

## **PRACTICAL DESIGN OF STEPPED COLUMNS**

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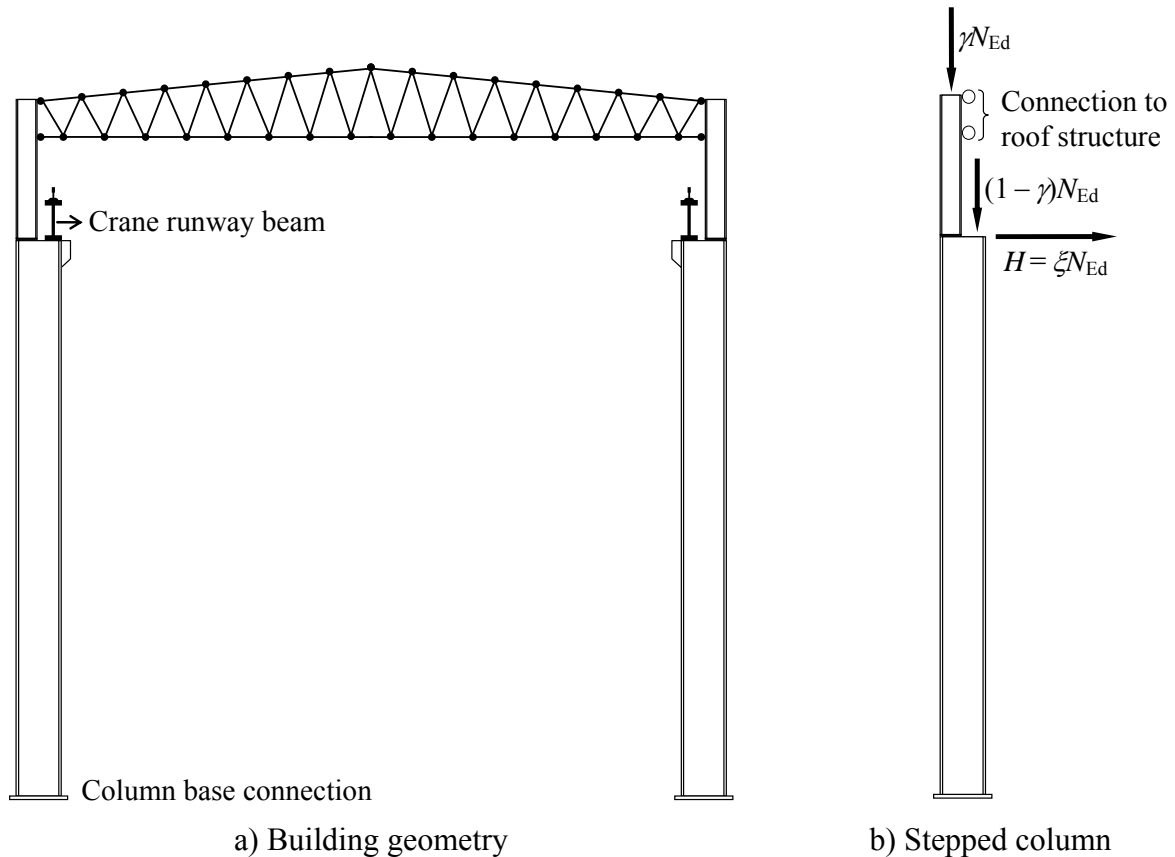
**Abstract:** This paper deals with buckling aspects of the design of stepped columns in heavy mill buildings. In these structures, columns have to carry significant axial loads that usually act eccentrically and strength reducing bending moments due to lateral loads. A simple physical model for buckling behaviour analysis is proposed and formulated using the differential equations of equilibrium. The exact solution of the governing equations is found by using symbolic computation. Effective buckling length coefficients and the corresponding critical loads are obtained and the results are presented in the form of design charts. The structural response is then evaluated and discussed for a number of practical cases for sway and sway prevented columns.

### **1 Introduction**

Mill buildings are heavy industrial structures within which machinery, materials and products are lifted and moved in a large work area by overhead travelling cranes. These industrial facilities are usually designed with rigid steel-framed structures and are characterized by long roof spans and high floor-to-floor and floor-to-roof heights. The structure comprises common steel components used in roof and wall framing (e.g. roof trusses), wall systems and crane runway beams supported by columns. Several different column configurations can be used for the crane carrying structure [1]. Designers frequently choose stepped columns, with a single heavy wide-flange section as the lower segment and a lighter wide-flange section that supports the roof structure. The upper segment has to carry the roof and upper wall loads. The lower segment has to be designed for the extra load from the crane beam reaction, lower wall loads and self-weight. A typical geometry for crane buildings is shown in Fig. 1a. The column is extended up to the top of the truss and connected to the top and bottom truss chords. The roof truss can be regarded as an infinitely stiff horizontal member and it provides lateral support to columns. The column base may be fully fixed although pinned bases are often adequate.

This paper is an analysis of the stability behaviour of stepped columns of such mill buildings. The model is formulated in the context of the classical equilibrium approach. A set of equations governing equilibrium along with the appropriate boundary conditions are derived. This derivation is based on the following assumptions:

1. The analysis is purely elastic (the stress-strain relationship is completely linear),
2. Residual stresses are ignored,



**Fig. 1:** Typical cross-section of a heavy mill building

3. No local type of instability occurs,
4. The column is considered transversely supported so that the possibility of buckling about the weak axis is precluded,
5. Lateral torsional buckling is prevented,
6. The cut surfaces at the column splice are in perfect contact, in the case of bearing splices,
7. No appreciable initial curvature exists,
8. The effect of transverse shear on deformations is negligible.

This physical model is first formulated and the critical buckling load of the stepped column is investigated using eigen-boundary-value analysis. The corresponding effective buckling length is also computed. A main interest of the present paper is to derive rational effective length charts that may be readily used as a simple design tool.

