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van Damme, Myron; Visser, Paul

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A SWOT ANALYSIS OF HYDRODYNAMIC MODELS WITH RESPECT TO SIMULATING BREACHING

M. VAN DAMME(1) & P.J. VISSER(2)

(1) *Delft University of Technology, Faculty of Civil Engineering and Geosciences, Department Hydraulic Engineering, Delft, The Netherlands*
e-mail: m.vandamme@tudelft.nl

(2) *Delft University of Technology, Faculty of Civil Engineering and Geosciences, Department Hydraulic Engineering, Delft, The Netherlands*
e-mail: p.j.visser@tudelft.nl

ABSTRACT

Deriving the bed shear stresses from hydrodynamic models in breach models is challenging due to the continuous changing hydraulic head over the breach in combination with horizontal and vertical flow contractions, and the continuous rapidly changing breach geometry. Three stages can be distinguished in breach flows. Stage 1 initiates when the embankment starts to overflow and is characterized by flows over the embankment crest and down the landside slope. Stage 2 initiates when the landside slope has retreated towards the waterside slope. The hydraulic head increases rapidly and the flow contracts both horizontally and vertically resulting in a fully 3-dimensional flow. During Stage 3 a full breach has developed and the flow contracts mainly horizontally. This paper presents a SWOT analysis of flow modelling methods applied in breach models to derive the bed shear stresses. The paper shows that for a number of cases analytical methods are more accurate than numerical methods due to the fact that they give a more accurate description of the shear stresses on the embankment surface, whereas for some numerical methods errors are found of the order of 50%.

Keywords: *Breach models, Hydrodynamic models*

1. INTRODUCTION

Singh (1996) states that "Cristofano (1965) was perhaps the first to have simulated gradual dam breach erosion". Since 1965 many models for breaching of embankments (dams, dikes, levees) have been developed. These models range from relatively simple empirical models, to semi-empirical models and process based models. Empirical models are based on fitting analytical equations to historical embankment breach data (for instance the model of Froehlich, 1995). Semi-empirical models use historical data about the final breach geometry and failure time and then describe the breaching process as a linear process and calculate the discharge through the breach using theory of fluid mechanics (examples are the DAMBRK model of Fread, 1984, and the model of Macchione, 2008). Process based models are based on the numerical simulation of the physical processes based on the principles of fluid mechanics, soil mechanics and sediment transport. Examples of physical models are the BEED model (Singh et al., 1988, Singh, 1996), the BREACH model of Fread (1988), the BRES model for sand dikes (Visser, 1998), the HR BREACH model (Mohamed, 2002), the BRES-clay model of Zhu (2006), the AREBA model (Van Damme et al., 2012), the model of Wu (2013), and the EMBREA model (Davison et al., 2013). This paper focusses on process based models.

A challenge in the development of process based breach models is how to accurately derive the shear stresses on the embankment surface from flow models. The shear stresses are used to determine the erosion rates and hence determine the breach growth. Accurately predicting these stresses is therefore of high importance. A distinction can be made between two erosion types: headcut erosion and surface erosion. During headcut erosion, the erosion process is governed by the normal impact of the flow on the embankment surface, whereby the flow, at occasions separates from the embankment surface. The landside slope thereby retreats towards the waterside slope due to undercutting and block failures. During surface erosion, the shear stresses dominate the erosion process whereby the flow remains attached to the embankment surface.

The continuous changing hydraulic head over the breach in combination with horizontal and vertical flow contractions form a challenge for modelling the flow. Three flow stages can be distinguished in the breaching process. Stage 1 initiates when the embankment starts to overflow and is characterized by flows over the embankment crest and down the landside slope. The flow mainly contracts vertically and rapidly accelerates along the landside slope whereby the vertical velocity component is significant (See Figure 1a). Stage 2 initiates when the landside slope has retreated towards the waterside slope. The hydraulic head increases rapidly and the flow contracts both horizontally and vertically resulting in a fully 3-dimensional flow (See Figure 1b). In Stage 3 a full breach has developed and the flow contracts mainly horizontally (See Figure 1c).

