Laminarisation in Flows of Concentrated Benthic Suspensions. Computations with a Low-Re Mixing-Length Model

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Abstract

In quiescent water bodies that form part of estuarine or coastal water systems, such as harbours and access channels, fine sediments may deposit to form concentrated benthic suspensions (CBS). The flow velocities in these mud layers may become low, and the viscosities may become large, so that the Reynolds number decreases below a critical value and a transition from turbulent to laminar flow, designated herein as laminarisation, takes place. In this report laminarisation of CBS, modelled as a Newtonian fluid, is simulated using the Prandtl mixing-length turbulence model supplemented with a modified Van Driest model of low-Reynolds-number flow, and a transition criterion based on the concept of a critical value of the turbulence Reynolds number. This criterion is calibrated with experimental data obtained from the literature. The model is shown to reproduce the vertical distribution of the mean velocity at low Reynolds numbers quite well. Laminarisation in a slowly decelerating layer of CBS is found to take place at a Reynolds number of 1020, where the Reynolds number is defined as the product of mean velocity and layer depth divided by the kinematic viscosity. The inability of mixing-length models of reproducing memory effects, which actually do occur in laminarising flows, and effects of non-Newtonian CBS properties are briefly discussed.

Contents

1 Introduction

Suspended fine sediment in coastal waters may be transported into estuaries, waterways and harbours by tidal currents and gravitational circulation. If in these more sheltered areas the hydrodynamic energy is insufficient to maintain a well-mixed suspension, the sediment will settle and form a layer of concentrated benthic suspension (CBS) or fluid mud. The molecular viscosity of such suspensions increases markedly with concentration (e.g., Parker and Hooper, 1994). At sufficiently high flow velocities, as in turbidity currents, the flow in a CBS layer may be turbulent. However, if on a (nearly) horizontal bed the velocity decreases and concentrations gradually increase because of continuing settling, viscous effects or buoyancy effects may become large so that a reverse transition from turbulent to laminar flow, that is, laminarisation, takes place. Eventually the CBS may come to a standstill (e.g., Kirby and Parker, 1983). Laminarisation may also occur in time-dependent or steady-state flows of CBS on a mild bed slope.

Reverse transition caused by viscosity and buoyancy can be characterised with critical values of the Reynolds number of turbulence, Re_T, and the flux Richardson number for shear flow, R_f, respectively. Ivey and Imberger (1991) define Re_T as

$$Re_T = \frac{u'L}{\nu} \tag{1.1}$$

where ν is the molecular viscosity of the CBS, u' the turbulence intensity and L the length scale of the energy containing eddies. Those authors reviewed literature on laminarisation and reported critical values, Re_{Tc}, of the turbulence Reynolds number for homogeneous shear flow in the range 9.3 to 18.2 with a mean value of about 15. The numerator in (1.1) is equal to the eddy diffusivity, which in turn equals the eddy viscosity, ν_T, divided by the turbulent Prandtl-Schmidt number, σ_T. The turbulence Reynolds number therefore can be written as

$$Re_T = \frac{1}{\sigma_T}\frac{\nu_T}{\nu} \tag{1.2}$$

Ivey and Imberger (1991) presented a second criterion for reverse transition. However, as shown in the Appendix, that criterion is a less severe one in the case of shear flow.

The flux Richardson number is defined as (e.g., Turner, 1973)

$$R_f = \frac{\Delta g \langle wc \rangle}{-\rho_b \langle uw \rangle \dfrac{\partial U}{\partial z}} \tag{1.3}$$

where $\Delta = (\rho_s - \rho_w)/\rho_s$, ρ_s and ρ_w are the densities of sediment and water, ρ_b is the bulk density, g the acceleration of gravity, <wc> the vertical turbulent mass transport, ρ_b<uw>

the turbulent shear stress and $\partial U/\partial z$ the vertical mean-velocity gradient. Transition to laminar flow will take place, if R_f increases beyond a critical value $R_{fc} \approx 0.15$ to 0.2.

Introducing the approximations $\langle uw \rangle \approx -u_*^2$, $\partial U/\partial z \approx u_*^2/\nu_T$ and, for near-equilibrium conditions, $\langle wc \rangle \approx W_s C$, and substituting from (1.2) gives

$$R_f \approx K\, Re_T \tag{1.4}$$

In these expressions u_* is the friction velocity at the fixed bed, W_s the settling velocity of the sediment, C the mean concentration, and K a parameter given by

$$K = \sigma_T \frac{C\, W_s}{\rho_b u_*} \frac{\Delta g \nu}{u_*^3} \tag{1.5}$$

The flow will be turbulent, if $Re_T > Re_{Tc}$ as well as $Re_T < R_{fc}/K$ (see Eq. 1.4), or

$$Re_{Tc} < Re_T < \frac{R_{fc}}{K} \tag{1.6}$$

A necessary condition for the flow to be turbulent, at least initially, therefore is $K < R_{fc}/Re_{Tc} \approx 0.01$. Using (1.5) it is easily shown that values of K this small are possible only if either the concentration C is small or the settling velocity W_s is small because of hindered settling (similar turbulence regimes were identified by Winterwerp (1996) for high-Reynolds-number flow). For CBS the latter case applies; in the sequel of this report the concentration is assumed high so that on the time scale considered settling is negligible, and only laminarisation caused by viscous effects is examined herein.

