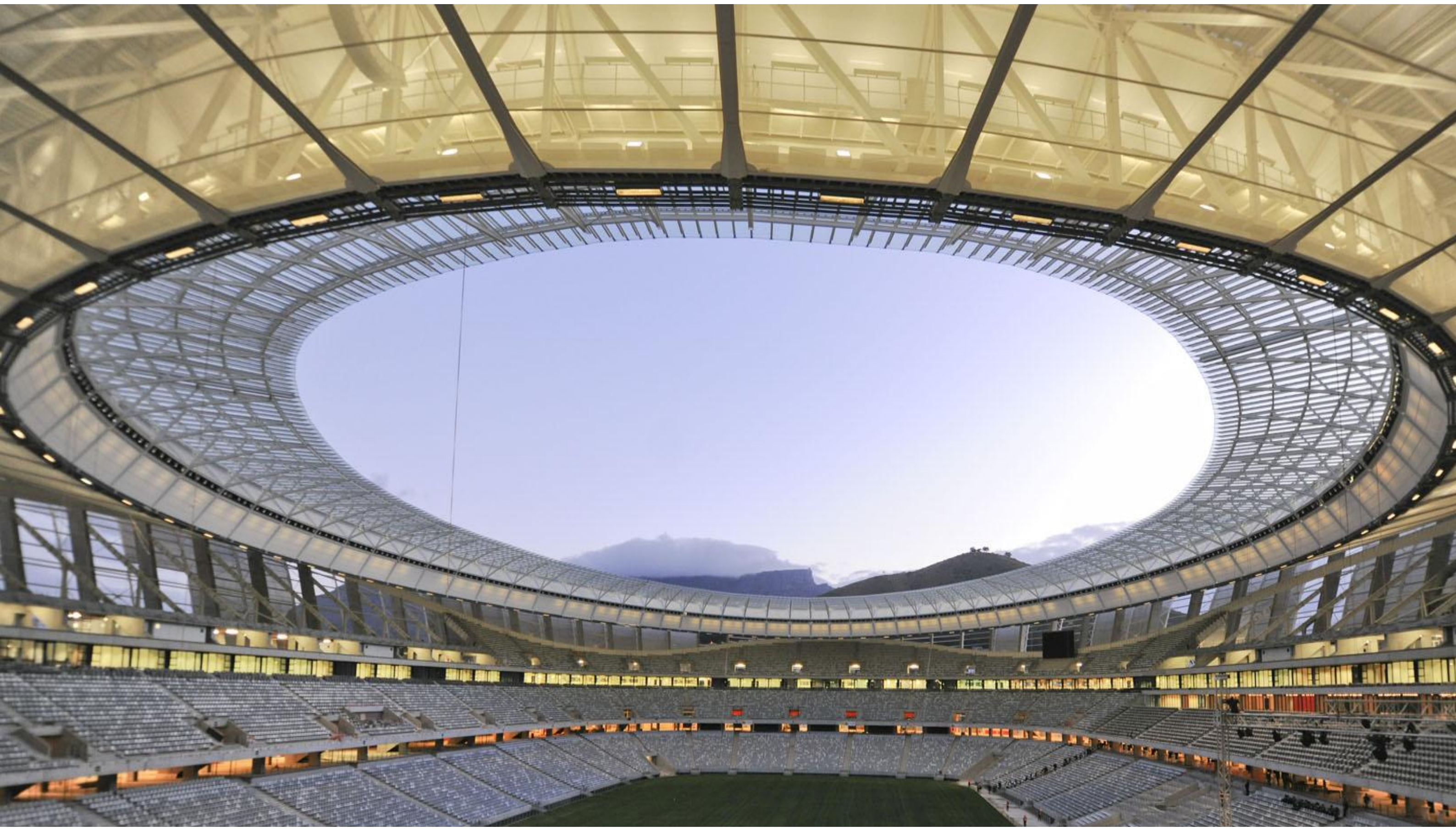
Tensile-compression ring

A study for football stadia roof structures



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Preface

This master thesis report is the result of the graduation work of BSc. Ivar Boom at the Faculty of Civil Engineering and Geosciences at the Delft University of Technology. The master thesis project is the final phase of the master Building Engineering to prove the graduate student is a worthy engineer.

The subject of this master thesis project is:

'Tensile-compression ring; a study for football stadia roof structures'

The goal of the subject is the investigation of the use of a tensile-compression ring structure to football stadia, how to use its benefits at full extend and to become an efficient roof structure for football stadia.
The thesis is written at Arcadis Building Division in Rotterdam. Due to the great amount of knowledge of the people at Arcadis about the structural design of stadia, I could do this graduation work under perfect conditions.
I would like to thank ir. André de Roo (Arcadis) for making it possible to conduct my graduation project at Arcadis. Next I would like to thank my daily supervisor at Arcadis, Jan Rodenburg, who supported me and my project with his knowledge about stadium design. Furthermore I would like to express my gratitude to prof. ir. R. Nijsse, prof. ir. F.S.K. Bijlaard and ir. S. Pasterkamp for supervising my graduation project.
At last I would like to thank ir. P.C. Kuiper and ir. M. Smith for supporting me with the FEM program Scia Engineer.

I hope everyone will enjoy reading this paper and will be interested and enthusiastic for the field of structural and building engineering.

Rotterdam, April 2012

Ivar Boom

Abstract

Part 1 Introduction

A tensile-compression ring is derived from the spoke wheel principle. The wheel is one of the greatest inventions in mankind. The use of the wheel has developed through time. Engineers found out that the use of the spoke wheel as a roof structure provides many benefits. With the spoke wheel principle a lightweight, cost-efficient roof structure can be made.
In recent years the spoke wheel principle has also been applied for oval shaped roofs in order to increase the application of the principle. An example is the use of the roof structure for football stadia. The problem is that the spoke wheel principle will only work at full extend when its shape is completely circular. The question rises if the use of a spoke wheel principle for football stadia roof structure is still attractive? In this thesis a study is made for the use of the spoke wheel principle for football stadia. The research question is as followed:

'To what extend is the application of the spoke wheel principle feasible and efficient for football stadia roof structures?'

For the research a design will be made of a spoke wheel roof structure, in order to gain knowledge and insight in the use of the spoke wheel principle for non-circular roofs. For the design the ground plan of a reference stadium has been used.

Part 2 Analysis

Spoke wheel principle
A spoke wheel consists of three elements: the rim, the hub and the spokes that connect the ring and hub. The interaction between these elements is what makes the spoke wheel so special.
The strength and stiffness of the wheel depends on the amount of ring action of the structure. To provide ring action the rim must be compressed, for instance by pre-tensioning the spokes. When radial tensile forces act on the rim, the rim becomes compressed due to curvature of the rim. The higher the pre-tension in the spokes, the more compression forces arise, the stronger and stiffer the wheel becomes.

The bicycle wheel is mainly radial loaded at the point where the wheel makes contact with the ground due to the dead load of the cyclist. In this load affected zone the spokes become compressed. By using pre-tensioned spokes, the high level of tension prevents the spoke from buckling. By pre-tensioning the spokes the wheel possess great strength for so little weight. The reason is that its principal elements, the spokes and rim, are loaded almost exclusively in normal forces, both tension and compression. The amount of bending in the wheel is minimal. The strength and stiffness of the wheel is influenced by several design parameters. The strength and stiffness increases with increasing pre-tension of the spokes, space flange of the hub (only in lateral direction), amount of spokes, tightness of the spokes, cross sectional area of the rim and load affected zone.

The conditions for a spoke wheel roof structure are different compared to a bicycle wheel. First the leading load is directed transverse to the roof. To withstand the dead load of the structure and additional variable loads (snow, wind, etc.) the roof needs to have sufficient transverse strength and stiffness. The second condition difference is the way the structure is supported. In a bicycle wheel only the hub is supported by the frame of the bicycle. In a roof structure the complete ring needs to be supported. The way of supporting the roof structure is very important. To provide ring action, the roof needs to be able to translate in its radial plane. Roll supports or rocker bearings needs to be applied to provide a free translation in this direction.

In the past, engineers have adapted the shape of the spoke wheel to increase its application, for instance for football stadium use. Engineers have first managed to create an opening in the roof by adding an extra inner ring to the structure. The following adaption was the deformation of the circular shape into an oval shaped roof. Although the structural efficiency of the spoke wheel decreases, the spoke wheel principle can be applied for a stadium roof structure. By adding an extra inner or outer ring, the strength of the structure could be increased to the oval shaped roofs.
Instead of using pre-tensioned spokes in the form of cables, it is also possible to use non pre-tensioned spokes with regular steel profiles. In case of using regular steel profiles beam action will play a role in the stiffness capacity of the roof structure. An example is the Feyenoord stadium in Rotterdam. A By using a spatial truss system, the amount of bending moments can be reduced.
Examples of cable structures (pre-tensioned spoke structure) that use the spoke wheel principle are the Commerzbank Arena in Frankfurt and the BayArena in Leverkusen. In these structures beam action does not play a role in the stiffness of the roof structure.

From the theory of the spoke wheel principle can be concluded that the ring action in a spoke wheel roof structure depends on four key factors. These are: curvature of the ring, loads acting on the ring, extensional rigidity of the ring and the ability to translate.
From the parameters that influence the bicycle wheel and the developments that have been made to increase the application of the spoke wheel principle, design variables can be determined that together will form a structural

design. In the following part, the influence of the design variables on the key factors (ring action) and the strength and stiffness of the roof are investigated. The relation between the design variables and the key factors is illustrated in figure 2.31.

The design variables are the following:

1. Shape of the (opening of the) roof
2. Double inner / outer ring
3. (Non) pre-tensioning of the spokes
4. Profile / elements
5. Supports / connections

Reference stadium
For the study of the use of the spoke wheel principle for football stadia a design of a spoke wheel roof is made. For the design a ground plan of a typical football stadium is used; the Amsterdam ArenA. Because no single shape of a stadium is equal, there has been chosen to investigate one typical ground plan of a football stadium. By using an existing ground plan, the shape of the perimeter of the roof and the place of the supports are fixed.
The roof must fulfil certain requirements and conditions. Besides the structural requirements (Eurocode) there are additional requirements. For instance: lines of sight, protection against the elements and quality of the grass. The requirements and conditions together form a design area in which the designer is free to move.

Part 3 Preliminary Design

Shape study
Before a design can be made, the shape of the ground plan of the reference stadium is investigated for structural problems and possibilities. The ground plan of the Amsterdam ArenA consists of an oval shaped outer perimeter, with straight sides and an inner rectangular roof opening (figure 6.1).

The possibilities of the structure can be determined by looking at the available ring action. The ring action depends on the four mentioned key factors: curvature, acting loads, extensional rigidity of the ring and translation. The only curvature is present in the corners of the outer perimeter. Only these areas can provide ring action. A condition is that the outer ring is able to translate.
The stress in the ring is provided by the radial load through the spokes due to the transverse load of the roof structure itself (permanent and variable loads) or due to possible pre-tensioning of the spokes. Due to the shape of the reference stadium there will be a non-constant load distribution. Most of the normal forces arise in the corner areas, where the loads are more able to be efficiently transferred due to the ring action. At the straight sides of the structure large deformations arise due to the inefficient load transfer. At these points the spokes are perpendicular connected with the rings (polygon of forces).
To withstand the large forces, the elements need to have sufficient extensional rigidity. The largest normal force will arise in the corner areas. In these areas the loads can be efficiently transferred and the largest normal forces will arise in the spokes and ring elements.

Design variables
The strength and stiffness of the roof structure can be improved by investigating the possibilities of the mentioned design variables above (1-5). The goal is to increase the use of the ring action by investigating the influence of the design variables on the key factors that determine the ring action (figure 2.31).

The first design variable is the shape of the roof structure. The shape of the outer ring or perimeter is fixed. Unlike the outer ring, for the shape of the inner ring there is room for improvement to increase the amount of curvature. It was possible to apply curvature with a maximum radius of 72,72 m at the short side and 2868,06 m at the long side. The amount of curvature at the long side therefore remains very little.

The next design variable is the choice of connection and support. These variables have a major influence on respectively the loads acting on the ring structure and the translation of the roof. For the roof structure rocker bearings are preferred as supporting system. Compared to roll supports, rocker bearings have financial benefits, can take up high tensile forces and it is possible to adapt the stiffness.
Between the ring and spoke elements it is possible to apply hinged and fixed connections. Although fixed connections show higher stiffness and strength results, hinged connection are applied. The reason is that the stiffness is provided by ring action instead of beam action. The loads are taken up by normal forces instead of bending moments and therefore will work more efficiently. Besides, smaller profiles can be used because the amount of bending moments is reduced.
The ring elements however, are fixed connected. The sides of the structure have a low stiffness capacity, large deformations will arise when hinged connections are applied and the structure becomes unstable.

The following design variable that has been investigated is the influence of the double outer or inner ring structure. This design variable is related to the use of the curvature in the structure and strength (EA) of the ring elements. The influence of the application of a double inner or outer ring structure depends on the use of non- or

pre-tensioned spokes. In this stage it is not possible to prove that the use of non- or pre-tensioned spokes provides a more efficient use of the spoke wheel principle for football stadia roof structures.
The main purpose of the use of a double inner or outer ring structure is to provide extra transverse support. The amount of transverse support depends on the amount of ring that can be provided.
Although the inner ring of the reference stadium possesses more curvature compared to the outer ring, a double outer ring is applied to the structure to increase the transverse support. The amount of ring action is not only dependent from the amount of curvature. Acting loads, extensional rigidity and translation is also of importance.
When a double inner ring structure is used, more loads will act on the outer ring structure. Because the outer ring structure has little stiffness that can be provided by ring action, large radial deformations arise at the sides of the outer ring. As a consequence, the inner ring will further deform radial as well as transverse. To prevent large deformations, the stiffness of the outer ring needs to be increased. By adding another ring to the outer ring, the available curvature will provide more stiffness to the structure. Besides, the extra ring increases the amount of extensional and bending rigidity.
The height of the double outer ring structure is 15,00m. This height has been determined by using the Feyenoord stadium as reference project. Roof covering will be placed at the bottom spoke level.

The last design variable is the choice of profile and elements which influences the strength (EA) and the stiffness of the elements itself. Steel is used as material for the design of the main bearing structure. Steel has the possibility to take up compression as well as tensile forces, has great structural properties and very suitable for optimization possibilities. Concrete is only preferable in case of high compression forces.
The choice of type of profile for the ring and spoke elements depends on the final design variable. The structural engineer has the option to use either pretensioned spokes or not. This choice has a great influence on the use of type of profiles. In this stage of the thesis one cannot say if a pretensioned spoke wheel roof shows better results and benefits compared to a non-pretensioned spoke wheel roof for the reference stadium.
For the steel elements either CHS profiles (S235) or cables (Y1770) elements are used for respectively non pre-tensioned spokes or pre-tensioned spokes.

The question that rises at the end of the Preliminary Design part is if a pretensioned or non-pretensioned spoke wheel roof will provide a stable structure and which design will provide the most efficient structure? Or in other words which design is able to fulfil all structural requirements and use the least amount of material and is most cost beneficial? In the following part, a study is made for a spoke wheel roof that uses a truss system and a cable system in order to find an answer to these questions.

Part 4 Detailed Design

Loads
The structural design need to fulfil all structural requirements described in the Eurocode regarding strength, stiffness and stability. However, there are no fixed regulations described in the Eurocode concerning the stiffness of a stadium roof. For the design is assumed that the total additional deformation can have a maximum value of a 1000 mm. For single bar elements, the maximum deformation is set at 1/100 of the total length of the bar. Due to the slope of the spokes, water accumulation is not leading in the design.

The leading variable loads are wind and snow. Dynamic wind loading is not taken into account, due to the lack of information. Fatigue has not been taken into account as well. Only static wind load is used for the determination of the design. For the calculations the test results of the reference stadium, the Amsterdam ArenA, are used for the wind pressure coefficients. For the load combinations wind from two directions are used.
Other loads that are taken into account are snow loads (0,56 kN/m^2) and permanent loads: roof covering (0,05-0,30 kN/m^2) and permanent installations (0,1 kN/m^2). For the calculations four load combinations are used; permanent, permanent + snow, permanent + wind situation 1 and permanent + wind situation 2.

Truss structure
For the detailed design, first a study is made for the use of a truss system for a spoke wheel roof. By means of a spatial truss system beam action will, next to ring action play a role in the amount of stiffness the structure can provide. Three different truss designs are analysed, each using the available ring action differently. The reason is that it is not possible to determine the most efficient structural design in an instant. By looking at three different designs, the chance of creating an efficient design is greater and more insight is gained in the way of using ring and beam action.

The idea for the first variant is to transport the loads to the corner areas. The shape study showed that in the corner areas most of the curvature is present to provide ring action. By directly supporting the weakest points of the roof structure, the inner ring elements at the straight sides, it is assumed that the available ring action is used at full extend (figure 10.11 and 10.12).
The idea for the second variant was to design a more regular, symmetric spatial truss system almost equal to the Feyenoord stadium. It is expected that the ratio ring : beam action is lower compared to variant 1. The amount of transverse support has been increased by applying strong vertical crosses at a slope of almost 45 degrees. The vertical crosses will also guide the forces to the corners of the ring elements and use a part of the available ring action as well (figure 10.19 and 10.20).

The third variant consists of a roof structure composed of crosses in the whole bottom and top spoke level, with the idea that the force finds the fastest or most efficient way to be distributed. The ring action is assumed to be used at full extend like variant 1. For this variant can be assumed that the force distribution is optimal. The elements need to be less strong and therefore smaller CHS profiles can be used. The disadvantage however, is that many elements have a low unity check value meaning that these elements become unnecessary (figure 10.25 and 10.26).

The three designs provide sufficient stability, stiffness and strength to fulfil all structural and additional requirements. The designs are further optimized using parametric modelling.
The first step is to divide the structure into group of elements that have equal purposes, for instance a group of elements that form the inner ring (example: figure 10.8). The following step is to determine the diameter of the CHS profile for that group of elements by looking at the leading element of the group. This process is iterative. When the diameter is known, the last step is to determine the thickness of every single element of that group. Not every element in a group is subjected to the same amount of loads. Material can be saved by applying elements with just enough of material (cross sectional area) to reach a unity check of around 0,90 – 1,00.
After optimization of the three designs, the results showed that the first variant uses the least amount of material: 79,90 kg/m². Variant 2 uses 83,70 kg/m² and variant 3 has an average dead load of 92,61 kg/m².

When a structure uses the least amount of material, it does not mean that the design is also structural efficient. By taking into account the deformation of the designs, one can determine the way of using the available ring action and the efficiency of the structure. The deformation, due to a constant load (snow), of the inner ring has been used to investigate (figure 10.48). Results (figure 10.50) show that the structures all have a maximum stiffness in the corners of the structure and that the stiffness is minimal at the centre of the long side.
Looking at the deformation differences in the ring one can see the use of the available ring action. In a complete circular roof the difference in deformation is zero. When the deformation difference in the roof structure, although the investigated reference stadium is non-circular, is little one conclude that the available ring action is used at great extend assuming the beam action is equal in the whole roof. Variant 3 has the smallest difference (table 10.17), followed by variant 1 and 2. The results confirm the expectations, variant 1 and 3 having the highest ratio ring : beam action. Variant 3 is able to transfer the loads to the curved outer ring elements more efficiently due to the greater amount of elements. The disadvantage is that this results in a higher average dead load compared to variant 1 and 2.
By taking into account the average amount of deformation of the inner ring due to a constant load and the average dead load of the structure, one can determine the efficiency of the variants. By multiplying these values, one knows the amount of steel a structure needs to only deform 1 m¹ at the centre of the long side at the inner ring in case of a constant additional variable load. The results (table 10.18) showed that variant 1 and 3 are the most efficient structural designs, where variant 3 shows slightly better results. The reason why the efficiency values of variant 1 and 3 are very close to each other is because both variants transfer the loads in the same way. The normal force distribution (figure 10.51 and 10.52) confirms this. It can be concluded that the available ring action is used at full extend when the loads from the weakest points of the structure (where there is a lack of curvature) are transported to the areas where sufficient ring action can be provided.

After the completion of the three variants, the designs are tested for structural resilience. A situation where an important element of the roof is damaged due to fire, bad assembly, etc. is simulated by removing leading elements.
Research has showed that none of the variants is able to remain stable when certain crucial elements are damaged. The reason is that the designs have been thoroughly optimized, both the geometry as well as the elements itself, to use as little material as possible. The designs fulfil all structural requirements, but have little reserve in strength and stiffness to provide sufficient structural resilience when crucial elements are damaged.
The leading elements are the centre outer ring elements in the tension ring and compression ring. When the inner tension ring is damaged (element nr. 1 in figure 10.39), the structure will not collapse in any of the three variants. To take up the loads from the compression ring, vertical sloped crosses need to be used (figure 10.41). For the tensile ring, horizontal crosses in the bottom spoke level (figure 10.42) have to be applied. The suggested structure that will provide sufficient structural resilience is illustrated in figure 10.43.

Cable structure
Unlike the truss structure, the cable structure will not provide stiffness due to beam action. The only additional stiffness can be provided by ring action. Because cables behave different from regular steel profiles, first the behaviour of a cable structure with the shape of the preliminary design has been investigated.
For the structural design a cable beam structure has been used. The design of a cable beam structure depends on certain design variables: curvature of the cables, type of cables, dimension of the cables, level of pretension and stiffness of the supporting structure.

The results show that the stiffness requirements are leading. The stiffness of the structure relies on the amount of ring action. Because no ring action is present at the straight sides, large deformations arise at these points. Due to the great deformations, the unity check values of other elements regarding the strength and stability increases. To provide a structural design that fulfils all requirements, the stiffness of the roof needs to be increased. The stiffness of the structure can be increased by increasing the pretension in the spokes and by increasing the stiffness of the supporting structure by using stronger elements.

After a numerous amount of iterative steps some conclusions could be made. Where curvature is present (corners), increasing of the pretension in the cables results in a decrease of the sag of the cables. The reason is that the increase of the pretension only has a small effect on the radial deformation of the outer ring. Because the outer ring possess curvature, it can take up the extra loads due to the ring action (polygon forces figure 11.18). When the pretension will be increased at the straight sides, it has an opposite, negative effect resulting in greater deformations.
Because there is no ring action at the straight sides, the supporting structure need to a very high stiffness. This can be provided by using very strong ring elements. The calculation results showed that the moment of inertia needs to be extremely high in order to fulfil all structural requirements. As a consequence a lot of steel is needed and the structural design becomes inefficient.

To increase the efficiency of the structural design more research is needed. An option is to differ the cable pattern to increase the use of the ring action (for example figure 11.19) or to find a solution to provide very high stiffness in the supporting outer ring structure and still use a low amount of material (figure 11.20). Another more rigorous option is to change the shape of the roof structure in order to create more curvature.

Construction
The way of constructing depends of where the structure is composed of. The construction of the truss structure differs from the way of construction for the cable structure.
For the construction of the truss structure, the roof is divided into 14 components. Outside the stadium, the components are preassembled on site. To support the inner ring, a temporary structure is made at the corners of every component. By using cranes, the components are lifted and are placed on the temporary structure. The deformation due to the dead load of every component at every corner needs to be taken into account to determine the height of the temporary structure. When the components are in place, the temporary structures are removed and the complete inner ring will sag until equilibrium has reached. The final step is the assembly of the roof covering.
For the construction of the cable structure, the roof is not divided into components. The first step is to bring the outer ring elements at its place. The second step is the assembly of the cable net structure. The radial cables and inner ring are laid out on the stands and on the field. By using jackets, the cables are simultaneously pre-tensioned and lifted to its final place. The last step is the assembly of the roof covering.
Reference projects have shown that the construction of a cable structure takes the least amount of time. For example the roof structure of the BayArena in Leverkusen only took two months of construction. Compared to regular stadia roofs, a truss structure still has a short building time.

Building costs
To determine the costs, it is assumed that the amount of costs is directly proportional to the amount of used material. Therefore, variant 1 of the truss structures is most financial beneficial. The estimated building costs for variant 1 is €10,97 mln. Aspects that not have been taken into account are construction, standardization, transport and the complexity of the structure. The truss structure consists of CHS elements each with different diameter/thickness in order to decrease the amount of needed material. This however has a negative effect on the production/assembly/standardization and thus the building costs.
For the cable structure it was not possible to determine the costs of the structure due to the lack of an efficient design. To design a cost attractive design compared to truss variant 1 the cable structure needs to have an average dead load of 48,41 kg/m^2, assuming the use ratio of RHS/cable is 1:1.

Part 5 Conclusion & Recommendations

Conclusion
At the end of the research an answer can be given to the research question. The application of the spoke wheel principle for football stadia, heavily depends on the presence of curvature. The feasibility of the spoke wheel principle for stadia roof structures depends on the shape conditions of the stadium in question.

Research in the thesis has shown that a spatial truss structure has the ability to use the spoke wheel principle even in case of stadium roofs with straight sides. Due to beam action these structures are still able to become an efficient structure. Compared to other type of roof structures (table 10.14) that have been used for football stadia, the designs proved that spoke wheel roofs composed of a truss system is an attractive type of structures even in case of non-circular roofs.
By means of parametric modelling it was possible to decrease the amount of needed material, resulting in a decrease of the building costs. However, the complexity of the structure (nodes), construction, standardization, transport, etc. has not been taken into account. More research is needed to determine the consequences of the complex design of the spoke wheel roof for the building costs.
The use of the spoke wheel principle has its limits. When only little curvature is present, less can be benefited from the structural strengths of the spoke wheel principle. The role of beam action comes more into play when the present curvature in the roof structure decreases. At a certain point, when the amount of curvature is minimal, the spoke wheel principle is no more applicable. But by using a spatial truss system it is possible to transport the loads to the areas that possess sufficient ring action and to eventually come to an efficient design for most of the football stadia.

A cable structure heavily relies on the amount of the available curvature and can only provide stiffness by means of ring action. A cable structure is still feasible for roofs with straight sides in a structural manner. However, looking from a financial and durability perspective the cable structure is not attractive for owners and is very inefficient. It is advised to use a cable structure only for circular or oval shaped spoke wheel roofs.
In table 14.1 a cable roof structure (Commerzbank Arena) is compared with other type of roof structures. The results show that this cable roof structure uses less material compared to conventional roof structures. What this roof and other cable spoke wheel roofs have in common, is that there is curvature present in the whole roof (circular or oval shaped roof). This way it is possible to still design a cable roof structure that uses a little amount of material. Otherwise measures need to be taken in order to become a stable structure.
More research is needed to determine what amount of curvature is needed to come to an efficient cable roof design. Besides, investigation is needed for the possibility to increase the stiffness of the structure and to still come to an efficient design. Possibilities are: using different cable patterns, transport the available ring action better or to increase the stiffness of the supporting structure by means of trusses.

Recommendations
The efficiency of a spoke wheel roof design depends on the amount of ring action the roof can provide. The leading key factor is curvature. The spoke wheel roof is therefore very suitable for circular shaped roofs and a very attractive choice compared to other type of roof structures. When the spoke wheel principle is used for non-circular shaped roofs it is advised to increase the use of the ring action in order to increase the efficiency. The best option is to adapt the shape of the roof. By using a self-supporting roof structure independent from the stands of the stadium, the shape of the roof can be adapted (figure 15.2). When this is not possible, the next best solution is to transport the loads to the areas which provide sufficient ring action (figure 15.3).

When designing a spoke wheel roof, the engineer has the option to use either a spatial truss system or a cable system. In a truss system beam action will play a role in non-circular roofs and will provide a part of the total stiffness. These type of systems are suitable for shapes that lack curvature. It is even possible to come to an efficient design for shapes that have straight sides. A condition is that there has to be sufficient curvature in the corners of the structure.
From a structural perspective, it is advised to use a cable system for circular or oval shaped roofs. Cable systems can only provide stiffness by ring action. In a complete circular roof, a minimum amount of material is needed and is therefore very attractive. Reference projects (chapter 3) showed that it is possible to come to an efficient design in case of oval shaped roofs. When straight sides are present, the cable structure becomes very inefficient. More research is needed to increase the application of the cable roof structure.

For the design of a spoke wheel roof the following aspects require further investigation:
- Use of fatigue and dynamic loading.
- Determination of the distance between the outer rings, angle of the spokes and the height of the rocker bearings (figure 15.1) in order to come to an efficient design.
- Influence of the use of different materials (other than S235 for steel profiles and Y1770 for cables) on the efficiency of the design.
- Interaction between additional requirements (protection against the elements, quality grass, etc.) and the design of an efficient spoke wheel roof.
- Design of efficient spoke wheel roof that has sufficient structural resilience.
- Building costs due to construction, transport, complexity of the structure (nodes), standardization, etc.