Imperfections in the form of column segments misalignment and load eccentricities are also considered. It is shown that the imperfections can produce highly unstable behaviour. These findings are important for designers aiming to achieve safer and more efficient and economic designs for stepped columns in mill buildings. Some guidelines to represent this information in a suitable form for subsequent inclusion in a maximum elastic strength column analysis are also proposed.

## 2 Model formulation

Consider an isolated stepped column in the frame illustrated in Fig. 1b. The crane loads dominate the design of the column. These are essentially axial compressive loads that act

eccentrically and thus produce moments in the column. The analysis of such columns requires a buckling analysis. By solving the equilibrium equations, the critical loads and the corresponding effective length coefficients are determined. The results depend on (i) the end fixities, (ii) the ratio of the end axial load to the intermediate axial load (parameter  $\gamma$ ), (iii) the ratio of the length and moment of inertia of the upper segment to the lower segment, and (iv) the splice mechanical properties [2,3].

The column ends are restrained by base and roof beam connections, respectively, that usually exhibit an elastic initial response (partially restrained connections). The roof truss framing into the top column segment also provides some kind of restraint against lateral deflection. In this paper, the two following cases are considered: (i) the top end is prevented from translating during buckling (sway prevented model) and (ii) the top end is free to translate during buckling (sway model). The two basic mechanical models are shown in Figs. 2 and 3. The stepped columns in Figs. 2 and 3 consist of two independent members, *I* and *II*, connected by a spring at point S.  $K_{0c}$  is the tangent elastic stiffness coefficient for this spring. Member *I* has a length  $L_1 = \alpha L$ , where  $L$  is the column length and  $0 \leq \alpha \leq 1$ . Member *II* has two segments of length  $L_2 = (1-\alpha)L - H_{tr}$  and  $L_3 = H_{tr}$ , where  $H_{tr}$  is the truss height. Each member has a constant bending stiffness  $EI_I$  and  $EI_{II}$ . The upper column segment is loaded axially by a compressive load  $\gamma N_{Ed}$ ,  $0 \leq \gamma \leq 1$  that is applied with an eccentricity  $e_0$ . The crane load  $(1-\gamma)N_{Ed}$  is eccentrically applied to the lower segment at a distance  $e_1$ . The column is also subjected to a concentrated lateral load  $H = \xi N_{Ed}$  at the step. The forces retain their direction as the column deflects.

This model offers a clear physical illustration and solid grounds in the mechanics of the problem. The fourth-order equilibrium equations for an initially straight stepped column loaded axially by compressive loads are derived below using the equilibrium method. In this classical approach, the problem is reduced to an eigen-boundary-value problem and the critical conditions are the eigenvalues. The governing differential equation of equilibrium is written in the following form [4]:

$$\frac{d^2 M_i}{dx_i^2} - N_i \frac{d^2 w_i}{dx_i^2} = 0 \quad (1)$$

where  $w$  is the lateral displacement and  $i = 1, 2, 3$ . For linearly elastic materials, the bending moment  $M$  and the column curvature are related as follows:

$$M_i = -EI_i \frac{d^2 w_i}{dx_i^2} \quad (2)$$

From Eqs. (1) and (2) we obtain the general fourth-order differential equilibrium equation:

$$\frac{d^2}{dx_i^2} \left( -EI_i \frac{d^2 w_i}{dx_i^2} \right) - N_i \frac{d^2 w_i}{dx_i^2} = 0 \quad \text{or} \quad w_i^{IV} + \mu_i^2 w_i'' = 0 \quad (3)$$

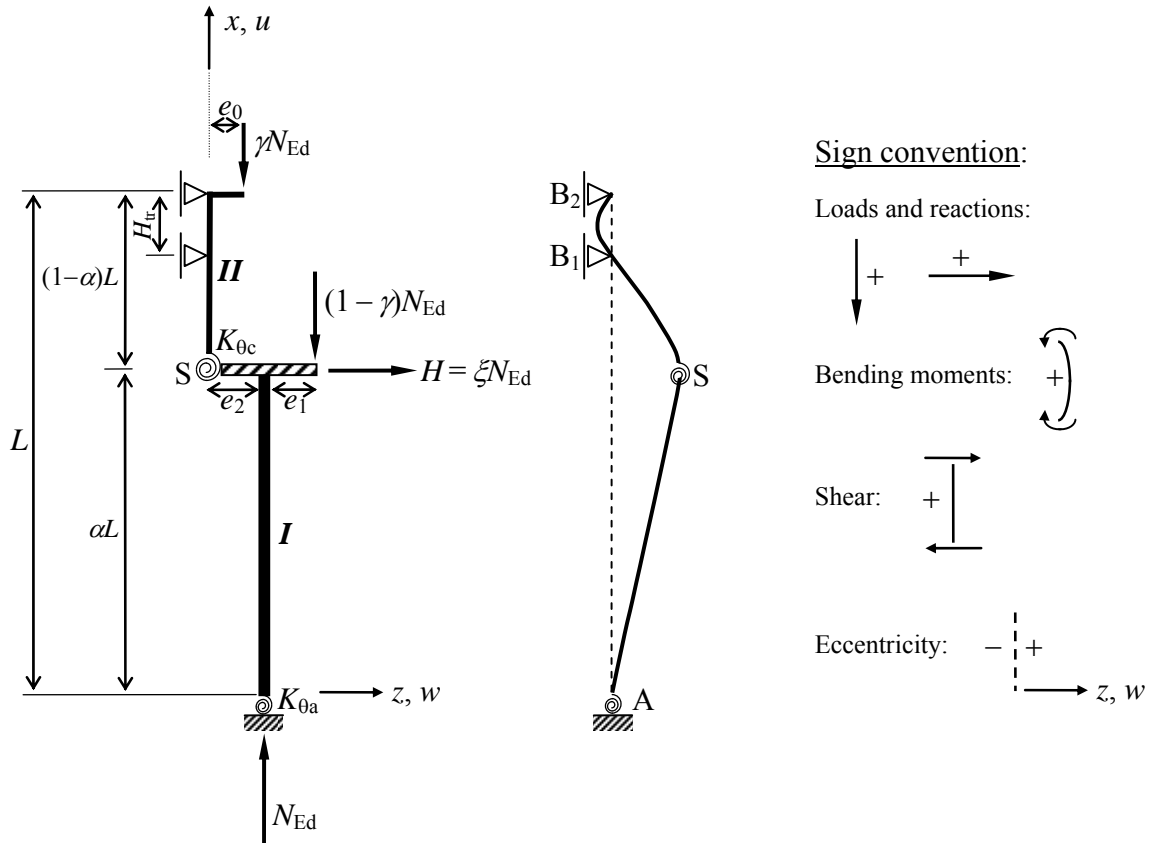
where  $\mu_i$  is given by:

