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Separation of Blended Data by Sparse Inversion Utilizing Surface-related Multiples

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SUMMARY

Blended surveys have recently appeared in production environments. This underlines the need for processing tools that will either process the recorded data directly or perform the separation into single source data (deblending). An inversion technique for the separation of such data is described here. The problem parameterization utilises the surface-related multiples in order to regularise the inversion. In this way, the separation and surface-related multiple elimination are performed in one step. Also, the physical meaning of the model space is exploited during the inversion by formulating the problem as a Basis Pursuit Denoise problem. The method has been applied on a synthetic dataset and it produced promising results.

Introduction

Methods that allow more than one source to fire simultaneously have been deployed for long time in land surveys. However, new success stories of great production increase due to blending, see Howe *et al.* (2009), Pecholcs *et al.* (2010) have captured the attention of the industry. In the marine case, the concept of blending is not so widespread. Although the idea was proposed years ago, see Beasley *et al.* (1998), no blended production surveys have been reported to date.

A major decision to be taken regarding processing of blended data is whether they will be processed directly or first separated and then processed. In the first case, all processing tools need to be redesigned in order to handle blended data. Such attempts aimed e.g. at developing specialized least-squares migration schemes, see Verschuur and Berkhout (2011). In the second case, an algorithm that will be able to 'deblend" the data is required, i.e., separate the blended records into shot records that contain the response of only one source. Then, a conventional processing flow can be applied to the data.

Deblending can be formulated as an inversion problem that estimates the unknown unblended data. Since this is an ill-posed problem, a regularization term is required. Akerberg *et al.* (2008) and Moore *et al.* (2008) use a sparsity constraint in the Radon domain in order to regularize the inversion. Similarly, Lin and Herrmann (2009) use an inversion approach that constrains the separated data to be sparse in the curvelet domain. Mahdad *et al.* (2011) and Abma *et al.* (2010) exploit the coherency of seismic data in order to constrain the inversion. van Groenestijn and Verschuur (2011) describe a sparse inversion process in which surface multiples provide unique information to the deblending process. However, their parameterization, being spikes in the space-time domain, was not adequate enough to provide noise-free deblending results. Also in the current paper, a sparse inversion approach is deployed to perform the separation. In contrast to the aforementioned inversion methods, sparsity is here promoted on an estimate of the subsurface reflectivity. This parameterization is also used in Kutscha and Verschuur (2010) for interpolation purposes. In this way, the model domain obtains a physical meaning. This proves to be crucial when it comes to extending this inversion technique to include other preprocessing steps as well, e.g., integrating deblending and surface multiple elimination.

Method

The formulation of our inverse problem is based on the feedback model, see Berkhout (1982). The feedback model describes a mechanism for modelling seismic data with surface-related multiples using a systems' theory approach. An important advantage of such a model is abstraction, is the fact that complex ideas can be expressed in an easy and comprehensive way. Following Berkhout (1982), seismic data (2D or 3D) can be arranged in the so-called data matrix $\mathbf{P}(z_0, z_0)$, where the first depth index z_0 indicates the depth level of the detectors and the second the one of the sources. Formulated in the temporal frequency domain, each element of $\mathbf{P}(z_0, z_0)$ is a complex-valued frequency component of a recorded trace. The position of each element in the $\mathbf{P}(z_0, z_0)$ matrix reveals the spatial coordinates of the source and receiver. The key advantage of this arrangement relies on the fact that many complex operations, e.g., extrapolation, cross-correlation, etc., can be trivially performed as matrix multiplications.