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Part I
Introduction

Introduction

During the introduction, the subject of the thesis will be introduced by motivating the choice of the subject and the background of the subject. After the introduction the problem and the aim of the thesis is described. The introduction is ended with the description of the scope and the thesis outline.

1. Introduction

At the Building Engineering section of the faculty of Civil Engineering and Geosciences there was a need for research on tensile-compression ring structures for large span stadia. Tensile-compression ring structures show major (non-) structural advantages for the use as a roof structure for large span stadia. The people of Arcadis showed interest in the research for the use of tensile-compression ring structures for football stadia roofs.

A tensile-compression ring structure is derived from the spoke wheel principle. The spoke wheel principle was first used for bicycles and later on found its application in the building industry. The structure contains of three components: a tensile ring, a compression ring and the elements that connect these rings, also called the spokes. Roofing structures relying on the structural behaviour of the spoke wheel principle have been used for many years and have developed further. First the structure was used for circular buildings such as the Madison Square Garden in New York. The spoke wheel principle has expanded with the construction of the Gottlieb-Daimler stadium in Stuttgart. It proved that the principle could also be used for non-circular shaped buildings [16].

When applying a tensile-compression ring roof (from now on called the spoke wheel roof) for football stadia, the structural characteristics are not as efficient as for circular shaped buildings. Despite the disadvantage of the inefficient structural behaviour of the spoke wheel principle for non-circular shaped roofs, this type of roof structure has some major advantages [7] for football stadia:

- Large spans are possible to cover grand stands
- The inner perimeter is completely column free, there are no obstructions between the spectators and the pitch
- The roof has a lightweight appearance
- This type of roof structures lends itself for constructing a new roof on existing stadia
- Transparent or translucent roof coverings are possible
- 'Floating' structure
- Special supporting structure is not needed, the roof can be supported by the framework of the stands
- Relatively low building costs

Recent years a vast amount of football stadia has been built in the world, despite the economic crisis. For coming football events new stadia are being build or old stadia are being reconstructed or redeveloped. An example is the coming European football tournament Euro 2012 in Poland and the Ukraine.
Future stadia are getting bigger and bigger and have to fulfil a constant rising comfort level of the spectators. Nowadays stadia have to provide shelter to all spectators in order to fulfil comfort requirements. To cover large spans often massive roof structures are designed. Examples of these types of roof structures are [13]:

- Spatial structures like domes (Astrodome)
- Arches (Amsterdam ArenA)
- Half cylinder shaped roofs (Gelredome)
- Free cantilevers (Philips Stadium)
- Roof structures supported by columns (Abe Lenstra Stadium)
- Roof structures supported by truss- or arch girders (Stadium Australia)
- Stayed structures (Manchester City Stadium)

The disadvantage of these types of roof structures is that a lot of material is needed, the construction is labour intensive and the building costs are high. Due to the mentioned advantages of a tensile-compression ring roof, there is an increase of interest in this type of roof structure for football stadia.

1.1 Problem description

The general shape of the ground plan of a regular football stadium is in contradiction with the spoke wheel principle. The structural benefits of the spoke wheel principle are used at full extend when the wheel or structural shape is completely circular. The shape of the ground plan of a regular football stadium however, has become more oval and even more rectangular shaped. This development can be explained.
In the past most football stadia had multiple purposes. Besides football, stadia were also used for track and field for instance. These stadia had a bowled or circular shape, due to the shape of the athletics track. Nowadays many football stadia around the globe are specifically used for football. The athletics track disappeared from the image of football stadia.
To offer the spectator a closer orientation to the playing field, the stands are built closer to the field [21]. The consequence is that the inner and/or outer perimeter of the stands of a football stadium is now more rectangular then the football stadia built in the past. The shape of the football stadium has evolved due to this development.

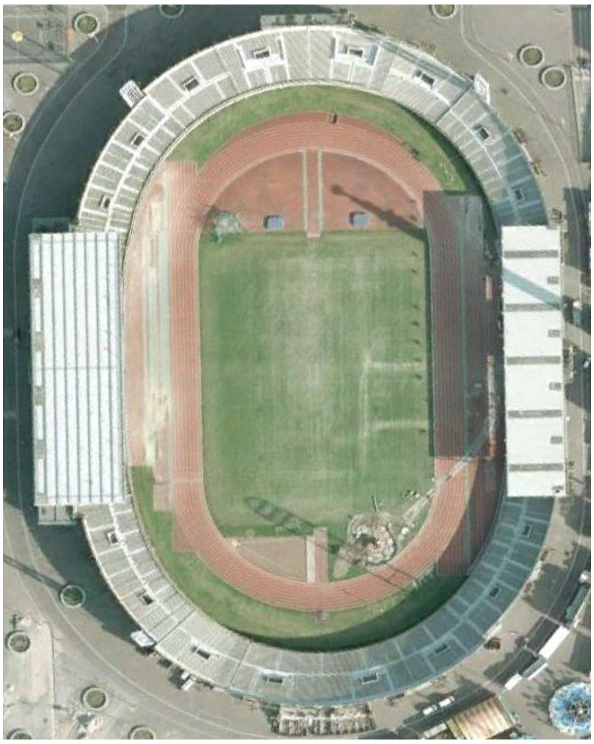


Figure 1.2 Olympic stadium in Amsterdam (1927)

Figure 1.1 Red Bull Arena in Salzburg (2003)

As mentioned earlier, the spoke wheel principle will work at full extend when this principle is applied to a complete circular structure. The question now rises if the use of the spoke wheel roof structure for football stadia is still attractive? Can the structural engineer still guarantee the mentioned advantages of the spoke wheel roof structure for the general (more oval or rectangular) shape of a present football stadium?

To find a structural solution that will solve the problem of the inefficient structural behaviour of a spoke wheel roof structure for football stadia, research is needed.

1.2 Aim of the thesis

The aim of the thesis is to find an answer to the problem described in the previous paragraph. The problem can be defined by the following research question:

To what extend is the application of the spoke wheel principle feasible and efficient for football stadia roof structures?

Efficient in this context is defined as: a spoke wheel roof structure that requires a minimum amount of material, costs and construction time.

To find an answer for the research question, a wide range of aspects need to be investigated. By dividing the research question into different sub-questions, the chance of providing a more complete and correct answer is greater.

The sub-questions that will be investigated are the following:

1. *What is the spoke wheel principle?*
   By investigating the working of the spoke wheel one must understand the principle. By knowing the properties of the spoke wheel, the strong and weak points of the wheel become familiar. This information is valuable for the design of a spoke wheel roof structure.

2. *How can the spoke wheel principle be applied as a roof structure for a football stadium?*
   What is the difference by using a spoke wheel as a bicycle wheel and as a roof structure for a football stadium? By using a spoke wheel as a roof structure, the spoke wheel will behave different and other structural problems can occur. By looking at reference projects, one can see how engineers have solved their structural problems with the use of the spoke wheel principle at present football stadia.

3. *What problems arise with the application of a spoke wheel roof structure to an existing stadium?*
   Before a design can be made, the arising structural problems must be investigated. For the structural problems, solutions can be found using the gained information about the spoke wheel principle.

4. *What are the design variables, what is the influence of the design variables and the consequence on the structural design?*
   There are different design variables that influence the final design of the structure and also each other, positive or negatively. These influences must be investigated to determine the consequences.

5. *Which design variables together form an efficient structural design of a spoke wheel roof?*
   From the different design variables a structural design can be made. Not only have the design variables an influence on the total design, also on each other. A design variable can strengthen or weaken another design variable. The right choice must be made to come to an efficient design.

6. *Is it possible to further optimize the structural design of a spoke wheel roof?*
   After a design has been determined after the investigation of the different design variables, an engineer has to look for more optimizations options for the structure. Is it possible to use the advantages of the spoke wheel principle in another way, or are there other possibilities to come to a more efficient design?

7. *Is a spoke wheel roof still efficient and attractive to be used for football stadia roofs?*
   To be an attractive structural option for football stadia it is important to know if the spoke wheel roof structure really is efficient compared to other type of stadia roofs.

To provide an answer as good as possible, an efficient structural design of a spoke wheel roof structure for a football stadium will be made. By designing such a structure, insight and knowledge will be gained about this typical roof structure.
With the design and research of the spoke wheel roof the sub-questions and eventually the main research question can be answered. The conclusions and recommendations can be used for the design of future stadia using a spoke wheel roof structure.

1.3 Scope of the thesis

To answer the research question a design will be made of an efficient spoke wheel roof structure. An efficient design however, depends on three key elements: the costs, construction time and the structural design itself.
These three key elements have an influence on each other and have shared values, which makes the design process for an efficient roof structure complex. For example: an advantage of using a cable system for the roof structure is the short construction time. The disadvantage however, is that a cable system is costly compared to regular steel structures.
The eventual spoke wheel roof structure depends on many factors that are all related to each other in some way. To come to a design of an efficient spoke wheel roof structure it is impossible to take all factors into account from scratch. There are too many unknown variables to start with. Instead, the problem of the thesis will be prioritised to the structural design of the spoke wheel roof. The first step in the thesis is to come to a structural design of an efficient spoke wheel structure. In a later stage of the thesis the other two key elements (building costs and construction time) will be taken into account. The relation between the key elements and the design parameters is presented in appendix A.1.

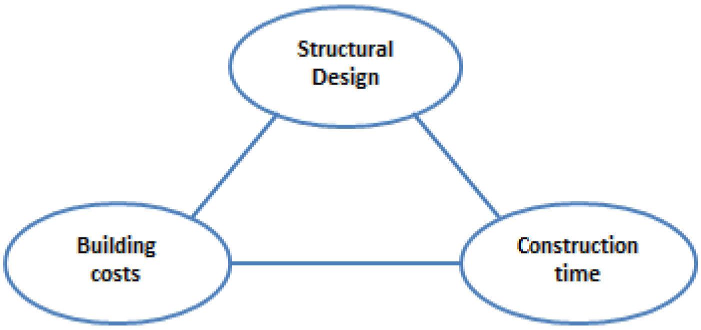


Figure 1.3 Relation between the key elements

The starting points and the project scope are defined as followed:

- To come to an efficient structural design of a spoke wheel roof structure, a basis ground plan of a reference stadium will be used. The reason to use a reference stadium is that no stadium is the same; this is also the case for football stadia. It is impossible to design spoke wheel roof structure that fits every football stadium. Therefore, in this thesis a present football stadium will be used as a reference and basis for the design of the roof structure.

- In practice, the final design of the roof is also dependent from the demands of the owner and the wishes of the architect. These parties can have a negative influence on the efficiency of the structural design of the tension-compression ring structure for the roof of a football stadium. In this project, this is not the case. That way, the most efficient structural design can be made.

- The structural calculations are based on the Eurocode. When the Eurocode is not sufficient, use is made of the Dutch NEN building codes.

- The structural design only concerns the roof structure. The structural design is based on the shape and dimensions of the reference stadium. The roof is supported on the stands of the roof structure, assuming the stands can carry these extra loads. For the design no secondary structures can be used.

- The research focuses on the structural design and analysis of the spoke wheel roof structure. The costs and construction are a smaller part of the research and will be taken into account last.

1.4 Thesis outline

To come to a good conclusion for the purpose of the thesis research about the spoke wheel principle, a structural design of a spoke wheel roof structure will be made. But what exactly is structural designing? Prof. Dr. Ir W. Kamerling from the University of Technology Delft described it as follows: 'Devising a system of elements, where the loads on a building can be transferred to the foundation, in considerations of the constraint arising from the concept for the building' [10].

In the field of structural engineering there are different methods to come to a structural design of a building. This thesis will make use of the method described by Prof. Dr. Ir. W. Kamerling.
The method contains of an analytical and a design component. The analytical component consists of the problem description, collecting of the needed information, determining the definition and classification of the design criteria and the analysis of the different variables that influence the design.
The design component is the creative part of the thesis where the structural design is formed. This component consists of the research for solutions for the different sub problems that arise. The sub solutions are combined and form alternatives. The alternatives will be tested, evaluated and further optimized [10,42].

The thesis is divided into four parts, excluding the introduction. These parts will be briefly described.

Analysis

In this part research is done about the spoke wheel principle (chapter 2). The theory of the spoke wheel principle, the application of the principle as a roof structure and the development as a roof structure over the years is described followed by the analysis of reference projects in chapter 3. The reference stadium that will be used for the structural design of the spoke wheel roof is described in chapter 4. The requirements and conditions concerning the design are determined last in this part.

Preliminary design

Before a final detailed design can be produced, a preliminary design has to be made. The application of the spoke wheel roof at the reference stadium is investigated in chapter 6 (Shape study). To come to an efficient structural design, different design variables need to be investigated. The influence of the design variables on the design are investigated in chapter 7.
After the research of the shape of the reference stadium and the design variables a conclusion can be made and a preliminary design can be determined.

Detailed design

After the preliminary design phase, the design can be further detailed. First the loads acting on the structure are described in chapter 9. The thesis is followed by the determination of the structural design of a spoke wheel roof structure by means of a spatial truss (chapter 10) and a cable net roof (chapter 11). The method of construction and the building costs are the final chapters.

Conclusion and recommendations

Finally a conclusion and recommendations are made where an answer is given to the main research question of the thesis.

Part 2
Analysis

Analysis

In this part of the thesis an analysis will be made about the spoke wheel principle and the way to eventually come to an efficient spoke wheel roof structure for football stadia.
The spoke wheel principle will be analysed as well as reference projects to see how engineers have used the spoke wheel principle on currently build stadia. For the structural design, the ground plan of a reference stadium will be used. The reference stadium will therefore be briefly described.
From the analysis of the original spoke wheel and the reference projects the fundamental requirements and additional requirements can be defined. Finally the conditions of which the design of the spoke wheel roof structure needs to meet are described.

2. Spoke wheel principle

2.1 Introduction

The spoke wheel principle started with the invention of the wheel. The human being has developed the wheel to transport goods, and later on to simplify the transport in general. The wheel is one of the little fundamental inventions that are pure invented by humans, unlike fire and electricity which are more discoveries.

The wheel has been developed through time to optimize its use. In the beginning, the wheel was just a disk that could rotate around its axes. These disks were heavy for transport. An important invention was to use spokes in a wheel to make the wheel lighter and therefore faster.
The first spoked wheels were made of wood. In a later stadium these wooden spokes were replaced by wired spokes. This was a major improvement in the optimization of the wheel. Tensioning the wires made these wheels possible, and with them came the lightweight bicycle that we know today. Wired spokes not only reduced weight, it also improved the durability of the wheel. Today's wire wheels can carry more than a hundred times their own weight [2].

In this chapter the theory of the spoke wheel principle will be explained. To get an understanding of the spoke wheel first the structural behaviour of the original spoke wheel, the bicycle wheel, is explained. It is important to understand why the spoke wheel is so efficient and how this structure can be used as a roof structure. By investigating the use as a roof structure, the strengths and weaknesses are known.

2.2 Bicycle wheel

The bicycle wheel we know today has wired spokes. Although wires are strong, they cannot directly replace wooden spokes that carry loads in compression. When the wheel is subjected to the slightest of loads, the rim would turn into an oval (figure 2.1). To prevent the wheel from buckling, the wires must be tensioned. The wires will buckle at the point where the compression force is larger compared to the amount of tension. The same loads that increase compression in wooden spokes reduce tension in wires.

A wheel with wire spokes works the same as one with wooden spokes. The difference lies in the force distribution in the spokes. In a wooden spoked wheel, force is transmitted from the ground to the hub by compressing the bottom spokes. As a result, this wooden spoke becomes shorter. The bottom spokes of a wired wheel become shorter as well. Instead of gaining in compression, the spokes lose tension.
It is important to keep the bottom spokes under tension in order to remain rigid. This is not possible without the help of the rest of the spokes. The bottom spokes are the only ones that change stress; they are being shortened and must remain rigid to provide enough support to the wheel [2].

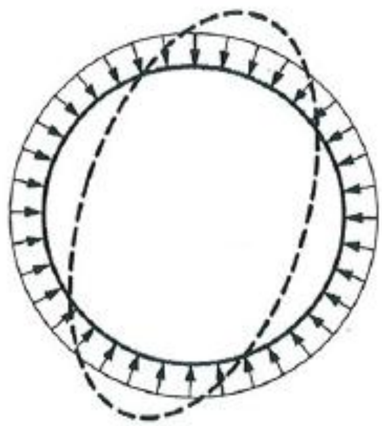


Figure 2.1 Collapse of the wheel without spokes

2.2.1 Ring action

The theory of the ring action is based on three types of equations to determine the force distribution for statically indeterminate structures; the kinematic, constitutive and statically equations [6]. The relationship between the three equations is displayed in figure 2.2.

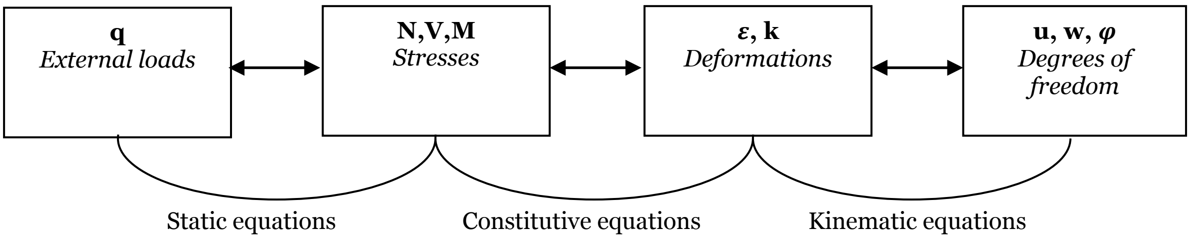


Figure 2.2 Mathematical determination of indeterminate structures. Reproduced from [4].

The ring action is an important structural characteristic of the spoke wheel. To understand the structural theory of the ring action, a curved element of a part of the ring structure is considered (figure 2.3). The ring has a radius r_0 and a normal force N along the longitudinal axis of the element. Following Bernoulli's theory, the rotation of the element on the right side is described as $\Delta d\alpha$. The deformation of the fibre QR to Q'R' is caused by the normal force and the transverse strain.

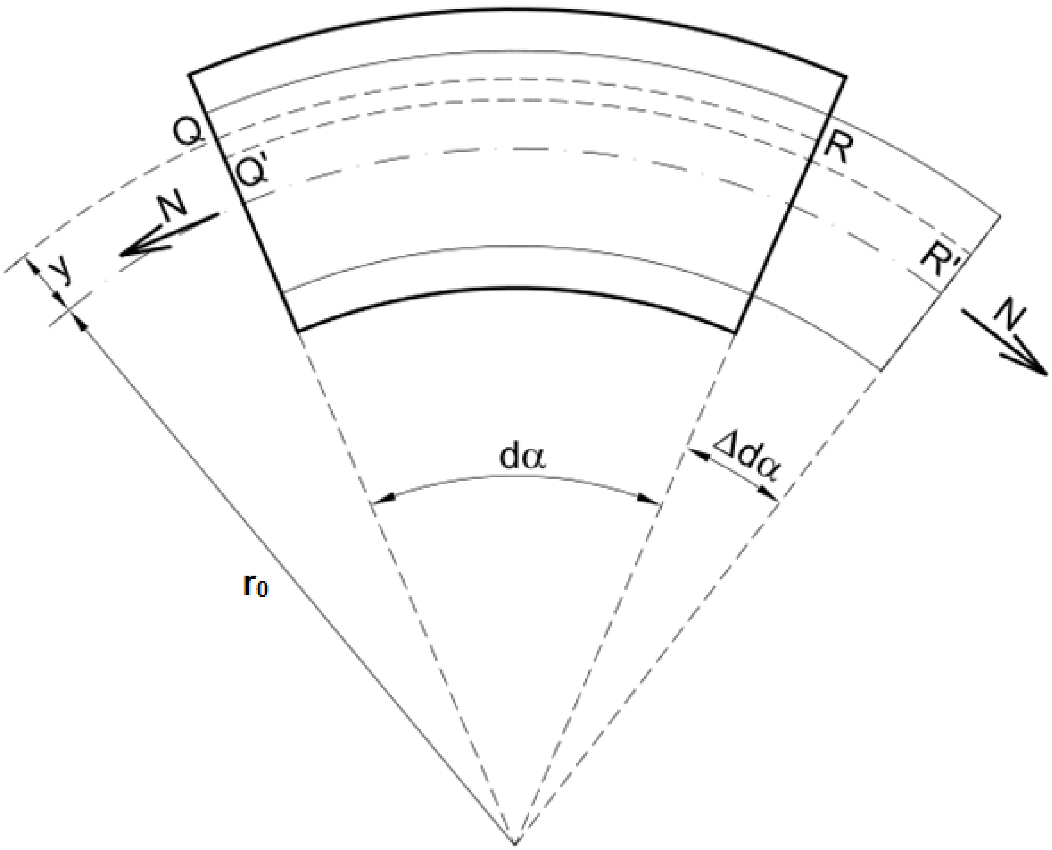


Figure 2.3 Section of a ring element

The strain of the fibre QR of this section of the ring can be calculated as followed:

$$\varepsilon = \frac{\overline{Q'R'} - \overline{QR}}{\overline{QR}} = \frac{(r_0+y)(d\alpha+\Delta d\alpha)-(r_0+y)d\alpha}{(r_0+y)d\alpha} = \frac{(r_0+y)\Delta d\alpha}{(r_0+y)d\alpha} = \frac{\Delta d\alpha}{d\alpha} \tag{2.1}$$

The transverse strain has been neglected with respect to the length $(r_0 + y)$. The calculation remains accurate when the radius r_0 is sufficiently large compared to the thickness of the ring element.

It can be concluded that the strain is independent from y, the position of the fibre with respect to the axis. The strain is equal in all fibres and so in the complete cross section of the ring. When the strain can be assumed as equal in the whole ring, the stress σ can also be assumed equal.

The stress and strain in a curved element can be determined in the same way as in the case of a straight element:

Stress: $$\sigma_x = \frac{N}{A(x)} \tag{2.2}$$

Strain: $$\varepsilon_x = \frac{\sigma_x}{E} = \frac{N}{EA(x)} \tag{2.3}$$

$$\varepsilon_y = \varepsilon_z = -\upsilon\frac{\sigma_x}{E} = -\upsilon\frac{N}{EA(x)} \tag{2.4}$$

The structural behaviour of the ring structure can now be further analysed. Assume a ring structure that is subjected to a distributed radial load (figure 2.4).

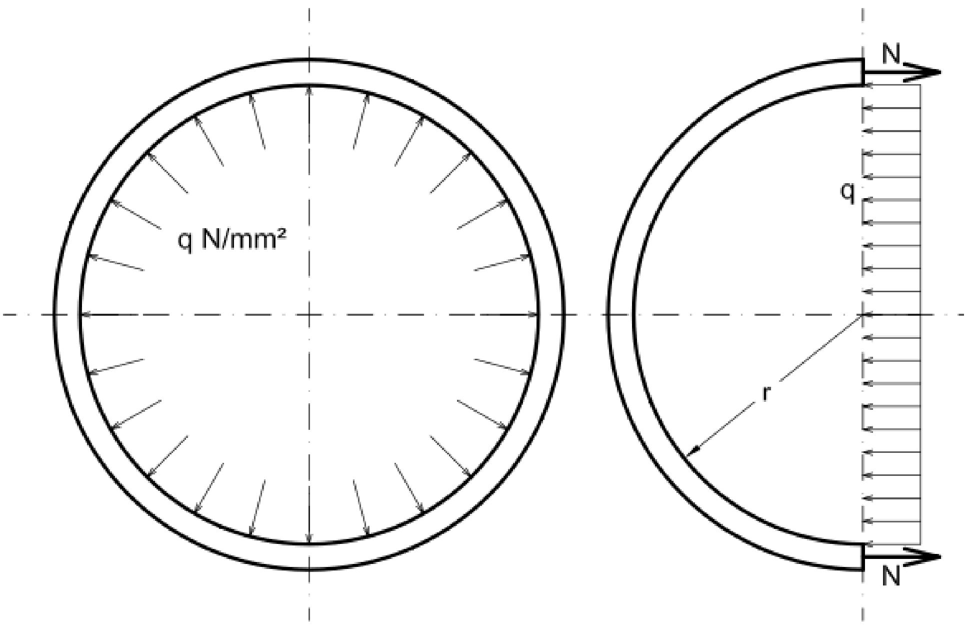


Figure 2.4 Constant load on a ring structure

In a ring with a constant radial load q working on it, the axial force in the ring is

$$N = \sigma \cdot A \tag{2.5}$$

The half ring, pictured on the right in figure 2.7, is in equilibrium when

$$2N = q \cdot D = 2 \cdot q \cdot r \tag{2.6}$$

From equations 2.5 and 2.6 the stress can be expressed as a function of the load, area and radius.

$$\sigma = \frac{N}{A} = \frac{q \cdot r}{A} \quad (2.7)$$

This formula is also called the formula of Barlow [6]. From equation 2.7 can be concluded that the ring action in a bicycle wheel depends on the radius of the rim (curvature), the area of the rim (assuming the rim consists of steel) and the load acting on the rim that all influence the stress value in a rim.

To prevent the ring from deforming to an oval shape, the ring structure must have a stress value unequal to zero. An option is to pre-stress the ring structure. In case of the bicycle wheel pre-tensioned spokes provide the stress in the ring to prevent the ring from deforming.

2.2.2 Loads

A bicycle wheel is loaded statically and dynamically, which can be divided in radial, lateral and torsion components (figure 2.5). The radial load is the leading load in the design of the wheel. A radial load is mainly caused by the weight of the cyclist, resulting in a displacement of the rim toward the hub. The area where the radial load acts on the rim is called the load affected zone. The load affected zone is the part of the rim above the ground contact area, the region that deforms the weight of the cyclist (figure 2.6).
Lateral loads are less decisive. Since a bicycle is ridden by balancing, lateral loads (that bend the wheel sideways) are usually small. Therefore, bicycle wheels need far less lateral strength than radial and torsion strength. In the case of the bicycle wheel torque is a dynamic load caused by pedalling or by a hub brake [2].

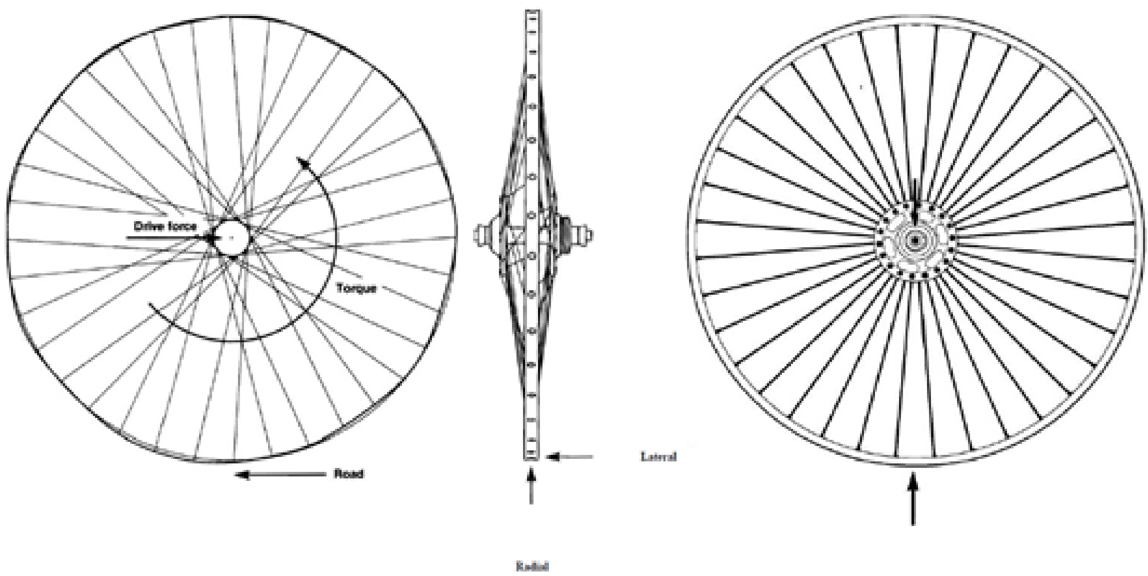


Figure 2.5 Radial, lateral and torsion load components working on the bicycle wheel. Reproduced from [2].

