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PUSHOVER ANALYSIS OF A FIXED STEEL OFFSHORE PLATFORM

part I

BEAM-COLUMN AND STRUT ELEMENTS IN THE INTRA PROGRAM

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PUSHOVER ANALYSIS OF A FIXED STEEL OFFSHORE PLATFORM

part 1: BEAM-COLUMN AND STRUT ELEMENTS
IN THE INTRA PROGRAM

part 2: PILES AND PILEGROUPS UNDER EXTREME LOADS

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SUMMARY

The computer program INTRA models the collapse behavior of trusswork offshore structures with the aid of two types of elements: 'Beam-column' elements, designed to simulate bending dominated failure of structural members, and 'strut' elements, that can represent normal force dominated failure.

The behavior of both types of INTRA elements has been investigated by comparison with the results of analytical solutions and of the MARC finite element program. A satisfying performance has been found for the beam-column element BEMC and the strut element ISTR.

A Fortran routine has been established to generate the required input parameters for INTRA strut elements, resulting in more exact input values than can be derived with the aid of the default property generation.

A parameter study has been carried out to the behavior of strut elements under conditions that may occur in the modelling of a large North Sea platform. It appears that lateral loads and imposed end rotations each may result in a maximum reduction of buckling strength of 15 %, but that, however, the average reduction is much lower. Also the post-buckling strength is only modestly influenced.

A plane frame pushover analysis has been performed with both the INTRA and the MARC program. The influence of schematizations in INTRA strut elements at the overall structural behavior appears to be acceptably small. However, serious numerical problems occurred during the INTRA analysis, resulting in a too low prediction of the ultimate load of the platform.

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NOMENCLATURE

A	area	[L ²]
\bar{B}	strain interpolation matrix	-
C	reduction factor	-
d	waterdepth	[L]
D	outer diameter	[L]
D_m	centre diameter = D-t	[L]
E	Youngs modulus	[F/L ²]
\bar{E}	matrix of material stiffness properties	-
\bar{f}	vector of nodal loads	-
F	force	[F]
F_E	Euler buckling load	[F]
F_{ult}	ultimate compressive load	[F]
F_y	squash load = A * σ_y	[F]
I	moment of inertia	[L ⁴]
K	effective length factor (fixity ratio)	-
\bar{K}	stiffness matrix	-
L, l	length	[L]
M	bending moment	[FL]
M_a, M_b	imposed end moments	[FL]
M_{pl}	plastic moment	[FL]
$M_{pl,red}$	plastic moment, reduced by normal force	[FL]
N	normal force	[F]
$N_y = N_p$	axial yield load or squash load = A * σ_y	[F]
P	force	[F]
q	lateral load per unit length	[F/L]
R	radius of gyration = $\sqrt{I/A}$	[L]
T, t	wall thickness	[L]
\bar{u}	vector of nodal displacements	-
V	element volume	[L ³]
w	displacement	[L]
x,y,z	coordinates	-

β	slope	-
γ	utilization ratio	-
δ	variational symbol	-
δ	lateral deflection	[L]
δ_0	intial lateral deflection	[L]
Δl	axial shortening	[L]
ϵ	strain	-
ϵ_{ax}	axial strain in centre of section	-
$\bar{\epsilon}$	vector of strains	-
κ	curvature	-
λ	load factor	-
λ	slenderness ratio = $K \cdot l / R$	-
λ_{mod}	modified slenderness ratio = $\frac{1}{\pi} * \sqrt{(\sigma_Y / \epsilon)} * \lambda$	-
ξ	amplification factor	-
σ_y	yield stress	[F/L ²]
$\bar{\sigma}$	vector of stresses	-
ϕ	angle	-

INTRA elements:

BEMC	beam-column element
ISTR	strut element
LANB	beam-column element
LTRS	linear truss element
NBEM	beam-column element (old)
NTRS	nonlinear truss element
STRT	strut element (old)

1. INTRODUCTION

The computer program INTRA is designed to analyse the nonlinear behavior of offshore platforms subjected to static loadings and earthquake excitations.

This report concentrates on some properties of INTRA with respect to static 'pushover' ultimate strength analysis.

The program models three dimensional frames with one element for each member. Several types of elements exist, and two important groups can be distinguished:

- Truss elements with axial stiffness but without bending stiffness, so-called 'strut' elements. An axial load versus axial displacement relationship, defining buckling and post-buckling behavior, has to be entered in advance. The elements should be used at places where normal-force dominated failure can be expected.
- Truss elements with both axial and bending stiffness, so-called 'beam-column' elements. The elements are designed to represent bending dominated failure. A description of a yield surface has to be input to take into account interaction between normal forces and bending moments. The usual INTRA schematization of an offshore platform consists of legs with axial and bending stiffness, connected by braces that have only axial stiffness, see figure 1.

Loadings are schematized as nodal forces and nodal moments. Static pushover analysis can be performed by an incremental increase of the forces of a certain load pattern, until collapse occurs.

The documentation on the input specifications and the theoretical background of INTRA is incomplete, while the reliability of some recently developed elements has not been sufficiently proven.

Three subjects have been studied to obtain some insight into those problems:

- 1) The behavior of strut elements.
- 2) The behavior of beam-column elements.
- 3) The behavior of a plane frame modelled with the help of the two element types.

Several comparisons have been made with analytical solutions, and with results of the MARC finite element program that can take material and geometrical nonlinearities into account in a sophisticated way.

This report will give an analytical description of the buckling and postbuckling behavior of struts and beam-columns, in chapter 2. A brief review will also be given of the properties of the INTRA and MARC elements involved in the three studies mentioned above, see chapters 3, 4 and 5. Subsequently the above three studies will be dealt with. More in detail, the following subjects will be described:

- 1a) The derivation of a computational method to calculate the required input (buckling) parameters for INTRA strut elements. (Chapter 6)
 - b) The relative importance of some parameters that influence buckling behavior in the specific situation of a North Sea fixed steel offshore platform. (Chapter 7)
 - c) The comparison of analytical, MARC and INTRA results. (Chapter 8)
- 2) An extension of an existing study on INTRA beam-column elements under combined axial and lateral loading. Analytical, MARC and INTRA results have been compared. (Chapter 9)
 - 3) A push over analysis of a two-dimensional schematization of a North Sea platform. MARC and INTRA results have been compared to obtain insight in the effect of assumptions and simplifications in the INTRA elements on the collapse behavior of a complete structure. (Chapter 10)

2. BUCKLING AND POST-BUCKLING BEHAVIOR

2.1 Axial force versus lateral deflection

Consider a pin-ended 'perfect' column, i.e. made out of perfect elastic material, straight, without residual stresses, etc, except for a virtual small initial imperfection.

(No difference will be made in this chapter between columns, struts or beam-columns)

Figure 2a depicts the axial force, F , versus the lateral deflection of the centre of the column, δ . Elastic buckling occurs at the Euler buckling load F_E , with

$$F_E = \frac{\pi^2 * EI}{(K*l)^2}$$

E = Youngs modulus

I = moment of inertia

l = length

K = fixity ratio

(equals one in case of pin-ended column)

'Perfect' columns do not actually exist, and therefore it is necessary to take initial imperfections into account. E.g., when the column has a sine-shaped initial deflection with a maximum amplitude δ_0 , its F versus δ characteristic is given by:

$$\delta = \xi * \delta_0 = \frac{1}{1 - \frac{F}{F_E}} * \delta_0$$

This expression results in the graph of figure 2b.

Similar expressions can be derived for the amplification factor ξ in the case of non-sine-shaped deflections, for example in case of a parabolic shape due to the lateral loading, see ref.[16]. However, the use of those alternative expressions will produce almost identical results, as long as small imperfections are considered.

The elasto-plastic behavior of a beam-column with initial deflection is depicted in figure 2c, starting from a bi-linear schematization of the moment-curvature relationship, see figure 3.

The shape of the descending curve can be expressed in a simple formula, starting from the 'plastic hinge' concept, i.e. assuming that all the plastic deformations are concentrated in infinite small areas, see appendix B. The required expression follows from the configuration of the buckled column in figure 2c, considering that after a plastic hinge has been formed in the centre of the column, the equilibrium requires:

$$F = \frac{M_{pl,red}}{\delta} \quad (*)$$

$M_{pl,red}$ represents the plastic moment of the section, M_{pl} , reduced by the normal force $N = F$, see appendix A.

Note that the above expression gives an upper bound, because it assumes a sudden development of the plastic hinge, due to the simplification of the M versus κ diagram. The actual F versus δ curve is shown in figure 4. Points A and C in that figure, representing first yield and full plastic hinge development respectively, can be established in an iterative way. However, establishment of point B in figure 4 requires a more complex calculation.

In routine design the value of F_{ult} (i.e. point B in figure 4) is estimated with the aid of a design curve, based on theoretical or experimental results, see appendix C.

The effect of lateral loading or end moments at the value of F_{ult} can be approximated with the help of a so called 'interaction formula', see also appendix C.

2.2 Axial force versus axial shortening

Consider again the initially deflected column with a bi-linear schematized $M - \kappa$ diagram, see figures 2c and 5a.

The shape of a graph representing the axial load F versus the axial shortening Δl , depends on the elasto-plastic behavior of the entire column, in a complex way as described in appendix B. However, simple closed form expressions can be derived using the plastic hinge concept. In this case four contributions can be distinguished (considering relatively small lateral deflections, i.e. $\sin \delta/2 \approx \delta/2$):

1) Shortening due to elastic axial compression:

$$\Delta l_1 = \frac{F \cdot l}{E \cdot A}$$

A = cross sectional area

2) Shortening due to the lateral displacement δ of the centre of the column, see figure 5b:

$$\begin{aligned} \Delta l_2 &= 2 * \left(\frac{1}{2}l - \sqrt{\left(\left(\frac{1}{2}l\right)^2 - \delta^2\right)} \right) \\ &= 2 * \left(\frac{1}{2}l * \left(1 - \cos(\arcsin \delta / \frac{1}{2}l)\right) \right) \end{aligned}$$

what can be written as a function of F with the aid of the above mentioned expression (*) that relates axial force to lateral deflection in the post-buckling situation:

$$F = \frac{M_{pl,red}}{\delta}$$

3) Shortening due to elastic curvature of the two parts of the column:

$$\Delta l_3 = 2 * \int_0^{\frac{1}{2}l} \frac{1}{2} * \left(\frac{dw}{dx}\right)^2 dx$$

x = coordinate along column axis, see figure 5c

w = displacement perpendicular to x-axis

This formula can also be written as a function of F with the aid of expression (*).

4) Shortening due to plastic strains in the plastic hinge.

The latter contribution, Δl_4 , can be established as follows:

The relationship between axial force, F, and bending moment, M, in a plastic hinge in a circular section is given by the equation of the yield contour: (see appendix A)

$$\frac{M}{M_{pl}} = \cos \left(\frac{\pi * N}{2 * N_y} \right)$$

M_{pl} = full plastic moment of section, see appendix A

$N_y = F_y = \text{squash load} = A * \sigma_y$

σ_y = yield stress

Theory of plasticity states that the vector of plastic strain increments is perpendicular to this yield contour.

The equation of the contour can be approximated with:

$$\frac{M}{M_{pl}} + \frac{N}{N_y} = 1, \text{ see figure 6}$$

This results in a simple expression relating Δl_4 to the curvature in the plastic hinge, and hence to the lateral deflection, δ , or to the axial force, F.

$$\Delta l_4 = 2 * \frac{M_{pl}}{N_y} * \arcsin(\delta / \frac{1}{2}l)$$

Figures 7 and 8 depict the F versus δ diagram and the F versus Δl diagram of a certain test column, derived with the above analytical approximations. Figure 9 represents again the F versus Δl results, showing the different contributions to the axial shortening in the post-buckling range. It can be seen that in the beginning the elastic shortening, Δl_1 , still dominates, whereas towards the end the change in geometry, Δl_2 , and the plastic shortening, Δl_4 , have the largest influence. Shortening due to elastic curvature, Δl_3 , is of minor importance.

NOTE: Considering relatively large deflections, it should be recognized that: $\Delta l_{global} = \Delta l_{local} * \cos \phi$, and $N = F * \cos \phi$,

$$\text{with: } \phi = \arcsin(\delta / \frac{1}{2}l)$$

NOTE: The above theory implicitly assumes that no local buckling will occur and that full plastic moment development is possible. It is also assumed that sufficient rotation capacity is available to achieve large post-buckling deformations. These are valid assumptions for tubular members with a D/T ratio upto about 50, see e.g. ref.[3]. Indeed actual platform members seldom exceed this value due to the regulations in the design codes.

The described methods will be applied in this report to check the results of several computer calculations. They will also be used in the establishment of a Fortran routine that calculates the input parameters for an INTRA strut element.

References: [3], [12], [18]

3. INTRA STRUT ELEMENTS

The linear truss element LTRS is the most simple element within INTRA. It has no bending stiffness and its axial force versus axial displacement characteristic is depicted in figure 10a.

Figure 10b shows the nonlinear variant NTRS. A different ultimate strength in tension and compression (buckling) can be defined.

The 'Marshall' strut element STRT has a more sophisticated behavior. It models post-buckling softening in the compression branch, see figure 10c. The shape of this so called 'backbone' curve has to be defined in advance, and therefore all the parameters influencing the shape should be known in advance too.

INTRA has the possibility to generate default values describing the backbone curve, as a function of input geometrical and stiffness properties. Figure 11 shows that in this way the response on multiple reversed loading can be modelled in a refined way, what is of great importance for earthquake analysis. Indeed, much efforts have been spent on a default generation of the degrading strength and stiffness in the subsequent loading cycles, see e.g. reference [10].

Recently a more refined strut element, ISTR, has been introduced, see figures 10d and 11, and at the same time the default input generation has been improved. Unfortunately however, no information is available on the theoretical background of the default generation, nor of the STRT, neither of the ISTR element.

4. INTRA BEAM-COLUMN ELEMENTS

Two elements will be treated, the BEMC and the LANB element, which can both take into account material and geometrical nonlinearity.

INTRA beam-column elements can resist nodal forces and moments. They can develop plastic hinges at both ends under the combined action of bending, normal force, and torsion.

A plastic hinge in an actual beam-column develops gradually because yielding starts in the extreme fibers of the section and then gradually proceeds towards the centre. INTRA approximates the plastic hinge development as follows:

Each beam-column element consists of several parallel elasto-plastic sub-elements that have a different axial yield strength and a different plastic moment, see figure 12. The sub-elements work together in the elastic range, but when the loading increases they subsequently 'fail', and in this way the total element loses its stiffness in a stepwise fashion, see figure 13.

The program requires the input of the moment-curvature diagram and the force-strain diagram of the section of the beam-column, and the description of a three-dimensional yield surface that defines the ultimate combinations of normal force, bending moment and torsion. From those data the program automatically calculates the properties of the sub-elements.

The BEMC element consists of three sub-elements, and the LANB element of four. Other differences between the elements are the shape of the yield surface and the length of the plastic hinge.

Geometric nonlinear effects are approximated with the aid of a 'string' matrix, a simple variant of a geometric stiffness matrix, see appendix E.

Both elements have an option to represent elastic shear deformations, end excentricities and initial forces.

5. THE MARC 'NO 14' ELEMENT

The MARC 'no 14' element is especially designed to model the elasto-plastic behavior of tubular beam-columns.

Plastic hinges can develop at the ends or in the centre of the element. Gradual plastic hinge modelling is achieved with the aid of a subdivision of the section in 16 segments that can yield individually, see figure 14.

Deformations perpendicular to the element axis (deflections) are described with a cubic interpolation. It can be shown that the buckling load of a pin-ended column can be calculated within 1 %, by using only two elements that use the above interpolation, see figure 15. For the fixed ended column and for the 'one side pinned - one side fixed' situation the deviations are less than 2 % and 3% respectively, see ref.[2] and ref.[13].

The above values can be obtained provided that the derivation of a consistent geometric stiffness matrix is implemented in the program, see appendix E.

Testruns were performed on members with modified slenderness ratio, λ_{mod} , upto 1.25 and with an initial out-of-straightness, δ_0 , of 0.001 L. Figures 16 and 17 show typical results of such a calculation, in comparison with the analytical results from chapter 2 (i.e. figures 7 and 8). An acceptable agreement is found, although the MARC results in the descending branch are a little above the theoretical upperbound.

6. STRut INput Generator, STRING

STRING is a Fortran program that calculates several input parameters for the 'backbone' curve of the INTRA strut element ISTR, see figure 18.

The program requires interactive input of geometrical and material properties of the strut and of parameters that influence the buckling behavior. It can take into account the effect of fixity ratio, lateral loads, imposed end moments and hydrostatic pressure.

- Slope β in figure 18 corresponds with EA , where E equals Young's modulus and A the sectional area. A deviation of the straight line because of an initial deflection cannot be taken into account in the INTRA formulation.
- The buckling load F_{ult} is derived from a design curve, see figure 19. This curve gives the value of F_{ult} as a function of the slenderness.
- The postbuckling slopes $E3$ and $E5$ and the values of $U34$, $P45$, and $U45$ are estimated with the aid of the analytical method described in chapter 1 and appendix B.
- The influence of lateral loads and end moments at the value of F_{ult} is approximated with the aid of the AISC interaction formula, see appendix C. Lateral loads also influence the slope of the post-buckling branch.
- The influence of hydrostatic pressure is calculated starting from reduced values for the plastic moment and the axial yield load according to formulas in reference [3], see appendix C. Those expressions take into account the reduction of the tension yield stress in axial direction, due to the presence of circumferential compressive stresses. The possible development of local buckling is not reckoned with.

NOTE: It is assumed that no local buckling will occur and that full plastic moment development is possible. It is also assumed that sufficient rotation capacity is available to achieve large post-buckling deformations. These are valid assumptions for tubular members with a D/T ratio upto about 50.

NOTE: STRING is only capable to generate parameters for static INTRA analyses. In case of dynamic calculations the INTRA default option is indispensable in establishing the parameters for reversed loading and the degradation effects.

7. PARAMETER STUDY

This chapter treats the relative importance of several parameters that influence buckling and post-buckling behavior of struts.

Consider the analytical solution depicted in figure 8. The shape of the curve is defined by the relationship between axial force F and axial displacement Δl . With the aid of dimensional analysis it is possible to derive a range of dimensionless constants that influence the shape of this curve, see appendix D. Trying to establish the mutual influence of those factors would be insurveyable, and a more pragmatic approach has been chosen:

- The quantities along the axes, F and Δl , are replaced by the dimensionless combinations σ/σ_y and $\Delta/\Delta l$.
- The geometrical parameters of the buckling strut that influence the shape of the curve can be chosen as L/D and D/T ratios. Those two parameters have been varied according to the combinations in figure 20, that depict the scatter in L/D and D/T ratios in the braces of an actual North Sea platform.
- The other parameters have been chosen either fixed, like the σ_y/E ratio, or have been varied one by one: A maximum lateral load has been estimated for the braces of the above mentioned platform, and a maximum platform rotation has been approximated, resulting in imposed end moments at the braces. The effects of the maximum waterdepth (hydrostatic pressure) and of a change in the effective length factor of the braces (end fixity ratio) have also been investigated.

Figure 21 shows 'backbone' curves, calculated with the aid of STRING, for combinations of L/D and D/T ratios analogue to figure 20. The values of lateral loading and depth are varied, influencing in this way the buckling load and the complete postbuckling behavior, and it can be concluded that:

- The influence of the lateral load, Q_{LAT} , increases with increasing L/D ratio and is scarcely influenced by a change in D/T ratio.
- The influence of hydrostatic pressure mainly depends on the D/T ratio. (The larger D/T , the larger is the effect)
- Both effects result in a maximum decrease of the ultimate load of about 15%, while the slope of the postbuckling branch is not strongly affected.

Note that the maximum hydrostatic pressure (seabottom level) and the maximum lateral load (wave top level) can never occur at once.

Figure 22 shows the influence of end moments due to imposed rotation, and of a change in fixity ratio, two parameters that influence only the values of the buckling load and of the forces in the beginning of the post-buckling behavior. It can be concluded that:

- The influence of imposed end rotations is the largest for members with low L/D ratios.
- To the contrary, a change in effective length factor shows the largest influence at members with high L/D ratios.
- Again, both effects result in a maximum decrease of the ultimate load of about 15 %.

Note that there exist several parameters that also influence the value of the buckling load, but that are not mentioned in this parameter study. Think of residual stresses, out-of-roundness and out-of-straightness.

Those effects are included in the buckling curve incorporated in 'STRING', see figure 19 curve 'A'. This curve is especially developed for offshore applications, see appendix C.

Reference [4] gives some statistical information about the magnitude of imperfections and residual stresses as they appear in actual platforms, showing a good agreement with the values used in the above curve 'A'.

An extensive parameter study concerning the influence of out-of-straightness and residual stresses in ref.[15] shows only small influences from imperfections in the range of the above mentioned values.

8. STRUTS: ANALYICAL, MARC AND INTRA RESULTS

Figure 23a upto 23c represent a comparison of strut 'backbone' curves generated in various ways. From chapter 7 it follows that the main geometric parameter influencing the buckling and post-buckling behavior of struts is the L/D ratio, and therefore only this parameter has been varied.

MARC and 'analytic' results turn out to agree reasonably well, while the STRING schematization results in an acceptable approximation as far as possible within the required five-point INTRA input.

The INTRA default values show a slightly different pattern but nevertheless they reasonably agree with the other curves. However, a considerable 'yield plateau' occurs in case of stocky members, see figure 23c. Such a behavior can be expected in case of axial yield of a perfectly straight column, but is unlikely to appear when initial imperfections and lateral loading are present.

Lines E in figure 23 represent the advised backbone curve for the old INTRA strut element STRT as mentioned in reference [9], showing an apparently too sudden post-buckling stiffness drop.

9. BEAM-COLUMNS: ANALYTICAL, MARC AND INTRA RESULTS

Two testproblems will be described in this chapter:

- The collapse behavior of a clamped column under a concentrated lateral load with an axial preload, see figure 24, showing geometrical and material nonlinearities. This problem has been analyzed previously by Kerstens, see reference [8].
- The behavior of another clamped column loaded by axial and lateral forces, showing geometric nonlinearities more in detail.

First testproblem, see figure 24.

The two straight lines represent the analytical upperbound of the deflection δ of the first test column as a function of the lateral load P . The MARC results reasonably agree, although their values are a little above the theoretical upperbound, i.e. they overestimate the value of the full plastic moment of the section.

The INTRA results are derived using the default option for generation of the moment-curvature diagram. The overall agreement is acceptable, although the plastic hinge develops in a stepwise fashion instead of gradually as with MARC.

The results of the recently developed INTRA LANB element (not depicted) show a large error in the value of the ultimate load, a discrepancy that is probably caused by a programming error (ref.[5]).

Figure 25 gives some results of Kerstens (ref.[8]) that clearly show the weakness of the old INTRA element NBEM, and a poor result of the INTRA BEMC element using manual - non default - input.

Second testproblem, see figure 26

Geometric nonlinearities in the INTRA beam-column elements are taken into account with the aid of a 'string matrix', see appendix E. This simplified geometric stiffness matrix results in an underestimation of the decrease in bending stiffness due to axial loads. Figure 26a shows the lateral load versus the lateral deflection of a simple clamped column, and figure 26b shows the lateral load versus the rotation at the top. In the situation without axial load the INTRA result completely agrees with the analytical solution, but in the case with axial preload a small difference occurs, see also appendix E. This seeming minor deviation has an important effect in

making it impossible to simulate a buckling strut with the aid of two INTRA elements (normal force dominated failure), a simulation that is very well possible with the MARC '14' elements, see chapter 5.

Note that the geometric nonlinearities in the post-collapse situation are taken into account without difficulties, i.e. the descending branch in figure 24 is described correctly. The change in geometry of the collapsing beam is satisfactorily described with the aid of two straight elements, whereas in the establishment of the ultimate load it is essential to take the deformed shape of the elements into account in the geometric nonlinearity. However, in a 'string matrix' the bending deformations of the element are neglected, see appendix E.