Upon viscous laminarisation the velocities in CBS flow tend to increase, at a constant driving force, because the bed friction coefficient decreases. However, if the Reynolds number continues to decrease, the bed friction coefficient will increase again (e.g., Van Kessel and Kranenburg, 1996).

Mathematical modelling of the flow process selected requires a turbulence model that takes low-Reynolds-number (low-Re) effects into account. The well-known Prandtl mixing-length (PML) model is adapted herein for simulating low-Re flows including laminarisation. As little seems to be known about the modelling of laminarisation with the PML model, a simple case is considered as a first step. The CBS is modelled as a homogeneous Newtonian fluid with time-independent properties. The flow is assumed to be quasi-uniform and quasi-steady, and the bed is horizontal. The flow is slowly decelerating owing to a decreasing driving force, so that transition occurs. The simplifications indicated allow for calibration and verification of the model with measurements reported in the literature. Extension to a more realistic behaviour of CBS is left for future work.

The mathematical model and its calibration are described in Section 2 of this report, and results of numerical computations are presented in Section 3. Some aspects of the model and the rheology of CBS are briefly discussed in Section 4.

2 A one-dimensional low-Re mixing-length model

2.1 Equations

The flow considered is a quasi-uniform, two-layer shallow water flow over a horizontal bed. The lower layer is a homogeneous CBS layer, and the upper layer is a water layer in which the flow velocities are small. Any mixing of water and CBS is disregarded.

Neglecting advective accelerations, the equation of motion in the direction of the flow becomes

$$\frac{\partial U}{\partial t} \approx -\frac{\rho_b - \rho_w}{\rho_b} g \frac{\partial h}{\partial x} - \frac{1}{\rho_b} \frac{\partial \tau}{\partial z} \tag{2.1}$$

where t is time, x the streamwise coordinate, z the vertical coordinate (positive upward, z = 0 at the bed), h the depth of the CBS layer and τ the total shear stress, that is, viscous shear stress plus turbulent shear stress.

A time-dependent flow rate q is imposed, which is given by

$$q = \int_0^h U(x,z,t)\, \mathrm{d}z \tag{2.2}$$

The pressure gradient term in (2.1) is eliminated by integrating this equation from z = 0 to z = h and substituting from (2.2). Neglecting the shear stress at the interface between water and CBS, the resulting equation can be written as

$$\frac{\partial U}{\partial t} = \frac{1}{h}\left[u_*^2 + \frac{\partial q}{\partial t} - U(x,h,t)\frac{\partial h}{\partial t}\right] - \frac{1}{\rho_b}\frac{\partial \tau}{\partial z} \tag{2.3}$$

where for a smooth bed $u_*^2 = -\tau(x,0,t)/\rho_b = \nu \partial U(x,0,t)/\partial z$.

In order to restrict the analysis to a clear-cut problem, the rigid-lid approximation is introduced, where the (imaginary) rigid lid is at the interface. The rigid lid is frictionless and horizontal, and its level does not vary with time. It then follows from the conservation of mass that the flow rate q becomes independent of x, and consequently that the x-dependence of all variables in (2.3) vanishes. Eq. (2.3) thus reduces to

$$\frac{\partial U}{\partial t} = \frac{1}{h}\left(u_*^2 + \frac{\mathrm{d}q}{\mathrm{d}t}\right) - \frac{1}{\rho_b}\frac{\partial \tau}{\partial z} \tag{2.4}$$

The boundary conditions at the bed and the interface are U(0,t) = 0 and $\tau(h,t) = 0$.

Adopting the Boussinesq hypothesis, the turbulent shear stress is given by

$$\frac{\tau}{\rho_b} = -(\nu + \nu_T)\frac{\partial U}{\partial z} \tag{2.5}$$

In a one-dimensional PML model the eddy viscosity follows from (e.g., Rodi, 1980)

$$\nu_T = l^2(z)\left|\frac{\partial U}{\partial z}\right| \tag{2.6}$$

where l(z) is the mixing length. Assuming the mixing length near the interface behaves as in the case of a free surface, empirical results for free-surface flow may be used. Nezu and Rodi (1986) show that for free-surface flows the distribution of the mixing length outside the viscous sublayer near the bed, herein denoted as $l_0(z)$, can be described by

$$l_0(z) = \frac{\kappa z\left(1 - \frac{z}{h}\right)^{1/2}}{1 + \Pi\left(\pi\frac{z}{h}\right)\sin\left(\pi\frac{z}{h}\right)} \tag{2.7}$$

