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THREE-DIMENSIONAL MODELLING OF MASONRY USING THE PARTITION OF UNITY METHOD

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Summary. This paper uses the Generalized Finite Element method for the introduction of the cohesive joint behaviour to model masonry. Cracked joints are modelled as displacement discontinuities and are introduced in the finite element model by additional degrees of freedom. These degrees of freedom are activated when the stress state in the joint reaches a critical level. The joint behaviour is governed by a rigid plasticity model.

1 INTRODUCTION

Since a few years, the modelling of displacement discontinuities within the framework of the partition of unity property of finite element shape functions has become popular. Using this property, enhanced nodal degrees of freedom can be added to the regular degrees of freedom during the computational process. The behaviour inside the discontinuity is governed by the enhanced degrees of freedom. The method is widely used for crack path prediction and failure prediction of quasi brittle materials[1,2,3,4].

Contrary to general structural problems, where the crack path is not known in advance, in masonry structures cracks often follow the joints. Consequently, in these structures the potential crack paths are known. Until now, interface elements, representing the joints, are placed on the brick interface for the so-called mesoscopic modelling of masonry[5,6]. However, this method shows several disadvantages. Firstly, a large number of additional degrees of freedom must be generated from the beginning of the computation. Secondly, the interfaces are present from the beginning, demanding a very high 'dummy' stiffness to correctly describe the 'elastic' behaviour of the joint. The high stiffness may lead to numerical problems such as oscillations and ill-conditioned problems.

In this work, a mesoscopic modelling technique using the Generalized Finite Element method[4] is presented. Stones are represented by three dimensional brick elements and remain elastic throughout the computation. Joints are situated on the stone boundaries. Initially the joints are not active. When the traction state inside a joint reaches a critical level, a displacement discontinuity is introduced. Joint separation is governed by a rigid plastic law.

2 ELEMENT TECHNOLOGY

An eight-noded three-dimensional brick element is used as underlying basis. Each side of the brick element is a possible position for a joint. A stone is defined as a collection of brick elements surrounded by joints. The total masonry structure is a collection of N_s stones. The displacement field $\mathbf{u}$ in an arbitrary point of the masonry structure is given by:

$$\mathbf{u} = \hat{\mathbf{u}} + \sum_{i=1}^{N_s} H_i \tilde{\mathbf{u}}_i \tag{1}$$

in which $\hat{\mathbf{u}}$ is the regular part of the displacement field, H_i is the Heaviside step function related to stone i and $\tilde{\mathbf{u}}_i$ is the enhanced part of the displacement field related to stone i. The Heaviside step function equals one when the point is situated inside stone i and zero otherwise. Equation (1) introduces an additional set of degrees of freedom for every stone. Simone et al.[4] showed that nodes on a tripple joint only need two independent additional sets of degrees of freedom. Taken this into consideration, a finite element mesh can be constructed such that the total number of additional degrees of freedom is limited to two.

In the beginning of the computation, only the regular displacement field is active. After each converged time step, the traction state within a joint is analysed. Whenever the traction state of a joint reaches a critical level, i.e. the joint starts to crack, the enhanced degrees of freedom describing the displacement jump of the joint are enhanced. The opening of the joint is further governed by a rigid plasticity law.

3 MATERIAL MODEL

In this paper, all non linearities are concentrated in the joint. The brick remains elastic throughout the whole computation. Consequently, cracks cannot run through bricks. The separation behaviour of the joint is governed by a rigid plastic material model, derived from a classical elasto-plastic model.

3.1 Rigid plastic model

When using interface elements for the modelling of the joint behaviour, a classical elasto-plastic model can be used. In order to suppress separations in the joint in the elastic region, high elastic stiffness are used. These stiffnesses can result in numerical difficulties during return mapping and the general non linear solution procedure.
Using the generalized finite element method, joint separation is introduced when the joint reaches a critical level. This means that the joint is not present in the elastic region and consequently no

artificial elastic stiffness is necessary. Therefore, the elasto-plastic model should degenerate in a rigid plastic model. To obtain the rigid plastic model, the tangential stiffness, relating traction and separation rate for a classical elasto-plastic model is written according to:

$$\mathbf{D} = \mathbf{D}^e - \frac{\mathbf{D}^e \mathbf{m} \mathbf{n}^T \mathbf{D}^e}{\mathbf{n}^T \mathbf{D}^e \mathbf{m} - h} \tag{2}$$

in which $\mathbf{D}^e$ is the elastic stiffness, $\mathbf{n}$ and $\mathbf{m}$ are the normal to the yield surface and the plastic potential, respectively, h is the softening modulus. Inverting equation (2) yields the tangential compliance :

$$\mathbf{C} = \mathbf{C}^e + \frac{\mathbf{m}\mathbf{n}^T}{h} \tag{3}$$

where $\mathbf{C}^e$ represents the elastic compliance. In a rigid plastic model, the deformations are totally plastic. In the elasto-plastic model, the elastic deformations can be eliminated by introducing an infinite value for the elastic stiffness, so that:

$$\mathbf{C}^R = \lim_{\mathbf{D}^e \to \infty} \mathbf{C} = \frac{\mathbf{m}\mathbf{n}^T}{h} \tag{4}$$

This relationship is only valid for softening or hardening behaviour, i.e; when $h \neq 0$. The obtained compliance can be directly implemented in the material model. For the traction update, the incremental traction separation law can be integrated.