$$\mu_1^2 = \frac{N_{Ed}}{EI_I} \quad \text{and} \quad \mu_2^2 = \mu_3^2 = \frac{\gamma N_{Ed}}{EI_{II}} \quad (4)$$

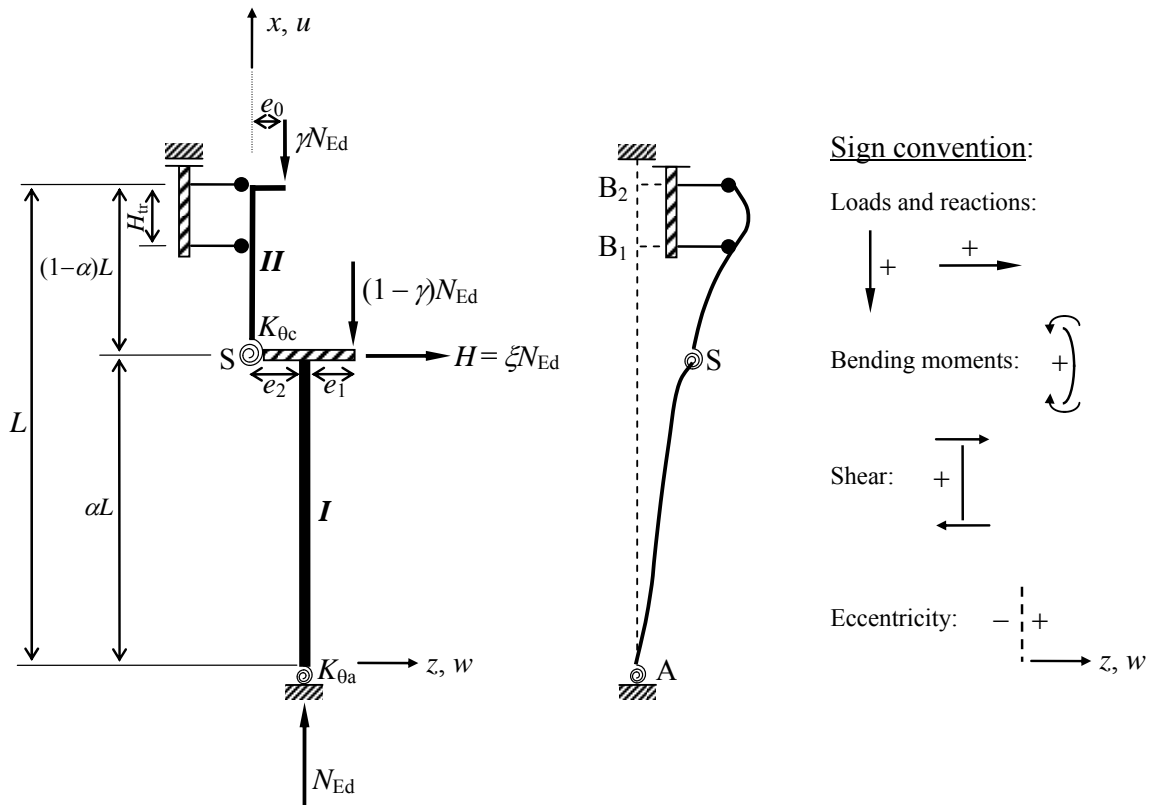
The general solution of this equation is:

$$\begin{aligned} \text{Member 1: } w_1 &= A_1 \sin \mu_1 x + A_2 \cos \mu_1 x + A_3 x + A_4 \\ \text{Member 2: } w_2 &= B_1 \sin \mu_2 x + B_2 \cos \mu_2 x + B_3 x + B_4 \\ \text{Member 3: } w_3 &= C_1 \sin \mu_3 x + C_2 \cos \mu_3 x + C_3 x + C_4 \end{aligned} \quad (5)$$

where  $A_j$ ,  $B_j$  and  $C_j$  are constants ( $j = 1, 2, 3, 4$ ). This solution must satisfy the prescribed boundary conditions, which are described in the following sections. This requirement leads to twelve linear algebraic equations in the twelve constants  $A_j$ ,  $B_j$  and  $C_j$ .



**Fig. 2:** Column model: sway prevented case



**Fig. 3:** Column model: sway case

## 2.1 Sway prevented column

The boundary conditions in this case are given by:

$$\begin{aligned}
 &\text{at section A} && w_1 = 0 \\
 &&& -EI_I w_1'' = -K_{\theta a} w_1' \\
 &\text{at the splice location} && w_1|_{x_1=\alpha L} = w_2|_{x_2=0} \\
 &&& -EI_I w_1''|_{x_1=\alpha L} + EI_{II} w_2''|_{x_2=0} + (1-\gamma)N_{Ed}e_1 + \gamma N_{Ed}e_2 = 0 \\
 &&& -EI_{II} w_2''|_{x_2=0} = -K_{\theta c} \left( w_2'|_{x_2=0} - w_1'|_{x_1=\alpha L} \right) \\
 &&& \left( -EI_I w_1''' - N_{Ed} w_1' \right)|_{x_1=\alpha L} = \left( -EI_{II} w_2''' - \gamma N_{Ed} w_2' \right)|_{x_2=0} + \xi N_{Ed} \\
 &\text{at section B}_1 && w_2|_{x_2=(1-\alpha)L-H_{tr}} = w_3|_{x_3=0} = 0 \\
 &&& w_2'|_{x_2=(1-\alpha)L-H_{tr}} = w_3'|_{x_3=0} \\
 &&& w_2''|_{x_2=(1-\alpha)L-H_{tr}} = w_3''|_{x_3=0} \\
 &\text{at section B}_2 && w_3 = 0 \\
 &&& -EI_{II} w_3'' = -\gamma N_{Ed} e_0
 \end{aligned} \tag{6}$$

