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Some aspects of hydrostatic restoring for elastic bodies

by

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XXIV International Workshop on Water Waves and Floating Bodies

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The International Workshop on Water Waves and Floating Bodies is an annual meeting of engineers and scientists with a par interest in water waves and their effects on floating and submerged bodies.

The Workshop was initiated by Professor D. V. Evans (University of Bristol) and Professor J. N. Newman (MIT) following informal meetings between their research groups in 1984. First intended to promote communications between workers in the UK and the US interest and participation quickly spread to include researchers from many other countries.

In the organization and conduct of the Workshop, particular emphasis is given to the participation of younger researchers, interdisciplinary discussion between engineers and scientists, and the presentation of preliminary work before it is published elsew Since its inception, the Workshop has grown from strength to strength and annually brings together marine hydrodynamicists, nav architects, offshore and arctic engineers and other scientists and mathematicians, to discuss current research and practical problen Attendance is restricted to the authors of submitted extended abstracts that are reviewed for acceptance by a small committee. Th Proceedings of each Workshop include Introductions with background information, copies of the extended abstracts, and recorded discussions. The success of the Workshops is due not only to the dedication of the participants, but also to the efforts of the host/organizers for each event and to the financial support of many government and industrial sponsors. These organizations and p are identified in each Introduction. Special sessions have been organized at some Workshops to honor individuals who have partici in the Workshops, as well as some mentors who predated the Workshops.

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24th International Workshop
Water Waves and Floating Bodies
Zelenogorsk, Russia

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Workshop Programme

Sunday 19th April 2009

8.00-8.50 Breakfast

9.00-10.00 Registration

10.00-12.00 Excursion around Zelenogorsk

12.00-13.00 Registration

13.00-14.00 Lunch

14.00-14.25 Introduction, welcome

Session 1 – Chair: Y. Kim

14.25-14.50 Grue, J. Modifications to the interfacial wave field moving over variable bottom topography in three dimensions

14.50-15.15 Gang, W., Xiao-bing, S., Xian-qi, Z Comments on no definitive trend for the amplitude of the transverse waves generated by a moving body in a two-layer fluid of finite depth

15.15-15.40 Bulatov, V.V., Vladimirov, Yu.V. Non-harmonic internal gravity wave packets in stratified media

15.40-16.00 Coffee Break

Session 2 – Chair: W.Y. Duan

16.00-16.25 Eatock Taylor, R., Taylor, P.H., Drake, K.R. Tank wall reflections in transient testing

16.25-16.50 Molin, B., Lecuyer, B., Remy, F. Hydrodynamic modeling of partial dikes

16.50-17.15 Farley, F.J.M., Chaplin, J.R., Hearn, G.E., Rainey, R.C.T. Persistent modes for water waves and a bulge tube in a narrow channel

17.15-17.30 Coffee Break

Session 3 – Chair: J .Grue

17.30-17.55 Delhommeau, G., Noblesse, F., Yang, C. Highly simplified Green function for steady flow about a ship

17.55-18.20 Chen, X.B., Lu, D.Q. Time-harmonic ship waves with the effect of surface tension and fluid viscosity

18.20-18.45 Greco, M., Bouscasse, B., Colicchio, G., Lugni, C. Weakly-nonlinear seakeeping model: regular/irregular wave interaction with a ship without/with forward speed

19.00 Welcome party

Monday 20th April 2009

8.00-8.50 Breakfast

Session 4 – Chair: J.N. Newman

8.50-9.15 Faltinsen, O.M., Timokha, A. Analytically-based solutions for linear sloshing

9.15-9.40 Lin, F., Ge, C., Li, E. Computation of sloshing loads by velocity potential analysis and CFD modeling

9.40-10.05 Kulczycki, T., Kuznetsov, N. High spots of the free surface for the fundamental sloshing mode

10.05-10.20 Coffee Break

Session 5 – Chair: E. Campana

10.20-10.45 Abrahamsen, B.C., Faltinsen, O.M. Decay of air cavity slamming pressure oscillations during sloshing at high fillings

