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Joint Doppler and DOA Estimation Using 2D MUSIC in Presence of Phase Residual

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***Abstract*— This paper investigates the joint Doppler and DOA (Direction-of-Arrival) estimation with a wideband phased array in presence of phase residual due to range-Doppler coupling appearing in pulse-Doppler radars. 2D MUSIC algorithm is applied and a compensation approach is developed to eliminate the influence of phase residual. Simulation data validate the improvement of joint Doppler and DOA estimation performance using the proposed method.**

I. Introduction

Moving targets detection is usually performed by Doppler processing over a number of transmitted waveforms which form the coherent processing interval (CPI). To estimate simultaneously both range and Doppler, FMCW waveforms are widely used especially in weather and automotive radars. Due to range-Doppler coupling in such waveform, a phase residual arises for moving targets, which decreases the resolution and increase estimation bias. This problem becomes especially severe for radars with large operational bandwidth and fast moving targets [1], [2], [3].

DOA, also called direction finding, angle estimation or bearing estimation, utilizes the phase delay between array elements. Many algorithms to estimate DOA have been proposed in the last decades. The beamforming methods, such as Bartlett, minimum-variance distortionless response (MVDR) can effectively perform filtering in the spatial domain by emphasizing certain DOAs while suppressing others [4]. However, the resolution of beamforming methods is decided by the size of the array, which limits its practical applications. Another approach called Maximum likelihood estimator (MLE) chooses the DOA which most closely fits the measured data. The problem limiting its use in real system is its computational intensity and the requirement for accurate initial estimation [5], [6]. One of the most popular approaches for DOA estimation currently is called subspace based methods, which exploit the eigenstructure of the covariance matrix. Subspace based methods, such as multiple signal classification (MUSIC) [7] and estimation of signal parameters via rotational invariance technique (ESPRIT) [8], take advantage of the orthogonality between signal subspace and noise space to estimate the angles of coming signals. As the subspace based methods can provide excellent performance on the resolution and accuracy, algorithms from this group are promising in real application. The subspace based methods can also be extended to 2 dimension [9], [10], in azimuth and elevation, or in azimuth and range. In addition, MUSIC-based method is able to image extended targets [11].

Alternative algorithms for joint estimation include IAA (Iterative Adaptive Algorithm)[1] which is bad on SNR (Signal Noise Ratio) tolerance compared with subspace based super resolution method, or RFT (Radon-Fourier Transform) algorithms [2], [3], whose performance highly relies on the hardware of the system, such as number of elements. Thus, the subspace based method seems a better choice for joint estimation for DOA and Doppler.

In this paper we present an algorithm to suppress the phase residual in joint estimation of Doppler and DOA using wideband pulse Doppler radar. 2D MUSIC is applied, and a compensation method for phase residual is integrated into the 2D MUSIC. The rest of the paper is organized as follows. In section II, the signal model for wideband phased array is introduced. In section III, 2D MUSIC is introduced to estimate jointly the Doppler and DOA. The simulation results are shown in section IV and finally, conclusions are drawn in section V.

II. Wideband Model

A. Element Received Signal Model

A pulse-Doppler radar with wideband waveform is considered herein. For a single element wideband radar, several pulses are transmitted in series by transmit antenna and the reflected signals are received by receive antenna. Then range compression is applied on received signals which are then transferred to fast-frequency/slow time domain by FT (Fourier Transform) on fast-time. After such pre-processing, the received signal can be modeled as $K \times M$ matrix and expressed as [1]:

$$\mathbf{Y} = \sum_{i=1}^{I} x_i \mathbf{A}_i + \mathbf{N} \tag{1}$$

where $i = 1, 2, ..., I$, x_i, represent the number index of scatters and ith complex amplitude, respectively, $\mathbf{A}_n$ is a $K \times M$ matrix containing target signature and $\mathbf{N}$ is additional white Gaussian noise with power σ^2.