## 2.2 Sway column

The boundary conditions are now given by:

$$\begin{aligned}
 &\text{at section A} && w_1 = 0 \\
 &&& -EI_I w_1'' = -K_{\theta a} w_1' \\
 &\text{at the splice location} && w_1|_{x_1=\alpha L} = w_2|_{x_2=0} \\
 &&& -EI_I w_1''|_{x_1=\alpha L} + EI_{II} w_2''|_{x_2=0} + (1-\gamma)N_{Ed}e_1 + \gamma N_{Ed}e_2 = 0 \\
 &&& -EI_{II} w_2''|_{x_2=0} = -K_{\theta c} \left( w_2'|_{x_2=0} - w_1'|_{x_1=\alpha L} \right) \\
 &&& \left( -EI_I w_1''' - N_{Ed} w_1' \right)|_{x_1=\alpha L} = \left( -EI_{II} w_2''' - \gamma N_{Ed} w_2' \right)|_{x_2=0} + \xi N_{Ed} \\
 &\text{at section B}_1 && w_2|_{x_2=(1-\alpha)L-H_{tr}} = w_3|_{x_3=0} \\
 &&& w_2'|_{x_2=(1-\alpha)L-H_{tr}} = w_3'|_{x_3=0} \\
 &&& w_2''|_{x_2=(1-\alpha)L-H_{tr}} = w_3''|_{x_3=0} \\
 &&& \left( -EI_{II} w_2''' - \gamma N_{Ed} w_2' \right)|_{x_2=(1-\alpha)L-H_{tr}} = 0 \\
 &\text{at segment [B}_1\text{B}_2] && w_3|_{x_3=0} = w_3|_{x_3=H_{tr}} \\
 &\text{at section B}_2 && -EI_{II} w_3'' = -\gamma N_{Ed} e_0
 \end{aligned} \tag{7}$$

## 2.3 Critical buckling

A nontrivial solution to Eq. (3) exists if any of the twelve constants is not equal to zero. This happens if the determinant of coefficients  $A_j$ ,  $B_j$  and  $C_j$  vanishes. The expansion of this determinant leads to the characteristic equation. The smallest positive root yields the buckling

load  $N_{cr}$  and the shape of the deflection curve, Eq. (5) (first eigenvector). Analysis is run within the algebraic manipulator Mathematica [5].

To demonstrate this procedure, consider a sway prevented column similar to that depicted in Fig. 2. The column has a pinned base. The step is located at  $\alpha=0.6$ . The roof truss has a height of  $0.1L$ . The two column segments are rigidly connected ( $K_{\theta c} = \infty$ ). The ratio between the applied loads is  $\gamma=0.1$  and the ratio between second moment of area  $I_I/I_{II}=2.5$ . The smallest root of the characteristic equation leads to the critical condition and  $N_{cr} = \pi^2 EI_I / 0.386L^2$ .

Critical loads and corresponding effective lengths are important parameters in the stability analysis of stepped columns in mill buildings, and also for estimating second order load effects by approximate methods. These methods are still very useful tools in the design office, despite the increased availability and capacity of computational methods. In order to provide the structural engineer with simple design tools, equivalent length charts are appended to this paper.

## 2.4 Effect of column moments

The eccentric loads and lateral forces produce end moments at each column segment. The maximum load sustained by the column is a function of the bending moment and is determined from a load-deflection approach to column analysis.

The load-deflection response corresponds to the solution to the fourth-order differential equilibrium equation, Eq. (3). The column now starts to deform laterally from the commencement of loading and the deflection increases progressively and rapidly with the load. The load-deflection curve approaches the critical buckling asymptotically.

Where the bending effect is secondary compared to the axial force effect, the structural analysis involves the features of a stability problem. In these cases and for design purposes, it is usual to express column strength directly by means of column curves based on elastic limit analysis [6,7]. If both bending and axial effects are significant, as in most practical columns in mill buildings, the column segments have to be treated as beam-columns. For in plane flexural bending, the interaction formula based on the attainment of first yield  $f_y$  in an initially stress-free member can be written as follows:

$$\frac{N_{Ed,i}}{N_{b,R,i}} + \kappa \frac{M_{Ed,i}}{M_{R,i}} \leq 1 \quad (8)$$

where  $N_{Ed,1} = N_{Ed}$ ,  $N_{Ed,2} = \gamma N_{Ed} = N_{Ed,3}$ ,  $N_{b,R}$  is the buckling resistance in the plane of the applied moments, given by:

$$N_{b,R,i} = \chi A_i f_y \quad (9)$$

$A$  is the area of cross section and  $\chi$  is the reduction factor for flexural buckling [8].  $M_{Ed}$  is the maximum bending moment and  $M_R$  is the in-plane flexural capacity.  $M_{Ed}$  is a function of the applied load producing moments (primary moments). In the context of an elastic limit strength criterion, the in-plane flexural capacity is taken as:

$$M_{R,i} = W_{el,i} f_y \quad (10)$$

$W_{el} = Ai^2/c$  is the section modulus corresponding to the fibre with maximum elastic stress ( $i$ : radius of gyration,  $c$ : distance from neutral axis to extreme fibre). The interaction factor  $\kappa$  is defined by the following general expression:

$$\kappa = \frac{C_{m,i}}{1 - N_{Ed,i}/N_{cr,i}} \quad (11)$$

that assumes that the out-of-plane displacements are prevented. For columns subjected to linear distribution of first-order moments, the interaction factor  $\kappa$  can be written as follows [9]:

$$\kappa = \frac{C_{m,i}}{\cos\left(\frac{\pi}{2} \sqrt{\frac{N_{Ed,i}}{N_{cr,i}}}\right)} \quad (12)$$

