Wave transmission at vertical breakwaters

December 1997 Final Report

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December 1997

Preface

This MSc-thesis is the result of a study performed between January and November 1997. The work on the thesis was done at the Delft University of Technology, Faculty of Civil Engineering, in order to obtain the Master of Science degree. The report deals with the subject "Wave transmission at vertical breakwaters".

I would like to thank the members of my thesis committee for their advise, support, critical comment and never failing motivation:

- prof. drs. ir. J.K. Vrijling Del
 - Delft University of technology
- ir. W.H. Tutuarima

• dr. ir. L.H. Holthuijsen

- Delft University of technology
- dr. ir. J.W. van der Meer
- Delft Hydraulics Delft University of technology

Finally I would like to thank my family for the support, motivation and trust in me to start and finish this study, and all the others who I do not mention, but who made the last 6 years a great time.

Delft, December 1997 Kenrick Heijn

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Summary

Recently, interest in vertical breakwaters has grown. The development of ports from natural small harbours to artificial large harbours facing the outer sea has demanded the construction of breakwaters in rougher seas. The vertical breakwater could play a role of importance in this development. To obtain general guidelines for the design of vertical breakwaters, an European research project has been started, called PROVERBS-Mast III. This study is part of this project.

Breakwaters are constructed to provide a calm basin for ships and to protect harbour facilities. For ports open to rough seas, breakwaters play a key role in port operations. One of the endangers of harbour tranquillity is wave transmission. Waves hitting the breakwater transfer part of their energy into the harbour, whether by energy through the breakwater, or by energy transfer over the breakwater if the run-up of the waves exceeds the top level of the breakwater.

Goda (1969) has proposed relations describing wave transmission at vertical breakwaters, based on regular wave tests. Since then, little research work has been addressed to this subject. This report is an attempt to contribute to the enhancement of the understanding of wave transmission at vertical breakwaters.

Data of various caisson breakwater types are discussed and analysed. The sloping top caisson breakwater gives more wave transmission than conventional and horizontally composite breakwaters. For horizontally composite breakwaters, wave transmission is in general less than for conventional breakwaters. The difference in wave transmission between conventional, parapet and perforated breakwaters is not significantly large.

The applicability of the relations derived by Goda (1969) for irregular waves using the significant height, has been verified. These relations can also be used to describe wave transmission for various types of caisson breakwaters by adaptation of the coefficients. A relation between the coefficients and a parameter characterising the type of caisson breakwater could not be found.

Wave transmission is due to overtopping and transmission through the structure. An effort is made to describe wave transmission due to overtopping with the percentage of overtopping waves. A relation between wave transmission due to overtopping and the percentage of overtopping waves, which depends on the crest freeboard, has been derived. A relationship for wave transmission through the structure is also given. These relations are based on data of a conventional caisson breakwater, i.e. caisson placed on a rubble mound foundation. The results are discussed and applied to sloping top caisson breakwaters.

Finally, wave transmission has been studied in a theoretical approach to get a better insight in the process. The method as described in this report, however, consequently overestimates the wave transmission coefficient, probably due to the non-linearity of the phenomenon wave transmission.

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List of symbols

Symbol	Description	Dimension
B _b	Width of rubble berm, at toe of wall	m
B _c	Width of caisson	m
b _e	Equivalent covering width, see section 3.1	m
Cr	Coefficient of reflection	-
с	Wave celerity	m²/s
Cq	Group velocity	m²/s
Ď	Dissipation factor	
d	Water depth over berm in front of wall	m
d _c	Depth of structure in foundation	m
Ed	Absorbed or dissipated wave energy	
Ei	Incident wave energy	
Er	Reflected wave energy	
Et	Transmitted wave energy	
F	Energy flux	2
g	Gravitational acceleration	m/s²
H _{1/3}	Mean height of highest 1/3 of waves in a record	m
$H_{2\%}$	Wave height exceeded by 2% of waves in a record	m
H	Incident wave height	m
H _{m0}	Significant wave height from spectral analysis, defined $4\sqrt{m_0}$	m
Ho	Wave height at deep water	m
H _r	Reflected wave height	m
Hs	Significant wave height, average of highest one-third of wave heights	m
H _{si}	Significant incident wave height, average of highest one-third of wave heights	m
H _{st}	Significant wave transmitted height, average of highest one-third of wave heights	m
H,	Transmitted wave height	m
h _b	Height of berm above sea bed	m
h _c	Height of caisson	m
h _f	Exposed height on caisson / crown wall	m
h _r	Depth of rubble core beneath caisson to sea bed	m
hs	Water depth at toe of structure	m
h'	Water depth from base of caisson (d + d _c)	m
k	η _{max} /Η _i	Set
Kt	Total wave transmission coefficient	-
K _{to}	Transmission coefficient (transmission through the structure)	-
K _{tt}	Transmission coefficient (transmission by overtopping)	-
L	Wave length	m
m	Cotangent of bedslope	
m	Discharge coefficient for the flow over the weir	
m ₀	Zeroth moment of the wave energy density spectrum	
Now	Number of overtopping waves	
N _w	Number of waves	3
Q	Mean overtopping discharge, per unit length of structure	m³/s/m
Q*	Dimensionless overtopping discharge	-

List of symbols (continued)

Symbol	Description	Dimension
Qn .	Quantity of overtopping per wave and per unit length of structure	m ³ /m per
10		wave cycle
q	Discharge per unit width	m³/s/m
R _c	Crest freeboard, level of crest above still water level	m
Ru	Run-up level, relative to static water level	m
R_{us}	Run-up level of significant wave	m
$R_{u2\%}$	Run-up level exceeded by 2% of run-up crests	m
Sp	Deep water wave steepness, related to peak wave period = $2\pi H/gT_p^2$	-
T _m	Mean wave period	S
Tp	Peak wave period	S
t	Time	S
у	Overflow depth	m
Z	z-axis, vertical direction	m
Z_{max}	Maximum amplitude of waves	-
ß	Direction of wave propagation relative to normal to breakwater alignment	0
r v	Freeboard reduction factor	-
, η(t)	Surface elevation of waves over the breakwater measured from SWL	m
σ(x)	Standard deviation of x	
σ_i^2	Variance of the incident wave	
σ_t^2	Variance of the transmitted wave	
ξ	Breaker parameter	

1. Introduction

1.1 General

For proper handling of vessels within a harbour basin a certain degree of basin tranquillity is required. The tranquillity of the harbour can be endangered by three phenomena:

- Disturbance of tranquillity by waves entering the harbour mouth (combined effect of diffraction and refraction).
- Waves hitting the breakwater transfer part of their energy into the harbour, whether by energy through the breakwater, or by energy transfer over the breakwater if the run-up of the waves exceeds the top level of the breakwater.
- Wave generation by local wind. When the direction of a strong wind is parallel to the longer harbour axis wind waves of some importance can be generated.

The endanger of the harbour tranquillity is illustrated in Figure 1.1.



Figure 1.1 Endanger of harbour tranquillity

Breakwaters are constructed to provide a calm basin for ships and to protect harbour facilities. They are also sometimes used to protect the port area from the intrusion of littoral drift. In fact, for ports open to rough seas, breakwaters play a key role in port operations.

The most common type of breakwater in the world is the rubble mound breakwater. Rubble mound breakwaters have a rubble mound and an armour layer that usually consists of shape-designed concrete blocks. Recently there is again interest in vertical breakwaters. The development of ports from natural small harbours to artificial large harbours facing the outer sea has demanded the construction of breakwaters in more rough seas. The vertical breakwater could play a role of importance in this development.

To obtain general guidelines for the design of vertical breakwaters, an European research project has been started, called PROVERBS-Mast III. PROVERBS (<u>Probabilistic Design Tools for Vertical Breakwaters</u>) is a multi-national and multi-disciplinary research project funded by the European Union within the MAST III (Marine Science and Technology)-Programme for a period of three years (1996-1999). Therefore, a studygroup has been formed at the Delft University of Technology. This study has been done within this project.

1.2 Problem description

Economic considerations often indicate that the structural integrity of the breakwater shall be such that the structure is able to survive severe weather conditions without major damage. The functional requirements, however, do not always require that absolute tranquillity is maintained under such conditions. Since the volume of material involved in the structure and thereby its costs, is dependent on its height, it is worthwhile to consider the minimum crest level of the breakwater.

Therefore it is necessary to give a good prediction for the wave transmission. However, little research work had been addressed to this subject in the past, since most attention had been paid to wave forces and breakwater stability. It may be noted that these aspects are interrelated since the breakwater crest elevation influences the amount of wave force. Goda (1969) is the main reference for wave transmission at vertical breakwaters.

Relations describing wave transmission at vertical breakwaters were derived by Goda (1969), based on regular wave tests. According to Takahashi (1996), these relations are applicable to irregular waves using the significant wave height. However, a verification has not been given.

Recently, studies have been performed on wave transmission for various types of caisson breakwaters, such as sloping top caissons (Takahashi, 1996) and horizontally composite breakwaters (Tanimoto et al., 1987). However an overall view of caisson breakwaters has not been found in literature.

A conclusion is that to little information on wave transmission is available in the literature to allow accurate estimates to be made during design. A factor which makes the establishment of allowable limits for wave transmission even more difficult is the simultaneous presence of waves which penetrate through the harbour entrance. The resulting total wave height is not simply the sum of the wave height components! Even a sum based upon wave energy proves to be unreliable. Large scale model tests can provide insight into the problem for specific harbours.

Also, the process of wave transmission seems to be a 'black box'. In general the problem of wave transmission has been attacked experimentally. The experimental results are mostly presented by creating a graph of the transmission coefficient versus a relative freeboard.

1.3 Thesis objective

This report deals with wave transmission at vertical breakwaters. The objective for this thesis is to analyse and discuss data of various types of vertical breakwaters and to come to a general design formula which relates the wave transmission coefficient with the relative freeboard. Also the phenomenon wave transmission will be studied in a more theoretical approach to get a better insight in the process.

1.4 Outline of the report

The outline of this report is as follows: In chapter 2, the design of breakwaters in general is described. A review of literature of wave transmission is given in chapter 3. Chapter 4 analyses and discusses data for various types of caisson breakwaters. It is tried to verify the applicability of the equation of Goda(1969) for irregular waves and to adapt this equation to various caisson breakwater types. In chapter 5 it is tried to derive a relation between transmission due to overtopping and the percentage of overtopping waves. The relation between the percentage of overtopping waves and relative freeboard is also given. The results are discussed and applied to sloping top caisson breakwaters. In chapter 6 wave transmission has been studied in a more theoretical approach in order to get a better insight of the process. The results are compared with data set M 2090 and H2137 of DELFT HYDRAULICS. Finally the conclusions and recommendations as can be drawn from this study are given in chapter 7.

2. Design of breakwaters

The following section describes the governing parameters for the design of breakwaters. Wave parameters as well as structural parameters are given. The hydraulic response will also be discussed.

2.1 Governing parameters

2.1.1 Wave parameters

The wave conditions, which are important for design of breakwaters, are given by:

- H_{1/3} mean height of highest 1/3 of waves in a record;
- H_{m0} significant wave height from spectral analysis, defined $4\sqrt{m_0}$;
- H_s significant wave height;
- m₀ zeroth moment of the wave energy density spectrum;
- T_p peak wave period;
- β direction of wave propagation relative to normal to breakwater alignment.

The parameters can be extended by adding characters and without changing their general meaning. For example, the significant wave height can be varied by adding "i" representing the incident significant wave height $H_{\rm si}$.

Wave height

At deep water the wave height distribution can be described using a Rayleigh distribution. However, in shallow and depth limited water the highest waves break and in most cases the wave height distributions can no longer be described by the Rayleigh distribution. In those situations, the actual wave height distribution may be important to consider, or another characteristic value than the significant wave height. Characteristic values often used are the 2% wave height, $H_{2\%}$, and the $H_{1/10}$, being the average of the highest ten percent of the waves. An advantage of using H_s , of the Rayleigh distribution, in shallow water is that it is a safe approach. The truncation of the wave height exceedance curve due to breaking is not taken into account.

Wave steepness

The influence of the wave period is often described using the deep water wave length related to the wave height at the toe of the structure, resulting in a fictitious wave steepness:

$$s_p = \frac{2\pi H_s}{gT_p^2}$$
(2.1)

However, most breakwaters are made in shallow water. In fact, the wave length at the structure will differ from the deep water wave length.

2.1.2 Structural parameters

There are a number of parameters which characterise a vertical breakwater. A list of notations and symbols have been produced by PROVERBS. The parameters are outlined below.

- B_b width of rubble berm, at toe of wall;
- B_c width of caisson;
- d water depth over berm in front of wall;
- d_c depth of structure in foundation;
- h_b height of berm above sea bed;
- h_c height of caisson;
- h_f exposed height on caisson / crown wall;
- h_r depth of rubble core beneath caisson to sea bed;
- h_s water depth at toe of structure;
- h' water depth from base of caisson (d + d_c);
- m cotangent of bedslope;
- R_c crest freeboard, level of crest above still water level (SWL).

Figure 2.1 indicates the notations to define the cross section of the breakwater.



Figure 2.1 Definition of geometric parameters

2.2 Hydraulic response

When waves act on breakwaters, part of the incident wave energy is dissipated. Part of the remaining energy, however, is reflected and generates reflected waves in front of the breakwaters. The remaining is transmitted and yields waves transmitted behind the breakwaters. The wave reflection is sometimes a problem because it makes another agitation in front of the breakwaters. On the other hand, wave transmission is essentially important in the design of breakwaters because the most principal function of breakwaters is to prevent wave propagation behind them and so create a calm water area there.

The following sections will give a short description of the hydraulic response of a breakwater to the wave conditions. Wave run-up, wave overtopping, wave transmission and wave reflection are discussed.

2.2.1 Wave run-up

Wave action on a structure will cause the water surface to oscillate over a vertical range generally larger than the incident wave height. The maximum level reached in each wave is termed run-up, R_u, and defined relative to the still water level. The design run-up can be used as an indicator of possible overtopping or wave transmission.