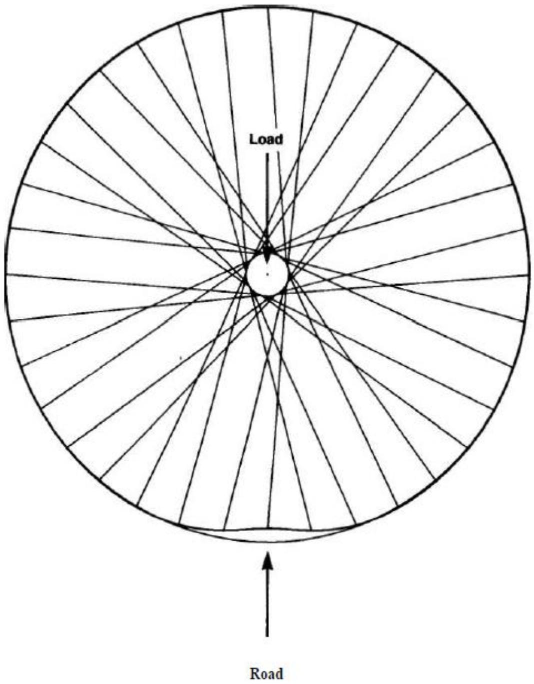


Figure 2.6 Load affected zone at the bottom of the wheel. Reproduced from [2].

2.2.3 Strength

The bicycle wheel has a great strength for so little weight. The reason is that its principal elements, the spokes and rim, are stressed almost exclusively in tension and compression. The amount of bending in the wheel is minimal. Structural engineers have already successfully used these specific properties of the wheel for aircrafts and triangulated bridge structures. All these structures rely on tension and compression, because these forces stress the material uniformly.

Wheels are loaded in radial, lateral and torsion direction. To withstand these loads, the wheel needs to have sufficient strength in these directions. All strength components will improve with increasing spoke tension.
The lateral strength depends on how far apart the hub flanges are spaced. If the spokes are sufficiently tight that they do not become slack from vertical loads, then both lateral and torsion loads are no concerns. These loads are relatively small and do not occur simultaneously with extreme vertical (radial) loads.
The overall strength can be further improved by increasing the load affected zone. A rigid rim combined with many thin spokes will give the longest load-affected zone and the best stress distribution. By lengthening the load-affected zone, a strong rim distributes loads over more spokes than a weaker rim can. Since thin spokes are more elastic than thick ones, they absorb larger rim deflections without becoming slack. The more spokes carrying the load, the stronger and more durable the wheel can be. Wheels used by professionals in classic road races have a good balance between strength and weight.

It can be concluded that the strength of a wheel depends on certain design parameters. A strong wheel has a large cross section rim and many thin spokes at high tension. A large, hollow cross section gives the rim bending and torsion rigidity as well as high resistance to buckling in compression.
The load limit of a wheel is the load at which its spokes buckle. The compressive strength of the rim allows it to carry the force of the many highly tensioned spokes that give high load-carrying capacity. The bridging effect of a rigid rim lengthens the region over which the load is distributed, and a greater number of spokes means that there will be more of them in this region to carry the load [2].

2.2.4 Stiffness

Stiffness is described as the ratio of load to displacement. The radial stiffness of a wheel is a measure of the force required to deflect the rim radial. It is primarily influenced by the number and thickness of spokes and by the depth of the rim. A stiffer rim extends the load-affected zone. With increasing of the load affected zone, more spokes are affected. This results in an increase of the wheel stiffness.
A wheel's lateral stiffness resists sideways deflections of the rim. The lateral stiffness is affected by flange spacing, rim strength, and the number and thickness of spokes.

Like lateral strength the lateral stiffness will improve when the space between the hub flanges increases. The angle between the spokes becomes larger and the lateral support increases. The more a spoke pulls to the side, the greater its lateral support.
Besides the lateral stiffness increases when the radius of the rim decreases (smaller diameter), more spokes take up the loads and the spokes are closely spaced along the rim [2].

2.2.5 Wheel collapse

Wheels can collapse from several causes. However the failure usually occurs the same way. The rim is forced to one side where the tire touches the ground, and the wheel takes on the shape of a saddle.
Another possible cause is the collapse of the wheel when the spokes in its load-affected zone become loose. The load that will cause collapse is roughly equal to the sum of the tension in four or five spokes. Therefore, the tighter its spokes are, the greater the load affected zone, the greater a wheel's load capacity. Another less common failure results from the rim breaking and releasing all spoke tension. This can happen when the wheel strikes a curb or falls into a grating in the road.
Spokes and rims not only fail as a consequence of high or sudden loads. Fatigue is an important fail possibility to taken into account. The fatigue limit is how often the metal can be stretched or bent back and forth in the elastic stress zone before it develops a crack and breaks. For most applications fatigue life is measured in millions of cycles. The closer a cyclic stress is to the elastic limit (the boundary between elastic and plastic zones) the sooner the material will break. The fatigue life of a metal depends both on average stress and stress change. Alone, static stress has no effect on fatigue, but combined with dynamic stress, it does [2].

2.2.6 Conclusion

From the analysis of the spoke wheel principle the first sub-question can be answered, which is described as followed:

'What is the spoke wheel principle?'

The spoke wheel principle is the principle where large loadings can be taken up by a very efficient lightweight structure; the spoke wheel. This lightweight structure has a great amount of stiffness and strength due to the ring action. The ring action depends on the curvature of the wheel, the stress in the rim and the geometry (area) of the rim. The interaction between the spokes and the rim is what makes the structure so strong and stiff for such a lightweight structure.
The structural characteristics of the bicycle wheel show that the wheel structure is very interesting for structural engineers. The lightweight structure has a great amount of strength and stiffness, which shows that the wheel structure is an attractive structure to apply in the building industry.

The properties of the spoke wheel roof can be divided into strong and weak points of the structure. These are summarized below.

Strong points	Weak points
• Completely stressed in tension or compression instead of bending. • High radial and lateral strength and stiffness.	• Small load affected zone, load divided over less spokes. • Rim must be compressed in order to provide ring action. • Subjected to fatigue. • High pre-tension of the spokes needed to prevent the spokes from buckling.

Table 2.1 Strong and weak points of the spoke wheel

Although it seems that the spoke wheel has more weak than strong points, the structure remains a very attractive structure. The lightweight structure and high strength and stiffness capacity is very valuable for the application of the spoke wheel in the building industry.

The strength and stiffness of the spoke wheel are dependent from the amount of ring action. As already mentioned, the ring action determines the structural efficiency of the spoke wheel. The ring action depends on the following key factors: curvature of the wheel, the loads acting on the rim and the area of the rim (assuming the rim is made of steel). The wheel is completely circular; therefore curvature will always be present.
A bicycle wheel has certain design parameters that influence these key factors and therefore directly influence the strength and stiffness of the wheel. The design parameters and their influence on the strength and stiffness of the spoke wheel are presented in table 2.2.

Increase of design parameter	**Strength**		**Stiffness**	
	Radial	Lateral	Radial	Lateral
Pre-tension in the spokes	+	+	+	+
Space flange hub	-	+	-	+
Amount of spokes	+	+	+	+
Space between the spokes	-	-	-	-
Cross sectional area rim	+	+	+	+
Load affected zone	+	+	+	+

Table 2.2 Influence of design parameter on strength and stiffness of the spoke wheel

The design parameters that influence the strength and stiffness of the bicycle wheel are useful for the design of a spoke wheel roof structure. The main difference is that the lateral (or transverse in case of a roof) load is leading instead of the radial load.
From table 2.2 can be concluded that increasing the mentioned design parameters could improve the radial or lateral strength and stiffness of the roof.

2.3 Spoke wheel roof

Engineers found out that the spoke wheel could be very suitable for the use as a roof structure. However, the conditions of a bicycle wheel and a spoke wheel roof structure are very different. The bicycle wheel is more of a structure on its own. A roof structure depends from its supporting structure and is subjected to other kinds of loads (transverse loads are now decisive) and load distributions.
For the investigation of the spoke wheel roof structure physical non-linear and geometrical non-linear performances are not considered.

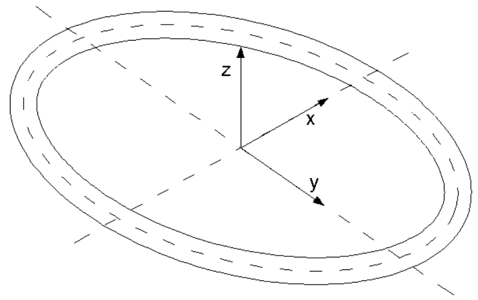


Figure 2.7 Global coordinate system spoke wheel roof

2.3.1 Ring action

The ring action within the ring structure of a spoke wheel roof is, unlike the bicycle rim, influenced by the shape of roof (stadium is more oval shaped, bicycle wheel is always completely circular) and the way the structure is supported. For now is assumed that the shape of the roof is perfectly circular and has no negative influence on the ring action. In a later stage of the thesis the influence of the shape on the ring action will be investigated.

Support of the spoke wheel roof

A bicycle wheel is only supported by the framework of the bicycle which is connected to the axis (central hub) of the wheel. The rim can translate in radial as well as in lateral direction. The situation for the spoke wheel roof is different. The ring of a spoke wheel roof may not translate transverse to its plane, due to structural requirements and to prevent the roof from collapsing.

The way of supporting the ring is very important in order to keep the efficiency of the ring action optimal. The translation transverse to its plane (z-direction) is blocked. In order to use the ring action at full extend the radial translation must be free. When the translation is blocked, there will be only beam action (bending) in the ring in order to withstand the loads. This can be prevented using roll supports. The ring can be schematized as illustrated in figure 2.8 and 2.9.

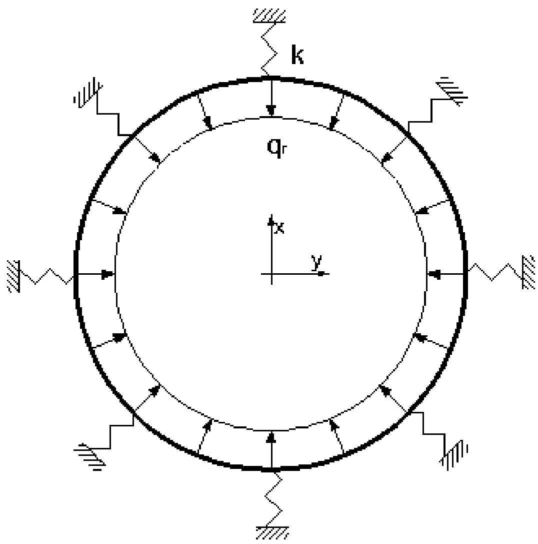


Figure 2.8 Schematization ring structure

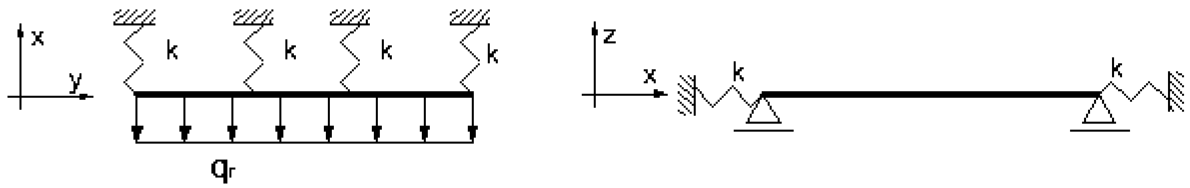


Figure 2.9 Schematization of a ring element in both planes

The ring is supported on roll supports in its plane and has a free translation. However, the ring action provides a certain resistance in this direction due to the stiffness of the ring itself. Therefore the ring has a maximum translation in its plane. The translation resistance in the plane of the roof structure can be schematized as a constant spring system (figure 2.9). The spring constant is set equal to the translation resistance of the ring in its plane.

A radial load q acts on the ring to provide ring action, like in case of a bicycle wheel. In order to find equilibrium, the force in the spring must be equal to the radial load caused by the spokes in the roof structure.

The spring constant is expressed as followed:

$$k = \frac{F_{spring}}{u} \tag{2.8}$$

Where F_{spring} is the spring force and u the deformation of the spring.

In order to find equilibrium in the structure it holds that:

$$q = F_{spring} = k \cdot u \tag{2.9}$$

Normally, a distributed load q is not equal to a spring force F. However, in this situation the spring force F represents a constant spring force over the whole ring. Both loads have the same units: N/m.

The deformation of the spring is equal to the extension of the radius of the ring. The spring constant can be expressed as a function of the radial load q:

With $k = \frac{q}{u}$

and $u = \delta_r = \frac{(2\pi q r^2)/EA}{2\pi} = \frac{qr^2}{EA}$

follows: $k = \frac{EA}{r^2}$ (2.10)

It can be concluded that the ring action is described by equation 2.9. The key factors that influence a spoke wheel roof are:

- Curvature
- Loads acting on the ring
- Extensional rigidity of the ring
- Translation of the ring

The increase of the extensional rigidity has a positive effect on the spring constant capacity and therefore on the strength and stiffness of the ring in its plane (radial). The increase of the radius however, has a negative effect on the spring constant and the strength and stiffness of the ring structure.

For the choice of the support can be concluded that roll supports in the plane of the ring structure should be applied. Due to the application of roll supports, the ring structure can translate in its plane (radial) and the ring action will not be obstructed. However, the structural engineer must take into account that the translation must be limited in order to fulfil structural requirements. The translation cannot be too large so that structural safety cannot be guaranteed any more.

When hinged supports are used, the ring structure cannot deform in its plane and no ring action will arise. The loads acting on the structure can only be taken up by beam action. Due to the radial load acting on the ring structure and the blocked translation, the amount of transverse forces (Vy) and bending moments (Mz) increases. In case of fixed supports there will only be beam action instead of ring action.

Besides roll supports, there is the possibility to use rocker bearings instead of roll supports. Using rocker bearings, the translation in the plane of the ring is not blocked. The rocker bearings provide some resistance, due to the wall bracings that need to be applied to keep the structure stable.

2.3.2 Loads

The principle of the spoke wheel is very suitable to apply to a roof structure. The difference with the bicycle wheel is that the transverse load will be decisive. The transverse load will cause radial loads on the roof structure itself. The radial load acting on the ring is caused by the vertical deformation (up- or downwards) of the central hub and the spokes and by the possible pre-tensioning of the spokes. The radial load is very important, in order to let the ring action work. The ring must be stressed to provide ring action, following equations 2.2 – 2.7.
Another difference between the bicycle wheel and the spoke wheel roof is that the load affected zone is variable. The load affected zone in the bicycle wheel is only the area that is in contact with the surface. In case of a spoke wheel roof, the load can be distributed over the whole structure.

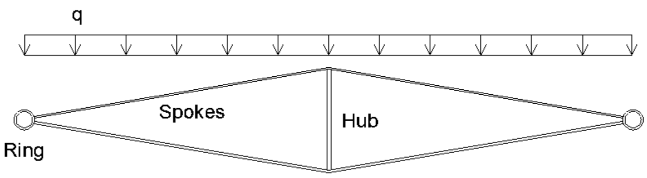


Figure 2.10 Transverse load on spoke wheel roof structure

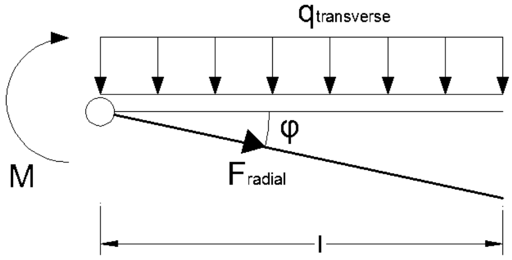


Figure 2.11 Transverse load causing radial load in spoke and bending in ring

In case of a spoke wheel roof, the radial load on the ring cannot be assumed as a constant distributed load. The distances between the spokes are too big to assume a distributed radial load is acting on the ring elements.

The distributed load q in a spoke wheel roof can be determined by dividing the normal force in the spokes by the distance between the spokes x (figure 2.12).

$$q = \frac{N}{x} \tag{2.11}$$

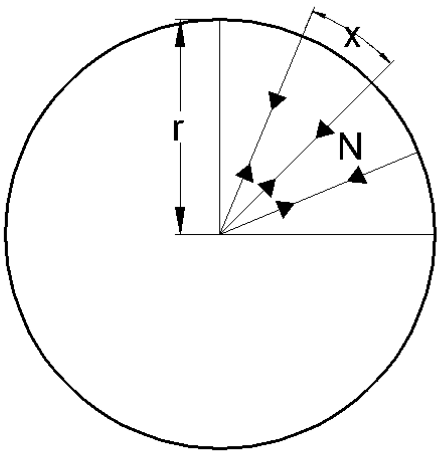


Figure 2.12 Geometry of the spoke wheel roof

With $= \frac{2\pi r}{n}$, n being the amount of applied spokes

The constant load is now expressed as: $q = \frac{Nn}{2\pi r}$ (2.12)

From this assumption and equation 2.5, 2.6 and 2.7 follows that

Stress: $\sigma = \frac{N}{A} = \frac{q \cdot r}{A} = \frac{Nn \cdot r}{2\pi r \cdot A} = \frac{Nn}{2\pi A}$ (2.13)

Due to the stress in the ring, the ring will deform. When tensile arises in the ring, the total ring will extend. In case of compression the ring is compressed. The total extension (in case of tension) of the perimeter of the ring can be determined by integrating the strain over the whole circumference of the ring.

Total extension of the ring: $\delta_o = \int_0^{2\pi r} \varepsilon dx = \frac{\sigma}{E} \int_0^{2\pi r} dx = \frac{2\pi q r^2}{EA} = \frac{2\pi N n r^2}{EA 2\pi r} = \frac{Nnr}{EA}$ (2.14)

Extension of the radius: $\delta_r = \frac{\delta_o}{2\pi} = \frac{Nn2\pi r}{EA}$ (2.15)

From formula 2.14 and 2.15 can be concluded that the extension of the ring increases when the radial load and radius have a higher value. When the extensional rigidity increases (i.e. the cross sectional area of the ring structure), the extension of the ring decreases.
The ring dimensions of a random football stadium are very large. With great dimensions come great loadings. To withstand the high radial loads acting on the ring structure, the ring needs to have a high extensional rigidity value when there is a lack of curvature.

Using the schematization of figure 2.8 and 2.9 the normal- and transverse force and bending moment behaviour in the ring can be explained.
The radial load should be schematized as a point load. As mentioned, the distance between the spokes in a spoke wheel roof structure for a stadium is too large to express the radial load as a distributed load.
The spring force in the xy-plane will be expressed as a constant distributed load, the ring action works as a spring over the whole roof structure. The force and moment distribution of a ring element between two spokes is illustrated in figure 2.14.

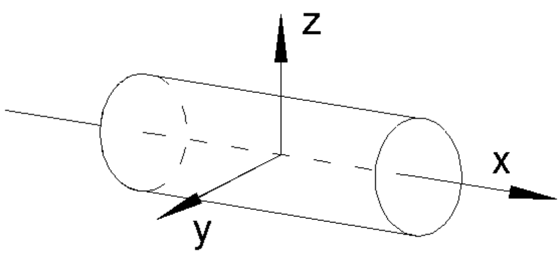


Figure 2.13 Local axes ring element

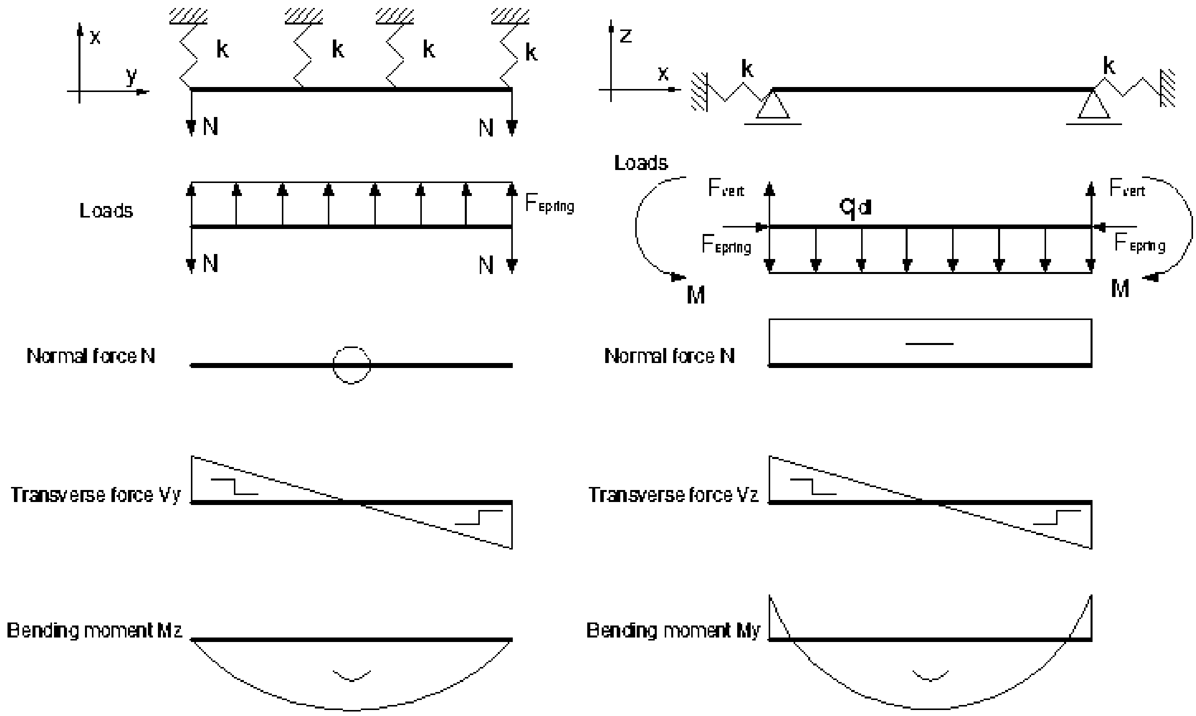


Figure 2.14 Force and moment distribution in the xy- and xz-plane of the ring structure.

In the ring structure torsion (Mx) will not arise in the elements. Because the shape is circular and symmetric in all directions, all spokes and ring elements transfer the same amount of loads. In the ring structure there is equilibrium in its plane. When there is a difference in load transfer in the spokes, torsion could arise.

2.3.3 Strength

Compared to the bicycle wheel, the transverse (or lateral) strength has become leading. One of the design parameters that influence the transverse strength is the space between the hub flanges. The diagonal spokes have a vertical and horizontal force component. The greater the space between spokes or hub flanges, the higher the vertical force component which provide transverse strength. The structural designer needs to find equilibrium to withstand both transverse as radial loads.

The following design variable that influences the transverse strength is the load affected zone. As mentioned in paragraph 2.3.2 the load affected zone in a roof structure is variable. When a distributed load acts on a perfectly circular roof, the load affected zone acts on the whole ring structure. The load is therefore equally distributed over the ring. The greater the load affected zone, the better the load distribution. The load affected zone can be increased by using more spokes and use ring elements with a large cross section.

A large, hollow cross section gives the ring structure bending and torsion rigidity as well as high resistance to buckling in compression. Other options are increasing the amount of spokes and use of pretensioning the spokes.

2.3.4 Stiffness

The roof structure is mainly transverse loaded; therefore the transverse stiffness is decisive in the design of the roof structure. Due to the transverse load, the roof structure will deform in vertical direction. The transverse load can be up or downwards. To fulfil structural requirements, the vertical deformation must be limited by the transverse stiffness of the roof structure. The influence of the ring action on the transverse stiffness will be further explained.

The spoke wheel roof structure can be schematized as displayed in figure 2.15. The ring elements are placed on roll supports to provide a free translation in the plane of the structure. The ring action of the outer ring works like a spring system. The schematization of the structure at mid-span of the structure is illustrated in figure 2.15 on the right. The structure is transverse loaded by a distributed load q, causing a deformation at the inner ring.

Besides a horizontal distributed load q a vertical force acts on the spoke. The vertical force is caused by the ring action and the possible pre-tension in the spokes. Due to this vertical force, the horizontal deformation (expressed as w) will decrease. This is called the 2nd order effect. Eventually a new equilibrium will be found. The final deformation is reached when the resistance against the deformation is equal to the acting force.
The ring action and pre-tension cause a positive effect on the deformation or sag of the spokes in the roof structure. To determine the exact deformation of the structure, special calculation software will be used in the design process.

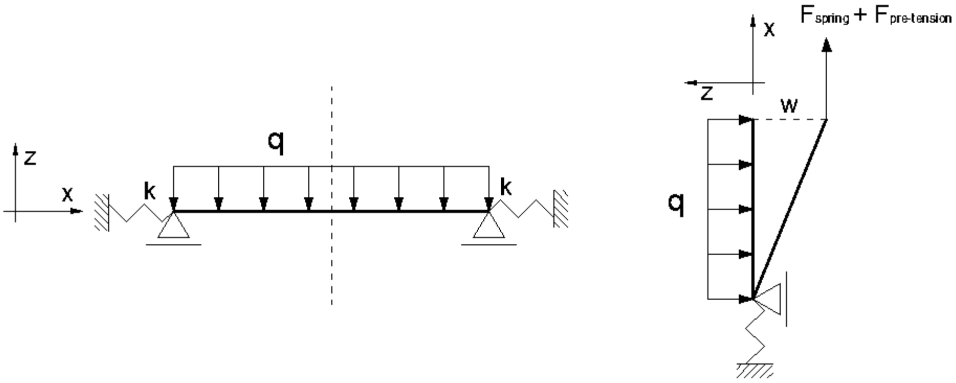


Figure 2.15 Schematization of ring – spoke connection (left) and schematization at mid-span (right)

The transverse stiffness is further influenced, like the transverse strength, by the space between the flanges or distance between top and bottom spokes, the strength of the ring structure and the number and thickness of the spokes.

The radial stiffness is primarily influenced by the number and thickness of spokes and also by the depth of the ring structure. A stiffer ring structure extends the load-affected zone so that more spokes are affected and increases the total stiffness of the wheel.

2.3.5 Roof collapse

The spoke wheel roof structure can collapse from different causes. The ring structure is subjected to high forces. The extension rigidity of the ring elements must be sufficient to prevent the ring from buckling in case of compression. Besides the ring should have sufficient torsion rigidity to withstand high bending moments. As a consequence of the transverse load, the roof will sag. This causes a bending moment in the ring elements (figure 2.16).
The spokes can collapse from fatigue and buckling. A roof structure is subjected to dynamic and static loads. Due to the dynamic loads the amount of stress is highly fluctuating in the spokes. The fatigue life of a metal depends on these stress changes, which can cause breaking of the spokes.
Besides fatigue, the spokes can be subjected to buckling when the amount of pre-tension is not sufficient. At extreme transverse loads, the spokes can be compressed. When the amount of tension is less compared to the arising compression, the spoke will buckle and the roof will collapse.

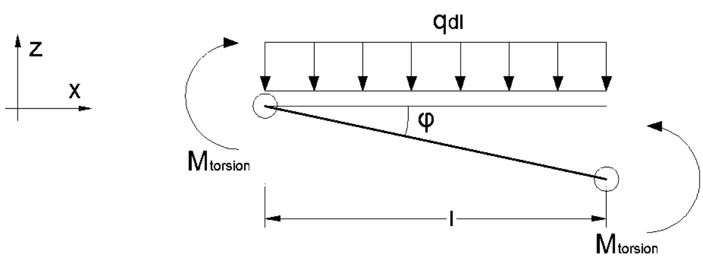


Figure 2.16 Bending moment in ring elements

2.3.6 Conclusion

The spoke wheel roof shows equal characteristics as the bicycle wheel. One of the main differences is the way the leading loads acts on the structure. Due to the transverse loads, the spokes and the central hub will sag. These will cause a radial load. From there the structure behaves similar to the bicycle wheel.
In case of a spoke wheel roof the load affected zone is larger compared to the bicycle wheel. The distribution of the transverse load due to dead weight, wind, snow etc. is generally better compared to the small load affected zone of the bicycle wheel. The load can be divided over more spokes. However, the distance between the spokes of a roof is larger compared to a bicycle wheel. As a consequence the radial load is not equally distributed over the ring.

The second main difference is the way the structure is supported. In a bicycle wheel only the central hub is lateral supported, in a spoke wheel roof this is the case for the complete ring.
To use the ring action at full extend, the radial translation of the ring must be free. When this translation is blocked, the transverse load will only be taken up by the beam action of the spokes. To prevent high bending moments, roll supports or rocker bearing need to be used as supporting structure for the ring.
The ring must provide sufficient ring action in order to prevent the ring from deforming too much. The free translation provided by the roll supports or rocker bearings must be limited to prevent high translations and to fulfil structural requirements.
From these differences compared to the bicycle wheel can be concluded that the ring action in a roof structure not only depends on the curvature, extensional rigidity of the ring and the loads, but also on the translation of the ring.