Assuming that each point in the subsurface contributes to the reflected wavefield that reaches the surface one should define a grid of points in the subsurface. The local velocity and density can be used to define a reflectivity operator $\mathbf{R}^{\cup}(z_m, z_m)$ for a certain depth level z_m. This operator turns the incident wavefield into a reflected wavefield. The incident wavefield to depth level z_m can then be computed by extrapolating the wavefield at the surface towards that depth level by using a propagation operator $\mathbf{W}(z_m, z_0)$. Similarly, the reflected wavefield can be extrapolated towards the surface using operator $\mathbf{W}(z_0, z_m)$. The total wavefield at the surface can be expressed as

$$\mathbf{P}(z_0, z_0) = \sum_{m=1}^{M} [\mathbf{W}(z_0, z_m)\mathbf{R}^{\cup}(z_m, z_m)\mathbf{W}(z_m, z_0)](\mathbf{S}(z_0) + \mathbf{R}^{\cap}(z_0, z_0)\mathbf{P}(z_0, z_0)), \qquad (1)$$

where the summation takes place over the depth levels and $\mathbf{R}^{\cap}(z_0, z_0)$ denotes the surface reflectivity

operator. An interesting interpretation of eq. 1 is that the source wavefield that excites a free-surface earth consists of the initial source wavefield plus the up going wavefield that is reflected by the surface. Note that no internal multiples or surface waves are contained in this model. However, the significance of eq. (1) relies on the fact that it provides a reflectivity-oriented way to look at seismic data.

Source blending can be introduced to eq. (1) as multiplication with a blending matrix $\mathbf{\Gamma}(z_0)$. This matrix contains the blending parameters in the form of amplitude and phase terms applied to the source matrix $\mathbf{S}(z_0)$. In the current implementation we focus, without loss of generality, on blending matrices that contain only phase terms. For example, time delays in the firing of the sources can be expressed as linear phase terms of the form $e^{j\omega\tau}$, whereas linear sweeps can be expressed as quadratic phase terms of the form $e^{j\omega\beta^2}$. The feedback model for blended sources now becomes

$$\mathbf{P}'(z_0,z_0) = \sum_{m=1}^{M}[\mathbf{W}(z_0,z_m)\mathbf{R}^{\cup}(z_m,z_m)\mathbf{W}(z_m,z_0)](\mathbf{S}(z_0)\mathbf{\Gamma}(z_0) + \mathbf{R}^{\cap}(z_0,z_0)\mathbf{P}'(z_0,z_0)), \qquad (2)$$

where $\mathbf{P}'(z_0,z_0) = \mathbf{P}(z_0,z_0)\mathbf{\Gamma}(z_0)$ denotes the blended up going wavefield. This equation has dual meaning; firstly, it provides a way to model the total up going wave field for given subsurface reflectivity, and secondly, given a proper inversion scheme, the recorded up going wavefield can be used to recover reflectivity. However, the focus of this paper is not on retrieving the exact reflectivity operators. Instead, we wish to perform certain preprocessing steps: deblending and surface multiple elimination. For such tasks it is not desired to aim at recovering reflectivity properties on a dense grid, since this can be very computationally intensive, but rather do it for a few depth levels.

The term macro boundaries is introduced here to describe this move from recovering reflectivity coefficients on a dense grid to recovering it only for a few depth levels. Essentially, the subsurface is divided into parts in the depth direction. It is crucial that each of these parts contain some strategic boundaries, meaning boundaries that contribute the most to the reflected wavefield. It becomes obvious that the true reflectivity is not recovered in this way. Instead, the *effective reflectivity*, $\hat{\mathbf{R}}^{\cup}(z_k,z_k)$, of a *macro layer*, i.e., area around a macro boundary, is recovered. This can be seen as the total impulse response of that part of the subsurface, hence, it contains apart from reflectivity, propagation information as well. Moreover, the exact propagation operators $\mathbf{W}(z_k,z_0)$ can be replaced by approximate operators, $\hat{\mathbf{W}}(z_k,z_0)$. Eq. 2 now becomes