where $\kappa \approx 0.41$ is Von Karman's constant, and Π is Coles' wake strength parameter. As Nezu and Rodi (1986) report that Π vanishes for low-Re flows, it is assumed herein that $\Pi = 0$. The resulting distribution is the well-known Bakhmetev profile. Close to the bed (2.7) reduces to $l_0 \approx \kappa z$. Van Driest (1956) modified this expression for the mixing length so as to take into account viscous effects near a smooth wall. Van Driest's expression reads

$$l(z) = \kappa z\left[1 - \exp\left(-\frac{1}{A}\frac{u_* z}{\nu}\right)\right] \tag{2.8}$$

where A is a constant, $A \approx 26$. For $z << A\nu/u_*$ Eq. (2.8) gives $l(z) \approx \kappa u_* z^2/(A\nu)$ and for $z >> A\nu/u_*$ the mixing length approaches the high-Re value, κz.

Because ν_T also tends to zero near the interface, viscous effects may also become dominant near z = h. However, these viscous effects must be much smaller than those near the bed, because there is free slip at z = h. As the shear stress vanishes at this level, the distribution of the mixing length near z = h is less important. Therefore any viscous effects near the interface are neglected, and (2.7) is combined with Van Driest's expression to give

$$l(z) = l_0(z)\left[1 - \exp\left(-\frac{1}{A}\frac{u_* z}{\nu}\right)\right] \tag{2.9}$$

This modified Van Driest formula is adopted in the mixing-length model under consideration. It reduces to (2.8) for $z/h << 1$.

2.2 Laminarisation

The Van Driest modification of the mixing-length distribution aims at correctly reproducing the viscous effects on the near-wall mean-velocity distribution in an otherwise fully turbulent flow. However, it does not predict laminarisation of flows at Reynolds numbers that are acceptable from a physical point of view. An additional criterion for laminarisation must therefore be introduced. As indicated in Section 1, such criterion should be based on the Reynolds number of turbulence. Because the only turbulence parameter in the PML model is the eddy viscosity, Eq. (1.2) is adopted as a starting point for a laminarisation criterion. As the turbulence in the flow under consideration is not homogeneous because of wall influence, a choice must be made as to a representative eddy viscosity on which to base the laminarisation criterion. The approach pursued herein is to select the maximum value, ν_{Tm}, of the vertical distribution of the eddy viscosity and to introduce an empirical proportionality coefficient, c, in (1.2). This coefficient also accounts for a possible influence of the inhomogeneity of the turbulence on the critical Reynolds number. The criterion for laminarisation thus becomes

$$\frac{\nu_{Tm}}{\nu} < c\,\sigma_T\,Re_{Tc} \tag{2.10}$$

This criterion is related to the velocity U through (2.6). The selected value of Re_{Tc} is 15, as before, and that of σ_T is 0.7 (e.g., Hinze, 1975). The coefficient c then should be of the order one.

A layer-depth-averaged eddy viscosity could be used instead of the maximum value. As these two values are more or less proportional to each other, the two approaches are likely to be equivalent.

Because the PML model is not capable of simulating memory effects in turbulence, the eddy viscosity is equated to zero when the laminarisation criterion is satisfied.

2.3 Model calibration

The criterion (2.10) for laminarisation seems to be new, and the mixing-length model proposed has to be calibrated to obtain correct critical mean-flow Reynolds numbers.

An experiment that is suitable to determine the coefficient c in (2.10) is described by Badri Narayanan (1968). That author examined laminarisation of air flow in a duct with rectangular cross-section and constant height. The flow was led through a narrow section, so that it was turbulent, after which it passed through a two-dimensional diffuser to reach a wide section of constant width. In the diffuser the mean-flow Reynolds number decreased to attain a constant value in the wide section. For Reynolds numbers, Re, in the wide section less than $Re_c = 1400 \pm 50$ the turbulence decayed gradually, and the flow became laminar at some distance downstream of the diffuser. Here Re is defined as $U_m h/\nu$, where U_m is the mean flow velocity in the wide section and h half the height of the duct.

To simulate the flow in the wide section of Badri Narayanan's set-up, Eq. (2.7) has to be modified for flow in a duct. Dean (1978) recommended, on the basis of a literature survey, an empirical mean-velocity distribution for high-Re flow in wide ducts that accurately fitted the data available at the time. The mixing length, $l_0(z)$, resulting from this velocity distribution is given by

$$l_0(z) = \frac{\kappa z \left(1 - \frac{z}{h}\right)^{1/2}}{1 + 3.68\left(\frac{z}{h}\right)^2 - 4.68\left(\frac{z}{h}\right)^3} \qquad \left(0 \leq \frac{z}{h} < 1\right) \qquad (2.11)$$

Again, h is half the height of the duct. This expression was substituted in (2.9) instead of (2.7).