With these conclusions a part of the second sub-question has been answered. The question was:

'How can the spoke wheel principle be applied as a roof structure for a football stadium?'

The spoke wheel system shows that it is suitable to use as a roof structure. To apply a spoke wheel roof, the engineer must find solutions to withstand transverse loads and the way of supporting the roof to come to an efficient design.

The spoke wheel roof structure can be improved by looking at the design parameters mentioned in paragraph 2.2. The design parameters that influence the stiffness and strength of the bicycle wheel (table 2.2) also accounts for the spoke wheel roof. These are presented in table 2.3. These parameters must be taken into account for the structural design of the spoke wheel roof.

Increase of design parameter	**Strength**		**Stiffness**	
	Radial	Transverse	Radial	Transverse
Pre-tension in the spokes	+	+	+	+
Space flange hub	-	+	-	+
Amount of spokes	+	+	+	+
Space between spokes	-	-	-	-
Cross sectional area rim	+	+	+	+
Load affected zone	+	+	+	+

Table 2.3 Influence of the design parameters on the strength and stiffness of the spoke wheel roof

The roof structure that has been analysed in this chapter is perfectly circular. The stadium roof that will be designed during this thesis had a different, less efficient shape. The influence of the shape of the reference stadium will be analysed in a later stage of the design process.

2.4 Development of the spoke wheel roof

The spoke wheel roof discussed in the previous paragraph, has little application possibilities due to its shape. To increase its application, engineers have adapted the shape over the years. The development and the consequences of these adaptations are described in this paragraph.

2.4.1 History

The spoke wheel finds it origin in the use for bicycles. Engineers found out that the principle of the bicycle spoke wheel could be used for the structural design of buildings. Due to the addition of spokes the wheel is capable bearing transverse loads. By using the wheel horizontally the spoke wheel principle is suited for the design of roof structures.
The spoke wheel principle has been used for multiple types of buildings as a roof structure. Over the years engineers developed the spoke wheel structure to increase the application scope of the structure.
Engineers found out that by using a double inner ring with columns under compression (figure 2.18) an opening in the structure could be created at the central node. The inner rings will transfer the loads through the spokes to the outer ring.

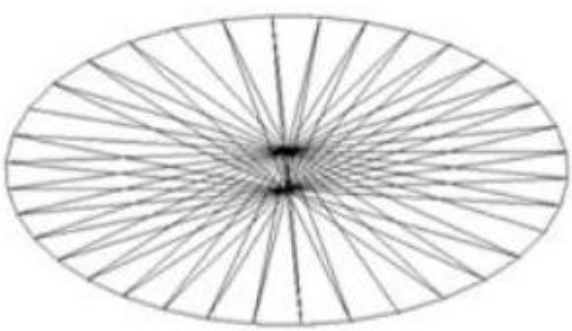


Figure 2.17 Spoke wheel with central node

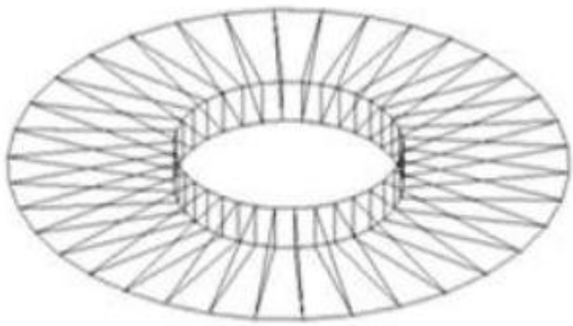


Figure 2.18 Spoke wheel with double inner tension ring

The first spoke wheel roof was built in 1958 for the U.S. pavilion during the World Expo in Brussels. Since then the spoke wheel principle has been used for many large span roof structures like stadia and halls.

A pioneer in the development of the spoke wheel system in the last thirty years was engineer Jörg Schlaich. An important project was the design of the Zaragoza arena roof built in 1988, were the spoke wheel roof was rediscovered and improved. The Zaragoza project was a starting point for other similar projects with more innovative contributions, for instance: oval and rectangular boundaries, different tension and compression ring arrangements and lightweight retractable roofs [22].
As seen in the figures the shape of the structure is still circular and its application in the building industry is still limited due to its shape. To increase the scope of the structure, for instance for the use of stadia, engineers found out with the design of the Gottlieb Daimler stadium that this structure can also be made in an oval shaped structure (figure 2.19).

In figure 2.18 the structure only consists of one outer ring and two inner rings. Instead of one outer ring it is also possible to use two outer rings. And instead of spreading the spokes from the outside to the inside of the structure it can also be vice versa (figure 2.20). Also with two outer rings and one inner ring it is possible to create a hole in the structure and to make an oval shape to increase the scope of the structure.

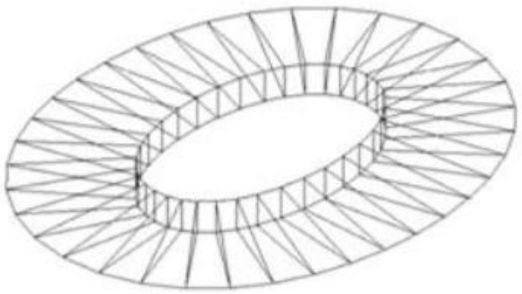


Figure 2.19 Oval shaped structure

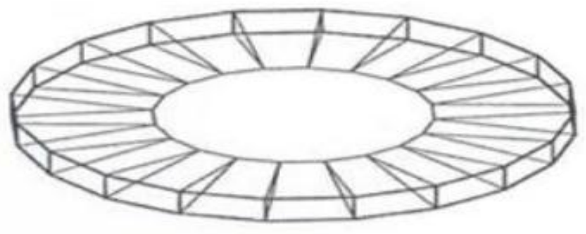


Figure 2.20 Spoke wheel with outer compression ring

The advantage of the use of the spoke wheel system for buildings is that it is a very efficient structure with minimal bearing forces. It is a very light structure and therefore will eliminate the need for costly supports, foundation and material use [21].

After the construction of the Gottlieb Daimler stadium, numerous other stadia have been built with similar spoke wheel roofing. Some examples are the Malaysian National stadium in Kuala Lumpur and the Nigerian National stadium in Abuja. These stadia roofs consist of one compression ring and two spread tensions, all with an oval form [18].

In 2006 the World Cup was held in Germany and the spoke wheel roof was developed further. Several new stadia were built for the World Cup as well as some renovations. One of them was the renovation of the Hamburg stadium were a new roof was built on top of the stadium. The roof of the Hamburg stadium was the first approximation of a rectangular opening for a spoke wheel system.

The latest examples of stadia with a spoke wheel roof are the new Greenpoint stadium in Cape Town and the stadium in Durban for the World Cup held in 2010.

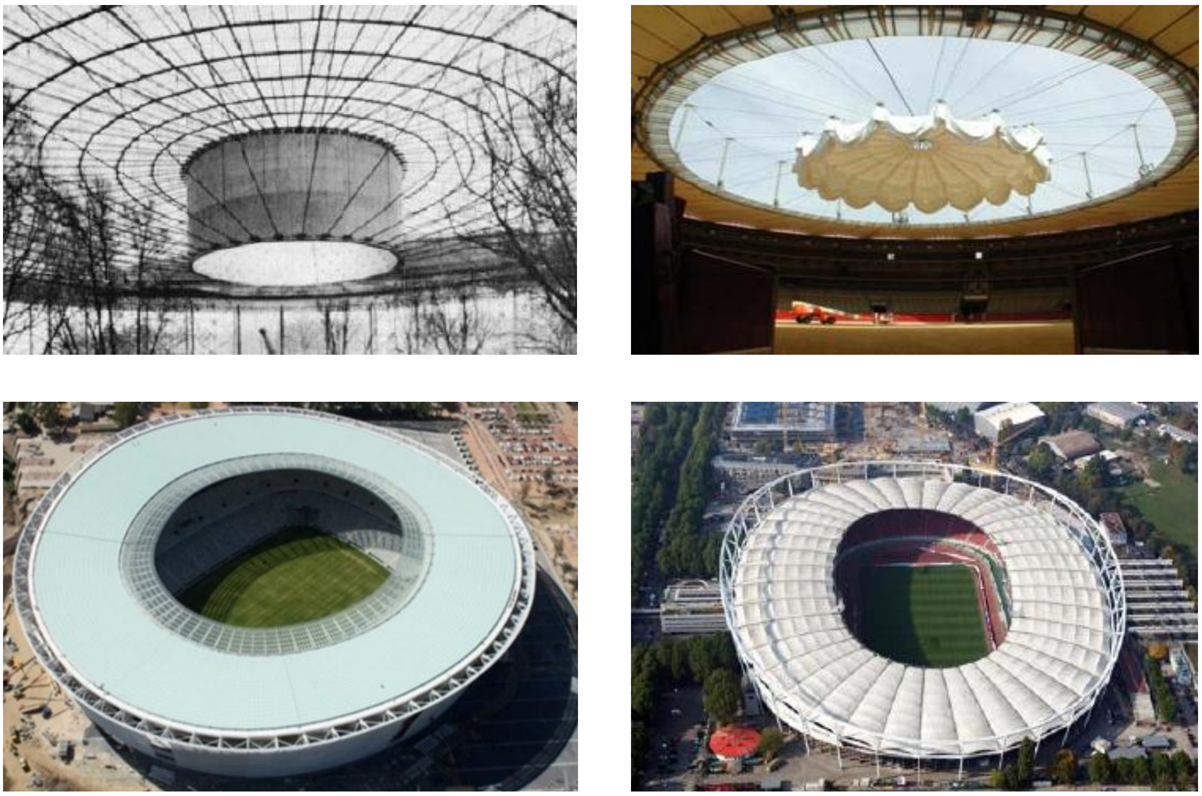


Figure 2.21 Clockwise: US Pavilion, Brussels 1958; Zaragoza Arena, Zaragoza 1988; Gottlieb Daimler Stadium, Stuttgart 1992; New Greenpoint Stadium, Cape Town 2010

2.4.2 Opening in the spoke wheel roof

The first step to increase the application of the spoke wheel roof for buildings was creating an opening in the roof (figure 2.18). By using a double inner ring with columns under compression an opening could be made in the spoke wheel roof.
The addition of an internal ring changes the structural behaviour of the roof structure. In the double inner ring tensile forces arises, assuming the spokes are under pre-tension. The pre-tension force is now indirectly transferred through the inner ring, instead of the direct interaction in case of using only a central hub.

To cover great spans a very high pre-tension value is needed, to fulfil the structural requirements for maximum deformations etc. The interaction between the inner rings, spokes and outer rings is of importance to transfer the forces efficiently. To transfer the forces efficiently, only normal forces should occur in the spokes.

It is important to provide equilibrium in the plane of the structure. When doing so, most of the load is transferred by normal forces and the structure will be very efficient. Equilibrium is provided when the angle between the spokes and ring elements in the outer and inner ring are equal to each other (figure 2.23). In this structure, there will be no loss of normal forces (see explanation polygon of forces paragraph 2.4.3).

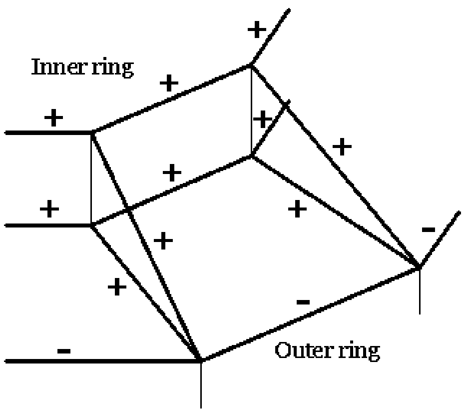


Figure 2.22 Normal force distribution within a pretensioned spoke wheel with double inner tension ring

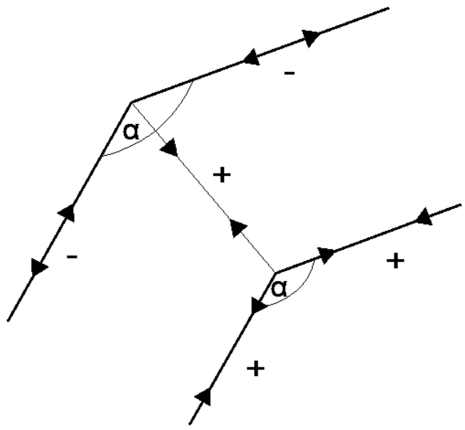


Figure 2.23 Equilibrium in plane

2.4.3 Oval shape

To increase the scope of the spoke wheel structure further, engineers saw the opportunity to deform the spoke wheel roof to an oval shape. This way, the structure is more suitable for example for stadia.
An oval shape however, has a negative influence on the most important structural characteristic of the spoke wheel; the ring action.
On the long sides of the oval, the curvature r is greater compared to the short sides of the structure. The influence on the ring can be explained using equation 2.7, 2.11 and 2.12.

Assume that the value of the loads and the profile of the ring (extensional rigidity) remain constant. When the radius in equation 2.7 increases, the stress in the ring increases as well:

$$\sigma = \frac{N}{A} = \frac{q \cdot r}{A}$$

The consequence of a higher stress value in a ring is that the extension (along and transverse to its longitudinal axes) of the ring increases as well. Using equation 2.11 and 2.12 this can be explained:

Total extension of the ring: $\delta_o = \frac{Nnr}{EA}$

Extension of the radius: $\delta_r = \frac{\delta_o}{2\pi} = \frac{Nn2\pi r}{EA}$

The ring action of the ring structure can be expressed as a constant spring force (figure 2.8). The ring action can be expressed as:

$q = F_{spring} = k \cdot u$

With $u = \delta_r = \frac{(2\pi q r^2)/EA}{2\pi} = \frac{qr^2}{EA}$ and $k = \frac{EA}{r^2}$

The formula describes that the greater the radius, the smaller the amount of ring action will be. On the long sides the radius has a great value, resulting in a decrease of the ring action. The amount of ring action that takes up the loads will decrease.

From earlier analysis has been concluded that the ring action not only depends on the curvature, but also from the extension rigidity of the ring, the loads acting on the ring and the possibility of translation. In contradiction to the original (circular) spoke wheel roof, the curvature of an oval shape is not constant over the whole edge. On the long sides of the oval, the curvature is smaller compared to the short sides of the structure. As a consequence the amount of stress in the ring is not constant and therefore also the ring action and the extension in the ring elements.
On the short sides of the oval ring the radius is smaller compared to the long sides, therefore the stress has a lower value and more ring action is present. Due to the linear relation between the stress and strain, the extension of the radius shows parallel results.

Assume a flat oval shaped model that consists of just a single inner and outer ring, which spokes are not pre-tensioned. The outer ring is placed on roll supports and the whole roof structure is subjected to a uniform distributed transverse load. The inner ring and spokes will sag and as a consequence tensile stresses arise in the spokes and inner ring. The inner ring is now subjected to extension.
Due to the oval shape, the extension at the short sides is less than on the long side. The same occurs in the compressed outer ring. Due to the compression, the outer ring elements deform inwards. Again the deformation on the short side is smaller compared to the long side. Figure 2.24 displays the deformation of the ring in its plane and the deformation difference.

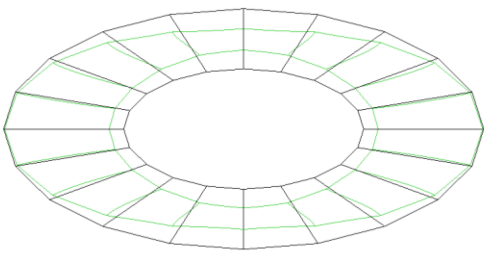


Figure 2.24 Horizontal deformation

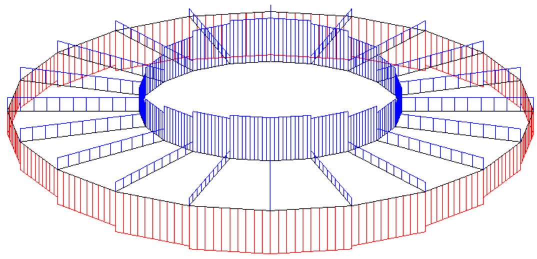


Figure 2.25 Normal force distribution in an oval shaped ring (red = compression, blue = tension)

Not only is there a stress difference in the rings, also in the spokes. Not every spoke – ring connection transfers the loads equally efficient. The reason therefore is the angle difference between the spokes and the rings.
In case of an original (circular) spoke wheel roof, the angles between the spokes and rings are constant. In an oval shaped spoke wheel, as can be seen in figure 2.24, the angle between the spokes and ring on the long sides is wider than on the short sides. Due to the wider angle the load is transferred less efficient. This can be explained using the rules of the polygon of forces illustrated in figure 2.26.

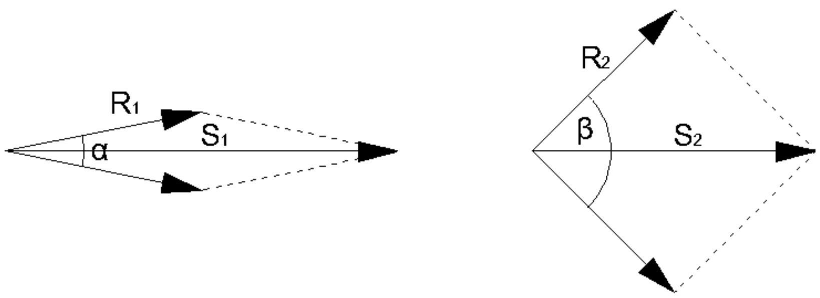


Figure 2.26 Polygon of forces

In figure 2.26 the angle between the ring elements has been drawn. The angle α between the ring elements is smaller compared to angle β. Following the rules of the polygon of forces, the smaller the angle between the ring elements, the greater the force resultant. Assuming that the forces in the ring $R_1=R_2$, the resultant in the spokes S_1 is greater than S_2.
On the short sides of the oval roof structure, the angle is smaller. The spokes on that side will therefore transfer the loads more efficient. As a consequence these spokes transfer most of the loads and higher normal forces arise here.

From the analysis can be concluded that where the curvature has the highest value, most of the loads are being transferred at that point. This point provides the greatest amount of ring action and therefore the most strength and stiffness.

2.4.4 Double tension/compression ring

With the addition of an extra ring to create an opening in the roof, the option arose to change the setting of the spoke wheel roof. First a double inner tension ring was used to create an opening in the roof. The possibility came to use a double outer compression ring instead. This setup has the advantage that the load of the inner ring decreased and that less pre-tension is needed to cover the same span. However, due to the lower dead load, the roof is more vulnerable for the upward lift. In case of wind suction the roof is lifted up and has less dead load to resist this lift.
The force distribution in the roof remains equal, only the set-up of the structural design has changed (2.27).

The greatest advantage of using a double tension or compression ring is that the transverse strength and stiffness of the roof will increase. Transverse forces are introduced due to the geometry of the spoke framework. The consequences for the structural behaviour are investigated in a later stage of the thesis. 3

NOTE: the terminology of double tension/compression ring can only be used when the spokes are pre-tensioned.

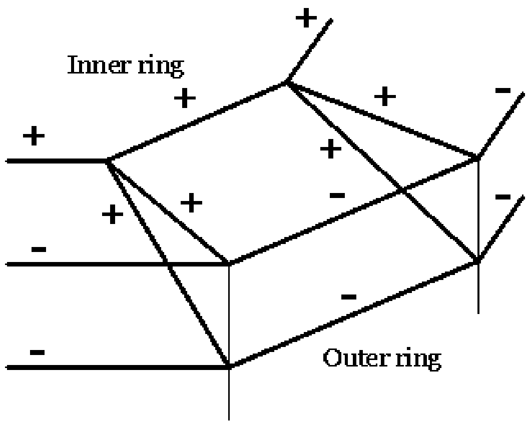


Figure 2.27 Normal force distribution within a pretensioned spoke wheel with double outer compression ring

2.4.5 Non pre-tensioned spoke wheel roof

Engineers found out that spoke wheel roof structures could also be designed without pre-tensioning the spokes. Projects in the past showed that without pre-tensioning the spokes, engineers were still able to design an efficient spoke wheel roof. An example is the Feyenoord stadium (chapter 3) in Rotterdam, the Netherlands.

The structural behaviour of a spoke wheel structure is significantly different when the spokes are not pre-tensioned. This will be explained using the spoke wheel roof with the double inner ring from figure 2.18.
As a consequence of any transverse load the spokes and the central hub will sag to a point where eventually equilibrium will be found. The sag of the hub causes tensile forces in the lower spokes. Because the spokes are not pre-tensioned, compression forces will arise in the upper spokes (figure 2.28).

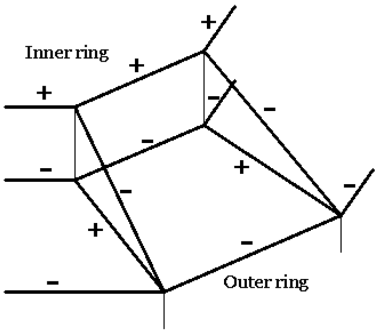


Figure 2.28 Bending in beam. Reproduced from [12]

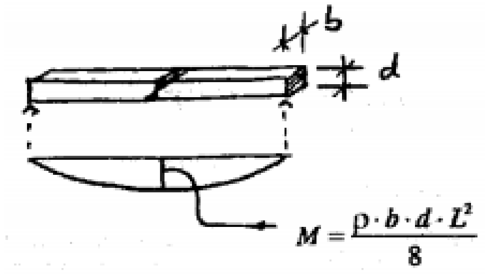


Figure 2.29 Spoke wheel roof without pre-tensioned spokes.

Now the spokes are not pre-tensioned, beam action will play a role in the strength and stiffness of the structure. Cables, unlike other steel profiles (for example CHS, RHS, HE, I), do not have a bending rigidity and therefore beam action will not occur in cable roof structures.

The spoke wheel roof structure of a stadium needs to cover a large span. For such structures, a traditional beam-column system is not very efficient. The bending moment in beams increases quadratic ally with the beam's span, thus requiring the beam's cross section to increase in the same order of magnitude. This can be easily illustrated mathematically using the beam in figure 2.29.

The bending moment in a rectangular beam can be expressed as:

$$M = \frac{\rho \cdot b \cdot d \cdot L^2}{8} \quad (2.25)$$

The maximum stress in the beam is expressed as:

$$\sigma_{max} = \frac{My}{I} \quad (2.26)$$

where $y = \frac{d}{2}, I = \frac{bd^3}{12}$

Follows that: $\sigma_{max} = \frac{\rho bdL^2}{8} \cdot \frac{d}{2} \cdot \frac{12}{bd^3} = \frac{6\rho L^2}{8d}$ (2.27)

The determination of d will be: $d = \frac{6\rho L^2}{8\sigma_{max}}$ (2.28)

Equation 2.28 shows that the depth of the beam, d, varies with the square of the span length L. With this increase in depth, the dead load of the beam increases as well. As a consequence the bending moment in the beam increases proportionally. An even deeper beam is needed to resist the bending moments. This illustrates that with increasing span, solid beams become more and more inefficient. The higher the dead load the more strength is needed, making it unsuitable to span a great length as in the case of a football stadium. The failure of a beam for wide span use demonstrates the ineffectiveness to resist load in this particular situation.
The solution is to resists load only through tension or compression and minimizing the amount of load taken up by beam action. Therefore a truss system is very suitable for a spoke wheel roof structure which has non pre-tensioned spokes [14, 15].

2.4.6 Conclusion

The spoke wheel roof structure could be applied as a football stadium roof structure due to the different adaptations made possible by engineers in the past. These adaptations are briefly summarized.

The application of the spoke wheel roof has been increased due to the different adaptations that have been applied to the roof structure over the years. The first adaption was creating an opening in the roof structure. Creating an opening in the roof causes a decrease of radial and lateral stiffness and strength of the structure, unless the amount of pre-tension is sufficient.
The following adaption had greater consequences for the structural behaviour of the roof structure. The oval shaped roof structure became less efficient compared to the original circular spoke wheel roof. Due to the oval

shape the radius of the ring structure is not constant over the whole roof. On the long sides the radius increased causing a decrease of the ring action. Therefore the amount of stress and deformation increased. A stronger ring structure (increase of extensional rigidity) is needed to provide sufficient stiffness and strength.
The use of a double compression ring instead of a double tension ring has consequences for the amount of sag and material use. The point of gravity is closer to the supports, resulting in smaller sag at equal pre-tension. The disadvantage is the greater use of material.
The last mentioned adaption is the use of non pre-tensioned spokes. Instead of cables, other steel profiles (CHS, HE, etc.) can be used. These profiles have a bending rigidity and can also take up loads by bending (beam action).

2.5 Conclusion

The spoke wheel principle is based on the amount of ring action a wheel can provide. The ring action depends on the amount of curvature, loads acting on the ring and the extensional rigidity of the ring. The higher the amount of ring action, the greater the strength and stiffness of the wheel. When the spoke wheel is used as a roof structure, the translation of the outer ring also plays an important role in the amount of ring action as well.

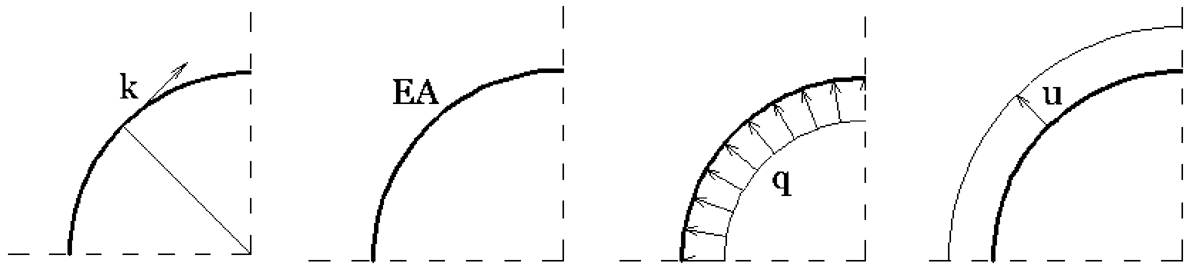


Figure 2.30 Key factors that influence the ring action in a spoke wheel roof: curvature, extensional rigidity, loads and translation

The spoke wheel has proven to be structurally very efficient as a roof structure. Although the decisive load is transverse directed instead of radial, the structural behaviour of the spoke wheel roof shows parallel results to the bicycle wheel.

The amount of ring action depends on the four mentioned key factors (figure 2.30). The design of the spoke wheel roof is determined by certain design parameters which have an influence on the key factors and thus on the ring action of the spoke wheel roof.
The influence of the design parameters on the strength and stiffness of the roof are presented in table 2.4. The design parameters are related to the way a bicycle wheel works. These design parameters are useful for the design of a spoke wheel roof.

Increase of design parameter	**Strength**		**Stiffness**	
	Radial	Transverse	Radial	Transverse
Pre-tension in the spokes	+	+	+	+
Space flange hub	-	+	-	+
Amount of spokes	+	+	+	+
Space between spokes	-	-	-	-
Cross sectional area of the ring	+	+	+	+
Load affected zone	+	+	+	+
Radius	-	-	-	-

Table 2.4 Influence of the design parameters on the strength and stiffness of the roof structure.

To extend the use of a spoke wheel as a roof structure, engineers have adapted the shape of the spoke wheel roof. However not all adaptations are an improvement, the spoke wheel roof is still very attractive to use for example stadia and large halls. An opening in the roof, the use of an oval shape and non pre-stressed spokes all have a major impact on the structural behaviour of the spoke wheel roof.

In this stage of the thesis the design of the spoke wheel roof is not known yet. Before the final spoke wheel roof structure designed, a preliminary design must be determined. With the investigation of the bicycle wheel (design parameters) and the development of the spoke wheel roof, the design variables that influence the structural design of a general football stadium roof can be determined.

The design variables are the following:

1. Shape of the (opening of the) roof
2. Double inner / outer ring
3. (Non) pre-tensioning of the spokes
4. Profile / elements
5. Supports / connections

The presented design variables are related to the design parameters from table 2.4 that influence the strength and stiffness of the bicycle wheel. The design variables all have a relation on the key factors that determine the ring action in a roof structure: curvature, loads, extensional rigidity and translation (figure 2.31).