10.45-11.10 Iafrati, A. Air entrainment and degassing process in breaking waves

11.10-11.35 Afanasiev, K., Rein, T. Numerical simulation of the dam break problem by general natural element method

11.35-11.50 Coffee Break

Session 6 – Chair: O.M. Faltinsen

11.50-12.15 Yoon, B.S., Semenov, Y.A. Flow separation at the initial stage of the oblique water entry of a wedge

12.15-12.40 Xu, G.D., Duan, W.Y., Wu, G.X. Time domain simulation of water entry of twin wedges through free fall motion

12.40-13.05 Halbout, S., Malleron, N., Remy, F., Scolan, Y.-M. Impact of inflated structures on a liquid free surface

13.05-14.00 Lunch

Session 7 – Chair: T.I. Khabakhpasheva

14.00-14.25 Meylan, M.H., Tomic, M. Resonances and the approximation of wave forcing for elastic floating bodies

14.25-14.50 Bennetts, L.G., Williams, T.D., Squire, V.A. An approximation to wave scattering by an ice polynya

14.50-15.15 Bonnefoy, F., Meylan, M., Ferrant, P. Non-linear higher order spectral solution of a moving load on a floating ice sheet

15.15-15.40 Sturova, I.V. Nonlinear hydroelasticity of a plate floating on shallow water of variable depth

15.40-16.00 Coffee Break

Session 8 – Chair: X.B. Chen

16.00-16.25 Checherin, I., Pustoshny, A. On the estimation of wash effect of ship waves system

16.25-16.50 Westphalen, J., Greaves, D., Williams, C., Drake, K., Taylor, P. Numerical simulation of an oscillating cone at the water surface using computational fluid dynamics

16.50-17.15 Ermanyuk, E.V., Gavrilov, N.V., Kostomakha, V.A. Impact of a circular disk with flat, concave and convex bottom on shallow water

17.15-17.40 Colicchio, G., Greco, M., Miozzi, M., Lugni, C. Experimental and numerical investigation of the water-entry and water-exit of a circular cylinder

17.50 Meeting of the Workshop Committee

19.00 Dinner

Tuesday 21st April 2009

8.00-8.50 Breakfast

Session 9 – Chair: N.G. Kuznetsov

8.50-9.15 Dobrokhotov, S. Complete and explicit asymptotics of solutions to the linearized shallow water equations generated by localized perturbations

9.15-9.40 Porter, R., Evans, D.V. Estimation of wall effects on floating cylinders

9.40-10.05 Voisin, B. Added mass for wave motion in density-stratified fluids

10.05-10.20 Coffee Break

Session 10 – Chair: I.V. Sturova

10.20-10.45 Andronov A.N. On the stability of bifurcating solutions in some problems about capillary-gravity waves

10.45-11.10 Evans, D.V., Peter, M.A. Reflection of water waves by a submerged horizontal porous plate

11.10-11.35 Motygin, O.V., McIver, P. Trapping of gravity-capillary water waves by submerged obstacles

11.35-11.50 Coffee Break

Session 11 – Chair: B. Molin

11.50-12.15 Kim, Y., Kim, K.H., Kim, Y.H. Linear and nonlinear springing analyses in time domain using a fully coupled BEM-FEM

12.15-12.40 Malenica, S., Molin, B., Tuitman, J.T., Bigot, F., Senjanovic, I. Some aspects of hydrostatic restoring for elastic bodies

12.40-13.05 Ten, I., Korobkin, A. Interaction of elastic structure with non-uniformly aerated fluid

13.05 – 14.00 Lunch

Session 12 – Chair: D.K.P. Yue

14.00-14.25 Ferreira, M.D., Newman, J.N. Diffraction effects and ship motions on an artificial seabed

14.25-14.50 Avni, R., Toledo, Y., Agnon, Y. Linear and nonlinear complementary mild slope equations

14.50-15.15 Aubault, A., Yeung, R.W. Multi-hull interference wave-resistance in finite-depth waters

15.15-15.40 Noblesse, F., Delhommeau, G., Yang, C. Bow waves of a family of fine ruled ship hulls with rake and flare