Figure 1: flow profiles over an embankment during erosion.

In this paper a SWOT analysis is presented of methods that are currently applied to simulating breach flows with respect to modelling the breach development. Both analytical (in Section 2) and numerical 1D, 2D and 3D, grid based methods (in Section 3) are discussed. Analytical methods are honed for their speed but are limited to describing the flow for a set of pre-defined flow geometries. The accuracy of the numerical method depends on the method of choice.

2. ANALYTICAL MODELS

The analytical based breach models discussed in this section all use the broad-crested weir formula to determine the breach discharge $Q_{br.}$ For a quasi-steady flow the energy balance equations gives the following broad-crested weir equation:

$$Q_{br} = mBh_2\sqrt{2g(H-h_2)} \quad [1]$$

where m is a discharge coefficient, g [m^2/s] is the acceleration of gravity, B [m] is the averaged breach width, H [m] is the outside water level relative to the breach bottom and h_2[m] is the water depth over the breach bottom. Eq. [1] can be applied in all stages of the breaching process. At the end of Stage 3, during which the breach grows mainly in lateral direction, the downstream water level has risen to such a level that it drowns the breach flow, a weir formula for subcritical discharge has to be applied. There are several formulae for broad-crested weirs for subcritical discharge, but these formulae are similar to [1] and differ only in the value for h_2. The cross-section of the breach, i.e. of the channel in the crest and in the landside slope is usually assumed to be rectangular or trapezoidal. For either case slight variations in the critical depth and hence discharge exist. For a rectangular weir $h_2 = \frac{2}{3}(H-z)$. One main advantage of using the broad weir equation is that the effects of non-hydrostatic pressure distributions due to horizontal and vertical flow contraction can be accounted for the in the weir factor m. Also the effects of friction could be accounted for by modifying m. By applying different weir coefficients for each breach stage the breach discharge could be predicted with high levels of accuracy.

From the discharge that follows from Eq. [1], a calculated depth profile, and assumed breadth B the flow velocities can be calculated along the embankment perimeter from which the shear stresses and erosion rates can be derived. With exception of the BRES model, the erosion rate over the crest during Stage 1 is calculated from the critical flow velocity that follows from the broad crested weir formula and the assumption of a critical depth everywhere along the crest. The momentum and energy balance equations give a critical depth over the crest under the assumption that the effects of friction are negligible. In reality this is not always the case and the critical depth is likely only to be reached nearby where the crest meets the landside slope. The flow velocities over the crest will thereby be predominantly sub-critical. Hence in assuming a critical velocity over the crest most of the analytical models will overestimate the erosion rate with exception of the BRES model which will underestimate the erosion rate at the crest.

The BEED model (Singh et al., 1988, Singh, 1996) and the BREACH model of Fread (1988) also describe the flow over the landside slope of the embankment analytically. However in these models the water and sediment flows through the downstream channel are not calculated to determine the erosion rate of the landside slope, but only to route the water and sediment flow. This latter is puzzling since the flow over the landside slope is supercritical with flow velocities significantly larger than in the overflow section (see Visser, 1998) where the breach erosion is calculated. Both in the BEED model and in the BREACH model it is assumed that the length of the back water curve (S2 profile) can be neglected and that the flow velocity on the landside slope is given by the Chézy law for normal flow. Inherent to these models is the assumption that equilibrium transport conditions are met along the landside slope. Again, this is in conflict with the general description of a breach formation as defined by Visser (1998), and Morris (2011) who identified the retreat of the landside slope towards the waterside slope to be the significant erosion process in breach formation.

In the BRES model (Visser, 1998), the BRES-clay model (Zhu, 2006), the AREBA model (Van Damme et al., 2012) and the model of Wu (2013) the flow over the landside slope of the embankment (in Stages 1 and 2) is described analytically to calculate the erosion rate of this slope in the case of surface erosion. These three models are based on the assumption that the retreat of the landside slope dominates the downwards erosion of the crest and that no equilibrium transport conditions are reached along the embankment perimeter, which has also been observed during field tests. The BRES-clay model, the AREBA model and the model of Wu include also breaching of cohesive embankments by headcut migration but in this case the process is described differently whereby semi-empirical equations are used.