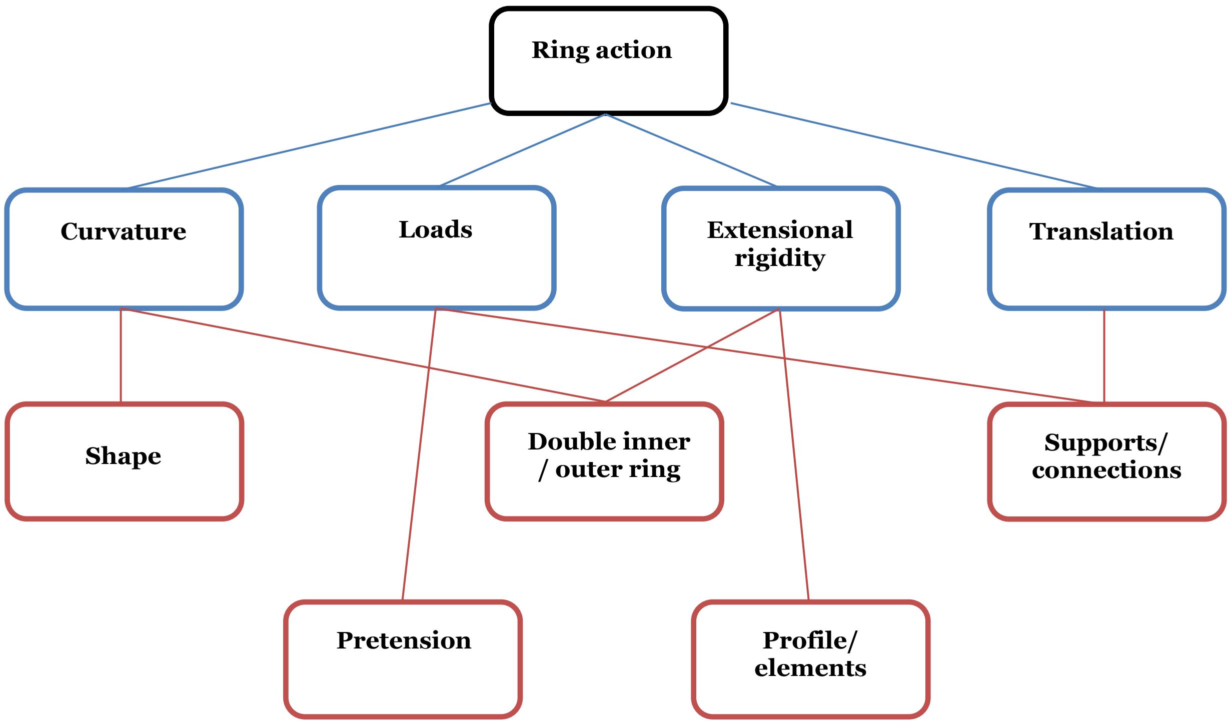


Figure 2.31 Direct relations of the design parameters on the key factors

Before the preliminary design can be determined, the influence of the choice of the different design variables on the ring action of the roof structure must be investigated. For example: Is it more efficient to use roll supports or rocker bearings; and is a structure with pre-tensioned spokes more (cost) efficient than the use of non pre-tensioned spokes. The relations are described in the next part of the thesis: Preliminary design. When these variables are investigated, the structural engineer can determine the preliminary design of the main bearing structure.

3. Reference projects

By looking at reference projects one can see how a spoke wheel roof has been applied for present football stadia. Due to the shape of football stadia the application of a spoke wheel roof has become more complex. The reference projects show how structural engineers solved this problem.

3.1 Feyenoord stadium, Rotterdam

In 1994 the Feyenoord stadium, also called 'De Kuip', was renovated. The quality of the accommodation had to increase to compete with the growing competition nationally and internationally.
The renovation included a new accommodation for players and press, the supplementation of business units, the construction of the 'Maasgebouw' (consists of restaurants, offices, museum and meeting rooms) and the addition of a roof on the stadium. Special attention will be given to the structural design of the roof.

Figure 3.1 Feyenoord Stadium (De Kuip), Rotterdam

3.1.1 Roof structure

Assumptions

When designing the roof structure for the Feyenoord Stadium the engineers had to stick to the following assumptions:

- *Detached*
  A primary condition for the roofing of the stadium was that the structure could not block the sightlines of the spectators. The roof structure had to be column free over the whole depth of the stands.
  At new stadiums a roof structure is most of the time integrated in the structure of the stands. By making a fixed connection between the cantilevered roof and the structure of the stand, you get a relatively simple structure. In this case it was not possible to use this solution. The structure of the stands could not handle the needed fixed bending moment. Engineers decided to design a roof structure that is independent from the stand structure.

- *Floating*
  An important aesthetical demand was that the roof should appear to 'float', just like the second ring of the stadium appears to be floating above the first ring.
  Another demand was that the supporting structure outside the stadium should not dominate the appearance of the stadium. Therefore also in the independent roof structure a fixed cantilever is avoided. To deliver the necessary needed amount fixed moment heavy structural elements would be needed and dilatations had to be applied to the roof.

- *Ground plan*
  The Feyenoord Stadium has a characteristic ground plan, with heavy curvatures in the four corners and light curvatures along the sides. The new roof had to follow this shape to preserve the characteristic shape of the stadium.

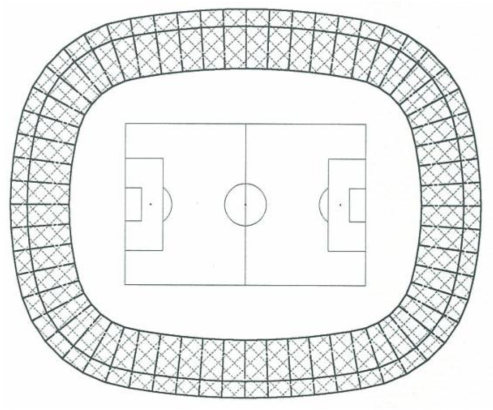


Figure 3.2 Ground plan of the Feyenoord Stadium. Reproduced from [25].

Spatial Structure

The solution of the mentioned assumptions was a spatial structure that transfers the loads three-dimensionally. By using engineering software (Diana) engineers were able to make a structural design step by step.
The first idea was to use triangular elements at a certain distance with one compression ring, two tensile rings and diagonals in two planes in between. This structure was imposed on bars on the outside of the stadium.

The structure was put in an engineering software program where the distance between the triangular elements first was held at 30 meters, the so called rough grid. After investigating this model one could conclude that:

- The structural system was spatially stable.
- The force transfer partially takes place through ring action and partially by beam action.
- The largest deflection occurs at the tensile ring in the middle of the long side of the roof structure.
- Diagonals in at least two planes are essential for the spatial load transfer and form fixing of the structure.

The perspective from the first idea was good and was ready to be worked out to a fine grid. The trusses were placed at a distance of 10 meters from each other. An important intervention was to replace the diagonals from the roof planes to a lower level. These planes with the diagonals give the structure more stiffness in horizontal and vertical direction. The horizontal trusses transfers the wind loads to the sides of the wind bucks: the upright tilted sloped trusses transfer the loads to the corners.
The vertical load transfer at the short and long sides of the stadium roof partially takes place by ring- and bar action. The reason is that at the sides the structures has not enough curvature to fully carry the loads by ring action. Because the deflection at the corners is limited, the stiff and sloped placed truss will transfer the loads to the corners.
At the wind bucks trusses are placed at the lower and upper level to carry the horizontal loads due to wind.
A structural characteristic of the stadium roof is that the forces in the rings at an equal load distribution on the entire roof surface are not constant due to the special shape of the ground map. To transfer the loads efficiently diagonals are also needed at an equally distributed load.

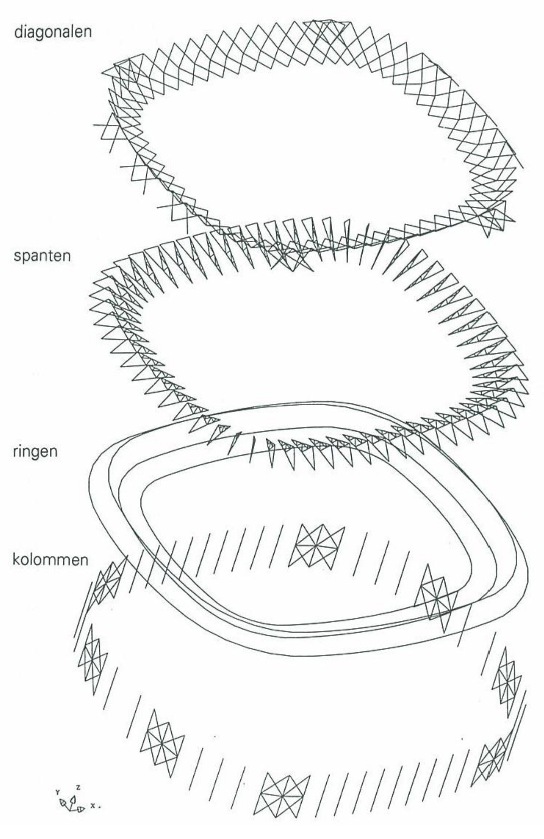


Figure 3.3 Elements of the roof structure. Reproduced from [25].

The diagonals have additional advantages:

- The form of the structure is fixed at asymmetrical loading.
- The compression ring is supported at every node, this way the buckling length will be limited at 10 meters (the maximum truss distance).
- The horizontal loads due to wind can easily be transferred to the wind bucks.

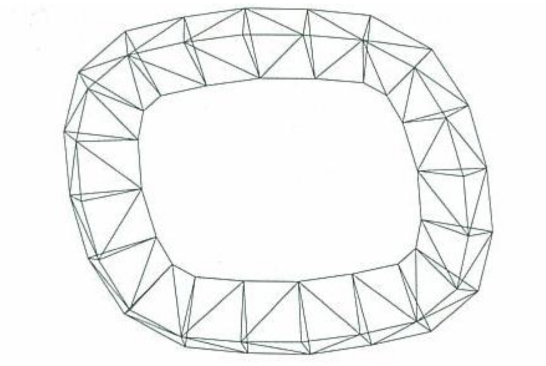


Figure 3.4 Rough grid of the first design [19]

Figure 3.5 Final spatial structure

The structure is mainly constructed from cold-formed tube profiles with S355 steel. Only the upper edges from the trusses, the purlins and the vertical bars behind the glass sloped planes are executed with hot-rolled HE profiles, S355. For the columns spiral welded tubes are used. All connections were executed with 8.8 and 10.9 bolts [25, 31].

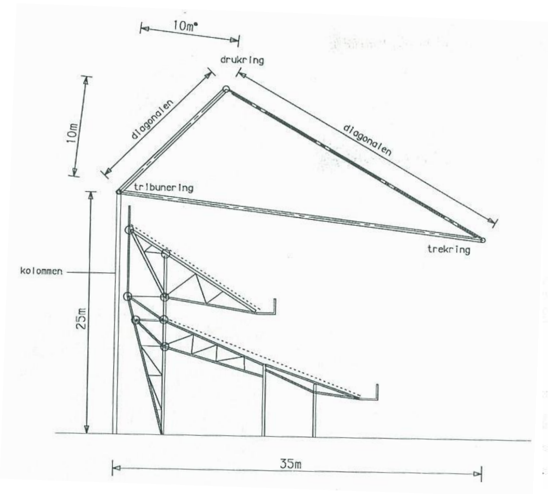


Figure 3.6 First design of the roof [25].

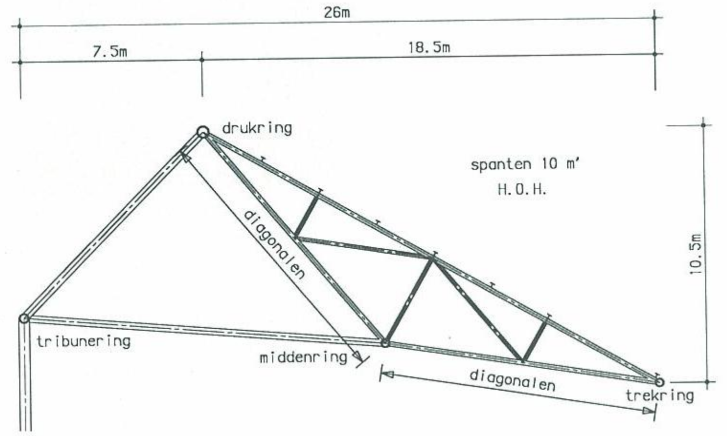


Figure 3.7 Final design of the roof [25]

3.1.2 Fabrication and assembly

The trusses were assembled in the factory and were transported to the building site as a whole. All other parts were transported as single elements. At the outside of the stadium 20 roof units were assembled.
For the assembly of the roof structure 58 temporary columns were placed at the inside. On the columns the roof units are connected with each other. After closing of the ring the temporary columns were removed step by step.
After removing the temporary columns the measured deflections were compared with the calculated deflections. The largest deflection (500mm) occurred at the long side. At that point the temporary columns were removed last [26].

3.2 Commerzbank Arena, Frankfurt

In 1998 Germany was appointed to host the FIFA World Cup 2006. Between 1998 and 2005 a series of new football arenas were built. Frankfurt was one of the hosting cities and the former Wald stadium was renovated and expanded to a capacity of 53.200 seats (figure 3.8 and 3.10 - 3.12). The roof structure of the stadium is based on the spoke wheel principle. What makes this stadium special is that this principle is used for a stadium with a rectangular opening and that the roof structure consists of two interlocked spoke wheels.

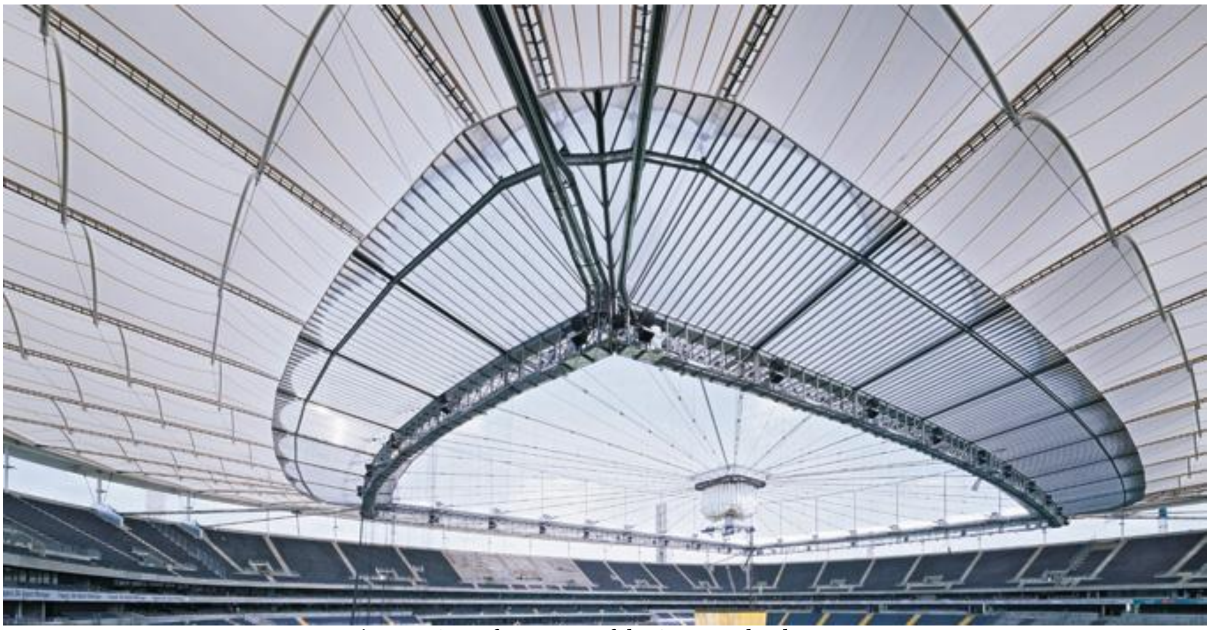


Figure 3.8 Roof structure of the Commerzbank Arena

3.2.1 Roof structure

The Commerzbank Arena is, like the Imtech Arena in Hamburg, one of the very few stadiums which have a rectangular opening for a roof structure based on the spoke wheel principle.
The compression ring along the circumference of the stadium bowl forms the rim of the roof structure. The central hub of the spoke wheel is replaced by a cable ring in which the tension forces are carefully balanced with the forces of the compression ring (figure 3.9). Radial cable girders make up the spokes and connect the rings.

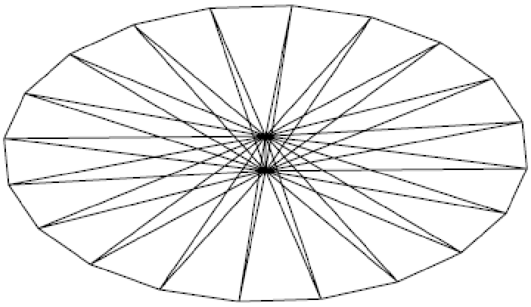


Figure 3.9 Spoke wheel roof

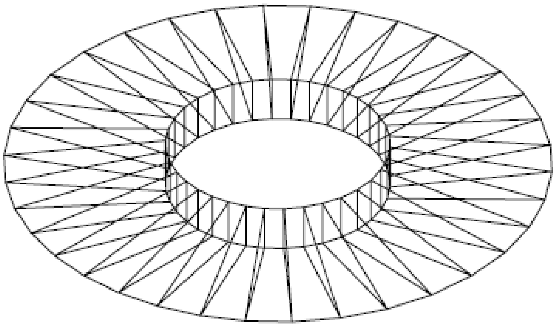


Figure 3.10 Double inner tension roof

The roof structure has one compression ring above the outer edge of the bowl with radial cables spreading out towards the inside of the stadium and ending in two parallel tension rings running on top of each other. From there the radial cables of the inner roof start and meet at the centre of the stadium at one point. The vertical components of the upper and lower tension rings are balanced by means of vertical struts.
The complete roof structure is supported by slender vertical columns of 8,5m height in each axis. The roof therefore appears to be floating almost weightlessly above the stands (figure 3.11).

The roof cladding consists of a combination of four different materials:

- Metal cladding at the outer edge
- PTFE for the fixed roof
- Polycarbonate, for the 15m wide sheets parallel to the tension ring.
- PVC, for the retractable roof.

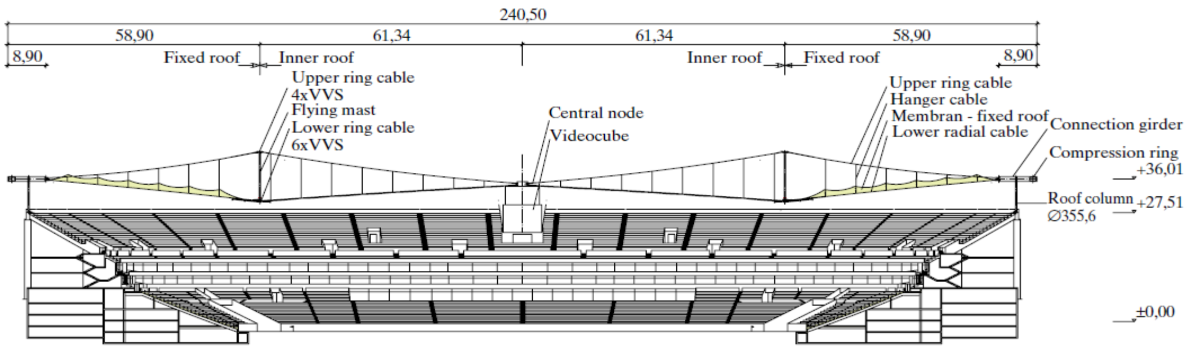


Figure 3.11 Section of the Commerzbank Arena. Reproduced from [21].

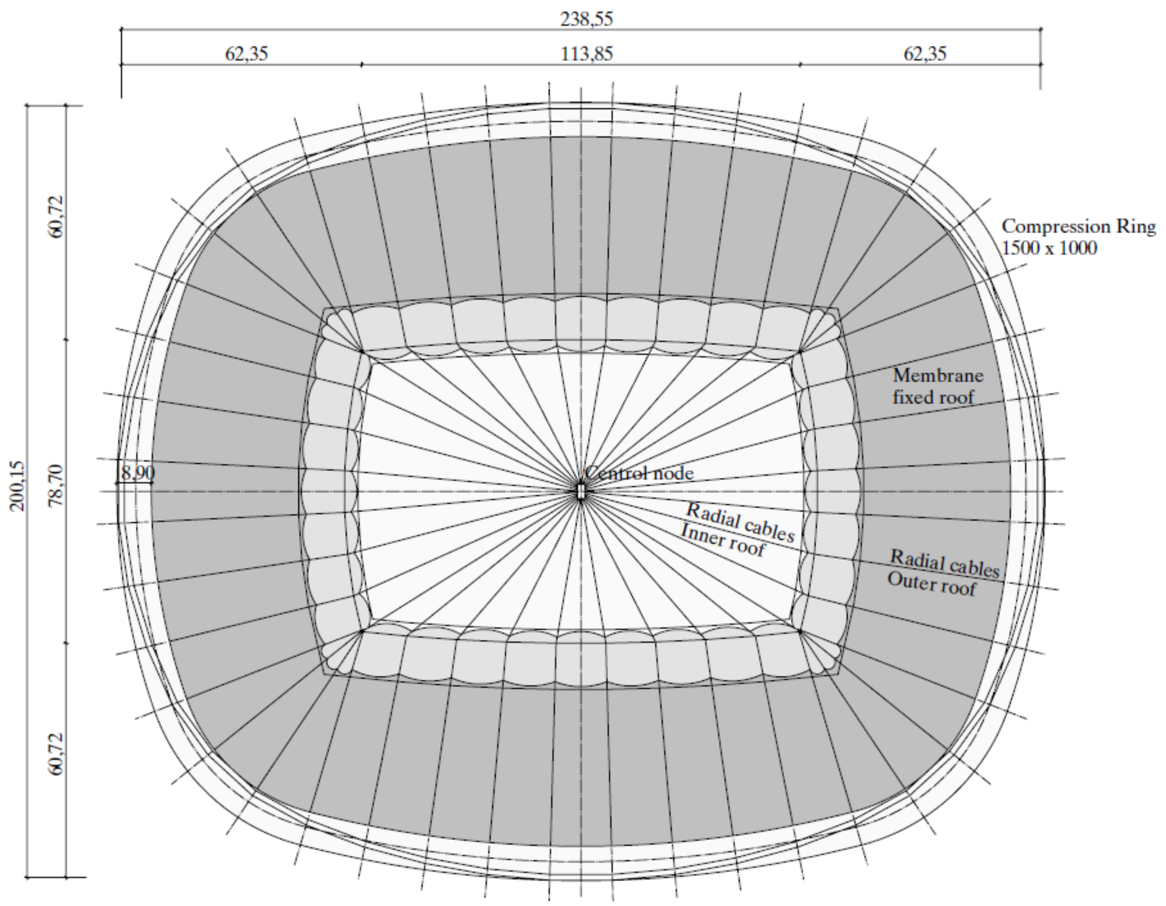


Figure 3.12 Plan view of the Commerzbank Arena. Reproduced from [21].

As mentioned before the roof structure of the stadium consists of two interlocked spoke wheel structures (figure 3.11. and 3.12).

In standard spoke wheel structures, ring geometries and ring forces are strongly dependent on each other and cannot be chosen freely. The inner roof of the Commerzbank Arena helps to overcome this design limitation.

The structure of the inner roof follows the same principles as the outer roof, resulting in the mentioned two interlocked spoke wheels with one outer compression ring, two tension rings and the central hub. This combination allows a choice of the geometry of the tension ring freely as the balancing forces for the compression ring could be assigned to the tension ring or the hub respectively. Not only the horizontal position could be adjusted but also the vertical shape of the roof, leading to several architectural options as well as the possibility of arranging the drainage in the corner areas of the roof.

Because of the shape of the roof one of the most challenging parts of the structural design was the stability check for the compression ring, because standards are hardly applicable. The process contained of calculating maximum stresses in the ring under factored loading including imperfections. The imperfections were adopted by using the scaled deflections under the first Eigen frequency of the system. As a final step standard buckling check was carried out for each compression ring element [21, 28, 39].

3.2.2 Fabrication and assembly

By increasing the amount of prefabrication one could decrease the required building time on the site. The compression ring had to be fabricated following strict tolerance requirements both in the length of each element and angular deviation of the end plates.

The compression ring elements were quickly erected one after the other and fixed together, after the columns and linking girders were installed and temporarily fixed. The placement was very precise and major surveying during erection could be avoided by the attention during fabrication, saving immense time and money at the site.

The complete compression ring had to be finished to start with the installation of the cable structure. The cables were completely pre-fabricated and connected with the end-sockets.

The cables were then laid out on the stands and the field. All cables were connected to each other with cast steel clamps and ring connectors, producing one single cable net. The radial cables were connected by means of temporary strands to lifting jacks fixed to the compression ring. By pulling these temporary strands, the complete cable net could be lifted off the floor towards its final position.

After the cable net was installed the arches and the fabric were erected bay by bay. The catwalk, including the polycarbonate roof, was erected parallel. The steel skeleton in the model of the roof was connected to the central hub of the cable structure [21, 39].

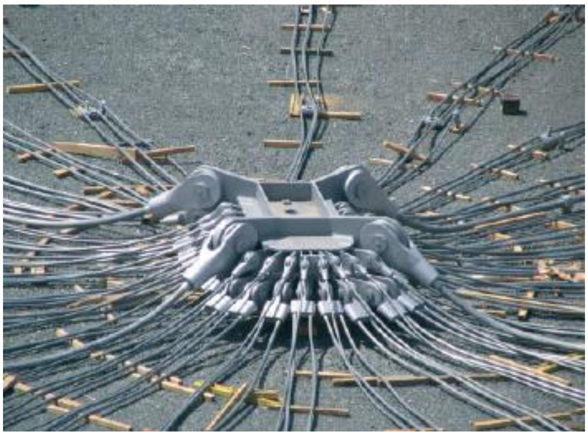


Figure 3.13 Central hub. Reproduced from [39].

Figure 3.14 Erection of the cable ring structure [39].

3.3 BayArena, Leverkusen

In 2007 the expansion and renovation of the BayArena started. The stadium could host 22,500 spectators at that time. The renovation- and expansion activities included the increase of the north-, west- and east stand of the stadium. The capacity would increase to 30,000 seats.
New VIP-boxes, hospitality areas and a new roof have been build. The total renovation cost around 70 million Euros.

Figure 3.16 Aerial view BayArena, Leverkusen

Figure 3.15 Aerial view BayArena, Leverkusen

3.3.1 Roof Structure

Structural highlight of the new BayArena was the complex roof structure. For the roof structure the spoke wheel principle has been used. The roof has a circular shape with a diameter of 217m. The roof reaches far beyond the extended platform areas, providing shelter from rain and sun for visitors even in the front rows of the arena.
The roof consists of two outer compression rings and one inner cable tension ring. The outer compression rings are connected with 72 upper and 36 lower radial steel cables (diameter is 70 mm) to the inner tension ring, forming the supporting structure of the roof. The roof is supported by eight 40-tons heavyweight V-shaped columns [39, 47].
For this roof structure 2,800 tons of steel and 12,800 m of steel cable has been used. The covering of the roof consists of Makrolon cellular sheets. These sheets have the ability to block enough sunlight for the spectators and yet let the grass have enough sunlight to grow [47].

3.3.2 Fabrication and assembly

Just like the Commerzbank Arena (Chapter 3.2) the roof of the BayArena was lifted up. The first phase consisted of lifting the steel cables simultaneously. In the following phase the cables were pulled together in groups and then each cable had to be aligned sings massive forces impacted in the entire roof. In the final phase the cables were secured.
The Makrolon sheets were prefabricated and assembled to the steel structure at ground level on the building site. With cranes the pre-assembled parts were lifted to its place, 30 m above ground. Special trained construction workers assembled the parts to the roof structure. The total construction and renovation of the stadium took only 20 months [39]

3.4 Conclusion

The reference projects show how engineers have found solutions to apply a spoke wheel roof structure at a football stadium. From all three analysed stadia, the most important conclusions that are relevant for the structural design of the spoke wheel roof are summarized.

Feyenoord Stadium, Rotterdam

- The spokes are non pre-tensioned and consist of CHS steel profiles
- To diminish bending moments, a spatial truss is applied.
- The largest deformations arise at the long side of the roof as a consequence of the low ring action at this point.
- The vertical load transfer at the short and long sides of the stadium roof partially takes place by ring- and beam action.
- The sides of the structure have not enough curvature to fully carry the loads by ring action. Stiff, sloped trusses are used to transfer the loads to the corners.
- The horizontal loads are transferred by horizontal trusses, which transfer the loads to the wind bucks.

Commerzbank Arena, Frankfurt

- A cable net system is applied for the roof structure
- Cables do not possess bending rigidity. No bending moments will arise and the load cannot be transferred by beam action.
- To take up the loads a very high pre-tension force is applied in the cables.
- Due to the shape of the stadium and the high pre-tension working on the ring, the stability check of the ring is very important and decisive.
- To provide more stability the roof consists of two interlocked spoked wheels.
- With the cable net system, the construction time of the roof is very short compared to conventional stadium roofs.

