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St. John's, Newfoundland
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An Experimental Study of the Dynamics of Catenary Mooring Lines

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Abstract

Catenary mooring systems are widely used in the ocean environment. This paper describes an experimental program set up to study the dynamic forces that may occur in these systems. The relation of these dynamic forces to the applied forcing function is examined and an example is given where the dynamic forces in the system would be significant.

1 Introduction

Catenary mooring systems provide a simple and relatively inexpensive way to position objects in the ocean environment. Because of the properties of the system, the moored object will be able to move when subjected to displacing forces, but this movement will induce a restoring force. The restoring force is non- linear and a small change in displacement can produce a large change in it.

Traditionally, the way of analyzing catenary mooring lines has been to use the quasi-static analysis. This paper examines the limitations on the quasi-static analysis and proposes that in some situations dynamic forces are important in shallow water applications.

2 Description of the Catenary System

There are two properties that are inherent in the analysis of the catenary. First it is assumed that there is zero bending stiffness, second it is assumed that the cable has a uniform mass per unit length. The first property is true for chains and almost true for wire rope. Note that in this paper, cable may mean either chain or wire rope.

These preceding assumptions are used to derive the equations that describe the catenary. The derivation involves setting up a second order differential equation and can be found in most texts on differential calculus. The equation that is finally derived is non-linear.

In descriptive terms, the catenary mooring system operates as follows. Using the basic catenary system as shown in Figure 1, it is seen that as the moored object

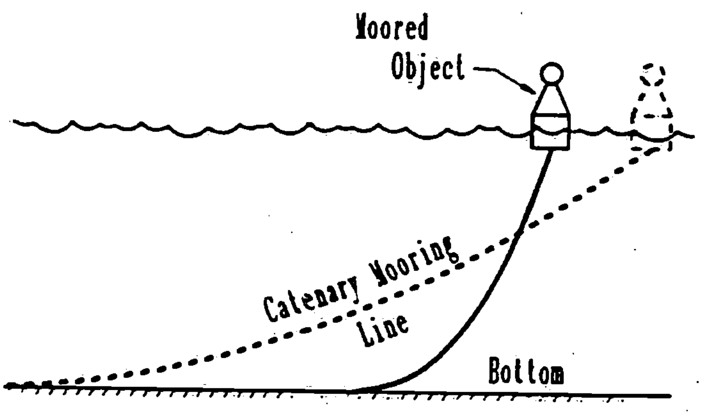

Figure 1: The Basic Catenary Mooring System

displaces, the cable is forced to assume a shape closer to a straight line between the moored object and the bottom. As more chain is picked up from the bottom, there is more vertical force acting on the object. However, the chain can only transmit force in a direction tangent to it thus the increased vertical force induces an increased horizontal force on the moored object.

If there is a constant displacing force on the object, the catenary will allow a horizontal displacement until the horizontal component of the force in the catenary is equal to the displacing force. When the system is in equilibrium it is defined as being static.

The static case is fairly easy to solve, so it makes sense that the first attempts to analyze the dynamic case were to superimpose the static forces on a time varying displacement. This method is called the quasi-static case and is still the most popular solution method. However there may be significant problems with the quasi-static solution.

In shallow water, the cable is affected by the forcing function it experiences at the top and the drag of the cable in water. For typical wave phenomena, the frequency of this function would be small enough that a quasi-static analysis would give reasonable results. However, if the frequency of the forcing function was higher, significant dynamic forces could result.

When it was realized that dynamic forces in cables could be significant, there were a number of experiments designed to examine these forces. Suhara et al (1981) oscillated the top end of a chain in the vertical and horizontal directions. They defined the response of the cable to fall into four stages: quasi-static, harmonic oscillation, snap and free-fall. In the harmonic oscillating condition, the tension is found to vary nearly sinusoidally. During the snap condition the chain goes slack and then comes up taut generating an impact force. In the free-fall condition the chain cannot keep up with the motion of the block and motions out of the plane are observed.

van den Boom (1985) also performed experiments where the free end of a cable was oscillated. He reported dynamic tension amplification factors (the ratio of the dynamic force to the static pre-tension) of 6 to 7.

