based design approach using cross-sectional reliability requirements for each failure mode. The main principle is that the total failure probability for the segment is translated to requirements for cross sections that each represent an independent dike section. If the requirements are met for each cross section, the overall target is also met. The requirement for an independent section for failure mode m ($P_{T,m,cs}$) is given by:

$$P_{T,m,cs} = \frac{\omega_m \cdot a_m \cdot P_{T,segment}}{b_m}, \qquad (15)$$

where $P_{T,segment}$ is the maximum failure probability for the segment, ω_m is the fraction of the total failure probability that is allocated for failure mode m, a_m and b_m are length effect factors. a_m represents the fraction of the dike segment that is sensitive to failure mode m, and b_m represents the length of an independent equivalent section in metres. For more details see [32]. Values for the modes considered are presented in Table 5, these are default values (see [32]). The basic assumption is that these requirements have to be fulfilled for a period of 50 years, and that after improvement all cross-sectional requirements have to be satisfied (i.e., all reinforcement works are done in 2025). Note that values for ω_m do not add up to 1 as there are other mechanisms that are also considered in the reinforcement project but not in the analysis in this paper.

In the case study we consider a specific set of measures as defined by the local water authority. We consider all measures in Set 6 in Table 2, and additionally we consider the option of soil based reinforcement at $t = 20$ (2045). Other assumptions are described in Section 4.1 and Appendix A.

5.2. Case study results

In order to assess the reliability deficit for the segment under consideration we first compute the current and predicted reliability

without taking any measures, which is displayed in pane (a) of Fig. 6 for the year 2075. Here we see that for most sections either piping erosion or inner slope instability has a reliability deficit, and that overall (bar chart on the right) the system reliability (black line) $\beta \approx 1$, which is extremely low, and much lower than the required value. Another thing that quite clearly emerges from the figure is that neighbouring sections often have similar issues: sections 34b-40 have a large deficit for inner slope instability, whereas sections 16–22 have a large deficit for piping erosion.

The reliability information from pane (a) of Fig. 6, together with information on different available measures, their effects on reliability and their respective costs can be used to generate design alternatives for the entire segment, both with a greedy search optimization and a reliability-based design approach based on cross-sectional requirements.

Panes (b) and (c) of Fig. 6 show the selected measures and resulting reliability in 2075, for an investment based on the greedy search algorithm. Panes (d) and (e) show the same for a target reliability based investment. Pane (f) shows the life cycle costs of both approaches. There are a few distinct differences between the two methods. First of all, the target reliability based investment results in a higher system reliability due to some conservatism in the cross-sectional target reliability values. Also, as for the target reliability based approach each section has to satisfy a target reliability for each mode, there are many sections where expensive diaphragm walls are needed to meet the requirements. When using the greedy search algorithm such investments are avoided by increasing reliability at other dike sections, or by using partial renewal measures (i.e., a Vertical Sandtight Geotextile (VSG, inversed triangle) or stability screen (SS, circle), see Appendix A for specifications). This is not feasible for the target reliability based investment as it becomes impossible or extremely expensive to meet requirements for other failure modes. For instance: dike section 38 is improved using a diaphragm wall when using the target reliability based approach at a cost of ≈ 17 M€. In order to prevent these large expenses in the optimized approach only a much cheaper stability screen (cost ≈ 3.5 M€) is constructed, resulting in lower reliability, but also only a fraction of the cost. This is then compensated by using more extensive measures at other dike sections (e.g., section 34a), where the benefit–cost ratio of extra investments is larger. This is a degree-of-freedom ignored by the cross-sectional target reliability approach. When looking at resulting system failure probabilities there are two main differences between the two methods:

- For the optimized approach the resulting system reliability for overtopping is highest, whereas for a target reliability based approach it is lowest. There are clearly more investments in crest
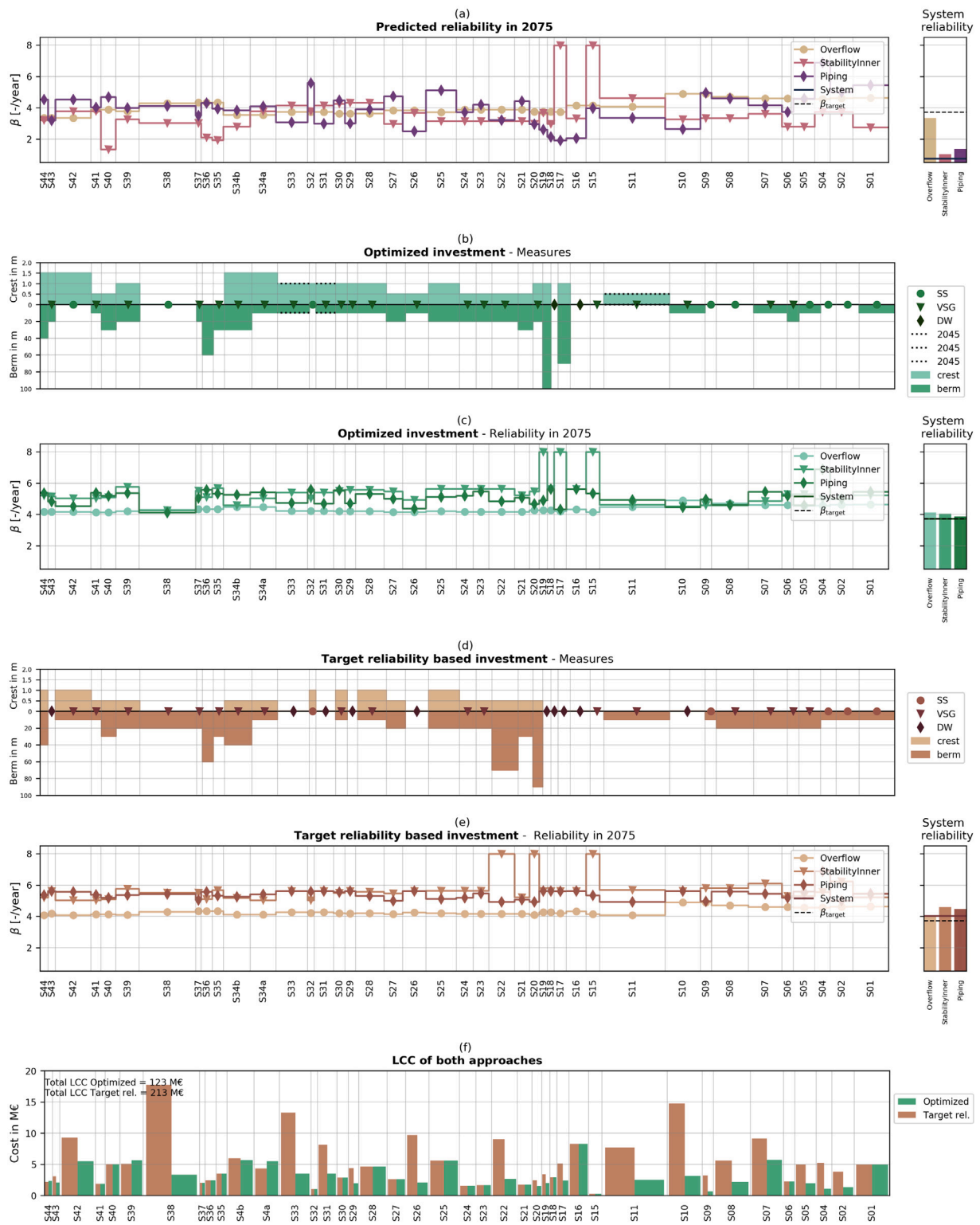
Fig. 6. Input and results for both planning approaches. Pane a shows the reliability in 2075 without investments, panes b and d show measures taken using the greedy optimization and target reliability approaches; c and e show resulting reliability in 2075. Pane f shows the resulting life-cycle cost of both approaches. Vertical grid lines indicate boundaries between sections, and the width of sections indicates their relative length (total length of the segment is 20 km).

height for the optimized approach (see Fig. 6b). Examples are sections S34a, S34b, where higher crest increases are planned.