The numerical model described in Section 3.1 was used to solve the governing equations in the domain (0,h). In order to determine the coefficient c the laminarisation criterion (2.10) was switched off, the flow rate $q = U_m h$ was kept constant, and the selected Reynolds numbers Re were 1350, 1400 and 1450. As an initial condition an approximate velocity distribution (linear in the viscous sublayer and logarithmic elsewhere) was prescribed. The computation continued until a steady-state situation had been reached. The coefficient c was then calculated from (2.10). The value of this coefficient thus obtained for $Re_c = 1400 \pm 50$ is 0.61 ± 0.02, which value indeed is of the order one, as required.

3 Numerical computations

3.1 Case examined

In order to simulate laminarisation in flows of CBS, Eqs. (2.4) through (2.6) and (2.9) were solved numerically together with the boundary conditions mentioned in Section 2.1for a prescribed time-dependent flow rate. The numerical scheme used was the explicit Euler scheme. The grid was equidistant and the grid size was less than the thickness of the viscous sublayer. For the results shown, a grid size of h/100 was found sufficient for convergence of the solutions.

The flow rate q prescribed is either constant, or is a decreasing function of time given by, see Figure 1,

$$q(t) = \frac{q_1 + q_2}{2} - \frac{q_1 - q_2}{2} \tanh \frac{2t - t_2 - t_1}{t_2 - t_1} \tag{3.1}$$

where q_1, $q_2 \leq q_1$, t_1 and $t_2 > t_1$ are positive costants (q_2 may be equal to zero). The flow rate decreases from a value less than q_1 to q_2; the time scale of the change in flow rate is $(t_2 - t_1)$.

Dimension analysis is introduced to order the numerical results. A suitable parameter to non-dimensionalise the velocity is the mean velocity. However, because the mean velocity depends on time, it is replaced with the maximum mean velocity, $U_1 = q_1/h$. The velocity then can be written as

$$\frac{U}{U_1} = f\left(\frac{z}{h}, \frac{U_1 t}{h}; Re_1, \frac{q_2}{U_1 h}, \frac{U_1 t_1}{h}, \frac{U_1 t_2}{h}\right) \tag{3.2}$$

where f is a function to be determined from the computations, and $Re_1 = U_1 h/\nu = q_1/\nu$.

3.2 Results

To examine whether the modified Van Driest model applies at Reynolds numbers near the critical value, a calculation was made for one of Badri Narayanan's (1968) experiments. The Reynolds number Re in this experiment was 865, well below the critical value. Therefore the flow just downstream of the diffuser, that is, when the turbulence was in an incipient state of decay, was simulated. To this end the laminarisation criterion (2.10) was switched off and the flow rate was kept constant. The computation continued until the velocity profile did not change anymore. As shown in Figure 2, the velocity profile thus computed compares well with Badri Narayanan's measurements.

This good agreement, even in the case of incipient decay, indicates that at Reynolds numbers above the critical value the modified Van Driest model suffices to represent viscous effects.

Figure 2 also shows the final distribution of the eddy viscosity. The vertical tangent in the origin of the plot of ν_T versus z results from the exponential function in (2.9). The decrease of ν_T near the centre-line ($z/h = 1$), which is a consequence of Dean's (1978) expression for the high-Re velocity distribution in ducts, illustrates the fact that there is no turbulence production at the centre-line.

An example of laminarisation in a slowly decelerating flow of a CBS layer, for which (2.7) applies, is shown in Figure 3. The values of the dimensionless parameters in (3.2) are $Re_1 = 1340$, $q_2/(U_1 h) = 0.756$, $U_1 t_1/h = 5.37$ and $U_1 t_2/h = 107$. The total simulation time is given by $U_1 t/h = 537$. In addition $c = 0.61$ and $\Pi = 0$.

Transition to laminar flow was found to occur at $Re = Re_c \approx 1020$ at $U_1 t/h \approx 220$. This value of Re_c is quite acceptable from a physical point of view. The final Reynolds number q_2/ν was chosen, by trial and error, slightly less than Re_c to ensure that the flow was in a quasi-steady state at the instant of transition. Also see Figure 1.

Figure 3 shows that after laminarisation, the velocity distribution gradually changes to the well-known parabolic profile for laminar flow. The left-hand panel reflects the assumption that $\nu_T = 0$ for $Re < Re_c$.

Results of a computation with a larger decrease in flow rate and, as a consequence, in Reynolds number are shown in Figure 4. The values of the dimensionless parameters in this case are $Re_1 = 3050$, $q_2/(U_1 h) = 0$, $U_1 t_1/h = 12.2$ and $U_1 t_2/h = 489$. The total simulation time is given by $U_1 t/h = 550$. The values of the coefficients c and Π are the same as before. Transition to laminar flow occurs at $U_1 t/h \approx 380$. The Reynolds number at that instant is about 980, which value differs from Re_c because of the unsteadiness of the flow. The left-hand panel of Figure 4 shows that in the turbulent regime the viscous sublayer gradually thickens. In the laminar regime, flow reversal would occur near the bed, if the computation were continued. However, this aspect of the flow process is beyond the scope of this work.

