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Master's Thesis in Embedded Systems

Energy Allocation Strategies for Micro-Grids

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Abstract

The advances of the information and communication technology (ICT) brought changes in the energy distribution domain, introducing the Smart Grid (SG). In SG, generators, distributors, and consumers communicate in a bidirectional way. SGs are envisaged to include micro-grids (MG) consisting of distributed control networks of consumers, prosumers, and the power grid. Two-way communication in MGs allows allocating the produced energy inside a community of consumers, to decentralize the energy flow. However, challenges arise regarding energy sharing, namely: (i) how to balance the demand and supply inside communities; (ii) are there policies that prioritize among the consumers while distributing the producers' excess of energy; and (iii) how to balance the economic benefit –under a policy– for everyone who participates.

In this thesis, we propose energy allocation strategies (EAS) for MG communities consisting of households that use renewable sources of energy (RSEs). Our objective is to maximize the energy usage and the cost reduction, under certain priority policies. Through an in-depth analysis of energy and socioeconomic data of the community, we form groups of households that share similar characteristics, and we channelize the energy flow at will. We present seven, simple and optimized, EASs and several consumer priority policies (CPPs). Our EASs and CPPs are scalable and can meet the specific needs of an MG community. We evaluate our algorithms and techniques using real data, acquired from a community of 443 households over a year. We show that the groups of households that we prioritize cover their needs of energy, sometimes completely, in periods of high energy production. We compare on economic basis trading energy within the MG and requesting energy from the grid (classic way). The expenses for prioritized groups of consumers under our EASs are decreased, up to 50% in certain cases. Further, it is shown that even the non-prioritized consumers are benefited economically by allocating the excess of energy.

Preface

When I first came here in the Technical University of Delft, in my mind I had a diploma thesis on networks. Having finished my previous master with a thesis on Delay Tolerant Networking (DTN), the track 'telecommunications and sensing systems' of the electrical engineering domain seemed suitable to continue my studies. However, while working as a researcher on DTN, I was particularly interested in platforms designed for embedded software scenarios, like IBR-DTN. In addition, the domain of the Internet of Things –basically the concept of IoT, to put it in its theoretical frame– was always intriguing for me. Thus, during the second year of my studies here, I decided to make an internship, in Athens, where I had the chance to fully build, program, and maintain an IoT-ecosystem of sensors. Before completing my internship, I had already decided to continue with a thesis in the embedded software group of TU. So, I contacted VP (Ranga Rao Venkatesha Prasad), and informed him about my educational background and my intentions. Then, he connected me with Akshay, and we started working on Smart Grids.

This report describes the methodology and the results of my project in the domain of energy allocation in a Micro-grid community of households. My intention was to provide knobs for the control of energy sharing inside the community, in a decentralized way, respecting at the same time the specific characteristics that it presents regarding the consumption and generation of energy.

A paper was written during this thesis, with the title "Energy Allocation Strategies for Micro-Grids: Algorithms, Policies and Economical Aspects". It is an article of 10 pages, supplemented with 5 pages of algorithms, and it is submitted for review for the ACM eEnergy 2017 conference.

Acknowledgements

I am sincerely grateful to Akshay and VP. Although they were in busy periods in their professional lives, they supported and guided me. I thank Akshay, for he did not provide any solution to my problems. But, he chose to guide me towards the solutions, to make me find them, and ultimately, to educate me. When I look back in retrospect, I know that I would have done the same if I wanted my student to learn, and then create. I thank VP, because, he believed in me from the beginning, and he sees my vision and my passion to contribute on the domain. Without their support and their *hard-way* of making me learn, I would not be able to create the algorithms and the policies that led us to the submission of a paper. After all, in my opinion, it is what you leave behind –your mark– on a scientific domain, that makes your work valuable.

Finally, I have no words to thank my mother, Fani, for she brought me up all alone, and without her I would not be who I am, and my girlfriend, Diana, because, after more than three years, she still tolerates me.

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Chapter 1

Introduction

This chapter introduces the SGs, and their relation to the RSEs and the ICT. The problem statement of this thesis is given and the research goals regarding its solution are stated.

1.1 Electricity generation and ICT

Massive production of electricity by centralized stations continues to grow since its first inception, in 1882. Small central grids (CG) have evolved into big international networks of power lines –controlled by several companies– which provide energy to industry and citizens. It is foreseen that by 2020 net electricity generation will be increased to the order of 25.8 trillion kWh globally [1]. The coal, which was used as a source of electricity production by the first CGs, continues to be used in many stations worldwide. It accounted for 33.17% of the electricity generation in the US in 2015 [2], despite its negative impact on the carbon footprint. Nowadays, that the effects of the greenhouse gases on the environment are reported, many developed countries pursue energy policies to reduce the use of coal and other fossil fuels, by incorporating to the electricity production energy sources like the sun, the wind or geothermal heat, which are eco-friendly.

The rise of the RSEs has brought in the spotlight several technologies, used not only by the industry, but also by individuals, to generate energy, like solar panels and wind turbines. Individual consumers that –except drawing energy from the CG– use RSEs to produce energy are called *prosumers*. Note that the RSEs are intermittent, usually requiring forecasting (e.g., sunlight for solar panels). The existing forecasting models [3, 4] study the power generation as a stochastic process, solved by artificial neural networks (ANN). Nevertheless, under situations of energy shortage, a prosumer is not guaranteed to be able to cover his/her needs based only on RSEs. Thus, the CG, being a stable electricity supplier, is often imperative.

The autonomous energy generation by RSEs led to the need for a system in which bidirectional communication, between the companies in the power industry (electric utilities) and the individuals, could take place. However,

traditionally the energy distribution network is centralized. CGs are primarily used to carry power from a few generators to a large number of consumers. Thus, the CG has the role of the master and households or industry buildings are seen as end-terminals. Because of lack of monitoring, and to improve energy efficiency, energy utilities modernized the traditional grid by employing intelligent monitoring, control, and communication to enhance efficiency, reliability, and sustainability of power generation and distribution networks. This is popularly referred to as Smart Grid (SG). SGs promote two-way communication between consumers and utilities by deploying a large number of smart meters. The smart meters collect fine-grained data from the consumers and provide real-time information regarding energy consumption, to improve awareness and efficiency in its regulation. The autonomous energy generation and the bidirectional scheme of communication decentralize the SG, making it a dynamic energy ecosystem. SG is a distributed control network of small communities of consumers and prosumers, which are called micro-grids (MG).

1.2 Problem statement and research goals

Recently, there have been few successful implementations of MGs [5, 6]. In these cases, the households share the excess of energy, trying to balance in that way electricity supply and demand, in an attempt to decentralize the energy market. Since then, several research works, which study the potential of energy negotiation in an SG community using RSEs, have been conducted, focusing also on the market response [7–10]. The goal of MGs is to allocate energy between consumers and prosumers while having some policy that prioritizes certain households (or targets). However, allocating excess of energy from prosumers to consumers is not trivial, because of the following reasons: (i) energy requirements for individual consumers vary over time, and hence allocation mechanisms need to be adaptive; (ii) prioritizing certain households causes bias in the community, hence there is a need to develop fair energy allocation schemes, (iii) the predictability of the amount of energy that is produced by prosumers has limits, because weather is variable. Any energy allocation strategy is affected by combinations of the aforementioned challenges. For example, consider a policy which prioritizes households that require high amounts of energy (highly deficient). If, for a particular day, the aggregated amount of generated energy is low, this policy will lead to only a few households served. Also, note that energy needs are connected to socioeconomic attributes like household-size. Thus, the allocation of energy should consider energy consumption/generation characteristics and socioeconomic attributes, like household-size, income, age, etc.

An EAS defines the way in which the excess of energy of the prosumers is distributed to (a group of) consumers. Several EASs, proposed recently, span from simple concepts, such as first in first out (FIFO) [11], to more complicated techniques, which use Game Theory aspects [12, 13]. These algorithms mainly focus either on allocating the excess to the maximum number of households

or on minimizing the cost of power that is purchased from the CG. This thesis focuses on developing novel allocation strategies that consider several energy and socioeconomic attributes of the consumers. Furthermore, the proposed EASs aim to achieve fairness, which can be defined for all (or a group of) consumers in the MG community. Specifically, we try to answer the following question:

In what way the energy allocation among the members of the community can be maximized under the constraints imposed?

We propose various algorithms for energy sharing between households and define policies that prioritize certain members inside the community. The objective of this work is to increase the potential of a consumer to receive energy from the prosumers, increasing, at the same time, the welfare of all the households in an MG.

In this thesis, we follow a three-step methodology. First, we characterize households, based on their energy consumption/generation and socioeconomic factors. Second, we present various policies, which prioritize households for energy allocation. Third, we present strategies (EASs) in order to enable fair energy allocation between prosumers and consumers. Further, to test the efficacy of our algorithms, we employ a real-world dataset from 443 households [14] for a year.

The contributions of this thesis are summed up in the following way:

1. Complete characterization of a household community regarding energy.
 - Using unsupervised clustering algorithms.
 - Defining temporal characteristics of its members regarding energy usage.
 - Connecting energy clustering with meta-data attributes.
2. Defining priority policies between the consumers with respect to different service goals.
3. Creating simple and sophisticated algorithms for energy allocation (EASs).
4. Proposing new metrics to evaluate the results of the EASs implementation in a community.
5. Evaluating our methodology using real, fine-grained energy data and socio-economic information of 263 consumers and 180 prosumers [14].
6. Comparing the cost between our approaches and the classical non-sharing approaches.
7. Creating and evaluating two-stage sharing approaches, based on grouping households that present similar energy behavior.

1.3 Organization

A thorough system model of an MG community is presented in [Chapter 2](#), along with the method to characterize its energy behavior. In [Chapter 3](#), the priority policies and the energy allocation algorithms are defined. In [Chapter 4](#), we create the metrics to evaluate our methodology and we present the results of its experimental implementation in a real community. Further, a comparison of the expenses between allocating the energy and requesting it from the CG is presented. Finally, in [Chapter 5](#), we draw the concluding remarks and propose future steps.

Chapter 2

System model and community characterization

In this chapter, we analyze the model of an MG community (Fig. 1) and describe the method that is followed to characterize it. Further, we implement this method in a real household community and the results are presented.

An MG community includes households which use the power line of the CG to transfer energy, under the dictations of a central controller (CC). The households present different energy needs which derive from the usage of electrical appliances. As the households differentiate in building size/type and the residents present heterogeneity in socio-economical terms and preferences, the energy needs are specific for each of them. Among the households, there is a group of prosumers, which present energy generation potential through RSEs, i.e., sunlight, the wind, geothermal heat, using photovoltaic (PV) panels and wind (or geothermal) turbines.

We assume bootstrapping using sensors that communicate with smart meters and offer fine-grained data on the consumption of household appliances and the generation of solar panels. These amounts of data are acquired at a constant frequency and can be used to characterize a household regarding its electrical energy attributes. In order to characterize a household completely, we connect our observations with social attributes which are related to several non-energy characteristics. These meta-data attributes usually affect the energy characterization of a household. For example, a family with three kids, probably, consumes more than a young couple.

2.1 Related works

The system model of an MG is proposed in several works [13, 15, 16] with variations on the entities –CG, controller, prosumers, and consumers– that compose it.

In [15], the MG community is composed of one provider and several consumers (or subscribers), which use the power line of the CG for energy dis-

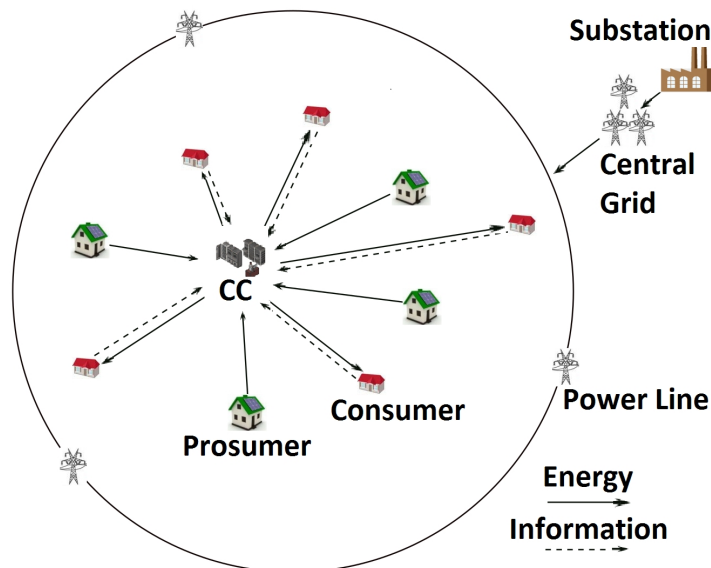


Figure 1: System Model

tribution. The consumers are equipped with controllers. They communicate with each other and with the provider through LAN and convey information on their energy needs. This infrastructure allows the development of algorithms, aiming to maximize the energy consumed by every subscriber under a fair policy.

In [16], a small MG community of 40 households, which use solar power is presented. Every household is equipped with PV panels, batteries (for energy storage), and sensors. In every household, sensors measure the amounts of the energy that is harvested and of the energy that is needed from the appliances. Energy measurements and battery state information are collected by an internal controller, located inside every household, which transmits them –along with predictions on future consumption and generation– to a central controller. The transmission happens in a separate distribution network from the CG. The controller is responsible for the priority policy in energy allocation among the households and for the pricing schemes that are to be imposed.

In [13], solar and wind RSEs are considered. However, the notions of consumer and prosumer are expanded from the household level to characterize whole MG communities. Thus, there are several MGs with generating potential, which could be characterized either as consumers or as providers, depending on the difference between their produced energy and their needs. The energy trading between consumer and provider-MGs takes place on the power line of the CG, and it is controlled by a central distributor. The distributor –which is also a storage for the excess generated by the provider MGs– accounts the historical data of each MG, regarding the amount of excess it has provided to the system of MGs, in order to assign priorities on energy allocation. The efficacy of this strategy is evaluated by numerical examples.

The importance of characterization as means of profiling sensor results, balancing supply-demand, and decision making regarding energy attributes in SGs, is recognized in the works of [17–21].

In [17], the households are characterized by the fine-grained data they offer in consumption terms (thirty-minute granularity over 1.5 years). Supervised machine learning algorithms are applied on data streams of appliances’ sensors to define each household’s profile. By data analysis, the authors infer meta-data characteristics of the households, which relate to housing attributes (building type), economical aspects (annual income of a family), and social characteristics (age of occupants).