$$\mathbf{P}'(z_0,z_0) = \sum_{k=1}^{K}[\hat{\mathbf{W}}(z_0,z_k)\hat{\mathbf{R}}^{\cup}(z_k,z_k)\hat{\mathbf{W}}(z_k,z_k)](\mathbf{S}(z_0)\mathbf{\Gamma}(z_0) + \mathbf{R}^{\cap}(z_0,z_0)\mathbf{P}'(z_0,z_0)), \qquad (3)$$

where $K << M$. Interestingly, both $\mathbf{R}^{\cup}(z_m,z_m)$ and $\hat{\mathbf{R}}^{\cup}(z_k,z_k)$ matrices have few large values in the *x-t* domain. This property is used to regularize the inversion. Implicitly, the surface-multiples-generating mechanism also acts as a regularization of the inversion. Moreover, certain explicit constrains can be applied to the physical domain of $\hat{\mathbf{R}}^{\cup}(z_k,z_k)$ as in the case of the focal transform, see Berkhout and Verschuur (2006). The focal transform is identical to the one described here with the difference that no assumptions were made on the surface multiples.

An inversion step has to be performed in order to retrieve the effective reflectivity from observed data. In this inversion problem, the effective reflectivity $\hat{\mathbf{R}}^{\cup}$ is the unknown and the up going wavefield $\mathbf{P}(z_0,z_0)$ is the observation. Eq. 3 describes the *forward model* of the problem, i.e., the process that gives an estimate of the observation for given input $\hat{\mathbf{R}}^{\cup}(z_k,z_k)$. Eq. 3 can be written in standard linear algebra notation by transforming $\hat{\mathbf{R}}^{\cup}(z_k,z_k)$ and $\mathbf{P}(z_0,z_0)$ to the *x-t* domain and arranging them as vectors $\mathbf{x}$ and $\mathbf{b}$ respectively. Matrix $\mathbf{A}$ is a matrix implementation of the forward model of eq. (3), hence, $\mathbf{Ax} = \mathbf{b}$. The objective of the inversion is to find the solution to this system of linear equations. However, this problem is under-determined, meaning that many solutions satisfy the equations. In such cases, an iterative scheme is usually deployed where a priori information about the solution is used to

constrain the problem. Here, the solution sought is the one with the smallest $l1$-norm, i.e., the sum of the absolute values. This stems from our observation that a few large values dominate in the effective reflectivity matrix. Hence, the optimization problem solved becomes

$$\text{minimize } \|\mathbf{x}\|_1 \text{ subject to } \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2 \leq \sigma. \tag{4}$$

In the optimization community, this problem is often referred to as the *basis pursuit denoise* problem. In this problem we do not try to precisely fit the modelled data to the observation, but rather keep the residue within a certain limit σ. The SPGL1 solver of van den Berg and Friedlander (2008) was designed to solve this problem. Deploying the solver yields an estimate of the effective reflectivity. The deblended primaries $\mathbf{P}_0(z_0, z_0)$ can then be obtained in a reconstruction approach by applying the following forward model to the estimated $\hat{\mathbf{R}}^{\cup}(z_m, z_m)$:

$$\mathbf{P}_0(z_0, z_0) = \sum_{k=1}^{K} [\hat{\mathbf{W}}(z_0, z_k)\hat{\mathbf{R}}^{\cup}(z_0, z_0)\hat{\mathbf{W}}(z_k, z_0)]\mathbf{S}(z_0). \tag{5}$$

Please note that the same propagation operators, $\hat{\mathbf{W}}(z_0, z_k)$, that were used during the inversion, are used in the reconstruction as well. This means that errors in the operators affect only the accuracy of $\hat{\mathbf{R}}^{\cup}(z_0, z_0)$ but not the final result, in this case the deblended primaries. In this way, two preprocessing steps have been performed on the acquired blended data: deblending and surface multiple removal.

Example

The inversion scheme is applied here to a numerical dataset. The input data are produced by a layered model with three reflectors. Figure 1 shows an unblended shot record of this dataset. Five sources fire with small time delays for each blended shot record. Figure 2(a) shows a shot record of this blended dataset. Please note that no internal multiples were modelled, only surface-related multiples.