As described above, analytical models inherently contain the assumption that (for Stages 1 and 2) the water depth is uniform along the landside slope and that the flow velocity is given by the Chézy law for normal flow. The bed shear stress is then calculated from this flow velocity. In both the BRES model and the AREBA model the flow over the landside slope is calculated with an analytical approximation for the solution of the Bélanger equation for gradually varied flows. The Bélanger equation applied in the BRES model and AREBA model is an analytical approximation of the 1D shallow water equations for quasi steady flows whereby the depth is defined perpendicular to the landside slope, with the bed level gradient, and breadth constant. The accuracy of this equation is $O(\Delta d)^2$ where Δd is the difference between the normal depth and the actual depth. Hence the accuracy of this equation increases as the difference in depth becomes smaller. However for the relatively small depths associated with breach flows and steep slopes, the solution found with the Bélanger equation is a good match for numerical models. Nevertheless the accuracy is directly dependent on the accuracy of the upstream depth boundary condition, and the assumption of a hydrostatic pressure distribution along the landside slope. A comparison of this approximation with the numerical solution of the momentum equations indicates that the analytical solution overestimates the length of the back water curve. A more accurate analytical approximation for the back water curve is foreseen for the near future and to be implemented in the BRES model.

During Stage 3 the weir flow is maintained due to horizontal contraction as the breach has fully formed. During this stage the horizontal contraction can be accurately accounted for in the weir formula. Nonetheless errors in discharge calculations can form due to assumptions in breadth. The breadth term in Eq. [1], indicates a breadth integrated approach whereby the water level does not significantly change in lateral direction. However to balance the effects of horizontal contraction the water level will most likely be higher in the center of the breach than near the sides. This leads to a complicated water surface profile and hence potential errors in the lateral integration process of the flow. Further study to these effects is recommended.

Hence analytical approaches can form a reasonably accurate method for breach flow predictions. Errors that follow from the assumption of a quasi-steady flow are insignificant as the time scale for the flow to travel the length of the embankment surface is small compared to the time needed for the erosion process to have a significant effect on the flow. The main assumptions that limit the analytical models are therefore the assumption of having a constant breadth geometry and a spatially constant bed gradient along the landside slope. It would be possible to extend the analytical approximation to dealing with slopes for which the bed gradient is not spatially constant (during Stages 1 and 2), however this would require a grid based approach and hence significantly increase the run-time of these models, in which case a purposely developed full 1D numerical model becomes a good alternative especially considering that 1D models also deal with variations in breadth geometry. Despite the limitations of the analytical solutions, the models do have the significant advantage that the depth profile along the breach geometry is calculated perpendicular to the embankment surface. Hence the flow velocity component that is calculated is the main flow velocity component parallel to the embankment surface. The shear stresses that follow from these calculations are therefore also the shear stresses on the embankment surface and hence the erosion rates can also be defined and calculated perpendicular to the embankment surface as is done in the BRES model and the AREBA model. Another advantage of analytical models is their speed, which can be crucial in Mont-Carlo simulations. If these models can be calibrated and validated with good breach data sets, then these models remain of importance. One disadvantage of the analytical approaches is that they are limited to modelling failure due to overflow. In the case of wave overtopping the quasi-steady assumption that underlies the broad-weir equation, and the calculations of the depth profiles is no longer valid. Hence, unless some new equations are developed for dealing with wave overtopping, sophisticated numerical breach models need to be applied to model this process.

3. GRID BASED MODELS

To overcome the limitations of analytical flow solvers in breach models of being solely applicable to simplified geometries, and to only being applicable to quasi steady problems, more sophisticated breach models have been developed which incorporate numerical flow models. It is thereby of importance that these flow models are not applied beyond sensible limits. Unfortunately on occasion this is the case with detrimental consequences for the accuracy with which the erosion process is described, as will be discussed in this section. Most of the problems identified with the numerical models in this section are applicable to both modelling failure due to overflow and modelling failure due to wave overtopping. Four different flow modelling methods have been selected for discussion on their applicability into breach models; shallow water equation solvers, the EMBREA flow solver (by HR Wallingford ltd.), 2D CFD models, and 3D CFD models. The generic properties of all these models is that they assume that the acceleration terms do not dominate over the viscous terms in the momentum balance equations, which makes all these models applicable to flows down a steep slope. The applicability of these flow models with respect to breach modelling will be judged based on their ability to deal with variations in flow geometry, their computational expense, and the accuracy with which equations describe the processes.