10. A PLANE FRAME ANALYSIS: MARC AND INTRA RESULTS

The main reason for performing a comparative plane frame analysis with MARC and INTRA has been the uncertainty about the effect of several properties of the INTRA elements at the behavior of the complete structure. Important differences between the two programs at element level are:

<u>INTRA</u> (strut elements)	<u>MARC</u> (two 'no. 14' elements)
- neglect of bending stiffness	- inclusion of bending stiffness
- predefined influence of lateral loads and end moments	- actual influence of lateral loads and end moments
- predefined influence of initial imperfections	- difficult representation of initial imperfections
- sudden changes in stiffness in the 'backbone' curve	- gradual plastic hinge development

Possible differences between the two programs at structure level are:

- A difference in failure loads of the braces that may result in a different sequence of failing members, i.e. in a different failure path of the models. Eventually this may lead to a different ultimate strength of the models.
- An underestimation of the over-all elastic stiffness with INTRA because of neglect of the bending stiffness in the struts, against a more exact estimation of the elastic stiffness with MARC.
- A different failure path due to the above neglect of the bending stiffness in the struts.

Figure 27 shows the jacket structure of the large North Sea platform already mentioned in chapter 7. A schematic representation of one of the end frames is given in figure 28. The foundation is replaced by fixed supports, and the deckloading is represented as constant vertical forces.

Two computer models have been generated:

- An INTRA model in which the legs consist of BEMC beam-column elements and the bracing of ISTR strut elements.
- A MARC model that represents each member with the aid of two MARC '14' elements.

Simulation of a 40 meter wave with the MARC wave loading option resulted in a loading pattern of nodal forces and moments. Additional loads have been generated to represent the influence of the wave loading at the (non-modelled) remaining part of the three-dimensional structure. The loads at the 'centre nodes' of the members in the MARC model could not be applied directly at the INTRA model (INTRA elements have no centre nodes) and therefore a transformation to loads at the 'end nodes' has been carried out.

Two sets of comparative calculations have been made:

- A comparison between INTRA and MARC, starting from a schematization with pin-ended braces. In the INTRA model this is realized by specifying an effective length factor of 1.0, and in the MARC model by defining hinges between legs and braces, see figure 29a.
- A comparison between the two programs starting from a schematization with fixed-ended members. In INTRA the effective length factor now simply equals 0.5, but in MARC two schematizations have been chosen: A model with completely moment carrying joints, see figure 29b, and a model with fixed-ended members but nevertheless with pinned connections between legs and braces, see figure 29c. The latter schematization agrees with the INTRA approach.

Figure 31 depicts the results of the first comparison with pin-ended members, by representing the load factor λ versus the horizontal displacement of one of the nodes at deck level.

The MARC calculation shows a gradual loss of stiffness of the structure, due to buckling and yielding of the diagonals as indicated in figure 30c.

Eventually a number of eight braces fail and any increase in λ results in infinite deformations, i.e. the calculation is terminated due to an ill condition of the stiffness matrix. This does not necessarily mean that a complete collapse mode (mechanism) has been detected, but that at some place in the structure a further increase in nodal loading cannot be resisted anymore, (partial mechanism).

Using the INTRA program the same braces fail, but the calculation terminates at a much earlier moment, after failure of two braces instead of eight.

Figure 32 represents the comparison with fixed-ended members. Note that two different MARC schematizations have been used. It follows from lines A and B in figure 32 that the neglect of bending stiffness in the braces

results in an underestimation of the overall elastic stiffness of 5 %. The comparison of the INTRA results with those of the 'INTRA-like' MARC schematization (line B) shows a behavior equivalent to the situation with pin-ended members: Eight braces fail in the MARC analysis against only three in the INTRA calculation.

Upto the premature termination of the INTRA calculation, the both programs show the same failure pattern. Figure 33 depicts the utilization ratios γ of the members in the INTRA model at the maximum calculated load, where

$$\gamma = F / F_y \text{ for members in tension, and}$$

$$\gamma = F / F_{ult} \text{ for members in compression.}$$

Generally spoken the values of γ agree well with those found in the MARC calculation, although the buckling load of the members in the INTRA model is about 8 % lower than of the corresponding members in the MARC model, due to the implicate formulation of initial imperfections in the INTRA struts.

It can also be seen from figure 33 that the compressive forces in the horizontals do not exceed half the ultimate compressive strength. Consequently for the present configuration and loading pattern it appears to be unnecessary to model horizontals with the aid of strut elements.

A possible explanation of the premature termination of the INTRA program may be found in different numerical solution procedures within MARC and INTRA.

11. CONCLUSIONS

* An approximate analytical description of the (post-) buckling behavior of beam columns is an appropriate tool to check the behavior of computer representations of tubular members.

* The INTRA strut element ISTR satisfactorily represents buckling and post-buckling behavior (normal force dominated failure) when using the default property generation. A more sophisticated property generation can be achieved with the aid of the Fortran routine 'STRING' described in this report.

The recommendations mentioned in reference [9] result in unrealistic post-buckling behavior.

* A parameter study of aspects influencing the (post-) buckling behavior of tubular members in the modelling of a large North Sea platform yields the following results:

- Lateral loads and imposed end rotations each may result in a maximum reduction of the buckling strength of 15 %. However, the average reduction is much smaller, while also the post-buckling behavior is only slightly influenced.
- Hydrostatic pressure (175 m waterdepth) or a 10 % change in effective length factor result in even smaller reductions of the buckling load.

* The INTRA beam-column elements BEMC can model bending dominated failure in an acceptable way.

However, it is not possible to model the buckling behavior of a column (normal force dominated failure) using BEMC elements, due to the poor representation of geometric nonlinearity.

* A comparison between a plane frame pushover analysis performed with both the INTRA and the MARC programs yields the following conclusions:

- Modelling of horizontals and diagonals in the INTRA model with the aid of strut elements results in an under-prediction of the overall elastic stiffness of 5 %, due to the neglect of bending stiffness in the strut elements.

- Buckling only occurs in diagonals while compressive forces in the horizontals do not exceed half the buckling strength. Consequently in the present analysis horizontals could as well have been modelled with the aid of beam-column elements.
- Both MARC and INTRA calculations terminate before a complete collapse mechanism has been developed. The MARC results show large plastic deformations and an almost complete loss of stiffness. However, the INTRA program terminates at a much earlier point, thus predicting a too low ultimate strength.
- Upto the premature termination of the INTRA calculation the both programs show the same failure pattern. The buckling load of the INTRA members appears to be 8 % lower than of the corresponding MARC members, due to the implicate formulation of initial imperfections in the INTRA strut elements.
- A possible explanation of the premature termination of the INTRA program may be found in different numerical solution procedures within MARC and INTRA.

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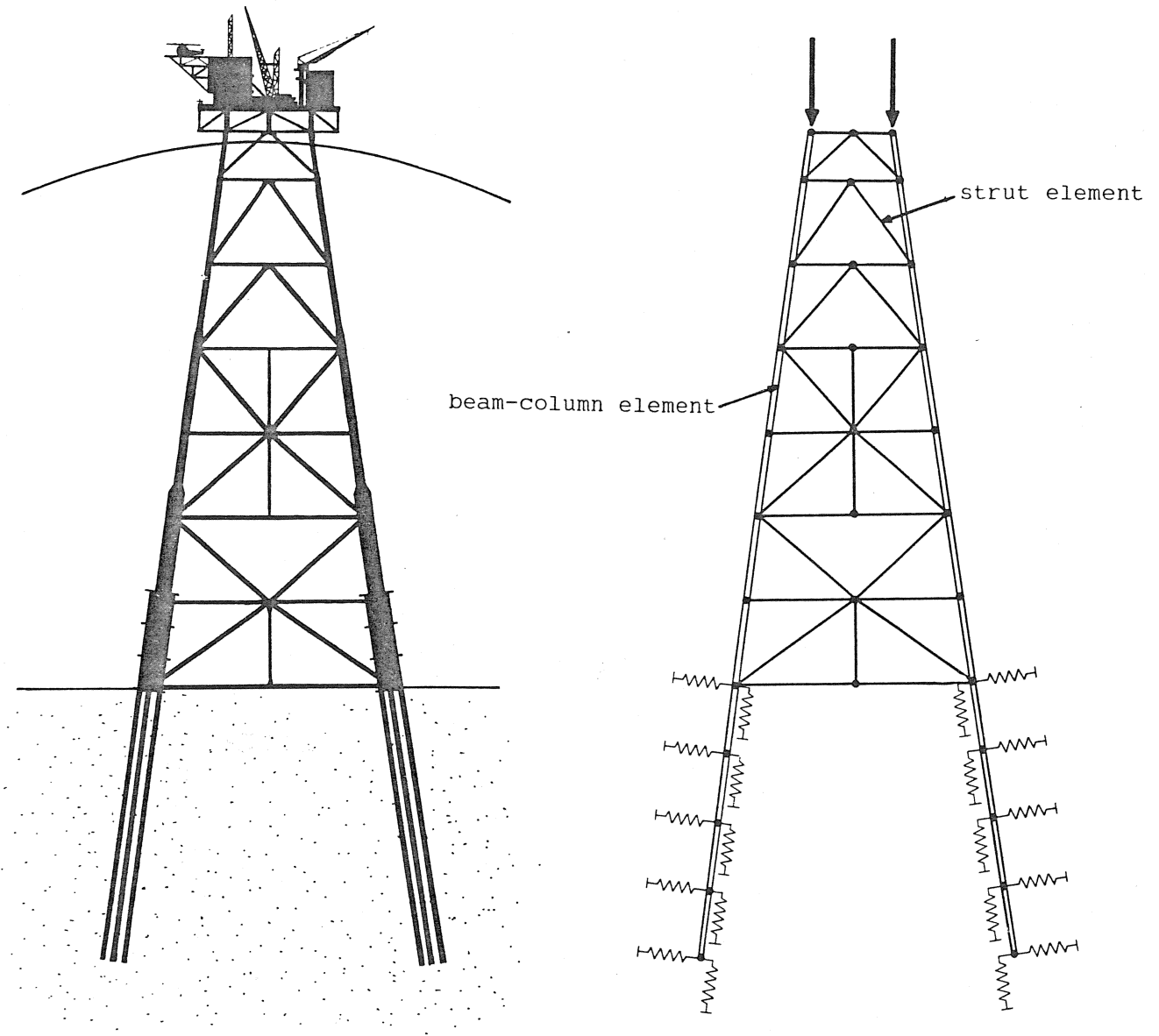


Figure 1

INTRA schematization of a fixed offshore platform

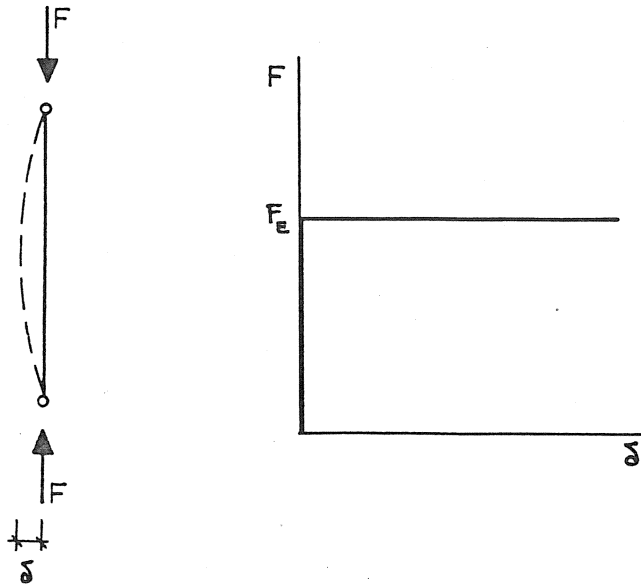


Figure 2a
'perfect' column

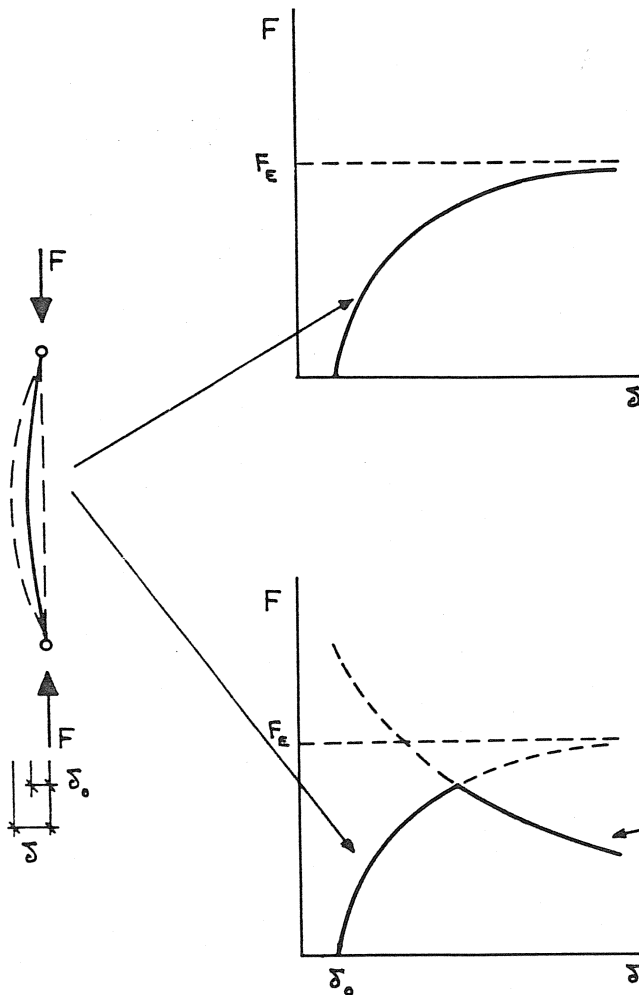


Figure 2b
- initial deflection
- elastic behavior

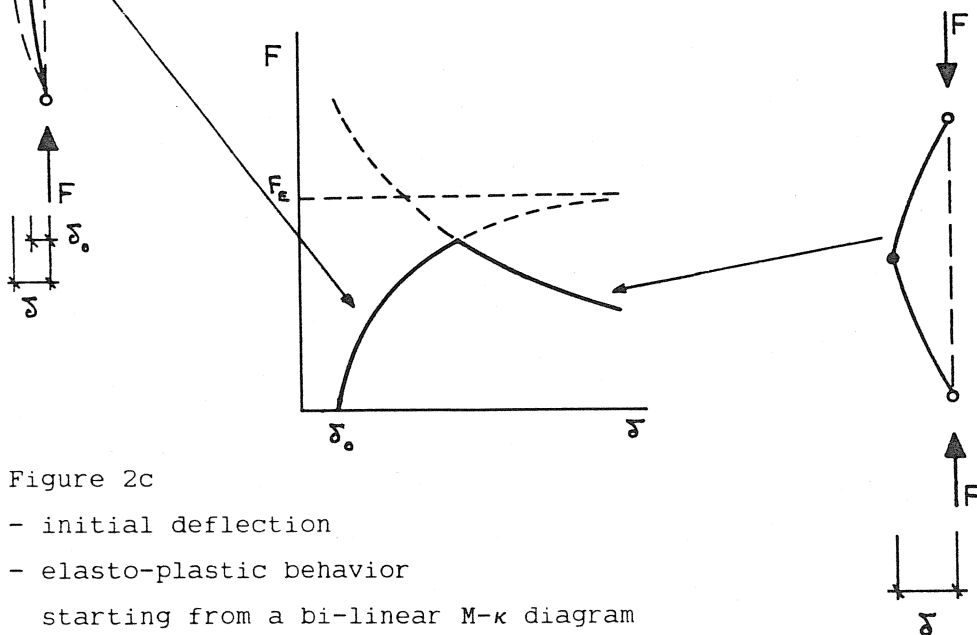


Figure 2c
- initial deflection
- elasto-plastic behavior
starting from a bi-linear $M-\kappa$ diagram

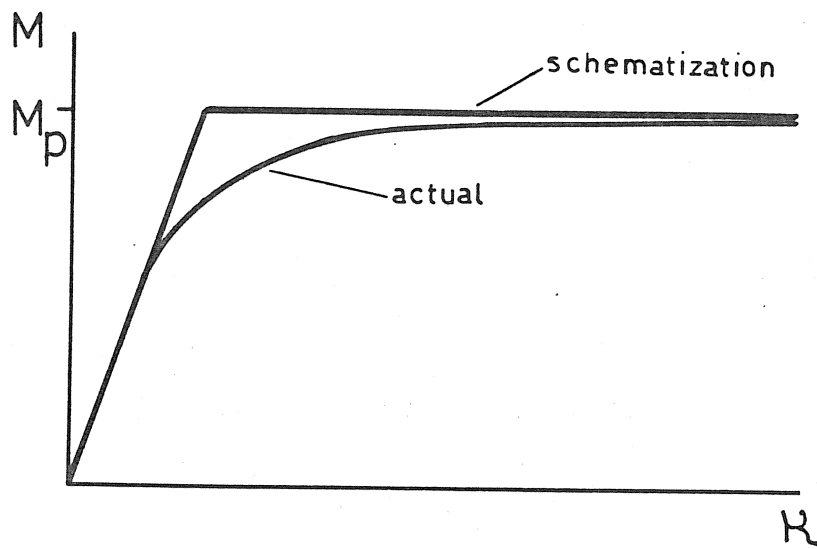


Figure 3
 Moment - curvature diagram;
 actual shape and bi-linear schematization

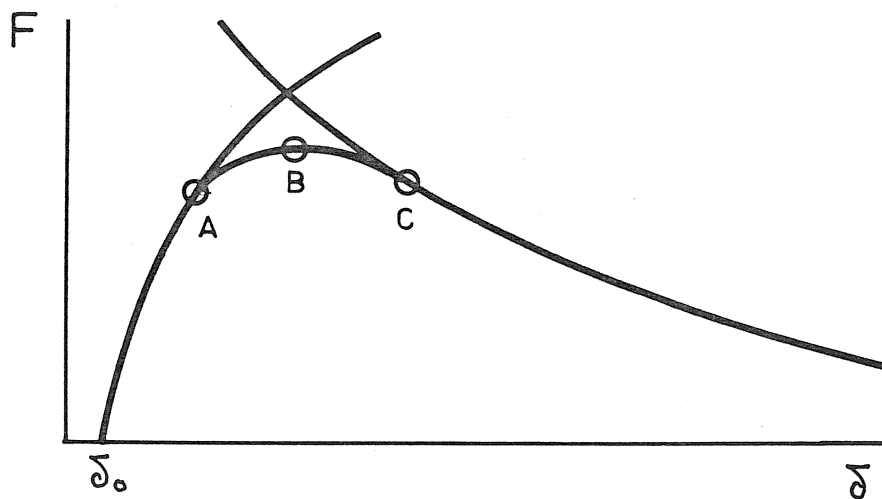


Figure 4
 Axial force versus lateral deflection;
 actual shape and 'plastic hinge' schematization.

