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Implications of undersampling in system identification

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1 Background

The exact reconstruction of continuous-time signals based on their samples is crucial when designing system identification methods. Undersampling may lead to information loss, limiting the applicability of identification methods without additional assumptions [1].

2 Problem Formulation

Consider a single-input single-output, linear time-invariant, continuous-time system

$$\begin{aligned} x(t) &= G_0(p)u(t) \\ y(kh) &= x(kh) + v(kh), \quad k = 1, \dots, N, \end{aligned}$$

where $v(kh)$ is white noise, and the input $u(t)$ is a known continuous-time multisine of ordered frequencies $\omega_0 < \omega_1 < \cdots < \omega_M$ ($\omega_0 = 0$). Importantly, the sampling period h does not satisfy the Nyquist-Shannon criterion for exact input reconstructability, i.e., $h > \pi/\omega_M$.

Our goal is to obtain explicit conditions for the identifiability of $G_0(p)$ and the consistency of identification methods for this sampling regime.

3 Nonparametric and parametric estimators

We analyze the statistical properties of nonparametric and parametric identification methods when undersampling occurs. To this end, the least-squares estimator of $\{G_0(\pm \mathrm{i}\omega_\ell)\}_{\ell=0}^{M}$ requires the input frequencies to satisfy the non-overlapping condition [2]

$$\begin{cases} \omega_\ell \pm \omega_\tau \neq \frac{2n\pi}{h} & \text{for all } \ell, \tau = 1, \dots, M; \ell \neq \tau; n \in \mathbb{Z}, \\ \omega_\ell \neq \frac{n\pi}{h} & \text{for all } \ell = 1, \dots, M; n \in \mathbb{Z}. \end{cases}$$

Assuming that Nh is a multiple of the least common multiple of $\{2\pi/\omega_\ell\}_{\ell=1}^{M}$ and non-overlapping holds, we show that the least-squares estimator of the frequency response for each input frequency is given by

$$\hat{\mathbf{G}}^{\mathrm{f}} = \left[\frac{Y[1]}{U[1]}, \frac{Y[e^{-\mathrm{i}\omega_1 h}]}{U[e^{-\mathrm{i}\omega_1 h}]}, \frac{Y[e^{\mathrm{i}\omega_1 h}]}{U[e^{\mathrm{i}\omega_1 h}]}, \dots, \frac{Y[e^{-\mathrm{i}\omega_M h}]}{U[e^{-\mathrm{i}\omega_M h}]}, \frac{Y[e^{\mathrm{i}\omega_M h}]}{U[e^{\mathrm{i}\omega_M h}]}\right]^{\top},$$

where $U[e^{\mathrm{i}\omega h}], Y[e^{\mathrm{i}\omega h}]$ are the DTFTs of $u(kh), y(kh)$, respectively. The frequency response estimates are mutually uncorrelated and unbiased for any $N \geq 2M+1$.

Moreover, in stationary state with Gaussian noise, the parametric maximum likelihood estimator is obtained by minimizing either of the following cost functions:

$$V_{\mathrm{f}}(\boldsymbol{\theta}) = \left(\mathbf{G}^{\mathrm{f}}(\boldsymbol{\theta}) - \hat{\mathbf{G}}^{\mathrm{f}}\right)^{\mathsf{H}} \left[\mathrm{Cov}\{\hat{\mathbf{G}}^{\mathrm{f}}\}\right]^{-1} \left(\mathbf{G}^{\mathrm{f}}(\boldsymbol{\theta}) - \hat{\mathbf{G}}^{\mathrm{f}}\right), \quad (1)$$

$$V_{\mathrm{t}}(\boldsymbol{\theta}) = \sum_{k=1}^{N} \left(y(kh) - \hat{y}(kh, \boldsymbol{\theta})\right)^2, \quad (2)$$

where $\hat{y}(kh, \boldsymbol{\theta})$ is the one-step-ahead predictor of $y(kh)$.

When input frequencies do not overlap after aliasing, $\dim(\boldsymbol{\theta}) \leq 2M+1$ ensures identifiability for standard parametrizations and consistency of the prediction error method. If frequency overlap occurs, consistency holds if $\dim(\boldsymbol{\theta})$ does not exceed the number of unique non-overlapping input frequency lines.

4 Simulation example

With a sampling frequency 100 times smaller than the standard sampling frequency for this system, the nonparametric estimator exhibits no noticeable bias at any frequency. In conclusion, provided the input frequencies do not overlap, the proposed method enables accurate identification beyond the Nyquist frequency.

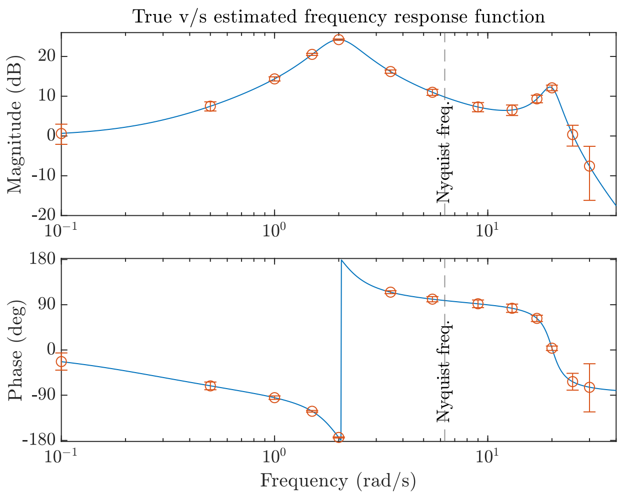

Figure 1: Bode plot of the system (blue), and the mean of the estimated frequency response via least-squares, with its 95% confidence interval (red).

References

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