In [18,19], clustering algorithms are used to categorize the energy behavior of customers inside groups. By energy profiling, the future needs of an SG regarding supply and demand can be forecasted, maintaining in that way the energy balance. In [19], clustering results characterize the consumption state of households and their potential in regulating their needs. By defining their temporal dynamics, the households can self-regulate their needs. Thus, they can improve their consumption level and their load factors, and be led to reduction in their expenses.

In [20], a holistic clustering attitude is proposed to characterize large data-streams that change dynamically, and are created by the numerous smart meters that are involved in an SG. The goal is to enable proper decision-making on grid sustainability issues, like anomalies, events, and trends. In [21], clustering is expanded from the level of households, to become the main means of consumption analysis in industrial parks.

2.2 Micro-grid

We study an MG community in which the entities that negotiate are not companies or technological centers, but households. Because of this, in our system model, we focus only on autonomous energy generation by PV, which is the main means of energy generation for households. Note that if the energy needs cannot be covered by the generated energy, the deficit is always drawn from the CG. Thus, without loss of generality, we only consider the excess of energy and its allocation. In a community of c consumers and p prosumers, let the group of consumers be $\mathbf{C} = \{C_1, C_2, \dots, C_c\}$ and the group of prosumers be $\mathbf{P} = \{P_1, P_2, \dots, P_p\}$.

The power line is mandatory for the functionality of our system model. Except for the intermittent nature of the RSEs, which leads to energy shortages, direct energy sharing presents obstacles regarding infrastructure. The energy cannot be transferred directly between \mathbf{C} and \mathbf{P} , as it would be expensive to have a separate transfer scheme between every possible pair of prosumer and consumer. Moreover, even if such a system was constructed for models of a few households, despite the cost, it would be impossible to scale it, as the possible combinations of pairs increase exponentially with the addition of new households in the system. The responsibility of applying communication

schemes on energy allocation between the households is passed to the CC, which utilizes the CG infrastructure for the actual energy transfer.

In Fig. 1, the CC has direct connection to all the households in order to route information between them. It keeps information about the energy needs of consumers and the amounts of energy produced by the prosumers. As a result, it creates prosumer-consumer pairs for the transfer of energy. The decision that is taken by the CC regarding any energy transaction is forwarded to the prosumers and consumers that are involved. However, apart from being a communication point, the CC can connect directly to the power line of the CG to store energy in sophisticated EASs, which involve (groups of) households from both the consumer and the prosumer sides. The total energy produced by \mathbf{P} during every time interval is stored in CC, and then distributed to \mathbf{C} , according to the selected EAS.

2.2.1 Paradigm

In Fig. 1, an optimized case of energy allocation in an MG community is presented. To begin with, the CC stores the excess of energy $-\mathbf{E}_e = \{E_e^1, E_e^2, \dots, E_e^p\}$ —that every individual prosumer of \mathbf{P} produces during a predefined time interval. The amount of excess that is stored in the CC is less than the total energy that is generated by \mathbf{P} , because prosumers present energy needs for their own electrical appliances, and thus they are not able to allocate the whole amount of generated energy to consumers. Since storage phase is complete, the CC accepts information by the consumers regarding their energy needs $-\mathbf{E}_a = \{E_a^1, E_a^2, \dots, E_a^c\}$ —and then it applies the dictated sharing strategy (EAS) on \mathbf{C} . The result is that every consumer-household $i \in [1, c]$ accepts an amount of energy, represented by $\mathbf{E}_g = \{E_g^1, E_g^2, \dots, E_g^c\}$, to cover its needs partially, $E_g^i < E_a^i$, or totally, $E_g^i = E_a^i$. This depends on the priority of every consumer when the EAS is applied.

However, the assignment of priority relations among the consumers depends on their status inside the community with regards to an energy attribute. This status is obtained by defining the relation of every household to the rest, inside the community. If their status is not defined, the consumers cannot accept the amount of excess that should correspond to them. Assume a situation, in which the excess of energy produced, \mathbf{E}_e , is to be allocated to the consumers, \mathbf{C} ,—which present energy needs \mathbf{E}_a —under a policy which prioritizes certain among them, according to an attribute. If the community is not characterized regarding this attribute, the status of its consumers cannot be created. Thus, the priority relation among them cannot be clear. Consequently, the decided amount of excess that is to be given to a consumer k , E_g^k , does not correspond to the real status of him/her in terms of the attribute of priority (e.g., a highly deficient consumer could accept large amounts of excess during a low deficient-priority policy).

By characterizing the households of a community, specific groups, with members that present similar energy trends, are created inside the community.

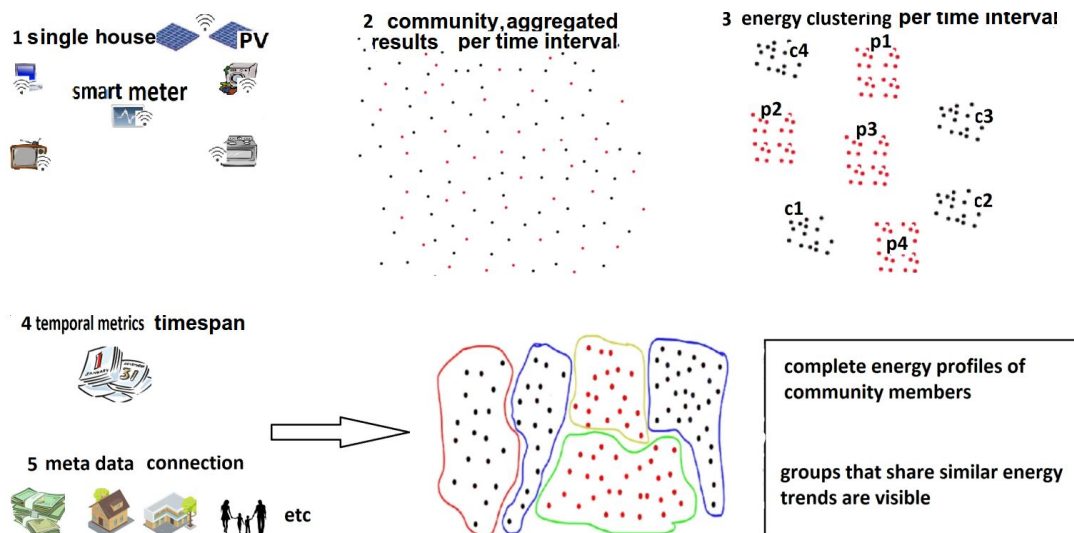


Figure 2: Community Characterization

The characterization, based on energy behavior, is important for the energy allocation procedure, in order to define a consumer priority policy (CPP) between the consumers or to describe an energy sharing relation between (a group of) consumers and (a group of) prosumers. After proper characterization, the produced groups present internal compactness (or homogeneity). They are consisted of households with low variance regarding an energy attribute.

2.3 Community profiling

In this section, we describe the procedure to transform a community of prosumers and consumers into groups with homogeneous members (see Fig. 2). The first step in this direction is to sort the energy usage of the households. The first categorization is made on their potential to generate energy. The households that are able to generate energy are separated from the rest, as prosumers. Using the consumption and generation data, we extract the deficiency and excess of energy for every individual household. For a household i , steady time of measurement t , consumption measurements Con^{it} , and generation measurements Gen^{it} , (i) supposing that the household can cover its needs by itself, the excess is defined as $E_e^{it} = \text{Gen}^{it} - \text{Con}^{it}$, while, (ii) supposing that it still needs energy, its deficiency is defined as $E_a^{it} = \text{Con}^{it} - \text{Gen}^{it}$. The smart meters can provide measurements at high frequencies –every few min–

utes to every minute. However, the patterns of a family's daily chores, which affect the energy consumption behavior of a household, tend to be different on weekdays and on weekends. Thus, to achieve convergence (and thus stability) of energy allocation, we smooth the differences in energy by averaging the electrical measurements over predefined time intervals. In this work, we use weekly time intervals. The average of an energy attribute, Att , over a time interval, T , is defined for every household, i , as $\overline{\text{Att}}_i = (\sum_{t=1}^{t_{\max}} \text{Att}_i^t) / t_{\max}$, where t_{\max} is the number of consecutive measurements that constitute a time interval T . The average cannot by itself characterize the energy behavior of a household, as there is no measure of comparison. The energy behavior has to be defined by comparing with the average results of the other households in the MG community. For example, a household is described as less or highly deficient, only related to the rest, and not in general. To associate every household with the others (inside its MG community), we perform clustering.

2.3.1 Clustering

Clustering is an unsupervised method of machine learning, which uses algorithms to classify households into different levels regarding their attributes. In our case, the distribution is based on the average values of the attributes of the households. The number of different clusters, in which the households are distributed, is dictated by the applied clustering algorithm, either internally –the algorithm computes them– or externally –we prefix them. The clusters are characterized by their centroids, which are central values. The households with values that are closer to a certain centroid than to the others are placed around it. Specifically, assume that the averaged values for an attribute Att , over a certain time interval T , are $\overline{\mathbf{C}}_{\text{Att}}^T = \{\overline{C}_{\text{Att},1}^T, \overline{C}_{\text{Att},2}^T, \dots, \overline{C}_{\text{Att},c}^T\}$, for the group of consumers \mathbf{C} . After clustering, the group is defined as $\overline{\mathbf{C}}_{\text{Att}}^T = \mathbf{Cl}_{\text{Att},1} \cup \mathbf{Cl}_{\text{Att},2} \cup \dots \cup \mathbf{Cl}_{\text{Att},m}$, where m is the total number of clusters. The same holds for the prosumers, $\overline{\mathbf{P}}_{\text{Att}}^T$. In this thesis we use the *Expectation-Maximization* (EM) algorithm to define the number of clusters for an electrical attribute [22], and to distribute every household to a cluster. A household is a member only of one cluster for every time interval T ; its cluster characterizes its relation to the rest of the households. For example, assuming that clusters regarding consumption are set in ascending order of their centroids, a household which is assigned to consumption-cluster 1 for week 1 is a low consumer, compared to the other households during that week. The same holds for all the households assigned to cluster 1 regarding consumption. The distribution of absolute differences, between the members of a cluster and its centroid, defines the cluster variance. Clusters with high internal compactness present low variances, as their members are close to their corresponding centroid.

The above characterization holds for each household only for the predefined time interval, T , in which the average calculation is made, and it cannot be used to obtain insights into the energy behavior of households during longer pe-

riods of time. As an example, assuming an hour-time interval, clustering would offer insights for the energy attributes of every household hourly. We would need to collect twenty-four continuous clustering results to make assumptions about the energy behavior of each household for a day. We consider the year as the ideal time period for characterization, because it includes the main reasons for which the energy patterns of a household vary –all the seasonal changes and the possible vacations. Although it was possible to average the values of the energy attributes on a year-basis, and then cluster them, we could not obtain insights about the temporal transitions of the households. In other words, we could not reproduce the temporal changes in consumption and generation, which are observed during smaller time intervals, and which are obtained by clustering. Thus, clustering was employed on week-basis in order to smooth the weekend and daily variability. To acquire energy consumption/generation perspective for the households over a year, temporal metrics were used. These metrics are the temporal membership and the temporal adaptability.

2.3.2 Temporal metrics

Cluster membership refers to the existence of a household in one of the clusters that are defined for an energy attribute and *cluster adaptability* refers to the transition that happens between different clusters of the same attribute between consecutive time intervals (periods of time, in which clustering happens). For example, a household in week 1 was in consumption-cluster 2 and in week 2 it was found to be in cluster 1. This household is said to have performed a $Cl_{con,2} \rightarrow Cl_{con,1}$ transition in adaptability terms, being a member of $Cl_{con,2}$ and then of $Cl_{con,1}$ in membership terms. The terms *temporal membership* and *temporal adaptability* assess the possibility for a household to be a member of a cluster or to perform a cluster transition. By using percentages, we are able to understand if a certain cluster membership or transition is more possible than other memberships or transitions for a household, because it took place more frequently during the year. Thus, for a household i , in a clustering scheme of a chosen time interval T , its temporal membership for cluster u over a year is defined as

$$Cl_{Att,u,i} = \frac{\sum_{T=1}^{T_{max}} Cl_{Att,u,i}^T}{T_{max}}, \quad (2.1)$$

$$Cl_{Att,u,i}^T = \begin{cases} 1, & i \text{ member of } Cl_u \\ 0, & \text{otherwise,} \end{cases}$$

where T_{max} is the number of consecutive time intervals, T , that constitute a year.

The yearly temporal adaptability of a household between clusters u and v is defined similarly as

$$(Cl_{Att,u} \rightarrow \dots \rightarrow Cl_{Att,v})_{i,z} = \frac{\sum_{T=1}^{T_{max}} (Cl_{Att,u}^{T-z} \rightarrow \dots \rightarrow Cl_{Att,v}^T)_i}{T_{max} - z}, \quad (2.2)$$

$$(\text{Cl}_{\text{Att},u}^{T-z} \rightarrow \dots \rightarrow \text{Cl}_{\text{Att},v}^T)_i = \begin{cases} 1, & \text{if the transition(s) took place} \\ 0, & \text{otherwise,} \end{cases}$$

where z is the number of consecutive cluster transitions. For example, the evaluation of the temporal adaptability for the immediate transition $\text{Cl}_3 \rightarrow \text{Cl}_1$ uses $z = 1$, while the evaluation of $\text{Cl}_3 \rightarrow \text{Cl}_2 \rightarrow \text{Cl}_1$ will be performed with $z = 2$. Without loss of generality, satisfactory insights on the temporal adaptability of a household could be drawn also by measuring the total number of cluster transitions regardless their type, noted as *hop-count adaptability*. The logic is the same with (2.2), but $z = 1$ always and u, v can vary throughout the year. Thus,

$$(cl_{\text{Att},u} \rightarrow cl_{\text{Att},v})_i^T = \begin{cases} 1, & \text{if } u \neq v \\ 0, & \text{otherwise} \end{cases}$$

Note here that, regarding adaptability, we are interested only in transitions that pose a beneficial impact on the household that performs them. In consumption and deficiency terms, a transition from high to lower clusters is considered beneficial, because the household saves expenses. Regarding generation and excess, low to higher cluster transitions are considered beneficial, as the prosumer shows his/her potential to generate higher amounts of excess of energy (than other prosumers inside the community), and this can lead to higher profit by selling them. Based on temporal membership and adaptability, the households are assigned to groups inside the MG community regarding their energy profile, e.g., a group of highly consuming households is one that encloses all the households that are members of the highest cluster in consumption terms during a large part of the year. To dictate periods of the year, under which the households are grouped, we define the *theta limits*, θ_m for membership and θ_t for adaptability. These are percentage limits –0.25, 0.5 and 0.75– that define four levels for our temporal metrics from 0% to 100% –low, moderately low, moderately high and high. By solving (2.1) and (2.2), the households of the community are distributed in these levels for their energy attributes.