Evidently, more advanced low-Re turbulence models are available nowadays, e.g., various low-Re k-ϵ models. Nevertheless, it may be concluded that the low-Re mixing-length model developed gives a good insight into the effect of viscosity on laminarisation, by which process turbulent exchange of mass and momentum reduces to zero.

4 Discussion

When compared to the transition from laminar to turbulent flow, the reverse transition from turbulent to laminar flow is a relatively slow process. The experiments of Badri Narayanan (1968) show that at Re = 865, for example, the turbulence had nearly decayed at a distance of about 45 times the height of the duct downstream of the diffuser. For Re = 625 this distance was 35 times this height. Decaying turbulence cannot be easily simulated with a mixing-length turbulence model. Models of this type assume local hydrodynamic equilibrium of the turbulence and disregard memory effects. As a result the transition is instantaneous. Mixing-length models could be modified so as to account for memory effects on an *ad hoc* basis, but a two-equation turbulence model seems more appropriate for this purpose.

In this report a constant effective molecular viscosity was introduced to model the rheological behaviour of CBS. However, rheological properties of CBS may be non-Newtonian in that the viscosity of high-concentrated cohesive sediment decreases markedly with increasing shear rate, because floc sizes decrease with increasing shear rate. In the case of turbulent flow, the question arises how an effective shear rate could be estimated. Assuming that the floc size is proportional to the Kolmogorov length scale of the turbulence, the appropriate shear rate parameter is $G \equiv (\epsilon/\nu)^{1/2}$, where ϵ is the dissipation rate (Camp and Stein, 1943). The parameter G is largest near the bed and decreases in the upward direction. As the time fluid parcels need to move across the layer depth, which time is of the order h/u_*, in most cases is larger than the few seconds flocs need to adapt to a new shear condition (Van Kessel and Blom, 1998), the viscosity is lowest near the bed and increases in the upward direction. The largest and smallest values of ϵ are about $0.25u_*^4/\nu$ and $0.8u_*^3/h$, respectively (e.g., Hinze, 1975). The largest and smallest values of G then are about $0.5u_*^2/\nu$ and $0.9(u_*^2/\nu)(u_*h/\nu)^{-1/2}$. Because near transition u_*h/ν is much larger than one, there is a considerable variation in G (and consequently in ν) across the depth of the CBS layer. Assuming, for example, $u_* = 0.01$ m/s, h = 0.5 m and $\nu = 10^{-5}$ m^2/s (bed) to 10^{-4} m^2/s (interface), the shear parameter G is found to vary from 5.0 s^{-1} at the bed to 0.13 s^{-1} at the interface.

As a next step in low-Re turbulence modelling, laminarisation of CBS flows including the variations in ν across the layer depth could be considered. However, empirical data on laminarisation in such flows seems to be non-existent. A simplified modelling approach would be to derive an overall shear rate parameter from the vertically averaged dissipation rate, which for the flow under consideration is about $u_*^2U_m/h$, and to estimate a constant viscosity using this parameter.

In a slightly more advanced approach the depth dependence of the viscosity could be included. Lumley (1978) argues that, because turbulence is dominated by inertia, turbulence properties should be the same for all media, provided the length and time scales of the turbulence are large compared to those characterising the medium. This condition is not satisfied, and inertia is not dominating, near a rigid bed. Because the laminarisation criterion (2.10) uses the maximum eddy viscosity, which is found in the inertia dominated region, Lumley's argument implies that this criterion would still apply. The molecular viscosity in this criterion then should be deduced from the shear rate parameter G at the location where the eddy viscosity is maximal. Alternatively, the location where ν_T/ν is maximal could be used. However, it may be necessary to modify the Van Driest model so as to account for non-Newtonian near-bed effects. A simple approach would be to adopt a near-bed value of the molecular viscosity in the Van Driest expression.

Another aspect of CBS rheology is yield stress. A criterion for transition from laminar to turbulent flow of a yield-stress fluid is known from the literature (e.g., Liu and Mei, 1989). However, this criterion is likely not to apply to reverse transition, that is, laminarisation. Network structures present in fluid mud, for example, are disrupted by turbulence-induced shear so that a suspension of flocs comes into existence. These flocs do have strength, but on a macroscale floc strength results in an increase in effective viscosity of the suspension rather than in a yield stress (Kranenburg, 1999). Therefore, the introduction of yield stress may not be correct in the case of turbulent flow of CBS.

Acknowledgements

The writer is grateful to Mr. A.M. den Toom of the Laboratory of Hydromechanics, who implemented the low-Re mixing-length model, and to Rob Uittenbogaard and Han Winterwerp of WL | delft hydraulics for valuable comments. This work was carried out as part of the MAST3-COSINUS Project. It was partially funded by the European Commission, Directorate General XII, under Contract No. MAS3-CT97-0082.