Recently, Faure (1989) and Papazoglou et al (1990) have shown that for deep water moorings, the elasticity of the cable becomes an important parameter. They both used springs to model the stiffness of a truncated cable for model tests.

In general, experimental work in this field is scarce.

3 Objectives

The investigations of dynamic analysis have been mainly focused on deep water applications and on analyzing the mooring systems of large offshore structures. The general feeling was that the quasi-static analysis was sufficient for shallow water systems. To study this an experimental program was started at Memorial University of Newfoundland.

The investigation had, as its objectives, to examine the reaction of catenary mooring lines to a sinusoidal forcing function applied at the free end of the cable. This function could be applied at various frequencies and angles, and to various conditions of pre-tension.

4 Dimensional Analysis of a Catenary

The first step in the program was to analyze the problem through a dimensional analysis and derive a set of model laws that govern the system.

To perform the dimensional analysis, the parameters important to the experiment had to be defined. These parameters were categorized into three groups: the first consisting of the quantity to be measured in the system, the second defining the input parameters to the system and the last group containing the parameters describing the properties of the system. Figure 2 shows the parameters of the system.

The parameter to be measured, T, is the tension in the chain at the top connection. The input parameters

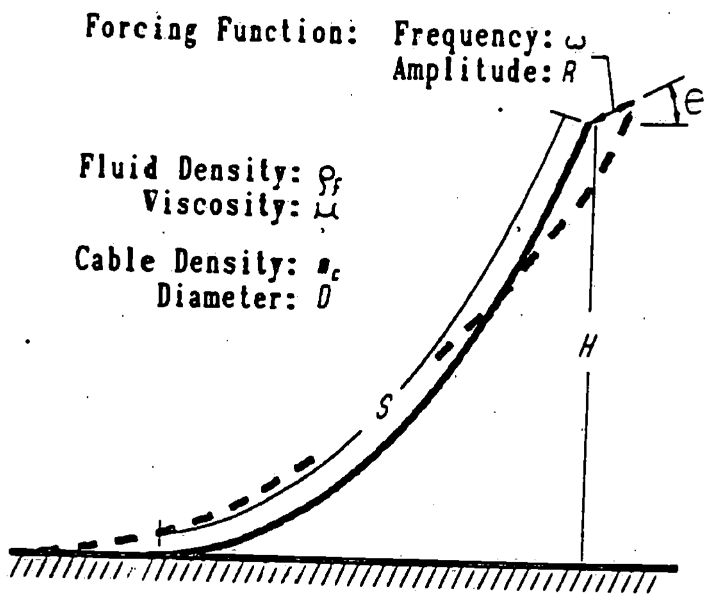

Figure 2: Test Parameters

that describe the system consist of the initial configuration of the system and the forcing function that acts at the top part of the cable. The initial configuration is described by the scope of the chain, S, and the depth of the water, H. The static pretension is an important consideration but for a specific cable it is defined by the scope of the cable and the depth of the water.

There are three parameters needed to describe the sinusoidal forcing function. The first two are the frequency, ω, and the amplitude of displacement, R. The third is the angle θ that the line of action makes with the water, (see Figure 2).

The system parameters describe various properties of the working fluid and the cable. For the fluid, the important parameters are the viscosity and density of the fluid (μ and ρ_f respectively). For the cable the parameters are a measure of its density, m_c, the effective Young's modulus, E and the cable diameter, D. m_c is a quantity equal to the density of the cable minus the density of the fluid and E is the Young's modulus for the cable as a whole. The diameter is straight forward for a wire strand cable, but for a chain it is defined as the diameter of a cylinder with the same length and volume as the chain. Another parameter that must be included is the gravitational constant, g. These parameters form the functional relation for the system.