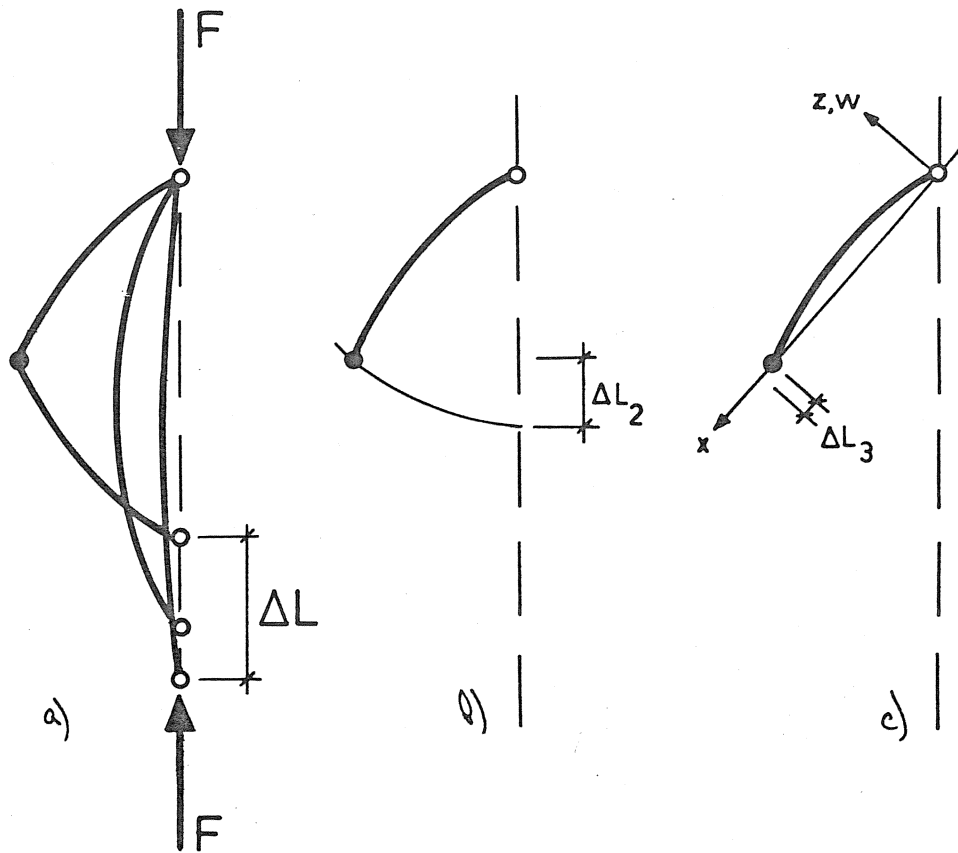


Figure 5
Axial force versus axial shortening

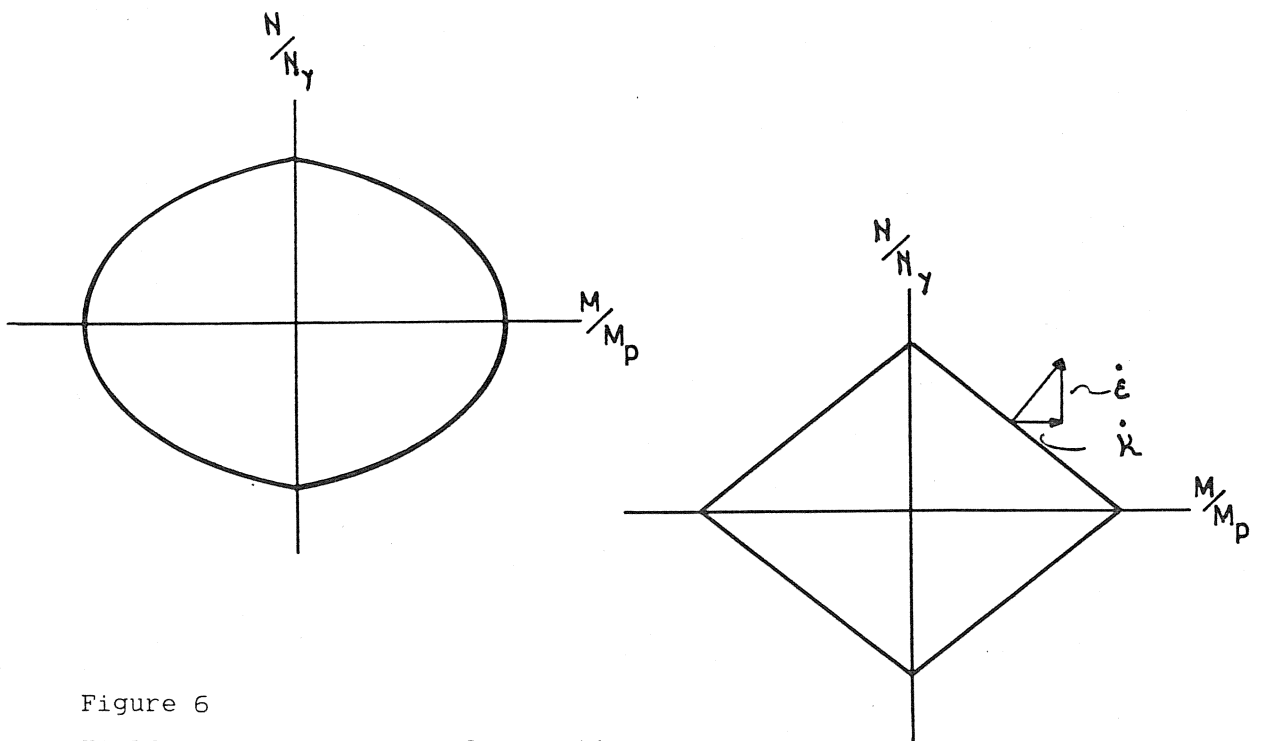


Figure 6
Yield surface of a circular section;
actual shape and schematization

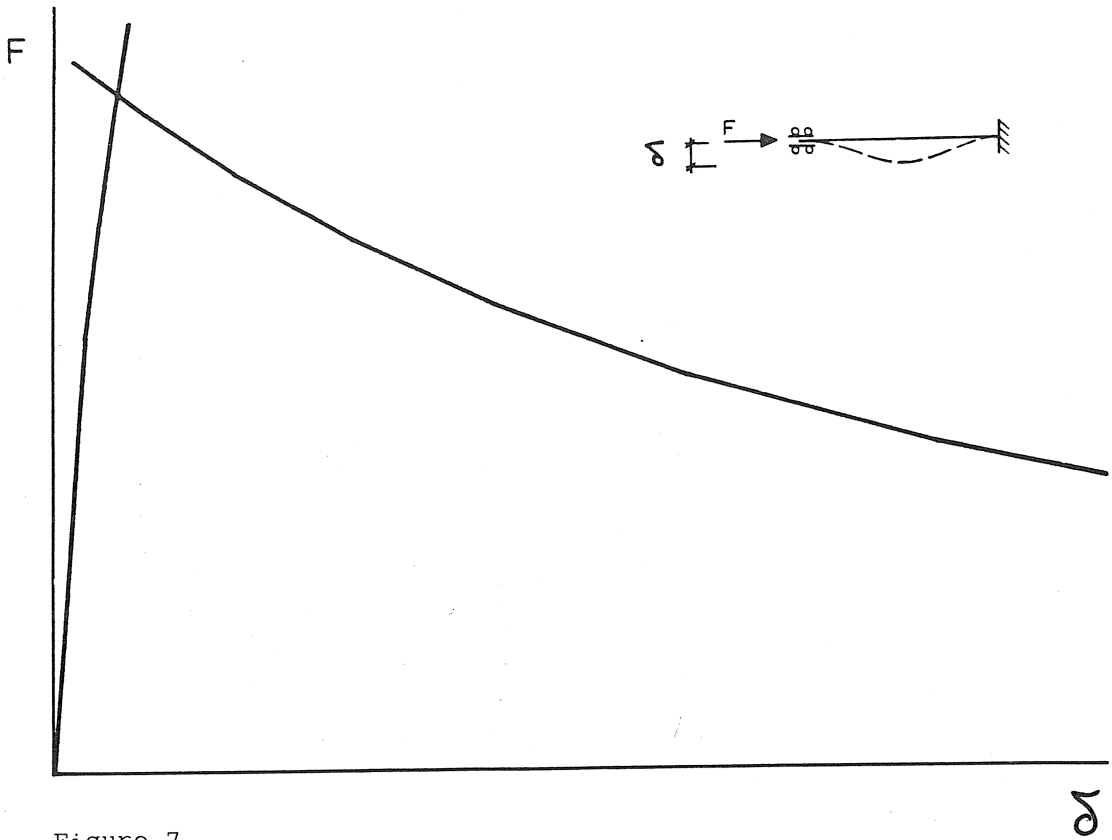


Figure 7
Axial force versus lateral deflection

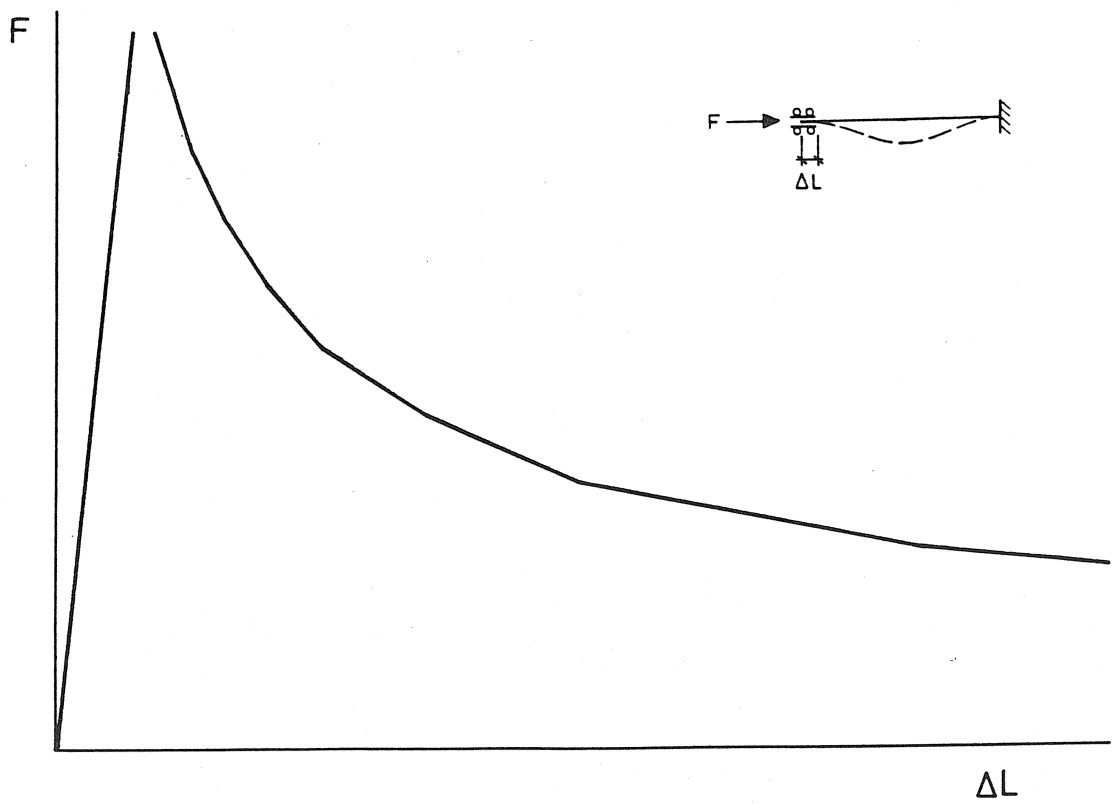
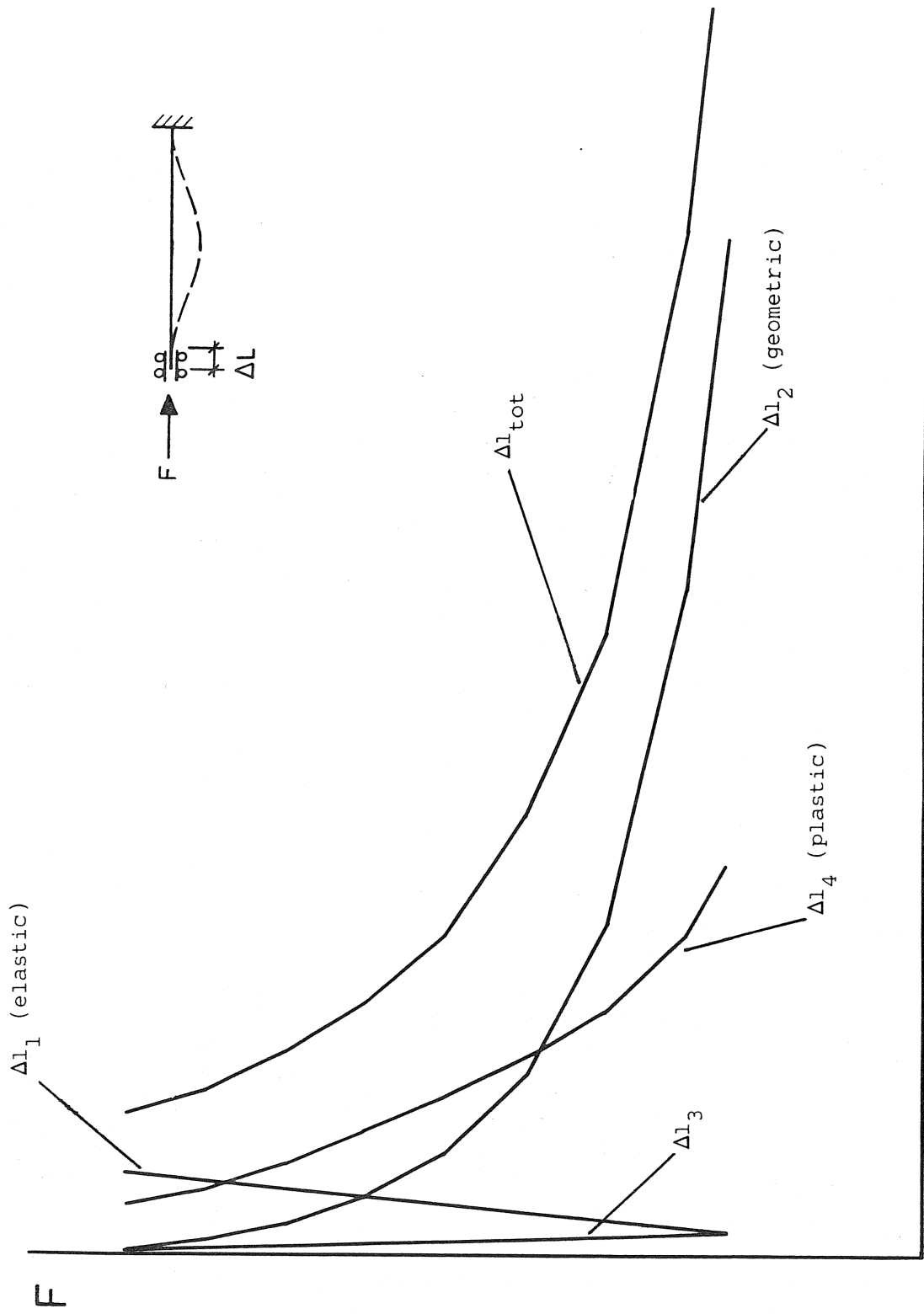


Figure 8
Axial force versus axial shortening



ΔL

Figure 9
Contributions to axial shortening (post-buckling)

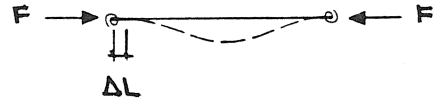
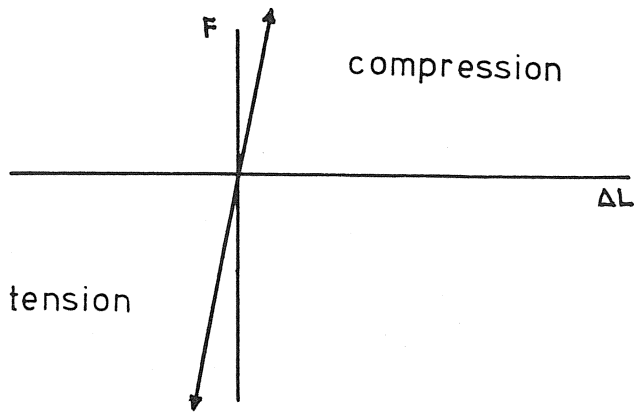


Figure 10a
Response of linear truss element LTRS

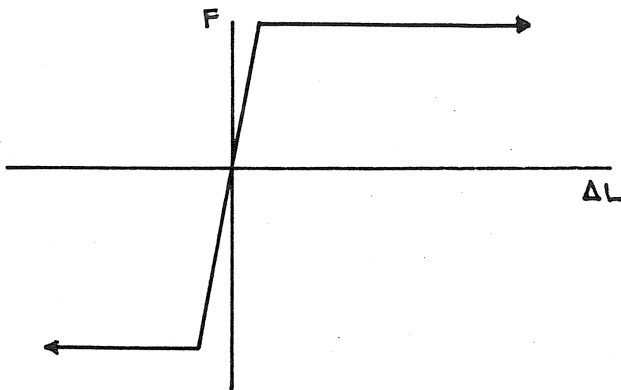


Figure 10b
Response of nonlinear truss element NTRS

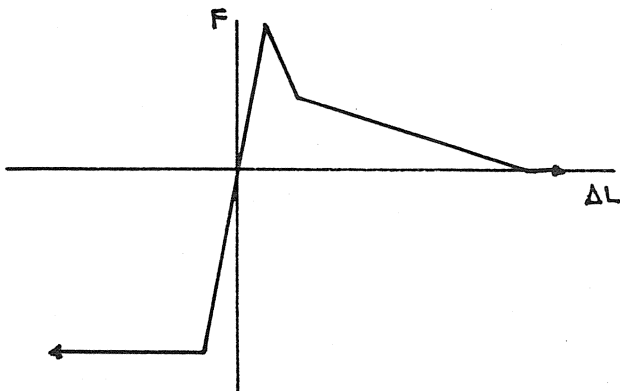


Figure 10c
Response of 'Marshall' strut element STRT

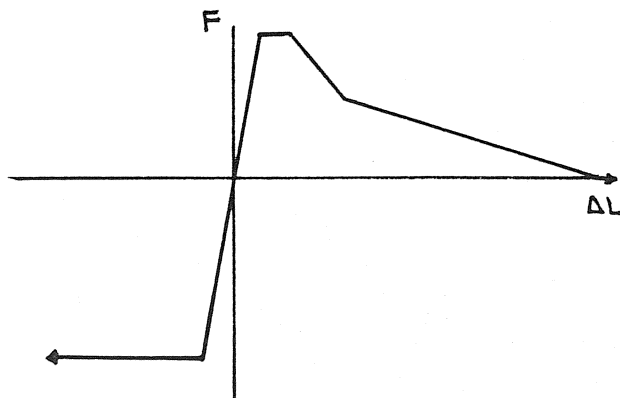


Figure 10d
Response of improved strut element ISTR

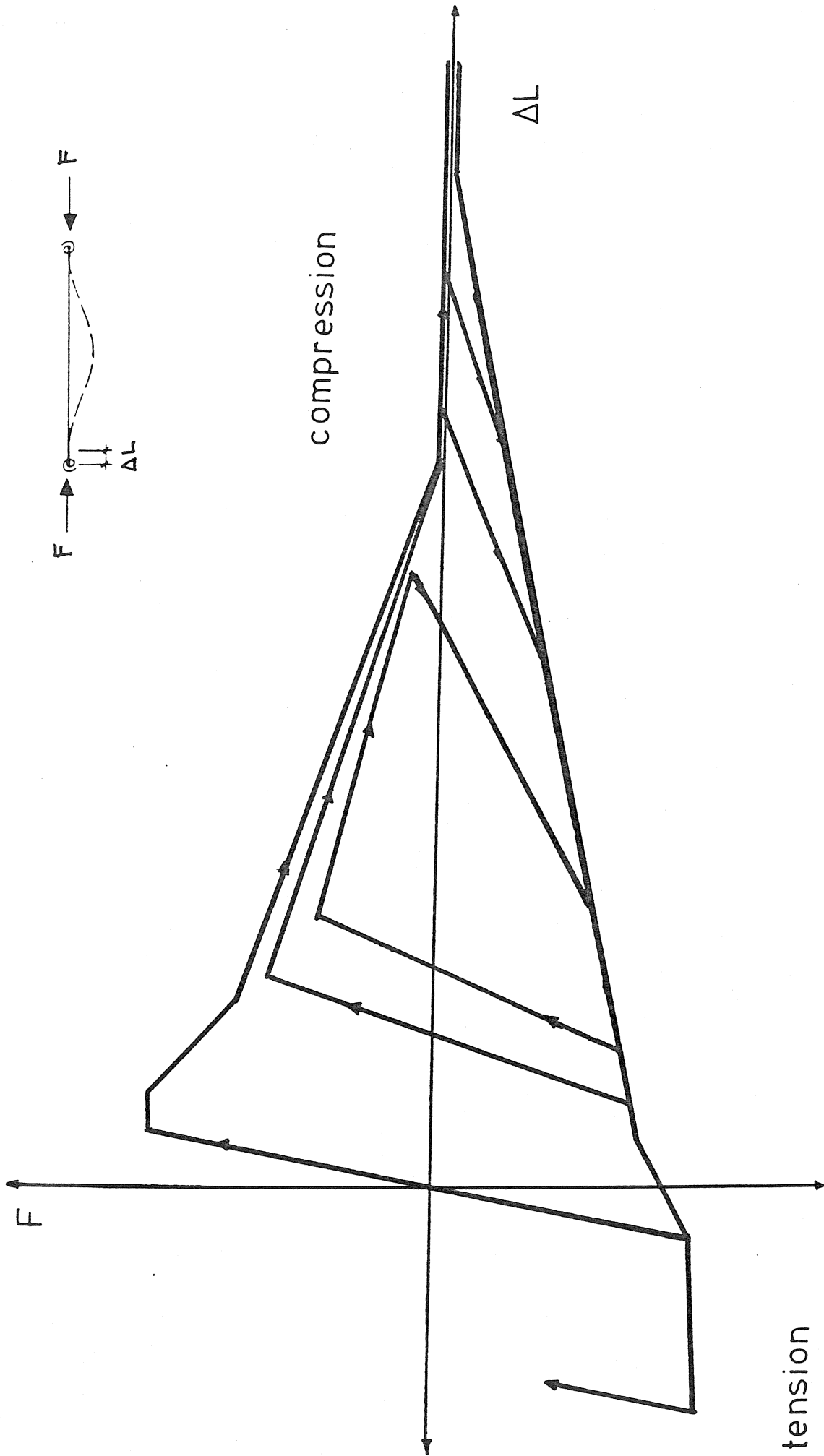


Figure 11
 Response of strut element to cyclic loading



Figure 12a
Schematic representation of beam-column element
divided in three sub elements (two-dimensional)

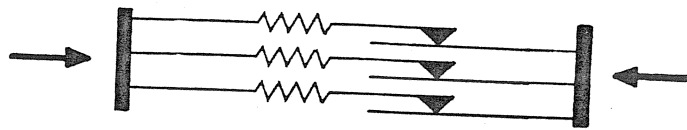


Figure 12b
One dimensional representation of beam-column element
divided in three eleasto-plastic sub elements

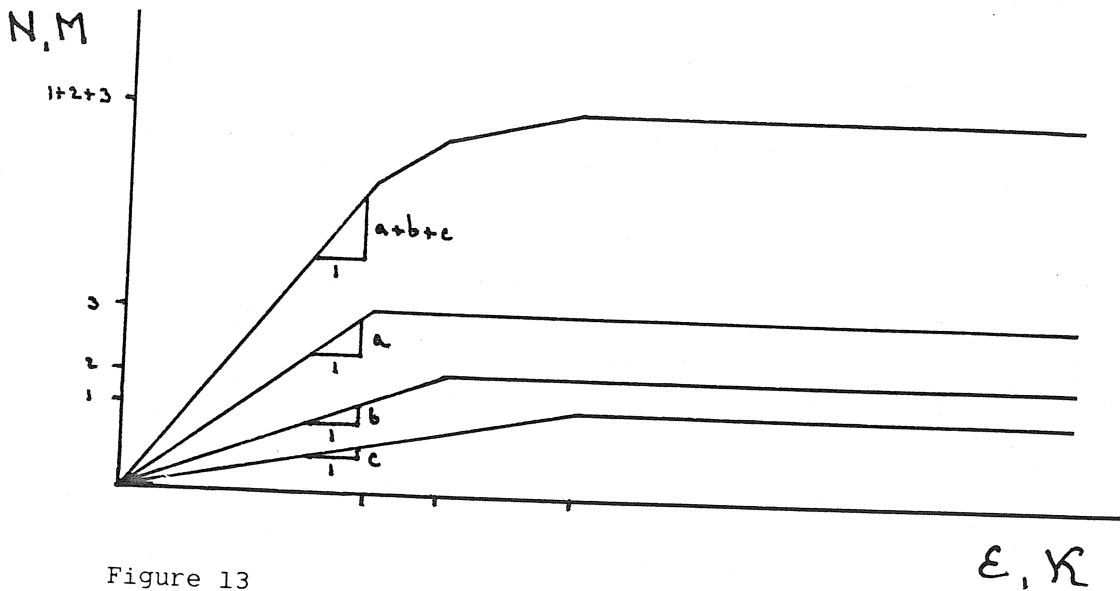


Figure 13
Multi-linear $N-\epsilon$ or $M-\kappa$ relationship,
composed of three bi-linear contributions

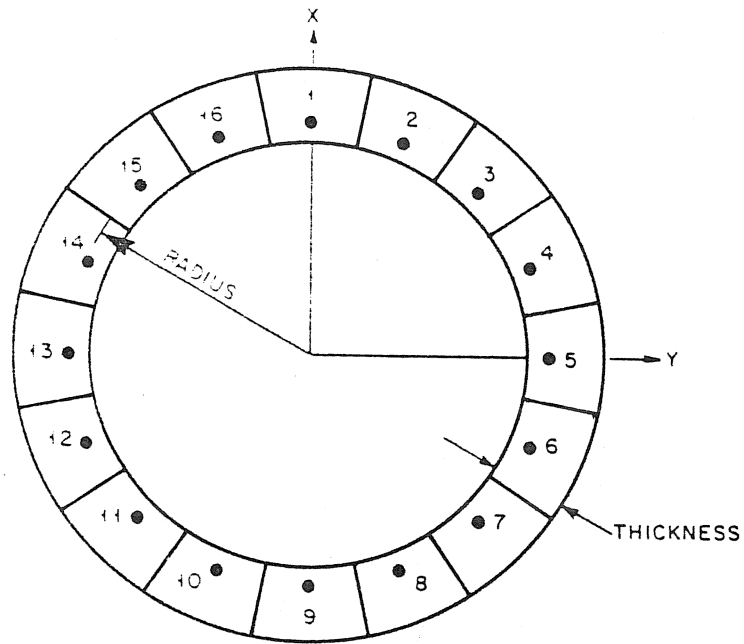


Figure 14
 MARC 'no 14' element:
 subdivision of cross section in 16 segments



Figure 15
 Schematization of a beam column
 with the aid of two elements

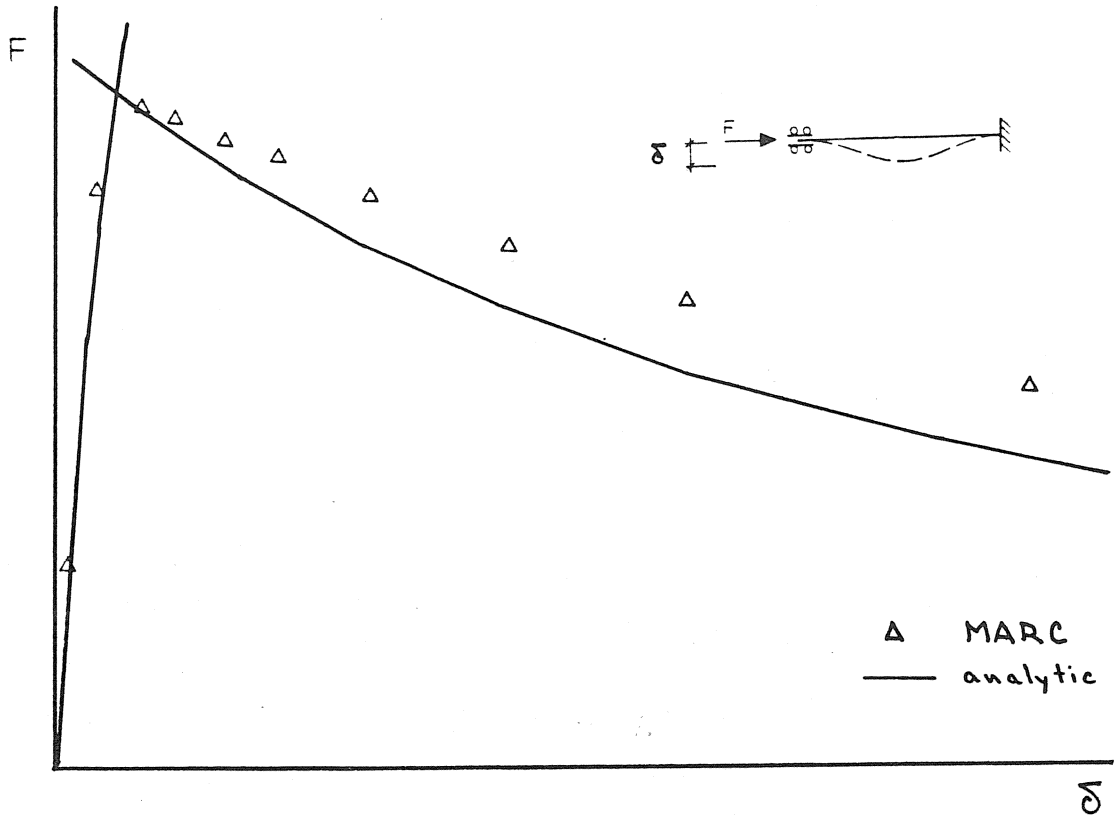


Figure 16
 Axial force versus lateral deflection.
 MARC results and analytical upperbound