3.2 Yield surface

The elastic domain of the joints is bounded by a combination of two yield surfaces[5]. This surface consists of a Rankine yield surface and a Mohr-Coulomb yield surface. The Rankine yield surface bounds the elastic domain in the tensile region. The yield surface is given by:

$$f_R = T_n - f_{t0} \exp\left[\frac{-f_{t0}}{G_{fI}} \Delta_n^{pl}\right] \tag{5}$$

where T_n is the traction component normal to the joint, f_{t0} is the initial tensile strength and G_{fI} is the mode I fracture energy. For the Rankine surface, an associative flow rule is adopted. Figure (1) shows the results for a tow brick system in tension.

The Mohr-Coulomb yield surface is given by:

$$f_{MC} = |\bar{T}_t| + T_n \tan\phi - c_0 exp\left[\frac{-c_0}{G_{fII}} \bar{\Delta}_t\right] \tag{6}$$

where $\bar{T}_t$ and $\bar{\Delta}_t$ represent the shear traction and separation in the plane of the joint, respectively, c_0 is the initial value of cohesion, G_{fII} is the mode II fracture energy and ϕ is the internal friction angle. The shear traction and shear separation can be written in local coordinates, connected to the plane of the joint. Figure (2) shows the results for a tow brick system in shear.

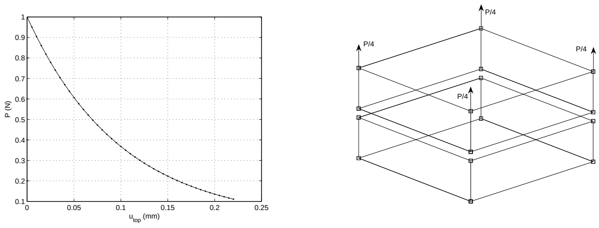

Figure 1: Load-deformation curve and deformed mesh for a two brick system subjected to tension

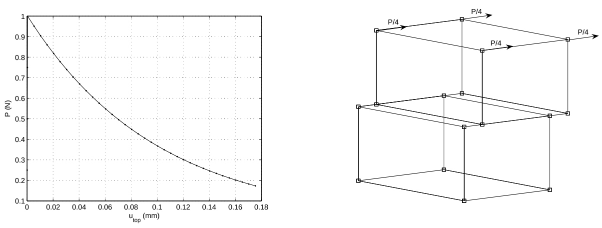

Figure 2: Load-deformation curve and deformed mesh for a two brick system subjected to shear

4 CONCLUSIONS

In this contribution, a three-dimensional mesoscopic model for masonry is introduced. The model uses the generalized finite element method to introduce displacement discontinuities in the joints. The stone behaviour remains linear elastic. Initially, the model is linear elastic. Every timestep, the tractions in the joints are evaluated. When a joint reaches a critical level, a displacement discontinuity is introduced, by defining additional degrees of freedom. The joint behaviour is governed by a rigid plastic material law, derived from an elasto-plastic material model.

REFERENCES

[1] G.N. Wells and L.J. Sluys. A new method for modeling cohesive cracks using finite elements. *Int. J. Num. Meth. Engng.*, **50**(12), 2667-2682,(2001).

[2] T. Belytschko and T. Black. Elastic crack growth in finite elements with minimal remeshing. *Int. J. Num. Meth. Engng.*, **45**, 601-620,(1999).

[3] K. De Proft. *A combined experimental-computational study to the discrete fracture of brittle materials.* Phd thesis, Free University Brussels,(2003).

[4] A. Simone, C.A. Duarte and E. Van der Giessen.A Generalized Finite Element Method for polycrystals with discontinuous grain boundaries. *Int. J. Num. Meth. Engng.*, **67**, 1122-1145,(2006).

[5] P.B. Lourenco. *Computational strategy for masonry structures.* Phd thesis, TU Delft,(1996).

[6] J. Alfaite and de Almeida. Crack evolution in confined masonry walls. *Proceedings of fourth World Congress on Computational Mechanics*, Argentina, Vol I,(1998).