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Appendix - On Ivey and Imberger's (1991) second criterion for the damping of turbulence

The second criterion Ivey and Imberger (1991) propose for the damping of turbulence, reads

$$\frac{\epsilon}{\nu N^2} < \text{about } 15 \tag{A1}$$

where ϵ is the dissipation rate and N the buoyancy frequency (see Turner, 1973). Those authors do not distinguish between shear flow and other types of flow. Restricting the analysis to shear flow, results of Schumann and Gerz (1995) can be used to rewrite (A1). Schumann and Gerz present a compilation of data and approximate algebraic correlations for homogeneous turbulence in shear flow. These data were obtained from experiments, large-eddy simulations and direct numerical simulations reported in the literature.

Eq. (10) of Schumann and Gerz (1995) reads, in the present notation,

$$K_T = c_h \frac{\epsilon}{N^2} \tag{A2}$$

where K_T is the eddy diffusivity and c_h a coefficient that depends on the gradient Richardson number characterising density stratification. As indicated in the Introduction, K_T equals $u'L$. Substituting from (1.1), Eq. (A2) therefore can be written as

$$\frac{\epsilon}{\nu N^2} = \frac{1}{c_h} Re_T \tag{A3}$$

The damping criterion (A1) then becomes

$$Re_T < \text{about } 15 c_h \tag{A4}$$

Table 3 of Schumann and Gerz (1995) shows that the coefficient c_h never becomes larger than 0.2. It is therefore concluded that for shear flow Ivey and Imberger's (1991) first criterion, that is, Re_T less than about 15, is the more severe one.

FIGURES

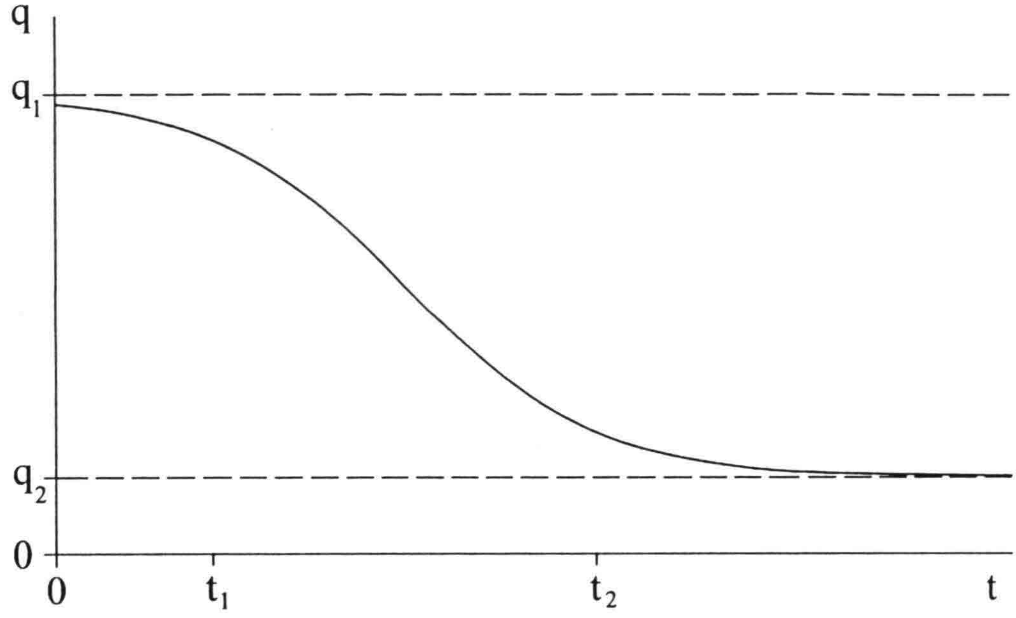

Figure 1. Prescribed flow rate according to Eq. 3.1.