2.3.3 Socioeconomic attributes

The temporal metrics results are combined with meta-data attributes to derive a complete characterization of the community. There is a variety of them for the households and they influence the energy attributes. Some of them refer to a household as a building –its building type or size– and others to social aspects of a household like the income of its occupants. Moreover, there are several programs for consumption regulations, designed by companies, which are signed by families. Some of these programs are designed for low-income families and others involve strategies like text-feedback or monetary

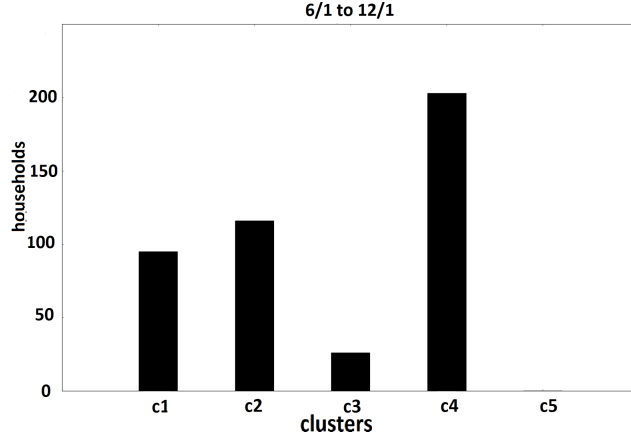


Figure 3: Consumption Clustering

incentives. The programs can be used as meta-data attributes, as they reveal social and energy aspects of the behavior of households. To be more accurate, we supplement our results with meta-data, obtained by a questionnaire, in which a part of the households participated. After connecting the meta-data with the results from clustering, the households are fully characterized, and prioritization policies can be applied on them.

2.4 Results on a real community

In this section, we evaluate the behavior of a real community of consumers and prosumers [14], using clustering algorithms and temporal metrics. In Fig. 3, the smart meter results for the second week of the year, regarding consumption, are clustered. The EM algorithm has distributed the consumer households into five clusters. That week, most of the households were in high levels of consumption; levels increase from c_1 to c_5 . Under the same procedure, for every week of the year, all the households are clustered on their energy attributes. Being in a low consumption-cluster is beneficial, denoting that a household can regulate its consumption efficiently. On the other hand, for the 180 prosumers, which –apart from consumption and deficiency– are also evaluated regarding generation and excess of energy, being in high clusters is beneficial, because it implies high amounts of energy produced (and possibly allocated inside the community).

Since weekly clustering has taken place, temporal behaviors are evaluated by the metrics of temporal membership and adaptability. In Fig. 4a and 4b, the x -axis shows the clusters in terms of consumption and generation respectively; c_1 represents low consumption/generation and c_5 represents high. Further, the position of the clusters on x -axis represents the cluster centroids. The membership ratio for a household being in a particular cluster, with respect to the total number of weeks, is θ_m . In Fig. 4a, about 400 households were low consumers, out of which about 115 households were in c_1 for more than 75%

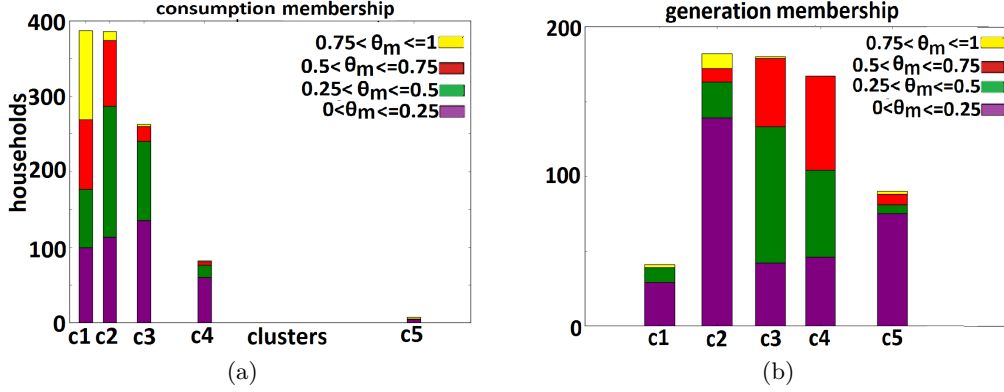


Figure 4: Temporal Membership Behaviors

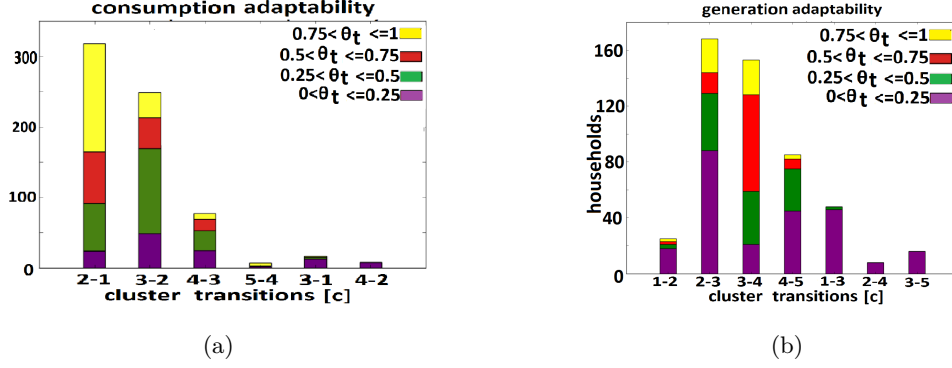


Figure 5: Temporal Adaptability Behaviors

of the year (yellow part). Furthermore, the number of households in c_4 and c_5 is much less and their overall time of staying in these clusters is also rare, as seen in the figure. Regarding generation, in Fig. 4b, around a hundred out of 180 prosumers manage to produce satisfactory amounts of energy – distributed over c_3, c_4 – for more than half of the year. This is the reason for the low consumption membership of many prosumers, as they are able to cover their needs with a part of their generated energy. Note that in generation and excess terms the first cluster centroid is zero and refers to those households that possess solar panels but do not generate (excess of) energy for a part of the year. On temporal adaptability, in Fig. 5a and Fig. 5b, the x -axis presents the beneficial cluster transitions, i.e., from high to lower clusters in consumption terms (e.g., c_2 to c_1) or the opposite in generation terms. As it is observed, immediate transitions characterized by two cluster difference are rare, because they demand high regulation potential from the households. As shown in Fig. 5a, most of the households regulate their consumption between c_1, c_2, c_3 most of the year, which explains the high number of households in these clusters in Fig. 4a. Similarly, for prosumers (Fig. 4b and Fig. 5b), we

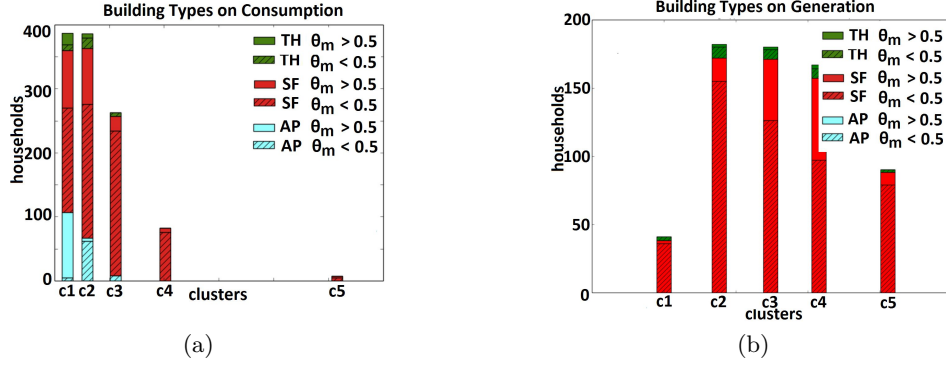


Figure 6: Energy and Socio-Economical Behavior – Building Type

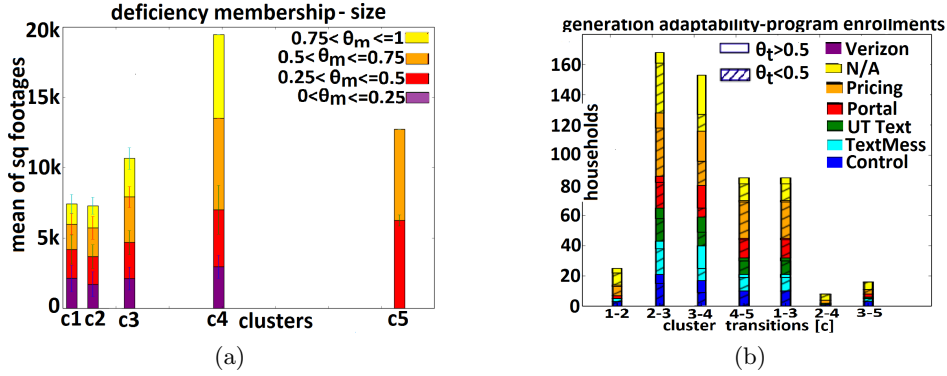


Figure 7: Energy and Socio-Economical Behavior – Size and Program Enrollment

infer that cluster transitions occur mostly between c_2, c_3 , and c_4 .

After considering the energy behavior of the households over the year, we take into account socioeconomic attributes, to add accuracy to the characterization. In Fig. 6a and Fig. 6b, three different building types of households –apartments, single family homes, and town homes– are studied on their temporal membership regarding consumption and generation. It is inferred that apartments (cyan) cannot generate energy and –along with the town homes (green)– constitute the lowest consumers in the community. These insights are complemented with results regarding the size of the households (see Fig. 7a). As the level of deficiency increases from c_1 to c_4 , the mean of square footages of the corresponding household members increases. We observed in Fig. 6a that in high deficiency clusters the vast majority of households are single-family homes, so we can infer that this is a large and energy consuming type of households. On the other hand, smaller types of households (apartments and town homes) are members of lower deficiency-clusters, especially town homes, as they have many members able to generate sufficient amounts of energy. As shown in Fig. 7b, households that are enrolled in Verizon –a program for

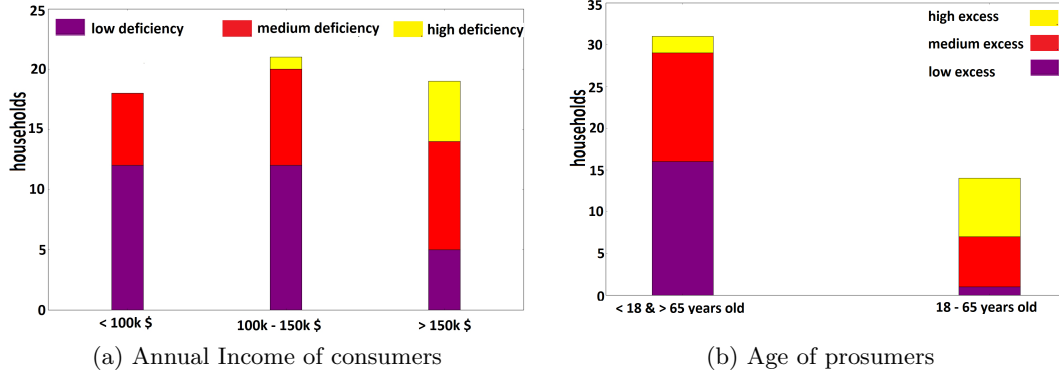


Figure 8: Household characterization with respect to questionnaire results

low-income families, in which all apartments are enrolled– did not manage to generate any amount of energy. By connecting this fact with the clustering results on apartments, it is implied that this type of building characterizes in general low-income, small, low-consuming households. In the same way, we study the whole community using graphs that connect energy attributes with meta-data attributes. For example, the households enrolled in pricing incentive programs (orange) generate energy more efficiently than the rest, thus they are more reliable for energy sharing. The same holds for those enrolled in portal programs (red), as they are able to observe the pricing incentive enrolled households and copy their behaviors. Text-feedback programs are proven inefficient, because their performances are similar to households that did not accept any feedback (see *control* category in Fig. 7b).

The relation between meta-data and energy attributes can be supplemented with the results of a survey in the form of a questionnaire in which a part of the households participated. For example, in Fig. 8a, it is observed that relatively rich residents tend to consume high amounts of energy. This can be connected to the fact that they dwell in large households, which in turn are connected with high consumption membership results. In Fig. 8b, it is seen that among prosumers that are members in high excess-clusters, the vast majority is composed of families with members between 18 and 65 years old. On the other hand, families with young children or elderly often produce low amounts of excess of energy. This is connected to the relatively high energy needs that the children and the elderly present.

2.5 Summary

The controller (CC) and the power line (CG) are imperative to any MG community. The first takes the decisions that decentralize the community from the classic grid and the second is used to carry out the energy transfer. However, without the proper understanding of the energy-behavior of the households in-

side the community, any decision of the CC regarding energy allocation does not have a stable base. This leads to energy allocation models working on priority policies which do not represent the needs of the community. By using energy clustering and temporal metrics –combined with social attributes–, the complete energy profile of an MG community is created, and thus, the priority policies that are imposed reflect the real needs of the community.

Chapter 3

Priority policies and energy allocation

Since the characterization of the MG community is complete, the energy profile of every household is obtained. Then, policies (CPPs) are developed, to prioritize among the consumers, those who should be served before others (see *order* in Fig. 9), or those who should accept higher amounts of energy than others (see *weights* in Fig. 9). A priority policy can be based either on energy attributes or on meta-data attributes. The energy based CPPs that we use prioritize the high or the less deficient households among the consumers. The meta-data based CPPs prioritize consumers on (i) the building size, (ii) the building type, (iii) the annual income of the residents, and (iv) their enrollment in consumption regulation programs. In addition, the priority policy that is applied can merge energy and meta-data attributes. In that way, the accuracy of the priority policy is augmented, as both energy and meta-data attributes are combined. This type of CPP is called *two-dimensional*, with the attributes seen as dimensions of the policy (e.g., prioritizing the less deficient and small households).