3.1 Shallow water equations based models

Numerous cases are known whereby breach model developers solve the shallow water equations to determine the flow through a breach (Fujisawa et al., 2014; Wu & Marooli, 2012). In the derivation of the shallow water equations it is inherently assumed that vertical accelerations of the flow are negligible small compared to the horizontal accelerations. The pressure distribution is thereby considered as hydrostatic (Falconer, 1993). An advantage of these models over analytical approaches is that the unsteady solutions could theoretically be used to derive the horizontal velocity components for the unsteady flow over the embankment during wave overtopping. The solution of the shallow water equations solvers consists of the horizontal velocity components. For the one-dimensional shallow water equations this is solely one component and for the 2D model, these are 2 components. The one-dimensional shallow water equations are given by:

$$\frac{\partial b d_x}{\partial t} + \frac{\partial Q_x}{\partial x} = 0$$
$$\frac{\partial Q_x}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q_x^2}{A_x}\right) + c_f \frac{|Q_x| Q_x}{A_x R_x} = -g A_x \frac{\partial h_x}{\partial x} \qquad [2]$$

where b [m] is the flow breadth, d_x [m] the water depth measured vertically upwards, c_f is a roughness coefficient, h_x [m] the water level, R [m] the hydraulic radius, Q_x [m^3/s] the horizontal velocity component multiplied by the vertical wetted cross sectional area bd_x which is equal to A_x [m^2].

During the early stages of breach development the flow lines contract vertically over the embankment crest and the flow accelerates in vertical and horizontal direction along the landside slope (See Figure 1a). As the landside slope retreats towards the waterside slope, either the embankment erodes according to the surface erosion process, or according to the headcut erosion process. In the case of headcut erosion flows separate from the bed at the downstream end of the base of the headcut and subsequently lands on the next headcut base. In this case the assumption of a hydrostatic pressure distribution inherent to the shallow water equations is invalid and modellers are forced to either apply a semi-empirical analytical approach or a more sophisticated numerical method. In the case of surface erosion the erosion is governed by the shear stresses and hence a depth integrated approach would be more viable. As the landside slope retreats towards the waterside slope due to surface erosion, erosion of embankment material takes place normal to the embankment surface.

The solution of the shallow water equations is given by the horizontal velocity component. Effects of flow contraction are not accounted for due to the assumption of a hydrostatic pressure distribution. However the effects of friction can be accurately accounted for. The vertical velocity component along the landside slope of the embankment is moreover significant (see Figure 1a) and may not be ignored. However in many breach models that apply the shallow water equations (Fujisawa et al., 2014; Wu et al., 2012) this is the case. The horizontal velocity component is then used to derive the shear stresses in the horizontal x,y-plane, which are in turn used to determine the erosion rate in z-direction. The significance of this error could easily be quantified using the following example. Assuming that the shear stresses are quadratically dependent of the flow velocities (Van Damme, 2014), the error made in the calculation of the shear stresses on an embankment slope of 1:2 is 25%. Assuming a further underestimation in flow velocity of 10% due to the fact that the shallow water equations do not account for flow contraction, the error could increase further by 20% .

Besides errors made in calculation of the shear stresses that act on the embankment surface, shallow water equations solvers moreover assume that the erosion gives a displacement in bed level in the vertical z-direction whereas erosion acts perpendicular to the embankment surface. Assuming a vertical bed displacement also conflicts with the generic description of a breach formation process whereby the landside slope retreats towards the waterside slope. To model this retreat accurately the significant horizontal component of the erosion rate should be accounted for to accurately describe breach growth.