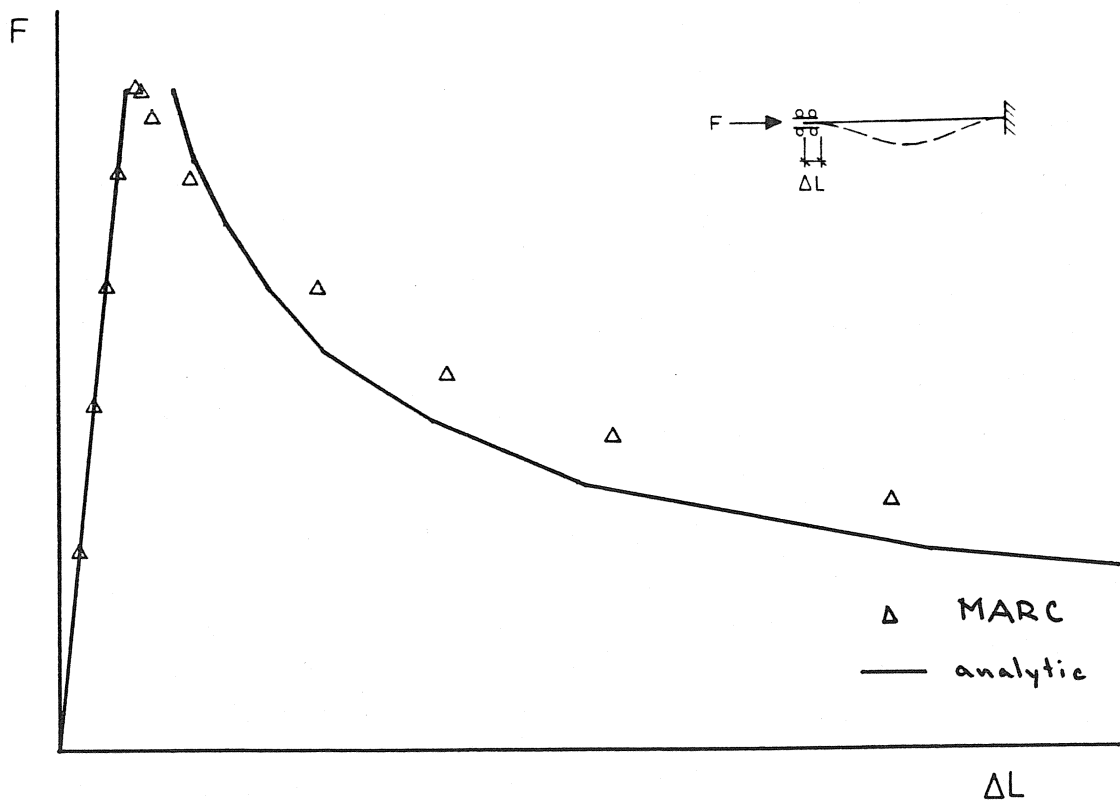


Figure 17
 Axial force versus axial shortening.
 MARC results and analytical approximation

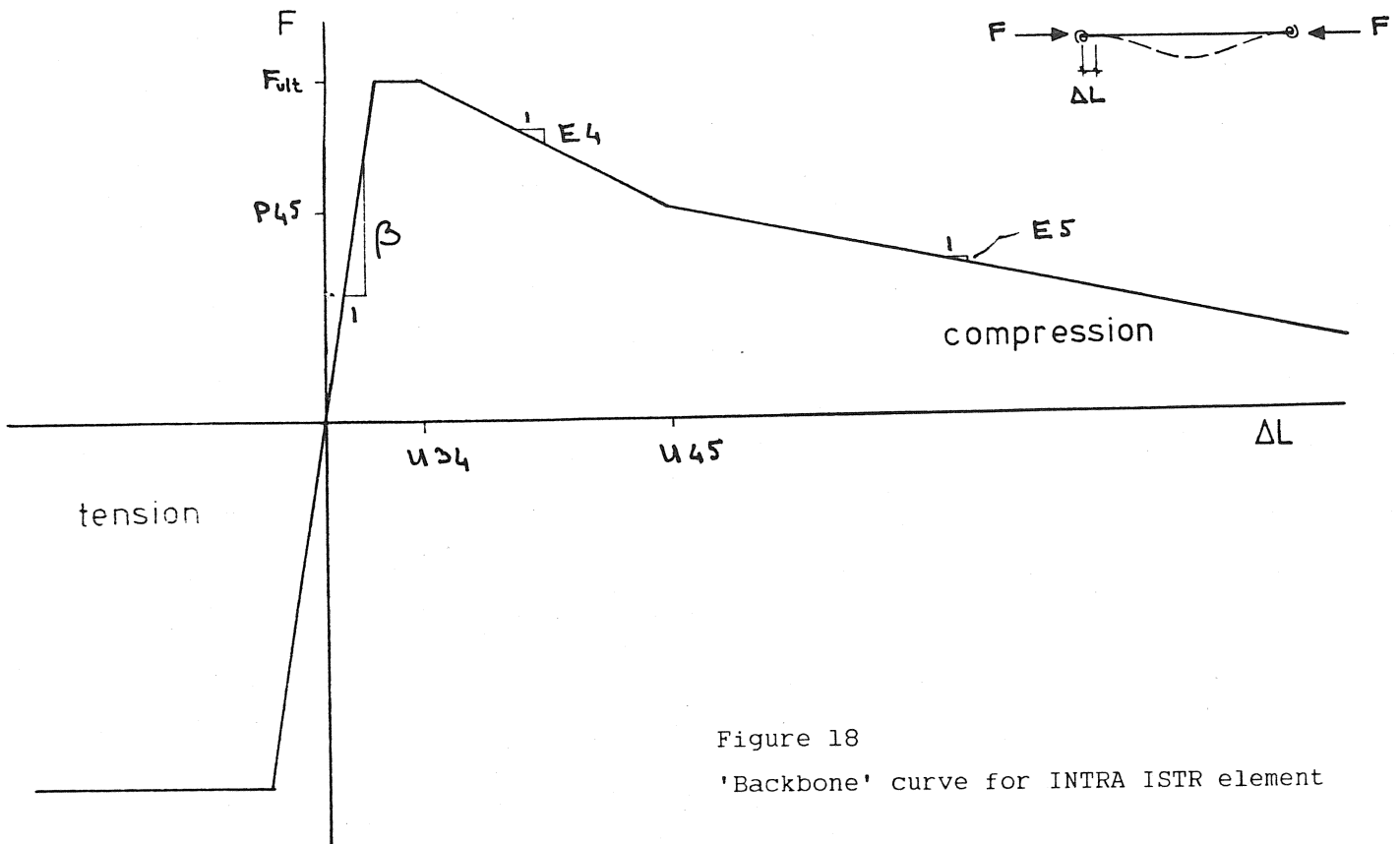


Figure 18
'Backbone' curve for INTRA ISTR element

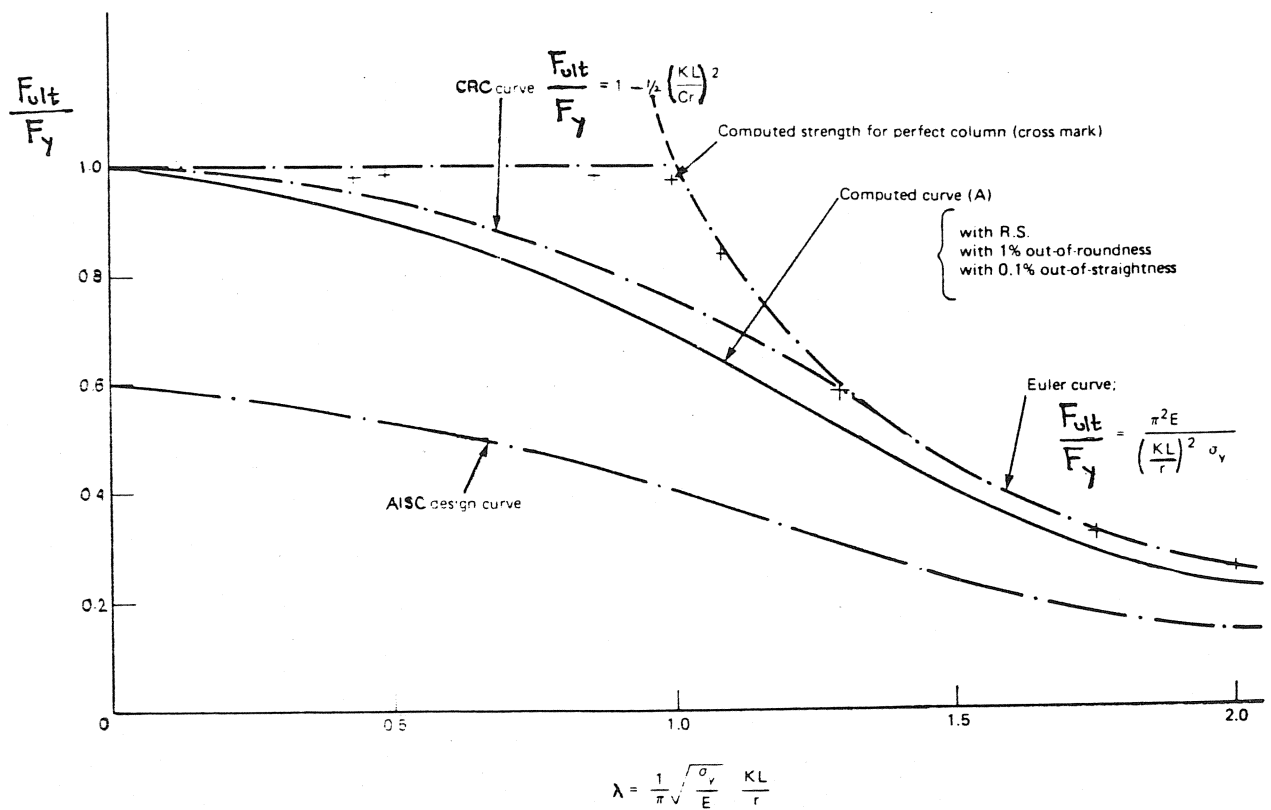


Figure 19
Column strength design curve (ref.[3])

● $A = 1.2$ or 1.4 ~
 ○ $A = 0.9$ or 1.0 ~
 + $A = 0.7$ ~
 △ $A = 1.1$ ~

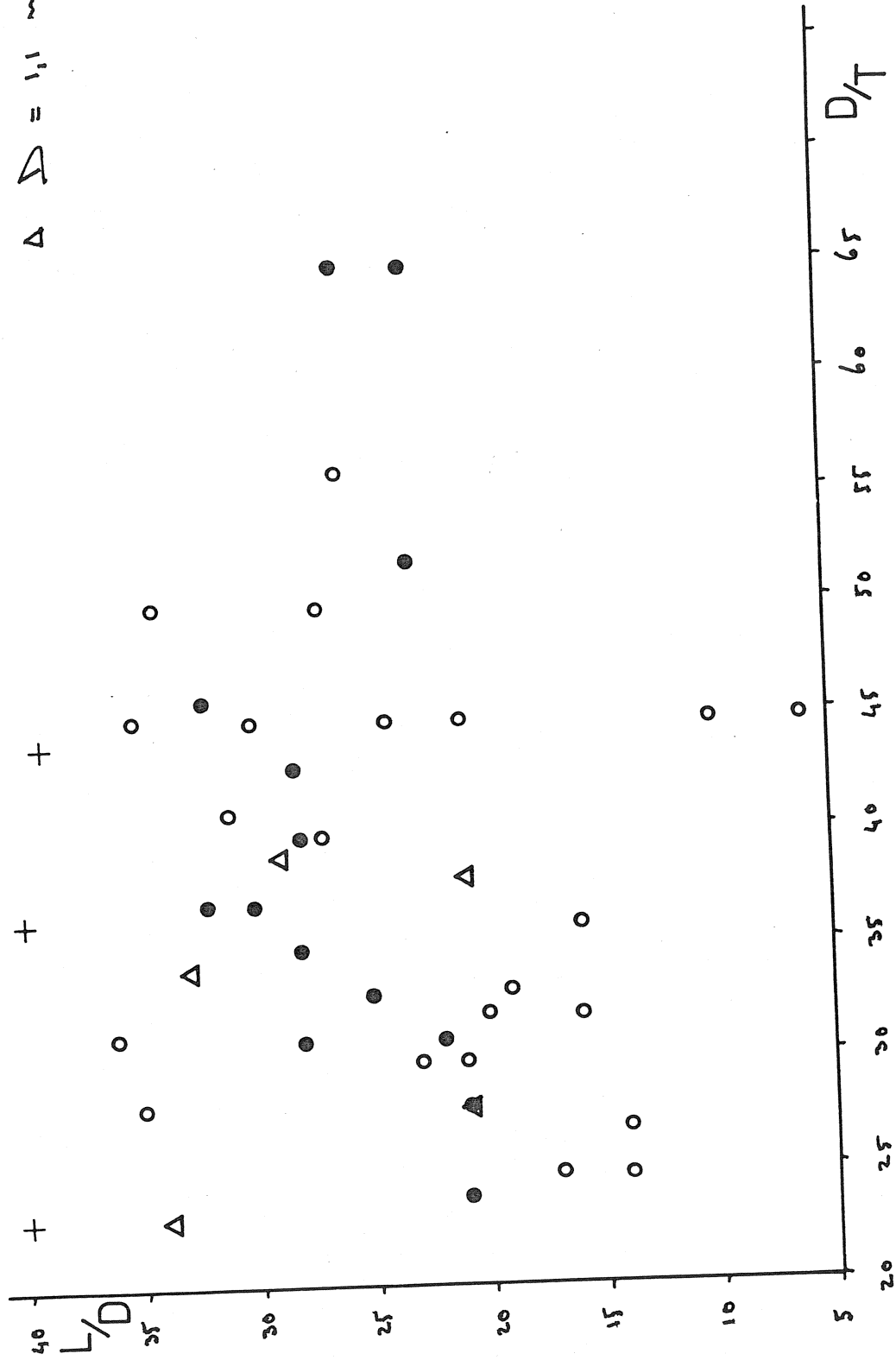


Figure 20
 Scatter in L/D and D/T ratios of the members of a large North Sea platform



$K = 0.7$

MEND = 0 kNm

QLAT = 0 kN/m or 7.2 kN/m

DEPTH = 0 m or 175 m

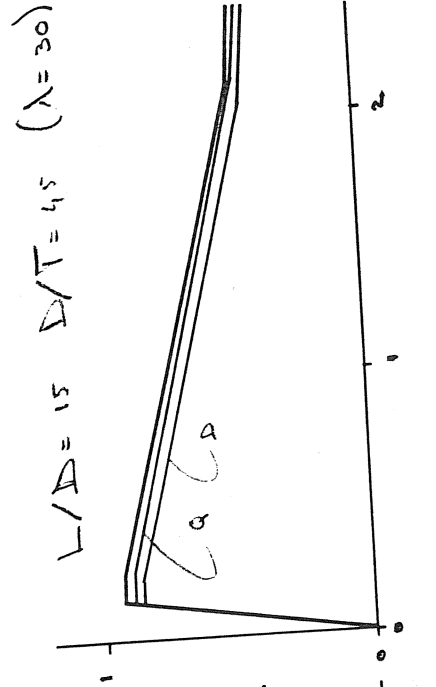
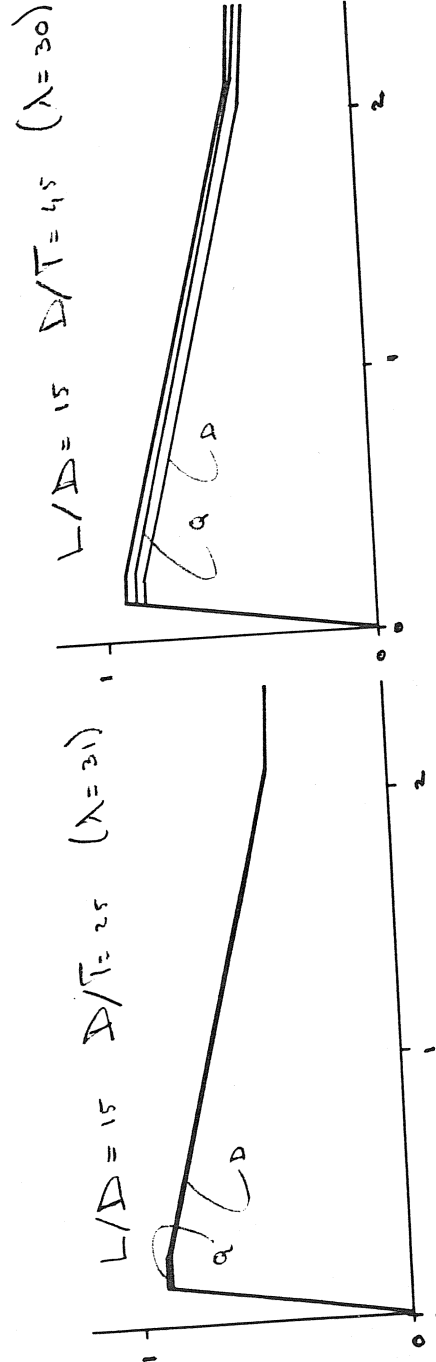
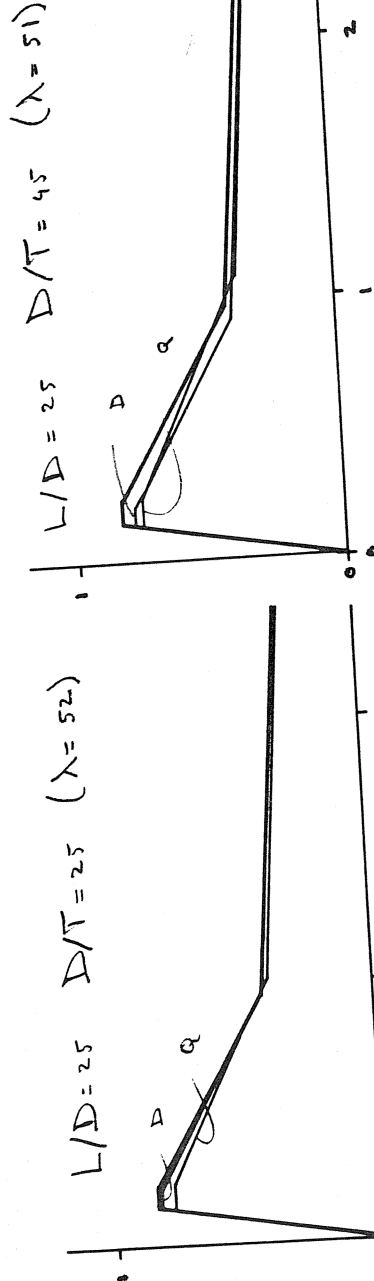
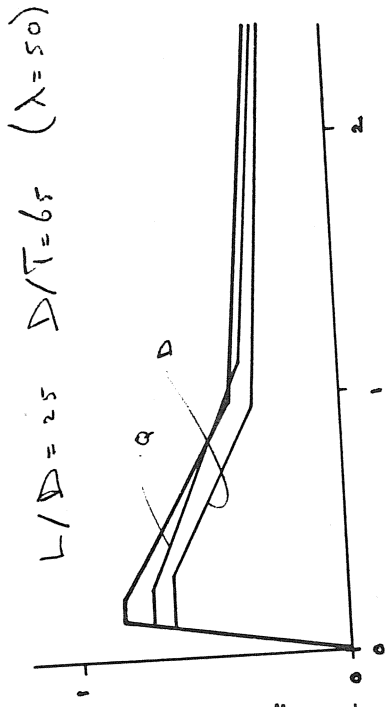
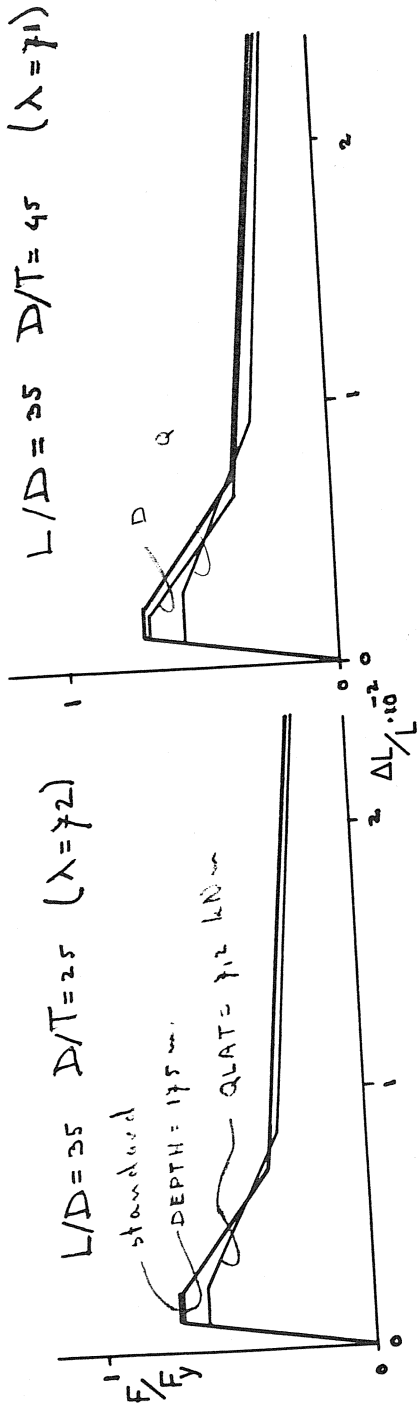