Figure 2.2 Wave run-up level

The wave crest height of the second order approximation solution for finite amplitude standing wave proposed by Miche (1944) is given by:

$$\frac{\mathsf{R}_{\mathsf{u}}}{\mathsf{H}_{\mathsf{i}}} = 1 + \frac{\pi}{4} \frac{\mathsf{H}_{\mathsf{i}}}{\mathsf{L}} \left\{ 3 \operatorname{coth}^{3} \left(2\pi \frac{\mathsf{h}_{\mathsf{s}}}{\mathsf{L}} \right) + \operatorname{tanh} \left(2\pi \frac{\mathsf{h}_{\mathsf{s}}}{\mathsf{L}} \right) \right\}$$
(2.2)

where R_u is the wave run-up height from the still water level, h_s is the water depth at the toe of the vertical breakwater, H_i is the incident wave height at h_s , L is the incident wave length at h_s .

Under the action of random waves, run-up levels will vary from wave to wave. It is therefore meaningful to determine a characteristic measure of run-up, which is typical of the sea state. Two measures of run-up may be considered in accordance with the following definitions:

R_{us} : the significant run-up (analogous to significant wave height H_s), i.e. the average of the highest one third run-up levels reached;

 $R_{u2\%}$: the run-up level exceeded by only 2% of the waves.

Run-up is often given in a dimensionless form, R_{ux}/H_s , where the subscript x describes the level considered (for instance 2%). However, for a vertical wall not much is known about irregular wave run-up.

2.2.2 Wave overtopping

Wave overtopping is the discharge Q over the top of the breakwater as a result of waves exceeding the crest freeboard.



Figure 2.3 Wave overtopping

Many formulae which describe wave overtopping can be found in literature. All formulae are expressed as a function of some sort of dimensionless crest freeboard. An exponential relationship is assumed according to Franco et al (1994):

$$\frac{Q}{\sqrt{gH_s^3}} = Q^* = ae^{\frac{bR_c}{H_s}}$$
(2.3)

in which:

Q is the mean discharge expressed by m³/s per m a and b are experimental coefficients

Franco et al. (1994) found that for vertical-face breakwaters b = 4.3 and a = 0.192, see also section 3.3.

2.2.3 Wave transmission

Wave transmission is the phenomenon that wave energy will overtop and pass through the breakwater. In general, the wave transmission coefficient K_t is expressed by the ratio of the transmitted wave height (H_t) to the incident wave height (H_i):

$$K_{t} = \frac{H_{t}}{H_{i}}$$
(2.4)

The incident wave height is measured in front of the structure, excluding the effects of reflection. The transmitted wave height is measured behind the breakwater.



Figure 2.4 Wave transmission

However, the wave transmission coefficient K_t can also be expressed in terms of energy.

$$K_{t} = \sqrt{\frac{E_{t}}{E_{i}}}$$
(2.5)

Raichlen et al. (1992) implies that it is important to view wave transmission in more basic terms (see section 3.1.4).

The governing parameters related to transmission are: structural geometry, the crest freeboard, crest width, water depth and the hydraulic parameters: wave height and wave period.

In general, wave transmission is looked at with a graph of the wave transmission coefficient K_t in which the abscissa represents the crest freeboard to the incident wave height, R_c/H_i , because the relative freeboard R_c/H_i greatly controls the wave overtopping.

2.2.4 Wave reflection

All coastal structures reflect some proportion of the incident wave energy. This is often described by a reflection coefficient, C_r , defined in terms of the incident and reflected wave heights, H_i and H_r , respectively, or the total incident and reflected wave energies, E_i and E_r :

$$C_{r} = \frac{H_{r}}{H_{i}} = \sqrt{\frac{E_{r}}{E_{i}}}$$
(2.6)

When considering random waves, values of C_r may be defined using the significant incident and reflected wave heights as representative of the incident and reflected energies.

The reflection coefficient of vertical breakwaters is generally high, but less than 1.0 due to the effects of rubble mound foundation and wave overtopping. In particular, it is reduced considerably when breaking waves act on the breakwaters.

3. Review of literature

In this chapter a review of previous studies on wave transmission is given. In section 3.1 a review of wave transmission at vertical breakwaters is given. Section 3.2 deals with wave transmission at impermeable breakwaters with a slope. While wave transmission is mostly due to overtopping, a study of overtopping is also given (section 3.3).

3.1 Wave transmission at vertical breakwaters

In this section a review of previous studies with regard to wave transmission at vertical and composite breakwaters is given.

Because transmitted waves are caused by transmission through the structure and by overtopping, Tanimoto et al. (1987) proposed the following procedure to estimate the wave transmission coefficient

$$K_{t} = \sqrt{K_{tt}^{2} + K_{to}^{2}}$$
(3.1)

in which

 K_{tt} , K_{to} = transmission coefficients due to overtopping and transmission through the structure, respectively.

Because transmitted waves by the overtopping are generated by the dropping of the water mass, they have a complicated form with high frequency components. According to Tanimoto, not only the wave height but also the wave period of transmitted waves are different from those of incident waves in general. Also some hydraulic characteristics of transmitted irregular waves, such as the distributions of wave height and period, will change as they propagate for a long distance.

Wave transmission at vertical breakwaters is mainly the result of waves generated at the lee by the impact of the fall of the overtopping water mass. The governing factor is the crest freeboard R_c of the breakwater relative to the incident wave height H_i . The height of the rubble mound is a secondary factor affecting the wave transmission coefficient. Goda (1969) proposed the following relations of the transmission coefficient for vertical breakwaters, based on regular wave tests:

$$K_{t} = \frac{H_{t}}{H_{i}} = \begin{cases} \sqrt{0.25 \left[1 - \sin \frac{\pi}{2\alpha} \left(\frac{R_{c}}{H_{i}} + \beta\right)\right]^{2} + 0.01 \left(1 - \frac{h'}{h_{s}}\right)^{2}} \\ & \text{for } \beta - \alpha < \frac{R_{c}}{H_{i}} < \alpha - \beta \\ 0.1 \left(1 - \frac{h'}{h_{s}}\right) & \text{for } \frac{R_{c}}{H_{i}} \ge \alpha - \beta \end{cases}$$
(3.2)

where α = 2.2, and β given in Figure 3.1.

The term with the sinus component represents wave transmission due to overtopping and the term $(1-h'/h_s)$ represents wave transmission through the structure. The term h' is the distance

from the design water level to the bottom of the caisson. As can be seen from figure 3.1, β depends on the ratio d/h_s, which affects the wave overtopping. Note the difference between h' and d, see Figure 2.1.



According to Tanimoto the relations proposed by Goda are applicable to the transmission coefficient of the significant wave height of irregular waves, although these relations are based on regular wave tests.

Most breakwaters in Japan are designed with a relative crest height $R_c/H_s = 0.6$, where H_s is the design significant wave height mostly corresponding to a return period of 50 years. Then, the transmission coefficient is calculated by Goda's formula as about 0.2 for the typical conditions of $d/h_s = 0.6$ and $h'/h_s = 0.7$.

Figure 3.1 Nomograph for parameter β

Figure 3.2 is a graphical representation of Goda's formula. The width B_c of the upright section tested, covers a range $B_c/h_s = 0.8 - 1.1$, which represents standard geometry of vertical breakwaters. The portion of empirical curves in the range $R_c/H_i > 1.5$ represents the estimation of wave transmission through the rubble mound foundation.



Figure 3.2 Transmission coefficient for regular waves at vertical breakwater (Goda, 1969)

Also in Tanimoto et al. (1987), the horizontally composite breakwater is discussed. A horizontally composite breakwater is a composite breakwater covered with wave dissipating blocks, see Figure 3.3.



Figure 3.3

$$b_{e} = 0.5(b_{e} + b_{b})$$

Figure 3.4 shows experimental results of irregular wave tests, where the significant wave heights of transmitted waves were measured. The solid curves in the figure indicate the average relations of the experimental data for the different values of $b_e/L_{1/3}$, where b_e is the equivalent covering width defined by the following relation:





Figure 3.4 Wave transmission coefficient for horizontally composite breakwaters (Tanimoto et al., 1987)

As indicated by these average relations, the value of K_t ranges from 0.10 to 0.16 for a relative crest freeboard of 0.6, being less than corresponding values of ordinary vertical walls (K_t = 0.2, see previous page). It can be easily seen that the wave transmission coefficient K_t decreases as the relative covering width $b_e/L_{1/3}$ increases.

Also some other characteristics of irregular transmitted waves, which have been obtained for horizontally composite breakwater, are reported in Tanimoto.

Takahashi (1996) investigated the wave transmission coefficient K_t for six sloping top caisson breakwaters. A sketch of a sloping top caisson breakwater is given in Figure 3.5. A sloping top caisson breakwater is a caisson breakwater which has a superstructure that is sloped to reduce the wave forces. The sloping top caisson is very stable, yet overtopping is large, and the crest freeboard R_c must be higher than that of the ordinary vertical wall type to obtain the same wave transmission coefficient K_t.

Beside the structural parameters outlined in section 2.1.2. there are two more parameters needed to characterise the sloping top caisson breakwater, i.e. α and d_c. α is the angle of the slope and d_c is the distance between the still water level and the point where the slope starts. For a semi-submerged sloping top caisson the value of d_c is negative and for a standard sloping top caisson, having a slope starting from the sill water level, d_c = 0 m (type 1).



Figure 3.5 Sketch of cross section sloping top caisson breakwater

In table 3.1 the values of the parameters for each type are given. In annex 1 the crosssections of the six sloping top caissons are given.

Type	R.	dc	α	d	hs
1	32	0	45°	80	1044
2	32	8	45°	80	1044
3	32	-21.8	45°	80	1044
4	16	-21.8	45°	80	1044
5	16	-27.6	45°	80	1044
6	16	-21.8	56°	80	1044

 Table 3.1
 Values of parameters for the six types of sloping top caissons (unit : cm)

Figure 3.6 shows experimental results which were used to investigate the wave transmission coefficient K_t for the six sloping top caisson breakwaters.



Figure 3.6 Wave transmission coefficient K_t for sloping top caissons (Takahashi, 1996)

It is obvious that the sloping top caisson has a relatively large wave transmission coefficient K_t compared to ordinary vertical walls, and also, that the wave transmission coefficient K_t becomes large when either the angle of the slope α is small and/or the value of d_c is negative and large. Taking such factors into account, K_t for sloping top caissons can be formulated as follows, after Takahashi (1996):

$$K_{t} = \frac{H_{t}}{H_{i}} = \begin{cases} \sqrt{0.25 \left[1 - \sin \frac{\pi}{4.4} \left(\frac{R_{c}}{H_{1/3}} + \beta + \beta_{s} \right) \right]^{2} + 0.01 \left(1 - \frac{h'}{h_{s}} \right)^{2}} \\ ;\beta + \beta_{s} - 2.2 < \frac{R_{c}}{H_{1/3}} < 2.2 - \left(\beta + \beta_{s} \right) \end{cases} \\ 0.1 \left(1 - \frac{h'}{h_{s}} \right) \qquad ;\frac{R_{c}}{H_{1/3}} \ge 2.2 - \left(\beta + \beta_{s} \right) \\ \beta_{s} = -0.3 \sqrt{\frac{R_{c} - 2d_{c}}{H_{1/3} \tan \alpha}} \end{cases}$$
(3.4)

where β is obtained from Figure 3.1. The lines drawn in Figure 3.6 represent equation 3.4 calculated for each type. According to Takahashi (1996) the calculated results in Figure 3.6 using equation 3.4 show good agreement with the experimental results.

In Funakoshi et al. (1994), model tests were conducted to obtain the hydraulic characteristics of the breakwaters with rear parapets. А rear parapet breakwater is a breakwater where the parapet of an ordinary breakwater superstructure is placed at the rear of the breakwater body. By a front parapet breakwater the parapet is placed at the front. The parapet breakwater is outlined in Figure 3.7, after Funakoshi et al. (1994).

Figure 3.8 shows the wave transmission coefficient Kt. The abscissa represents the relative freeboard the at parapet to the incident significant wave height, R_c/H_{si}. In the figure, the
mark represents the rear parapet breakwater, and the O mark the front parapet breakwater. This figure indicates that the transmitted waves are generally higher for the rear parapet breakwaters than the front breakwaters. Therefore, the crest freeboard at the rear parapet must be increased by

breakwater.





Parapet breakwater Figure 3.7

about 10 to 20% to maintain the same transmission coefficient as the front parapet

3-5



Figure 3.8 Wave transmission coefficient for vertical breakwater (Funakoshi et al., 1994)

Figure 3.9 shows the wave transmission coefficient K_t for the case where the rear parapet of the improved shape is made on the vertical breakwater. In the figure, the × mark represents the improved shape, the \bullet mark the unimproved breakwater (here called "basic shape"). This figure indicates that the transmitted waves for the improved shape are reduced by 10 to 20% from the basic shape, and that the rear parapet breakwater with improved shape can be nearly maintained the same wave transmission coefficient as the front parapet breakwater.



Figure 3.9 Effect of parapet shape on transmission coefficients (Funakoshi et al., 1994)

The conclusion drawn by Funakoshi et al. (1994) is that the increase in the transmitted waves by the rear parapet breakwater, in comparison with the front parapet breakwater, can be minimized by improving the shape of the parapet.

For several types of caisson breakwaters hydraulic experiments were made to investigate among other things wave transmission. In Lee et al. (1994) the experimental results are reported. Figure 3.10 gives the sketches of the cross-sections of the four types of caisson breakwaters. For more details see Annex 1.



Figure 3.10 Sketches of the four types of tested breakwaters

In Figure 3.11, the wave transmission coefficients K_t are presented in terms of the ratio between the crest freeboard R_c and incident significant wave height H_{si} . For the caisson type breakwater with wave energy dissipating blocks in front of the caisson (type I) the transmitted waves are higher than for the other types. The difference between the perforated-wall breakwater (type II + III) and conventional caisson breakwater (type IV) is small, but the former gives slightly less transmitted waves. Furthermore in Lee et al. (1994), figures are given of the energy density spectra of the transmitted waves for breakwaters of Type I and II, the ratio of the maximum wave height to the significant wave height of the transmitted waves and the change in period of transmitted waves for breakwaters of type II and III.



Figure 3.11 Comparison of transmission coefficient for different types of breakwaters (Lee et al., 1994)

Wave transmission by overtopping is dealt with by Raichlen et al. (1992). An overtopping jet is formed above the breakwater. This jet plunges into the water on the shoreward side of the breakwater generating the transmitted waves. Perhaps because of this impact, the wave generation process appears to be quite violent. The relatively short wave length for the transmitted waves is evident, but is equally evident that there must be some remaining energy at the original wave period in the spectrum of the overtopped waves.