Since priority relations are assigned to the households, an energy allocation strategy (EAS) is applied, to dictate the procedure, in which the CC distributes the excess of energy among the consumers. Several EASs are proposed in this thesis. They vary (i) on the priority relations under which the

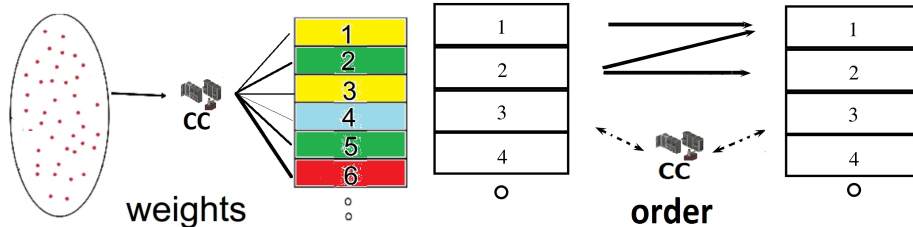


Figure 9: Priority relations and energy allocation

excess of energy is allocated and (ii) on the responsibilities of the CC. The less complex EASs serve the consumers one by one and use the CC only for communication purposes –transmitting deficiency or excess information. The complex EASs first store the excess of energy in the CC, and then distribute it to (groups of) consumers.

3.1 Related Works

The subject of energy sharing in SGs is approached by different scopes.

The work of [23,24] focuses on managing the prosumers, to define a clear role for them during the allocation of energy. By analyzing how direct (internal) parameters –like energy consumption/generation– or indirect (external) parameters –like the weather forecasting or the energy market– affect prosumers’ sharing potential, this series of works aims to create energy-sustainable prosumer communities that exchange excess of energy in an inter-community level in a smart way.

On the other hand, [15,25] focus on consumption regulation mechanisms for utility maximization. The work of [25] refers to a sole consumer, as a part of a broader group of households. It defines an optimal strategy for demand response, which relies on regular changes/adaptations of the consumer load, as a response to the changes in the electricity prices. Quality of demand response relies on real-time, two-way communication between the consumer and the energy supplier. The work of [15] refers to a consumer community, defining an optimal real-time pricing scheme, to maximize consumers’ utility in a way beneficial to both the community and the energy supplier. This work evaluates the proposed algorithm using simulation results.

Regarding the allocation strategies followed, the related works span from algorithms created on simple FIFO concepts [11] to algorithmic schemes of high complexity, utilizing Game Theory aspects [12,26,27].

In [11], an algorithm based on a FIFO philosophy was created to perform energy sharing between a group of suppliers and a group of consumers, by using energy data and supplier-consumer geographical distances. Its goal was to reduce the energy transmission loss by creating short-distance pairs of supplier-consumer.

Among several works –referred in [26]–, which relate energy sharing in SG communities with Game Theory, an interesting approach in the domain of non-cooperative games is introduced in [27], in which the energy transfer –from a controller that stores excess of energy to residential units– takes place as a Stackelberg game. The Stackelberg equilibrium, which guarantees the highest cost-reduction for both the controller and the group of residential units, is proven to be reached under a unique distributed algorithm that is proposed in this work.

Another approach in the field of non-cooperative games involves a group of consumers, which are seen as opponents, trying to acquire the largest possible portion from a common energy source –which is controlled by a utility

company—, in order to cover their needs [12]. The energy allocation procedure is transformed into a Nash non-cooperative game, and it is proven that the electricity costs, for a community of consumers seen as opponents, are decreased to their lowest value at the equilibrium point of the game. By using simulation results, [12] confirms that electricity peaks, energy costs, and individual charges are diminished.

3.2 Consumer Priority Policies (CPP)

Every CPP is created to serve a particular goal. High deficiency policies serve the problematic households that cannot regulate efficiently their consumption. Low deficiency policies aim to serve as many households as possible, because, being less deficient than others, a household can cover its needs easier. CPPs which are focused on size are created to connect it with deficiency, and subsequently, to serve households for the same reasons as deficiency-CPPs. It is interesting to observe the connection between size and energy, and compare the efficiency of policies based on clustering, like the deficiency ones, against policies based on meta-data attributes. Also, there is the possibility to follow a random policy of prioritization, which is more straightforward and works efficiently in groups of consumers with similar energy attributes.

3.2.1 Relations of priority

The relations of priority that are defined by the CPPs are of two types: (i) order (RoO) and (ii) weight (RoW).

A RoO arranges \mathcal{C} into a sequence, in which the members are served in the way the sequence dictates. The order-related CPPs impose a FIFO concept regarding service. The low-complex EASs use RoO because of their simplicity, as they do not complicate the sharing procedure.

A RoW, on the other hand, is applied when a CPP assigns weights to the members of \mathcal{C} . The consumers are served with the amounts of excess that are dictated by their weights. In that way, everyone is served (except if a consumer's weight is zero), so this type of policies define flexible priorities between the members of \mathcal{C} . The complex EASs, that first store, and then distribute the excess of energy, are usually applied on consumer groups that are sorted by RoW. Also, a combination of RoO and RoW can be used.

The CPPs that develop RoO between the consumers tend to serve the same households during every time interval, because of the FIFO concept that characterizes them. This leads to unfairness in service in the community. The problem is augmented in priority policies that are designed for highly deficient (groups of) households, and in weeks of low excess. To ensure fair energy allocation, we propose two measures; the λ *level of service* and the *round-robin mechanism*.

The λ level is a percentage limit of service, posed to every household. When this limit is reached, the next household in the RoO will be served.

Consequently, more households can accept energy under the same amount of excess, because the excess of energy is not distributed to the households until they are fully served, but until a predefined limit.

The round-robin mechanism augments the complexity in energy allocation, but increases the service diversity, in return. The households that are served are rotated to the end of the service sequence (moved to the end of the queue). Their previous positions are taken by the households which followed them in the previous time interval. For example, assuming a week time interval and a priority sequence of five consumers, $\{C_1, C_2, C_3, C_4, C_5\}$, if at week_{*w*} three consumers are served, at week_{*w*+1} the priority sequence under the round-robin mechanism becomes $\{C_4, C_5, C_1, C_2, C_3\}$.

In weight relations (RoW), the exact amount, for which a consumer is entitled, is defined by the weight of this specific consumer in relation to all the other consumers of the community. The ratios between the weights of the consumers dictate the differences in the amount of excess of energy that they accept. The procedure is dictated by the following equation,

$$\sum_{i=1}^p E_{\mathbf{e}}^i = x \sum_{j=1}^c w_j. \quad (3.1)$$

where, at first, the total amount of excess from the prosumers, during a time interval, is collected. Then, by using the weights \mathbf{w} that are assigned to every consumer of the group \mathbf{C} , the *single unit of excess*, x , is computed, and every consumer, j , accepts an amount of energy which corresponds to xw_j .

Assume two consumers; C_1 and C_2 . Suppose that we want to prioritize C_1 . The weights are higher or equal to 1, because zero weight gives no energy to the consumer and a weight between 0 and 1 or a negative weight has no physical value in (3.1). Let us assume two cases. Case 1: $w_1/w_2 = B$ and case 2: $w'_1/w'_2 = A$, and also $A > B$. In other words, the prioritization in the second case is stricter. We will prove that the difference in the accepted energy between the two consumers in the second case will be higher, for the same amount of excess. After weight normalization and without loss of generality, the weights in the first case are B and 1, and in the second case are A and 1, with $A > B \geq 1$. The case $A > B = 1$ is obvious, because the difference between the accepted energies, for weights w_1 and w_2 , is 0 (the two consumers accept the same amount of energy excess). We focus on the case $A > B > 1$. Let us assume the produced amount of excess is the same for both cases, sumExc. Thus by (3.1), –assuming x and x' the corresponding single units of excess– we have

$$Ax' + x' = Bx + x = \text{sumExc} \Rightarrow x' = x \frac{B+1}{A+1}.$$

The difference between the accepted energy excesses of C_1 and C_2 is

$$\begin{cases} Bx - x, & \text{case 1} \\ Ax' - x', & \text{case 2 (higher ratio)}. \end{cases}$$

$$\begin{cases} (B-1)x \\ (A-1)x' \end{cases} \Rightarrow \begin{cases} (B-1)x \\ (A-1)x \frac{B+1}{A+1} \end{cases}$$

If we divide case 2 over case 1 we have

$$\frac{(A-1)\frac{B+1}{A+1}}{(B-1)} = \frac{(A-1)(B+1)}{(A+1)(B-1)} = \frac{AB + A - B - 1}{AB - A + B - 1}$$

Since $A > B > 1$ the following hold:

1. $A - B > 0$
2. $B - A < 0$
3. $AB + A - B - 1 > 0$
4. $AB - A + B - 1 > 0$
5. $A - B > B - A$

Finally we obtain,

$$\frac{AB + A - B - 1}{AB - A + B - 1} > 1.$$

This confirms that in the second case, where the weight ratio is higher, the prioritization is stricter, because the difference in the accepted amounts of energy, between the prioritized and non-prioritized household, is higher than in the first case. This can be scaled for c consumers, by comparing every individual ratio of all the possible consumer pairs.

3.3 Energy Allocation Strategies (EAS)

There are two main types of energy sharing strategies regarding complexity: (i) simple and (ii) optimized.

Simple strategies are less complex and more straightforward in communication and energy allocation terms. Energy distribution involves pairing prosumers and consumers. CC is used only for communication purposes. The excess of energy is transferred through the power line.

Optimized allocation strategies are more complex mechanisms of energy allocation, where CC –besides routing– also stores energy. Thus, extra infrastructure is required for the transfer of energy between the CC and the power line. At the initialization phase of an optimized EAS, weights are assigned by the CC to every consumer, according to the CPP that is followed. Then, the CC performs computations to define the portion of the excess of energy that is to be sent to every consumer, and redistributes the collected amount of excess accordingly.

The cooperation of the CC with every separate household is important. The households that do not communicate with the CC cannot participate in the sharing process, because they cannot send information about their needs or their excess of energy. The EASs –apart from complexity– differ on the impact that their application has on the community and on every household specifically. An appendix with all the EASs is given at the end of the thesis report (see Algorithms).

3.3.1 Simple EASs

Random

In this EAS, all the households are served randomly (Alg. 1), through the power line. Every prosumer sends information about its excess to the CC, and the CC picks randomly the consumer to be served. If the consumer is covered fully, the remaining energy is sent randomly to another. This approach is the least complex as it does not use any priority policy for the consumers. It is useful when the consumers share similar energy trends, and thus their prioritization does not offer any advantage.

Greedy

The greedy algorithm exploits the characteristics of RoO policies. Under this algorithm (Alg. 2), the CC applies the designated CPP on the consumers, listing them in a priority sequence in accordance with the goal of the CPP. The excess of energy is transferred by every prosumer to its corresponding pair under a FIFO concept, as it is observed in Fig. 9 (*order* relation). The λ level of coverage can delimit the deficiency coverage, and thus allow more consumers to accept energy. The greedy approach on energy negotiation is the least complex algorithm, among those that work under a priority policy in order to serve the households. However, the order of service is the same for every time interval, so the *service diversity*, i.e., the number of different households served, stays at low levels. If the intention is to serve only certain households and the excess of energy is in adequate levels, then applying the greedy approach is satisfactory.

Round Robin

Often, we need to broaden the group of served households, without changing the greedy service concept, which is a simple allocation approach. Under these circumstances, the round robin algorithm maintains the priority given by the RoO of the CPP, and, at the same time, augments the service diversity (Alg. 3). This EAS uses the notion of time interval (T) to reshape the sequence of households that have to be served, and is initially created by the CPP, in $T = 1$. In $T = 2$, the algorithm pushes the previously-served households at the end of the service sequence and the sequence is redefined. The greedy algorithm is then reapplied on the new consumer sequence. This mechanism is

applied until a fixed number of consecutive time intervals, called *Time-Limit*, (TL), passes. This limit dictates the reinitialization of the service sequence. The value of TL is important for the algorithm, as it dictates the number of service rounds until reinitialization, and, consequently, the depth of the service variety. When $\{T \bmod TL = 0\}$, the reinitialization of the consumer sequence takes place. If TL is close to one, the CPP that is imposed on the consumers affects the energy sharing in a high degree, because of the frequent reinitialization of the consumer priority sequence, which leads to the sequence that was defined by the CPP at the beginning. The extreme case in which $TL = 1$ is the greedy approach.

3.3.2 Optimized EASs

Weighted

The weighted algorithm has the lowest complexity among the optimized EASs (Alg. 4). During every time interval, the total excess of energy, produced by the prosumers, is gathered by the CC, which forms \mathbf{C} into N subgroups, $\mathbf{C} = \cup_{n=1}^N \mathbf{C}_n$, and assigns the same weights, w_n , to the consumers of every subgroup, n . The highest weights are given by the CPP to the groups of prioritized consumers. The priority policy for this approach is based both on meta-data (size) and on energy clustering (deficiency) attributes, for augmented accuracy. The total number of groups depends on the cardinality of groups regarding size and deficiency, in the following way: $N = SD$, where N , S , and D , respectively denote the total number of groups, the number of groups regarding size similarities, and the number of groups regarding deficiency clustering. Then, the amassed excess of p prosumers is distributed to c consumers, as defined by the equation

$$\sum_{i=1}^p E_{\mathbf{e}}^i = x \sum_{n=1}^N (w_n \mathbf{C}_n), \quad (3.2)$$

that is a product of (3.1), for

$$\sum_{j=1}^c w_j = \sum_{n=1}^N (w_n \mathbf{C}_n).$$

The weighted approach is more fair, regarding service, than the previous approaches, because the energy accepted by each household is correspondent to its weight, which is defined by its level of priority.

Nash

Following the use of RoW in order to dictate priorities, in the Nash algorithm, a different weight is assigned to every consumer, and all the consumers request energy according to their weights from the CC, simultaneously and continuously. They withdraw only when they are fully served (see Alg. 5).

The concept behind the Nash EAS relies on Game Theory, and specifically on the existence of an *equilibrium* on the choices of the consumers on the allocation of energy. Assuming (a group of) consumers without personal interest in cooperating to share the excess of energy, their natural approach towards the aggregated excess is standard, in order to receive the maximum possible amount of excess they can be entitled to. Also, supposing that a consumer decides not to ask for excess during a certain time interval, the part of energy that would (possibly) be allocated to him/her is distributed to the rest of the consumers. Thus, any other choice besides asking for energy harms a consumer. Seeing the consumers as players in a non-cooperative game of energy allocation, the situation in which they are led –where everyone is bound to a certain decision– was described by John Nash as an equilibrium [28]. At the initialization phase, different weights, \mathbf{w} , are assigned to every consumer according to the CPP that is imposed. Then, the CC, which holds information on the deficiency of every consumer, defines their ratios of deficiency over weight, called *heights of service*, \mathbf{H} ,

$$\mathbf{H} = \frac{\mathbf{E}_a}{\mathbf{w}} \quad (3.3)$$

In this approach, all the households have different weights, which are assigned according to the CPP –contrary to the weighted approach, where all the members of the same group have the same weight. In that way, the excess of energy that is accepted by each individual household fits to its needs and reflects its specific energy and socioeconomic characterization. In addition, the behavior of the community under this approach is the most realistic, because the households tend not to cooperate in energy sharing terms.