Figure 21

'Backbone' curves of struts with various L/D and D/T ratios

Lateral load (QLAT) and waterdepth (DEPTH) have been varied

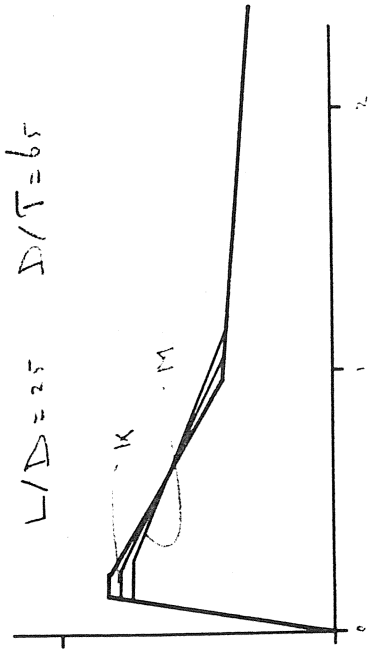
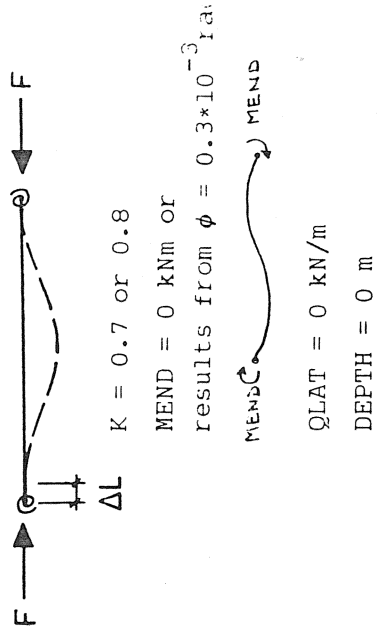
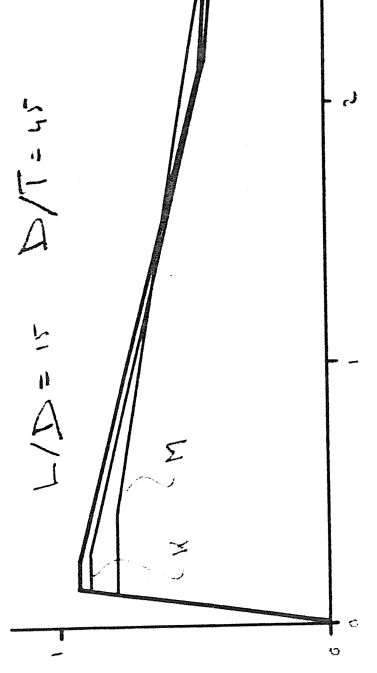
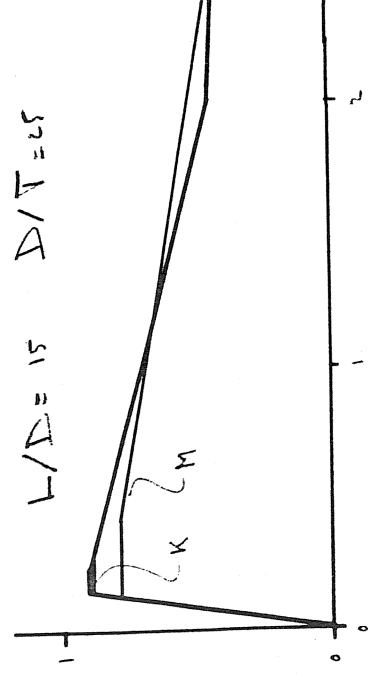
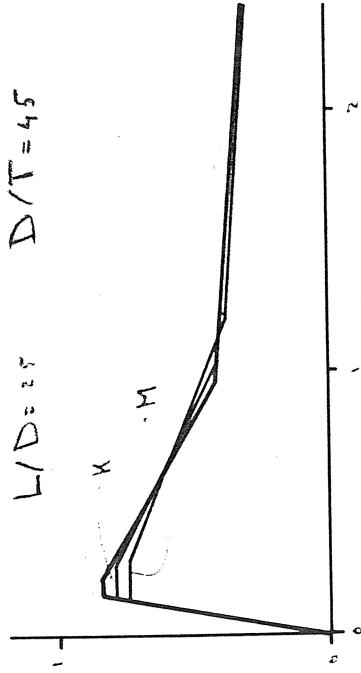
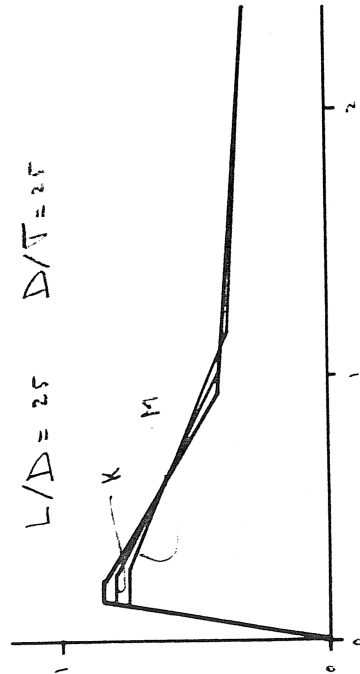
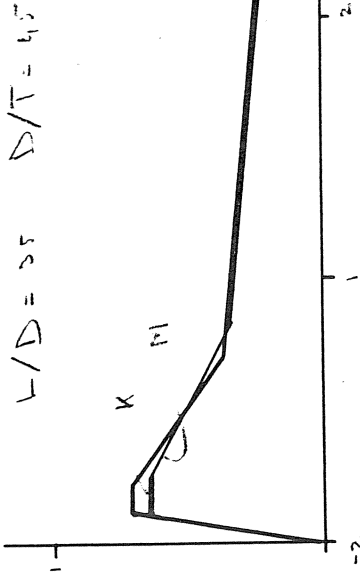
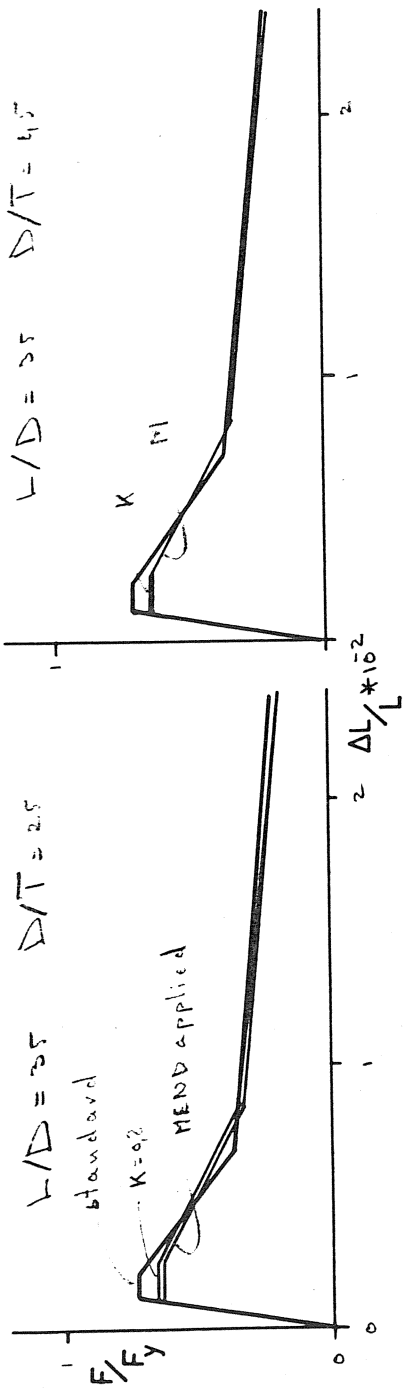


Figure 22
 'Backbone' curves of struts
 with various L/D and D/T ratios
 Fixity ratio (K) and moments due to
 imposed end rotations (MEND)
 have been varied

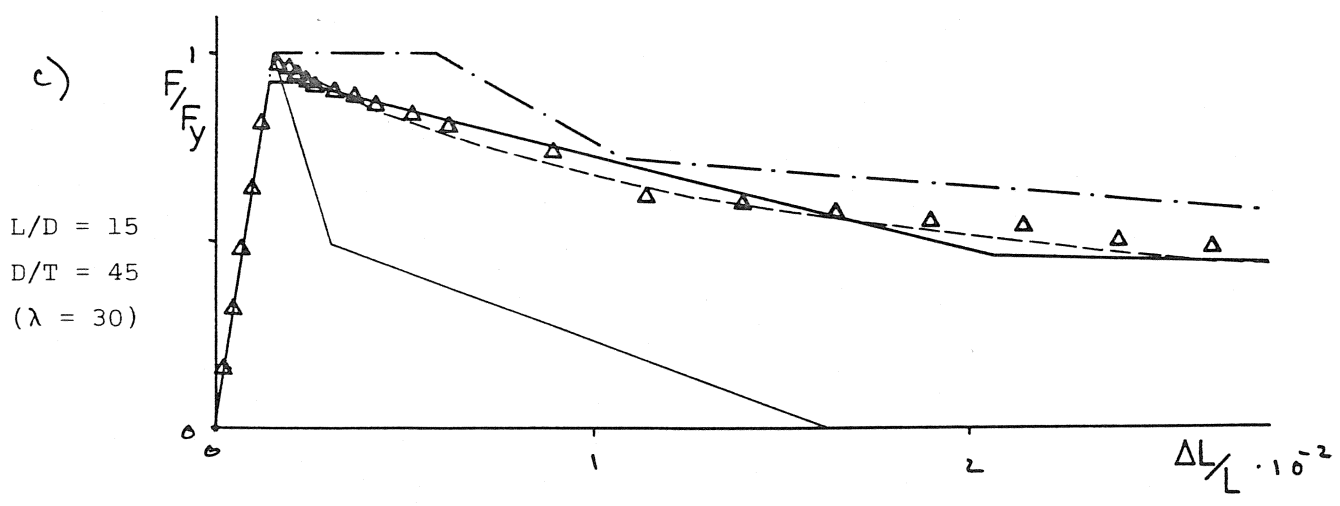
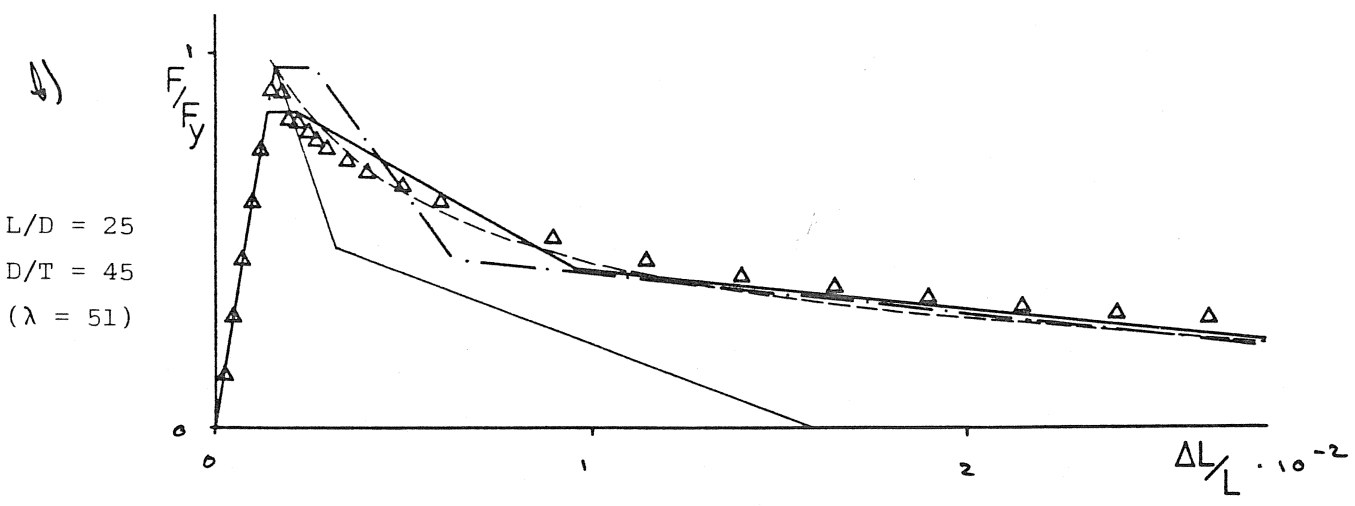
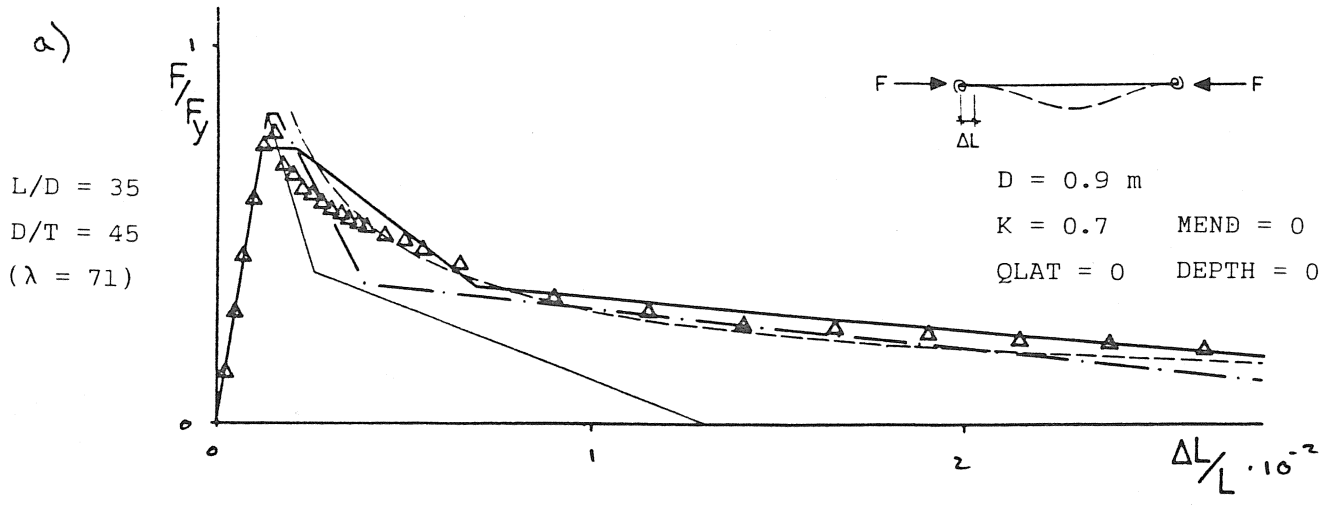


Figure 23
Comparison of
strut 'backbone' curves

A	----	Analytical approximation
B	————	STRING
C	△△	MARC
D	-.-.-.-	INTRA default ISTR element
E	————	INTRA recommendation (old)

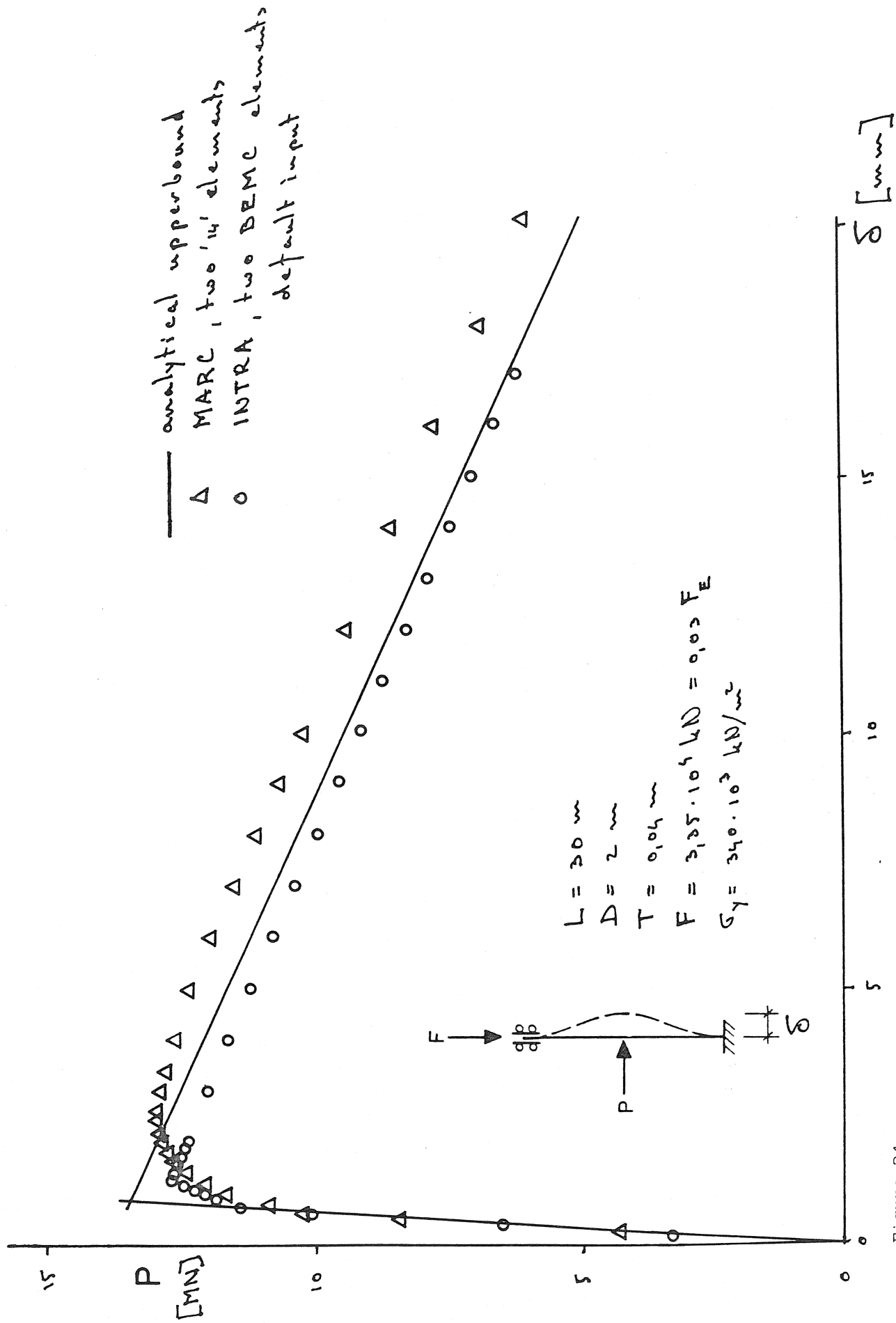


Figure 24
 Testcolumn, analytical, MARC and INTRA results

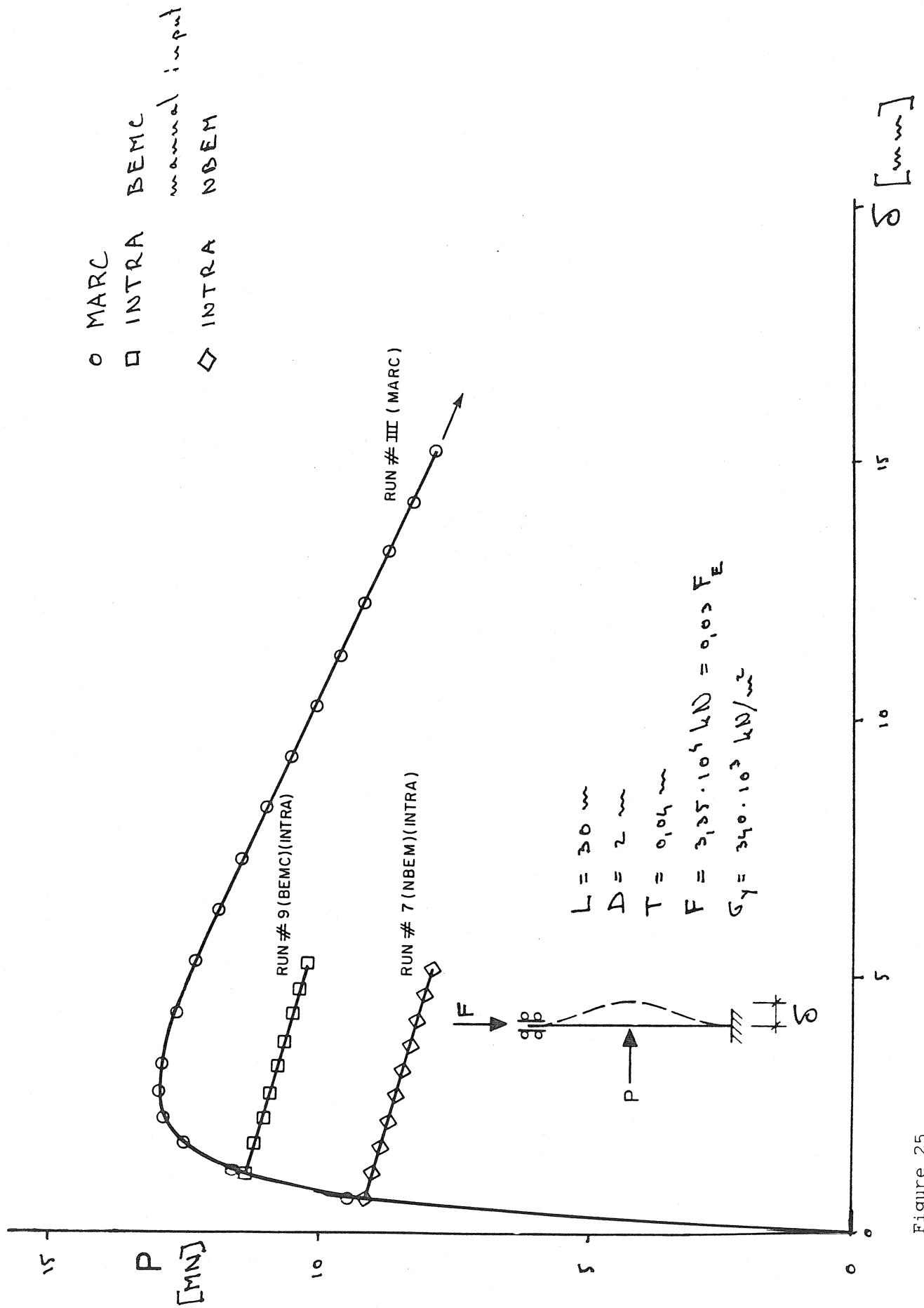
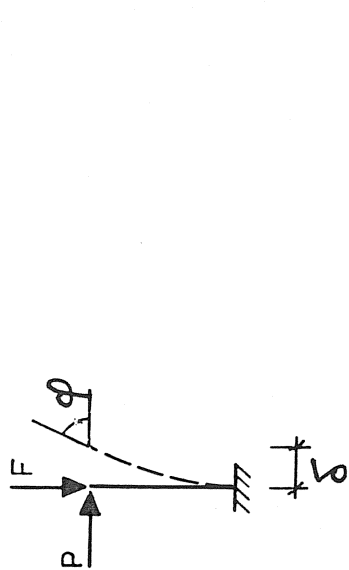