Figure 3.12 Sketch of breakwater sections Raichlen et al. (1992)

Two different definitions were used for the transmission coefficient by Raichlen et al. (1992) depending upon whether the structure was exposed to regular or irregular waves. Figure 3.12 gives a sketch of the breakwater section, see also Annex 1.

For regular waves, the transmission coefficient was defined as the ratio of the significant wave height of the transmitted waves to the wave height of the incident waves:

$$K_{t} = \frac{H_{st}}{H_{i}} = \frac{4.004 \,\sigma_{t}}{2\sqrt{2}\sigma_{i}} \cong \sqrt{2} \,\frac{\sigma_{t}}{\sigma_{i}} \tag{3.5}$$

in which

 σ_i^2 , σ_t^2 = variances of incident and transmitted, respectively.

Raichlen et al. (1992) defined the regular incident wave height as: $H_i = 2\sqrt{2}\sigma_i$. This will be discussed more fully later.

For irregular waves the expression used for the transmission coefficient was based on the ratio of the square root of the energy in the wave, i.e., the square root of the ratio of variances. If the wave height of the incident and the transmitted wave are distributed in the same manner, this corresponds to the ratio of the significant wave heights:

$$K_{t} = \frac{H_{st}}{H_{si}} = \frac{\sigma_{t}}{\sigma_{i}}$$
(3.6)

To demonstrate the relationship between the transmission of regular and irregular waves past section C and D, the variation of the transmission coefficient with relative freeboard is presented in Figure 3.13 for all data. However, to compare these waves it is necessary to redefine both the transmission coefficient and the relative freeboard in terms which are common to both types of waves. Since it is unrealistic to use the significant wave height to describe a regular wave train, the square root of the variance, i.e., the root-mean-square of the water surface time history, is used. Thus, the transmission coefficient is defined as the square root of the ratio of the transmitted wave energy to the incident wave energy, σ_t / σ_i , and the relative freeboard is defined as the ratio of the crest freeboard R_c to the square root of the variance of the incident waves σ_i .



Figure 3.13 The variation of the transmission coefficient with relative freeboard for regular waves and irregular waves, sections C and D (Raichlen et al., 1992)

The data in Figure 3.13 shows a reasonably well defined common variation of the transmission coefficient of both regular and irregular waves. The transmission coefficient corresponding to irregular waves is somewhat less than that for regular waves for the same relative freeboard.

3.2 Wave transmission at impermeable breakwaters with a slope

In the following sections a review on previous studies with regard to wave transmission at impermeable breakwaters with a slope are given. The regeneration of waves at the lee side of the breakwater is for this type of breakwater the same as for vertical breakwaters.

In Hamer and Hamer (1982), Laboratory experiments with regular waves were used to investigate wave transmission by overtopping for a smooth, impermeable breakwater with 1:4 slope. To define the wave transmission coefficient, Hamer and Hamer (1982) analysed the transmitted waves behind the breakwater. In spite of periodic incident waves, the transmitted waves showed irregularities. Due to the overtopping phenomena, higher harmonics arise. The wave height distribution of the transmitted waves was estimated by drawing histograms. In

case of little overtopping, a single-topped histogram was found. When overtopping increased $(H_{t1/2}/H_i \text{ is more than about 20\%})$ a double-topped distribution resulted, see Figure 3.14.





When overtopping increased, the higher harmonics are fairly clearly distinguished. One top represents the higher harmonics, the other top represents the first-order waves (basic waves). The mean value of the second top coincided with the significant wave height H_{t1/3}, taking all wave heights in account. Because of the fact that the significant wave height in cases of little overtopping also (nearly) coincides the top of the histogram concerned, this parameter was chosen to characterise the wave motion behind the breakwater.. To have a (dimensionless) wave transmission coefficient, Hamer and Hamer (1982) have chosen the parameter H_{t1/3}/H_i.





3-10

From the experiments a high correlation ($\gamma = 0.96$) has been found between the wave transmission coefficient H_{t1/3}/H_i and the normalized breakwater freeboard, when using Hunt's formula for the theoretical periodic wave run-up height. The following relation has been found:

$$\frac{H_{t1/3}}{H_i} = -0.69 \frac{R_c}{R_u} + 0.51$$
(3.7)

In Figure 3.15 the test results are given using the parameters R_c/R_u and $H_{t1/3}/H_{i}$. A disadvantage is the lack of data when the breakwater freeboard becomes zero.

In De Jong (1996), a basic transmission formula for rubble mound breakwaters has been derived, which includes the influences of crest width and slope angle. This formula has been adapted to impermeable structures with a smooth slope. The formula predicting K_t for impermeable structures is found to be, based on curve fitting:

$$K_{t} = -0.4 \frac{R_{c}}{H_{si}} + \left(\frac{B_{c}}{H_{si}}\right)^{-0.31} * \left(1 - e^{-0.5\xi}\right) * 0.80$$
(3.8)

in which

$$\xi = \frac{\tan \alpha}{\sqrt{s_p}} \tag{3.9}$$

For a vertical wall, the breaker parameter ξ is infinite and the influence of ξ has reached a constant value of 0.80. However at vertical walls there is much more wave reflection than for a quite steep slope, e.g. cot α = 1.5. Therefore the wave transmission should be lower (energy balance). According to De Jong (1996), a slightly declining slope for values of ξ going to infinite should be used and not the proposed constant value of 0.80.

3.3 Wave overtopping on vertical and composite breakwaters

Wave overtopping on vertical and composite breakwaters is dealt with by Franco et al. (1994). The results of an extensive model test investigation on the overtopping performance of caisson breakwaters have been analysed to produce a new comprehensive conceptual design method. A design formula and graph has been derived which relate the mean discharge $Q^* = Q/\sqrt{(g \cdot H_s^3)}$ with the relative freeboard R_c/H_s . The general equation is given by:

$$Q^* = a \cdot exp\left(-\frac{bR_c}{H_s}\right)$$
(3.10)

Franco et al. (1994) found that for vertical-face breakwaters b = 4.3 and a = 0.192, which is close to the one found by van der Meer and Janssen (1994) for sloping structures (a = 0.2); the value a = 0.2 was then kept constant for the successive regressions with different geometry's which generally showed a high correlation coefficient (Figure 3.16).

Then the influence of structural modifications with reference to the vertical-face breakwater can be described by suitable freeboard reduction factors (γ), which are the ratios between the reference value b = 4.3 and the various b coefficients fitted by equation 3.10 as given in Figure 3.16.






Figure 3.17 Wave overtopping on vertical and composite breakwaters (Franco et al., 1994)

All the data can be plotted together (Figure 3.17) after correction of the R_c/H_s values for each geometry with the corresponding γ , the general equation 3.10 thus becoming:

$$Q^{*} = 0.2 \cdot \exp\left(-\frac{4.3}{\gamma} \frac{R_{c}}{H_{s}}\right)$$
(3.11)

which can be effectively used for the preliminary design of vertical breakwaters. The reliability of the formula can be given by taking the coefficient 4.3 as a normally distributed stochastic variable with a standard deviation $\sigma = 0.3$.

4. Adaptation of equation Goda (1969) for various caisson breakwater types

Section 4.1 tries to verify the applicability of the equation of Goda (1969) for irregular waves. Furthermore in this chapter the various types of caisson breakwaters, as described in chapter 3, will be discussed and analyzed.

4.1 Applicability of equation of Goda (1969) for irregular waves

According to Takahashi (1994) the equation proposed by Goda (1969), based on regular wave tests, is applicable to the transmission coefficient of irregular waves with a significant wave height. However, the verification of the adaptability of the equation to irregular waves has not be found in literature.



Figure 4.1 Sketch of breakwater Raichlen et al. (1992)

Experiments were conducted with regular and irregular waves by Raichlen et al. (1992). In Figure 4.1 a sketch is given of the breakwater section. Figure 4.2 shows the transmission coefficient K_t versus the relative crest height R_c/H_i for regular and irregular waves. The transmitted wave height used for regular and irregular waves corresponds to the significant wave height. The incident wave height used for irregular waves corresponds also to the significant wave height. The wave transmission coefficient K_t for regular waves seems a bit too large around the value 0.3 of the relative freeboard R_c/H_i .

Wave transmission coefficient Kt versus Rc/Hi, Data Raichlen



Figure 4.2 Wave transmission coefficient K_t for irregular and regular waves, data Raichlen et al. (1992). Transmitted wave height used for regular waves corresponds to the significant wave height

From Figure 4.2 it can be seen that it is unrealistic to use the significant wave height to describe a regular wave train. However, the applicability of the equation of Goda (1969) for

irregular waves can be explained as follows. Instead of the significant transmitted wave height, Goda used the average value of the transmitted waves to describe the wave transmission coefficient. The relationship between the significant wave height and the average wave height, assuming a Rayleigh distribution, is $\overline{H} = 0.61 \cdot H_s$. In Figure 4.3 also a plot is made of the wave transmission coefficient K_t versus R_c/H_i for regular and irregular waves. However, for regular waves the average value of the transmitted waves is used instead of the significant wave height.



Wave transmission coefficient Kt versus Rc/Hi, Data Raichlen

Figure 4.3 Wave transmission coefficient K_t for irregular and regular waves, data Raichlen et al. (1992). Transmitted wave height used for regular waves corresponds to the average value of transmitted waves

The wave transmission coefficient corresponding to irregular waves is somewhat less than that for regular waves for the same relative freeboard. This difference could be due to the limited number of waves which can be used in the laboratory before waves reflected from the offshore face of the structure affect the incident wave, and the difference in the overtopping characteristics associated with a limited number of regular waves and the same number of irregular waves.

In Figure 4.5 the wave transmission coefficient K_t versus the relative freeboard R_c/H_s is given for the data of M 2090, 1985 (conventional breakwater) and Funakoshi, 1994 (front parapet breakwater). Also the equation of Goda (1969) is drawn in Figure 4.5. Figure 4.4 gives the sketches of the two breakwaters.



Figure 4.4 Sketch of the two breakwaters

The incident and transmitted wave height corresponds to the significant wave height. As can be seen from Figure 4.5 the equation of Goda (1969) gives good agreement with the data.

Eq. Goda applicable for irregular waves?



Figure 4.5 Applicability of equation of Goda (1969) for irregular waves

4.2 Discussion of various types of caisson breakwaters

The various types of caisson breakwaters described in chapter 3, are divided into three groups, i.e. sloping top caissons, horizontally composite and conventional + parapet + perforated breakwaters, to make it more convenient. Sketches of the various types of caisson breakwaters are outlined in Figure 4.7. Figure 4.6 shows the wave transmission coefficient K_t versus the relative freeboard R_c/H_s for various types of caisson breakwaters. It can be easily seen that the sloping top caisson has a relatively large transmission coefficient K_t compared to conventional and horizontally composite breakwaters, which is quite obvious.







Conventional + parapet + perforated breakwaters



Horizontally composite breakwaters



Figure 4.7 Sketches of the various types of caisson breakwaters, see also Annex 1

Also it can be seen from figure 4.6 that the value of K_t for horizontally composite breakwaters in general is less than corresponding values for conventional breakwaters. This could be due to the wave energy dissipation caused by the concrete blocks in front of the caisson section of the horizontally composite breakwater. However in Lee et al. (1994), see also chapter 3, the wave transmission of the caisson type breakwater with wave energy dissipating blocks in front of the caisson (type I) has a larger wave transmission coefficient K_t than the conventional type breakwater (type IV). This is not in accordance with Figure 4.6. A reason has not be found.

For each group, i.e. sloping top caissons, horizontally composite and conventional + parapet + perforated breakwaters, the different types are discussed in the following sections.

4.2.1 Sloping top caisson breakwaters

In Figure 4.7 the cross sections of the six types of sloping top caissons are given, see also chapter 3 (Takahashi et al., 1996). As can be seen from Figure 4.8, looking at the data for type 1 and 3, the wave transmission coefficient K_t is larger for the type where the slope of the caisson starts below the still water level. Also K_t is less for the caisson where the slope starts above the still water level (type 2). K_t becomes also large when the angle of the slope becomes small (type 4 and 6). It is interesting to look at type 3 and 4. For both types the angle of the slope is the same and also the point where the slope of the caisson starts (d_c = -21.8cm). However the crest freeboard R_c is 32cm for type 3 and 16cm for type 4. The influence of the crest freeboard R_c is taken into account with the dimensionless parameter R_c/H_i. So it should be expected that the wave transmission coefficient K_t is the same for both types. However, for large values of the relative freeboard R_c/H_i type 3 has a larger wave transmission coefficient K_t than type 4. It can be concluded that the angle of the slope, the point where the slope starts (d_c) in combination with the crest freeboard R_c, especially for large values of R_c/H_i, have an important influence on wave transmission for sloping top caissons.



Data sloping top caissons

Figure 4.8 Data sloping top caissons

Also can be seen from Figure 4.8 that the wave transmission coefficient K_t is around 0.10 - 0.15 for a relative crest height R_c/H_i of 1.7, which is quite large (especially type 3). So it is

interesting to look at wave transmission for the region $2 < \frac{R_c}{H_i} < 3$. However there is no data

available for this region.

4.2.2 Horizontally composite breakwaters

In Figure 4.7 the sketches are given of the cross sections of the horizontally breakwaters. All the breakwaters have concrete blocks in front of the caisson. Figure 4.9 shows the wave transmission coefficient K_t versus the relative freeboard R_c/H_s for this type of breakwater. The data of Tanimoto et al. (1987) is shown for the different values of the relative covering $b_e/L_{1/3}$, see also chapter 3. It can be easily seen that the wave transmission coefficient K_t decreases as the relative covering $b_e/L_{1/3}$ increases. For the data of Lee et al. (1994) and Raichlen et al. (1992) the relative covering $b_e/L_{1/3}$ is not known.



Data horizontally composite breakwaters

Figure 4.9 Data horizontally composite breakwaters

The difference between the two sections of Raichlen et al. (1992) is negligible. The value of wave transmission coefficient K_t for the sections of Raichlen et al. (1992) is less than corresponding values for the sections of Tanimoto et al. (1987). The less wave transmission could be due to the height of the caisson which is larger than the covering blocks in front of the caisson.

As can be seen from Figure 4.9 the wave transmission for Lee et al. (1994), type I, is rather large. This could be due to the sloping face of the caisson, see Figure 4.7. However it is a bit odd that this type of breakwater is the only horizontally breakwater which has a larger wave transmission than the conventional breakwater.