The moment magnifier  $C_m$  depends on the loading type and end conditions. Appropriate expressions for this equivalent moment factor are adopted in EN 1993 [10]. Consider again a column subjected to linear distribution of first-order moments. Let  $M_1$  and  $\psi M_1$  be the column segment end moments, with  $1 \leq \psi \leq 1$ . In this case, the EN 1993 adopts the following expression:

$$C_{m,i} = 0.79 + 0.21\psi + 0.36(\psi - 0.33)N_{Ed,i}/N_{cr,i} \quad (13)$$

The margin between the actual maximum bending moment –  $M_{actual}$ , Eq. (2), and the design amplified bending moment,  $M_{max}$ ,

$$M_{max,i} = \kappa M_{Ed,i} \quad (14)$$

is illustrated in several practical cases in the following section.

### 3 Applications

Having formulated the equilibrium equations, the critical buckling load of the stepped column is first investigated using the design tables in the Appendix. The equilibrium paths are then evaluated with respect to the additional column moments. The two basic configurations considered below are shown in Fig. 4. The examples are chosen so as to represent the isolated effect of (i) the magnitude of the horizontal load acting at the step and (ii) roof truss height. They are considered to be influential factors for the stepped columns.

#### 3.1 Column configurations and parameters

The geometries of the analysis configurations utilized in this research are derived from the geometry of the heavier-loaded column of the steel framework of the Turbine House at Lidell Power Station, New South Wales, Australia [11]. Different analysis configurations are created by changing some of the attributes. Important geometric and mechanical parameters are varied over a practical range of interest in order to evaluate the behaviour of a stepped mill column. Specific characteristics and attributes modelled in these studies are as follows (see also Fig. 4):

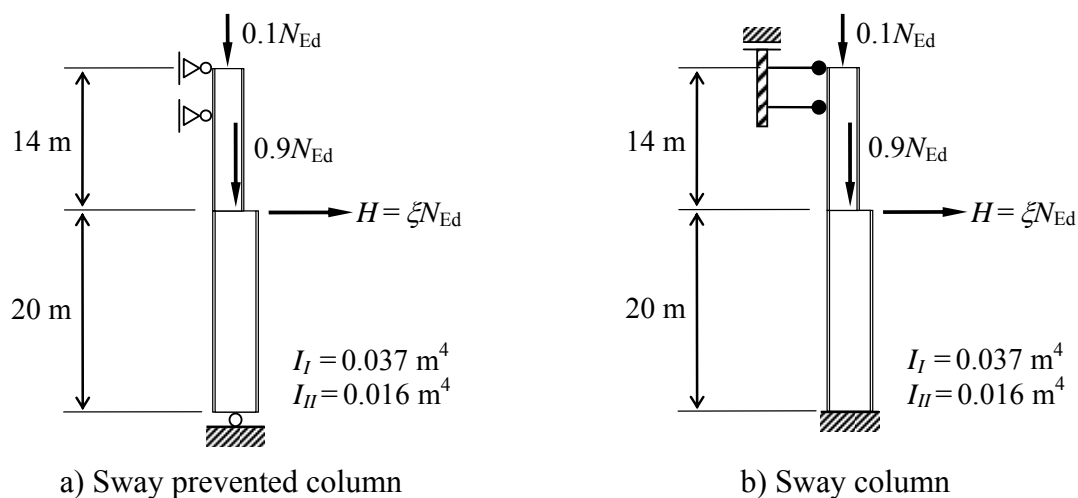


Fig. 4: Basic stepped column configurations

Column length	$L = 34$ m	
Columns base conditions	Sway prevented case	Pinned end (Fig. 4a)
	Sway case	Fully fixed end (Fig. 4b)
Member cross-section	Member I: flanges $1067 \times 38$ mm <sup>2</sup> and web $686 \times 38$ mm <sup>2</sup> Member II: flanges $610 \times 82.5$ mm <sup>2</sup> and web $686 \times 38$ mm <sup>2</sup>	
Splice location	$\alpha = 0.588$ ( $L_1 = 20$ m)	
Splice rotational stiffness	$K_{\theta c} \rightarrow \infty$ (rigid splice)	
Roof truss height	$H_{tr}/L = 0.1, 0.2$	
Loading coefficients	$\gamma = 0.1$ and $\xi = -0.05, 0, 0.05$	
Load eccentricities	$e_0 = 0$ $e_1 = 0$ $e_2 = -0.1905$ (segments aligned vertically to the flanges)	
Young's modulus	$E = 210 \times 10^3$ N/mm <sup>2</sup>	
Steel grade	S355 ( $f_y = 335$ N/mm <sup>2</sup> )	

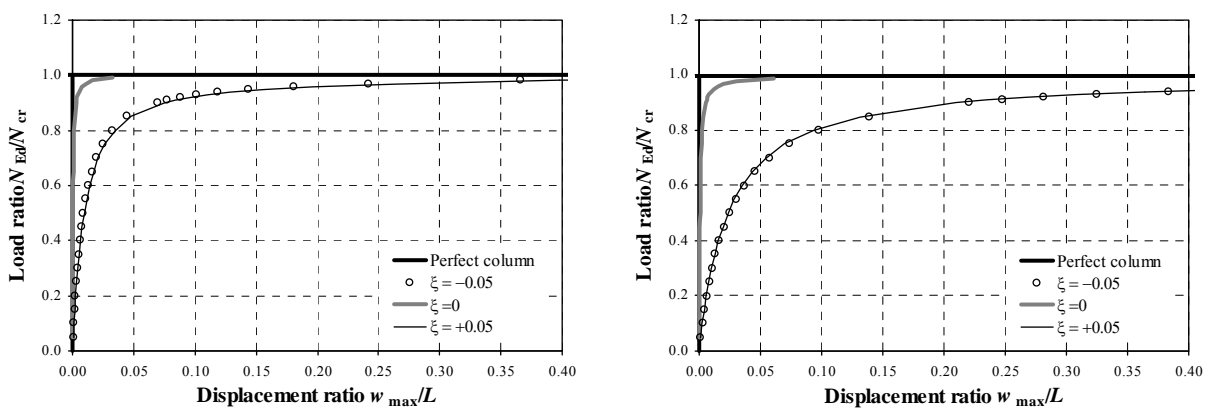
### 3.2 Analysis results

The effect of the magnitude of the horizontal force  $H$  and the roof truss height on the column carrying capacity are analysed in this section. The effect of the load eccentricity  $e_0$  and  $e_1$  and the column segments eccentricity  $e_2$  are not considered in this work. This subject has been discussed in previous work of the authors [12].