Figure 25
 Testcolumn, results Kerstens (ref.[8])



$$L = 31 \text{ m}$$

$$D = 3, \text{ m}$$

$$T = 0,045 \text{ mm}$$

$$F = 122957 \text{ kN} = 0,5 F_E$$

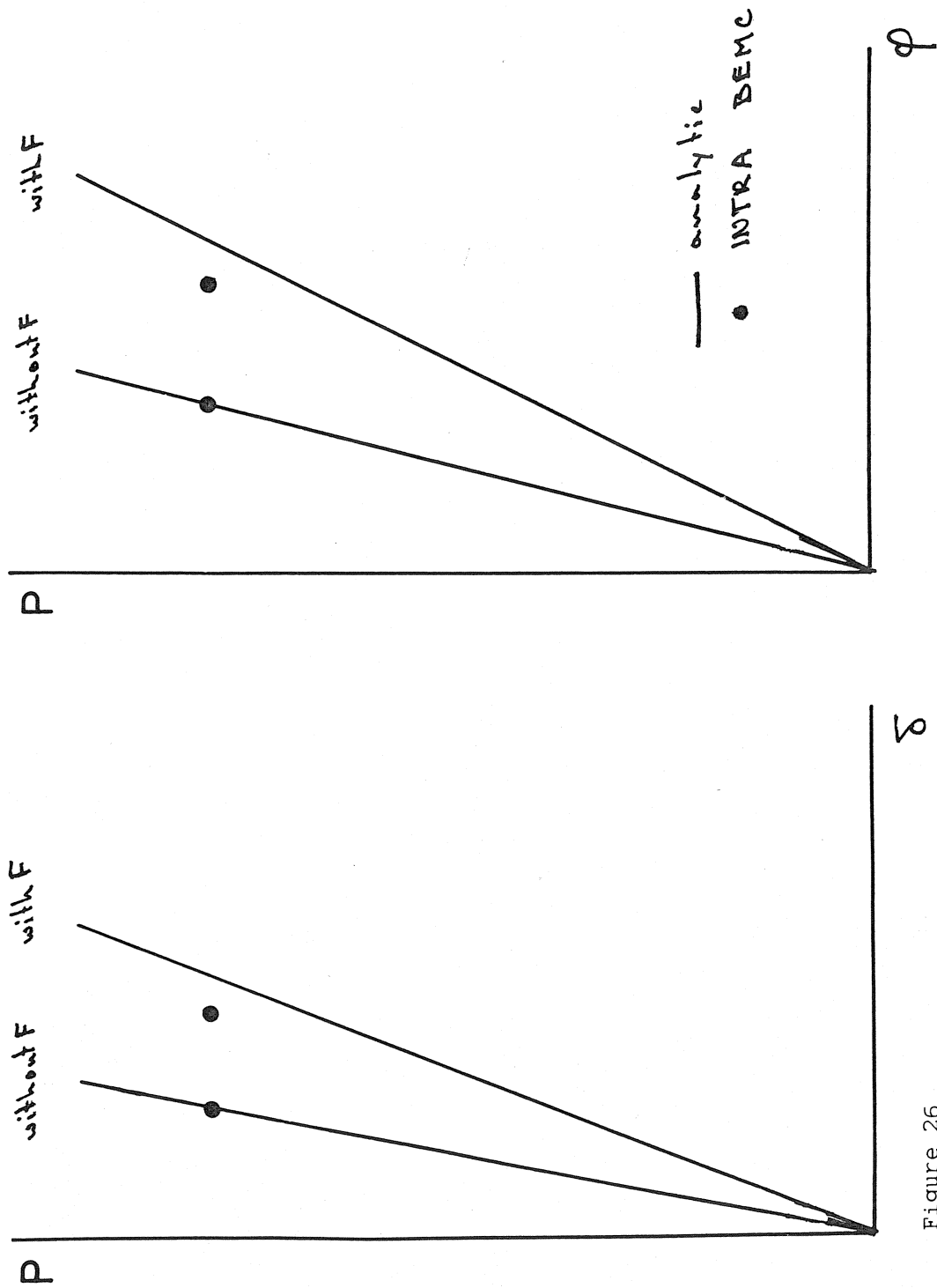


Figure 26

Testcolumn, geometric nonlinear behavior in the elastic range

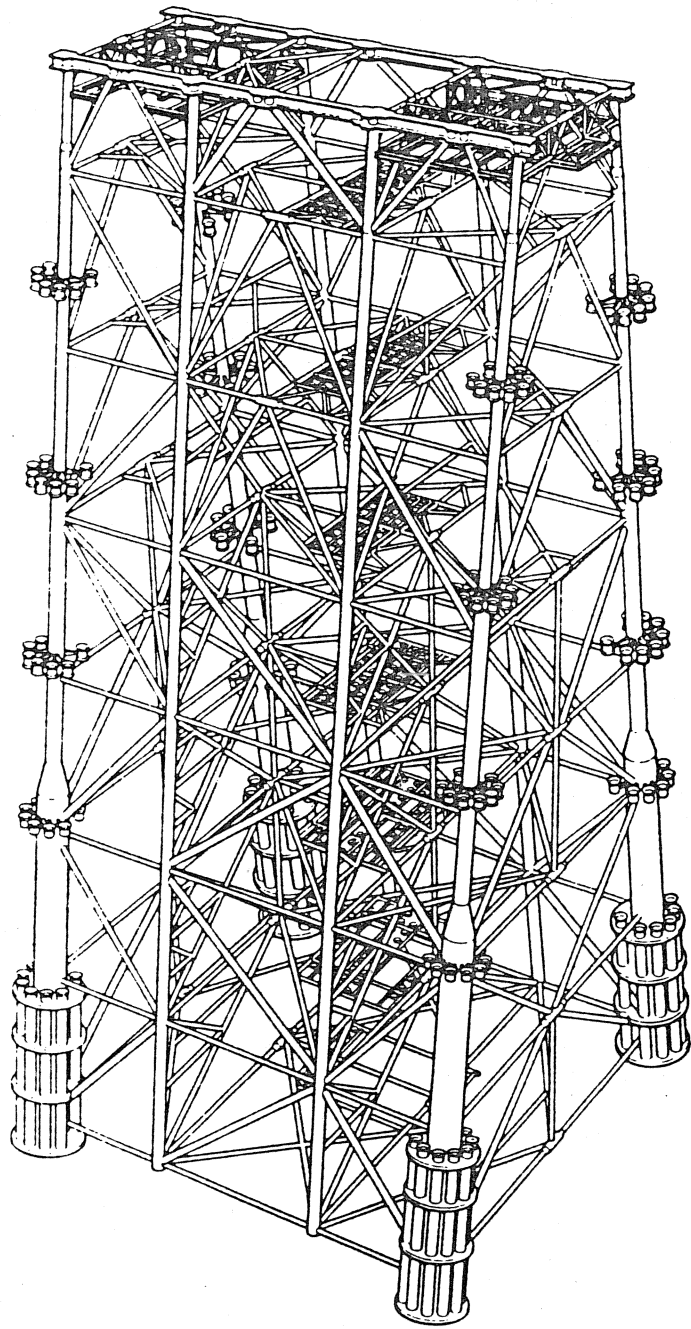


Figure 27

Jacket structure of a large North Sea platform

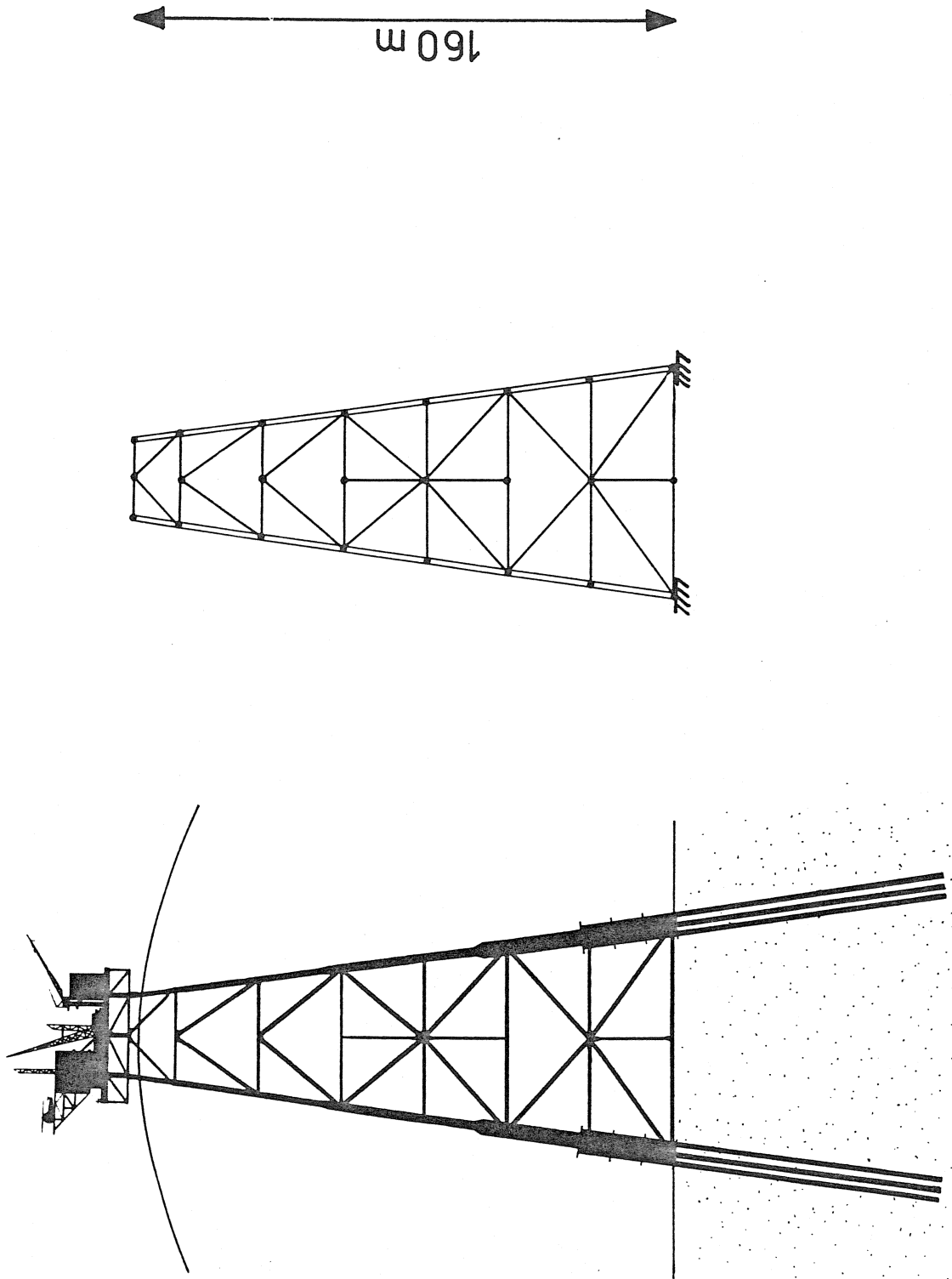


Figure 28
Large North Sea platform, end frame with schematic representation

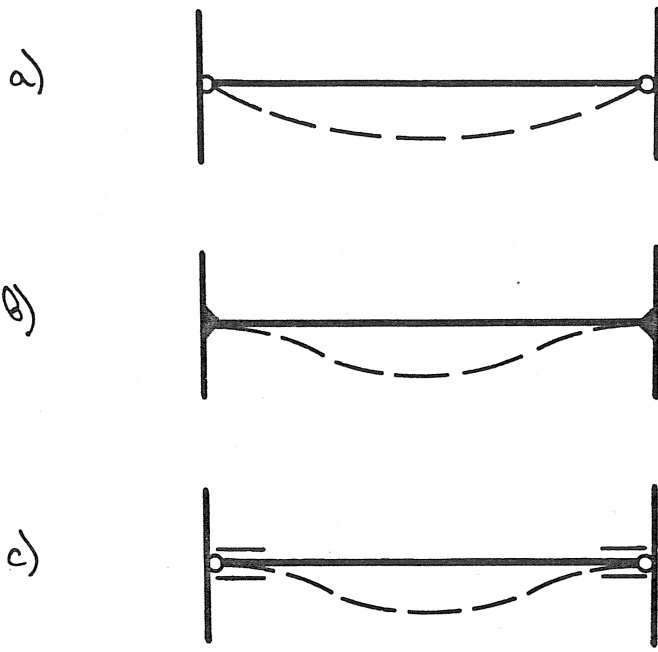


figure 29

- a) pin-ended brace ($K = 1$)
- b) fixed-ended brace ($K = 0.5$)
moment carrying joints
- c) fixed-ended brace ($K = 0.5$)
no moment carrying joints
(INTRA-like schematization)

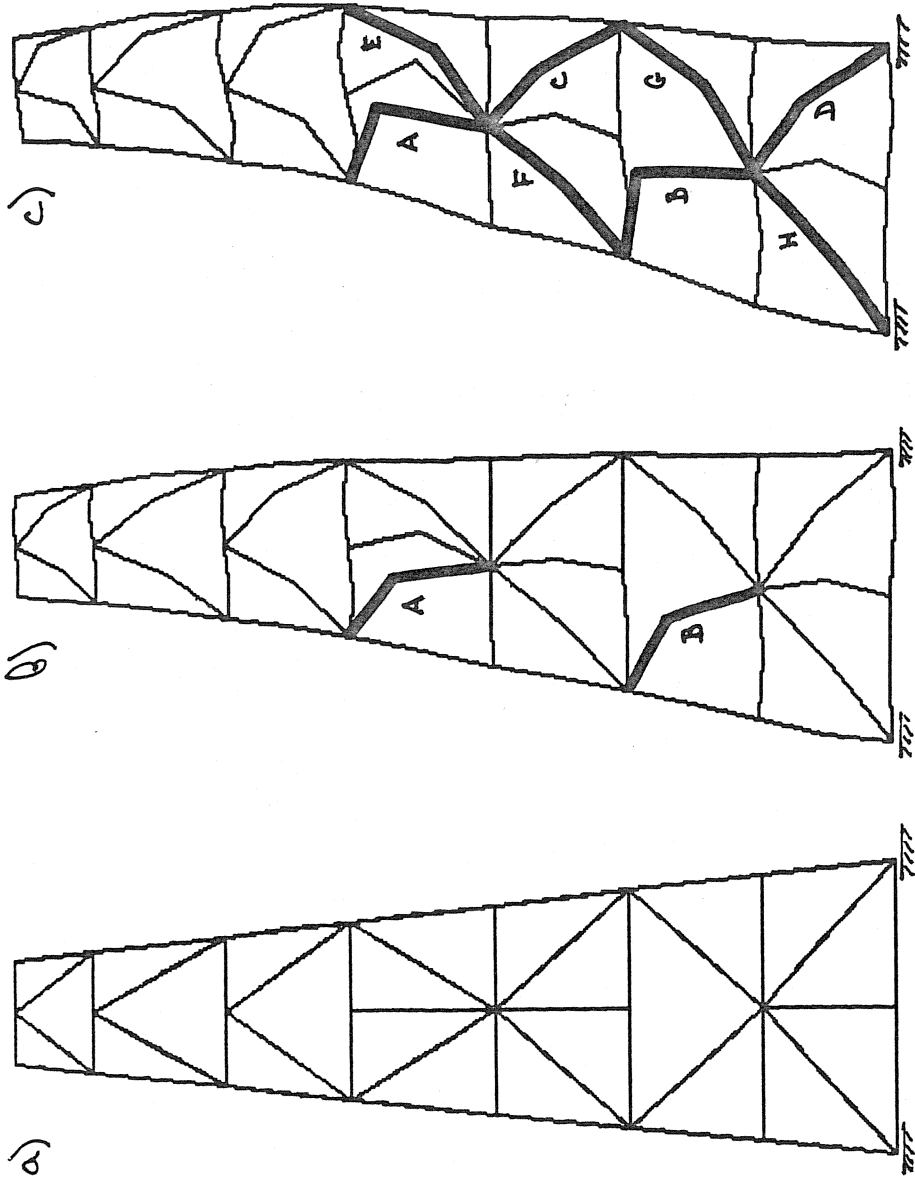


Figure 30
 Plane frame pushover analysis

- a) undeformed shape
- b) INTRA result: members A and B buckle
- c) MARC result: members A upto D buckle, members E upto H yield

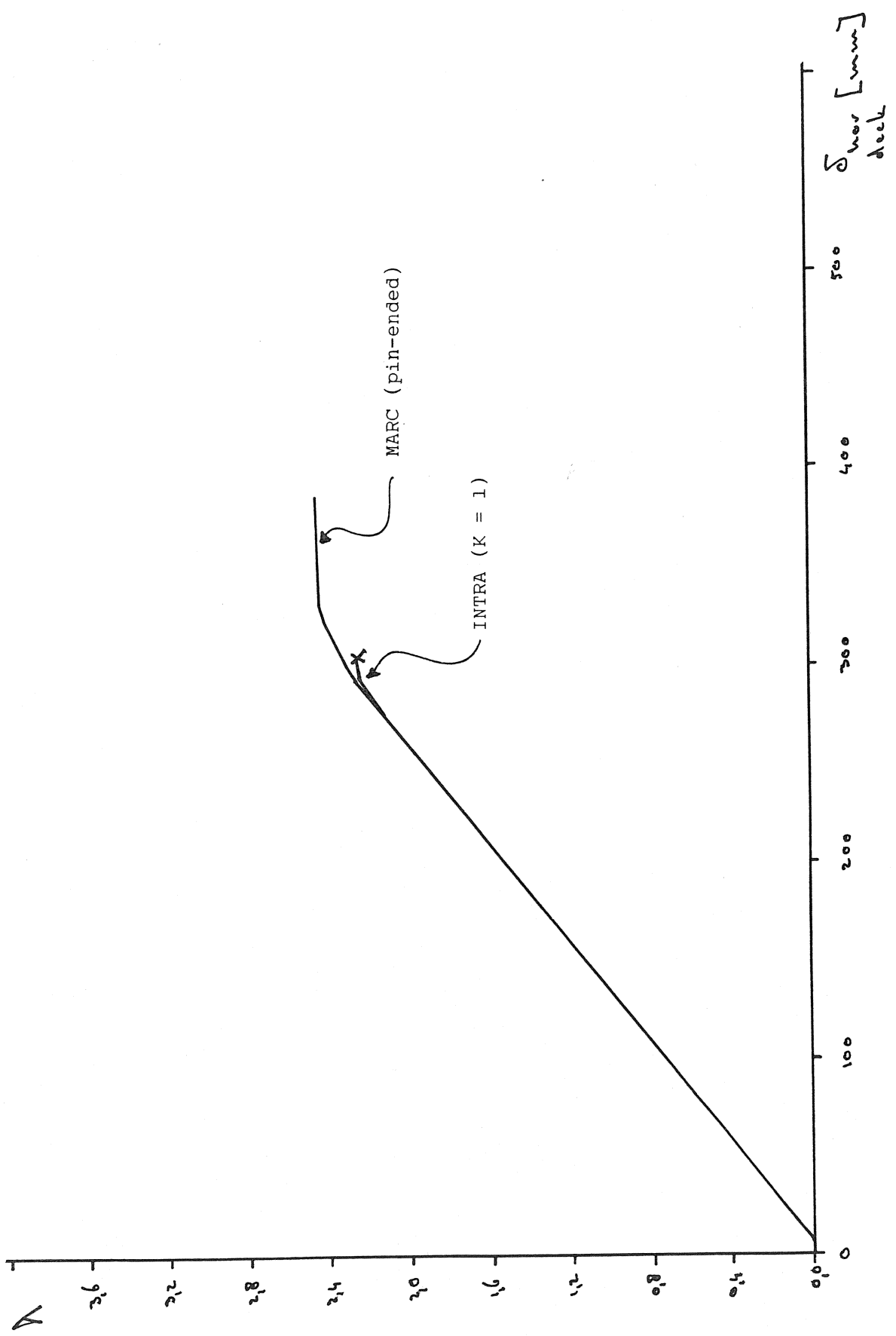


Figure 31
Comparison with pin-ended members

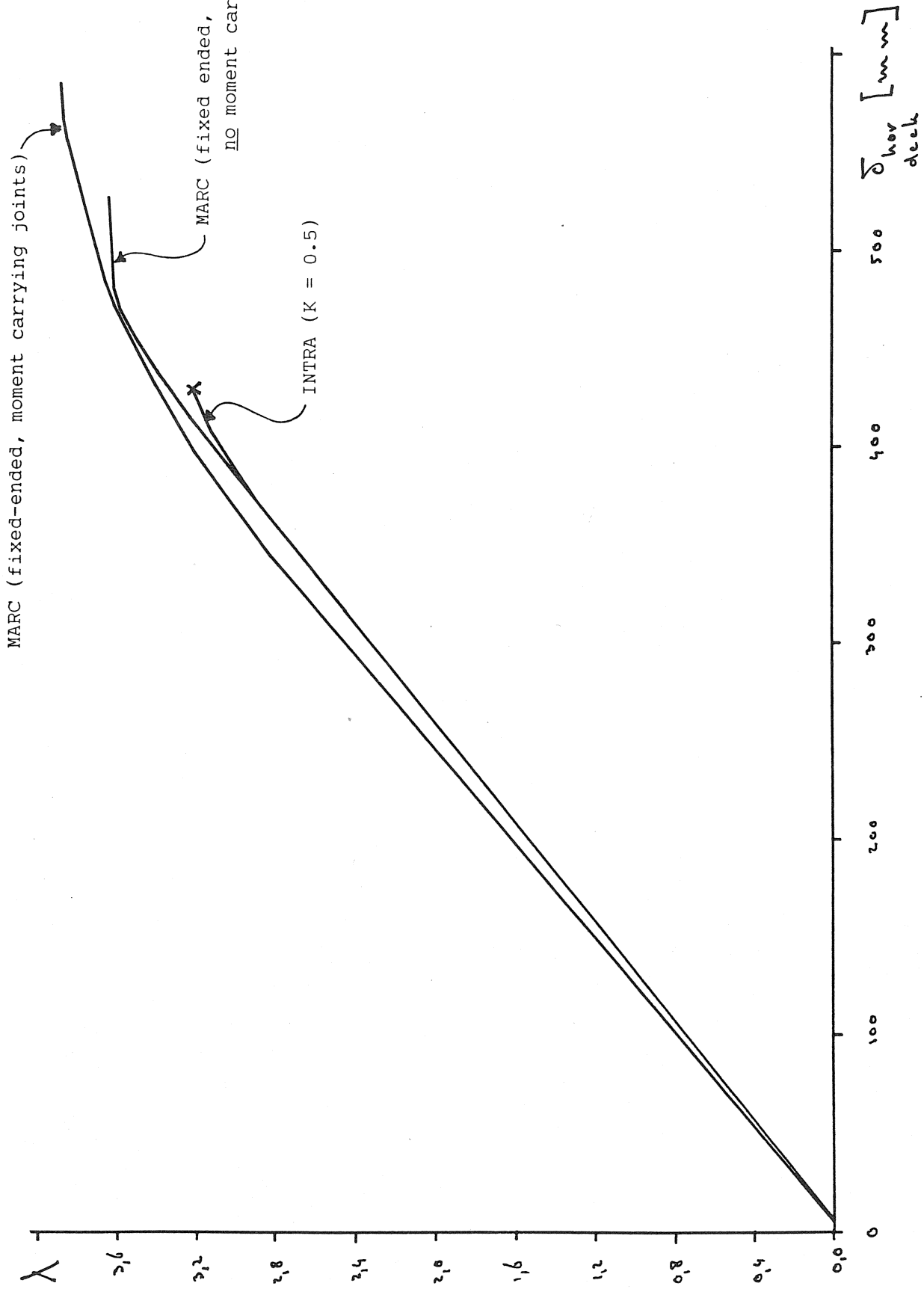


Figure 32
Comparison with fixed ended members

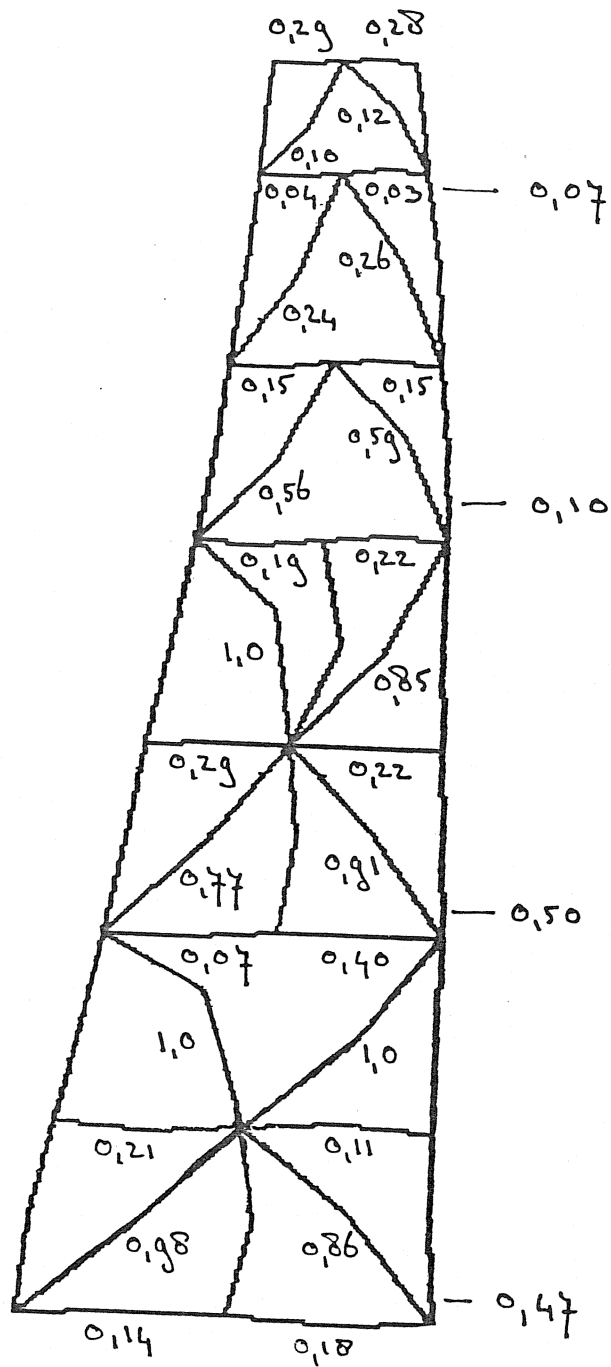


Figure 33
 Utilization ratios
 INTRA schematization, struts with $K=0.5$
 Two braces buckle, one yields

PROPERTIES OF CIRCULAR SECTIONS

Property	Full expression	Approximation
A	$\frac{\pi}{4} (D^2 - (D-2t)^2)$	$\pi D_m t^*$
I	$\frac{\pi}{64} (D^4 - (D-2t)^4)$	$\frac{\pi}{8} D_m^3 t$
$R = \sqrt{I/A}$	$\frac{\sqrt{D^2 - (D-2t)^2}}{4}$	$0.35 D_m$
$N_Y = N_{pl}$	$\frac{\pi}{4} (D^2 - (D-2t)^2) \sigma_Y$	$\pi D_m t \sigma_Y$
M_Y	$\frac{\pi}{32} \frac{(D^4 - (D-2t)^4)}{D} \sigma_Y$	$\frac{\pi}{4} D_m^2 t (1 - \frac{t}{D_m}) \sigma_Y$
M_{pl}	$\frac{D^3 - (D-2t)^3}{6} \sigma_Y$	$D_m^2 t \sigma_Y$
yield - surface	$\frac{M}{M_Y} = \frac{4}{\pi} \sin \left(\frac{\pi}{2} \left(1 - \frac{N}{N_Y} \right) \right)$ $\frac{M}{M_{pl}} = \cos \left(\frac{\pi}{2} \frac{N}{N_{pl}} \right)$	$\frac{M}{M_{pl}} + \left(\frac{N}{N_{pl}} \right)^{1.7} = 1$

A = cross sectional area
 D = outer diameter
 D_m = centre diameter = D-t
 I = moment of inertia
 $N_Y = N_{pl}$ = axial yield load

M_Y = 'first yield' bending moment
 M_p = full plastic moment
 R = radius of gyration
 t = wall thickness
 σ_Y = yield stress

$$\frac{M_Y}{M_{pl}} \approx 1.27$$

$$M_{pl_{red}} = M_{pl} \cos \left(\frac{\pi}{2} \frac{N}{N_{pl}} \right)$$

*) Note that in this particular case the 'approximate' expression exactly equals the full expression.