4.2.3 Conventional, parapet and perforated breakwaters

In Figure 4.10 the wave transmission coefficient K_t versus the relative freeboard R_c/H_s is plotted for conventional, parapet and perforated breakwaters. The differences in wave

transmission between the various types are not large. The conventional breakwater of Lee et al. (1994), type IV has a relatively large wave transmission coefficient K_t compared to the conventional breakwater of M 2090 (1985). Also the perforated breakwaters, type II and III of Lee et al. (1994) have large wave transmission coefficient compared to the other breakwaters, especially considered that for a perforated breakwater the wave transmission coefficient K_t should be reduced due to the wave absorbing behavior of this type of breakwater. It seems that for each type of the breakwaters of lee et al. (1994) the value of the wave transmission coefficient K_t is a bit too large, see also section 4.2.2.



Data conventional + parapet + perforated breakwaters

Figure 4.10 Data conventional, parapet and perforated breakwaters

The wave transmission coefficient K_t of the other types, M 2090 (1985) and Funakoshi et al. (1994), is what could be expected. For Funakoshi et al. (1994) see chapter 3.

The data of the UoN test (University of Naples) are the results of experiments for wave transmission (only) over the breakwater (conventional type) in the range covered by

 $1 \leq \frac{R_c}{H_i} \leq 3$. The transmitted wave height is about 1-2% of the incident wave height and not

zero as seems suggested by Goga (1969). The effect of wave transmission through the rubble mound of a breakwater in that range have a much stronger influence than wave transmission over the breakwater.

4.3 Derivation of α and β in equation of Goda (1969) for various types of caisson breakwaters

In this section for each type of breakwater given in Figure 4.7, the value of α and β will be derived. The aim of this section is to find any relation between the type of breakwater and the value of α and β . The equation of Goda (1969) is now written as follows:



Figure 4.11 Relation between coefficient β_s and the parameter (R_c + d_c)/d



Figure 4.12 Coefficient α_x versus coefficient β_x

$$K_{t} = \begin{cases} \sqrt{0.25 \left[1 - \sin \frac{\pi}{2(\alpha + \alpha_{x})} \left(\frac{R_{c}}{H_{s}} + \beta + \beta_{x} \right) \right]^{2} + 0.01 \left(1 - \left(\frac{h'}{h_{s}} + \gamma_{x} \right) \right)^{2}} \\ ;\beta + \beta_{s} - (\alpha + \alpha_{x}) < \frac{R_{c}}{H_{s}} < \alpha + \alpha_{x} - (\beta + \beta_{x}) \end{cases}$$

$$(4.1)$$

$$0.1 \left(1 - \left(\frac{h'}{h_{s}} + \gamma_{x} \right) \right) \quad ;\frac{R_{c}}{H_{s}} \ge \alpha + \alpha_{x} - (\beta + \beta_{x})$$

where $\alpha = 2.2$ and β is obtained using Figure 3.1. The coefficients $\alpha_{x_1} \beta_{x_2}$, and γ_x depend on the type of breakwater. The coefficient γ_x has been introduced because for horizontally composite breakwaters the transmission coefficient for large values of the relative freeboard R_c/H_s is larger than the predicted value by $0.1 \cdot (1-h'/h_s)$.

Equation 4.1 has been fitted with the least square method to each data set. The results are given in table 4.1.

Type of breakwater	αχ	βχ	γ _x	Region of validity, R _c /H _s
Sloping top caissons	αs	βs	γs	
Takahashi, type 1	3.1	1.05	0	0.66 - 1.80
Takahashi, type 2	3.4	1.33	0	0.64 - 1.86
Takahashi, type 3	2.6	0.33	0	0.64 - 1.74
Takahashi, type 4	0.8	-0.19	0	0.42 - 1.66
Takahashi, type 5	0.7	-0.28	0	0.42 - 1.66
Takahashi, type 6	1.0	0.01	0	0.32 - 1.79
Horizontally composite breakwaters	ahc	β _{hc}	γhc	
Tanimoto, b _e /L _{1/3} = 0.05-0.07	-0.4	-0.03	-0.3	0 - 0.82
Tanimoto, b _e /L _{1/3} = 0.07-0.10	0	0.16	-0.3	0 - 2.40
Tanimoto, b _e /L _{1/3} = 0.10-0.15	-0.2	0.23	-0.08	0 - 2.64
Raichlen, section C + D	-0.8	-0.01	0	0.26 - 0.71
Lee, hor. composite (type I)	-0.6	-0.57	0	0.95 – 1.55
Conv. + parapet + perforated breakwaters	αc	βc	Ϋ́c	
Lee, perforated (type II)	-0.3	-0.22	0	0.95 - 1.55
Lee, perforated (type III)	-0.4	-0.31	0	0.95 - 1.55
Lee, conventional (type IV)	-0.1	-0.14	0	0.95 – 1.55
Funakoshi, front parapet	0.2	0.13	0	0.61 - 1.19
Funakoshi, rear parapet (basic shape)	0.2	0.05	0	0.61 - 1.19
Funakoshi, rear parapet (improved shape)	-0.6	-0.25	0	0.59 – 1.19
M 2090, conventional	-0.9	-0.34	0	0.30 - 1.66
Table 4.1 Table for coef	ficients α_{x} ,	β_x , and γ_x	· · · ·	

The wave transmission coefficient K_t can now be calculated with the values of α_x , β_x , and γ_x as given in table 4.1. In Annex 3, Figure A3.1 till A3.18, the calculated results using equation 4.1 are plotted with the experimental results.

For the sloping top caisson breakwater a relation has been found between coefficient β_s and the parameter ($R_c + d_c$)/d. This relation is shown in Figure 4.11. Figure 4.12 shows coefficient α_x versus coefficient β_x . For the other types no relations could be found.

A graph of the calculated versus measured wave transmission coefficient K_{t} is given in Figure 4.13.

The standard deviation, σ , of the scatter around the line $K_{t_calculated} = K_{t_measured}$ is a criterion for the reliability of the equation. The scatter has been described by a Normal distribution with an average of zero and a certain standard deviation. A σ of 0.016 is obtained, which means a 90% confidence level of $K_t \pm 0.027$.



Figure 4.13 Calculated versus measured wave transmission coefficient

The wave transmission coefficient K_t is calculated with equation 4.1 using the values of the coefficients as given in table 4.1. It should be noted that these values are determined for a specific type of breakwater and not valid for other breakwater types.

5. Development of a new model based on percentage overtopping waves

In this chapter it is tried to describe wave transmission due to overtopping with the percentage of overtopping waves. More overtopping waves will give more wave transmission. The data set of M 2090 contains besides the wave transmission coefficient also the percentage of overtopping waves. A relation between wave transmission due to overtopping and the percentage of overtopping waves is derived and also a relationship for the wave transmission through the structure is given. The data set H 2137 is used to validate the formula derived from data set M 2090. In section 5.5 the coefficients in this formula have been modified to the sloping top caisson.

5.1 Description of model tests

DELFT HYDRAULICS has carried out various model tests where overtopping and transmission were measured, i.e. report no. M 2090 (1985) and report no. H 2137 (1995). In Annex 1 the various cross sections of the breakwaters are given. Annex 2 gives the various test data.

Report no. M 2090 (1985)

Most of the tests were performed on a rubble mound breakwater and part of the tests, tests 12 till 18, were performed on a caisson breakwater. For tests 12 till 18 a complete caisson was placed on a berm at MSL–5.0 m with a foreshore depth of MSL–6.0 m (or –10.0 m, test 12), see Figure A1.1. A wave gauge was placed on top of the caisson to determine the number of overtopping waves. Waves generated by overtopping were measured at the harbour side of the caisson. Waves were generated at MSL–13 m and propagated to the breakwater-sections along the constructed foreshore. In Figure A1.1 the foreshore geometry is given.

The results for overtopping and transmission are shown in Figure 5.1. Test 13, 14 and 16 were performed with the annual storm wave height of 1.6 m. The percentage of overtopping waves was 5% for tests 13 and 14 and 15% for the lower crest height. The transmitted wave height for all these three tests was less than 0.1 m. Test 15 (low water level) and test 18 (clients design) were run for the design wave height of the caisson. Generated wave heights were 0.13 m for the low water level and 0.32 m for test 18. The tropical storm gave 85% overtopping waves and a wave height at the harbour side of 1.35 m. Compared with the rubble mound this wave height is a little bit higher (test 6, step 6, 1.05 m).



5-1



% overtopping versus Rc/Hmoi Data M 2090, Rubble mound breakwater

Figure 5.2 Correlation between the percentage of overtopping waves and the relative freeboard, rubble mound breakwater (Data M 2090)



Figure 5.3 Correlation between the percentage of overtopping waves and the relative freeboard, caisson breakwater (Data M 2090)

For the annual conditions the generated wave height at the harbour side is smaller for the caisson than for the rubble mound. Test 12 give a transmitted wave height of 0.34 m and 40% overtopping waves.

Report no. H 2137 (1995)

DELFT HYDRAULICS has carried out model tests on a shifted caisson breakwater for a site specific location. For four tests wave transmission was measured and for three tests wave overtopping. In Figure A1.2 the cross section of the breakwater is given.

Overtopping was measured at two levels, the total overtopping over the crest-wall, and at the lower level of the wall at the lee-side of the service road on top of the breakwater. Part of the overtopping water flowed away through drain pipes and did not overtop the wall at the lee-side of the service road. With a deep water wave of $H_s = 4$ m (P001 and P002), the total overtopping was between 5 and 10 l/s/m, of which only 2 to 4 l/s/m overtopped the wall of the service road. With a deep water wave of $H_s = 2$ m (P003), the overtopping was negligible (q < 0.01 l/s/m).

The wave transmission in the harbour was mainly measured at some 350 m from the breakwater head at a place that was protected from parasitic waves (e.g. diffraction at the breakwater head. With an incident wave of $H_s = 4$ and 2 m, the wave transmission at this location was about 3% (P002 and P003) of the deep water wave. With the deep water design wave of $H_s = 8.7$ m the wave transmission increased to 4% ($H_s = 0.26$ to 0.35 m).

5.2 Relation between the percentage of overtopping waves and relative freeboard

Overtopping events occur unevenly both in time and amount, often just a few waves overtopping among the thousands. Franco et al. (1994) gives a relationship between the percentage of overtopping waves (N_{ow}/N_w) and the relative freeboard R_c/H_s . The percentage of overtopping waves N_{ow}/N_w is assumed to be Rayleigh distributed and can be expressed by the following equation, Franco et al. (1994):

$$\frac{N_{ow}}{N_{w}} = \exp\left(\frac{1}{k}\frac{R_{c}}{H_{s}}\right)^{2}$$
(5.1)

where k = 0.91 for caisson breakwaters, Franco et al. (1994).

In Figure 5.2 the percentage of overtopping waves versus the relative freeboard are plotted for rubble mound breakwaters (data M 2090). The data of rubble mound breakwaters are plotted to see if there is any influence of wave conditions or geometry of breakwater. The influence of wave conditions and geometry of breakwater could not be seen from data of caisson breakwater because of the few data points (only 7 measurements) available. For the caisson breakwater, the percentage of overtopping waves versus the relative freeboard are plotted in Figure 5.3.

As can be seen from Figure 5.2 the geometry of the breakwater and also wave conditions has influence on the percentage of overtopping waves. A lower crest height, test 5, gives less overtopping waves while a low water level gives more overtopping waves (test 3). Also the slope and the permeability of the rubble mound breakwater has influence on the percentage of overtopping waves. Also it can be seen that a longer wave period gives more overtopping

waves. This can be explained as follows. A longer wave period means a lower wave steepness, which will increase the run-up and therefore the overtopping rate.

Looking at Figures 5.2 and 5.3, the tendency of low water level (test 3 and 15) and low crest height (test 5 and 16) with regard to clients design (test 1 and 12-13, 17-18) is the same for both rubble mound and caisson breakwaters. For caisson breakwaters, a long wave period seems to have no influence on the percentage of overtopping waves, as far as one can conclude from one data point!

The percentage of overtopping waves N_{ow}/N_w is now assumed to be Weibull distributed, and not Rayleigh distributed as assumed by Franco et al. (1994). The Weibull distribution of N_{ow}/N_w can be expressed as follows:

$$\frac{N_{ow}}{N_{w}} = exp - \left(\frac{1}{k}\frac{R_{c}}{H_{s}}\right)^{n}$$
(5.2)

Equation 5.2 has been fitted with the least square method to all the data of rubble mound breakwater and to the caisson breakwater, test 12-13, 17-18 (Client design and tropical storm). The value n varies between 2.7 and 3.9 and k between 0.5 and 0.98 for the rubble mound breakwater, see table 5.1. For the caisson breakwater, test 12-13, 17-18, the value k = 0.73 and n = 2.0. From table 5.1 it can be seen that value n is around 3 for the rubble mound breakwater with the same geometry (see cross-section in Figure A1.1), except for test 2 (long wave period). A steeper slope (test 4) gives a lower value of n, i.e. n = 2.7, and a more permeable structure gives a larger value of n (test 6, n is 3.5). The tendency of test 2 and 6 are the same which is not so strange while waves with a lower wave steepness propagate much easier through the structure and waves also propagate easier to a more open structure. It is concluded that the value n is constant for an impermeable structure. A value of n = 2.0 is found for the caisson for clients design and is assumed constant. With n = 2.0 the value of k has been derived for the different situations, see also table 5.1.

Type breakwater	Test	Description	k	n
	1a – e	Clients design	0.60	3.0
	2a – e	Long wave period	0.79	3.9
Rubble	3a – e	Low water level	0.98	2.9
mound	4a – e	Slope 1 : 1.5	0.68	2.7
	5a – e	Low crest height	0.50	2.9
	6a – f	Open core	0.76	3.5
	12-13, 17-18	Clients design, tropical storm	0.73	2.0
Caisson	14	Long wave period	0.74	2.0
	15	Low water level	1.12	2.0
n sen en mangement sen en sen sen sen sen A legis chulture, statistice en sen sen sen sen sen sen sen sen sen	16	Low crest height	0.64	2.0

Table	5.1	Values	of k and n

The difference in values of k related to the k value of clients design is the same for rubble mound and caisson breakwaters. A low water level gives a larger value of k and a lower crest height gives a lower value of k. A longer wave period has no influence on the caisson breakwater. It seems that the value k depends on the crest freeboard R_c and the water depth d over berm in front of the breakwater. A graph is made where k is plotted versus the crest freeboard R_c divided by the water depth d. for both caisson and rubble mound breakwaters. As can be seen from Figure 5.4 the value k increases as R_c/d increases.