The scatter signature $\mathbf{A}$ involved in (1) has been studied in [12], [13] and is shown to be the product of a two-dimensional (2D) cisoid with cross-coupling terms.

$$A(k,m) = \exp\left[j2\pi\left(-\tau\frac{B}{K}k + \frac{2vf_c}{c}T_r m + \frac{2v}{c}\frac{B}{K}T_r km\right)\right] \tag{2}$$

where the parameters are:

- τ initial round-trip delay
- v the velocity of the scatter in the range direction
- B bandwidth
- K sampling point number in fast time
- f_c carrier frequency
- T_r the pulse repetition interval
- k fast-time/frequency index, $k = 1, 2, ..., K$
- m slow time/frequency index, $m = 1, 2, ..., M$

In (2), the first two components represent a fast-time frequency sampled at a rate B/K and a Doppler frequency $2vf_c/c$ associated with slow time sampling T_r. The third term is the cross-coupling between fast-time and slow-time which is brought by phase residual.

B. Array model

After defining the received signals for one element, we consider an array model. Here we use the uniform linear array, which the steering vector is

$$\mathbf{a}(\theta) = [1, e^{j2\pi\frac{d}{\lambda}\sin\theta}, ..., e^{j2\pi\frac{(L-1)d}{\lambda}\sin\theta}] \tag{3}$$

where l, L, d, λ, and θ, respectively, represent the index of elements, number of elements, the inter-space of the adjacent elements, wavelength of center frequency and the angle of coming direction.

Then we can get the array model as (4).

$$\begin{aligned} s(l,k,m) &= \sum_i^I x_i \left\{\exp\left(j2\pi\frac{ld}{\lambda}\sin\theta_i\right)\right. \\ &\times \exp\left[j2\pi\left(-\tau_i\frac{B}{K}k + \frac{2v_i f_c}{c}T_r m\right)\right] \\ &\times \left.\exp\left(j2\pi\frac{2v_i}{c}\frac{B}{K}T_r km\right)\right\} \end{aligned} \tag{4}$$

We use notation (5) to simplify the model as (6)

$$\begin{aligned} \omega_{\theta,i} &= 2\pi\frac{d}{\lambda}\sin\theta_i \\ \omega_{v,i} &= 2\pi\frac{2v_i f_c}{c}T_r \\ \omega_{r,i} &= -2\pi\tau_i\frac{B}{K} \\ \omega_{c,i} &= 2\pi\frac{2v_i}{c}\frac{B}{K}T_r \end{aligned} \tag{5}$$

$$s(l,k,m) = \sum_i^I x_i e^{j\omega_{\theta,i}l + j\omega_{r,i}k + j\omega_{v,i}m + j\omega_{c,i}km} \tag{6}$$

Then the new model can be rewritten as (7)

$$y(l,k,m) = s(l,k,m) + n(l,k,m) \tag{7}$$

where $n(l,k,m)$ represent the discrete noise.

III. 2D MUSIC and Phase Residual Compensation

To apply 2D MUSIC algorithm, steering vector is formulated by searching both on velocity and angle domain. Here, the velocity term is chosen for the first loop because the coupling term is function of velocity and it needs one calculation for coupling compensation in per velocity loop. First the 3 dimensional data need to be reshaped to 2 dimensional one by stacking element and slow time dimension together as(8). Then we get $LM \times K$ matrix $\mathbf{y}$.

$$\mathbf{y} = [\mathbf{vec}(y(:,:,1))^T \ \mathbf{vec}(y(:,:,2))^T \ ... \ \mathbf{vec}(y(:,:,K))^T]^T \tag{8}$$

where $\mathbf{vec}$ means vectorization operation, and T means the transpose of the matrix.

The compensation for the coupling term at the velocity in loop is written as conjugate coupling component as (9).

$$s_{com}(l,k,m) = e^{-\omega_{c,p}km} \tag{9}$$

where

$$\omega_{c,p} = 2\pi\frac{2v_p}{c}\frac{B}{K}T_r \tag{10}$$

and vectorize as

$$\mathbf{S}_{com} = [\mathbf{vec}(s_{com}(:,:,1))^T \ ... \ \mathbf{vec}(s_{com}(:,:,K))^T]^T \tag{11}$$

This term in fact does not depend on element index l, we multiply this term for each element as (12).

$$\tilde{\mathbf{y}} = \mathbf{y} \circ \mathbf{S}_{com} \tag{12}$$

where $\circ$ means Hadamard product.