15.40-16.05 Alam, M.-R., Mei, C.C. Ships advancing near the critical speed in a shallow channel with a randomly uneven bed

16.05-16.35 Coffee Break

Session 13 – Chair: Y. Agnon

16.35-17.00 Hara, T., Kukulka, T. Wave spectrum and breaking wave statistics of growing and mature seas

17.00-17.25 Yan, H., Liu, Y., Yue, D.K.P. Water surface impact of axisymmetric bodies

17.25-17.50 Oh, S.H., Kwon, S.H., Chung, J.Y. A close look at air pocket evolution in flat impact

17.50-18.15 Joncquez, S.A.G., Bingham, H.B., Andersen, P.

A comparison of methods for computing the added resistance of ships using a high-order BEM

18.15-18.40 Aranha, J.A.P. Asymptotic approximation of the flow around a slender cylinder: the Ginzburg-Landau equation

19.00 Banquet

Wednesday 22nd April 2009

8.00-8,50 Breakfast

Session 14 – Chair: D.V. Evans

8.50-9.15 Zhao, B.B., Duan, W.Y., Chen, X.B., Webster, W.C. Tsunamis simulations by using Green-Naghdi theory

9.15-9.40 Kimmoun, O., Scolan, Y.-M. Generation of focalized wave packet

9.40-10.05 Duan, W.Y., Zhang, T.Y. Non-reflecting simulation for fully-nonlinear irregular wave radiation

10.05-10.30 Dingemans, M.W., Klopman, G. Effects of normalisation and mild-slope approximation on wave reflection by bathymetry in a Hamiltonian wave model

10.30-10.50 Coffee Break

Session 15 – Chair: R. Eatock Taylor

10.50-11.15 Chatjigeorgiou, I.K., Mavrakos, S.A. Hydrodynamic diffraction by multiple elliptical cylinders

11.15-11.40 Engsig-Karup, A.P., Bingham, H.B. Boundary-fitted solutions for 3D nonlinear water wave-structure interaction

11.40-12.05 Teng, B., Gou, Y. A time-domain model of internal wave diffraction from a 3D body in a two-layer fluid

12.05-12.30 Peter, M.A., Meylan, M.H. Water-wave scattering by vast fields of bodies such as ice floes in the Marginal Ice Zone

12.30 Closing Workshop

13.00-14.00 Lunch

14.00 Tour to the Krylov Shipbuilding Institute

Transfers to the airport and Saint- Petersburg

Abstract for 24th IWWWFB, Saint Petersbourg, Russia, 2009

Some aspects of hydrostatic restoring for elastic bodies

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(3) Delft University of Technology, Delft, Netherlands

(4) Faculty of Mechanical Engineering and Naval Architecture, Zagreb,Croatia

Introduction

The paper deals with the evaluation of the linear hydrostatic restoring matrix for elastic body. In spite of quite important work on the subject (eg see [1, 2, 3, 4, 5, 6, 7]), the problem still seems to not be fully clear and different expressions proposed in the literature do not match each other!? On the other hand the application of, what seems to be the correct method, leads to some strange results for the internal loads!? The main purpose of the paper is to discuss and compare different methods.

Direct perturbation method

Before continuing, let us just recall the definition of the restoring coefficient which can be briefly stated as the ratio in between the reaction force and the displacement which produces it when the body is moved from initially equilibrated position in calm water. This means that the hydrostatic restoring will be composed not only of the pure hydrostatic pressure part but from all the forces which participate to the initial equilibrium of the body (gravity, concentrated external forces, ...) General situation is shown in Figure 1 (bold letters are used to denote the vector quantities). The instantaneous position of one point on the body is described by the vector $\boldsymbol{r}$ and the corresponding position at rest by the vector $\boldsymbol{R}$:

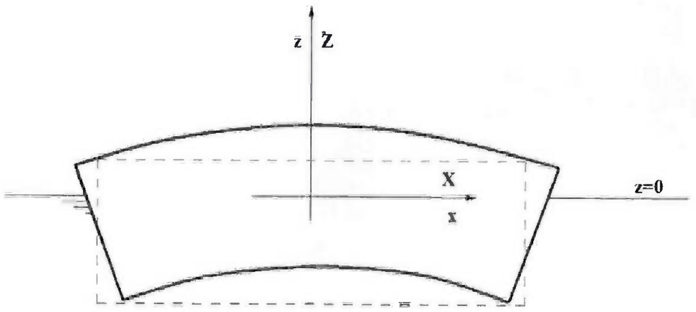

Figure 1: *Generalized body motion.*

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} \quad , \quad \boldsymbol{R} = X\boldsymbol{i} + Y\boldsymbol{j} + Z\boldsymbol{k} \tag{1}$$

The displacement vector for mode j is described by the vector $\boldsymbol{h}^j$:

$$\boldsymbol{h}^j = h^j_X\boldsymbol{i} + h^j_Y\boldsymbol{j} + h^j_Z\boldsymbol{k} \tag{2}$$

where h^j_X, h^j_Y, h^j_Z are the arbitrary functions of X, Y, Z.
Within these notations, the following relation is valid:

$$\boldsymbol{r} = \boldsymbol{R} + \boldsymbol{h}^j \tag{3}$$

We define the generalized hydrostatic pressure force, on mode i (note that the coefficient ϱg is omitted throughout whole the paper):

$$\tilde{\boldsymbol{F}}^H_{ij} = -\iint_{\tilde{S}_B} z\tilde{\boldsymbol{h}}^i\tilde{\boldsymbol{n}}d\tilde{S} \tag{4}$$

where the tilde sign " ˜ " denotes the instantaneous value of the corresponding quantity.
The final goal of the analysis is to extract, from the above equation, the leading order term with respect to the small modal displacement $\boldsymbol{h}^j$. In order to do that, we need to rewrite the above expression at the initial body position. We write:

$$\tilde{\boldsymbol{F}}_{ij}^{H} = -\iint_{S_B+\delta S_B} (Z+\delta Z)(\boldsymbol{h}^i + \delta \boldsymbol{h}^i)[\boldsymbol{n}dS + \delta(\boldsymbol{n}dS)] \tag{5}$$

where δ denotes the change of the corresponding quantity due to the body motion/distortion.
It can be shown that the integral over δS_B is of higher order so that only the integral over the initial wetted position S_B remains. The change of each quantity can be obtained using the notion of the deformation gradient [the subscript $_X$ means that the derivatives are to be performed with respect to the coordinate system (X, Y, Z)]:

$$\underline{\underline{\nabla_X \boldsymbol{h}^j}} = \begin{bmatrix} \dfrac{\partial h_X^j}{\partial X} & \dfrac{\partial h_X^j}{\partial Y} & \dfrac{\partial h_X^j}{\partial Z} \\ \dfrac{\partial h_Y^j}{\partial X} & \dfrac{\partial h_Y^j}{\partial Y} & \dfrac{\partial h_Y^j}{\partial Z} \\ \dfrac{\partial h_Z^j}{\partial X} & \dfrac{\partial h_Z^j}{\partial Y} & \dfrac{\partial h_Z^j}{\partial Z} \end{bmatrix} \tag{6}$$

The differential change of the different quantities become:

$$\delta Z = \underline{\underline{\nabla_X(Z\boldsymbol{k})}} \cdot \boldsymbol{h}^j = h_Z^j \quad , \quad \delta \boldsymbol{h}_i = \underline{\underline{\nabla_X \boldsymbol{h}^i}} \cdot \boldsymbol{h}^j \quad , \quad \delta(\boldsymbol{n}dS) = \nabla_X \boldsymbol{h}^j\, \boldsymbol{n} - (\underline{\underline{\nabla_X \boldsymbol{h}^j}})^T \cdot \boldsymbol{n} \tag{7}$$

where overscript ^{T} denotes the transpoze operation.
The final expression for the generalized restoring coefficient becomes:

$$C_{ij}^{H} = \iint_{S_B} \left\{ h_Z^j \boldsymbol{h}^i \cdot \boldsymbol{n} + Z(\underline{\underline{\nabla_X \boldsymbol{h}^i}} \cdot \boldsymbol{h}^j) \cdot \boldsymbol{n} + Z(\nabla_X \boldsymbol{h}^j)\boldsymbol{h}^i \cdot \boldsymbol{n} - Z[(\underline{\underline{\nabla_X \boldsymbol{h}^j}})^T \cdot \boldsymbol{n}] \cdot \boldsymbol{h}^i \right\} dS \tag{8}$$