To start the derivation, tension is assumed to be a function of the other parameters.

$$T = \phi\{\rho_f, \mu, m_c, E, D, S, H, g, R, \omega, \theta\} \tag{1}$$

Then the method of synthesis using linear proportions as outlined in Sharp (1981) is used to derive a dimensionless equation. The equation in its final form is as follows:

$$\frac{T}{m_c g D^2 H} = \phi\left\{\frac{\nu^{2/3}}{g^{1/3}D}, \left(\frac{E}{m_c g S}\right), \frac{\omega^2 R}{g}, \frac{R}{D}, \frac{S}{H}, \frac{D}{S}, \frac{m_c}{\rho_f}, \theta\right\}. \qquad (2)$$

Several parameters in this equation are familiar. The tension term and the ratio S/H are used in the static analysis of a catenary. The term $\nu^{2/3}/g^{1/3}D$ is a form of the Froude-Reynolds number. Also the term R/D is similar to the Keulegan-Carpenter number.

There was assumed to be no current in this analysis. However the addition of a current would just introduce another term which would result in a Froude number.

It is also important to note that owing to its definition, the forcing function (oscillating force) can be defined as either an amplitude and frequency, velocity and frequency or even an acceleration and frequency.

For their analysis Suhara et al (1981) used the parameters wX_0/T_{HO}, $Z_m\omega^2/g$ and Z_m/D_C. The first is a measure of the pretension in the chain (w is the weight per unit length). The second and third are similar to the groups $\omega^2 R/g$ and R/D except that the term Z_m is used where Z_m is the vertical displacement of the center of gravity of the catenary.

Papazoglou et al (1990) based their analysis on the method of governing equations. They produced similar results to those derived here except they came up with a term $w^2 L^2/(T/\rho_c A)$ instead of the term $g/w^2 R$ where L was the length of cable, ρ_c was its density and A was an equivalent cross-sectional area.

5 Experimental Study

The experimental work was carried out during September, 1990 in the wave/towing tank at Memorial University of Newfoundland. The tank had the dimensions 100m x 4m x 2m deep and was used because it provided the necessary depth and length needed to lay out the chain. A PC computer using a Keithley 5000 A/D converter was used to take measurements. The data was then transferred to a Vax 8530 computer for processing.

A schematic of the apparatus is shown in Figure 3. Figure 3a shows a plan view and Figure 3b shows it mounted under the catwalk over the wave tank. The block travelled on grooved wheels which ran on rails. The support could be inclined relative to the water.

Instrumentation consisted of an accelerometer and a two way force transducer. The accelerometer was used to monitor the motion of the chain to make sure it was sinusoidal. The two way force transducer was used to measure the tension at the chain connection.

It was necessary to use a two way force transducer because the chain was free to rotate about the connection point. By measuring force in mutually perpendicular directions, the total force could be found for any orientation.