- The overall system reliability in 2075 is higher for the target reliability based investment, most notably due to the fact that the reliability for inner slope stability and piping erosion is higher.

The main reason is that for many sections a diaphragm wall or Vertical Sandtight Geotextile is applied. These measures have a reliability that is much higher than the required reliability, yet there is no cheaper alternative that also meets the requirement. This explains the higher system reliability for these failure modes.
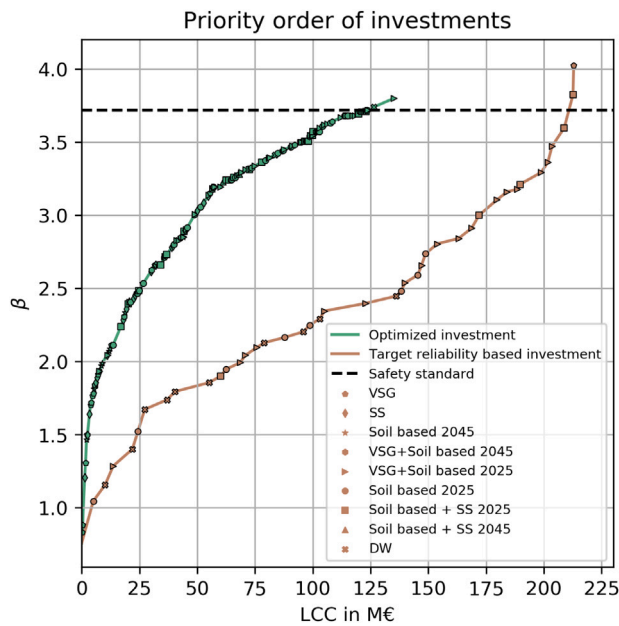
Fig. 7. Priority order for optimized investments (green) and investments based on target reliability (brown). Markers denote different types of measures at different dike sections, β is the reliability index in 2075. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 6.f shows the LCC for both approaches. Here we observe that for the target reliability based approach some sections have very high costs (e.g., 10, 33 and 38), whereas the costs for the optimized approach are much more evenly spread across the different sections. The difference in total LCC is very large: using the greedy search optimization as a basis for planning reduces the total investment from 213 M€ to 123 M€, a reduction of about 42%.

One of the major advantages of the greedy search approach that was mentioned is that it also yields a priority order of measures based on the search path. This order can give decision makers insight into the priorities for improving a dike segment, and can help making a risk-informed selection of parts of the project, if not all budget is readily available. The priority order is given in Fig. 7, which shows the LCC relative to the reliability index β in the year 2075. The green line denotes the search path for the optimized investments, whereas the brown line denotes the investments based on target reliability, ordered by the initial reliability of the sections (so the weakest is displayed first). For the greedy search path it can be observed that many small increments are taken, and that especially in the beginning the line is very steep, meaning that large improvements in reliability are achievable for a limited amount of money, but as the overall reliability increases, the marginal risk reduction for additional investments decreases.

One particularly important challenge in flood defence reinforcement projects is to properly deal with long term uncertainty in for instance economic growth and increase in hydraulic loads. Fig. 8 shows the resulting measures for a case where the increase in water level has been multiplied by a factor 3 for all sections. By comparing with pane (b) of Fig. 6 we can observe that investments in crest height increase are larger, in order to cope with the higher water levels. However, the influence on both the priority order and the investments in geotechnical measures is very similar, which demonstrates that for this case study the added value of including multiple scenarios for hydraulic load increase is limited, both in terms of the prioritized measures as well as the potential for future extension.