Water Filling (WF)

The WF algorithm is designed to combine the given RoW and the deficiency needs of every consumer, to create RoO between the households. At the initialization phase, different weights are assigned to every consumer by the CC, according to the CPP that is imposed. Then, the CC, which has information on every consumer's deficiency, defines their heights of service, \mathbf{H} . However, in this EAS, the CC arranges them in ascending order, which becomes the order of service for the consumers. The difference between this algorithm and the Nash EAS is that not all the consumers are permitted to make energy requests simultaneously. Assume that the transferred excess of energy is added on top of the height of service, \mathbf{H} , of every consumer as *additional height*, $\mathbf{h} = \mathbf{E}_g/\mathbf{w}$. As the CC commences the energy sharing procedure with the first consumer in the order of service, it transfers excess to it. So, its h_1 level increases until a point, in which $h_1 = H_2 - H_1$. At that point of time, and assuming there is enough excess of energy, the CC starts transferring excess also to the second consumer in the order, until the moment when $h_2 = H_3 - H_2 = h_1 - (H_2 - H_1) \Rightarrow h_1 = H_3 - H_1$. The procedure continues until every need is covered or until the total energy excess is depleted. A consumer

j is withdrawn of service only when he/she is fully covered ($h_j = H_j$). For two consumers, j and l , with $H_l > H_j$, it is also possible that $H_l - H_j \geq H_j$, and thus the consumer j is fully covered before l starts requesting for energy. The number of consumers that are served simultaneously at any time instance is $l + 1 - j$, where j is the most and l the least prioritized among them. These consumers are served simultaneously as long as they present equal sums of H and h . The generalized form of the way of service of WF EAS, for a number of c consumers by p prosumers, is described by Alg. 6. The WF approach is the most complex and demanding among the EASs, as it requires constant cooperation between the consumers, more computations from the CC, and updates of the consumer sequence of service at the beginning of every time interval. As it is inferred by Alg. 6, many consumers are fully covered before others even commence their energy requests. The WF algorithm is influenced by the domain of Information Theory, and specifically by power allocation in parallel Gaussian channels of different capacities [29, 30]. The major difference in our case is that the households present finite needs of energy, while the channels are receptive to an infinite amount of power –if it existed. Every consumer household has a priority in relation to all the other consumers in the community, regarding both its deficiency and its weight. This approach combines the service accuracy of approaches that use RoW and the households-cooperation of the approaches the use RoO. Thus, it guarantees high levels of community welfare.

Pareto

In Pareto EAS, there is no CPP applied among consumers. This EAS uses the λ parameter which was defined to pose a limit in deficiency coverage. However, the usage of λ in this EAS is to set a limit of service, under which, every consumer k covers the same percentage of its needs, E_g^k/E_a^k , (Alg. 7). During every time interval, the CC acquires information on each household's deficiency and gathers the total amount of excess. In a game, an outcome is characterized *Pareto optimal* when it cannot be improved, unless at least one of the players is harmed [31]. The Pareto EAS guarantees equal percentages of deficiency coverage to the consumers, by utilizing the whole gathered amount of energy excess. Since there is no more energy to distribute for a certain time interval, any other transfer, apart from the dictated by the λ level, leads to harming at least one consumer, by lowering his coverage percentage. Pareto EAS is a low complexity algorithm that could not be characterized optimized by itself. However, it is never used solely on a consumer group. It is employed in cooperation with other optimized algorithms like the Nash or the WF EAS, in two-stage approaches on energy allocation, which will be analyzed further in this section.

Table 1: Simple EASs in ascending complexity

| Random | Greedy | Round-robin |
|--------|--------|-------------------------------|
| | RoO | Served households Rotation |
| | | Service list Reinitialization |

Table 2: Optimized EASs in ascending complexity

| Pareto | Weighted | Nash | WF |
|--------|---------------|------------------|------------------------|
| | RoW | Separate Weights | RoO on Service Heights |
| | Group Forming | Service Heights | |

3.3.3 Complexity

The decision of the proper EAS for a community is influenced by the complexity in its mechanism. In Table 1 and Table 2, the EASs are presented in an order of increasing complexity (from left to right), with the basic characteristics that augment it being stated.

3.3.4 Two-stage approaches

Until this point, the EASs are applied between \mathbf{P} and \mathbf{C} inside the MG-community. In two-stage approaches, we focus on the same EASs. However, after community characterization, we create groups of consumers and prosumers, regarding similarities in energy and socio-economical attributes, before the commence of the energy sharing phase. Then, either we form pairs of prosumer-consumer groups, and EASs are applied strictly between them, or we apply one EAS-CPP to distribute the total excess of energy to the consumer groups (inter-group distribution) and another EAS-CPP to distribute the corresponding accepted excess inside every group (intra-group distribution). The advantage of grouping approaches is the application of different EASs, which fit the needs of every consumer group. By forming groups in a heterogeneous community of households, we create small teams with high internal compactness. Due to the homogeneity of these teams, applying a CPP in energy sharing is possible to be redundant; thus a random or a Pareto EAS would perform efficiently. Ultimately, this is beneficial in complexity terms, because of the simplicity of sharing mechanisms that do not prioritize among the consumers.

3.4 Summary

The insights and implications about the energy behavior of an MG community (see Chapter 2) offer the proper base for deployment of priority policies among the groups of consumers. These policies define the impact of service of the prioritized households, and are used by strategies that allocate the excess of energy inside the community. Since the energy needs inside an MG community differ, heavily affected by the socioeconomic background of its members, there is no optimal strategy for energy allocation. However, we have developed

seven strategies (EASs) –simple to highly optimized. After the complete characterization of an MG community, we are able to choose the proper strategy of energy allocation. The one that: (i) fits the priority policy we need to impose, (ii) maintains the level of complexity we prefer, and (iii) suits the needs of a household community.

In the next chapter, we will define metrics to measure in quantifiable terms the efficacy of each EAS under different aspects of energy allocation, simplifying in that way the procedure of choosing the ideal strategy for different MG communities.

Chapter 4

Experimental evaluation and implementation

In this chapter, we experimentally evaluate the performance of the algorithms and policies that were analyzed in [Chapter 3](#). To achieve this, we introduce a real community of households, Pecan Street, located in Texas, Austin [\[14\]](#). On this community, we apply the aforementioned policies and EASs, based on the energy characterization that happened in [Chapter 2](#). The results are presented, and they are evaluated by metrics that we introduce in order to compare the efficacy of our CPPs and EASs. In addition, by comparing the expenses of electricity between allocating the energy and requesting it from the CG, we confirm the economic benefits of energy sharing.

4.1 Pecan Street

The community we selected is composed of 443 households. Among them, there are 180 prosumers, which generate energy using PV. Consumers and prosumers communicate solely with the CG for energy transfer. Prosumers request energy from the CG only when the generated is not enough to cover their needs completely. In case the amount of energy that is produced by the solar panels is more than sufficient for his/her own energy needs, a prosumer presents an excess of energy. In Pecan Street, the prosumers neither possess the means for energy storage (batteries, capacitors etc) nor have the infrastructure for intra-community energy allocation. Thus, the excess of energy is sold back to the CG.

The electrical appliances of the households are loaded with sensors, which convey the consumption measurements to a central smart meter, using short-range wireless protocols. Prosumers possess also sensors on their PV panels to measure the produced energy and transmit the results to smart meters. We used the aggregated electrical consumption and generation data from the smart meters of the households during 2014 to define the deficiency and the excess of energy for every household. The smart meters logged the data every

minute, offering them in a fine-grained form for accurate analysis. We selected households that had data for more than 300 days.

4.2 Metrics

To evaluate the performance of the deployed EASs, we defined specific metrics, which cover the following issues:

- Deficiency coverage potential of EASs.
- Efficiency in prosumers utilization.
- Service diversity offered by each EAS.
- Social welfare on group and community level.

At first, we analyze the metrics that focus on households as entities being served. These metrics refer to (a group of) c consumers of a MG community. We use vector-parameters \mathbf{C}_{par} , for $\text{par} \in \{\text{served}, \text{notServed}, \text{unique}\}$. These parameters accept 1 or 0, based on the Boolean answer to the following statements for a consumer $k \in [1, c]$: (i) is k served fully?, (ii) is k not served at all?, and (iii) is it the first time that k is served?

To quantify the ability of a strategy to cover the needs of (a group of) c consumers completely inside a community under a certain CPP, we define the *Served Ratio* (SR) metric, for a constant time interval T , as follows:

$$\text{SR} = \frac{\sum_{k=1}^c C_{\text{served},k}}{c} \quad (4.1)$$

If SR is averaged, we can draw insights into the serving potential of an EAS during the whole year.

During a time interval, an important performance attribute of an EAS is its ability to serve (a group of) consumers with the prosumers that are active, i.e., able to produce excess of energy. The evaluation of the efficiency of a number of active prosumers, p_{act} , during T is given by the *Prosumers Beneficialness Ratio* (PBR), defined as

$$\text{PBR} = \frac{\sum_{k=1}^c C_{\text{notServed},k}}{p_{\text{act}}} \quad (4.2)$$

If PBR is averaged, we can draw insights into the way an EAS utilizes prosumers during the whole year. The PBR value is zero for sharing approaches that offer energy to all the consumers, like the weighted, the Nash and the Pareto EAS.

For the EASs that do not use (solely) RoW, it is also important to evaluate to what extent their approach serves different households under a CPP. This can be measured by *Uniqueness Ratio* (UR), which quantifies the service

diversity of a negotiation strategy for (a group of) consumers, for any set of consecutive time intervals, denoted as $T_b - T_a$, where $T_a, T_b \in [1, T_{\max}]$.

$$\text{UR} = \frac{\sum_{T=T_a}^{T_b} \sum_{k=1}^c C_{\text{unique},k}^T}{c} \quad (4.3)$$

High UR is not necessarily beneficial for a CPP, as it indicates that energy is sent also to households not intended to be served by the CPP. For example, assuming a CPP that prioritizes a group of less deficient consumers and $\lambda = 0.5$, it is possible that, during weeks of large amounts of excess produced, all the members of the less deficient group were served. This implies that consumers from other groups had received energy, too. Thus, there was enough excess of energy to cover all the members of the less deficient group with $\lambda > 0.5$. This, in turn, means that it was possible for the prioritized households to accept higher amounts of energy, but they did not.

With respect to fairness in service under a CPP –either it refers to a group of consumers or to the whole community–, metrics that involve households in absolute terms (1 and 0), like SR, PBR, and UR, do not offer the full picture, because they cannot discriminate among the households that accepted excess of energy, but were not fully covered. Irrespective of its type, the fairness can be measured by the ratio of the amount of excess given to a household (or a group) and its energy needs. This ratio is called *Energy Ratio* (ER), and for a consumer k , during T , the ER is defined as $\text{ER}_k = E_g^k / E_a^k$. When $\text{ER} = 1$, the consumer k is fully covered, while when $\text{ER} = 0$, no energy is accepted. However, if we need to evaluate service in priority terms (CPP), the deficiency coverage of every household has different impact on the community. The prioritized households are more important to serve, and this importance is quantified by assigning RoO or RoW to the consumers. Under this concept, for a consumer k , applying a weight that mirrors its significance to the group turns ER to its weighted form, $\text{ER}_{w,k} = w_k \text{ER}_k$. To evaluate it properly, we use the \log_2 relation to define the *Social Welfare* (SW) for any consumer k ,

$$\text{SW}_k = w_k \log(1 + \text{ER}_k) \quad (4.4)$$

However, SW_k cannot be characterized as high or low. It has to be related to the maximum possible value of SW that k could reach. Obviously, when a consumer is fully served, $\text{ER}_k = 1$. This leads to $\text{SW}_{k,\max} = w_k$. Thus, the metric to characterize every consumer regarding fairness is the *Social Welfare Ratio* (SWR), defined as $\text{SWR}_k = \text{SW}_k / w_k$. Following the same concept, to expand the individual SW into SW for groups or whole communities of c consumers, (4.4) gives $\text{SW}_c = \sum_{k=1}^c \text{SW}_k$, that reaches its maximum when there is enough excess of energy to serve all the consumers completely – $\text{SW}_{\max,c} = \sum_{k=1}^c w_k$. Thus, to define the community (or group) SWR,

$$\text{SWR}_c = \frac{\text{SW}_c}{\sum_{k=1}^c w_k}. \quad (4.5)$$

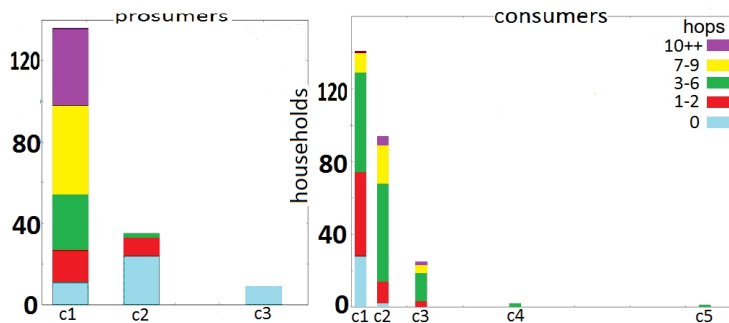


Figure 10: Prosumers-Consumers quality groups

4.3 Implementation results

In this section we compare the results from the application of the EASs on the Pecan St. dataset, using the aforementioned metrics. By using the evaluation results, we can decide on the combinations of EASs and CPPs that serve our energy allocation goals.

4.3.1 Ideal models of consumers and prosumers

The characterization of households in Pecan St. community enabled us to segment consumers and define policies, based on their energy consumption/generation and socioeconomic features. In social terms, we directly created CPPs to prioritize households. In energy terms, we first defined the *ideal* consumer and prosumer, and then we grouped the households by their proximity to the ideal models (see Fig. 10). Both the ideal prosumer and consumer present the lowest possible deficiency membership. Naturally, the ideal prosumer performs many transitions from low to higher excess-clusters while the ideal consumer is able to regulate its deficiency efficiently and thus hovers around low deficiency-clusters. Grouping prosumers and consumers, based on their ideal models, offers the opportunity (i) to evaluate the efficacy of EASs, by applying our metrics in each group separately, and (ii) to define groups for the two-stage energy sharing approach (see Subsection 3.3.4).