BayArena, Leverkusen

- A cable net system is applied for the roof structure
- To prevent problems due to the shape of the stadium itself, a circular roof has been applied. The ring action is used at full extend.
- To provide enough shelter, the shape of the roof covering is equal to the stands of the stadium.

With the conclusions from the three reference projects, an additional answer can be given to the second sub-question. The second sub-question is as followed:

'How can the spoke wheel principle be applied to a roof structure of a football stadium?'

The application of a spoke wheel roof structure at a stadium has a lot of consequences for the design. A stadium is not perfectly circular shaped. The lack of curvature on the sides of the stadium causes a decrease of ring action.
The reference projects show that the spokes are made out of regular steel profiles or cable elements and that they are pre-tensioned or not. This choice has a major impact on the structural design. In case of using non pre-tensioned spokes, the load is taken up by ring action and beam action. To prevent high bending moments, a truss system can be applied.
Another option is to use, like a bicycle wheel, pre-tensioned spokes with the help of a cable system. Cables do not provide any beam action. The total load has to be taken up by ring action and the pre-tension of the spokes. To carry the loads a very high pre-tension force is needed. This has a great effect on the ring elements. It is a challenge for the engineer, to find a solution to withstand these high forces.

4. Reference stadium

For the thesis, the research of efficient use of the spoke wheel roof system for football stadia, a reference stadium will be used for the design of the roof. The Amsterdam ArenA has been chosen as reference stadium. The reason why this stadium is chosen is the accessibility of the drawings and calculations of the structural design of this stadium at Arcadis. Arcadis is the official structural design partner of the Amsterdam ArenA and therefore Arcadis has a lot of information and knowledge about this stadium.

4.1 Amsterdam ArenA

4.1.1 General information

The Amsterdam ArenA was the first multifunctional stadium in the Netherlands accomplished on August 14th, 1996. It is home to the famous football club AFC Ajax and has a capacity of 52.690 spectators. Besides football, the stadium can be used for music concerts, indoor dance festivals, etc.
The Amsterdam ArenA was the first stadium in Europe which had a retractable roof. The roof can be closed in 25 minutes and consists of two giant panels of 37m x 118m, with a weight of 520 tons each which move between two arch girders [11].

Figure 4.1 Inside view of the Amsterdam ArenA

4.1.2 Roof structure

The roof of this stadium is a steel structure and the rest of the structure is made of concrete. The steel roof structure basically consists of a primary and a secondary structure.

The primary structure of the roof consists of two steel arch girders with a triangular cross section, two support girders, also with a triangular cross section, and four supporting structures (figure 4.2).
Between the two arch girders the retractable roof can move on the top chord of the arch girders. The support girders are connected to the arch girders at the same level as the tension tie of the arch girders. The support girders are not able to give enough lateral support to the arches. To make the arch girders stable, stiff frames were placed between the upper and lower chords of the arch.
The four supporting structures are made of concrete and are kept free from the steel structure, to avoid problems that can arise due to large differences in stiffness.

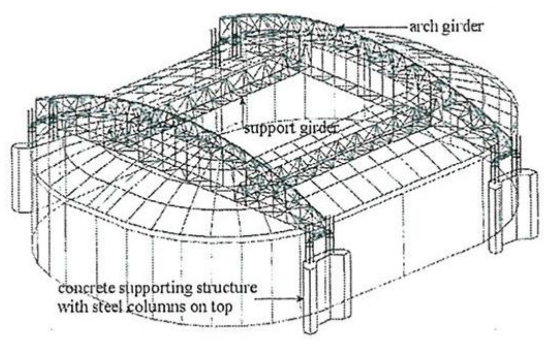


Figure 4.3 Arch girder. Reproduced from [13].

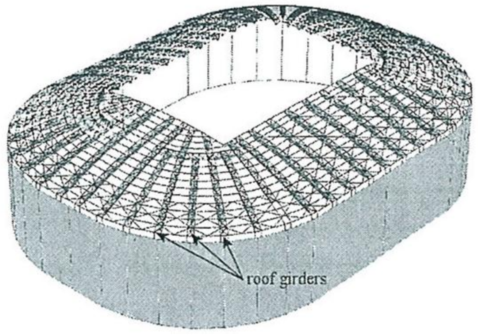


Figure 4.2 Roof girder. Reproduced from [13].

The secondary roof structure is formed by 50 steel roof girders, which are also triangular shaped. These girders are 50 to 70m long and a bracing around the perimeter of the roof makes sure that that the roof acts as a rigid diaphragm. These steel roof girders are supported by the 50 concrete frames, where in between the stands are placed. The loads from these roof girders are transferred to the support girders and then transferred to the arch girders and further to the foundation of the stadium.

Due to the wind loads a horizontal force acts on the roof structure (chapter 9, Loads). These horizontal forces are transferred by a bracing to the columns on top of the four supports. These columns consist of steel CHS columns of 11m high. These form an unbraced portal structure, which is not very stiff. The reason therefore is that the structure had to accommodate the forces and displacements due to the lengthening and shortening of the tension tie as a consequence of the loads and temperature effects. The piles that form the foundation of the staircases can be seen as springs [8, 13].

4.2 Design area

The Amsterdam ArenA roof will be used as a reference stadium for the investigation of the application of the tensile-compression ring structure for football stadia. The shape of the perimeter of the spoke wheel roof will be equal to that of the Amsterdam Arena. However, there are still shape variables left. The structural design can differ in height and span.

When designing a roof for a football stadium certain design requirements need to be met. Besides requirements regarding the structural aspects, there are also requirements to provide a certain comfort to the spectators. These requirements behold the lines of sight and protection from natural elements like sunlight, wind and rain. The requirements can be found in the stadium guidelines of the FIFA.

4.2.1 Lines of sight

There are certain design variables that are related to the lines of sight of the spectators. These factors are:
- Influence of columns
- Roof height, in case of blocked sight at a certain height.

A characteristic of a spoke wheel roof is that there are no columns needed to support the structure. This way the sight of the spectators will not be blocked by columns in the stands.
Unlike the influence of columns, the roof height is something to take into account. Figure 4.4 shows the maximum height of the ball that can be seen by the spectators that are most limited by the edge of the roof. These are the spectators that are present at the top row.
Also the height of sight cannot be less than 3 meters above the furthest side- or backline or 1 meter above the bar of the goal [19, 35].

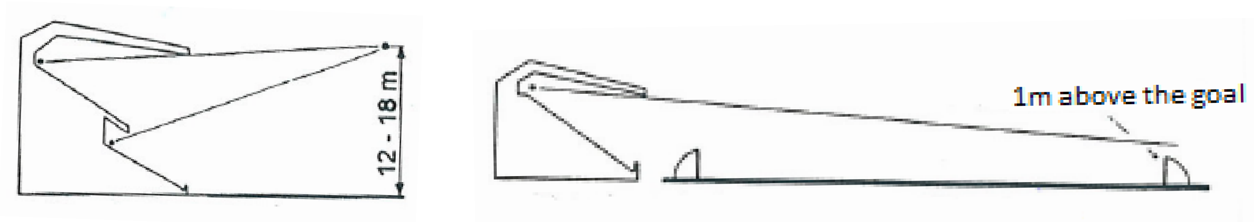


Figure 4.4 Effect of the edge of the roof on the sight of a high ball. Reproduced from [19]

4.2.2 Protection against the elements

Covered stands cannot guarantee that all spectators are protected against the influences of the weather. The vulnerability of a certain place on the stands of the elements is determined by five factors [19, 35]:
- Height of the front of the roof with respect to a certain place on the stands
- The overhang of the roof with respect to a certain place on the stands
- The direction of the wind with respect to the orientation of the stands
- The chance that the wind (and so the rain and snow) blows in the direction of the spectators
- The wind flow, especially for stadia with open corners.

The last three factors have to be taken into account when a complete stadium must be designed from scratch. In the case of this thesis the place and shape of the stadium is fixed. Therefore these factors will not be taken into account.

The height and overhang of the roof are factors that influence the design space of the roof structure. In figure 4.5 one can see a cross section of a stand with different angles from the edge of the roof with respect to the stand. These angles can be seen as a guideline of the way of protection for the spectators against the elements.

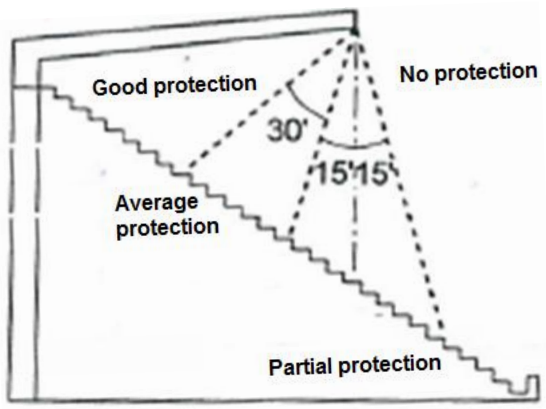


Figure 4.5 Protection of the roof. Reproduced from [19].

At the Amsterdam ArenA, the angle from the edge of the opening of the roof with respect to the front row of the stand at the short side is 26,27 degrees. At the long side of the stadium the angle is 13,45 degrees. The drawings of the Amsterdam ArenA can be found in appendix A.2.
For the design of the spoke wheel roof is assumed that the minimum angle between the edges of the roof with respect to the front row of the stand is 15 degrees. By using this angle an average protection against the elements is provided for the spectators. This way the comfort level of the structural design of the new spoke wheel roof is equal to that of the Amsterdam ArenA.

The requirements to increase the comfort of the spectators are in contradiction to with the requirement of providing an optimal quality of the grass. The roof structure influences the amount of sunlight on the pitch as well as the airflow in the stadium. Roof structures present at the east, west and south side cause shade on the pitch. Too much shade will slow the grow of the grass, natural dry of the grass and melting of the snow. The areas of the pitch that are subjected to shade vary every season of the year. With the help of computer software the shade in the stadium can be determined for every time of the year. The amount of shade can be diminished using transparent roof coverings.
For the determination of the design area is assumed that the angle used for the roof opening of the Amsterdam ArenA provides sufficient sunlight and airflow to remain the quality of the grass. As mentioned, the angle between the edge of the roof compared to the front row of the stand at respectively the short and long side is 26,27 and 13,45 degrees. The angle at the long side is smaller than the 15,00 degrees of the protection line. Because the difference is very small, it is assumed that 15,00 degrees is sufficient to provide enough sunlight and airflow.

4.2.3 Design boundaries

The design area for the structural design of the spoke wheel roof can be determined by drawing the lines of sight, the protection lines (min. of 15 degrees) and the angle between the edge of the original Amsterdam ArenA roof and the front row of the stand in the cross sectional drawings of the Amsterdam ArenA. Assuming the shape of the roof is straight (not curved) in its cross section, the edge of the roof must be present in the design area to fulfil the requirements regarding the lines of sight and protection. The drawings can be found in the appendix. In figure 4.6 the design area of the new roof is displayed.

The design boundaries of the shape of the roof are summarized below:

- Min. angle between edge roof and front row stands is 15,00 degrees.
- The edge of the roof must be present in the design area, assuming a straight roof.
- The number and place of the supports are equal to the plan of the Amsterdam ArenA.
- Only adaptations to the original roof of the Amsterdam Arena can be made. The design, dimensions, etc. of the stands, pitch, etc. remain fixed.

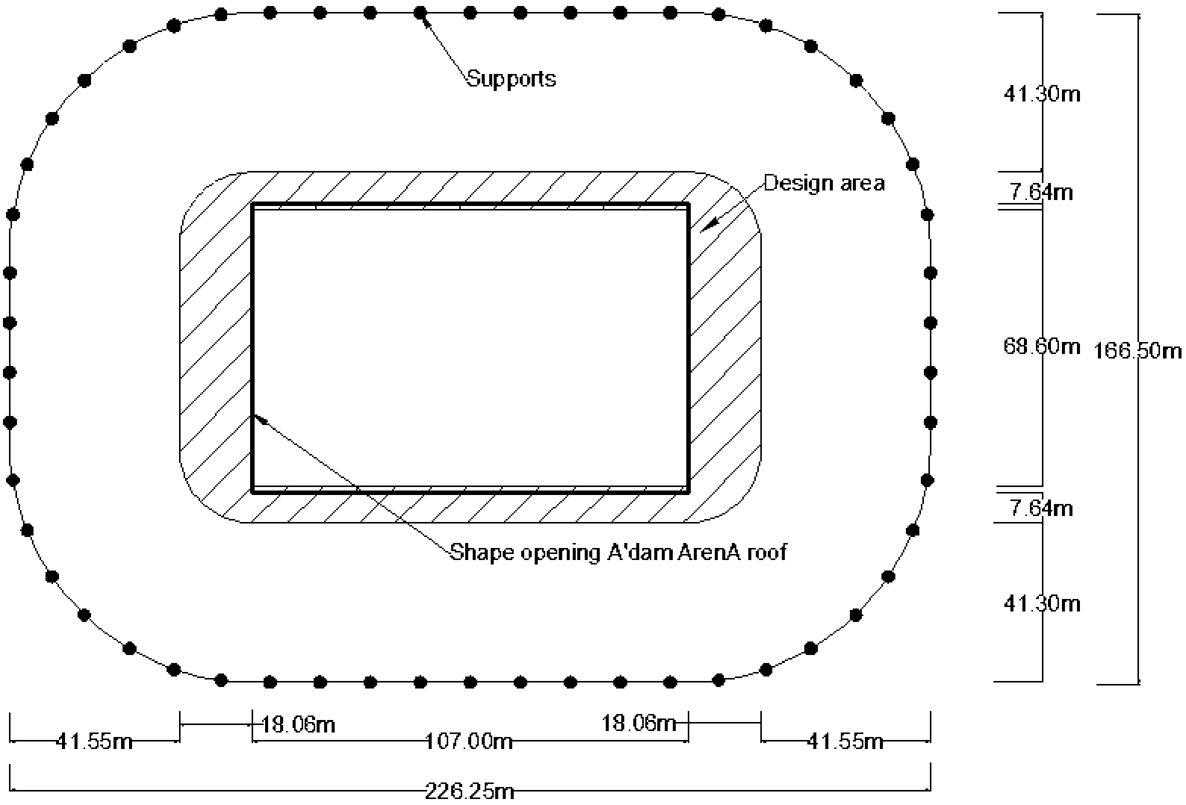


Figure 4.6 Design area of the roof over the whole height

4.3 Conclusion

The determination of a preliminary design is the first step to come to a final and efficient design for the spoke wheel roof. In chapter 2 (Spoke wheel principle) is explained that the preliminary design depends on certain design variables.

These design variables are the following:

1. Shape of the (opening of the) roof
2. Double inner / outer ring
3. (Non) pre-tensioning of the spokes
4. Profile / elements
5. Supports / connections

With the choice of the reference stadium, it became clear that the shape of the outer perimeter of the stadium is fixed. The outer perimeter of the stadium is oval shaped with straight sides. The shape of the opening of the roof is not yet determined. The rest of the design variables are still unknown in this phase of the design process.

Due to the fixed shape of the outer perimeter, some design parameters (table 2.4) are also already known. The curvature (radius) of the outer ring will be equal to the outer perimeter. The place of the supports is fixed, meaning that the space between the spokes is fixed as well.

5. Requirements & conditions

A structural design of a roof structure for a football stadium has to fulfil certain requirements. In the building industry the requirements depends on the owner of the project and the structural regulations described in the Eurocode. Besides the requirements there are conditions that need to be met as good as possible.

5.1 Requirements

The requirements can be divided into fundamental and additional requirements.

5.1.1 Fundamental requirements

The fundamental requirements are mainly about the structural design. These requirements can be found in Eurocode NEN-EN 1990:2002 and will be briefly described.

A structure has to be designed and calculated in such a way that it sufficiently contains it
- Structural resistance,
- Usability and
- Durability

Structural resistance

The structure has to resist every load and influence that can arise during construction and use (except for war and terrorism). The structural requirements which the structural design must fulfil can be found in chapter 9.

Usability

The structure has to keep its usability for its purposes where it is intended for. A structure has to be designed that it keeps its usability for its determined life expectancy. For a stadium roof structure this is 50 years.

Durability

The structure has to be designed and calculated in a way that the declination during its design life expectancy not undermines the performance of the structure below the planned quality level, taking into account its environmental conditions and the planned maintenance.

5.1.2 Additional requirements

The additional requirements for this thesis are requirements which are applied to provide the comfort of the spectators, the quality of the pitch, etc. The following requirements have to be fulfilled for the structural design of the roof structure [5, 9]

Shading from the sun

When designing a roof a minimum of spectators should look into the sun, especially when matches are played in the afternoon. Engineering firms study the shading on the spectators and the pitch for every time of the day and year by using computer modelling. Often these models are parallel with wind tunnel testing, to take the grass conditions into account. The amount of shading, sunlight and wind is crucial for the durability and the quality of the pitch.

Shelter from wind and rain

To provide more comfort, the shape of the roof must be designed in a way all spectators can shelter from wind and rain. The problem is when the wind and rain coming in the stadium is brought to a minimum; this will have a negative influence on the quality of the grass. Too little airflow in wet climates may give inadequate drying of the pitch after rain. These contradictory factors must be taken into account. When an engineering firm designs a roof structure for a stadium, the following factors are investigated:

- Air temperatures (rain or snow)
- Wind directions and velocities
- Local air turbulence due to the surrounding buildings or the design of the stadium itself.
- Conflict between needs of the spectators and the quality of the grass.

For the final designs of the roof structure of the thesis, the assumption is made that the roof opening must lie in the given design area (figure 4.6) to fulfil the requirements of shading from the sun and shelter from wind and rain. Besides the translucency of the roof must be met with the same amount of translucency the current roof of the Amsterdam ArenA has. This is to provide the grass enough sunlight.

View obstructions

To give the spectators an optimal view on the pitch, no obstructions can be placed between the spectators and the pitch. For instance, the roof edge must be high enough that the majority of the spectators keep sight of the ball when it raises high in the air. Another often made mistake is the place of the supporting roof columns. A characteristic of tensile-compression ring roof structures is that it has no supporting columns in the stand. Therefore this problem will not occur.

Maintenance

To provide good maintenance conditions, the roof structure should be well accessible. When reparation or cleaning of the roof is necessary, one should have good access to the place of work. Often a catwalk is applied; other solutions that provide accessibility are also possible.
An engineer has to keep in mind, to use structural elements that do not have sharp corners, to prevent dirt collecting on these elements.

5.2 Conditions

For the design there are some conditions that need to be met to come to an efficient spoke wheel roof structure.
For the thesis some conditions of the Amsterdam ArenA will be used as a basis for the structural design.

Roof dimensions

The structural design is based on the dimensions and shape of the roof of the Amsterdam ArenA. The dimensions of the outer perimeter and the place of the support are fixed. The design area of the roof opening has been determined in chapter 4. The edge of the spoke wheel roof must be present in the design area, in order to fulfil the requirements regarding the sightlines, the protection against the elements and still provide a good quality of the grass.
The detailed drawings of the cross sections of the Amsterdam ArenA can be found in the appendices.

Spoke wheel characteristics

To design an actual spoke wheel roof certain (structural) characteristics of the spoke wheel need to be the same. In chapter 2 the structural behaviour of an original spoke wheel and the application of the spoke wheel as a roof has been analysed and compared. Certain conditions of spoke wheel characteristics must be fulfilled to come to an actual spoke wheel roof design. These are the following:

- The inner perimeter is completely column free; there are no obstructions between the spectators and the pitch.
- The roof has a lightweight appearance.
- This type of roof structures lends itself for constructing a new roof on existing stadia.
- Transparent or translucent roof coverings are possible.
- ‘Floating’ appearance of the structure
- Special supporting structure is not needed; the roof can be supported by the framework of the stands.

Conclusion Analysis

The conclusions from the part Analysis are briefly summarized for each chapter.

Chapter 2 – Spoke wheel principle

Bicycle wheel
- The high radial and lateral stiffness and strength of the bicycle wheel is provided by the ring action.
- Ring action depends on three key factors: curvature of the rim, the area of the rim (assuming rim is made of steel) and the loads acting on the rim.
- Radial load is the leading load component. The wheel has a small load affected zone (4-5 spokes).
- The bicycle wheel is completely stressed in tension or compression, instead of bending.
- Bicycle wheel collapses due to: fatigue, sudden loads, a small load affected zone, loosen of the spokes, extreme compression loads.
- The design variables that influence the stiffness and strength of the bicycle wheel are presented in table 2.2.

Spoke wheel roof
- The translation of the ring plays an important role in the capacity of the ring action. The translation is the fourth key factor that influences the ring action.
- Transverse load is the leading load component and the load affected zone is spread over to the whole ring.
- Radial load is not a distributed load, due to larger spoke distances. Causing a decrease of efficient load distribution.
- Applying roll supports do not influence the ring action. Rocker bearings only have little influence.
- Stiffness and strength depends on the design variables mentioned in table 2.3.
- Transverse stiffness and strength is decisive for the design to fulfil the structural requirements.
- Roof structure is assumed to be perfectly circular. A stadium roof has a less efficient shape. The influence of the shape must be further investigated.
- The influences of the design variables (oval shape, etc.) are not always an improvement for the efficiency of the structure. However, it increases the application possibilities of the spoke wheel roof.

Chapter 3 – Reference projects
The reference projects show how engineers have found solutions to apply a spoke wheel roof structure at a football stadium. From all three analysed stadia, the most important conclusions that are relevant for the structural design of the spoke wheel roof are summarized.

- The general oval shape of a football stadium causes a decrease of the ring action. The weakest point is present at the long side of the structure.
- When regular steel profiles are used, the loads can be taken up by beam action as well.
- A solution to increase the stiffness at the long side is to transfer the vertical loads acting on the long side to the corners of the structure where most of the curvature (ring action) is present.
- To prevent or decrease the amount of beam action/bending moments, it is advised to use a truss or cable structure.
- A cable does not possess bending and torsion rigidity. No bending moments and beam action will arise.
- The construction time of a cable structure is very short compared to a truss structure and other conventional roof structures.

Chapter 4 – Reference stadium
The reference stadium that will be used for the thesis is the Amsterdam ArenA. The following design variables are already fixed using this reference stadium:

- Design area of the new roof (see appendix A.4).
- Amount and place of the supports
- Shape of the outer ring.

Chapter 5 – Requirements and conditions
The requirements and conditions determine the boundaries of the structural design. The requirements and conditions are listed in chapter 5.

In the following part of the thesis, the influences of the shape of the reference stadium and the design variables on the structural behaviour of the spoke wheel roof are investigated.

Part 3
Preliminary Design

Preliminary Design

In the Preliminary Design the first step to a structural design of a spoke wheel roof for the reference stadium will be made. Before the preliminary design will be determined, the application of a spoke wheel roof for the reference stadium will be investigated. The arising structural problems have to be determined, followed by possible solutions.
The preliminary design depends on certain design variables. The influence of the design variables on the design of the roof structure is further investigated. The goal is to increase the efficiency of the design of the spoke wheel roof structure.
At the end of the Preliminary Design part a conclusion is made, and the first sketches of an efficient design of the roof structure are presented.

6. Shape study

The Amsterdam ArenA has been chosen as a reference stadium for this thesis. Reference projects showed that the application of the spoke wheel to a football stadium can cause structural problems. Before a design for this reference stadium can be made, the model must be investigated for possible structural problems and possibilities concerning the ring action.

6.1 Reference shape

The shape of the Amsterdam ArenA (from now on called the reference shape) is very different compared to the original spoke (bicycle) wheel. The bicycle wheel can be assumed as the perfect efficient spoke wheel structure. In chapter 2 the theory of the spoke wheel has been explained. Using this theory and the knowledge gained from the analysis, the structural problems within the reference shaped roof (figure 6.1) can be determined.

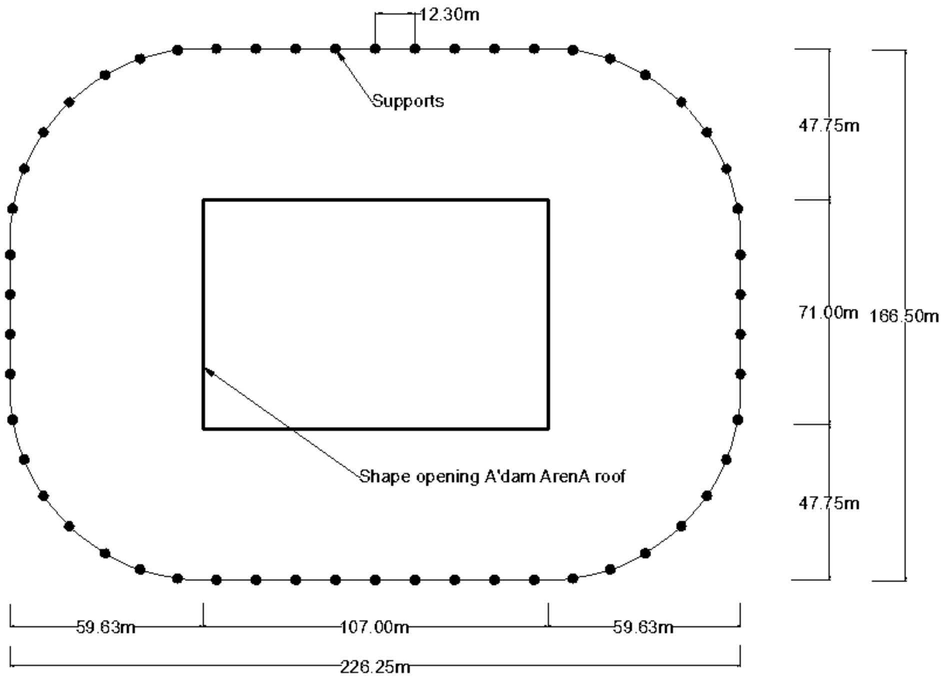


Figure 6.1 Reference shape

To investigate the structural behaviour of a spoke wheel roof with the shape of figure 6.1, a model with the software program Scia Engineer 2010 has been made. Using this FEM program it is possible to make 2nd order calculations. Further information about the calculation method is presented in appendix A.3. The reference model is presented in appendix A.2.

6.1.1 Ring action

The efficiency of the ring structure can be determined by looking at the amount of ring action. In case of a roof structure, the translation also influences the ring action. When the ring is not stressed, the ring will turn into an oval shape when it is subjected to the slightest of loads. The theory of the ring action can be found in chapter 2. The reference stadium is assumed to be placed on roll supports. The supports will have no influence on the ring action.

The amount of stress can be calculated using equation 2.7. The stress depends on the radial load, the radius and the cross-sectional area of the ring:

$$\sigma = \frac{N}{A} = \frac{q \cdot r}{A} \qquad (6.1)$$

With the stress, the strain is determined using equation 2.3 and 2.4:

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{N}{EA(x)} \quad (6.2)$$

$$\varepsilon_y = \varepsilon_z = -v\frac{\sigma_x}{E} = -v\frac{N}{EA(x)} \quad (6.3)$$

The amount of ring action can be expressed as a spring force (chapter 2). The spring force depends on the amount of deformation. In the case of the roof structure, the deformation is equal to the transverse strain multiplied with the perimeter of the ring.

$$u = \Delta l = \varepsilon_y \cdot l \quad (6.4)$$

The spring constant is equal to:

$$k = \frac{EA}{r^2} \quad (6.5)$$

Forming the total spring force (ring action) by the equation:

$$F_{spring} = k \cdot u \quad [\frac{kN}{m}] \quad (6.6)$$

Using these formulas the efficiency of the outer and ring can be determined.

For the investigation of the shape of the reference stadium, the roof is placed on roll supports. Assuming the extensional rigidity is equal in the whole structure, the ring (action) of the roof structure can be investigated by looking at the radius (curvature) of the ring and the amount of radial load (stress) acting on the structure.

Outer ring

The outer ring of the roof can be described as an oval shape with straight sides, both at the short and long side of the roof structure. The problem of straight sides, is that there is no curvature and therefore the radius has an infinite value;$r = \infty$.