COMBINED BENDING AND AXIAL LOADING

The stresses and strains in a tubular section due to combined bending moments and normal forces can be described with the aid of moment-normalforce-curvature, M-N- κ , relationships.

Three sets of M-N- κ relationships can be established, corresponding to three different states of strain, see figure B1:

- 1) elastic strains only,
- 2) plastic strains in the compression zone (primary plastic),
- 3) plastic strains in both the compression and the tension zone (secondary plastic).

Algebraic M-N- κ expressions for each of the three situations can be found in e.g. ref.[3].

The boundaries of the above three states are depicted in figure B2, and can be expressed as follows:

Primary plastic behavior starts to occur when:

$$\frac{M}{M_{pl}} + \frac{N}{N_{pl}} \geq 1$$

The boundary between primary and secondary plastic behavior can only be determined in an iterative way, see ref.[3].

The state of full plasticity is reached when :

$$\frac{M}{M_{pl}} = \cos \left(\frac{\pi}{2} \frac{N}{N_{pl}} \right)$$

This expression is the well known 'yield contour' of a circular section, see appendix A.

To establish the axial shortening of a beam-column under combined bending and axial loading, the centroidal axial strains, ϵ_{ax} , have to be known that occur in each of the above three stress states. To this purpose ref.[3] gives M-N- ϵ_{ax} relationships that, however, require an iterative procedure to derive the explicit values of ϵ_{ax} .

This problem is avoided by assuming that all the plastic strains are concentrated in infinite short areas: the plastic hinge concept as described in chapter 2. In fact all plastic shortening and curvature are concentrated

in points, defining the relationship between those magnitudes with the aid of the yield contour of a circular section. In this way plastic shortening is related to the change in rotation in the plastic hinges.

In chapter 2 a second assumption is introduced by simplifying the shape of the yield contour. This makes it possible to establish closed form expressions relating axial shortening to axial force.

Both assumptions result in an acceptable small error as long as members are considered with relatively low slenderness and small initial imperfections, see e.g. the results from chapter 8.

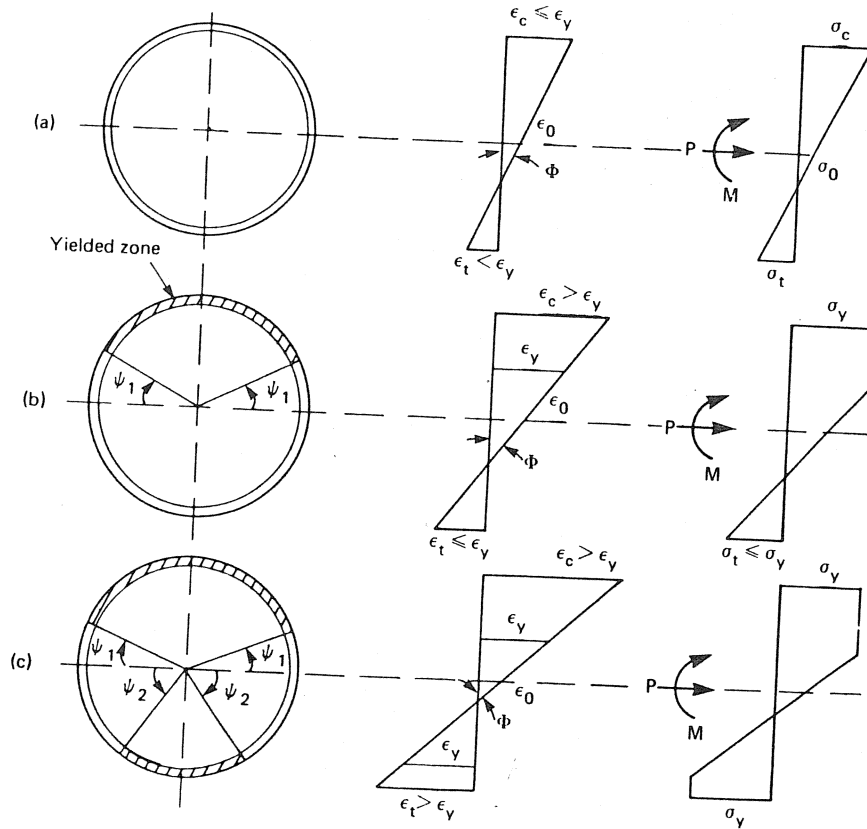


Figure B1
 Elastic, primary plastic, and secondary plastic states of strain
 in a tubular section due to combined bending moments and normal forces
 (ref.[3])

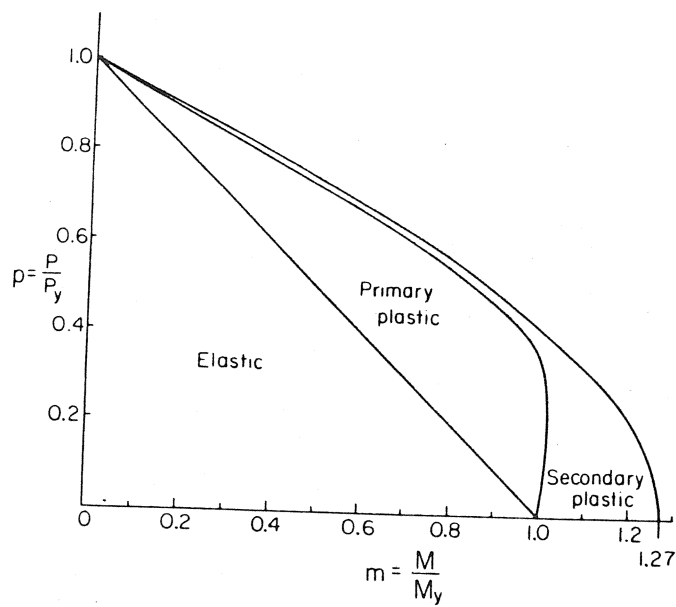


Figure B2
 Boundaries between elastic, primary plastic,
 and secondary plastic states of strain
 (ref.[3])

STRING, THEORY AND LISTING

STRut Input Generator (STRING) calculates several input data for the INTRA strut element ISTR, see figure C1.

Pre-buckling shortening (upto PCR, see figure C1) results from axial compression only.

The value of $PCR = F_{ult}^*$ is derived from the column strength curve 'A' from Toma and Chen (ref.[3]). This curve is established starting from numerical results taking into account longitudinal and circumferential residual welding stresses, 1.0 % out of roundness of the tube diameter and 0.1 % out of straightness of the tube length, see figure C2. This curve, especially designed to predict the ultimate compressive strength of offshore platform members, shows slightly lower ultimate loads than the frequently used CRC column strength curve.

The reduction of the ultimate load due to lateral loading and end moments is calculated with the aid of the AISC interaction formula:

$$\frac{F_{ult}^*}{F_{ult}} + \frac{C M}{M_{pl} (1 - (F/F_E))} = 1$$

C = reduction factor depending on the ratio of the end moments, given by:

$$C = 0.6 - 0.4 \frac{M_a}{M_b} \geq 0.40$$

- F_{ult}^* = ultimate load reduced by lateral loads or end moments
- F_E = Euler buckling load
- F_{ult} = ultimate load derived from column strength curve
- M = bending moment resulting from lateral loads or end moments
- M_a, M_b = end moments
- M_{pl} = plastic moment

A further reduction, due to hydrostatic pressure, can be taken into account with the aid of expressions derived in reference [3]. Those expressions are

based on a two-dimensional (plane stress) description of the stress state in the wall of the submerged tubular member. They describe a reduction of the tension yield stress in axial direction due to the presence of circumferential compressive stresses resulting from the hydrostatic pressure.

The postbuckling shortening (from U34, see figure C1) is established from three contributions: elastic axial strains, change of geometry, and plastic axial strains. It is assumed that the plastic strains are concentrated in three plastic hinges, using a simplified expression for the yield surfaces in those regions, see chapter 2 and appendix B.

NOTE: It is assumed that no local buckling will occur and that full plastic moment development is possible. It is also assumed that sufficient rotation capacity is available to achieve large post-buckling deformations. These are valid assumptions for tubular members with a D/T ratio upto about 50.

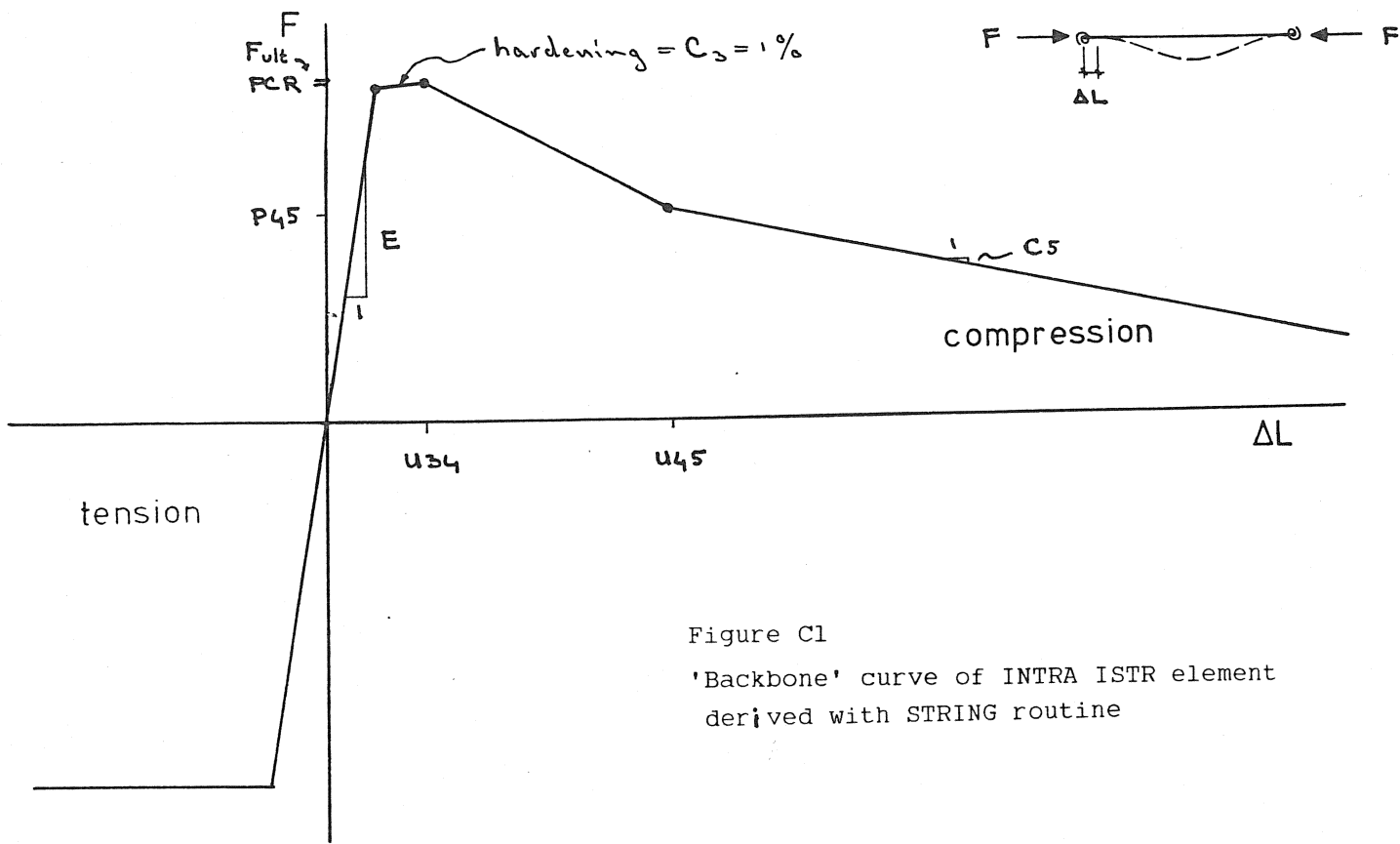


Figure C1
 'Backbone' curve of INTRA ISTR element
 derived with STRING routine

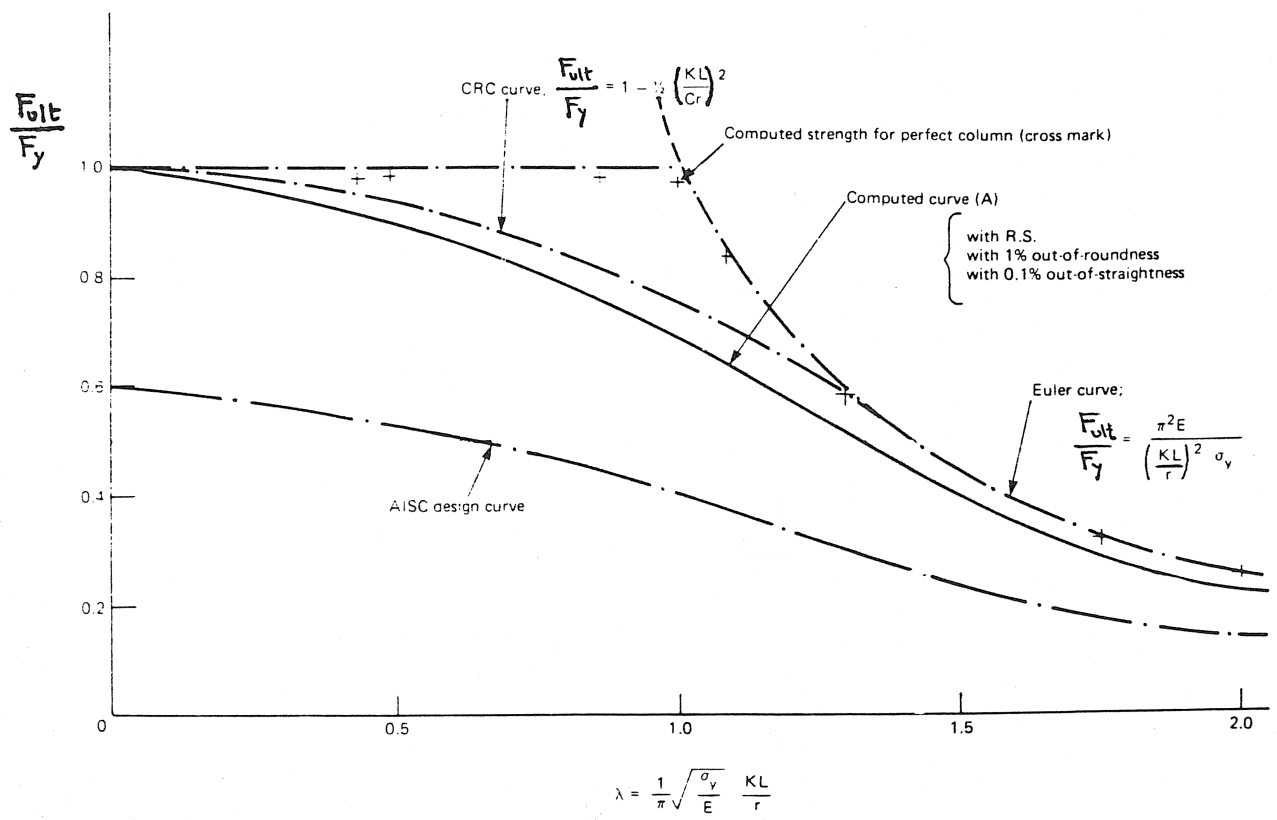


Figure C2
 Column strength design curve (ref.[3])

PROGRAM STRING

C STRUT INPUT GENERATOR (STRING) CALCULATES
 C SEVERAL INPUT DATA FOR THE
 C INTRA "BUCKLING STRUT" ELEMENT "ISTR".

C THE PROGRAM CAN BE USED TO ESTABLISH THE DATA FOR
 C FIXED ENDED OR PARTIALLY FIXED (SPRING) ENDED STRUTS.
 C A LATERAL (WAVE) LOADING CAN BE TAKEN INTO ACCOUNT,
 C WETHER DEFINED AS LATERAL LOAD / UNIT LENGTH, OR
 C AS EQUIVALENT NODAL MOMENTS.
 C ALSO END MOMENTS, REDUCING THE ULTIMATE AXIAL CAPACITY,
 C CAN BE TAKEN INTO ACCOUNT
 C THE EFFECTS OF HYDROSTATIC PRESSURE ON BUCKLING AND
 C POST BUCKLING BEHAVIOUR ARE RECKONED WITH.

C NOTATION:

C	A	SECTION AREA	[M**2]
C	C	REDUCTION FACTOR	[-]
C	C3, C5	INTRA INPUT PARAMETERS: HARDENING MODULI AS A PROPORTION OF E	[-]
C	CHECK	HELP VARIABLE TO CHECK CONVERGENCE	[KN]
C	D	OUTER DIAMETER	[M]
C	DEFL	LATERAL DEFLECTION AT CENTRE OF STRUT	[M]
C	DELTA(*)	AXIAL SHORTENING (FUNCTION STATEMENT, SEE SUBPROGRAM AT BOTTOM OF LISTING)	[M]
C	DELTA1	AXIAL SHORTENING DUE TO ELASTIC COMPRESSION	[M]
C	DELTA2	AXIAL SHORTENING DUE TO CHANGING GEOMETRY	[M]
C	DELTA3	AXIAL SHORTENING DUE TO PLASTIC STRAINS	[M]
C	DEPTH	DEPTH BELOW WATER SURFACE	[M]
C	E	YOUNG'S MODULUS	[KN/M**2]
C	F	AXIAL LOAD	[KN]
C	FEUL	EULER BUCKLING LOAD	[KN]
C	FU	ULTIMATE AXIAL LOAD	[KN]
C	FUH	ULTIMATE AXIAL LOAD REDUCED BY HYDROSTATIC PRESSURE	[KN]
C	FUHM	ULTIMATE AXIAL LOAD REDUCED BY HYDROSTATIC PRESSURE, BENDING MOMENTS AND LATERAL LOAD	[KN]
C	FY	AXIAL YIELD FORCE (SQUASH LOAD)	[KN]
C	FYH	AXIAL YIELD FORCE REDUCED BY HYDROSTATIC PR.	[KN]
C	HELP	HELP VARIABLE	[KN*M]
C	HYP	HYDROSTATIC PRESSURE	[KN/M**2]
C	HYPCRIT	CRITICAL HYDR. PR. CAUSING COLLAPSE OF TUBE	[KN/M**2]
C	I	MOMENT OF INERTIA	[M**4]
C	ICOUNT	COUNTING VARIABLE	[-]
C	K	EFFECTIVE LENGTH FACTOR	[-]
C	L	LENGTH	[M]
C	LAMBDA	MODIFIED SLENDERNESS RATIO	[-]
C	M1, M2	END MOMENTS DUE TO LATERAL LOAD	[KN*M]
C	M3, M4	END MOMENTS DUE TO IMPOSED ROTATION	[KN*M]
C	MA, MB	TOTAL END MOMENTS	[KN*M]
C	MPL	PLASTIC MOMENT	[KN*M]
C	MPLH	PLASTIC MOMENT REDUCED BY HYDROSTATIC PR.	[KN]
C	MPLHRED	PLASTIC MOMENT REDUCED BY HYDROSTATIC PRESSURE AND AXIAL LOAD	[KN*M]
C	NU	POISSONS RATIO	[-]
C	PCR, P45	INTRA INPUT PARAMETERS: FORCE COORDINATES AT INTERSECTION OF ZONES	[KN]
C	QLAT	LATERAL LOAD / UNIT LENGTH	[KN/M]
C	R	RADIUS OF GYRATION	[M]
C	SIGY	YIELD STRESS	[KN/M**2]
C	T	WALL THICKNESS	[M]
C	U34, U45	INTRA INPUT PARAMETERS: DISPLACEMENT COORDINATES AT INTERSECTION OF ZONES	[M]