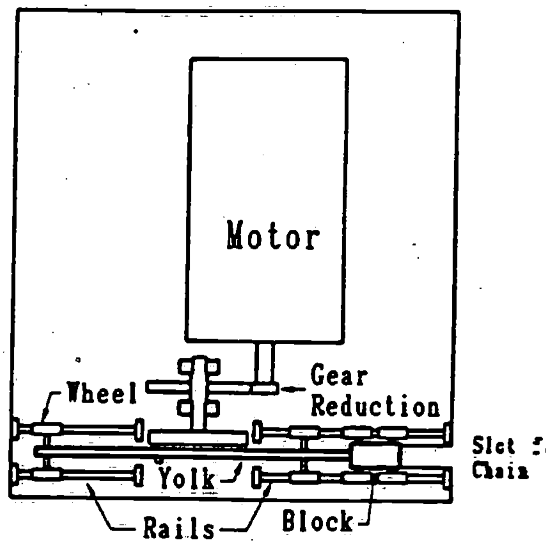

Figure 3a: Top View of Equipment

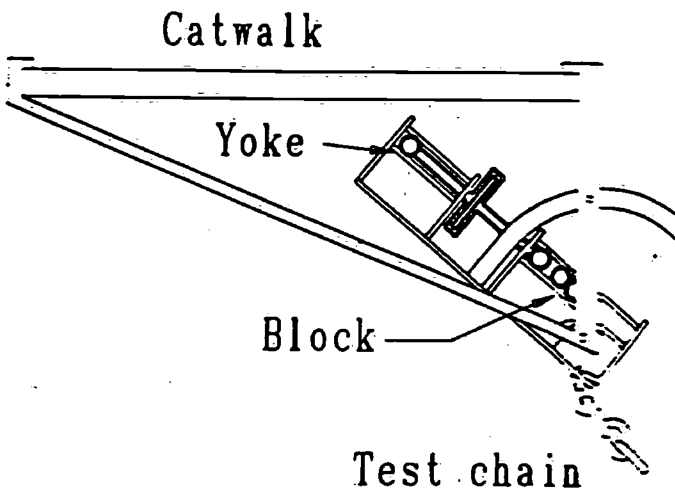

Figure 3b: Side View of Equipment

The parameters changed during the test were the static pre- tension, the frequency of the forcing function and the angle that the forcing function made with the horizontal. Pre-tension was set by moving the position of the anchor and values of 15, 20 and 30 N were used for the experiment. Angles of 0°, 30°, 60° and 90° were used and the frequencies ranged from about .4 Hz to 4 Hz.

The test chain was an open linked steel chain with a weight of .786 kg/m and a wire diameter of 6.35mm. The R value was .038 m.

Another series of tests was run to see how long it would take for the dynamic forces to reach their full magnitude.

6 Results

The data acquisition and processing routine is shown in Figure 4. The data was processed on a Vax 8530 computer and the sampling rate for the tests was 250 Hz. Sample plots from the test series are shown in Figure 5.

The ratio of the maximum dynamic force to the static force was obtained and is plotted against the frequency for the various pre-tensions and forcing functions (see Figure 6). The ratio was also plotted against the acceleration parameter ($\omega^2 R/g$) and the plots are shown in Figure 7.

All the results show the same basic shape. The dynamic force amplitude starts out relatively flat indicating that a quasi- static analysis would be acceptable for this section. However, very soon dynamic forces become important. The graphs eventually reach a peak, after which a further increase in frequency causes a decrease in force.

The same type of results have been reported by Suhara (1981), van den Boom (1985) and Papazoglou (1990).

7 Discussion

From the results it can be seen that dynamic forces can sometimes be 5 times the static forces. The static pretension seem to have only a moderate influence on the dynamic force, however it does change the frequency at which the maximum force will occur.

It is also seen that the angle of inclination of excitation has a considerable influence on the dynamic force. The more in line the forcing function is with the top of the mooring line, the greater the dynamic force.

Finally, it is seen that the largest effect is caused by the frequency. As the frequency increases, the dynamic force increases up to a maximum.

The four regimes in which the chain responds were also observed, although they occurred at different frequencies for the different pre-tensions. The ranges are shown in Figure 6. Generally, up to a frequency of 0.5 Hz the maximum tension in the chain could be predicted by using a quasi-static analysis for all pre-tensions. However, for frequencies in the range of 0.5 to approximately 2 Hz the chain is in the harmonic oscillating range and dynamic forces are observable. Past this point, the force time plots are no longer sinusoidal in nature. From 2 Hz to 2.5-4 Hz (depending on the pre-tension) the chain is in the snap condition and, although the maximum force keeps increasing, the minimum force has gone to zero. It is in this area that the largest dynamic tensions are recorded. Eventually, the chain enters the free fall range and dynamic forces start to decrease. The frequency range did not go high enough to allow the free fall condition to be observed for all pre-tensions.

The large forces found in the snap and the free fall condition are due to internal impact forces of the chain. These forces are caused by impacts between the links. As the tension in the chain goes to zero, the links are able to assume a motion, to a certain extent, independent of their neighbors. However, when the line comes under tension again, the links must take up a certain position and orientation. The forcing of the links into this set position causes the impact forces.

The chain also reacted quickly when the forcing function was first applied. In the harmonic condition, the steady state dynamic force was achieved after only 1 cycle of the forcing function. In the snap and free fall ranges, it took up to 5 oscillations before the maximum dynamic force was observed.

It is important to examine how the modelling of catenary mooring cables applies in the ocean environment. The first step in this is to examine scale errors.