6. Discussion

In this study we demonstrate how the cost effectiveness of dike reinforcements can be improved using a system optimization. We use

a greedy search algorithm with heuristics based on system reliability rules and benefit–cost ratio of measures that yields near-optimal plans for reinforcement of dike segments, at a much lower cost than the commonly applied approach based on cross-sectional target reliability. From a comparison with a Mixed Integer Programming approach it is shown that in the majority of cases the algorithm finds the optimal solution, and in the other cases it is close (i.e., very small differences in Total Cost). Even for the systems where the algorithm performs relatively bad (difference in Total Cost > 1%), this inaccuracy is minor in comparison to other major uncertainties in the design, such as estimates of improvement cost and geotechnical strength parameters.

It has to be noted that in this study we only consider dike sections as part of the segment, but in practice there can also be hydraulic structures (e.g. inlet sluices) that are part of the flood defence. As typically the reliability of these structures can be computed, these can also be included in the analysis. Thus, the approach is not solely applicable to dikes, but to flood defences in general.

The method can also be used with more advanced methods such as probabilistic reliability calculations and more advanced methods for computing system reliability including correlation between sections and mechanisms, such as the Equivalent Planes method [33]. In our schematization of the dike system we use semi-probabilistic estimates for reliability and relatively simple approaches to model the correlation between different dike sections. As our case study concerns the early planning phase of a reinforcement project it is not yet sensible to use more advanced methods due to uncertainties in for instance geotechnical strength parameters. The method is useable with more advanced computation methods, the only requirement is that for each failure mode per section the reliability (in time) can be computed, and that this can be translated to a system reliability estimate. Of course, when changing major underlying assumptions a re-evaluation of the accuracy of the greedy algorithm advisable.

In the considered case study we do not explicitly deal with the wide variety of potential future scenarios for increasing hydraulic load or socio-economic conditions as has been done in other studies (e.g., [17, 18,48]). For the case study it is shown that, mainly due to the dominance of geotechnical failure modes, different scenarios for hydraulic loads have little bearing on the priority order and type of investments. However, this might not always be the case, and in such cases including these future uncertainties is recommended. This can be done either by using probabilistic estimates of future uncertainty [e.g. 49], or by considering multiple scenarios for which the different dike segment designs can be evaluated (in line with e.g. Kwakkel et al. [18]). The latter is quite feasible as the current computation time is still relatively limited (approximately 10 min for 1 evaluation of the case study). It has to be noted however, that such an extension and the subsequent increase in computation time makes the approach less useable in the design process, where it is often desired to have a practical tool that can be used to quickly evaluate various design considerations.

In our approach we also assumed that the reinforcement costs for different dike sections are independent. In reality this is not necessarily the case: especially if a large project is cut up in different fragmented small projects overhead costs may rise, resulting in higher overall costs. In our case, as there are only 2 moments of investment this is not relevant, but it might be for other cases. Correlations between the cost of different otherwise independent sections cannot yet be dealt with in the algorithm. It has to be noted that such correlations are not common practice in cost estimates for dike improvements [44].

The approach has clear benefits compared to the commonly applied reliability based design approach using cross sectional requirements: the overall life-cycle costs are about 42% lower when using an optimized design based on the greedy search algorithm, while the reliability requirement is met in both cases. It has to be noted that in practice the target reliability based approach is also refined throughout a reinforcement project, for instance by slightly altering the different cross-sectional requirements between mechanisms or sections. Then
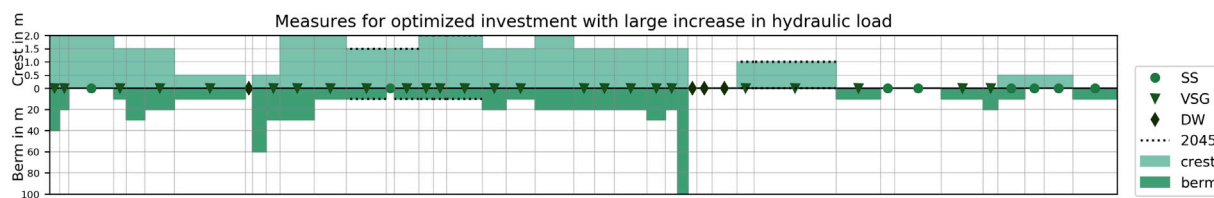
Fig. 8. Measures for optimized investment with 3 times higher water level increase rates.

the difference would become slightly smaller, albeit in an ad hoc manner. It has to be noted that the cost savings depend strongly on the accuracy of the input, specifically the reliability estimates. For this and other methods for planning flood defence reinforcements it is therefore advised to ensure that trustworthy reliability estimates are available.