The value of k of 0.73 (clients design) is less than the value of 0.91 found by Franco et al. (1994), equation 5.1. This could be due to range of validity for the relative freeboard R_c/H_s of the data set of Franco et al. (1994), i.e. $R_c/H_s > 1$. Also the range of k is quite large, from k =

0.64 for a low crest height till k = 1.12 for a low water level. The assumption that the percentage of overtopping waves is Rayleigh distributed (n = 2.0) is only correct for caisson breakwaters.



Influence of Rc/d on k

Figure 5.4 Influence of R_c/d on the value k

More data is recommended to verify the values of k and n. The influence of a low water level and a low crest height should be more thoroughly investigated.

The relation between the percentage of overtopping waves for caisson breakwaters as given by equation 5.2 (n=2) can also be derived theoretically. Under the assumption that the irregular incident waves follow a Rayleigh distribution and overtopping will take place when the wave height H_i is larger than the crest freeboard R_c, the percentage (probability) of overtopping waves N_{ow}/N_w is expressed by the following equation:

$$\frac{N_{ow}}{N_{w}} = e^{-2\left(\frac{R_{c}}{H_{s}}\right)^{2}}$$
(5.3)

With n = 2 and k = $1/\sqrt{2} \approx 0.71$ equation 5.2 will give equation 5.3. As can be seen from table 5.1, the values are of the same order of magnitude.

5.3 Relation between transmission and percentage of overtopping waves

Transmitted waves are caused by wave overtopping and transmission through the breakwater. The transmission coefficients by both causes are respectively denoted as K_{to} and K_{tt} , with the total transmission coefficient K_t being expressed as, after Tanimoto (1987):

$$\mathbf{K}_{\mathrm{t}} = \sqrt{\mathbf{K}_{\mathrm{to}}^{2} + \mathbf{K}_{\mathrm{tt}}^{2}} \tag{5.4}$$

When there are no waves overtopping the breakwater, $N_{ow}/N_w = 0$, transmission is only due to transmission through the breakwater. Creating a graph of the wave transmission coefficient caused by overtopping waves K_{to} versus the percentage of overtopping waves N_{ow}/N_w , a linear regression line y = ax + b should have a value for parameter b of zero. K_{to} is now approximated by the following relation:

$$K_{to} = \alpha \cdot \frac{N_{ow}}{N_{w}}$$
(5.5)

where α has to be derived from measurements.

From the total amount of overtopping waves a certain amount will overtop the breakwater to fall into the water at the lee side to generate waves. The width of the breakwater as well as the roughness or any obstacle on top of the breakwater will have influence on this process. These factors are represented in the parameter α , see equation 5.5.

Cause the total transmission coefficient K_t has been measured, the wave transmission coefficient caused by transmission through the structure K_{tt} has to be known to be able to calculate the wave transmission coefficient caused by overtopping K_{to} . According to Goda (1969) wave transmission through the structure is strongly influenced by the rubble mound height. The wave transmission coefficient caused by transmission through the structure K_{tt} is expressed as follows:

$$K_{tt} = \beta \cdot \left(1 - \frac{h'}{h_s} \right)$$
(5.6)

The height and permeability of the rubble mound will have influence on transmission through the structure. These factors are represented in the parameter β , see equation 5.6.



Wave transmission caused by overtopping versus % overtopping

Figure 5.5 Wave transmission coefficient K_{to} versus percentage of overtopping waves

When β is a known value, the transmission coefficient K_{to}, transmission caused by overtopping, can be calculated with equation 5.5. The values of K_{to} can be plotted versus the percentage of overtopping waves and a linear regression analysis can be performed. β is now chosen in such a way that the regression line fulfil equation 5.5. This procedure has be done with the data of M 2090 for the caisson breakwater. A value of 0.15 for β has been found which gives a mean value of 0.35 for α . Parameter α can be assumed normally distributed with a variation coefficient of 10% for α . The linear regression line with the mean value of α is shown in Figure 5.5.

More data points are recommended to verify the relation between the transmission coefficient caused by overtopping K_{to} and the percentage of overtopping waves. Because there is only one data point for large values of percentage of overtopping waves ($N_{ow}/N_w > 60 - 70\%$), a linear regression analysis should be handle with care.

Also can be said that there are data points needed of only wave transmission through the structure to verify and/or approve the relation of the wave transmission coefficient K_{tt} as given in equation 5.5.

5.4 Discussion

Wave transmission is due to overtopping and transmission through the structure. In this chapter the following expression is found for the wave transmission coefficient K_t :

$$K_{t} = \sqrt{\left[\alpha \cdot \exp\left(-\frac{1}{k} \frac{R_{c}}{H_{s}}\right)^{n}\right]^{2} + \left[0.15 \cdot \left(1 - \frac{h'}{h_{s}}\right)\right]^{2}}$$
(5.7)

where for conventional type of breakwater α = 0.35 and n = 2. The value k depends on the crest freeboard and the water depth over the berm in front of the breakwater, see Figure 5.4.



Wave transmission Typical conditions: d/hs = 0.6 and h'/hs = 0.7



Figure 5.6 shows the wave transmission coefficient K_t for vertical wall breakwaters for the typical conditions $d/h_s = 0.6$ and $h'/h_s = 0.7$ using equation 5.3. Also the equation proposed by Goda (1969) is plotted.

As can be seen from Figure 5.6 the value k, and therefore R_c/d , has a large influence on the wave transmission coefficient K_t . So, a good prediction of the value k is necessary. Because the value k proposed in this paper is based on a small data set, more data on caisson breakwaters are needed to allow for a good prediction of the value k and thus the wave transmission.

To allow for some information regarding the reliability of the derived formula, the standard deviation of the difference between measured and calculated wave transmission should be known. It is assumed that the scatter around the line $K_{t_measured} = K_{t_calculated}$ can be described by a Normal distribution with an average of zero and a certain standard deviation. A σ of 0.015 is obtained, which means a 90% confidence level of $K_t \pm 0.025$. Figure 5.7 shows the measured versus the calculated wave transmission coefficient. The approach predicts wave transmission for H 2137 satisfactory.



Measured vs. calculated wave transmission Data M 2090 and H 2137

Figure 5.7 Measured versus calculated wave transmission coefficient for conventional breakwater

The approach as described above can be applied to other types of vertical breakwaters. The approach for wave transmission through the structure has not to be modified, while for wave transmission due to overtopping the coefficients k and n in equation 5.2 and α in equation 5.5 has to be derived for each type of breakwater.

In the following section it is tried to derive the coefficients k, n and α for sloping top caisson breakwaters. The coefficients k, n and α have not been derived for the other types of breakwaters because the data set of these breakwaters contains only the wave transmission coefficient and the relative freeboard, and not the geometry of the breakwater.

5.5 Derivation of coefficients k, n and α for sloping top caissons

The cross sections of the six types of sloping top caissons are given in Annex 1, see also chapter 3 (Takahashi et al., 1996). For each type of sloping top caisson the coefficients k, n and α have been derived. It is assumed that the coefficient α in equation 5.5 only depends on the angle of the slope and therefore constant for type 1 – 5. Equation 5.7 has been fitted with the least square method to each data set. The results are given in table 5.2.

Туре	α	k	n	Region of validity, R _c /H _s
1	0.47	0.95	0.74	0.66 - 1.80
2	0.47	0.83	0.71	0.64 - 1.86
3	0.47	1.47	0.92	0.64 - 1.74
4	0.47	1.16	1.27	0.42 - 1.66
5	0.47	1.22	1.36	0.42 - 1.66
6	0.43	1.12	1.18	0.32 - 1.79

 Table 5.2
 Coefficients α, k and n for sloping top caissons

The coefficient α is assumed to be dependent on the angle of the slope. For type 1 – 5 the angle is 45°, type 6 has an angle of 56° and for conventional type 90°. In Figure 5.8 the relation is given between the coefficient α and the angle of the slope.



Coefficient alpha versus the angle of the slope

Figure 5.8 Relation between coefficient α and the angle of the slope

For the conventional type of breakwater coefficient k depends on the parameter R_c/d . For the sloping top caisson coefficient k depends on the parameter ($R_c - d_c$)/d, see Figure 5.9.

A constant value of coefficient n has been assumed for the conventional type of breakwater, i.e. n = 2. For the sloping top caisson, different values of n has been found. It has been found that the parameter ($R_c + d_c$)/d has influence on the coefficient n. In Figure 5.10 coefficient n has been plotted versus parameter ($R_c + d_c$)/d.

More experiments are needed to verify the values of the coefficients k, n and α .



Influence of (Rc-dc)/d on k

Figure 5.9 Relation between k and $(R_c - d_c)/d$

Influence of (Rc+dc)/d on k



 $\label{eq:Figure 5.10} Figure \ 5.10 \qquad \mbox{Relation between coefficient n and } (R_c + d_c)/d$

Figure 5.11 shows the measured versus the calculated wave transmission coefficient. The scatter around the line $K_{t_measured} = K_{t_calculated}$ can be described by a Normal distribution with an average of zero and a certain standard deviation. A σ of 0.018 is obtained, which means a 90% confidence level of $K_t \pm 0.030$.



Measured vs. calculated Kt

 Figure 5.11
 Measured versus calculated wave transmission coefficient for sloping top caisson breakwater

The wave transmission coefficient K_t is calculated with equation 5.7 using the values of the coefficients as given in table 5.2.

The reliability of the wave transmission coefficient for sloping top caissons calculated with the new model as well as with the equation of Goda (1969) by adaptation of the coefficients α and β , is about the same. However, for the new derived model relations have been found between the coefficients and parameters characterising the sloping top caisson breakwater, which is not the case for the equation of Goda (1969).

6. Theoretical approach

The run-up, overtopping and transmission are mutually interrelated through the medium of wave energy. This chapter tries to formulate the relations between the wave run-up height, wave overtopping and wave transmission at vertical breakwaters to give a better insight in the phenomenon wave transmission due to wave overtopping. Section 6.1 will deal with regular waves, while section 6.2 gives an estimation for irregular waves. In section 6.3 the theoretical approach will be discussed.

6.1 Regular waves

6.1.1 Overtopping

This section will describe a theoretical approach of wave overtopping. The method looks at wave overtopping by applying the "calculation method for discharge of overflow weirs". Namely, overtopping waves on vertical breakwaters are regarded as complete overflows at sharp crest weirs. This method is called the "weir model". The weir model was originally proposed by Kikkawa, Shi-igai and Kono (1969) to estimate wave overtopping rate. In Figure 6.1 a definition sketch is given of the weir model.



Figure 6.1 Definition sketch of the weir model

The well known formula which expresses the discharge over a sharp edged weir is as follows:

$$q = \frac{2}{3}m\sqrt{2g} y^{\frac{2}{3}}$$
(6.1)

where q is the discharge per unit width, m is the discharge coefficient and y is the overflow depth. It is usually admitted that equation 6.1 is only valid for steady flow. However, assuming that y does not change very rapidly with respect to time, equation 6.1 may be used for the analysis of wave overtopping. Writing that

$$y = \eta(t) - R_c \tag{6.2}$$

where $\eta(t)$ is the surface elevation of waves over the breakwater measured from SWL, and R_c is the crest freeboard, the following quasi-steady equation for the overtopping discharge can be obtained:

$$q = \frac{2}{3}m\sqrt{2g}\left(\eta(t) - R_{c}\right)^{3/2}$$
(6.3)

Further writing $\eta(t)$ as:

$$\eta(t) = \eta_{\text{max}} \cdot F(t) \tag{6.4}$$

where F(t) is a non-dimensional function of time which expresses the wave profile at the breakwater. η_{max} is approximately equal to the wave run-up height R_u but not the same one, because normally the wave run-up height R_u is measured without overtopping. If there is overtopping, the reflection rate must change and R_u and η_{max} cannot be of the same value.

Introducing the relationship $k = \eta_{max}/H_i$

$$q = \frac{2}{3}m\sqrt{2g} \left(kH_{i}\right)^{3/2} \left(F(t) - \frac{R_{c}}{kH_{i}}\right)^{3/2}$$
(6.5)

If m and k are constant for a wave period, q can be integrated with respect to time.

$$q_{0} = 2 \int_{t_{1}}^{t_{0}} q \, dt = \frac{4}{3} m \sqrt{2g} \left(k H_{i} \right)^{3/2} \int_{t_{1}}^{t_{0}} \left[F(t) - \frac{R_{c}}{k H_{i}} \right]^{3/2} \, dt$$
(6.6)



Figure 6.2 Definition sketch

where $F(t) > \frac{R_c}{k\,H_i}$ for $t_1 < t < t_0. \ q_0$

is the total discharge of overtopping for a period per unit width of the breakwater. As shown in Figure 6.2, t_1 is the time when $z = R_c$ and t_0 is the time when $z = \eta_{max}$.

In case of sine waves,

 $F(t) = sin \frac{2\pi}{T} t , \quad \text{equation} \quad 6.6$ becomes, in non-dimensional form:

$$\frac{Q}{\sqrt{gH_i^3}} = \frac{4\sqrt{2}}{3} m k^{3/2} \frac{1}{T} \int_{t_1}^{T/4} \left[\sin \frac{2\pi}{T} t - \frac{R_c}{k H_i} \right]^{3/2} dt$$
(6.7)

where $t_1 = \frac{T}{2\pi} \arcsin \frac{R_c}{k H_i}$ and T is the wave period.

The term m is the discharge coefficient for the flow over the weir and it will take the value of 0.5. The term k is a characteristic parameter and it is found experimentally to be a function of the relative freeboard R_c/H_i and is expressed as follows (for non-breaking waves):

$$k = 0.594 \frac{R_c}{H_i} + 0.590 \tag{6.8}$$

The effect of wind can be expressed by the change of k, while the breakers' effect may not.

6.1.2 Wave transmission due to overtopping

In this section it is tried to give a relation between the wave overtopping discharge and wave transmission. It is assumed that the mean wave overtopping discharge is a steady flow over a sharp crest weir with an energy flux equal to the energy flux of the transmitted waves.