By using the identity $(\underline{\underline{\nabla_X \boldsymbol{h}^j}}^T \cdot \boldsymbol{n}) \cdot \boldsymbol{h}^i = (\underline{\underline{\nabla_X \boldsymbol{h}^j}} \cdot \boldsymbol{h}^i) \cdot \boldsymbol{n}$ the above expression can be rewritten as:

$$C_{ij}^{H} = \iint_{S_B} \left\{ Z[\nabla_X \boldsymbol{h}^j \boldsymbol{h}^i \cdot \boldsymbol{n} + (\underline{\underline{\nabla_X \boldsymbol{h}^i}} \cdot \boldsymbol{h}^j - \underline{\underline{\nabla_X \boldsymbol{h}^j}} \cdot \boldsymbol{h}^i) \cdot \boldsymbol{n}] + h_Z^j \boldsymbol{h}^i \boldsymbol{n} \right\} dS \tag{9}$$

Molin's formulation

Molin's [3] used quite different method involving the integral transformations in order to represent the restoring in terms of the volume integrals. First we rewrite the general hydrostatic effort in the form:

$$\tilde{\boldsymbol{F}}_{ij}^{H} = \iint_{\tilde{S}_B} z\tilde{\boldsymbol{h}}^i \tilde{\boldsymbol{n}} d\tilde{S} = \iint_{\tilde{S}_B+\tilde{S}_F} z\tilde{\boldsymbol{h}}^i \tilde{\boldsymbol{n}} d\tilde{S} - \iint_{\tilde{S}_F} z\tilde{\boldsymbol{h}}^i \tilde{\boldsymbol{n}} d\tilde{S} = \tilde{\boldsymbol{F}}_{ij}^{H1} + \tilde{\boldsymbol{F}}_{ij}^{H2} \tag{10}$$

where $\tilde{S}_F$ denotes the instantaneous waterline surface.
The first part of the generalized force $\tilde{\boldsymbol{F}}_{ij}^{H1}$, is transformed into the volume integral:

$$\tilde{\boldsymbol{F}}_{ij}^{H1} = \iiint_{\tilde{V}_B} \nabla_x(z\boldsymbol{h}^i)d\tilde{V} = \iiint_{\tilde{V}_B} (z\nabla_x \boldsymbol{h}^i + h_z^i)d\tilde{V} \tag{11}$$

In order to transform the above integral from the instantaneous position $\tilde{V}(x, y, z)$ into the initial one $V(X, Y, Z)$, the following relations are used:

$$d\tilde{V} = (1 + \nabla_X \boldsymbol{h}^j)dV \quad , \quad z = Z + h_Z^j \quad , \quad \tilde{\boldsymbol{h}}^i = \boldsymbol{h}^i + \underline{\underline{\nabla_X \boldsymbol{h}^i}} \cdot \boldsymbol{h}^j \tag{12}$$

$$\nabla_x \tilde{\boldsymbol{h}}^i = \nabla_X \boldsymbol{h}^i + \boldsymbol{h}^j \cdot \nabla_X(\nabla_X \boldsymbol{h}^i) \quad , \quad h_z^i = h_Z^i + \boldsymbol{h}^j \cdot \nabla_X h_Z^i \tag{13}$$

After inserting the above expressions into (11), the following expression is obtained at leading order:

$$C_{ij}^{H1} = \iiint_{V_B} \left\{ Z[\nabla_X \boldsymbol{h}^i \nabla_X \boldsymbol{h}^j + \boldsymbol{h}^j \cdot \nabla_X(\nabla_X \boldsymbol{h}^i)] + h_Z^j \nabla_X \boldsymbol{h}^i + h_Z^i \nabla_X \boldsymbol{h}^j + \boldsymbol{h}^j \cdot \nabla_X h_Z^i \right\} dV \tag{14}$$