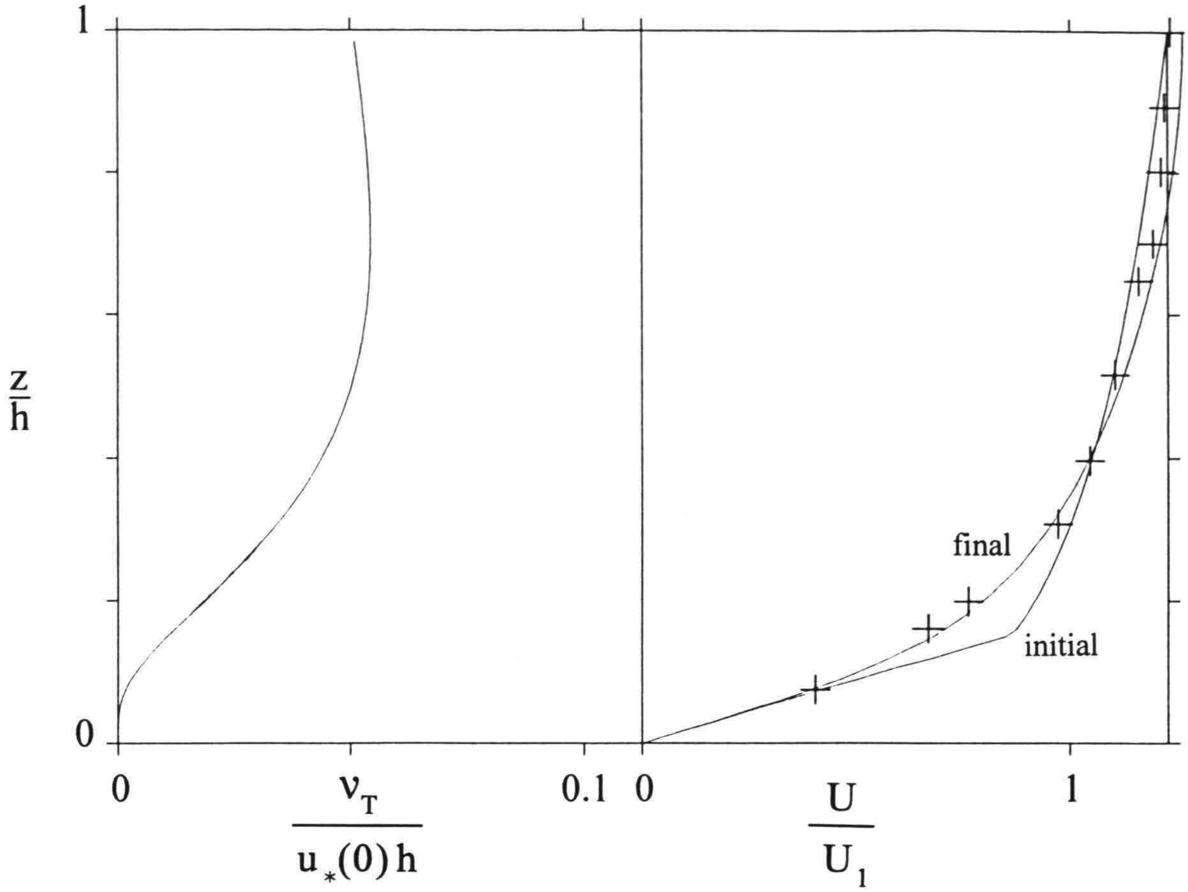

Figure 2. Simulation of flow in incipient state of decay at Re = 865. The initial velocity distribution consists of a linear part near the bed and a logarithmic profile above, representing fully laminar and fully turbulent flow, respectively. Crosses represent Badri Narayanan's (1968) mean-velocity measurements.