Since the compensation term is just a phase shift, it will not increase the noise power. Then we get a new data which the coupling term is removed for the velocity v_p of current loop. After that, we can do the angle search in current velocity of current loop.

Now 2D MUSIC algorithm can be formed. First, the covariance is computed using the data with the removed coupling term by (9) and (12) for the velocity in the current loop iteration.

$$\mathbf{R_x} = \mathbf{E}(\tilde{\mathbf{y}}\tilde{\mathbf{y}}^H) \tag{13}$$

where H operation means the conjugate transpose of the matrix.

Eigen decomposition is then used to decompose the covariance matrix, and the noise subspace associated to noise eigenvectors can be retrieved as $\mathbf{U_n}$ [10]. Here, a threshold for eigen values can be set to decide the number of targets. The steering vector is formulated as:

$$\mathbf{a} = [e^{-j\omega_{\theta,p}\times 0-j\omega_{v,p}\times 1},..,e^{-j\omega_{\theta,p}\times 0-j\omega_{v,p}\times M},\\ e^{-j\omega_{\theta,p}\times 1-j\omega_{v,p}\times 1},..,e^{-j\omega_{\theta,p}\times 1-j\omega_{v,p}\times M},\\ \dots\\ e^{-j\omega_{\theta,p}\times L-j\omega_{v,p}\times 1},..,e^{-j\omega_{\theta,p}\times L-j\omega_{v,p}\times M}] \quad (14)$$

After that, the MUSIC spectrum can be calculated using the following formula:

$$S_{music} = \frac{1}{\mathbf{a}\mathbf{U_n}\mathbf{U_n^H}\mathbf{a^H}} \quad (15)$$

Here, the alogorithm calculates the eigen decmoposition multiple times which is not time economic. The low complexity MUSIC algorithm in [14] can be applied for acceleration.

The algorithm is illustrated in Table I

TABLE I
2D MUSIC FOR JOINT DOA AND DOPPLER ESTIMATION

```
Algorithm
for v_p in [V_min, V_max]
  reshape data y to 2D dimension y as (8)
  compute s_com as (9) and multiply to raw data as (12)
  for θ in [θ_min, θ_max]
    compute covariance matrix as (13) and extract noise subspace
    formulate steering vector as (14)
    compute MUSIC spectrum as (15)
  end
end
```

IV. SIMULATION

In this section the influence of bandwidth and target radial velocity on estimation error is analysed and the ability to jointly estimate Doppler and DOA using the proposed method is evaluated. The parameters of the simulation are shown in Table II. Here the range resolution of the system is $\frac{c}{2B} = 0.0375m$.

TABLE II
PARAMETERS OF SYSTEM

Parameters	Values
Carrier frequency	77 GHz
Number of elements	12
Distance between elements	1.899 mm
Bandwidth	4 GHz
PRI	0.1 ms
CPI	1.6 ms
SNR	0 dB

A. Velocity Estimation Error with Velocity and Bandwidth

In the first part, we analyse the estimation error with the velocity. According to the coupling term in (5), $\omega_{c,i}$ is the function of the bandwidth of the system and the velocity of the target. And obviously, with wider bandwidth and higher velocity the coupling is larger, which mean the influence on the estimation results is larger.