The mean overtopping discharge will be considered as a steady flow over a sharp crest weir with an energy flux F_{flow} , see Figure 6.3. The expression for F_{flow} is given by:

$$\mathsf{F}_{\mathsf{flow}} = \rho \, \mathsf{g} \, \mathsf{Q} < \mathsf{H} > \tag{6.9}$$

in which Q = mean overtopping discharge, per unit length of structure [m³/s] <H> = head of energy [m], see Figure 6.3.



Figure 6.3 Application of the method of energy flux

The head of energy <H> can be calculated by equation 6.10,

$$\langle \mathsf{H} \rangle = \mathsf{R}_{\mathsf{c}} + \mathsf{E}_{\mathsf{k}} \tag{6.10}$$

in which E_k can be calculated from the discharge formula (equation 6.11) when the flow is critical,

$$Q = m \frac{2}{3} E_k \sqrt{\frac{2}{3} g E_k}$$
(6.11)

where m is the discharge coefficient.

The energy flux of the transmitted wave $F_{t,wave}$ can be given by:

$$F_{t,wave} = \frac{1}{8}\rho g H_t^2 c_g \tag{6.12}$$

where H_t is the transmitted wave height and c_g is the group velocity, expressed as follows:

$$c_{g} = nc = \frac{1}{2} \left[1 + \frac{2kh_{s}}{\sinh 2kh_{s}} \right] * c$$
(6.13)

in which c is given by:

$$c = \frac{L}{T} = \frac{gT}{2\pi} \tanh kh_s$$
(6.14)

According to Goda (1967), the transmitted waves produced by wave overtopping are composed of many wave trains having the periods of T, T/2, T/3 etc.. In Raichlen et al. (1992), a graph is given of the ratio of the average transmitted wave period with incident wave period versus the relative freeboard R_c/H_i . To give an estimation of the average transmitted wave period, the following relation is given, based on Figure 6.4:



Figure 6.4 The variation of the average transmitted period relative to the incident period with relative freeboard for regular waves (Raichlen et al., 1992)

The transmitted wave height can now be obtained as follows:

$$F_{flow} = F_{t,wave} \tag{6.16}$$

With equations 6.9 and 6.12 this becomes:

$$H_{t} = \sqrt{\frac{8Q < H >}{nc}}$$
(6.17)

The transmission coefficient can now be calculated with the following equation:

$$K_{t} = D \cdot \frac{H_{t}}{H_{i}} = D \cdot \frac{\sqrt{\frac{8Q < H >}{nc}}}{H_{i}}$$
(6.18)

In equation 6.18 parameter D has been introduced, called dissipation factor. This has be done to take into account energy losses. Also it is considered that errors caused from above assumptions are included in D. The value of D should be determined by experimental results.

6.1.3 Determination of dissipation factor

In this section the dissipation factor D will be determined making use of the data set of Goda (1969). The dissipation factor is now written as follows, using equation 6.18:

$$D = \frac{K_{t_exp erimentGoda}}{\sqrt{\frac{8Q < H >}{c_g}}{H_i}}$$
(6.19)

The dissipation factor D has been investigated on the influence of relative freeboard R_c/H_i , wave steepness H_i/L , relative water depth h_s/L and relative crest width B_c/H_i . In Annex 5, figures (A5.1 till A5.4) are given of the dissipation factor versus the dimensionless parameters mentioned before. Figure A5.3 shows the dissipation factor D versus the relative water depth h_s/L with the proposed function (data fitting). This function is:

$$D = 1.14 \cdot \left(1 - \frac{h_s}{L}\right) \tag{6.20}$$

To investigate what the influence of the remaining parameters is, the following action has been taken: the coefficient 1.14 is neglected and assumed to be a function of either R_c/H_i or H_i/L or B_c/H_i . This function is called A:

$$A = \frac{D}{\left(1 - \frac{h_s}{L}\right)}$$
(6.21)

Figures A5.5 till A5.7 show the results for known influence of the relative water depth h_s/L , so for A. It is chosen to describe the function A with the relative freeboard R_c/H_i as follows:

$$A = 1.15 \cdot e^{-1.12 \cdot \frac{R_c}{H_i}}$$
(6.22)

The coefficient 1.15 in equation 6.22 has been investigated on influences of the remaining parameters H_i/L and B_c/H_i . However no influences are found. The following relation is now proposed to describe the dissipation factor D:

$$D = 1.15 \cdot \left(1 - \frac{h_s}{L}\right) \cdot e^{-1.12 \cdot \frac{R_c}{H_i}}$$
(6.23)

The dissipation factor D has been determined by experimental results. The wave transmission coefficient can now be calculated when the incident wave height H_i , crest freeboard R_c , wave period T and relative water depth h_s/L are known values.



Influence of wave steepness on wave transmission hs/L=0.15

Figure 6.5 Influence of wave steepness



Influence of relative waterdepth on wave transmission Hi/I =0.02

Figure 6.6 Influence of relative water depth

6.1.4 Comparison theoretical approach with equation of Goda (1969)

In Figure 6.5 and 6.6 the influences on wave transmission of respectively wave steepness H_i/L and relative water depth h_s/L are shown. Also the equation of Goda (1969) is plotted in the figures. An increase of wave steepness will give an increase of wave transmission, see Figure 6.5. When the relative water depth decreases, the wave transmission will increase (Figure 6.6). The theoretical approach gives good agreement with the equation of Goda (1969). The difference between the theoretical approach and the equation of Goda (1969) is that the theoretical approach takes into account the relative water depth h_s/L and the wave steepness H_i/L .

In Figure 6.7 the calculated wave transmission coefficient K_t, calculated with the theoretical approach as well as with the equation of Goda (1969), versus the measured wave transmission coefficient K_t is given. As can be seen from Figure 6.7, the scatter around the line y = x is almost the same for both approaches.



Figure 6.7 Calculated versus measured wave transmission

6.2 Irregular waves

In this section irregular waves will be discussed. The irregular incident waves are assumed to follow a Rayleigh distribution. The Rayleigh distribution is characterised by the single value of H_{si} and can be described by:

$$f(H_{i}) = e^{-2\left(\frac{H_{i}}{H_{si}}\right)^{2}}$$
(6.24)

where $f(H_i)$ is the probability of exceedance of wave height H_i .

Though the phenomenon wave transmission is highly non-linear, wave transmission in this section is treated as a linear phenomenon. For each incident wave, the transmitted wave height can be calculated with equation 6.25.

$$H_{t} = K_{t}(H_{i}, R_{c}) \cdot H_{i}$$
(6.25)

where $K_t(H_i, R_c)$ is wave transmission coefficient for regular waves.

The expected value Ht can now be calculated with:

$$E(H_t) = \int_0^\infty K_t(H_i, R_c) \cdot H_i \cdot f(H_i) dH_i$$
(6.26)

The variance Ht will be:

$$Var(H_t) = E(H_t^2) - E(H_t)^2$$
 (6.27)

in which

$$E(H_t^2) = \int_0^\infty (K_t(H_i, R_c) \cdot H_i)^2 \cdot f(H_i) dH_i$$
(6.28)

The standard deviation H_t is given by:

$$\sigma(\mathbf{H}_{t}) = \sqrt{\mathsf{Var}(\mathbf{H}_{t})} \tag{6.29}$$

In Figure 6.8 a sketch is given of the described approach.



Figure 6.8 Sketch of approach for irregular waves.

With the method described above, also the significant wave height $H_{t1/3}$ can be estimated. Using the theoretical approach for regular waves as described in section 6.1 and the method described above, the expected value of H_t , the standard deviation $\sigma(H_t)$ and the significant wave height $H_{t1/3}$ are determined with the data of M 2090 and H 2137. The wave steepness has been considered constant. Furthermore it is assumed that the waves will break when $H_i > 0.78$ *h_s. The broken waves will keep the wavelength they had before breaking. In table 6.1 the results are given.

		Measured	Calculated				(1)/(2)
Test		K _{to} (1)	E(H _t)	σ(H _t)	H _{t1/3}	K _{to} (2)	factor
M 2090	12	0.130	0.270	0.300	0.600	0.240	0.542
	13	0.034	0.060	0.086	0.150	0.107	0.314
	14	0.034	0.060	0.093	0.160	0.116	0.295
	15	0.048	0.091	0.105	0.220	0.086	0.557
	16	0.049	0.093	0.116	0.218	0.156	0.312
	17	0.324	0.922	0.824	1.880	0.448	0.718
	18	0.124	0.282	0.323	0.641	0.249	0.497
H 2137	P001	0.051	0.127	0.148	0.310	0.090	0.565
	P002	0.027	0.130	0.149	0.315	0.089	0.298
	P003	0.022	0.015	0.028	0.039	0.023	0.981
	P004	0.052	0.457	0.395	0.963	0.173	0.302



The method described above calculates the transmitted wave height caused by overtopping. Transmission through the structure has not been taken into account. There the total wave transmission coefficient K_t has been measured, the transmission coefficient due to overtopping K_{to} has been calculated with equation 3.1 using the equation for the wave transmission coefficient caused by transmission through the structure K_{tt} proposed by Goda (1969), see equation 3.2. In Figure 6.9 the wave transmission coefficient due to overtopping K_{to}, calculated and measured, has been plotted versus the relative freeboard R_c/H_{si}.



Wave transmission due to overtopping Data M 2090 and H 2137



As can be seen from Figure 6.9 the general tendency is in agreement. However, the calculated values of K_{to} are always greater than those of the measured values. This could be due to the fact that wave transmission is highly non-linear.

6.3 Discussion

For regular waves, the theoretical approach describes wave transmission satisfactory. However, for irregular waves the method as described in section 6.2, consequently over estimates the wave transmission coefficient K_t (= H_{st}/H_{si}), although the general tendency is in good agreement. So it can be concluded that wave transmission of irregular waves at a vertical breakwater can <u>not</u> be treated as a linear phenomenon.

It also should be noted that in case of the regular waves, the transmitted waves were characterised by the average wave height and wave period. However, as indicated by Hamer and Hamer (1982), transmitted waves do show irregularities. Especially when overtopping increase, higher harmonics will clearly distinguish.

7. Conclusions and recommendations

In this last chapter the conclusions and recommendations, which can be drawn from the study wave transmission at vertical breakwaters, are mentioned.

7.1 Conclusions

- The relations describing wave transmission at vertical breakwaters derived by Goda (1969), based on regular wave tests, are applicable to the wave transmission coefficient K_t of irregular waves with a significant wave height.
- The equation of Goda (1969) can be used to describe wave transmission for various types of caisson breakwaters by adaptation of the coefficients α and β (see table 4.1 on page 4-9). A relation between the coefficients and a parameter characterising the type of caisson breakwater could not be found.
- The sloping top caisson breakwater has a relatively large wave transmission coefficient K_t compared to conventional and horizontally composite breakwaters. The value of K_t for horizontally composite breakwaters is in general less than corresponding values of K_t for conventional breakwaters. The difference in wave transmission between conventional, parapet and perforated breakwaters is not significantly large.
- Wave transmission is due to overtopping and transmission through the structure. In this report the following expression is found for the wave transmission coefficient K_t :

$$K_{t} = \sqrt{\left[\alpha \cdot \exp\left(\frac{1}{k} \frac{R_{c}}{H_{s}}\right)^{n}\right]^{2} + \left[0.15 \cdot \left(1 - \frac{h'}{h_{s}}\right)\right]^{2}}$$
(5.7)

where for conventional type of breakwater $\alpha = 0.35$ and n = 2. The value k depends on the crest freeboard and the water depth over the berm in front of the breakwater (see Figure 5.4 on page 5-5). However, the influence of the berm has <u>not</u> been taken into account for wave transmission due to overtopping. Also for sloping top caisson breakwaters the coefficients α , n and k are determined (see table 5.2 and Figures 5.8 till 5.10 on page 5-9 and 5-10).

• The phenomenon wave transmission has been studied in a theoretical approach to get a better insight in the process. The method as described in this report, however, consequently overestimates the wave transmission coefficient K_t, probably due to the non-linearity of wave transmission.
7.2 Recommendations

- More tests are needed to verify the coefficients α, n and k of equation 5.7 for conventional and sloping top caisson breakwaters. Measurements of percentage of overtopping waves and transmitted waves caused by the overtopping are recommended. Also the wave transmission coefficient caused by transmission through the structure should be more thoroughly investigated.
- Additional analysis should be performed to verify the influence of other less important structural and hydraulic parameters, such as width of the breakwater, permeability of the rubble mound and wave period.
- Additional analysis should also be performed to evaluate the frequency characteristics of transmitted waves.
- Further work is necessary to take into account the effects of wave obliquity and directional spreading.

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ANNEX 1

Figures of cross sections of the various types of breakwaters













measures in metres





(a) Type I: Caisson type breakwater with wave dissipating blocks in front of caisson



(b) Type II: Dual-wavechamber perforated-wall caisson breakwater



(d) Type IV: Conventional caisson type breakwater





Figure 1a. Section of Old Breakwater C; Super-Structure Built 1889.