At the same time, the leading order term of the second part of the generalized hydrostatic force is easily obtained as:

$$C_{ij}^{H2} = -\iint_{S_F} h_Z^i h_Z^j dS \tag{15}$$

Newman's formulation

In Newman's formulation [4], the restoring coefficient is defined by the following expression:

$$C_{ij}^{H} = \iint_{\tilde{S}_B} z\tilde{\boldsymbol{h}}^i \tilde{\boldsymbol{n}} dS - \iint_{S_B} z\boldsymbol{h}^i \boldsymbol{n} dS = \iiint_{\Omega} \nabla_X (Z\boldsymbol{h}^i) d\Omega = \iiint_{\Omega} (Z\nabla_X \boldsymbol{h}^i + h_Z^i) d\Omega \tag{16}$$

where Ω denotes the volume in between the instantaneous wetted surface $\tilde{S}_B$ and the initial one S_B. Under the small displacement assumptions we can write $d\Omega = \boldsymbol{h}^j \boldsymbol{n} dS$ so that the final expression for the restoring coefficient becomes:

$$C_{ij}^{H} = \iint_{S_B} (Z\nabla_X \boldsymbol{h}^i + h_Z^i)\boldsymbol{h}^j \boldsymbol{n} dS \tag{17}$$

Equivalence of different expressions

Newman to Molin

In order to compare Newman's formulation to Molin's formulation, first we subdivide the expression (17) in the following way:

$$C_{ij}^{H} = \iint_{S_B} Z\nabla_X \boldsymbol{h}^i \boldsymbol{h}^j \boldsymbol{n} dS + \iint_{S_B} h_Z^i \boldsymbol{h}^j \boldsymbol{n} dS = C_{ij}^{Ha} + C_{ij}^{Hb} \tag{18}$$

The first part is now transformed into volume integral:

$$\begin{aligned} C_{ij}^{Ha} &= \iint_{S_B+S_F} Z\nabla_X \boldsymbol{h}^i \boldsymbol{h}^j \boldsymbol{n} dS = \iiint_V \nabla_X (Z\nabla_X \boldsymbol{h}^i \boldsymbol{h}^j) dV \\ &= \iiint_V \left\{ Z[\nabla_X \boldsymbol{h}^i \nabla_X \boldsymbol{h}^j + \boldsymbol{h}^j \nabla_X (\nabla_X \boldsymbol{h}^i)] + h_Z^j \nabla_X \boldsymbol{h}^i \right\} dV \end{aligned} \tag{19}$$

The second integral is transformed into:

$$\begin{aligned} C_{ij}^{Hb} &= \iint_{S_B+S_F} h_Z^i \boldsymbol{h}^j \boldsymbol{n} dS - \iint_{S_F} h_Z^i h_Z^j dS = \iiint_V \nabla_X (h_Z^i \boldsymbol{h}^j) dV - \iint_{S_F} h_Z^i h_Z^j dS \\ &= \iiint_V (h_Z^i \nabla_X \boldsymbol{h}^j + \nabla_X h_Z^i \boldsymbol{h}^j) dV - \iint_{S_F} h_Z^i h_Z^j dS \end{aligned} \tag{20}$$

It is now easy to see that $C_{ij}^{H} = C_{ij}^{H1} + C_{ij}^{H2} = C_{ij}^{Ha} + C_{ij}^{Hb}$.

Direct to Molin

The original expression (9) is subdivided into two parts:

$$C_{ij}^{H} = \iint_{S_B} Z[\nabla_X \boldsymbol{h}^j \boldsymbol{h}^i \cdot \boldsymbol{n} + (\underline{\underline{\nabla_X \boldsymbol{h}^i}} \cdot \boldsymbol{h}^j - \underline{\underline{\nabla_X \boldsymbol{h}^j}} \cdot \boldsymbol{h}^i) \cdot \boldsymbol{n}] dS + \iint_{S_B} h_Z^j \boldsymbol{h}^i \boldsymbol{n} dS = C_{ij}^{Hc} + C_{ij}^{Hd} \tag{21}$$