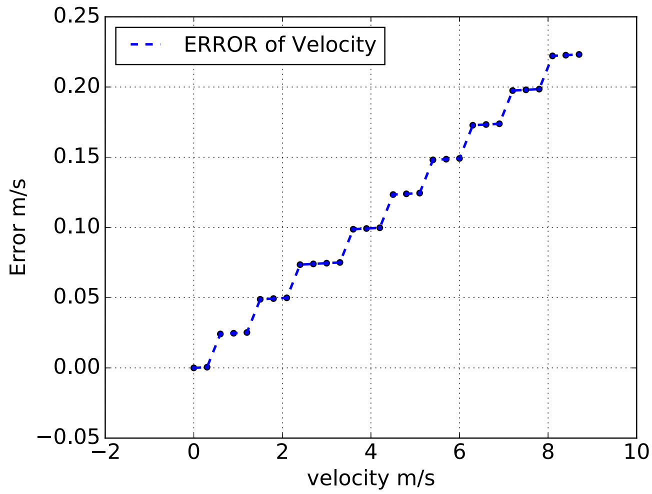

Fig. 1. Velocity estimation error target velocity

Using the parameters of the system as Table II, we compute the velocity estimation error for target velocities in Figure 1.

B. Compensation Algorithm Performance

For the second simulation, to simulate complex environment we add some targets with different velocities and DOAs. 6 targets are simulated with high velocities and random DOAs. The simulation results using 2D MUSIC without and with compensation are shown in Figure 2 and Figure 3, respectively. From Figure 2, estimation peaks appear at the biased position, with overestimated radial velocities. In the Figure 3, all the targets peaks appear in the right position with high resolution. The simulation results successfully validate the improvement of the proposed method on joint estimation of Doppler and DOA in presence of the phase residual.

V. CONCLUTIONS

In this paper, we have presented the phase residual compensation method for 2D MUSIC algorithm for joint estimation of Doppler and DOA using wideband radar. Firstly, we remove the phase residual influence on the parameters estimation by applying a compensation process for each velocity loop. Then we reformed the data model from 3 dimension–angle, slow-time and fast-time, to 2 dimension by stacking angle and slow-time dimension. Finally, the 2D MUSIC algorithms is used to estimate the Doppler and DOA jointly. The simulation results validate the proposed methods with high improvement in joint Doppler and DOA estimation performance. In the near future, the algorithm will be examined on the real data from automotive radar.

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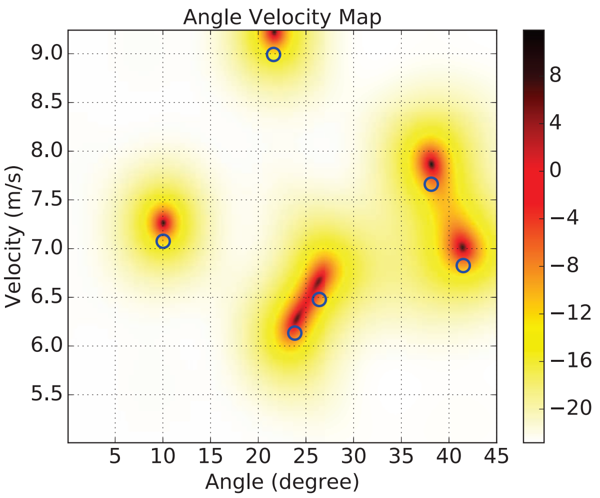

Fig. 2. Target position estimation without movement compensation for ten targets

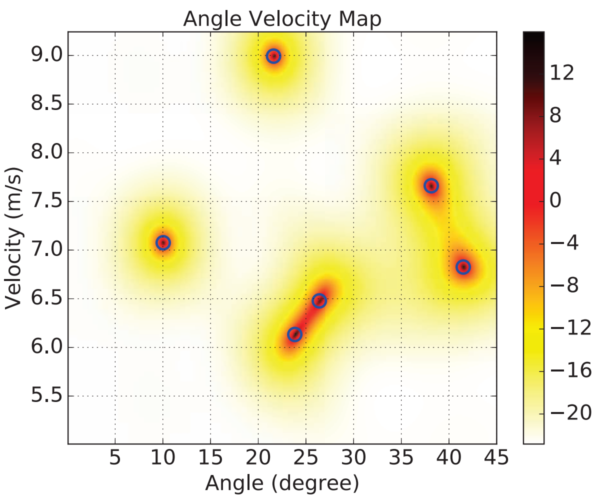

Fig. 3. Target position estimation with target movement compensation for ten targets

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