The term θ is an angle and thus transform identically. Geometric similarity can be achieved and thus the terms R/D, S/H and D/S will be modelled correctly. Also, since the experiments take place in water and the material is steel, the ratio m_c/ρ_f will be correct.

The term $\nu^{2/3}/g^{1/3}D$ accounts for the hydrodynamic properties of the cable. In this experiment, the Reynolds number placed operation well into the turbulent range, thus scaling should not have much effect on these properties. This is borne out by the results of van Sluijs and Blok (1977).

The term $E/m_c gS$ accounts for elasticity in the cable. Although for deep water, elasticity is an important parameter, this study is aimed more at waters with a depth measured in tens of meters rather than hundreds. Thus elasticity should not have a great influence on the system.

This leaves the parameter $\omega^2 R/g$ as the most important. This is the ratio of the acceleration that the top end of the chain will undergo to gravitational acceleration.

An example shows that for a given situation, significant dynamic forces will occur. If there is a stretch of water 10 km long with an average depth of approximately 15 meters and a wind of 70 km/hour blowing for 1 hour, waves with a period of 3.0 seconds and a significant height of 1 meter will be produced. A can buoy in this wave field would respond at the same frequency with approximately the same amplitude of motion. The $\omega^2 R/g$ parameter for this scenario is 0.23. From Figure 7, it can be seen that this corresponds to a dynamic amplification factor of about 1.5.

8 Conclusions

It is seen that, although the quasi-static analysis is valid for many shallow water conditions, the designer should be aware that significant dynamic forces may occur. The frequencies that these forcing functions operate at provides an indication of whether or not allowance for dynamic forces should be made.

Acknowledgments

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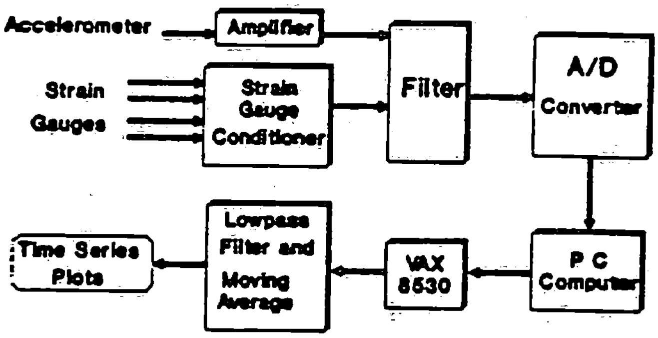