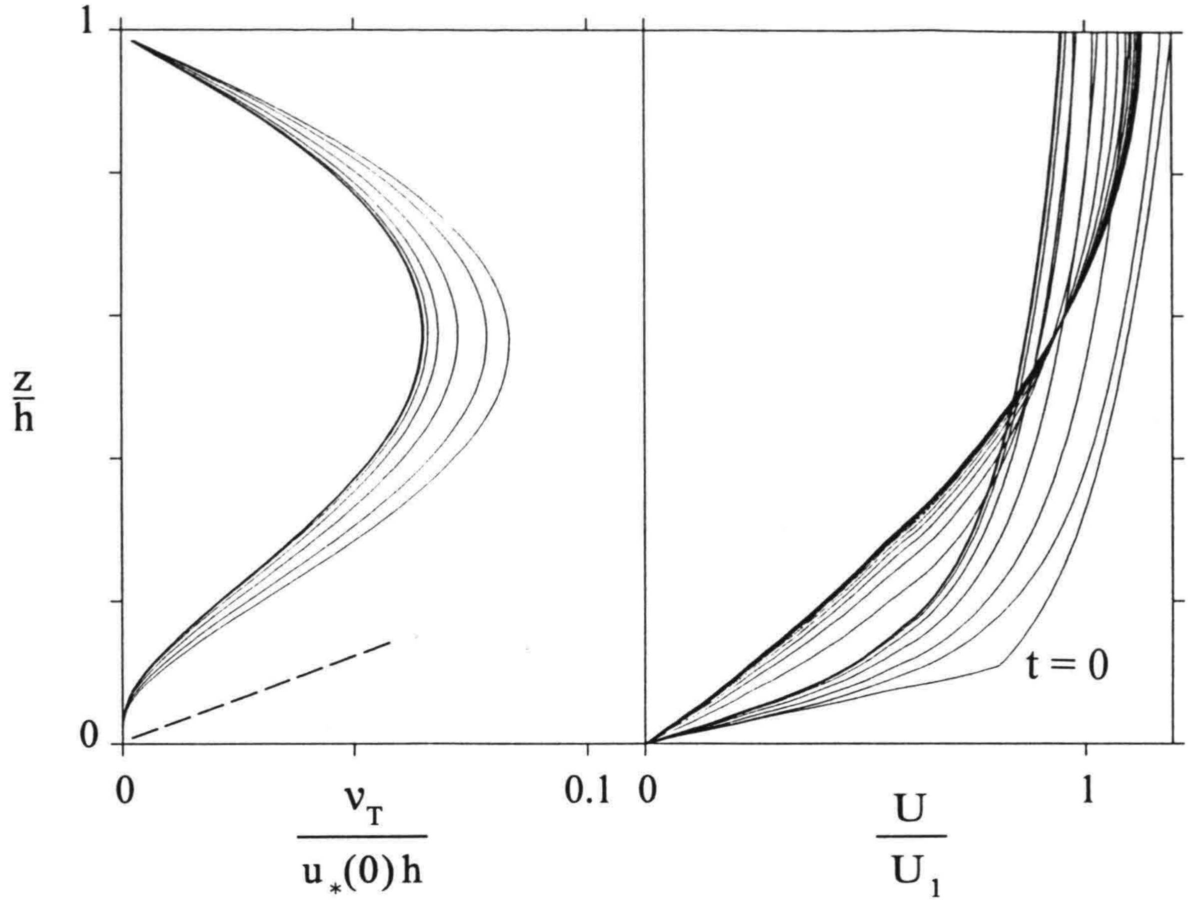

Figure 3. Simulation of slowly decelerating flow of a CBS layer for determining the critical Reynolds number. The time interval between plots is given by $U_1\Delta t/h$ = 26.9. The dashed line represents a high-Re eddy viscosity given by $\nu_T = \kappa u_*(0)z$.

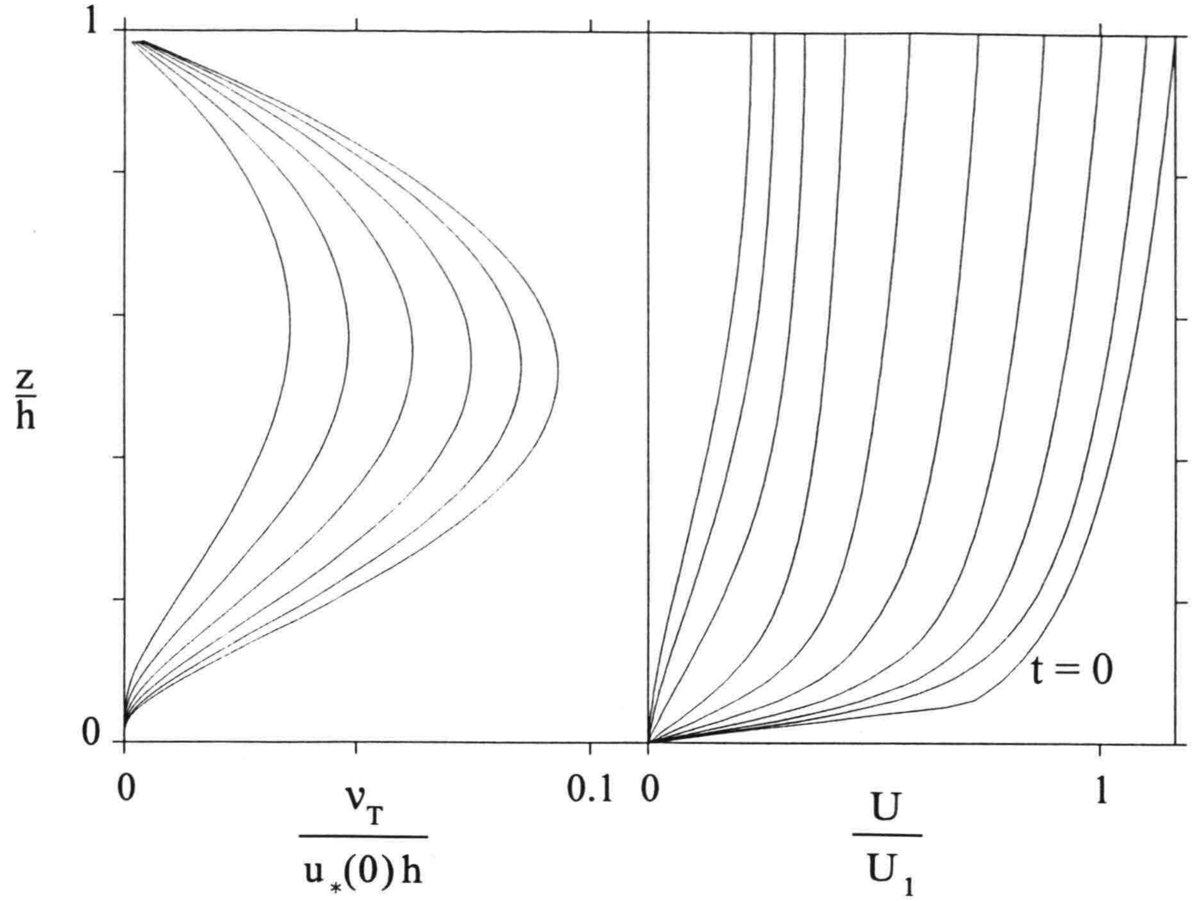

Figure 4. Example of decelerating CBS flow. The time interval between plots is given by $U_1\Delta t/h = 61.1$.