As in the previous section, the first integral is transformed into the volume integral:

$$C_{ij}^{Hc} = \iiint_V \nabla_X \left\{ Z[\nabla_X \boldsymbol{h}^j \boldsymbol{h}^i \cdot \boldsymbol{n} + (\underline{\underline{\nabla_X \boldsymbol{h}^i}} \cdot \boldsymbol{h}^j - \underline{\underline{\nabla_X \boldsymbol{h}^j}} \cdot \boldsymbol{h}^i) \cdot \boldsymbol{n}] \right\} dV \tag{22}$$

After rearranging different terms, the above expression can be rewritten in the following form:

$$C_{ij}^{Hc} = \iint_{S_B+S_F} \left\{ Z[\nabla_X \boldsymbol{h}^i \nabla_X \boldsymbol{h}^j + \boldsymbol{h}^j \nabla_X (\nabla_X \boldsymbol{h}^i)] + h_Z^i \nabla_X \boldsymbol{h}^j + \boldsymbol{h}^j \nabla_X h_Z^i - \boldsymbol{h}_i \nabla_X h_Z^j \right\} dV \tag{23}$$

At the same time, the second term is rearranged into:

$$\begin{aligned} C_{ij}^{Hd} &= \iint_{S_B+S_F} h_Z^j \boldsymbol{h}^i \boldsymbol{n} dS - \iint_{S_F} h_Z^j h_Z^i dS = \iiint_V \nabla_X (h_Z^j \boldsymbol{h}^i) dV - \iint_{S_F} h_Z^i h_Z^j dS \\ &= \iiint_V (h_Z^j \nabla_X \boldsymbol{h}^i + \nabla_X h_Z^j \boldsymbol{h}^i) dV - \iint_{S_F} h_Z^i h_Z^j dS \end{aligned} \tag{24}$$

By summing up the two terms we can easily show that $C_{ij}^H = C_{ij}^{H1} + C_{ij}^{H2} = C_{ij}^{Ha} + C_{ij}^{Hb} = C_{ij}^{Hc} + C_{ij}^{Hd}$.

Huang & Riggs formulation

Apparently, the Huang & Riggs [5] formulation is the same as the direct approach except that the term:

$$\delta \boldsymbol{h}_i = \underline{\underline{\nabla_X \boldsymbol{h}^i}} \cdot \boldsymbol{h}^j \tag{25}$$

is omitted. This makes the Huang & Riggs formulation different from the others.

Discussions

The above expressions represents the hydrostatic pressure part only and the gravity related part should be added in order to obtain the final expression for the restoring. This gravity related part should be the same for all the approaches, and can be derived in the following form:

$$C_{ij}^g = g \iiint_{V_B} (\boldsymbol{h}^j \nabla_X) h_Z^i dm \tag{26}$$

One possibility to check the validity of different formulations is to calculate the well known restoring matrix for rigid body modes of motions. Indeed, the six rigid body modes of motion can be defined as:

$$\boldsymbol{h}^1 = \boldsymbol{i} \quad , \quad \boldsymbol{h}^2 = \boldsymbol{j} \quad , \quad \boldsymbol{h}^3 = \boldsymbol{k} \quad , \quad \boldsymbol{h}^4 = \boldsymbol{i} \wedge (\boldsymbol{R} \wedge \boldsymbol{R}_G) \quad , \quad \boldsymbol{h}^5 = \boldsymbol{j} \wedge (\boldsymbol{R} \wedge \boldsymbol{R}_G) \quad , \quad \boldsymbol{h}^6 = \boldsymbol{k} \wedge (\boldsymbol{R} \wedge \boldsymbol{R}_G) \tag{27}$$

where $\boldsymbol{R}_G$ denotes the vector position of the center of gravity.
When applying the above discussed formulations, to these modal functions, the classical restoring matrix for rigid body is recovered by all the formulations except the one given by Huang & Riggs.
It is however not fully clear if the Huang & Riggs formulation should be compared directly to other formulations, since their formulation includes also some other terms such as the internal geometric stiffness. At the same time, the other formulations still have some problems in evaluating the internal loads!? All this will be discussed more in details at the Workshop.

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