Figure 1b. Section of Old Breakwater D; Sub-Structure Built 1869-1893, Super-Structure Built 1887-1924



ANNEX 2

Tables of data of various data sets

Experimental Data on Vertical Wall breakwaters Regular waves

	hs (cm)	T (s)	hs/L	Rc (cm)	Hi_corrected (cm)	Ht (cm)	Kt=Ht/Hi	Kr'=Hr/Hi	Hi/hs	Rc/Hi
Case I	50	0,8	0,5	10	11,33	0,85	0,075	0,462	0,2265	0,8829
Bc=40cm	50	0,8	0,5	5	11,69	1,56	0,133	0,352	0,2338	0,4278
	50	0,8	0,5	0	12,17	3,41	0,280	0,314	0,2433	0,0000
	50	0,8	0,5	30	9,83	0	0,000	0,538	0,1966	3,0518
	50	0,8	0,5	20	10,42	0	0,000	0,543	0,2085	1,9187
	50	0,8	0,5	10	9,37	0,75	0,080	0,548	0,1874	1,0671
	50	0,8	0,5	5	9,05	1,45	0,160	0,502	0,1810	0,5526
	50	0,8	0,5	0	9,34	3,25	0,348	0,433	0,1868	0,0000
	50	0,8	0,5	30	5,22	0	0,000	0,639	0,1043	5,7500
	50	0,8	0,5	20	5,71	0	0,000	0,587	0,1141	3,5052
	50	0,8	0,5	10	6,18	0	0,000	0,496	0,1236	1,6186
	50	0,8	0,5	5	5,85	0	0,000	0,42	0,1170	0,8547
	50	0,8	0,5	0	5,95	1,46	0,245	0,469	0,1190	0,0000
Case II	50	1.05	03	30	20.68	Δ	0.000	0 409	0.4136	1 4507
Bc=40cm	50	1,00	0,0	20	21 71	18	0.083	0.468	0.4343	0.9211
Do room	50	1.05	0.3	10	12.86	1.55	0.121	0.689	0 2571	0,7779
	50	1.05	0.3	5	13.83	33	0,239	0,636	0.2765	0.3616
	50	1.05	0.3	0	13.50	4.63	0.343	0.557	0.2701	0.0000
	50	1.05	0.3	10	11.84	0.68	0.057	0.609	0.2368	0,8445
	50	1.05	0.3	5	9.43	2.06	0.218	0.624	0,1887	0.5300
	50	1.05	0.3	0	9.33	4.33	0.464	0.534	0.1866	0,0000
	50	1,05	0,3	30	7,17	0	0.000	0,752	0,1435	4,1816
	50	1.05	0.3	20	7.37	0	0.000	0,738	0.1473	2,7148
	50	1,05	0,3	10	8,92	0	0,000	0,438	0,1784	1,1211
	50	1.05	0,3	5	6,98	1	0,143	0,46	0,1396	0,7163
	50	1,05	0,3	0	7,91	2,49	0,315	0,499	0,1582	0,0000
Case III	50	1,36	0,2	30	22,36	1,74	0,078	0,566	0,4473	1,3415
Bc=40cm	50	1,36	0,2	20	23,16	4,15	0,179	0,612	0,4632	0,8635
	50	1,36	0,2	10	23,13	4,82	0,208	0,663	0,4625	0,4324
	50	1,36	0,2	5	24,93	8,46	0,339	0,661	0,4985	0,2006
	50	1,36	0,2	0	24,55	9,54	0,389	0,543	0,4911	0,0000
	50	1,36	0,2	10	18,37	4,97	0,270	0,702	0,3675	0,5442
	50	1,36	0,2	5	19,00	5,62	0,296	0,503	0,3799	0,2632
	50	1,36	0,2	0	17,23	6,07	0,352	0,567	0,3446	0,0000
	50	1,36	0,2	5	15,30	2,88	0,188	0,586	0,3059	0,3269
	50	1,30	0,2	0	11,73	4,//	0,407	0,475	0,2346	0,0000
1	50	1,30	0,2	0	9,50	2,97	0,313	0,388	0,1900	0,0000
Case IV	50	1,8	0,14	30	19,90	0,88	0,044	0,417	0,3981	1,5072
Bc=40cm	50	1,8	0,14	20	19,72	1,21	0,061	0,483	0,3944	1,0142
[50	1,8	0,14	10	17,91	2,39	0,133	0,6	0,3582	0,5584
[50	1,8	0,14	5	17,25	5,25	0,304	0,623	0,3450	0,2899
[50	1,8	0,14	0	17,04	7,65	0,449	0,569	0,3408	0,0000
[50	1,8	0,14	30	11,16	0,65	0,058	0,517	0,2231	2,6889
	50	1,8	0,14	20	13,61	0,65	0,048	0,485	0,2722	1,4697
Ĺ	50	1,8	0,14	10	15,08	1,06	0,070	0,634	0,3016	0,6632
L	50	1,8	0,14	5	14,34	2,62	0,183	0,652	0,2868	0,3486
L	50	1,8	0,14	0	11,62	<u>5,05</u>	0,435	0,624	0,2324	0,0000
Ĺ	50	1,8	0,14	30	11,07	0	0,000	0,638	0,2215	2,7089
	50	1,8	0,14	20	13,03	0	0,000	0,562	0,2607	1,5345
	50	1,8	0,14	10	8,73	0,45	0,052	0,659	0,1745	1,1458
	50	1,8	0,14	5	7,60	0,54	0,071	0,753	0,1519	0,6582
L	50	1,8	0,14	0	8,45	3,5	0,414	0,716	0,1690	0,0000
	50	1,8	0,14	10	5,67	0	0,000	0,669	0,1135	1,7627
Ļ	50	1,8	0,14	5	5,55	0	0,000	0,669	0,1110	0,9009
	50	1,8	0,14	0	5,04	1,46	0,289	0,686	0,1009	0,0000

	hs (cm)	T (s)	hs/L	Rc (cm)	Hi_corrected (cm)	Ht (cm)	Kt=Ht/Hi	Kr'=Hr/Hi	Hi/hs	Rc/Hi
Case V	35	1,5	0,14	10	15,29	2,3	0,150	0,534	0,4368	0,6542
Bc=40cm	35	1.5	0.14	5	15,12	3,19	0,211	0,478	0,4319	0,3307
	35	1.5	0,14	0	14,18	5,41	0,381	0,494	0,4052	0,0000
	35	1.5	0,14	10	11,12	1,12	0,101	0,493	0,3178	0,8990
	35	1,5	0,14	5	10,51	1,74	0,166	0,572	0,3003	0,4758
	35	1,5	0,14	0	8,85	2,65	0,299	0,552	0,2529	0,0000
	35	1,5	0,14	10	8,82	0	0,000	0,303	0,2520	1,1338
	35	1,5	0,14	5	6,98	0,2	0,029	0,362	0,1994	0,7163
	35	1,5	0,14	0	5,83	1,69	0,290	0,465	0,1666	0,0000
Case VI	35	2	0,1	10	16,67	4,18	0,251	0,39	0,4762	0,6000
Bc=40cm	35	2	0,1	5	16,60	5,48	0,330	0,433	0,4743	0,3012
	35	2	0,1	0	13,83	5,45	0,394	0,441	0,3953	0,0000
	35	2	0,1	10	9,53	0,45	0,047	0,479	0,2722	1,0497
	35	2	0,1	5	13,10	1,66	0,127	0,379	0,3743	0,3816
	35	2	0,1	0	10,18	2,64	0,259	0,465	0,2909	0,0000
	35	2	0,1	10	4,05	0	0,000	0,69	0,1156	2,4709
	35	2	0,1	5	4,43	0	0,000	0,626	0,1265	1,1290
	35	2	0,1	0	3,46	0,97	0,280	0,608	0,0988	0,0000
Case VII	35	2,76	0,07	10	12,88	3,3	0,256	0,409	0,3680	0,7765
Bc=40cm	35	2,76	0,07	5	14,00	3,84	0,274	0,361	0,4000	0,3571
	35	2,76	0,07	0	11,69	4,74	0,406	0,393	0,3340	0,0000
	35	2,76	0,07	10	11,15	1,77	0,159	0,381	0,3186	0,8968
	35	2,76	0,07	5	11,80	2,03	0,172	0,334	0,3371	0,4238
	35	2,76	0,07	0	9,16	3,08	0,336	0,341	0,2618	0,0000
	35	2,76	0,07	10	5,33	0	0,000	0,546	0,1522	1,8772
	35	2,76	0,07	5	4,80	0	0,000	0,582	0,1372	1,0413
	35	2,76	0,07	0	3,78	1,19	0,315	0,571	0,1080	0,0000
Case VIII	50	1,8	0,14	0	20,16	7,63	0,378	0,573	0,4032	0,0000
Bc=40cm	50	1,8	0,14	0	19,46	7,44	0,382	0,544	0,3893	0,0000
	50	1,8	0,14	0	17,47	6,33	0,362	0,591	0,3494	0,0000
	50	1,8	0,14	0	15,76	5,81	0,369	0,586	0,3152	0,0000
	50	1,8	0,14	0	13,96	5,08	0,364	0,624	0,2791	0,0000
	50	1,8	0,14	0	12,82	4,87	0,380	0,621	0,2564	0,0000
	50	1,8	0,14	0	10,99	3,92	0,357	0,63	0,2197	0,0000
	50	1,8	0,14	0	9,34	3,24	0,347	0,656	0,1869	0,0000
	50	1,8	0,14	0	8,11	1,92	0,237	0,678	0,1621	0,0000
	50	1,8	0,14	0	6,31	1,5	0,238	0,692	0,1262	0,0000
	50	1,8	0,14	0	4,93	1	0,203	0,774	0,0986	0,0000
	50	1,8	0,14	0	3,00	0,5	0,167	0,667	0,0600	0,0000

Experimental Data on Horizontally composite breakwaters

For all data $h'/h_s = 0.6$

Irregular waves

b _e /L _{1/3} = 0.05 -	0.07	b _e /L _{1/3} = 0.07 -	0.10	b _e /L _{1/3} = 0.10	- 0.15
K _t	R _c /H _{si}	Kt	R _c /H _{si}	Kt	R _c /H _{si}
0,123	0,82	0,07	2,4	0,042	2,64
0,121	0,74	0,061	2,22	0,052	2,505
0,145	0,64	0,067	1,91	0,058	2,37
0,16	0,58	0,076	1,56	0,036	2,22
0,166	0,53	0,075	1,47	0,048	2,19
0,21	0,48	0,07	1,38	0,045	2,04
0,2	0,47	0,068	1,29	0,061	1,71
0,19	0,45	0,072	1,23	0,054	1,49
0,22	0,39	0,07	1,12	0,056	1,425
0,23	0,385	0,085	1,12	0,036	1,395
0,223	0,37	0,086	1,07	0,042	1,38
0,26	0,3	0,078	1,04	0,048	1,305
0,35	0	0,088	1,02	0,036	1,305
0,358	0	0,091	0,98	0,047	1,23
0,364	0	0,102	0,97	0,055	1,22
0,369	0	0,091	0,91	0,035	1,29
0,375	0	0,091	0,86	0,052	1,19
0,381	0	0,108	0,92	0,055	1,18
0,388	0	0,12	0,84	0,05	1,17
		0,15	0,76	0,073	1,07
		0,14	0,74	0,054	1,06
		0,145	0,73	0,066	1,05
		0,121	0,68	0,055	0,97
		0,2	0,67	0,073	0,94
		0,13	0,62	0,0697	0,88
		0,145	0,58	0,085	0,833
		0,175	0,52	0,091	0,02
		0,22	0,41	0,005	0,79
		0.21	0,39	0,005	0,09
		0.215	0,30	0,000	0,03
		0,23	0,30	0,003	0,73
		0,31	0	0,007	0,74
		0.32	0	0.097	0,60
		0.32	0	0,097	0.55
		0,33	0	0,12	0,50
		0.336	<u>0</u>	0.13	0.54
		0.34	0	0 11	0.52
		0.345	0	0,127	0.5
		0,352	0	0,15	0,49
		, <u> </u>		0,115	0,48
				0,12	0,41
				0,17	0,39
				0,31	0,02
				0,26	0,01
				0,25	0
				0,26	0
				0,28	0
				0,29	0
				0,295	0
				0,297	0
				0,3	0
				0,31	0

Experime	ental Data o	n sloping t	top caisso	n breakwa	Irregular waves			
	h _s (cm)	d (cm)	R _c (cm)	α (°)	d _c (cm)	Kt	R _c /H _{si}	
Type 1	1044	80	32	45	0	0,1125	1,8	
	1044	80	32	45	0	0,136	1,604	
	1044	80	32	45	0	0,122	1,58	
	1044	80	32	45	0	0,11	1,58	
	1044	80	32	45	0	0.127	1,09	
	1044	80	32	45	0	0,169	0,98	
	1044	80	32	45	0	0.15	0.96	
	1044	80	32	45	0	0.21	0.75	
	1044	80	32	45	1 0	0.253	0.66	
Type 2	1044	80	32	45	8	0.098	1.86	
iype L	1044	80	32	45	8	0.12	1 64	
	1044	80	32	45	8	0,12	1.62	
	1044	80	32	45	0	0,034	1,02	
	1044	80	32	45	0	0,1125	1,10	
	1044	00	32	45	0,	0,16	1,02	
	1044	80	32	45	8	0,145	0.75	
	1044	80	32	45	8	0,178	0,75	
	1044	80	32	45	8	0,23	0,64	
Гуре 3	1044	80	32	45	-21,8	0,145	1,74	
	1044	80	32	45	-21,8	0,164	1,74	
	1044	80	32	45	-21,8	0,173	1,64	
	1044	80	32	45	-21,8	0,164	1,49	
	1044	80	32	45	-21,8	0,21	1,21	
	1044	80	32	45	-21,8	0,225	1,13	
	1044	80	32	45	-21,8	0,197	1	
	1044	80	32	45	-21,8	0,27	0,79	
	1044	80	32	45	-21,8	0,32	0,64	
Type 4	1044	80	16	45	-21.8	0.098	1,66	
	1044	80	16	45	-21.8	0.13	1.51	
	1044	80	16	45	-21.8	0 155	1.32	
	1044	80	16	45	-21.8	0.2	0.91	
	1044	80	16	45	-21.8	0.234	0,01	
	1044	80	16	45	21,0	0.26	0,5	
	1044		10	4J	-21,0	0,20	0,07	
	1044	00	10	43	-21,0	0,23	0,63	
	1044	80	16	45	-21,8	0,32	0,6	
	1044	80	16	45	-21,8	0,3	0,53	
	1044	80	16	45	-21,8	0,34	0,59	
	1044	80	16	45	-21,8	0,36	0,415	
Type 5	1044	80	16	45	-27,6	0,1125	1,66	
	1044	80	16	45	-27,6	0,13	1,55	
	1044	80	16	45	-27,6	0,15	1,39	
	1044	80	16	45	-27,6	0,24	0,96	
	1044	80	16	45	-27,6	0,21	0,91	
	1044	80	16	45	-27,6	0,25	0,79	
	1044	80	16	45	-27,6	0,31	0,68	
	1044	80	16	45	-27,6	0,35	0,64	
	1044	80	16	45	-27.6	0,33	0,53	
	1044	80	16	45	-27,6	0,38	0,42	
vpe 6	1044	80	16	56	-21.8	0.084	1.79	
	1044	80	16	56	-21.8	0.108	1.51	
	1044	80	16	56	-21.8	0 141	1 36	
	1044		16	56	-21.0	0 17	0 02	
	1044	00	10	50	-21,0	0.25	0,82	
		00	10	50	-21,0	0,20	0,09	
	1044	- 00	- 10	00	-21,8	0,22	0,85	
	1044	80	16	56	-21,8	0,19	0,83	
	1044	80	16	56	-21,8	0,26	0,6	
	1044	80	16	56	-21,8	0,336	0,41	
	1044	80	16	56	-21,8	0,35	0,32	