C REFERENCE [3]: W.F.CHEN AND D.J.HAN
C TUBULAR MEMBERS IN OFFSHORE STRUCTURES
C PITMAN, LONDON, 1985

REAL I, K, L, LAMBDA, M1, M2, M3, M4, MA, MB, MPL,
\$ MPLH, MPLHRED, NU
CHARACTER DUMMY*1

C INTERACTIVE INPUT

WRITE(6,*) ' '
WRITE(6,*) ' '
WRITE(6,*) ' '
WRITE(6,*) 'BEGIN PROGRAM STRING'
WRITE(6,*) ' '
WRITE(6,*) 'TYPE IN DATA AND PRESS "RETURN" '
WRITE(6,*) ' '
WRITE(6,*) ' '
WRITE(6,*) 'LENGTH ? [M] '
READ(5,*) L
WRITE(6,*) 'OUTER DIAMETER ? [M] '
READ(5,*) D
WRITE(6,*) 'WALL THICKNESS ? [M] '
READ(5,*) T
WRITE(6,*) 'EFFECTIVE LENGTH FACTOR "K" ? [-] '
READ(5,*) K
WRITE(6,*) 'DEPTH BELOW WATER SURFACE ? [M] '
READ(5,*) DEPTH
WRITE(6,*) 'LATERAL LOAD / UNIT LENGTH ? [KN/M]; '
WRITE(6,*) 'PUT IN AN ARBITRARY NEGATIVE VALUE IF EQUIVALENT '
WRITE(6,*) 'NODAL MOMENTS HAVE TO BE SPECIFIED INSTEAD OF
\$ A LATERAL LOAD '
READ(5,*) QLAT
IF (QLAT .LT. -0.1E-6) THEN
WRITE(6,*) 'FIRST NODAL MOMENT DUE TO LATERAL LOAD ? [KN*M] '
WRITE(6,*) 'NEGATIVE VALUE '
READ(5,*) M1
WRITE(6,*) 'SECOND NODAL MOMENT DUE TO LATERAL LOAD ? [KN*M] '
WRITE(6,*) 'POSITIVE VALUE '
READ(5,*) M2
END IF
WRITE(6,*) 'FIRST END MOMENT DUE TO IMPOSED ROTATION ? [KN*M] '
READ(5,*) M3
WRITE(6,*) 'SECOND END MOMENT DUE TO IMPOSED ROTATION ? [KN*M] '
READ(5,*) M4

C CALCULATION OF SOME USEFUL DATA

PI = 4. * ATAN(1.0)
SIGY = 345.E3
E = 204.E6
NU = 0.3
I = PI/64 * (D**4 - (D-2*T)**4)
A = PI/4 * (D**2 - (D-2*T)**2)
HYP = 0.001 * 1024. * 9.81 * DEPTH
HYPCRIT = ((2*E) / (1-NU*NU)) * ((T/D)**3)
FY = A * SIGY
FYH = FY * (1.-((8.6/33.)*(HYP/HYPCRIT)**1.2))
MPL = (D**3 - (D-2*T)**3) * SIGY/6
MPLH = MPL * (1.- 0.2*(HYP/HYPCRIT))
R = (I/A)**0.5
LAMBDA = (1./PI) * ((SIGY/E)**0.5) * (K*L)/R
FEUL = ((PI*PI)*E*I) / (K*K*L*L)

C CALCULATION OF THE EQUIVALENT NODAL MOMENTS FROM

```

C THE INPUT LATERAL LOAD

IF (QLAT .GT. 0.1E-6) THEN
  M1 = -1 * (QLAT*L*L)/ 12
  M2 = (QLAT*L*L)/ 12
END IF

C- CALCULATION OF THE LATERAL LOAD FROM
C THE INPUT EQUIVALENT NODAL MOMENTS

IF (QLAT .LT.-0.1E-6) THEN
  QLAT = ((M2-M1) / 2) * (12. / L*L)
END IF

C CALCULATION OF THE ULTIMATE AXIAL LOAD "FU"
C BASED ON THE COLUMN STRENGTH CURVE -A- FROM TOMA AND CHEN (1979).
C THIS CURVE IS ESTABLISHED FROM NUMERICAL RESULTS,
C TAKING INTO ACCOUNT LONGITUDINAL AND CIRCUMFERENTIAL RESIDUAL
C STRESSES, 1.0 % OUT OF ROUNDNESS OF THE TUBE DIAMETER,
C AND 0.1 % OUT-OF-STRAIGHTNESS OF THE TUBE LENGTH.
C SEE REFERENCE [3], PAGE 82.

IF (LAMBDA .LE. 1.41) THEN
  FU = FY * (1.0 - 0.091*LAMBDA - 0.22*LAMBDA*LAMBDA)
ELSE
  FU = FY * (0.015 + 0.834/(LAMBDA*LAMBDA))
END IF

C CALCULATION OF THE REDUCED ULTIMATE AXIAL LOAD "FUH",
C TAKING INTO ACCOUNT HYDROSTATIC PRESSURE.
C SEE REFERENCE [3], CHAPT. 7

FUH = FU * (1. + 0.125*(HYP/HYPCRIT)*LAMBDA) * (FYH/FY)

C CALCULATION OF THE REDUCED ULTIMATE AXIAL LOAD "FUHM",
C TAKING INTO ACCOUNT HYDROSTATIC PR. AND NODAL MOMENTS.
C THE REDUCTION IS BASED ON THE MODIED AISC DESIGN FORMULA.
C SEE REFERENCE [3], PAGE 88 AND 236.

MA = M1 + M3
MB = M2 + M4
IF (ABS(MA) .GT. 0.1E-6 .OR. ABS(MB) .GT. 0.1E-6) THEN
  IF (ABS(MB) .GT. ABS(MA)) THEN
    HELP = MA
    MA = MB
    MB = HELP
  END IF
  C = 0.6 - (0.4*(MB/MA))
  IF (C .LT. 0.4) THEN
    C = 0.4
  END IF
  IF (C*ABS(MA) .GT. MPL*(1-(FUM/FEUL))) THEN
    WRITE(6,*) ' '
    WRITE(6,*) ' '
    WRITE(6,*) 'END MOMENTS DUE TO LATERAL LOAD OR
$ IMPOSED'
    WRITE(6,*) 'DISPLACEMENTS TOO LARGE'
    WRITE(6,*) ' '
    WRITE(6,*) ' '
    STOP 'PROGRAM FINISHED'
  END IF
  ICOUNT = 0
  FUHM = FUH
  CHECK = FUHM
  FUHM = FUH * (1.-((C*ABS(MA))/(MPL*(1-(FUHM/FEUL))))))
  IF (ABS(CHECK-FUHM) .GT. (0.01 * ABS(FUHM))) THEN

```

```

C      *** CONVERGENCE TOLERANCE = 0.01 * FUHM ***
      ICOUNT = ICOUNT + 1
      GOTO 10
      END IF
      ELSE
      FUHM = FUH
      END IF

```

C CALCULATION OF INTRA INPUT PARAMETERS

```

      U34 = DELTAL(A, E, FUHM, FYH, L, MPLH, PI, QLAT)
      C3 = 0.01
      P45 = 0.5 * FUHM
      U45 = DELTAL(A, E, P45, FYH, L, MPLH, PI, QLAT)
      C5 = (0.5*FUHM - 0.25*FUH) / (U45 -
$      DELTAL(A, E, 0.25*FUH, FYH, L, MPLH, PI, QLAT))
      PCR = (FUHM - U34 * C3 * (E*A)/L) / (1.-C3)

```

C OUTPUT OF INTRA PARAMETERS

```

      WRITE(6,*) ' '
      WRITE(6,*) ' '
      WRITE(6,*) ' '
      WRITE(6,20)
20     FORMAT(' INTRA INPUT PARAMETERS IN KN AND M OR DIMENSIONLESS:')
      WRITE(6,*) ' '
      WRITE(6,30) E
30     FORMAT(' YMOD', (E10.3))
      WRITE(6,40) SIGY
40     FORMAT(' SLYD', (E10.3))
      WRITE(6,50) C3
50     FORMAT(' C3', (E10.3))
      WRITE(6,60) C5
60     FORMAT(' C5', (E10.3))
      WRITE(6,70) -U34
70     FORMAT(' U34', (E10.3))
      WRITE(6,80) -PCR
80     FORMAT(' PCR', (E10.3))
      WRITE(6,90) -U45
90     FORMAT(' U45', (E10.3))
      WRITE(6,100) -P45
100    FORMAT(' P45', (E10.3))

```

C CALCULATION AND OUTPUT OF ADDITIONAL DATA

```

      U13 = PCR * L/(E*A)
      UMAX = U45 - P45/C5
      WRITE(6,*) ' '
      WRITE(6,*) ' '
      WRITE(6,*) ' PRESS "RETURN" TO CONTINUE'
      READ(5,105) DUMMY
105    FORMAT(A1)
      WRITE(6,110)
110    FORMAT(' ADDITIONAL DATA')
      WRITE(6,*) ' '
      WRITE(6,120) U13, U13/L
120    FORMAT(' U13, U13 / L', (2(3X,E10.3)))
      WRITE(6,130) PCR, PCR/FY
130    FORMAT(' PCR, PCR / FY', (2(3X,E10.3)))
      WRITE(6,140) U34, U34 / L
140    FORMAT(' U34, U34 / L', (2(3X,E10.3)))
      WRITE(6,150) FUHM, FUHM / FY
150    FORMAT(' FUHM, FUHM / FY', (2(3X,E10.3)))
      WRITE(6,160), U45, U45/L
160    FORMAT(' U45, U45 / L', (2(3X,E10.3)))
      WRITE(6,170), P45, P45 / FY

```

```

170  FORMAT(' P45, P45 / FY',2(3X,E10.3))
      WRITE(6,180) UMAX, UMAX / L
180  FORMAT(' UMAX, UMAX / L',2(3X,E10.3))
      WRITE(6,*) ' '
      WRITE(6,190) FY
190  FORMAT(' FY',(3X,E10.3))
      WRITE(6,195) FYH / FY
195  FORMAT(' FYH / FY',(3X,E10.3))
      WRITE(6,200) FEUL
200  FORMAT(' FEUL',(3X,E10.3))
      WRITE(6,210) FU
210  FORMAT(' FU',(3X,E10.3))
      WRITE(6,220) FUH / FU
220  FORMAT(' FUH / FU',(3X,E10.3))
      WRITE(6,225) FUHM / FUH
225  FORMAT(' FUHM / FUH',(3X,E10.3))
      WRITE(6,226) MPL
226  FORMAT(' MPL',(3X,E10.3))
      WRITE(6,227) MPLH / MPL
227  FORMAT(' MPLH / MPL',(3X,E10.3))
      WRITE(6,228) LAMBDA
228  FORMAT(' MODIFIED SLENDERNESS RATIO',(3X,E10.3))
      WRITE(6,229) K*L/R
229  FORMAT(' SLENDERNESS RATIO K*L/R',(3X,E10.3))
      WRITE(6,230) ICOUNT
230  FORMAT(' NUMBER OF ITERATIONS',(3X,I3))
      WRITE(6,*) ' '
      WRITE(6,*) 'END OF THIS PROGRAM'

```

END

```

REAL FUNCTION DELTAL(A,E,F,FYH,L,MPLH,PI,QLAT)
REAL L, MPLH, MPLHRED

```

```

C SUBPROGRAM TO CALCULATE THE AXIAL SHORTENING OF A STRUT
C IN A POST-COLLAPSE SITUATION (THREE PLASTIC HINGES HAVE
C BEEN FORMED).
C SEE REFERENCE [3], CHAPT.5

```

```

MPLHRED = MPLH * COS((PI/2)*(F/FYH))
DEFL = (2 * MPLHRED - 1./8 * (QLAT) * L*L) / F
IF (1./8 * (QLAT) * L*L .GT. 2*MPLHRED) THEN
  WRITE(6,*) ' '
  WRITE(6,*) ' '
  WRITE(6,*) 'LATERAL LOAD TOO LARGE,'
  WRITE(6,*) 'NO EQUILIBRIUM POSSIBLE'
  WRITE(6,*) ' '
  WRITE(6,*) ' '
  STOP ' PROGRAM FINISHED'
END IF
IF (((L/2)*(L/2) - DEFL*DEFL) .LT. 0.) THEN
  WRITE(6,*) ' '
  WRITE(6,*) ' '
  WRITE(6,*) 'LATERAL LOAD TOO LARGE; LATERAL DEFLECTION'
  WRITE(6,*) 'EXCEEDS 1/2 * L, TO MAINTAIN EQUILIBRIUM'
  WRITE(6,*) ' '
  WRITE(6,*) ' '
  STOP 'PROGRAM FINISHED'
END IF
DELTAL = F * L/(E*A)
DELTA2 = 2 * (L/2 - ((L/2)*(L/2) - DEFL*DEFL)**0.5)
DELTA3 = 4 * (MPLH/FYH) * ASIN(DEFL*2./L)
DELTAL = DELTAL + DELTA2 + DELTA3

```

END

DIMENSIONAL ANALYSIS

If a physical phenomenon can be described with the aid of n parameters, and if those parameters are defined with the aid of m fundamental dimensions, it will be possible to rearrange the n parameters in such a way that $m-n$ dimensionless parameters will remain, see de Vries, ref.[17].

It follows from chapter 2 that the post-buckling behavior of a strut can be described with the aid of 7 parameters, starting from the plastic hinge concept:

l	length	$[L]$,
D	diameter	$[L]$,
t	wall thickness	$[L]$,
Δl	axial shortening	$[L]$,
σ	normal force / sectional area	$[F/L^2]$,
σ_y	yield stress	$[F/L^2]$,
E	Youngs modulus	$[F/L^2]$.

In chapter 7 the influence of two other parameters at the post-buckling behavior has also been investigated:

q	lateral load per unit length	$[F/L]$,
d	waterdepth	$[L]$.

Notice that the effective length factor K , and the imposed end moments do not influence the post-buckling behavior as described within the three-plastic-hinges model.

Statics of physical phenomena can be described using two fundamental dimensions, force $[F]$, and length $[L]$, hence $7+2-2 = 7$ dimensionless constants can be derived. Systematic rearrangement according to ref.[17] results in the following: $\Delta l/l$, σ/σ_y , l/D , D/t , σ_y/E , $q/\sigma_y * l$ and d/l .

A parameter study has been carried out starting from those 7 expressions, see chapter 7. It was decided to investigate the relationship between σ/σ_y and $\Delta l/l$ (axial force versus axial shortening) while varying only the geometrical properties l/D and D/t and choosing appropriate fixed values for E/σ_y , $q/\sigma_y * l$ and d/l .

The main parameter in the description of buckling is the slenderness ratio λ . In case of a column with a circular section λ is related to the l/D ratio almost linearly:

$$\lambda = \frac{Kl}{\sqrt{I/A}} \approx \frac{Kl}{0.35 D} = \text{constant} * l/D$$

Indeed it appears that the l/D ratio shows the largest influence in a parameter analysis concerning the buckling behavior of tubular columns, see e.g. ref.[10] or ref.[15].

GEOMETRIC NONLINEARITY

Consider the equilibrium of a single element in the displacement based finite element method, described with the principle of virtual work:

$$\int_V \delta \bar{\epsilon}' \bar{\sigma} dV = \delta \bar{u}' \bar{f} \quad (1)$$

δ = variational symbol, denoting virtual changes

$\bar{\epsilon}$ = vector of strains

$\bar{\sigma}$ = vector of stresses

\bar{f} = vector of nodal loads (forces and moments) with respect to one element

\bar{u} = vector of element nodal degrees of freedom (displacements and rotations) in local coordinate system

Writing $\delta \bar{\epsilon} = \bar{B} \delta \bar{u}$, this can be expressed as:

$$\int_V \bar{B}' \bar{\sigma} dV = \bar{f} \quad (2)$$

\bar{B} = strain interpolation matrix

In case of linear behavior follows:

$$\int_V \bar{B}' \bar{E} \bar{B} dV \bar{u} = \bar{f} \quad (3)$$

\bar{E} = matrix of material stiffness properties

Transformation to global coordinates and summation of the element contributions results in the system of simultaneous equations describing the equilibrium of the total element assemblage:

$$\bar{K}_o^* \bar{u}^* = \bar{f}^* \quad (4)$$

\bar{K}_o^* = system linear stiffness matrix

\bar{u}^* = vector of system nodal degrees of freedom

\bar{f}^* = vector of system nodal loads

In case of geometric nonlinear behavior, the strains and stresses in the element are nonlinear functions of the element degrees of freedom. The equilibrium equations now have to be written in an incremental fashion, using a linearized expression. Starting from equation (2) this yields:

$$\delta \int_V \bar{B}' \bar{\sigma} dV = \delta \bar{f} \quad (5)$$

δ = variational symbol, denoting incremental changes

Considering that the strain interpolation matrix \bar{B} now consists of a linear part, \bar{B}_1 , and a nonlinear part, \bar{B}_{nl} , and considering that $\delta \bar{B} = \delta \bar{B}_{nl}$, the following can be derived: (see ref.[20])

$$\begin{aligned} \delta \int_V \bar{B}' \bar{\sigma} dV &= \int_V \delta \bar{B}' \bar{\sigma} dV + \int_V \bar{B}' \delta \bar{\sigma} dV = \\ &= \int_V \delta \bar{B}'_{nl} \bar{\sigma} dV + \int_V \bar{B}'_1 \bar{E} \bar{B}_1 dV \delta \bar{u} + \left\{ \int_V (\bar{B}'_1 \bar{E} \bar{B}_{nl} + \bar{B}'_{nl} \bar{E} \bar{B}_1 + \bar{B}'_{nl} \bar{E} \bar{B}_{nl}) dV \right\} \delta \bar{u} \end{aligned} \quad (6)$$

The first term of the right-hand-side of (6) can generally be written as: $\int_V \delta \bar{B}'_{nl} \bar{\sigma} dV = \bar{K}_\sigma \delta \bar{u}$. The matrix \bar{K}_σ is called the 'element initial stress matrix' or the 'element geometric stiffness matrix'.

The second term of the right-hand-side of (6) can be recognized in equation (3), and can be written as: $\int_V \bar{B}'_1 \bar{E} \bar{B}_1 dV \delta \bar{u} = \bar{K}_0 \delta \bar{u}$, being \bar{K}_0 the standard 'element linear stiffness matrix' for small displacements and rotations.

Finally, the third term of the right-hand-side can be expressed as: $\left\{ \int_V (\bar{B}'_1 \bar{E} \bar{B}_{nl} + \bar{B}'_{nl} \bar{E} \bar{B}_1 + \bar{B}'_{nl} \bar{E} \bar{B}_{nl}) dV \right\} \delta \bar{u} = \bar{K}_\epsilon \delta \bar{u}$, where \bar{K}_ϵ is known as 'element initial strain matrix', 'element large displacement matrix' or 'element large rotation matrix'.

The matrix \bar{K}_ϵ represents the change in element stiffness due to the changing position and changing shape of the element in a deforming structure, and its entries are a function of the current element nodal displacements and nodal rotations. The matrix \bar{K}_σ represents the additional element stiffness that results from the forces that act at the deformed elements in the deformed structure, and its entries are a function of the current stresses in the element.

Transformation to global coordinates and summation of the element contributions now results in a system of equations of the incremental variables:

$$(\bar{K}_0^* + \bar{K}_\epsilon^* + \bar{K}_\sigma^*) \delta \bar{u}^* = \bar{K}_t^* \delta \bar{u}^* = \delta \bar{f}^* \quad (7)$$

\bar{K}_t^* = system tangent stiffness matrix

The nonlinear load-deformation behavior of a structure can be established in a stepwise fashion starting from the above equation (7). Usually an additional Newton-Raphson type iteration procedure is applied, based on the same expression.

NOTES:

- Stresses and strains in large displacement analysis can be defined in several different ways, resulting in different expressions for the nonlinear part of the strain interpolation matrix, \bar{B}_{nl} .
- The incremental nodal displacements and nodal rotations of a deforming structure can be defined referring to the original configuration (Total Lagrangian description), or to a configuration in a previous (usually the latest) increment (Updated Lagrangian description). An effective application of the two different descriptions requires different definitions of stress and strain, what may result in different nonlinear parts of the strain interpolation matrices. However, if the appropriate definitions are applied, identical results are obtained, and the difference only appears in the numerical effectiveness of the both procedures, see ref.[1].
- The establishment of the 'element large displacement matrix' \bar{K}_ϵ is often neglected in element descriptions that refer nodal displacements and nodal rotations to a co-rotating local coordinate system (i.e. in an Updated Lagrangian description). In this case the change in element stiffness due to the change in shape of the element (referred to the co-rotated local coordinate system) is neglected, whereas the change in element stiffness due to the total change in position of the element (referred to the global coordinate system) is taken into account, performing the transformation of the 'element linear stiffness matrix' \bar{K}_0 to the global coordinate system, starting from the current (updated)

transformation matrix. Note that in the determination of the 'element geometric stiffness matrix' \bar{K}_0 the change in shape of the element still has to be taken into account.

A much stronger simplification is made when the 'large displacement matrix' \bar{K}_ϵ is neglected, describing the nodal displacements etc. referring to the original coordinate system. In this case only the geometric nonlinear effect of small total displacements and rotations can be described appropriately.

Geometric nonlinearity in INTRA beam-column elements ;

The INTRA beam-column element BEMC uses a Total Lagrangian description, without the establishment of a 'large deformation matrix' \bar{K}_ϵ . It has been mentioned above that this simplification implies that the element should only be used to model structures that experience relatively small total deformations. An additional approximation is made in the derivation of the geometric stiffness matrix \bar{K}_0 , as will be described in the following.

The INTRA beam-column element LANB starts from an Updated Lagrangian description. However, also in this element the approximate description of the geometric stiffness matrix \bar{K}_0 is applied.

Consider a two dimensional beam element with local coordinate system as depicted in figure E1. The vector of nodal degrees of freedom reads:

$$\bar{u}' = \{ u_1, v_1, \phi_1, u_2, v_2, \phi_2 \}$$

The axial displacements $u(x)$ within the element can be described with a linear interpolation, whereas the lateral displacements $v(x)$ can be represented with a third degree (cubic) polynomial. Assuming small strain conditions, a second order strain definition can be applied: $\epsilon = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial v}{\partial x} \right)^2$. The 'element geometric stiffness matrix' \bar{K}_0 can be established as follows, starting from classical beam theory and integrating over the original volume, see e.g. ref.[2] or ref.[13]:

$$\bar{K}_\sigma = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6N}{51} & \frac{N}{10} & 0 & -\frac{6N}{51} & \frac{N}{10} \\ 0 & \frac{N}{10} & \frac{2NI}{15} & 0 & -\frac{N}{10} & -\frac{NI}{30} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{6N}{51} & -\frac{N}{10} & 0 & \frac{6N}{51} & -\frac{N}{10} \\ 0 & \frac{N}{10} & -\frac{NI}{30} & 0 & -\frac{N}{10} & \frac{2NI}{15} \end{bmatrix}$$

N = normal force in element

A strongly simplified version of this 'element geometric stiffness matrix' can be derived starting from a linear interpolation of the lateral element displacements $v(x)$:

$$\bar{K}_\sigma = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{N}{1} & 0 & 0 & -\frac{N}{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{N}{1} & 0 & 0 & \frac{N}{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This matrix \bar{K}_σ is often referred to as 'string matrix'. An inconsistent situation occurs if it is used in combination with a linear stiffness matrix \bar{K}_0 that is derived starting from the cubic interpolation of lateral element displacements.

The INTRA beam-column elements BEMC and LANB can take into account geometric nonlinear effects only in such an inconsistent fashion, using the above 'string matrix'. In chapter 9 it is shown that this simplification results in an underestimation of the decrease in bending stiffness of an axially loaded beam-column. Therefore it is impossible to simulate a buckling strut with the aid of two INTRA BEMC elements.

References: [1], [2], [13], [14], [20].

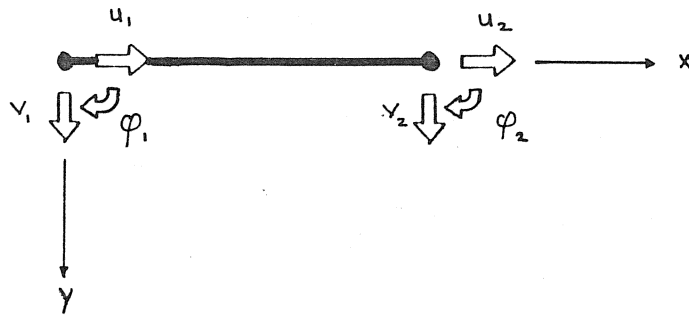


Figure E1

Two-dimensional beam-column element