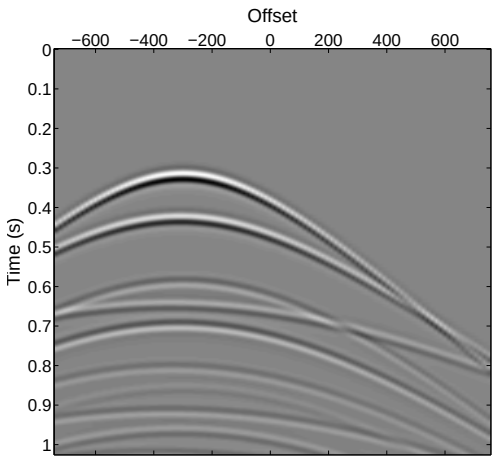


Figure 1 *Unblended shot record.*

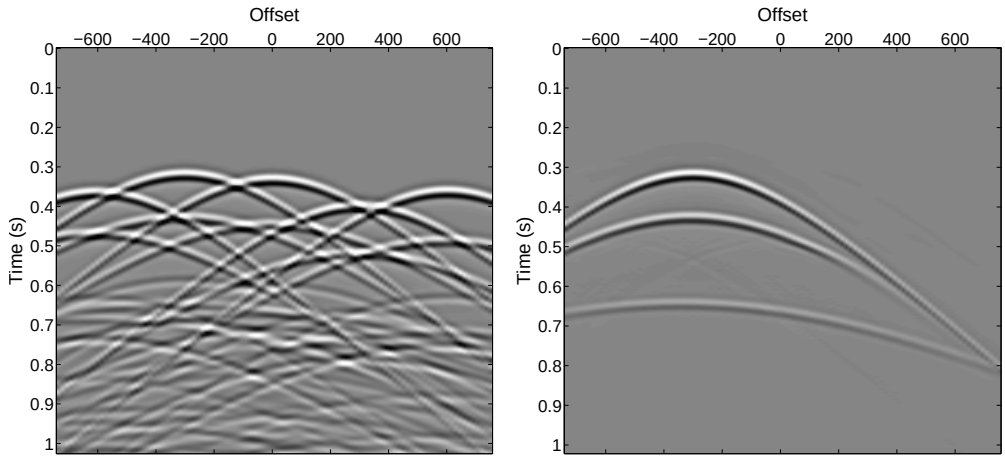
The first step for the estimation of the effective reflectivity is to define the macro layers. For this example, the earth is divided into two macro layers. The first macro layer contains the first two reflectors whereas the second layer contains the third reflector. Next, the surface reflectivity $\mathbf{R}^{\cap}(z_0, z_0)$ was taken -1. This is not very far from reality in marine environments. The SPGL1 solver converged after about 100 iterations. The last stage includes computing the reconstructed deblended primaries. The forward model described in eq. 5 is deployed and a shot record of the output is displayed in figure 2(b). The deblended primaries have been reconstructed successfully with only some small artefacts remaining in the record.

Conclusions

A sparse inversion approach to the separation of blended sources has been presented in this paper. The unique features of this method are that 1) it uses surface multiples as extra information to guide the deblending process and that 2) the sparsifying domain is physically meaningful. This means that a priori information can be introduced to help steering the inversion towards the correct solution. Furthermore, the deblended output is also free of surface multiples. The sparse inversion process was performed by the SPGL1 package, which was able to produce promising results for our simple numerical example.

Acknowledgements

The authors thank the DELPHI consortium sponsors for their support and thank the authors of SPARCO (van den Berg *et al.*, 2007) and SPGL1 (van den Berg and Friedlander, 2008) for providing their codes.



(a) (b)

Figure 2 *(a) Input: blended shot record, and (b) Output: deblended shot record with only primaries.*

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