Experimental Data on sloping top caisson breakwaters

Experimental Data on parapet breakwaters

Irregular waves

For all data d/h_s = 0,6 - 0,7

Front para	Front parapet		bet	Rear parapet		
		(basic sha	be)	(improved	shape)	
K _t	R _c /H _{si}	K _t	R_c/H_{si}	K _t	R _c /H _{si}	
0,0725	1,19	0,08	1,19	0,0625	1,19	
0,115	0,85	0,1325	0,85	0,09	0,91	
0,17	0,64	0,205	0,64	0,075	0,905	
0,1625	0,63	0,1575	0,63	0,1125	0,85	
0,19	0,61	0,215	0,61	0,1025	0,79	
<u>k </u>				0,18	0,64	
				0,14	0,62	
				0,22	0,61	
				0,19	0,585	

Data of Funakoshi (1994)

Table A2.4

Tvi	pe I	Typ	be II	Тур	e III	Type IV	
Kt	R _c /H _{si}	K _t	R _c /H _{si}	Kt	R _c /H _{si}	Kt	R _c /H _{si}
0,175	0,95	0,11	0,95	0,125	0,95	0,12	0,95
0.11	1,14	0,069	1,14	0,075	1,14	0,081	1,14
0.055	1,35	0,035	1,35	0,04	1,35	0,04	1,35
0,125	1,1	0,096	1,1	0,085	1,1	0,096	1,1
0.071	1,3	0,05	1,3	0,054	1,3	0,06	1,3
0.03	1,55	0.03	1,55	0,04	1,55	0,04	1,55

Data of Lee (1994)

Table A2.5

Experimental Data on horizontally composite breakwaters

0,376

0,28

0,502

0,24

0,26

0,248 0,248

0,18

0,12 0,063 0,46

0,57

0,312

0,75

0,75 0,79

0,81

0,85 0,93

1,05

	Regula	r waves		Irregular waves				
secti	ion C	secti	ion D	secti	ion C	sect	ion D	
K _t	R _c /H _i	Kt	R _c /H _i	Kt	R _c /H _{si}	K _t	R _c /H _{si}	
0,555	0,31	0,405	0,31	0,045	0,645	0,185	0,255	
0,546	0,31	0,402	0,33	0,06	0,595	0,175	0,265	
0,53	0,31	0,39	0,33	0,07	0,595	0,17	0,265	
0,48	0,31	0,375	0,35	0,04	0,588	0,1325	0,4	
0.44	0,33	0,35	0,4	0,075	0,705	0,1675	0,305	
0.41	0,33	0,333	0,46	0,07	0,705	0,161	0,305	
0,37	0,33	0,29	0,54	0,055	0,71	0,145	0,423	
0.395	0,35	0,15	0,81	0,2075	0,258	0,1175	0,49	
0,38	0,4			0,195	0,265			
0,347	0,46			0,1425	0,4			
0,3	0,54			0,15	0,305			
0,14	0,81			0,208	0,316			
0,505	0,28			0,11	0,423			
0,42	0,31	1		0,09	0,49			
0,39	0,36							
0,373	0,44	1						
0,29	0,57							
0,502	0,31							
0,492	0,34							
0,425	0,39							

Experimental Data on conventional breakwater

Irregular waves

test	H _{si} (m)	H _{st} (m)	R _c (m)	K _t	R _c /H _{si}	h (m)	T _p (s)	Sp	B _c /L ₀	B _c /H _{si}	H _{si} /h _s
1a	0,9	0,08	2	0,0889	2,2222	7	4,8	0,025	0,083	3,3333	0,129
1b	1,48	0,1	2	0,0676	1,3514	7	6,2	0,025	0,05	2,027	0,211
1c	2,1	0,18	2	0,0857	0,9524	7	7	0,027	0,039	1,4286	0,3
1d	2,52	0,28	2	0,1111	0,7937	7	7,3	0,03	0,036	1,1905	0,36
1e	3,08	0,51	2	0,1656	0,6494	7	7,9	0,032	0,031	0,974	0,44
2a	1,05	0,09	2	0,0857	1,9048	7	6,7	0,015	0,043	2,8571	0,15
2b	1,42	0,16	2	0,1127	1,4085	7	7,6	0,016	0,033	2,1127	0,203
2c	2,1	0,3	2	0,1429	0,9524	7	9,1	0,016	0,023	1,4286	0,3
2d	2,38	0,4	2	0,1681	0,8403	7	9,2	0,018	0,023	1,2605	0,34
2e	2,86	0,56	2	0,1958	0,6993	7	10,1	0,018	0,019	1,049	0,409
3a	1,18	0,03	2	0,0254	1,6949	5	4,9	0,032	0,08	2,5424	0,236
3b	1,6	0,05	4	0,0313	2,5	5	6,1	0,028	0,052	1,875	0,32
3c	2,2	0,08	4	0,0364	1,8182	5	7	0,029	0,039	1,3636	0,44
3d	2,65	0,09	4	0,034	1,5094	5	7,5	0,03	0,034	1,1321	0,53
3e	3,08	0,1	4	0,0325	1,2987	5	7,6	0,034	0,033	0,974	0,616
4a	1,03	0,09	2	0,0874	1,9417	7	4,9	0,027	0,08	2,9126	0,147
4b	1,46	0,13	2	0,089	1,3699	7	6,2	0,024	0,05	2,0548	0,209
4c	2,08	0,21	2	0,101	0,9615	7	7,1	0,026	0,038	1,4423	0,297
4d	2,52	0,29	2	0,1151	0,7937	7	7,5	0,028	0,034	1,1905	0,36
4e	2,84	0,36	2	0,1268	0,7042	7	7,9	0,029	0,031	1,0563	0,406
5a	1,02	0,16	1	0,1569	0,9804	7	4,9	0,027	0,08	2,9412	0,146
5b	1,46	0,27	1	0,1849	0,6849	7	6	0,026	0,053	2,0548	0,209
5c	2,08	0,47	1	0,226	0,4808	7	6,9	0,028	0,04	1,4423	0,297
5d	2,46	0,63	1	0,2561	0,4065	7	7,6	0,027	0,033	1,2195	0,351
5e	2,8	0,76	1	0,2714	0,3571	7	7,9	0,029	0,31	1,0714	0,4
6a	1,03	0,13	2	0,1262	1,9417	7	4,8	0,029	0,083	2,9126	0,147
6b	1,46	0,16	2	0,1096	1,3699	7	6,2	0,024	0,05	2,0548	0,209
6c	2,08	0,25	2	0,1202	0,9615	7	7	0,027	0,039	1,4423	0,297
6d	2,52	0,34	2	0,1349	0,7937	7	7,5	0,029	0,034	1,1905	0,36
6e	2,84	0,42	2	0,1479	0,7042	7	7,9	0,029	0,031	1,0563	0,406
6f	4	1,05	2	0,2625	0,5	7	10	0,026	0,019	0,75	0,571
13	1,4	0,05	1,75	0,0357	1,25	7,5	5,8	0,027	0,05	6,0714	0,187
14	1,38	0,05	1,75	0,0362	1,2681	7,5	7,4	0,016	0,039	6,1594	0,184
15	2,56	0,13	4,25	0,0508	1,6602	5	7,6	0,028	0,034	3,3203	0,512
16	1,4	0,07	1,25	0,05	0,8929	7,5	5,8	0,027	0,031	6,0714	0,187
17	4,2	1,35	1,25	0,3214	0,2976	8	10,2	0,026	0,052	2,0238	0,525
18	2,57	0,32	1,75	0,1245	0,6809	7,5	7,4	0,03	0,1	3,3074	0,343
12	2,5	0,34	1,75	0,136	0,7	7,5	7,4	0,029	0,1	3,4	0,333

Data of M2090

Table A2.7

Irregular waves

Experimental Data on conventional breakwater

test	H _{si} (m)	T _p (s)	R _c (m)	R_c/H_{si}	H _{st} (m)	K _t
P001	3,464	10,28	5,9	1,703	0,1833	0,053
P002	3,52	10,13	5,9	1,676	0,1083	0,031
P003	1,728	6,96	6,1	3,53	0,0467	0,027
P004	5,574	12,98	5,4	0,968	0,3033	0,054

Data of H2137

Table A2.8

ANNEX 3

Figures for Chapter 4







Sloping top caisson, type 2 Data Takahashiet al., 1996









Sloping top caisson, type 4 Data Takahashiet al., 1996







Sloping top caisson, type 6 Data Takahashiet al., 1996





Horizontally composite breakwaters, be/L1/3 = 0.05-0.07 Data Tanimoto et al., 1987

Figure A3.7





Figure A3.8







Horizontally composite breakwaters



Figure A3.10





Figure A3.11









Figure A3.13

TRITT





Figure A3.14

Front parapet breakwater Data Funakoshi et al., 1994



Figure A3.15





Figure A3.16








Figure A3.18

ANNEX 4

Figures for Chapter 5



Sloping top caisson, type 1 Data Takahashi et al., 1996



Figure A4.1



Sloping top caisson, type 2 Data Takahashi et al., 1996





Sloping top caisson, type 3 Data Takahashi et al., 1996









Sloping top caisson, type 5 Data Takahashi et al., 1996







Conventional breakwater Data M 2090



Figure A4.7

ANNEX 5

Figures for Chapter 6

Dissipation factor D versus relative water depth Rc/Hi



Figure A5.1

D



Annex 5

Figure A5.2

0,14

0,12

0,1

0,08

0,06

0,04

0,02

Hiľ

Dissipation factor D versus relative water depth hs/L



Dissipation factor D versus relative water depth Bc/Hi



Influence relative freeboard Rc/Hi with known influence of relative water depth hs/L







Figure A5.6

Influence relative freeboard Bc/Hi with known influence of relative water depth hs/L



ANNEX 6

Table of Normal distribution

k	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0080	0.0160	0.0239	0.0319	0.0399	0.0478	0.0558	0.0638	0.0717
0.1	0.0797	0.0876	0.0955	0.1034	0.1113	0.1192	0.1271	0.1350	0.1428	0.1507
0.2	0.1585	0.1663	0.1741	0.1819	0.1897	0.1974	0.2051	0.2128	0.2205	0.2282
0.3	0.2358	0.2434	0.2510	0.2586	0.2661	0.2737	0.2812	0.2886	0.2961	0.3035
0.4	0.3108	0.3182	0.3255	0.3328	0.3401	0,3473	0.3545	0.3516	0.3688	0.3759
0.5	0.3829	0.3899	0.3969	0.4039	0.4108	0.4177	0.4245	0.4313	0.4381	0.4448
0.6	0.4515	0.4581	0.4647	0.4713	0.4778	0.4843	0.4907	0.4971	0.5035	0.5098
0.7	0.5161	0.5223	0.5285	0.5346	0.5407	0.5467	0.5527	0.5587	0.5646	0.5705
0.8	0.5763	0.5821	0.5878	0.5935	0.5991	0.6047	0.6102	0.6157	0.6211	0.6265
0.9	0.6319	0.6372	0.6424	0.6476	0.6528	0.6579	0.6629	0.6680	0.6729	0.6778
1.0	0.6827	0.6875	0.6923	0.6970	0.7017	0.7063	0.7109	0.7154	0.7199	0.7243
1.1	0.7287	0.7330	0.7373	0.7415	0.7457	0.7499	0.7540	0.7580	0.7620	0.7660
1.2	0.7699	0.7737	0.7775	0.7813	0.7850	0.7887	0.7923	0.7959	0.7995	0.8029
1.3	0.8064	0.8098	0.8132	0.8165	0.8198	0,8230	0.8252	0.8293	0.8324	0.8355
1.4	0.8385	0.8415	0.8444	0.8473	0.8501	0.8529	0.8557	0.8584	0.8611	0.8638
1.5	0.8664	0.8690	0.8715	0.8740	0.8764	0.8789	0.8812	0.8836	0.8859	0.8882
1.6	0.8904	0.8926	0.8948	0.8969	0.8990	0.9011	0,9031	0.9051	0.9070	0.9090
1.7	0.9109	0.9127	0.9146	0.9164	0.9181	0.9199	0.9216	0.9233	0.9249	0.9265
1.8	0.9281	0.9297	0.9312	0,9328	0.9342	0.9357	0.9371	0.9385	0.9399	0.9412
1.9	0.9426	0.9439	0.9451	0.9464	0.9476	0.9488	0.9500	0.9512	0.9523	0.9534
2.0	0.9545	0.9556	0.9566	0.9576	0.9586	0.9596	0,9606	0.9615	0.9625	0.9634
2.1	0.9643	0,9651	0.9660	0.9668	0.9676	0.9684	0,9692	0.9700	0.9707	0.9715
2.2	0.9722	0.9729	0.9736	0.9743	0.9749	0.9756	0.9762	0.9768	0.9774	0.9780
2.3	0.9786	0.9791	0.9797	0.9802	0.9807	0.9812	0.9817	0.9822	0.9827	0.9832
2.4	0.9836	0.9840	0.9845	0.9849	0.9853	0.9857	0.9861	0.9865	0.9869	0.9872
2.5	0.9876	0.9879	0.9883	0.9886	0.9889	0.9892	0.9895	0.9898	0,9901	0,9904
2.6	0.9907	0,9909	0.9912	0.9915	0.9917	0.9920	0.9922	0.9924	0.9926	0.9929
2.7	0.9931	0.9933	0.9935	0.9937	0.9939	0.9940	0.9942	0.9944	0.9946	0.9947
2.8	0.9949	0.9950	0.9952	0.9953	0.9955	0.9956	0.9958	0.9959	0.9960	0,9961
2.9	0.9963	0.9964	0.9965	0,9966	0.9967	0.9968	0,9969	0.9970	0.9971	0.9972
3.0	0.9973	0.9974	0.9975	0.9976	0.9976	0.9977	0,9978	0.9979	0.9979	0.9980
3.1	0.9981	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986
3.2	0.9986	0.9987	0,9987	0.9988	0.9988	0.9988	0,9989	0.9989	0,9990	0.9990
3.3	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0,9992	0.9992	0.9993	0.9993
3.4	0.9993	0.9994	0,9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995

For example: $P\{k \le 1.64\} = 0.8990$