An added advantage of the optimized approach is that it helps focusing attention on the reinforcement of the most important parts of a large and complex dike segment. This structures the technical challenge which aids in adding risk-informed information to the multi-objective task of improving dike systems, where also other aspects of spatial planning have to be dealt with. For instance, in our case study (Section 5) we selected measures such that in both moments of investment considered (now and in 20 years), only one type of investment with a major impact could be done in order to limit nuisance to inhabitants. Other secondary objectives could also be included in the choice of measures, such that appropriate risk information is obtained, also for other stakeholders with different objectives. The approach facilitates such considerations, first of all by enabling quick evaluation of the influence of such restrictions on the overall solution, and secondly by providing insight into the importance of different measures through analysis of the priority order of investments.

While in this study the approach was applied to an investment decision for a dike segment, it is also useable for other types of decisions, such as optimizing investments over multiple dike segments. For instance, in the Netherlands about 1500 km of flood defences have to be improved, resulting in (currently) a list of over 50 projects. The greedy search algorithm could also be used to find a total cost optimal prioritization that balances national flood risk and reinforcement costs.

Compared to other optimization approaches such as Mixed Integer Programming, the greedy search routine encounters less issues with state space explosion, making it more suitable for systems with many components. Of course one could apply for instance a Mixed Integer Programming approach, whilst expanding the available hardware. However, the dimensions of especially the constraint in Eq. (10) increase almost quadratically with the number of sections (and solutions), so this would still be challenging for segments of over 30 independent sections. An added advantage of the greedy search routine is that it is very explainable, also to stakeholders with different expertise. This is an advantage compared to Mixed Integer Programming, and for instance genetic algorithms.

7. Conclusions

In this paper we have proposed a design approach to optimize reinforcements of a dike system. To make this computationally possible we used a greedy search algorithm, for which we derived heuristic rules that can be used for planning dike reinforcement projects in flood defence systems with a large number of independent elements. It was demonstrated that for a real world dike system the approach results in a 42% reduction of investment costs compared to the method that is typically applied in the same phase of a reinforcement project. An additional advantage is that a priority order of measures is determined, which is useful in making and explaining risk-informed decisions during the planning and design of dike reinforcement projects.

The greedy search algorithm employs two main heuristic rules: use of the benefit–cost ratio to select the local optimal investment, and relations between system and component reliability to translate investments at a dike section to risk reduction for the system as a whole. The approach is very useful in dike reinforcement projects as it offers good accuracy compared to a Mixed Integer Programming implementation, while it hardly suffers from state space explosion. From an analysis of 2800 different realistic dike segments the average difference in objective value was only 0.04%, which is negligible compared to other uncertainties in dike reinforcement projects.

The case study in this paper concerned an analysis in the early planning phase of a reinforcement project. Future developments could focus on validating and using the method in later design stages where typically more advanced reliability models are used. In principle the method is already suited for such an extension. Another interesting direction for future development is to use the approach for investments at multiple large dike segments such that national investments in flood protection can be optimized by balancing national flood risk and investment costs.

CRediT authorship contribution statement

Wouter Jan Klerk: Conceptualization, Methodology, Analysis, Software, Validation, Writing - original draft, Writing - review & editing. **Wim Kanning:** Conceptualization, Supervision, Writing - original draft, Writing - review & editing. **Matthijs Kok:** Project administration , Funding acquisition, Conceptualization, Supervision, Writing - review & editing. **Rogier Wolfert:** Conceptualization, Supervision, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ress.2020.107344.

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