Figure 4: The Data Acquisition and Processing Routine

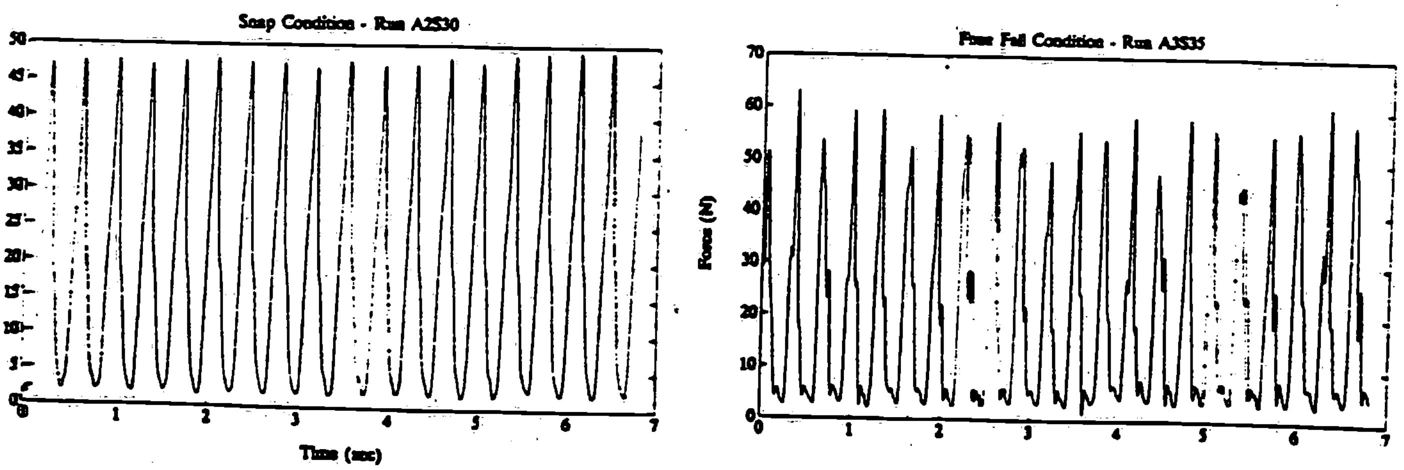

Figure 5: Sample Time Series Plots of Measured Force

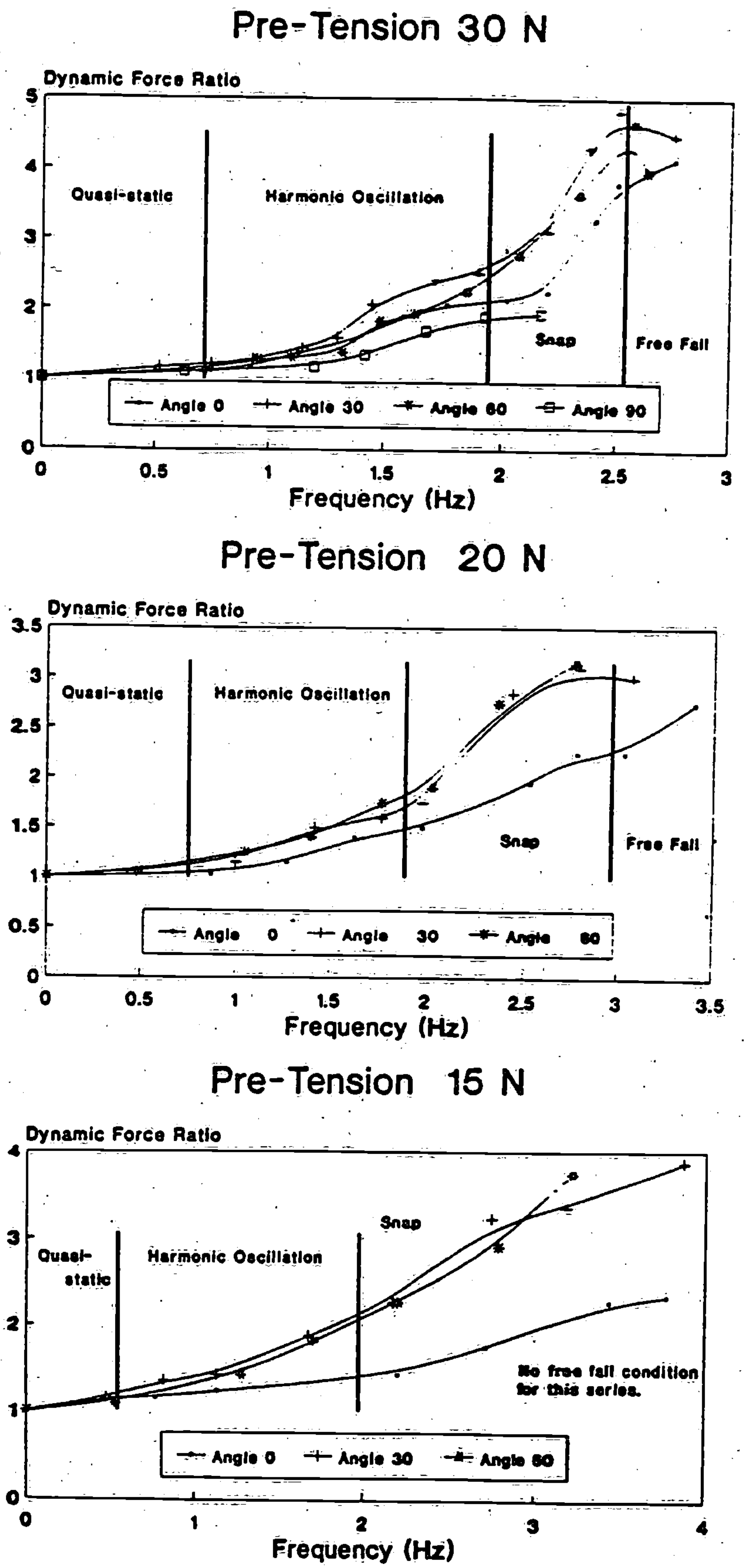