Fig. 5 shows load-deflections curves that help in assessing the isolated effect of a lateral force by means of a variable load factor  $\xi$ . (Naturally, the eccentric axial loads would also produce moments in the column.) The plots in Fig. 6 show the effect of the truss height.

The following trends can be observed from these plots:

1. When the column is subjected to a combination of axial loads and primary bending moments, the maximum load carrying capacity falls below the critical load.
2. The variations in the factor load  $\xi$  have a smaller influence on the load-deflection response in the case of sway-prevented columns.
3. The restraining effects of the truss height have a smaller influence on the load-deflection response in the case of sway prevented columns.



a) Sway prevented column

b) Sway column

**Fig. 5:** Effect of factor load  $\xi$  on the column carrying capacity ( $H_{tr}/L = 0.1$ )



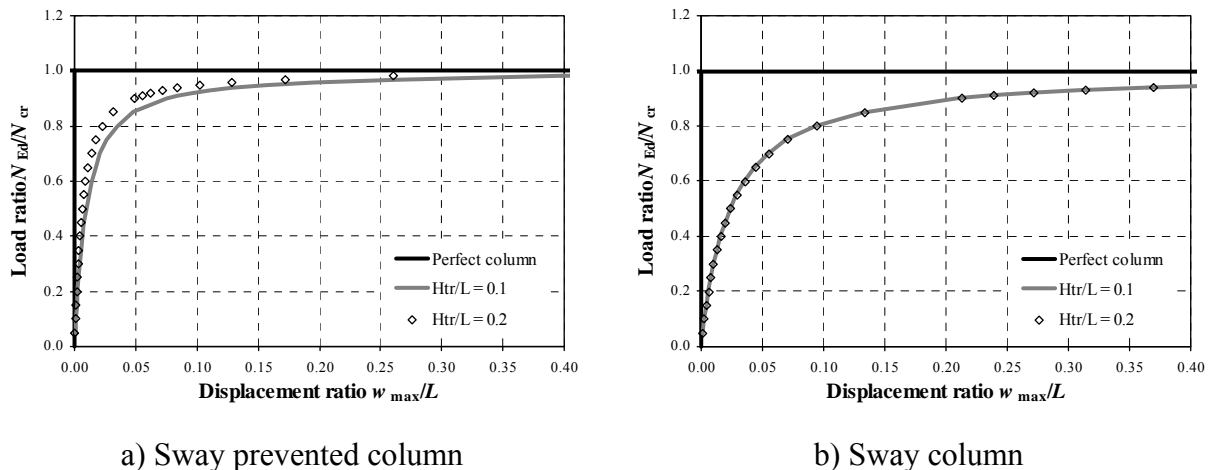


Fig. 6: Effect of roof truss height on the column carrying capacity ( $\xi = +0.05$ )

## 4 Design implications

The most relevant aspect in design is the practical application of the interaction formula that has the following form [10]:

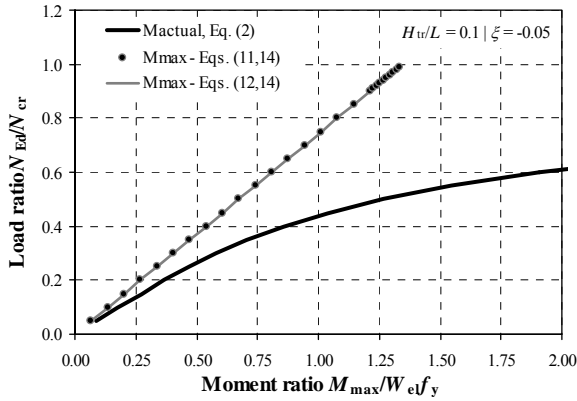
$$F(N, M) = \frac{N_{Ed,i}}{\chi A_i f_y} + \frac{M_{max,i}}{W_{el,i} f_y} \quad (15)$$

According to EN 1993-1-1, the designer has to ensure  $F(N, M) \leq 1$ , see Eq. (8). The actual column moment distribution along the length of the member is obtained from Eq. (2). The margin between the actual maximum bending moment ( $M_{actual}$ ), Eq. (2), and the design amplified bending moment, Eqs. (11), (12) and (14), is illustrated in two representative cases selected from the parametric study. Fig. 7 plots the moment vs. axial force response in no dimensional form for the selected examples, for the critical column member  $I$ . Key points are:

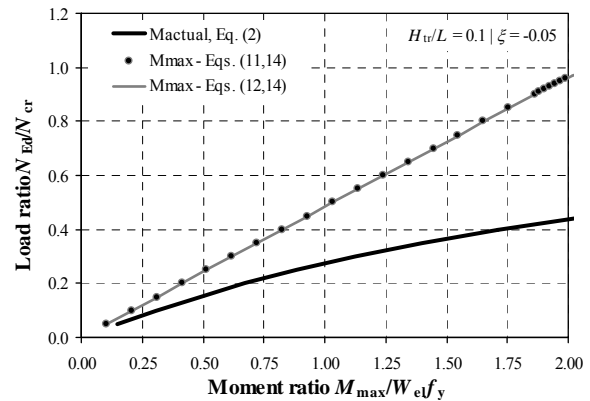
1. Column moments are more significant in the case of sway-permitted columns.
2. There are no significant variations in the prediction of the interaction factor  $\kappa$  given by Eqs. (11) or (12).
3. The design amplified moment computed by Eqs. (11) and (14) or by Eqs. (12) and (14) gives a good estimation of the actual bending moment for low axial load levels.
4. For axial load levels of  $20\% N_{Ed}/N_{cr}$  and above, the design amplified moments are not good predictors of the actual bending moment acting at the column segment.