4.3.2 CPP comparison

In Fig. 11, we compare the CPPs –defined under RoO– of the greedy EAS. It is observed that the policy that prioritizes the less deficient consumers manages to serve more households than the policy that gives priority to the highly deficient. Low deficiency CPP aims to serve a high number of households, which is achieved, because the prioritized households are easily served. High deficiency CPP aims to serve those in high-needs of energy, which require large amounts of excess. In addition, it is observed that the results regarding small size-CPP follow those that regard the low deficiency policy, and the results

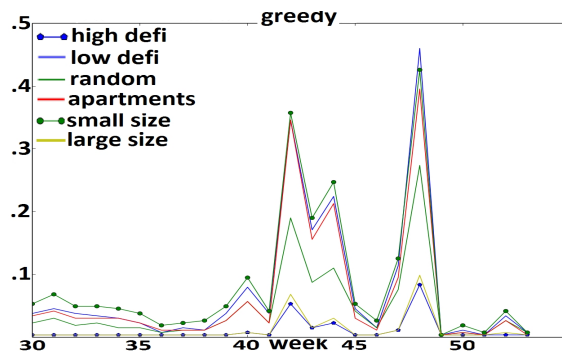


Figure 11: Served percentage of households, $\lambda = 0.5$ according to different Priority Policies

regarding large size-CPP follow those that regard the high deficiency policy. This confirms the characterization insight that large households tend to need high amounts of energy, while small households regulate their consumption effectively. As expected, the apartment-CPP follows the low deficiency and the small size policies, and it is a socially aware policy, as the residents of the apartments are enrolled in Verizon program, which is designed for low income families. The performance of the random policy stays in the middle of the other policies as it gives priority to no-one.

The importance of the combination of energy and meta-data characteristics to define priority relations between the consumers will be stated by an example. In the greedy EAS of Fig. 11, a CPP defined by the meta-data attribute of size –small size priority policy–, during certain weeks, manages to serve more households than the low deficiency-CPP, which is designed to serve as many households as possible. This happens because the households that are put in a RoO for their general behavior in deficiency terms are not guaranteed to be in that exact order every week, regarding deficiency. Because of the relation between deficiency and size, the order priority that is defined by size sometimes predicts the deficiency order better than the genuine deficiency-policy. The insight from this situation is that, even both the policies manage to serve a high number of consumers, a third policy, using the combination of size (meta-data) and deficiency (energy), would predict the RoO more accurately.

4.3.3 Service ratio and λ level

In Fig. 11, $\lambda = 0.5$, in an attempt to offer service diversity. Thus households are not fully served, and the SR metric has no value. The SR is measured only for $\lambda = 1$, which guarantees complete deficiency coverage. In Fig. 12a and Fig. 12b, different target groups of consumers, which are served for three consecutive months of 2014 –week 38 to 49– by the round-robin and by the greedy EASs, are presented, along with their number of members. The consumers were grouped regarding energy and meta-data attributes. In energy

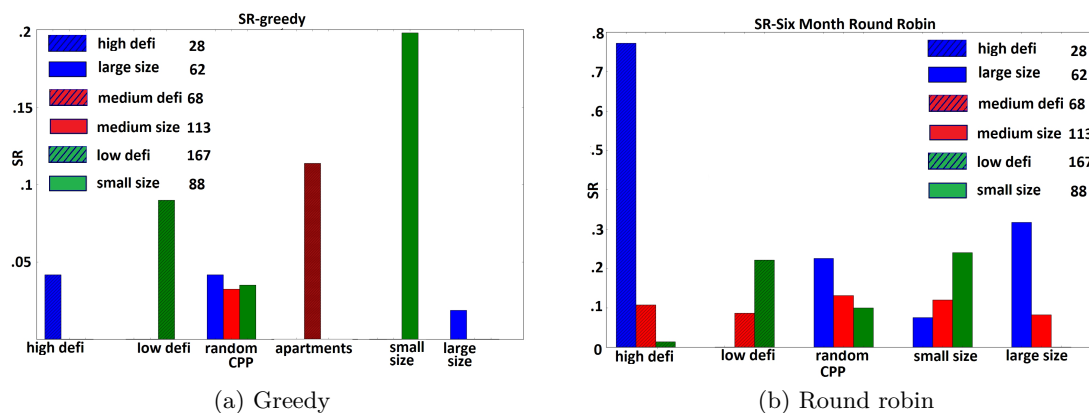
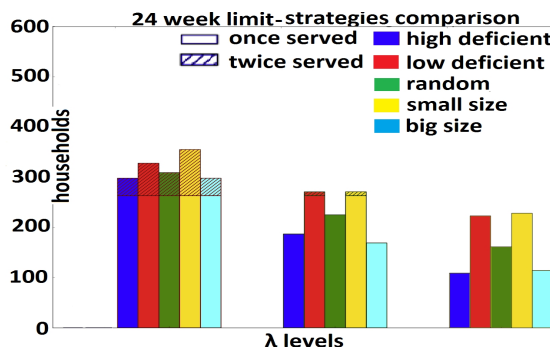


Figure 12: Served Ratio

Figure 13: Served percentage of households, λ 0.5, 0.75 and 1

terms, the insights of the ideal prosumers and consumers (see Fig. 10) were used to derive three different deficiency groups: low, medium, and high. Regarding meta-data, the information referring to the size of the households was used to define small, medium and big consumers. As it is seen in Fig. 12b, a round-robin EAS manages to serve, also, different groups of households, apart from the prioritized ones. As the TL increases, the impact of the priority policy lessens. Moreover, because of the repositioning of hard-to-serve consumers at the end of the service sequence (RoO), policies that prioritize highly deficient and large sized households serve more households in total under a round-robin EAS, than under a greedy EAS. The opposite happens for the policies that prioritize the small and the less deficient consumers. The impact of the combination of different λ levels and round-robin approaches in energy sharing is seen in Fig. 13, where the number of households that accepted energy once or twice throughout the year is presented for different values of λ . As the λ levels decrease, more households are served. However, at the same time, the connection between size and deficiency in CPPs, that was first observed in Fig. 11, remains.

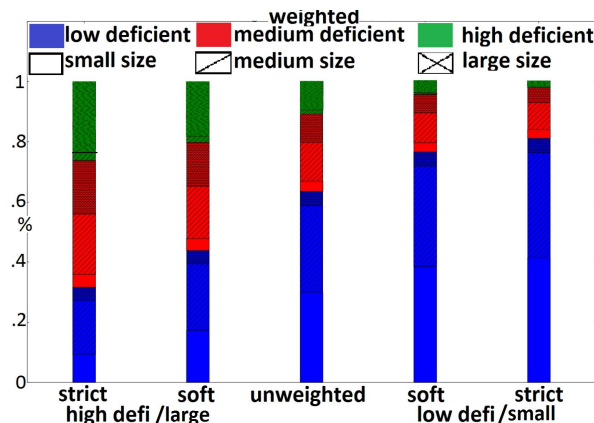


Figure 14: Excess percentage accepted weekly by different groups - Weighted
 High ratio between weights(strict): $1 \rightarrow 8$ (step 1.0), Low ratio between weights(soft): $1 \rightarrow 3.33$ (step 0.333), high deficient/small size: 0 members

4.3.4 Energy allocation in optimized EASs

The previous figures refer to simple EASs, which use RoO between the consumers for prioritization. In Fig. 14, the weighted approach, which has the lowest complexity among the approaches that utilize RoW, is presented. As it is observed, eight groups are created by the compilation of the deficiency characteristics of the households –defined in Fig. 10– and their meta-data attributes (two-dimensional priority). These groups receive stable percentages of the weekly excess, and this amount is shared equally by their members, as they have the same weight. According to the CPPs and the differences between the ratios of the imposed weights, every group accepts a different percentage of energy-excess. As seen in Fig. 14, high weight-ratios among the groups prioritize the groups strictly, while low ratios distribute the excess in a way more similar to the unweighted approach, where all the consumers accept the same percentage of excess every week, confirming in that way the proof of Subsection 3.2.1 (-RoW), about ratios between the consumers weights.

Contrary to the weighted approach, the other optimized approaches that use weights –Nash and WF EAS– assign a different weight to every consumer, according to the priority policy that is to be imposed. In Nash EAS, there is no cooperation between the consumers –they simultaneously request for energy–, while in WF EAS the households with the lowest H accept excess of energy before the others, creating in that way a RoO. In Fig. 15a and Fig. 15b, the energy allocation differences of these two approaches, during a week of low excess, are presented, for the same CPP, amount of excess, and group of consumers. As it is seen in Fig. 15a, the Nash approach does not permit high SR values; the excess is depleted before someone manages to cover completely his/her needs, because of the simultaneous demands for energy. In Nash EAS, to serve a number of consumers completely under a produced amount of excess, their assigned weights have to be high, in relation

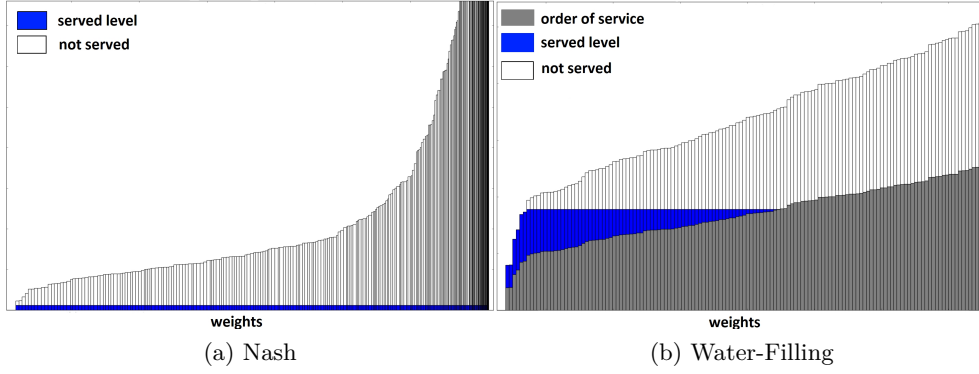


Figure 15: Conceptual structure of Nash and Water Filling – week 47

to the weights of the other households, in order to produce low heights of service –easily reached–, according to (3.3). On the other hand, in Fig. 15b, the WF approach serves completely the most prioritized households, because of the existence of the RoO, which was defined by the combination of weights and consumer-deficiencies. Nevertheless, there is a big group of households –not prioritized ones– that do not manage to cover even partially their needs.

4.3.5 Prosumers usage and uniqueness

For the energy allocation approaches that do not involve (solely) RoW between the consumers, it is important to evaluate their efficacy on the domains of prosumer utilization and service diversity. In Fig. 16a, note that the lowest values present the most efficient behaviors, as the metric is related to the consumers *not* served weekly, over the active prosumers according to (4.2). Among the EASs that use CPPs, it is seen that the WF sharing approach manages to utilize the prosumers in the most efficient way and, at the same time, it keeps a satisfactory UR ($\simeq 0.5$, Fig. 16b). The random EAS presents a much lower PBR than the other EASs (see Fig. 16a), because its mechanism obtains no priority in serving. That is the main reason of its usefulness in communities where the consumers present similar attributes and there is no need for priority policies. Its high diversity in consumer-service is reconfirmed by Fig. 16b, where the random EAS manages to serve almost 85% of the consumers throughout the year. By Fig. 16a and Fig 16b, we remark that, in service terms, for augmented uniqueness and efficient utilization of the prosumers, low λ values and low deficiency CPPs should be applied. The round-robin approaches are able to offer high diversity –even $UR = 1$ – as the TL increases, because of the continuous repositioning of consumers at the bottom of the sequence of service (see Fig. 13). However, this does not imply efficacy in prosumers utilization, for PBR is a week measure while UR is applied over consecutive weeks –even a year.

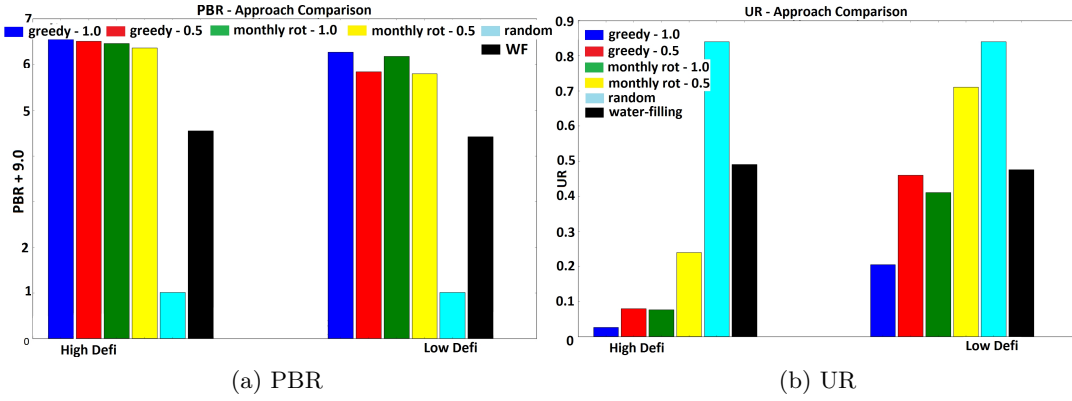


Figure 16: Service diversity - prosumer usage (week 38-49)

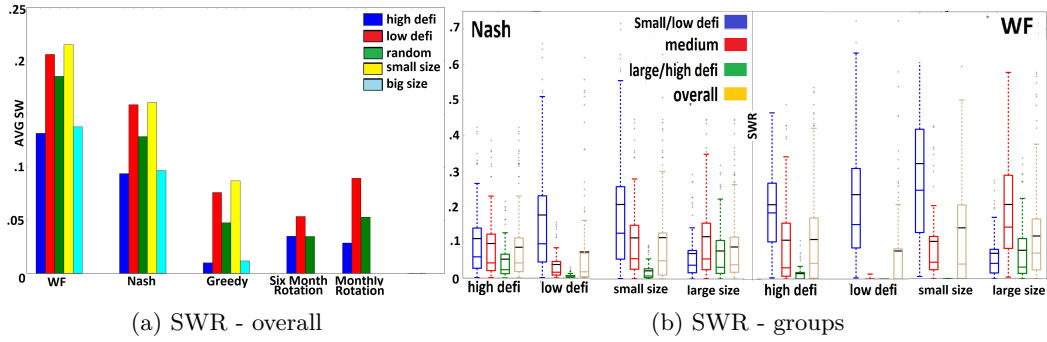


Figure 17: Community social welfare (yearly)