Figure 6: Dynamic Force Ratio Plotted Against Frequency

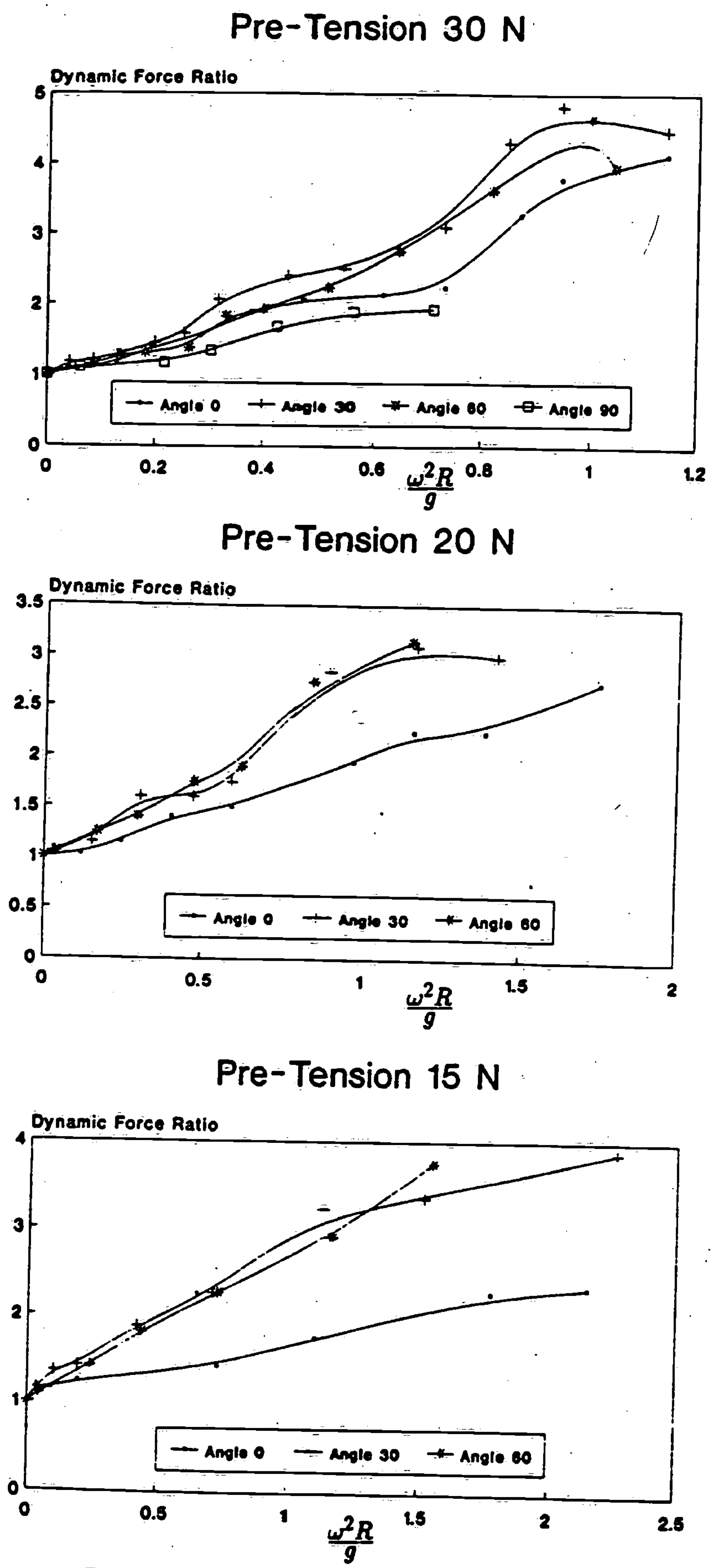

Figure 7: Dynamic Force Ratio Plotted Against $\omega^2 R/g$