Hence for an embankment with a landside slope of 1:2 the error made by using the 1D or 2D shallow water equations could easily become as large as 50%. In these cases solutions of models using an analytical approach would be more accurate since, although the flow itself is predicted with less accuracy, the shear stresses on the embankment surface are predicted with higher accuracy. When the retreat of the landside slope has reached the waterside slope the flow contraction drastically increases (see Figure1b) further increasing the error made by the shallow water equations. Only when a full breach has formed, the breach growth is governed by lateral erosion, and the flow lines through the breach are predominantly horizontal. At this moment do the shallow water equations give a more accurate prediction of the flow. Computationally the shallow water equations are much more computationally intensive to solve than analytical approaches, whereas less accuracy is achieved unless special measures are taken.

1.2 Modified grid based approach

One way to incorporate the effects of flow contraction into a numerical model is to add information to the model as is done in the EMBREA model by HR Wallingford ltd. In EMBREA a critical section at the furthest downstream end of the embankment crest is assumed. As the landside slope retreats towards the waterside slope, this point moves upstream. At this critical section, the water depth is assumed to have reached the critical depth. Based on the cross sectional shape of the breach, the model determines an appropriate weir coefficient and using the broad weir equation the breach discharge is determined (Mohamed, 2002; Morris, 2011). The advantage of using the broad weir equation is that the effects of flow contraction can be accounted for in the model. The cross sectional embankment profile is then divided into sections and based on either a 1D energy balance equation, or 1D momentum balance equation the flow depths and flow velocities are determined at each section. Energy and momentum are thereby assumed to be transferred to the embankment due to friction. Because of the 1D approximation, the solution of the equations still exists of the horizontal velocity component. This velocity component is therefore corrected for the local slope using the average of the water level gradient and bed level gradient. The corrected velocity components are then used to determine the bed shear stresses from which the erosion rates perpendicular to the embankment surface are determined. The vertical component of the erosion rate is accounted for in lowering of the bed level. The horizontal component of the erosion rate is accounted for by allowing for grid points to move by the magnitude of the horizontal erosion component.

The spatial scale at which these calculations are performed is initially quite large in EMBREA but due to small fluctuations in water level, the flow calculations are expected to be reasonably accurate. A disadvantage of this method is that the use of the weir equation limits it to quasi-steady solutions. Hence wave-overtopping cannot be modelled with this flow model. The computational expense of running this model is thereby approximately equal to solving the one-dimensional shallow water equations. Despite the fact that EMBREA artificially accounts for flow contraction, and accurately accounts for a vertical velocity component, the depth integrated flow modelling approach limits the use of this flow solver to the cases where the surface erosion process is modelled. Hence for simulating the case of headcut erosion an semi-empirical approach is applied in EMBREA. Hence EMBREA combines the ability of analytical approaches to deal with flow contraction, with grid based methods to account for more complicated geometries and hence is thereby quite versatile in approach and applicable to every breach stage.

1.2 CFD models

To arrive at a method which can be applied irrespective of the type of erosion, the flow needs be solved in the vertical 2D plane. The problem with solving the flow in the vertical domain is that no direct relationship between the pressure gradient and velocity can be given and hence in addition to the mass, and momentum balance equations, the pressure-Poisson equation needs to be solved to enforce a divergent free solution. Moreover is an additional equation required to represent the effects of turbulence. These equations make the flows in the vertical 2D domain more computationally intensive to solve. One could limit the computational times by using a quasi-steady approach to limit the times that the equations need to be solved, and the unsteady approach in the case of failure due to wave overtopping. Current 2D CFD models moreover assume a constant breadth geometry limiting the models in their applicability to problems with a spatially constant breadth, to which analytical approaches are also limited. Moreover is it not possible to account for the effects of horizontal flow contraction which become significant when the retreat of the landside slope reaches the waterside slope. However, unlike the analytical approaches, and depth-integrated models, CFD models are able to accurately model the flow for any type of geometry in the cross section, including for example embankments with a berm, and/or top wall, and can they be applied for simulating headcut erosion processes and surface erosion processes. Although currently CFD models do not account for the flow contraction in the horizontal direction a potential area of development exists there for example by modifying standard CFD solvers to deal with variable breadth geometries (Van Damme, 2014).