4.3.6 Social welfare

Fairness regarding service cannot be seen properly by SR, because it denotes absolute service (1 or 0). When every household is assigned a weight or a sequence position by a CPP, the impact of its service becomes differentiated from the rest of the households. For example, covering half of the deficiency of the highest-weighted household in a community is more important, in fairness terms, than serving completely the lowest-weighted one. Service-fairness in a community is described by the SWR metric (4.4). In Fig. 17a, the advantage of the optimized against the simple approaches regarding social welfare is clear. The augmented complexity –because of the weight assignment, the energy storage, and the computations for the distribution of energy– is counterbalanced by high social welfare ratios, irrespective of the strategy. Specifically for the WF and Nash EASs, under specific CPP, weights, deficiency, and excess of energy, WF EAS manages higher SWR results. Focusing only on these two EASs, in Fig. 17b, their impact on different groups of households –which are defined by their closeness to the ideal models– is observed under the same RoW. The WF EAS prioritizes stricter among the households than the Nash EAS, because the households are prioritized with relations of both weight and

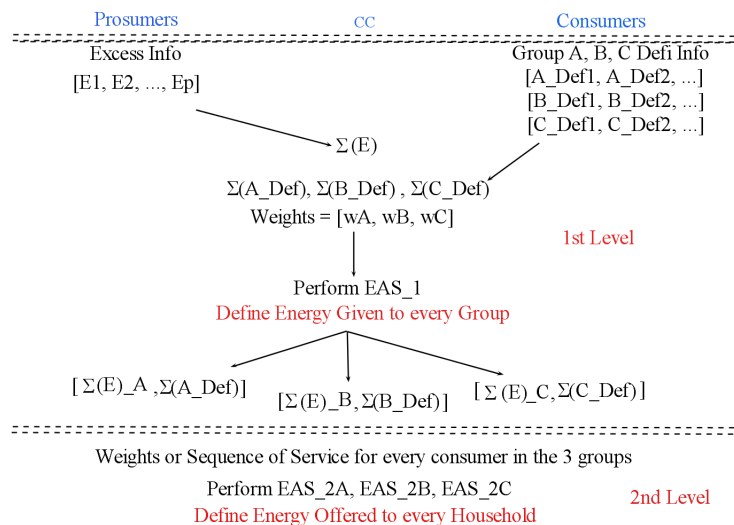


Figure 18: Working schema of two stage energy sharing based on grouping

order. On the other hand, in Nash EAS, because of the simultaneous demand, the groups' social welfare results present closer values. Note here that groups like the big sized or the high deficient households present deficiencies which are hard to cover. Thus, the impact of their priority weights in (4.4) is lower than the impact of the weights of other groups, when they are prioritized. The WF approach presents higher SWR results not only in the prioritized household-categories but also in the overall SW results, which confirms Fig. 17a. Further, in WF we observe more outliers (see Fig. 17b), which leads to the conclusion that the Nash algorithm is more stable than the WF.

4.3.7 Results of two-stage approaches

In two-stage energy allocation approaches, the excess of energy that is produced by P passes through a phase, in which it is divided in portions; not to be distributed to every single consumer, but to groups of consumers. The division of excess in portions can happen either by grouping prosumers according to their ideal models, and collecting the groups' amounts of excess, or in an EAS-CPP combination scheme, seen in Fig. 18. This figure describes schematically the mechanism of a two-stage approach, which permits several combinations of EASs and CPPs, in the way analyzed in Subsection 3.3.4.

Pairs of groups negotiation

Pairs of groups, from the consumer and prosumer sides, that negotiate on excess allocation can be created depending either on energy –see Fig. 10– or on meta-data. In Fig. 19, we present the results of pair-grouping in terms of meta-data (size). Three groups of consumers and prosumers were created, and the energy sharing was permitted only between households of the same group

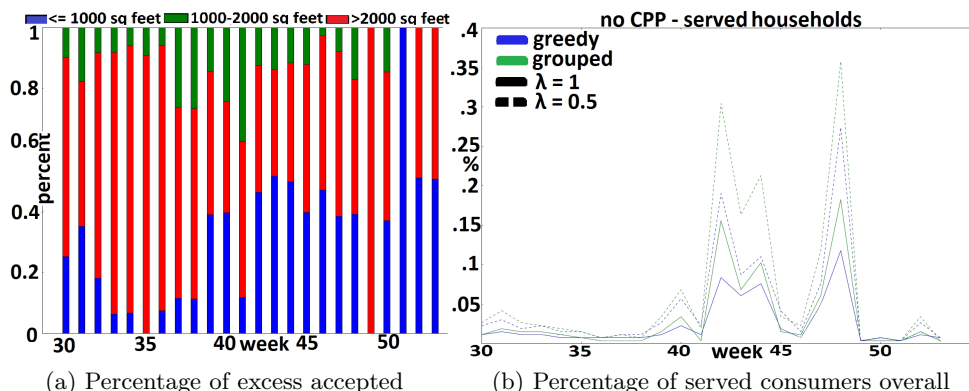


Figure 19: Size grouping on households; greedy vs grouping on $\lambda = 1$ and $\lambda = 0.5$ (right). Consumers: 88 small, 113 medium, 62 big. Prosumers: 64 small, 65 medium, 51 big.

—e.g. small prosumers transfer excess of energy only to small consumers. As it is observed, the size-grouping outperforms the classic greedy approach when there is no priority policy (random CPP). This result confirms the advantage of grouping as a low complexity energy allocation approach and the importance of connecting size and energy demand.

EASs and CPPs combinations

Regarding priority policies and allocation algorithms combinations, we will state the one that balances group and community-fairness in the most efficient way. We use Nash EAS in the first stage of energy sharing, for distributing energy to the groups of consumers, and WF EAS between the members of every group (second stage). More specifically, Nash and WF are the most optimized approaches and both present the best results in SWR terms. Nash EAS reflects the reality in energy service between households, as, most of the times, they do not cooperate. Similarly, in our case, the different groups of consumers, having different needs and characteristics, would simultaneously request for amounts of energy, according to their weights. However, in intra-group energy allocation, households would be eager to cooperate under a common strategy, because they share the same needs, and thus WF can be adopted. The results of this strategy are presented in Fig. 20.

Note here that in Fig. 20b (large CPP on groups), when WF is applied inside the group of large households, and the largest among them are prioritized, they are so highly deficient, that the excess allocated to the group is depleted mostly by them. The result is that, although prioritized by Nash EAS in the first level, the group of large households ends up with lower SWR than the medium and small households-groups. This implies high variance in deficiency terms inside this specific group, and thus further segmentation is needed.

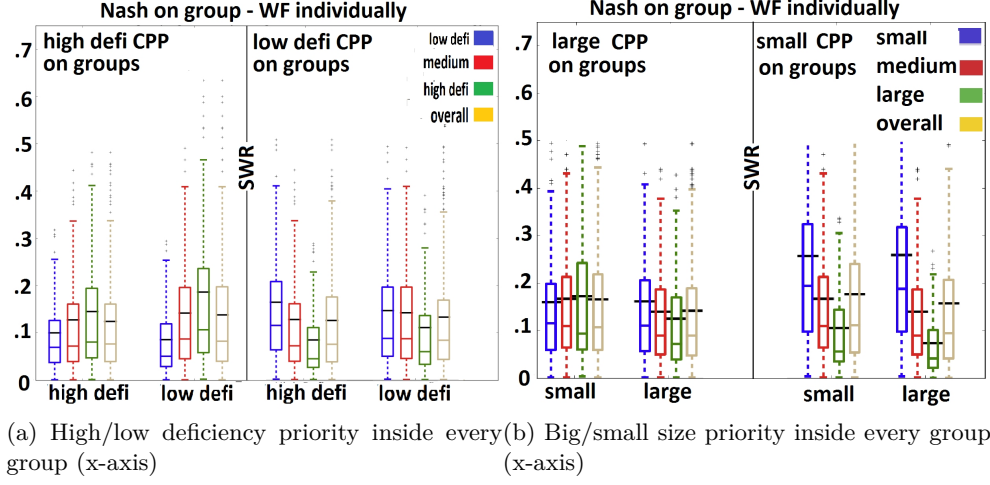


Figure 20: Two stage energy sharing - Nash and Water Filling approaches combination

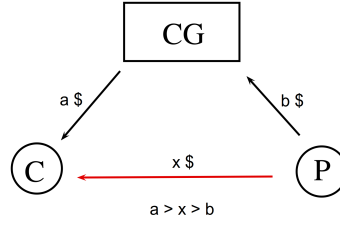


Figure 21: Cost schema for Energy Sharing

4.3.8 Cost analysis

In this section, we employ a standard pricing day from the ‘Power Smart Pricing’ site [32], with hourly costs per kW. Using this day’s pricing scheme, we evaluate the differences in the expenses when applying energy sharing from 10/13/2014 to 10/26/2014, a period in which relatively high production of energy was observed on the prosumers side. By allocating the energy excess, consumers experience price-reduction for their coverage of deficiency as they buy energy at lower prices than they would do from the CG. Further, prosumers profit from sharing because they sell their excess in higher prices to the consumers than to the CG (see Fig. 21). In Fig. 22a, the cost reduction using Nash EAS is observed for the prioritized groups of households for any corresponding CPP. We do not claim that the reduction would be exactly at the levels indicated in the figure. This depends on the prices per kWh which would be imposed by the prosumers. However, (i) these are the lowest limits that the costs could reach through the decentralization of energy distribution, and (ii) the costs cannot be higher or equal to the expenses results for no coordination (black), because this would imply that the prosumers sell their excess of energy at higher prices than the grid –which is against the economical con-

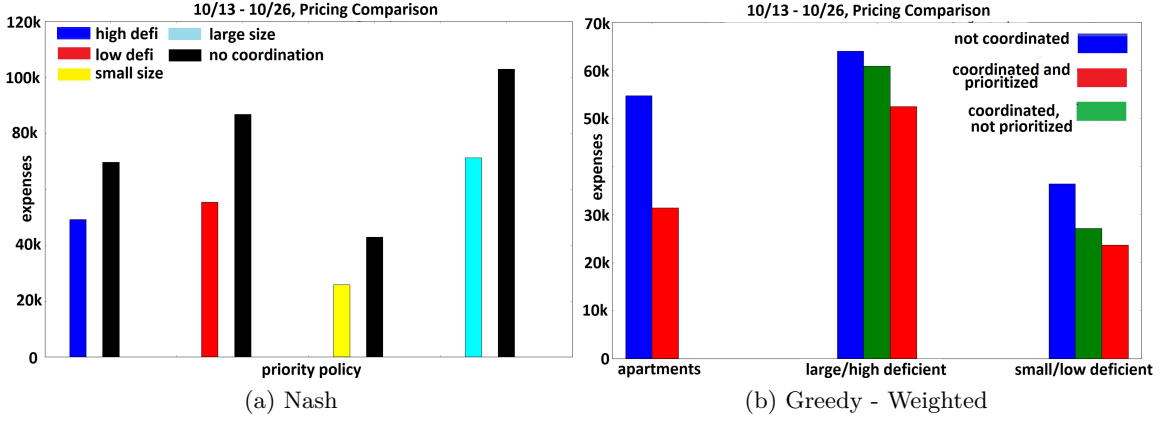


Figure 22: Expenses reduction

cept of energy sharing (see Fig. 21)–, and no one would buy energy from them. In addition, note that energy sharing is beneficial for every consumer, either prioritized or not. In Fig. 22b, and in terms of weighted EAS (mid and right bar-groups), although the cost reduction is higher for the prioritized groups of households (red), there is still a considerable reduction in expenses for the less prioritized consumers that accepted lower amounts of excess (green).

4.4 Summary

In this chapter, the impact of seven different energy allocation strategies, deployed on a real community of prosumers and consumers, was presented. The strategies were applied under different priority policies that channelize the excess of energy to the preferred (groups of) households. The metrics that were developed cover a broad range of energy sharing aspects and offer the opportunity to evaluate the efficacy of our strategies. In general, homogeneous communities share the excess of energy efficiently under simple algorithms. The use of the round robin mechanism and the λ level of service augments the service diversity among the consumers and improves the prosumers-usage efficacy. Allocation schemes of high complexity are intended for communities of varying energy behaviors. In these communities, water-filling EAS is the strategy that guarantees the highest social welfare ratios, but encloses the highest levels of complexity in its mechanism. Low complexity algorithms that do not use CPP –random and Pareto– work efficiently under two-stage approaches, which group the households based on their energy behavior. Regarding electricity expenses, energy allocation is proven beneficial, even for consumers that are not prioritized. It should be stated that community characterization of Section 2.3 was crucial, being the cornerstone of several approaches and energy negotiation scenarios that work with RoO or RoW (either one-stage or two-stage). The plurality of combinations of EASs and CPPs allows any characterized MG community to share the excess of energy among its members efficiently.

Chapter 5

Conclusions and future work

In this chapter we wrap up our work and present proposals for future works on the Smart Grids domain.

5.1 Conclusions

Smart Grids are rapidly becoming a reality. There is already awareness amongst consumers to also generate energy, using solar, wind, and other RSEs, which makes them prosumers. In this modernized version of the distribution grid, two-way communication between consumers, prosumers, and utility controllers is established. Bidirectionality in communication introduces interaction in energy allocation among the members of an MG community. This will avoid energy losses and also benefit prosumers, compared to selling energy to the central grid.

In this thesis we created, implemented, and evaluated simple and optimized algorithms (EASs), which control the sharing of energy in an MG community of households, consisting of prosumers and consumers. The EASs were implemented in accordance with prioritization policies (CPPs), which dictate the impact of covering the needs of each consumer inside the MG. However, CPPs are accurate on their priority targets when they are designed with knowledge of the characteristics of the community, on which they are applied. Thus, by using fine-grained energy data (for clustering) and socioeconomic attributes, we gain insights into the behavior of the MG community regarding energy consumption and generation. The results of the temporal energy dynamics of an MG community reveal its complexities in energy usage.