The moment calculations presented above are now included in the design interaction formula  $F(N, M)$ . Fig. 8 shows plots of the axial load level against  $F(N, M)$ . It can be appreciated that the design approach indicates satisfactory results in assessing the column carrying capacity. Although the preceding analysis has shown that the amplified moment anticipated by EN 1993 – Eqs. (11) and (14) – does not give an accurate prediction of the actual column moments, the design interaction expression does provide a reliable assessment of the column carrying capacity, as observed in the (limited) range of tests analysed above. The following observations are also made:

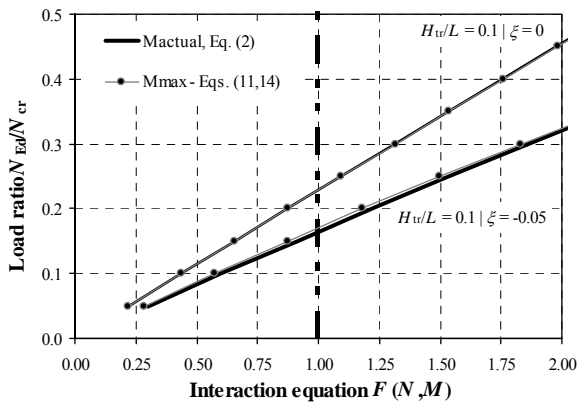
1. The member axial force is limited to  $22\% N_{Ed}/N_{cr}$  in the case of sway prevented columns when the primary bending moments are not significant ( $\xi = 0$ ).
2. This percentage increases to  $46\%$  in the case of columns that are able to sway.
3. The predominant influence of primary bending moments is observed in the significant load capacity reduction, that can be as high as  $20\%$  in the case of sway columns.



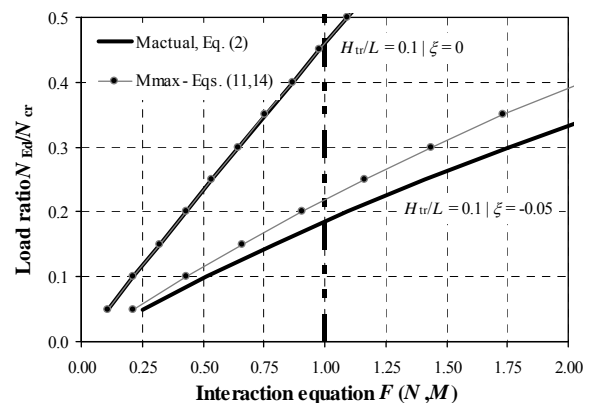
a) Sway prevented column



b) Sway column

**Fig. 7:** Column moments: member I

a) Sway prevented column



b) Sway column

**Fig. 8:** Interaction formula: member I

## 5 Concluding remarks

The main conclusions are briefly summarized:

1. In spite of the widespread of computer-assisted design techniques, there is still a role for simple hand calculation methods. This paper is a contribution to that philosophy with respect to the evaluation of the critical buckling loads of stepped crane columns in mill buildings (see Appendix).
2. The restraining effect of the truss has a beneficial effect on the critical load of the column. Comparisons between the above results and the results presented in [12] support this statement.
3. The adequacy of the simple strength design method adopted in EN 1993 to predict the stepped column capacity is assessed and verified. In the light of this discussion it follows that proper consideration of the interaction effect between both column segments has to be incorporated in the design approach.
4. Designers and steel fabricators would potentially be interested in the outcomes of this study and the authors are further extending this topic to set up sound design criteria regarding the requirements for stiffness and strength of column splices in this type of construction.

## Appendix. Design tables

The design table presented below contain effective length factors  $K_1$ ,  $K_2$  and  $K_3$  for stepped columns, in terms of practical values of the ratio  $I_I/I_{II}$ , the step location  $\alpha$ , the ratio between the applied axial loads at the top of the column and at the stepped level  $\gamma$ , the ratio  $H_{tr}/L$  and the splice non-dimensional rotational stiffness  $k_{0c} = K_{0c}L/EI_I$ . The column base conditions are those encountered in practical non-sway and sway mill building systems: pinned base for sway prevented columns and fully fixed connections for sway columns.

The following definitions are adopted ( $C$  is the global end-fixity factor):

Critical load of the system	$N_{cr} = \frac{C\pi^2 EI_I}{L^2}$
Critical load of the lower segment	$N_{cr,1} = \frac{\pi^2 EI_I}{(K_1 L_1)^2} = N_{cr}$
Critical load of the upper segments	$N_{cr,2} = \frac{\pi^2 EI_{II}}{(K_2 L_2)^2} = N_{cr,3} = \frac{\pi^2 EI_{II}}{(K_3 L_3)^2} = \gamma N_{cr}$
Effective length factor of the lower segment	$K_1 = 1/\alpha\sqrt{C}$
Effective length factor of the upper segments	$K_2 = L\left[(1-\alpha)L - H_{tr}\right] / \left[H_{tr}^2 \sqrt{\gamma C I_I / I_{II}}\right]$
	$K_3 = L / \left[H_{tr} \sqrt{\gamma C I_I / I_{II}}\right]$

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Table: Equivalent length chart (example)