Full 3D CFD models are capable of modelling the flow for any type of geometry and for any type of erosion. However the computational challenge and expense of modelling a breach using a full 3D CFD model is significant. However it could proof useful for research purposes and to serve as a benchmark test for the simpler models.

4. DISCUSSION

As can be seen from the SWOT analysis to hydrodynamic models in breach models provided in this paper, analytical approximations are limited to simulating the flow through a simplified geometry with a constant breadth and a near spatially constant landside slope gradient. Moreover can they only be used to describe the depth profile along the embankment in the case of surface erosion, and quasi-steady flows. Nevertheless are analytical models strong in the sense that they are fast to run and that they account for the effects of flow contraction through the use of the weir equation. Moreover do they calculate the shear stress component in the direction parallel to the embankment surfaces. Unfortunately no friction is accounted for in calculating the flow depth profile along the crest leading to a possible underestimation of the flow depth, and an overestimation of the flow velocities along the crest. Last but not least are analytical methods limited to solving breach flows for those cases whereby the breadth geometry is constant along the length of the breach.

Although one would expect vertical two dimensional CFD models to provide a much more accurate description, like analytical methods two dimensional CFD models are currently also limited in their description by having to assume a constant breadth geometry along the length of the breach. Unlike analytical approaches CFD models are able to provide much more detail on the flow profile which could be used to gain better insights into the processes of erosion. Moreover would CFD models be able to model breach progression due to headcut formation as well as breach progression due to surface erosion. Also is it be possible to model flows over more complex structures, like an embankment with a top wall.

Shallow water equations have often been applied to modelling breach formation in embankments, however due to the fact that they do not account for flow contraction, nor for the vertical velocity components, is the error made in the calculations of the shear stresses on the embankment, substantial. However the flow profile itself could quite accurately be described by the horizontal velocity component and water depths. When solely the shear stress components in the horizontal plane are used to calculate the erosion rate in vertical direction, this error could easily become as large as 50% for a landside slope gradient of 1:2. The fact that the horizontal erosion component is not accounted for by the model moreover leads to an incorrect description of the erosion process, identified by the horizontal retreat of the landside slope towards the waterside slope.

Finally the modified grid based approach which was developed by HR Wallingford Ltd. (Mohamed, 2002; Morris, 2011) for modelling breach flows in EMBREA has the advantages of the analytical approach of being able to account for flow contraction in both horizontal and vertical direction. Moreover does it have the advantage of being able to account for the shear stresses in determining the water level and flow velocities at several grid points. Also is the model not restricted to a fixed breadth geometry. Last but not least does EMBREA account for the effects of vertical flow velocities along the landside slope. All combined does this make the predictions with EMBREA potentially more accurate than models based on analytical approaches and more accurate than breach models based on the shallow water equations. Table 1 gives a summary of the SWOT analysis.

Table 1. Summary SWOT analysis of hydrodynamic models in breach models.

	Analytical	SWE	EMBREA	2D CFD	3D CFD
Applicable to many flow geometries	xx	xxxx	xxxx	xxx	xxxxx
Accurate description of the flow	xx	xxxx	xxxx	xxxx	xxxxx
Calculation of the bed stresses and erosion behaviour	xxx	x	xxx	xxxx	xxxxx
Computationally friendly	xxxxx	xxx	xxx	xx	x

(x) Very poor, (xx) Poor, (xxx) OK, (xxxx) Good, (xxxxx) Very Good

5. CONCLUSIONS

The SWOT analysis presented in this paper shows that much improvement can be made in developing ways of modelling the flow through a breach. Moreover does it show that analytical approximations are relatively accurate in their description of the erosion process. The flow modelling method applied in EMBREA (by HR Wallingford ltd.) is currently the most versatile method as it is applicable to variable breadth geometries, accurately accounts for the erosion perpendicular to the embankment surface, and has an acceptable run time. Further development of 2D CFD models for use in breach models by making them applicable to variable breadth geometries is moreover recommended.

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