To satisfy the specifics of every community, we developed seven strategies, based on social and energy characteristics. These strategies span from simple random algorithms, for homogeneous communities, to highly sophisticated ones, for less compact communities. We analyzed one year of data from 443 houses (Pecan St., Texas) to test the impact of our algorithms. This work provides many knobs to control a decentralized allocation of energy, under various scenarios with a different grid focus, and to reduce the overall expenses of buying energy from the central grid.

5.2 Future works proposals

The triplet of characterization-CPP-EAS that this work proposes is a novel approach to the new domain of intra-community energy allocation. There is space for research both in the domains of characterization and energy allocation.

Regarding characterization and prioritization, it would be interesting to consider also geographical data, which we were not able to obtain. With this type of data, grouping could be done more efficiently.

In addition, as a large SG community would consist of several smaller ones, its system model would be described as a distributed system of MG-communities (like Pecan St.) –with their own CC, consumers and prosumers. In that SG model, inter-community communication schemes on energy sharing would emerge, seeing every MG community as a separate entity, with a specific energy and socio-economical profile, and its own balance between supply and demand of energy. A study on the communication between CCs and on the way the excess of energy would be stored in them, in order to be distributed, would profit the Smart Grids domain. Since we expect that every household would be a prosumer in the near future, it would be interesting to evaluate the scaling potential of our EASs.

The deployment of our algorithms and policies on the industrial domain, where the energy profiling is different than the household communities, and where other RSEs are also used (the wind, geothermal, etc.), would offer interesting insights.

Further, plug-in hybrid electric vehicles (PHEV) should be integrated in our system model. Their mobility can improve the balance between supply and demand, if used properly.

Nowadays, in the era of cloud computing, large amounts of fine-grained data can be transferred to controllers via cloud servers. Allocation strategies which involve controllers located far from the MG communities should be studied, along with the types of the two-way communication that will emerge between the controllers and the households.

Nomenclature

Abbreviations

| | |
|-----|------------------------------------|
| MG | Micro-Grid |
| SG | Smart Grid |
| RSE | Renewable Sources of Energy |
| CG | Central Grid |
| CC | Central Controller |
| EM | Expectation-Maximization algorithm |
| CPP | Consumer Priority Policy |
| EAS | Energy Allocation Strategy |
| RoO | Relation of Order |
| RoW | Relation of Weight |
| WF | Water-Filling algorithm |

Symbols

| | |
|---------------|--|
| c | Number of consumers |
| p | Number of prosumers |
| \mathcal{C} | Group of consumers |
| \mathcal{P} | Group of prosumers |
| E_a | Energy needs of \mathcal{C} |
| E_g | Covered energy needs of \mathcal{C} |
| E_e | Excess of energy of \mathcal{P} |
| Con | Consumption of \mathcal{C} and \mathcal{P} |

| | |
|---|--|
| Gen | Generation of P |
| Att | Energy attribute, Con , Gen , E_a , E_e |
| t | Time of (i) single measurement of Con and Gen , and (ii) single computation of E_a and E_e |
| T | Time interval |
| t_{\max} | Number of consecutive t that constitute T |
| T_{\max} | Last T of interest, total number of T |
| $\text{Cl}_{\text{Att},u,i}$ | Membership of consumer i in cluster u for an Att |
| z | Number of time intervals with consecutive cluster transitions |
| $(\text{Cl}_{\text{Att},u}^{T-z} \rightarrow \dots \rightarrow \text{Cl}_{\text{Att},v}^T)_i$ | Consecutive transitions of consumer i , from Cl_u at $T - z$, to cluster Cl_v at T , for an Att |
| θ_m | membership ratio of a household, being in a particular cluster, with respect to total number of weeks |
| θ_t | ratio of number of transitions of a household from a particular cluster to another over all its transitions over the time intervals considered |
| λ | Level of deficiency coverage |
| TL | Time-Limit for RoO re-initialization |
| S | Groups of consumers inside C that present similarities regarding size |
| D | Groups of consumers inside C that present similarities regarding deficiency (clustering) |
| N | Subgroups inside C , created by S and D combinations |
| w | Weights imposed on C |
| x | Single unit of excess |
| H | Heights of service, ratio of E_a over w |

| | |
|------------------------|---|
| h | Additional served heights, ratio of $\mathbf{E_g}$ over w |
| C_{served} | Fully covered consumers over a T |
| $C_{\text{notServed}}$ | Consumers who accepted no excess of energy over a T |
| C_{unique} | Consumers served at least once over $T_b - T_a$, with $T_b, T_a \in [T_1, T_{\max}]$ |
| p_{act} | Active prosumers –who generated (excess of) energy– over a T |
| SR | Served ratio |
| PBR | Prosumer beneficialness ratio |
| UR | Uniqueness ratio |
| \mathbf{ER} | Energy ratio, ratio of $\mathbf{E_g}$ over $\mathbf{E_a}$ |
| \mathbf{ER}_w | Weighted \mathbf{ER} |
| \mathbf{SW} | Individual social welfare, \log_2 form of \mathbf{ER}_w |
| \mathbf{SW}_c | Social welfare of community (or group) of c households |

Energy allocation algorithms

Algorithm 1 Random Algorithm

i prosumer index
 P, C is set

On every time interval

Initialization phase:

- 1: $i = 1$
- 2: A consumer C_{r1} with deficiency E_a^{r1} is picked randomly by the CC

Main phase:

- 3: **while** $i \leq p$ **do**

Communication phase:

- 4: P_i sends info on its excess E_e^i to the CC
- 5: **if** $E_a^{r1} = 0$ **then**
- 6: C_{r1} is removed from C
- 7: **if** $\text{length}(C) = 0$ **then**
- 8: **Break**
- 9: **end if**
- 10: A new consumer C_{r2} is picked randomly
- 11: $r1 \leftarrow r2$
- 12: **end if**
- 13: C_{r1} sends info on its deficiency E_a^{r1} to the CC

Allocation phase:

- 14: P_i transfers energy to C_{r1} through the power line
 - 15: **if** $E_e^i > E_a^{r1}$ **then**
 - 16: $E_e^i \leftarrow E_e^i - E_a^{r1}$
 - 17: $E_a^{r1} = 0$
 - 18: **else**
 - 19: $E_a^{r1} \leftarrow E_a^{r1} - E_e^i$
 - 20: $i \leftarrow i + 1$
 - 21: **end if**
 - 22: **end while**
 - 23: C is reset
-

Algorithm 2 Greedy Algorithm

k consumer index, i prosumer index

C defined by the CPP (RoO)

P, λ are set

On every time interval

Initialization phase:

1: $i = 1, k = 1$

Main phase:

2: **while** $i \leq p$ **do**

Communication phase:

3: P_i sends info on its excess E_e^i to the CC

4: **if** $\lambda E_a^k = 0$ **then**

5: $k \leftarrow k + 1$

6: **if** $k > c$ **then**

7: **Break**

8: **end if**

9: **end if**

10: C_k sends info on its delimited deficiency λE_a^k to the CC

Allocation phase:

11: P_i transfers energy to C_k through the power line

12: **if** $E_e^i > \lambda E_a^k$ **then**

13: $E_e^i \leftarrow E_e^i - \lambda E_a^k$

14: $\lambda E_a^k = 0$

15: **else**

16: $\lambda E_a^k \leftarrow \lambda E_a^k - E_e^i$

17: $i \leftarrow i + 1$

18: **end if**

19: **end while**

Algorithm 3 Round-robin Algorithm

k consumer index, i prosumer index

Initialization phase:

- 1: Initial consumer list C_{ini} defined by the CPP (RoO)
- 2: $C \leftarrow C_{\text{ini}}$
- 3: P , λ , TL are set
- 4: $T = 1$

Main phase:

- 5: **while** $T \leq T_{\text{max}}$ **do**
- 6: **if** $(T \bmod \text{TL}) = 0$ **then**
- 7: $C \leftarrow C_{\text{ini}}$
- 8: **end if**
- 9: Perform **Greedy** Algorithm on P , C , λ for initialized indexes $i = 1, k = 1$
- 10: Since **Greedy** has finished, k defines the number of served consumers
- 11: *Consumer list C rotates k times to become C'
- 12: $C \leftarrow C'$, $T \leftarrow T + 1$
- 13: **end while**

* By rotating $k - 1$ times we favor the last served consumer-household of every time interval T because it is on the top of the service list for $T + 1$ while it was partially served on T . By rotating k times we are unfair to the last served consumer because it is pushed at the bottom of the service list while it was not fully served.

Algorithm 4 Weighted Algorithm

k consumer index, i prosumer index

At the beginning:

- 1: CC arranges \mathbf{C} into N consumers subgroups, $\mathbf{C} = \cup_{n=1}^N \mathbf{C}_n$, with $N = SD$
- 2: CC assigns weights $w_n \forall n \in [1, N]$ according to CPP (RoW)

On every time interval

Initialization phase:

- 3: CC amasses the individual prosumer excesses $\sum_{i=1}^p E_e^i$
- 4: CC computes the single unit of energy x by (3.2)

Communication phase:

- 5: Consumers \mathbf{C} send info on their deficiency \mathbf{E}_a to the CC
- 6: CC checks every consumer's k corresponding subgroup, $k \in \mathbf{C}_n \in \mathbf{C}$, and corresponding weight, $w_k \in \mathbf{w}_n \in \mathbf{w}$, $\forall k \in [1, c]$

Allocation phase:

- 7: $E_a^k \leftarrow E_a^k - xw_k$, $\forall k \in [1, c]$
-

Algorithm 5 Nash Algorithm

k consumer index, i prosumer index

At the beginning:

- 1: CC assigns weights $w_k \forall k \in [1, c]$ according to a CPP (RoW)

On every time interval

Initialization phase:

- 2: CC amasses the individual prosumer excesses, $\sum_{i=1}^p E_e^i$
- 3: Consumers \mathbf{C} send info on their deficiencies \mathbf{E}_a to the CC
- 4: CC defines the heights of service \mathbf{H} by (3.3)

Energy Allocation phase:

- 5: **while** $\sum_{k=1}^c H_k > 0$ **do**
 - 6: CC chooses non-zero minimum height of service, $\min(\mathbf{H})_{\text{nz}}$
 - 7: **if** $(\min(\mathbf{H})_{\text{nz}} \sum_{k=1}^c w_k) \leq \sum_{i=1}^p E_e^i$ **then**
 - 8: $E_a^k \leftarrow E_a^k - \min(\mathbf{H})_{\text{nz}} w_k, \quad \forall k \in [1, c]$
 - 9: $\sum_{i=1}^p E_e^i \leftarrow \sum_{i=1}^p E_e^i - (\min(\mathbf{H})_{\text{nz}} \sum_{k=1}^c w_k)$
 - 10: Consumer with $\min(\mathbf{H})_{\text{nz}}$ is fully served
 - 11: $w_{\min(\mathbf{H})_{\text{nz}}} = 0$
 - 12: $\mathbf{H} \leftarrow \mathbf{H} - \min(\mathbf{H})_{\text{nz}}$
 - 13: **else**
 - 14: $\min(\mathbf{H})_{\text{nz}} \leftarrow \frac{\sum_{i=1}^p E_e^i}{\sum_{k=1}^c w_k}$
 - 15: $E_a^k \leftarrow E_a^k - \min(\mathbf{H})_{\text{nz}} w_k, \quad \forall k \in [1, c]$
 - 16: **Break**
 - 17: **end if**
 - 18: **end while**
-

Algorithm 6 Water-Filling (WF) Algorithm

i prosumer index

j, l : corresponding indexes of the most and least prioritized consumer being served simultaneously

At the beginning:

- 1: CC assigns weights $w_k \forall k \in [1, c]$ according to a CPP (RoW)

On every time interval

Initialization phase:

- 2: CC amasses the individual prosumer excesses, $\sum_{i=1}^p E_e^i$
- 3: \mathbf{C} send information on their deficiencies \mathbf{E}_a to the CC
- 4: CC defines initial heights of service by (3.3) and forms them in ascending order, \mathbf{H}_{ini} , (RoO)
- 5: $j = 1, l = 1$
- 6: $\mathbf{H} \leftarrow \mathbf{H}_{ini}$

Energy Allocation phase:

- 7: **while** $j \leq c$ **do**
- 8: ** Perform **Nash** algorithm energy allocation phase on the following:

$$\left\{ \begin{array}{l} \text{group of } (l + 1 - j) \text{ consumers} \\ \text{with weights assigned in step 1} \\ \text{with } \sum_{i=1}^p E_e^i \\ \text{with additional heights } \mathbf{h} \text{ defined as follows} \\ \left\{ \begin{array}{ll} \text{if } l + 1 \leq c, & h_k = \begin{cases} H_{l+1} - H_k, & \text{if } H_{l+1} < 2H_{ini,k} \\ 2H_{ini,k} - H_k, & \text{otherwise} \end{cases} \\ \text{else } & h_k = 2H_{ini,k} - H_k \\ \text{for } & k : [j, l] \end{array} \right. \end{array} \right.$$

After Nash algorithm:

- 9: Total excess decreased, $\sum_{i=1}^p E_e^i$ updated
- 10: Individual deficiencies of $(l + 1 - j)$ households decreased or covered, E_a^k updated $\forall k \in [j, l]$
- 11: WF Heights of service updated
- 12: $a \leftarrow j$ ▷ for algorithmic reasons only
- 13: **for** $k : [a, l]$ **do**
- 14: $H_k \leftarrow H_k + h_k,$
- 15: **if** $H_k = 2H_{ini,k}$ **then**
- 16: $j \leftarrow j + 1$
- 17: **end if**
- 18: **end for**
- 19: **if** ($H_l = H_{l+1}$ or $H_l = 2H_{ini,l}$) and $l + 1 \leq c$ **then**
- 20: $l \leftarrow l + 1$
- 21: **end if**
- 22: **end while**

** If $\sum_{i=1}^p E_e^i = 0$ during Nash algorithm, the procedure stops, and the individual households of the group of $(l + 1 - j)$ that are served, update their deficiencies with the corresponding part of the last portion of excess that was left.

Algorithm 7 Pareto Algorithm

k consumer index, i prosumer index

On every time interval

Initialization phase:

- 1: CC amasses the individual prosumer excesses, $\sum_{i=1}^p E_e^i$
- 2: CC amasses the individual consumer needs, $\sum_{k=1}^c E_a^k$
- 3: CC defines the percentage coverage level (λ)
- 4: $\lambda = \frac{\sum_{i=1}^p E_e^i}{\sum_{k=1}^c E_a^k} < 1$

Energy Allocation phase:

- 5: $E_a^k \leftarrow E_a^k(1 - \lambda), \forall k \in [1, c]$
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