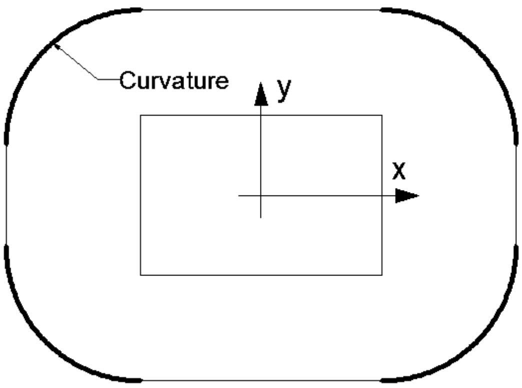


Figure 6.2 Curvature in the corner outer ring elements

When assuming the radial load and cross-sectional area remains fixed, in theory an acting radial distributed load q unequal to zero, would lead to an infinite value of the strain (equation 6.2 and 6.3). Although only in theory this is possible, the formulas show that great deformations will arise in the straight sides for any given value of the radial load and cross-sectional area. It can be concluded that the ring provides no ring action in these points of the outer perimeter.

The corners of the outer perimeter do have a radius value unequal to infinite: $r \neq \infty$. Meaning that the corners provide certain stiffness and strength in the plane of the ring, due to the ring action. The amount of stress and strain will be lower compared to the straight sides of the structure.

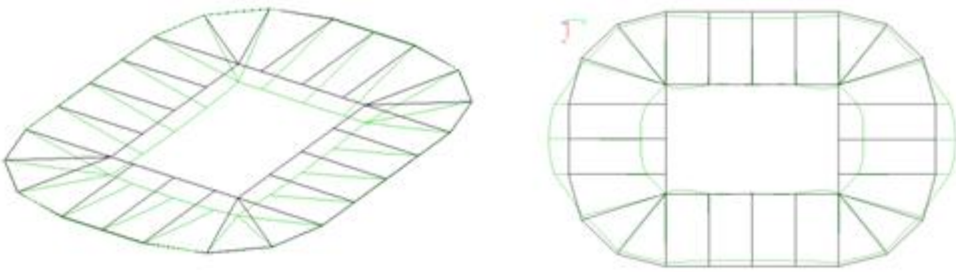


Figure 6.3 Deformation of the reference structure

Inner ring

The inner perimeter is completely rectangular shaped. The radius is infinite and therefore provides no ring action. The deformation of the inner ring shows parallel results to the straight sides of the outer perimeter. Using equation 2.3, 2.4 and 2.7, the stress and strain would have, in theory, an infinite value.
The inner perimeter will not provide any transverse resistance. No normal forces will arise due to the ring action of the inner ring.

6.1.2 Loads

Load affected zone

In a roof structure the leading load component is the transverse load. The radial load acting on the ring is caused by the vertical deformation (up- or downwards) of the central hub and the spokes as a consequence of the transverse load (figure 6.4). When pre-tension is applied in the spokes, this will cause an extra radial load on the ring structure.

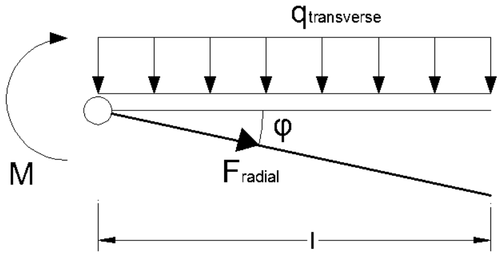


Figure 6.4 Transverse load causing radial load in spoke and bending in ring

As explained in chapter 2 (Spoke wheel principle), the load affected zone influences the efficiency of the spoke wheel structure. In a roof structure the load affected zone is present in the whole outer ring. Although it is positive to increase the load affected zone, in case of the reference shape this is not. Radial load acts on the straight sides of the structure, which possess only little stiffness due to the lack of ring action. This causes large deformations at the sides of the roof.

Non-constant load distribution

In paragraph 2.4.3 the consequences for the spokes in an oval shaped spoke wheel have been explained. Due to the shape, the angle between the spokes and ring elements is not constant. This causes a non-constant load transfer distribution in the spokes of the structure. The angle between the spokes and ring elements determines the amount of force a spoke can transfer. Using the rules of the polygon of forces the situation for the reference shaped roof can be explained.

The radius of the ring determines the efficiency of the load transfer in the structure. The smaller the radius, the smaller the angle between the spokes and ring elements, the better the load transfer. This is illustrated by figure 2.16 in chapter 2.
The corners of the roof of the reference shape have a smaller radius compared to the sides of the structure. The situation for both spoke-ring connections is displayed in figure 6.5.

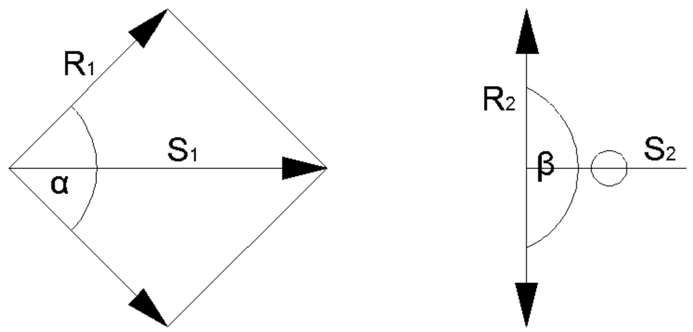


Figure 6.5 Polygon of forces; situation in the corner (left) and on the sides (right)

The figure describes the load transfer difference between the corners of the roof (figure 6.5) and the sides. In theory the spokes on the straight sides transfer no loads (nl: nulstaven), due to the perpendicular connection with the inner and outer perimeter.
In this case all load acting on the roof structure is carried by the spokes and rings in the corners of the roof structure. The normal force distribution is illustrated in figure 6.6 when the structure is subjected to downward transverse loads.

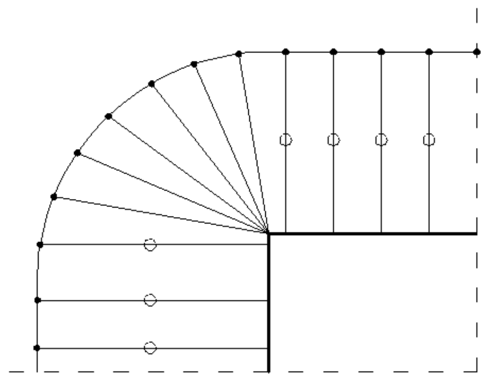


Figure 6.6 Normal force distribution (Red = compression, blue = tension)

When looking more detailed to the corners of the reference model (figure 6.7), the angles at the spoke-ring connection between the inner and outer perimeter are not equal to each other. The polygon of forces illustrated that the smaller the angle, the more normal force can be efficiently transferred. Because there is an angle difference, the amount of normal forces in the spokes will also differ. Meaning that the force in spoke $S_1 \neq S_2 \neq S_3 \neq S_4$.
The difference of normal forces in the spokes has an effect on the transverse forces (Vy) and bending moments Mx (torsion) and Mz in the ring elements as well as in the spokes itself.

Figure 6.8 illustrates an example of the consequences of different normal forces working on a single outer ring element. One must keep in mind; this is an example and can differ from the results showed in the appendix.
The difference in the normal force ($N_1 \neq N_2$) causes a different transverse force (Vy) and bending moment (Mz) distribution compared to the spoke wheel roof discussed in chapter 2.

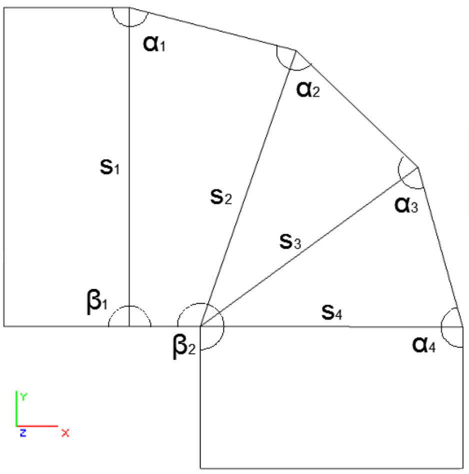


Figure 6.7 Corner of the reference model

In the example the single ring element is only influenced by the loads acting on the element itself. In case of the reference structure, the force and moment distribution also depends from the adjacent ring elements.
It can be concluded that the corners of the reference structure provide most of the stiffness of the structure.

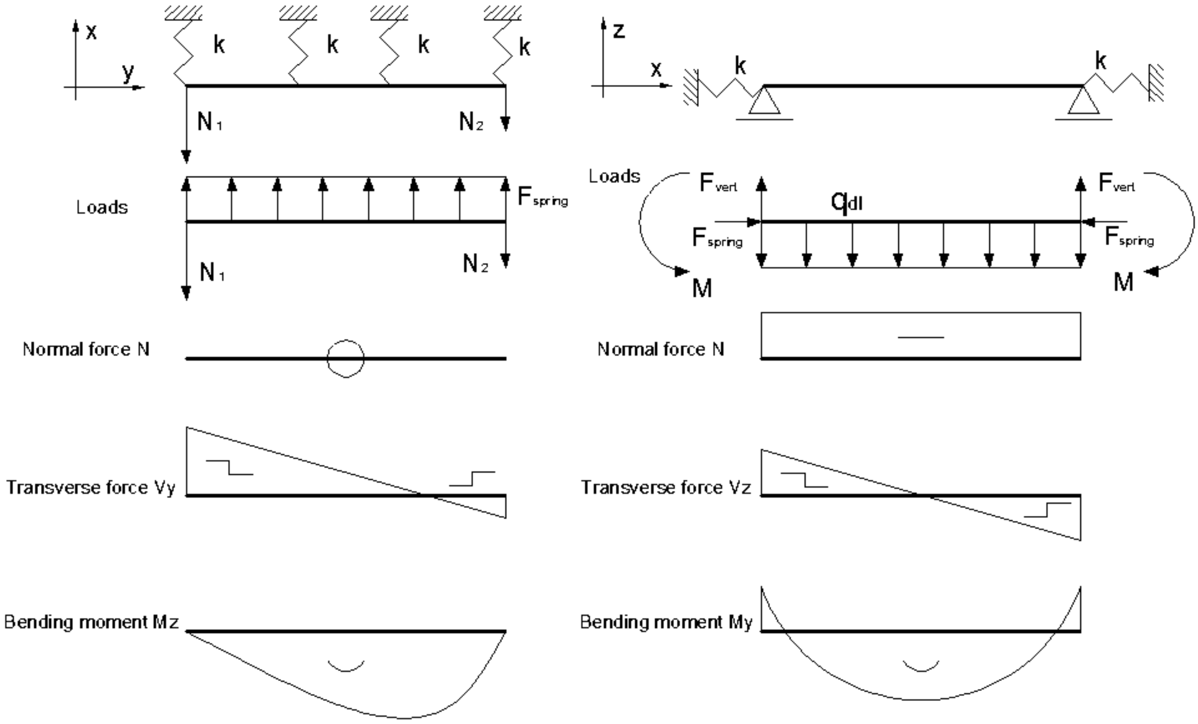


Figure 6.8 Example of the structural behaviour of a single outer ring element

6.1.3 Strength

At the sides of the structure there is a lack of curvature. At these points there will be little ring action, leading to large transverse deformations of the inner ring. As a consequence of the large deformations, very large transverse forces and moments will arise in the connected outer ring elements. To withstand these high forces and moments, the ring structure must have a large cross section. A large, hollow cross section (for instance RHS/CHS profiles) gives the ring structure bending and torsion rigidity.
The highest normal force values will arise in the corner outer ring elements. To provide sufficient strength, a large hollow cross section is needed to prevent the ring from buckling.

The transverse strength of the roof structure is the leading direction in order to fulfil the strength requirements. The strength in this direction can be greatly improved by increasing the space between the spokes, or in other words, using a double inner or outer ring structure. Due to the geometry, transverse forces are introduced and the transverse strength of the structure will increase.
The transverse strength can be further improved by pre-tensioning the spokes. Another option to increase the transverse strength is to improve the load affected zone. The possibility to improve the load affected zone within the reference shape is however limited. The load affected zone can be increased by using stronger ring profiles or by using more spokes with a smaller distance between the spokes. However, no extra spokes can be applied, because the amount and place of the supports is fixed. The only possibility to improve the load affected zone is to use stronger profiles for the spokes and ring elements.

The radial strength of the structure can be improved by using stronger profiles (high value of the extensional rigidity). As already mentioned, there is no possibility to apply more spokes to increase the radial strength.

6.1.4 Stiffness

The roof structure is mainly transverse loaded; therefore the transverse stiffness is decisive in the design of the roof structure. Due to the transverse load, the roof structure will deform in vertical direction. The transverse load acts up or downwards. To fulfil structural requirements, the vertical deformation must be limited by the transverse stiffness of the roof structure.
In chapter 2 has been explained that the transverse stiffness is influenced by the ring action. Due to the second order effect, the ring action (figure 6.9, illustrated as spring force) and possible pre-tension decreases the amount of deformation.

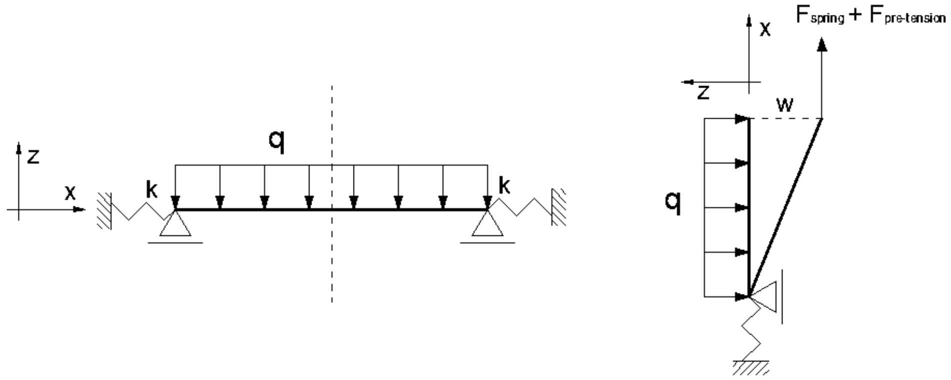


Figure 6.9 Deformation of spoke wheel roof – second order effect.

However, the sides of the structure have no curvature and therefore provide no ring action. Besides, the shape of the roof causes a non-constant stress distribution in the ring as a consequence of the difference in curvature. The largest transverse and radial deformations arise on the straight sides. To decrease the amount of deformation, an option is to provide extra transverse stiffness by pre-tensioning the spokes, apply stiffer ring elements on the sides or by using a double inner or outer ring structure.

The radial stiffness is primarily influenced by the number and thickness of spokes and also by the depth of the ring structure. A stiffer ring structure extends the load-affected zone so that more spokes are affected, and it increases the radial and transverse stiffness of the roof. In the design it is however not possible to increase the number of spokes. To increase the radial stiffness stronger spokes can be used and the depth of the ring structure can be increased.

6.1.5 Roof collapse

The spoke wheel roof structure can collapse from different causes. The most decisive failure will occur at the sides of the roof structure, where no curvature is present. To prevent the roof from large transverse and radial deformations, the lateral and transverse strength and stiffness need to be increased greatly.
The roof will most likely collapse as a consequence of failure of the elements present at the sides of the roof. Especially in the ring elements very large bending moments (Mz and torsion Mx) and transverse forces (Vy) will arise due to the large deformations. The ring elements need to have sufficient extensional, bending and torsion rigidity to prevent the ring from collapsing. The corner outer ring elements need to be strong enough to withstand the high amount of normal forces and to prevent the ring from buckling.
Other causes of roof collapse can be due to fatigue or breaking of the spokes or other elements. In the design process of the spoke wheel roof fatigue will however not been taken into account. The dynamic loads acting on a stadium roof are assumed as static loads.

6.2 Conclusion

The shape study showed that efficiency of the spoke wheel roof structure for the reference stadium has decreased. With the analyses of the shape of the reference stadium, an answer can be given to the third sub-question, which describes the following question:

'What problems arise with the application of a spoke wheel roof structure to an existing stadium?'

From the analysis can be concluded that the shape of the reference stadium causes different structural problems. By looking at the consequence of the shape of the reference stadium on the key factors that determine the ring action, the problems can be determined. One must keep in mind that the investigation of the shape has been done for a structure that is assumed to be placed on roll supports. Pretension has not been taken into account and the choice of material has not yet been made.

Curvature
- At the sides of the roof structure there is no curvature present. As a consequence no ring action can be activated.
- The inner ring does not possess any curvature and will not provide ring action.
- The only curvature is present in the corner outer ring elements. The ring action can only be provided in these areas of the roof structure.

Loads
- The load distribution on the outer ring is non-constant. Looking at the polygon of forces, only the spokes in the corner areas will transfer the loads efficiently to the outer ring elements. In the corner ring elements the largest normal forces will arise. The non-constant load distribution causes extra transverse forces (Vy) and bending moments (Mx and Mz) in the outer ring elements, especially at the straight sides.
- The corner ring elements can take up large stress values due to the presence of curvature (ring action). The straight sides can only take up little stress values, which can be resisted by the stiffness, provided by the ring elements itself.

Extensional Rigidity
- In the corner ring elements, the largest normal forces will arise. These elements need to have sufficient strength to prevent the ring from buckling.

Translation
- The spoke wheel roof structure is assumed to be placed on roll supports. The radial translation of the roof structure is limited due the stiffness provided by the ring elements itself and the ring action. The largest radial deformations arise at the straight sides of the inner and outer ring. The largest sag is present at the centre of the long side of the inner ring.

The problem of the shape of the reference stadium is that it is not possible to provide ring action in the whole roof structure. The largest deformations arise at the points where the ring action is very little: at the straight sides, especially on the long side of the structure. The inner ring will sag; as a consequence large radial deformations arise in the connected outer ring elements. To prevent the ring from deforming, the ring has to have sufficient stiffness. The stiffness at this point cannot be provided by the ring action. To prevent large deformations, the ring elements need to have sufficient bending rigidity themselves (beam action). To provide the needed amount of bending rigidity, probably very large profiles are needed which results in a great use of needed material. To come to an efficient structural design can be concluded that the structural designer needs to use the available ring action at full extend to save material and reduce building costs.

For the mentioned problems solutions have to be found in order to use the available ring action better and to come to an efficient structure. The challenge is to decrease the deformations (to fulfil the structural requirements from the Eurocode), the amount of forces and moments and still minimize the amount of material and costs.

To increase the strength and stiffness of the structure there are solutions which have been discussed in chapter 2 (Spoke wheel principle). The solutions are certain design variables that influence the key factors and therefore the strength and stiffness of the structure (either negative or positive).

To come to an efficient preliminary design, first the influence of the following variables must be investigated:

1. Shape of the (opening of the) roof
2. Double inner / outer ring
3. (Non) pre-tensioning of the spokes
4. Profile / elements
5. Supports / connections

In chapter 4, Reference Stadium, has been concluded that the shape of the outer ring and place of the supports is fixed. However, there are some design variables left to investigate. These design variables will be applied to the reference model and further analysed in the following chapter.

Shape Inner ring

Unlike the outer ring of the reference shape, the conditions prescribe that the inner ring can be adapted. The amount of curvature that can be applied to the ring will be investigated.

Support and connections

The choice of type of support and connection determines the structural behaviour of a structure. In chapter 2 is explained that the spoke wheel roof need to be supported by either a roll or rocker bearing, in order to provide a free translation in its plane and to use the ring action at full extend. The connection between the ring elements and the ring and spokes are investigated next.

Double inner/outer ring

The choice of applying a double inner or outer ring will be investigated. The influence on the structural behaviour will be explained.

Structural material and elements

In the following paragraph different materials and elements are discussed. The (dis-) advantages of all are described. The ring and spokes are subjected to different kind of stresses and need different materials or elements.
The type of profile is decisive for the structural behaviour of the roof structure. For instance cables can only take up tensile forces where regular steel profiles (CHS/RHS, I/H, etc.) can take up both compression and tension. The choice of profile has a large influence on the structural design.

(Non) pre-tensioning of the spokes

The influence of pre-tensioning the spokes or not is investigated at a later stage of the design process.

At the end of the Preliminary Design part, the analysis and investigations will be concluded and the preliminary design will be determined.

7. Design variables

To improve the structural design of the spoke wheel roof there are some design variables that influence the strength and stiffness of the design. The available design variables can be categorized in the following categories:

- Geometry of the main bearing structure
- Structural elements

First the design variables regarding the geometry of the main bearing structure is investigated. The geometry of the main bearing structure beholds the shape of the inner ring, the choice of supports and connections, followed by the application of a double inner or outer ring. At last the possible structural elements are analysed.

7.1 Geometry of the main bearing structure

For now the geometry of the roof structure has three design variables that influence the preliminary design. These are the shape of the inner ring (paragraph 7.1.1), the choice of supports and connections (7.1.2) and the possibility to apply a double tension or compression ring (paragraph 7.1.3) in order to increase the transverse support.

7.1.1 Shape inner ring

A spoke wheel's most important structural property is the use of ring action. The greater the curvature, the better the ring action, the stiffer and stronger the structure will be. The reference model appeared to have no curvature in the inner ring of the roof. Due to the lack of curvature, the structure will not provide stiffness through ring action.
To provide more stiffness in the ring, the ring will be more curved. In chapter 4 the design area of the edge of the roof has been determined. The drawings can be found in appendix A.4. In this phase of the thesis the final shape of the roof structure has not been determined yet. To determine the shape of the inner ring there are some dilemmas the structural engineer has to deal with. The drawings in appendix A.4 show that the higher the edge of the roof will be, the more curvature can be applied to the ring. However, the higher the roof top, the larger the roof structure will become. In that situation the amount of material and costs will increase.
For the preliminary design of the spoke wheel roof structure is assumed that the edge of the roof is in line with the top of the stands. At this height there is some room left for applying curvature at the inner ring. This is illustrated in figure 7.1. The total design area at this height of the roof is illustrated in figure 7.2.
The design area is set by boundary conditions described in chapter 4. The lines of sight of the spectators, protection against the elements (wind, snow, rainfall, etc.) and quality of the grass are the conditions that together eventually form the design area of the ring.

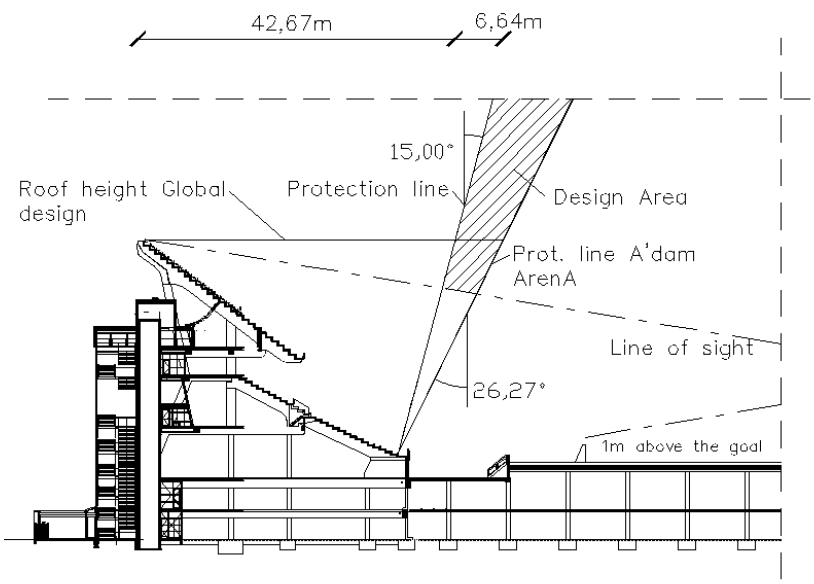


Figure 7.1 Assumption of the roof height for the preliminary design

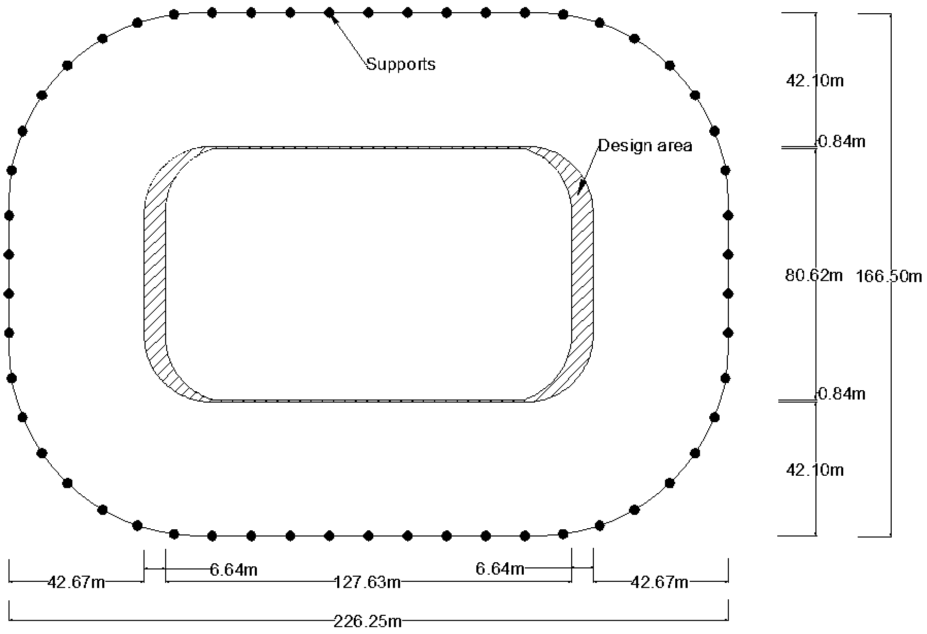


Figure 7.2 Design area of the inner ring

The design area illustrated in figures 7.1 and 7.2 shows the area in which curvature can be applied to ring. To decrease the amount of deformation of the inner ring, the opportunity to apply curvature in the ring must be used at full extend. The more curvature in the ring, the more ring action and the stiffer the ring will be.
Figure 7.3 shows the maximum curvature possible for the given design area. The sides of the inner ring are equal to an ellipse with radial value $t = 0 - \pi$.
An ellipse however, does not have a constant radius. The largest radius is present at the centre of the ellipse ($t = \frac{1}{2}\pi$). On the short side the largest radius is 72,72m and on the long side 2868,06m. These radiuses are decisive to determine the stiffness of the spoke wheel at the sides of the structure.
The total area of the roof structure is 21.960m² or approximately 22.000m². The determination of the shape of the inner ring and the total area of the roof is presented in appendix A.4.

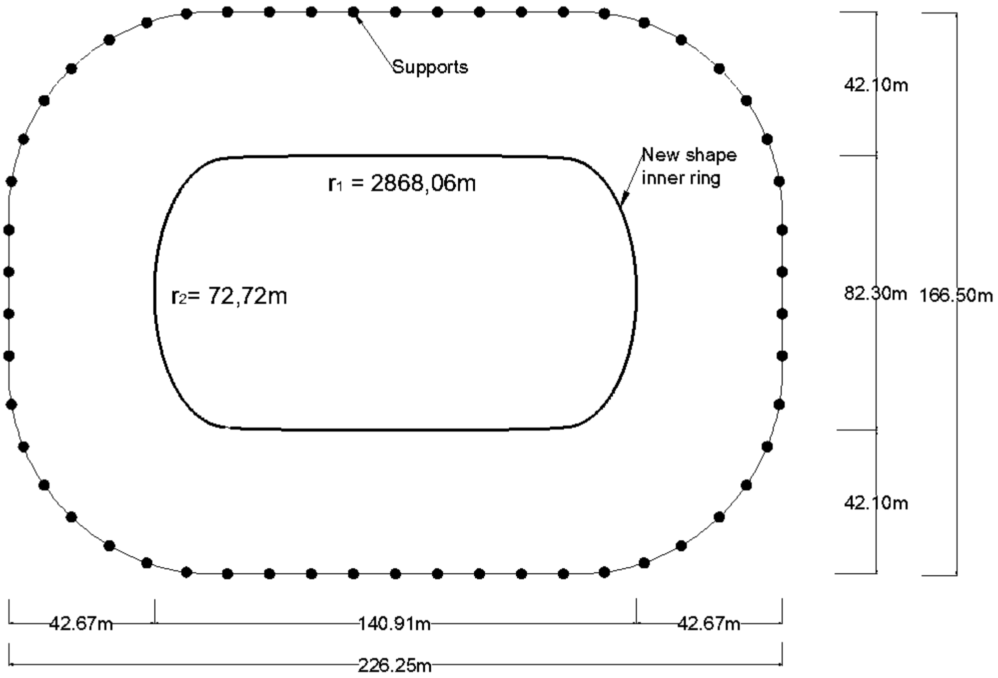


Figure 7.3 Shape of the preliminary design of the inner ring.

With the application of the maximum curvature possible for the given design area, the inner ring structure will provide, unlike the original shape of the inner ring of the reference stadium, ring action.
The greatest ring action is present in the corners of the structure. At $t = 0$ and $t = \pi$ (in figure A4.4) the ellipse has the greatest curvature and therefore the most ring action capacity. The weakest points remain at the centre of the sides of the inner ring, where the ring action is minimal.