$I_I/I_{II}$	$\alpha$	$\gamma$	$H_{tr}/L$	$k_{\theta c}$	Sway prevented column				Sway column			
					$C$	$K_1$	$K_2$	$K_3$	$C$	$K_1$	$K_2$	$K_3$
2	0.6	0.10	0.1	1	2.63	1.03	41.38	13.79	0.88	1.78	71.58	23.86
				5	2.66	1.02	41.13	13.71	1.02	1.65	66.52	22.17
				10	2.66	1.02	41.10	13.70	1.05	1.63	65.46	21.82
			$\infty$	2.66	1.02	41.10	13.70	1.09	1.60	64.23	21.41	
			0.2	1	2.88	0.98	6.59	6.59	0.90	1.76	11.82	11.82
				5	2.91	0.98	6.55	6.55	1.07	1.61	10.79	10.79
		10		2.92	0.98	6.55	6.55	1.12	1.57	10.56	10.56	
		$\infty$	2.92	0.97	6.54	6.54	1.18	1.53	10.29	10.29		
		0.20	0.1	1	2.47	1.06	30.21	10.07	0.87	1.78	50.77	16.92
				5	2.57	1.04	29.61	9.87	1.00	1.67	47.47	15.82
				10	2.58	1.04	29.55	9.85	1.03	1.64	46.80	15.60
			$\infty$	2.58	1.04	29.50	9.83	1.06	1.62	46.04	15.35	
	0.2		1	2.84	0.99	4.69	4.69	0.89	1.76	8.37	8.37	
			5	2.85	0.99	4.68	4.68	1.06	1.62	7.67	7.67	
		10	2.86	0.99	4.68	4.68	1.10	1.59	7.52	7.52		
	$\infty$	2.86	0.99	4.68	4.68	1.16	1.55	7.35	7.35			
	0.7	0.10	0.1	1	2.18	0.97	30.26	15.13	0.69	1.72	53.90	26.95
				5	2.27	0.95	29.65	14.83	0.86	1.54	48.26	24.13
				10	2.29	0.94	29.53	14.76	0.91	1.50	47.01	23.50
			$\infty$	2.32	0.94	29.38	14.69	0.97	1.45	45.50	22.75	
			0.2	1	2.23	0.96	3.75	7.49	0.71	1.70	6.66	13.31
				5	2.42	0.92	3.59	7.19	0.93	1.48	5.80	11.60
		10		2.48	0.91	3.55	7.11	1.00	1.43	5.59	11.19	
		$\infty$	2.55	0.89	3.50	7.00	1.09	1.37	5.34	10.69		
0.20		0.1	1	2.18	0.97	21.44	10.72	0.69	1.72	38.15	19.08	
			5	2.25	0.95	21.09	10.54	0.85	1.55	34.26	17.13	
			10	2.27	0.95	21.01	10.51	0.90	1.51	33.40	16.70	
		$\infty$	2.28	0.95	20.93	10.47	0.95	1.46	32.38	16.19		
	0.2	1	2.22	0.96	2.65	5.30	0.70	1.70	4.71	9.42		
		5	2.41	0.92	2.55	5.10	0.93	1.48	4.11	8.22		
10		2.46	0.91	2.52	5.04	0.99	1.43	3.97	7.93			
$\infty$	2.53	0.90	2.49	4.98	1.09	1.37	3.79	7.59				
3	0.6	0.10	0.1	1	2.33	1.09	35.92	11.97	0.85	1.81	59.55	19.85
				5	2.50	1.05	34.63	11.54	0.93	1.73	56.83	18.94
				10	2.52	1.05	34.52	11.51	0.95	1.71	56.32	18.77
			$\infty$	2.53	1.05	34.42	11.47	0.97	1.70	55.75	18.58	
			0.2	1	2.79	1.00	5.47	5.47	0.87	1.79	9.81	9.81
				5	2.79	1.00	5.47	5.47	0.98	1.68	9.23	9.23
		10		2.79	1.00	5.47	5.47	1.00	1.66	9.11	9.11	
		$\infty$	2.79	1.00	5.47	5.47	1.04	1.64	8.97	8.97		
		0.20	0.1	1	2.09	1.15	26.80	8.93	0.84	1.82	42.32	14.11
				5	2.38	1.08	25.10	8.37	0.91	1.75	40.66	13.55
				10	2.41	1.07	24.95	8.32	0.92	1.74	40.36	13.45
			$\infty$	2.44	1.07	24.81	8.27	0.94	1.72	40.03	13.34	
	0.2		1	2.70	1.01	3.93	3.93	0.86	1.80	6.96	6.96	
			5	2.71	1.01	3.92	3.92	0.96	1.70	6.58	6.58	
		10	2.71	1.01	3.92	3.92	0.99	1.68	6.50	6.50		
	$\infty$	2.71	1.01	3.92	3.92	1.01	1.66	6.42	6.42			
	0.7	0.10	0.1	1	2.14	0.98	24.95	12.48	0.66	1.75	44.78	22.39
				5	2.18	0.97	24.74	12.37	0.78	1.62	41.38	20.69
				10	2.18	0.97	24.71	12.35	0.81	1.59	40.68	20.34
			$\infty$	2.19	0.96	24.66	12.33	0.84	1.56	39.87	19.93	
			0.2	1	2.20	0.96	3.08	6.15	0.69	1.73	5.51	11.03
				5	2.32	0.94	3.00	5.99	0.85	1.55	4.96	9.92
		10		2.35	0.93	2.98	5.95	0.89	1.51	4.84	9.68	
		$\infty$	2.40	0.92	2.95	5.89	0.95	1.47	4.69	9.39		
0.20		0.1	1	2.12	0.98	17.72	8.86	0.66	1.76	31.72	15.86	
			5	2.15	0.97	17.62	8.81	0.77	1.63	29.41	14.71	
			10	2.15	0.97	17.61	8.80	0.80	1.60	28.95	14.47	
		$\infty$	2.15	0.97	17.59	8.79	0.82	1.57	28.43	14.21		
	0.2	1	2.19	0.96	2.18	4.36	0.68	1.73	3.90	7.81		
		5	2.30	0.94	2.13	4.25	0.84	1.56	3.52	7.04		
10		2.33	0.94	2.11	4.23	0.88	1.52	3.44	6.87			
$\infty$	2.36	0.93	2.10	4.20	0.93	1.48	3.34	6.68				