7.1.2 Supports & connections

The choice of type of support and connection influences the structural behaviour of the roof structure. The supports and connections have a direct influence on the two key factors translation and loads (figure 2.31) in the ring structure. The influence of the type of support and connection is investigated in this paragraph. For the analysis the reference shape model from chapter 6 is used.

7.1.2.1 Supports

The spoke wheel roof structure will be supported at the nodes of the outer ring of the reference stadium. The reference stadium has 50 support nodes were the roof structure is connected to the rest of the stadium structure. The place of the support nodes is fixed and equal to the place of the supports of the reference stadium (figure 7.4).

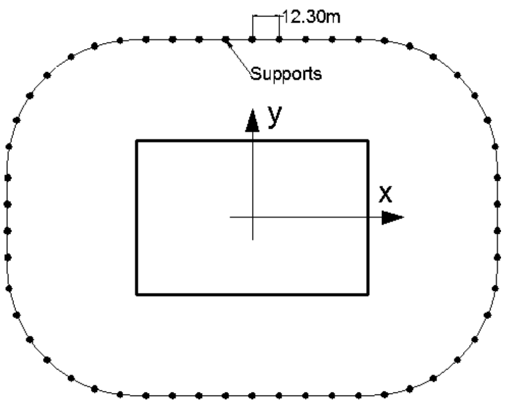


Figure 7.4 Place of the supports

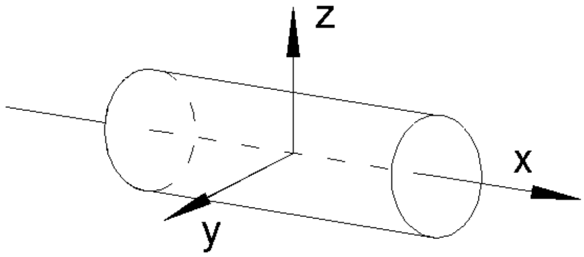


Figure 7.5 Local axes single element

The structural engineer has the option to apply the following supports:

- Roll supports
- Rocker bearings
- Hinged supports
- Fixed supports

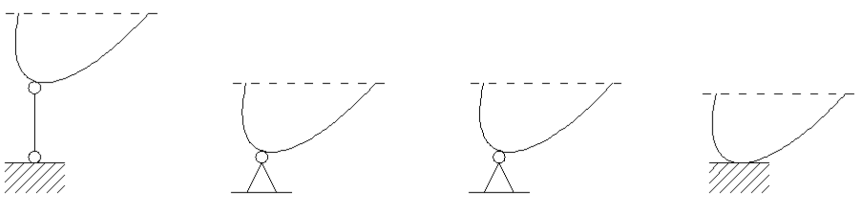


Figure 7.6 Types of supports: Rocker bearing, roll, hinged and fixed

The influence of the translation on the ring action has been described in chapter 2. The ring action is activated by a radial load working on a ring structure. The curvature of the ring structure provides stiffness of the structure to withstand the radial load. The ring action can be illustrated by a spring force working on a ring structure.

Equation 2.15 expresses the spring constant:

$k = \frac{EA}{r^2}$

Followed by the spring force: $F = k \cdot u$

With u is the transverse extension of the ring. This extension or translation is important to provide ring action in the ring structure. In order to let the ring action work, the ring structure must be able to translate/extend/deform in its plane.

With a hinged or fixed support, the translation in all directions is blocked. When a hinged or fixed support is applied to the outer ring of the roof structure, there will be no ring action. The ring structure will become a simple beam-column system and will resist loads only through beam action.
Besides, the radial load acting on the ring structure will then be taken up by the supports and not by the ring action of the outer ring. When applying hinged or fixed supports, the loads acting on the roof are taken up by the supporting structure of the roof. In the case of the reference stadium these are the stands of the stadium.
The structural properties of the stands are unknown in this thesis. If the roof structure is hinged or fixed supported the stands of the stadium must be strong enough to withstand the (horizontal) loads from the roof structure acting on it. Because these properties are unknown, the option to apply hinged or fixed supports is left out of consideration.

In the following paragraphs the influence of the use of roll supports and rocker bearings for the reference stadium is investigated. At the end a conclusion and a recommendation is made to eventually come closer to a final, efficient spoke wheel roof design.

Roll supports

'Within a roll support the mass can translate parallel to the roll course and rotate around its support.'

Due to the free translation in the xy-plane, the roll support is also called a sliding bearing. With the help of table 7.1 and figure 7.7 the degrees of freedom are clearly visible.

Translation	Value	Rotation	Value
Ux	Unknown	Rx	Unknown
Uy	Unknown	Ry	Unknown
Uz	0	Rz	Unknown

Table 7.1 Values of the local translations and rotations of roll supports

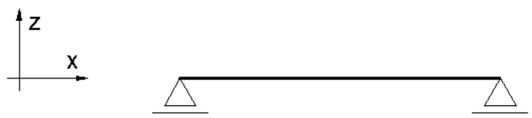


Figure 7.7 Roll support of the outer ring

The application of just roll supports that can translate free in the xy-plane is not enough to stabilize the roof structure. The sliding bearings with the characteristics of table 7.1 cannot take up tensile forces. When the structure is horizontally loaded, the structure will collapse. To provide horizontal stability some roll supports need to be rigid in the local x- direction (figure 7.5). The roof structure will be stable when the translation in the global x- and y- direction is blocked (figure 7.4). This can be accomplished by blocking the translation of the ring elements in the local x-direction. The roof structure cannot rotate around its global z-axes and can only translate in its local y-direction (table 7.2). This way, stability is guaranteed for the whole structure.

To provide enough stability, it is not necessary to block the translation of all roll supports in the local x-direction. As mentioned, the structure remains stable when the translation in the global x- and y direction is blocked. This can be accomplished by applying a blocked translation in the local x-direction of a single support on each side of the roof structure (figure 7.8). The advantage is that it saves costs and the stability of the structure is still guaranteed.

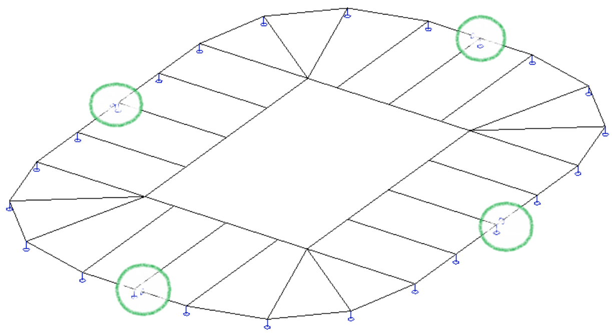


Figure 7.8 Place of the roll supports with blocked translation in local x-direction

The structure will not only be loaded by wind or its dead load, also loads due to temperature change can arise. Especially in large structures, like stadium roof structures, extension of the material is an important design factor to take into account. When roll supports are applied, the roof structure can translate in its horizontal plane. As a consequence of the free translation, no forces due to prevented extension will arise.

Translation	**Value**	**Rotation**	**Value**
Ux	Unknown	Rx	Unknown
Uy	0	Ry	Unknown
Uz	0	Rz	Unknown

Table 7.2 Values of the local translations and rotations of roll supports with horizontal stability

Rocker bearings

'A rocker bearing is a straight bar (or column) that is hinged connected at both ends which is only loaded at its ends. A rocker bearing prevents translation in the direction of the bar. Only translation perpendicular can arise, as well as rotation around the ends.'

Rocker bearings have the same degrees of freedom as the roll supports (table 7.3).

Translation	**Value**	**Rotation**	**Value**
Ux	Unknown	Rx	Unknown
Uy	Unknown	Ry	Unknown
Uz	0	Rz	Unknown

Table 7.3 Values of the local translations and rotations of rocker bearings.

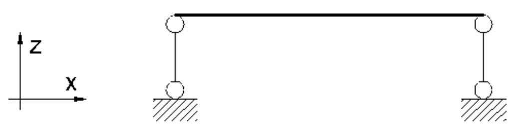


Figure 7.9 Local axes rocker bearings at outer ring

Like roll supports, rocker bearings are sliding bearings. When applying rocker bearings, the spoke wheel roof structure can translate in the global x- and y-direction. This way the structure can slide in its horizontal plane. Due to this characteristic, rocker bearings can take up loading due to temperature changes (extension of the elements).Therefore no undesirable forces and moments will arise of preventing the material to extend.

As in the case of roll supports, the rocker bearings do not provide enough horizontal stability. To provide stability, diagonal wall bracings are applied. The advantage of the rocker bearing compared to the roll supports is that it can better take up tensile forces and has the ability to influence the horizontal stiffness of the structure. The choice of the profile for the wall bracings, the amount and place of wall bracings determines the horizontal stiffness.

The structure can already be stabilized with only two diagonal wall bracings in the global x- and y-direction of the roof structure when the correct type of profile is applied (enough stiffness). The disadvantage of only applying wall bracings at only two points of the structure is the unequally distributed horizontal stiffness. This has major consequences for the way the roof structure deforms; extra transverse forces (Vy) and bending moments (Mx and Mz) can arise. An option to avoid this is to apply wall bracings between every rocker bearing. This way the roof structure still can translate in its plane. However, the disadvantage is that the material use and the costs will increase rapidly.
A possible solution is to apply less bracings which are symmetrically placed. For example four wall bracings, each on every side of the reference structure (figure 7.10). The values of the local translation and rotations of the rocker bearing support with bracings are presented in table 7.4.

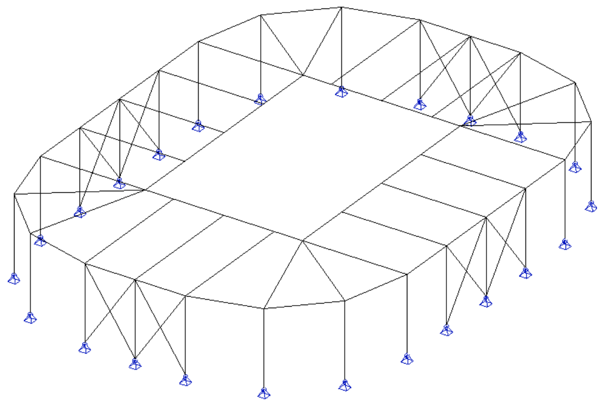


Figure 7.10 Place of the rocker bearings

Translation	Value	Rotation	Value
Ux	Unknown	Rx	Unknown
Uy	0	Ry	Unknown
Uz	0	Rz	Unknown

Table 7.4 Values of the local translations and rotations of rocker bearings with wall bracings

An advantage of rocker bearings compared to roll supports is that the stiffness in the plane of the roof can be adapted. As mentioned, the choice of profile, place and amount of wall bracings determines the stiffness.
The amount of stiffness in the horizontal plane influences the amount of ring action in the roof structure. Assume the stiffness infinite, the structure will not deform in its plane. Like in case of hinged and fixed supports, the translation is blocked in the plane of the structure.

The dilemma when determining the right profile, place and amount of bracings is that the structure need to be strong and stiff enough to fulfil the stability requirements from the Eurocode. However, it cannot be too strong that it prevents the structure from providing ring action. Equilibrium has to be found and this must be investigated in a later stadium of the design process.

Conclusion

The choice of support has a great influence on the structural behaviour of the roof structure, meanly on the stress in the ring structure and the translation. The application of hinged or fixed supports is left out of consideration. When these supports are applied the supporting structure of the roof, the ring is not able to translate in a radial direction. Looking at the equations regarding the ring action in chapter 2, the ring needs to be able to deform in order to create ring action. When there is no deformation possible in the plane of the ring, the stiffness is created by beam action. The structure will need to have stronger structural elements in order to provide sufficient stiffness and to withstand high stress values, causing an inefficient structural design.
Besides, the structural properties of the stands are unknown in this thesis. If the roof structure is hinged or fixed supported the stands of the stadium must be strong enough to withstand the loads from the roof structure acting on it.

Between the rocker bearings and the roll supports there are some structural differences. Rocker bearings can better take up tensile forces and have the ability to influence the horizontal stiffness.

The analysis showed that the amount and place of supports has a large influence on the structure. One can choose to equally spread the transfer of the loads to the underground or concentrate it at certain places (for example at the four sides of the structure). By concentrating the transfer of the loads less material is needed. A requirement is that the stadium itself can also transfer the horizontal loads at the point where the roof structure is supported.

Besides the structural point of view, there is the perspective from financial and architectural point of view. Looking at the costs, the use of roll supports is more expensive compared to the use of rocker bearings. The reason is that roll supports need to be adapted to provide a blocked translation in the local x-direction. From an architectural point of view, the use of rocker bearing will strengthen the floating character of the spoke wheel roof.

It can be concluded that generally the application of rocker bearings is more suitable for spoke wheel roof structures then the use of roll supports. For the structural design rocker bearings will be applied. To stabilize the roof, wall bracings will take up the horizontal loads. In order to transfer the loads as efficient as possible, the wall bracings need to have a slope of 45 degrees. The distance between the supports is 12,30m. Therefore, the height of the rocker bearings will be equal to the distance between the supports.

Type of support	Advantage	Disadvantage
Rocker bearing	Efficient load transfer Free horizontal translation Adaptable horizontal stiffness Relatively cheap Ability to take up high tensile forces	Wall bracings needed for horizontal stability Large deformations possible
Roll	Efficient load transfer Free horizontal translation	Large deformations possible Adaption of supports needed to provide horizontal stability

Table 7.5 (Dis-)advantages of roll support and rocker bearing.

7.1.2.2 Connections

Within the spoke wheel structure different connections can be applied. The spoke wheel roof structure can be divided into two connections that have a great influence on the stress distribution of the roof structure: the connection between the spokes and rings; and the connection between the ring elements itself.

Spokes – Rings

The connection between the spokes and rings can be either hinged or fixed connected. The influence of both types of connections on the structure is investigated and explained in this chapter.
The reference model from chapter 6 has been used for the analysis. The ring elements in the outer- and inner ring are assumed to be fixed together.

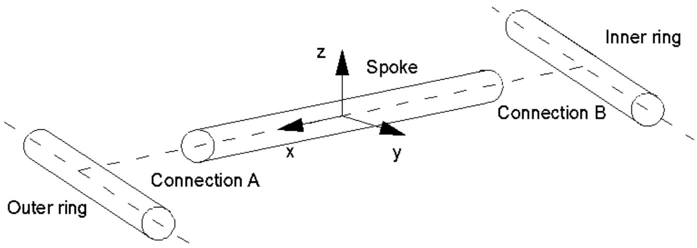


Figure 7.12 Connection A and B between spokes and ring

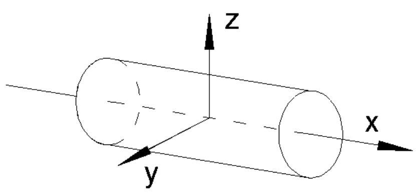


Figure 7.11 Local axes single element

Hinged connection

The spoke is hinged connected at both ends. The values of the translations and rotations of both nodes are presented in table 7.6. The schematization of the hinged connection between the spokes and ring is illustrated in figure 7.11. The hinged connection between the spokes and ring is applied at the reference model from chapter 6.

Connection A		**Connection B**	
Translation/rotation	Value	Translation/rotation	Value
Ux	Unknown	Ux	Unknown
Uy	0	Uy	Unknown
Uz	0	Uz	Unknown
Rx	0	Rx	0
Ry	Unknown	Ry	Unknown
Rz	Unknown	Rz	Unknown

Table 7.6 Degrees of freedom hinged connection

The hinged connection between the ring and spokes, with the translation and rotation described in table 7.6, can be schematized as presented in figure 7.13. The outer ring node (connection A) is placed on roll supports, as explained in the previous paragraph. The ring action in the inner and outer ring is illustrated as a spring in connection A (outer ring) and B (inner ring). And due to the hinged connection, the spokes can rotate around node A and B.

The structure is kinematic indeterminate, but remains stable due to the second order effect. This can be explained by looking at the right schematization in figure 7.13 which is equal to the original schematization.
The deformation of the spoke here is caused by the horizontal distributed load q. In an equilibrium situation the final deformation is reached when the resistance against the deformation is equal to the acting force. The horizontal deformation can be determined using linear elastic calculations.
However, besides a horizontal distributed load q, a vertical force acts on the spoke. The horizontal deformation will decrease, as a consequence of the ring action. This is called the 2nd order effect. Eventually a new equilibrium will be found, the horizontal deformation is expressed as w.
The ring action causes a positive effect on the deformation or sag of the spokes in the roof structure.

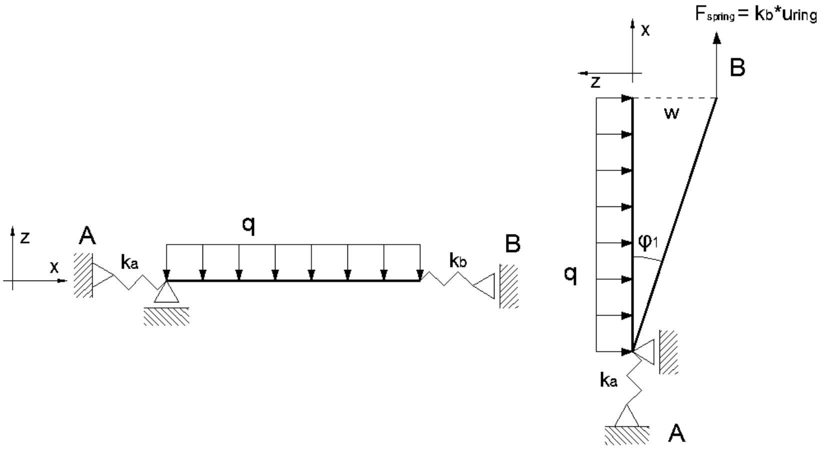


Figure 7.13 Schematization of hinged connection (left) and deformed schematization (right).

For the schematization of figure 7.13 it is not possible to determine the forces, moments and deformations of the structure by hand. There are too much unknown values for this kinematic indeterminate schematization. For the calculations of the structure FEM computer software is needed.

However, it is possible to describe the structural behaviour of this part of the roof structure. The most important structural property is the ring action. When a load acts on the roof structure, a radial load will work on the ring structure. This load 'activates' the ring action, which is represented as a spring force in the schematization. The spring force prevents the inner ring from large sagging and eventually equilibrium will be found.
In order to come to a structurally safe design for the roof structure, the amount of deformation is one of the important factors to take into account. The deformation of the structure depends on many factors: the load acting on the structure, properties of the used elements (extensional and torsion rigidity), spring force of the rings (depends on radius and transverse deformation of the rings), etc. To prevent high values for the deformation, the ring action is most important in this structural system.

To understand the influence of the ring action on the structure, the ring action will be further investigated. By describing the relation of the ring action with the other mentioned factors, one can understand the structural system better. Because there are too many unknown values, assumptions are made.
The ring action is expressed for a deformed situation. In this situation equilibrium of the structure is found and the structure is no more kinematic indeterminate. The schematization is illustrated in figure 7.13 and 7.14. The modules of elasticity E is assumed to be equal to an infinite value $E = \infty$. As a consequence, the bending and extensional rigidity are also infinitely large. The spoke is not subjected to bending or extension, which also influence the deformation of the ring and indirect the amount of ring action.

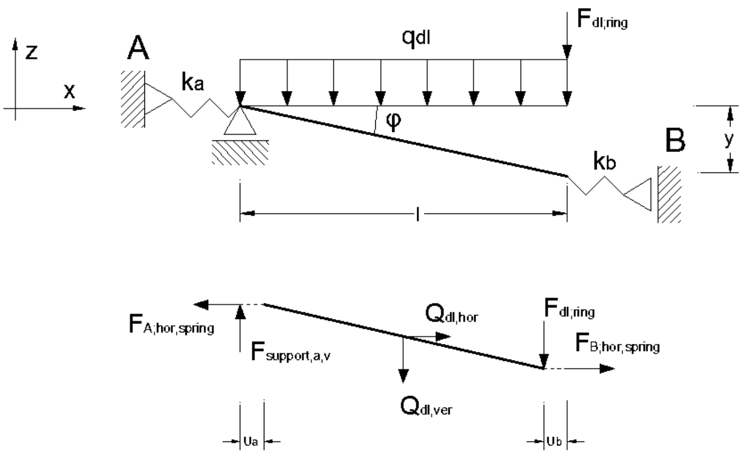


Figure 7.14 Deformed situation

In the schematization the sag of the inner ring is expressed as y. Due to the deformed situation the dead load can be divided in a vertical and horizontal component. With the help of the displacement method the ring action can be described. In appendix A.5 the calculation is presented.

The formula of the ring action is equal to $F = k \cdot u$ with $k = \frac{EA}{r^2}$. For this situation the modulus of elasticity is set equal to ∞. To describe the relation of the ring action, the relation is expressed for the horizontal deformation u for both connections A and B.

The horizontal deformation in node A and B can be expressed as a relation of spring constant k:

$$u_A = \frac{ql^2 + F_{dl;ring}l - \frac{1}{2}ql^2 + \frac{qly}{2\tan(\varphi)}}{k_A \cdot y + ql - F_{dl;ring} - \frac{1}{2}ql} \tag{7.1}$$

$$u_B = \frac{\frac{1}{2}ql^2 - \frac{ql}{tan(\varphi)} \cdot \frac{1}{2}y + F_{dl;ring} \cdot l}{\frac{1}{2}ql + F_{dl;ring} + k_B \cdot y} \tag{7.2}$$

Equation 7.1 and 7.2 show that the deformation of the node at the inner and outer ring depends from the amount of load acting on the structure. The load 'activates' the ring action of both rings and provide a transverse and radial support.

As mentioned, for the determination of the relation between the horizontal deformation and the ring action is assumed that the modulus of elasticity of the spokes is infinitely large.
In equations 7.1 and 7.2 the influence of the extensional and torsion rigidity of the spokes has been left out of consideration. The actual horizontal deformation also depends from the bending and extension of the spokes. These cause an extra deformation, see figure 7.15.

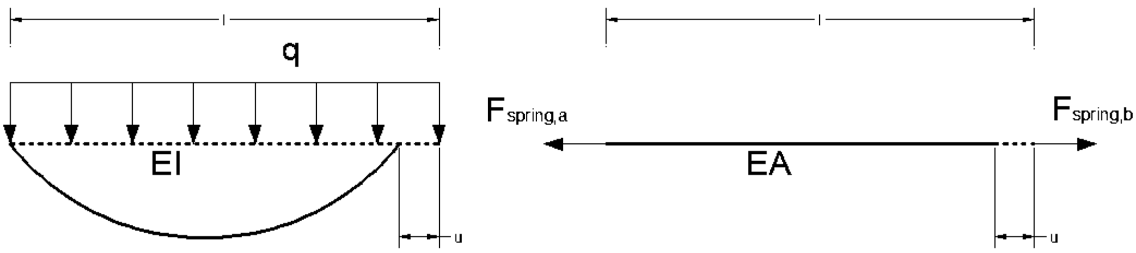


Figure 7.15 Horizontal deformation of the spoke due to bending and extension

The hinged connection between the spokes and rings provide structural characteristics of the roof equal to those of the spoke wheel. A spoke wheel roof mainly takes up loading by normal forces due to the interaction between the spokes and the rings.
Due to the hinged connection there will be no high transverse forces and bending moments in the connections and the elements. Most of the load will be transferred by normal forces. With the application of the hinged connection undesirable loads (Vy, Mx and Mz) are minimized and the roof structure can transfer the loads efficiently. The force and moment distribution of the deformed structure is illustrated in figure 7.16.

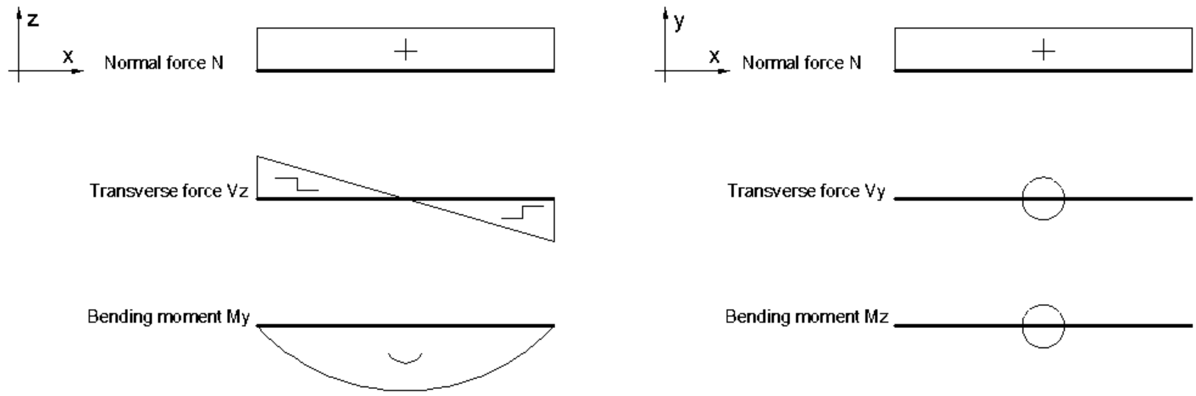


Figure 7.16 Force and moment distribution of the hinged connection in the deformed situation

Fixed connection

In the table below the values of the translations and rotations of a fixed connection between spoke and ring elements are presented using figure 7.11.

Connection A		**Connection B**	
Translation/rotation	Value	Translation/rotation	Value
Ux	Unknown	Ux	Unknown
Uy	0	Uy	Unknown
Uz	0	Uz	Unknown
Rx	0	Rx	0
Ry	0	Ry	0
Rz	0	Rz	0

Table 7.7 Degrees of freedom fixed connection spokes-rings

The schematization of the fixed connection between the spoke and rings is illustrated in figure 7.17.
The spoke and inner ring will sag due to the transverse load, the ring action of the outer and inner ring provide transverse and radial support. The horizontal ring force will decrease the sag of the structure. The final sag can be determined using 2nd order effect. As in the case of the hinged connection there are too much unknown values to determine the relation between the sag and the ring action by hand.

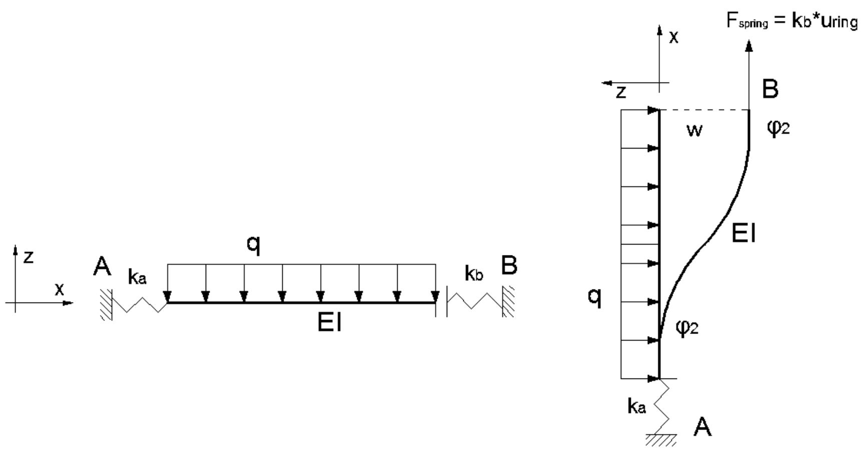


Figure 7.17 Schematization of fixed connection (left) and deformed schematization (right)

Like in the case of the hinged connection, the ring action of the fixed connection can be described. The modulus of elasticity is again assumed to be infinite. This way the bending and extension of the spoke is left out of consideration. Using the boundary condition that the rotation is zero in both nodes, with the displacement method the horizontal deformation can be expressed as follows:

$$u_A = \frac{M_B + ql^2 + F_{dl;ring} l + \frac{1}{2} qly(\tan(\varphi))^{-1} - \frac{1}{2} ql^2}{k_A \cdot y + ql - F_{dl;ring} - \frac{1}{2} ql} \tag{7.3}$$

$$u_B = \frac{M_A - \frac{1}{2} qly(\tan(\varphi))^{-1} + \frac{1}{2} ql^2 + F_{dl;ring} \cdot l}{\frac{1}{2} ql + F_{dl;ring} + k_B \cdot y} \tag{7.4}$$

The equation shows that the ring action is influenced by the fixed rotation of the connections. Due to the blocked rotation, bending moments arise in the ring. Unlike the hinged connection, a fixed connection takes up the loading not just by normal forces. Due to the small transverse stiffness of the roof structure, large transverse forces and bending moments arise in the connections. As a consequence fewer loads are being transferred by normal forces. A greater part of the transverse stiffness and strength is provided by beam action instead of ring action.
The force and moment distribution of the deformed situation is illustrated in figure 7.18.

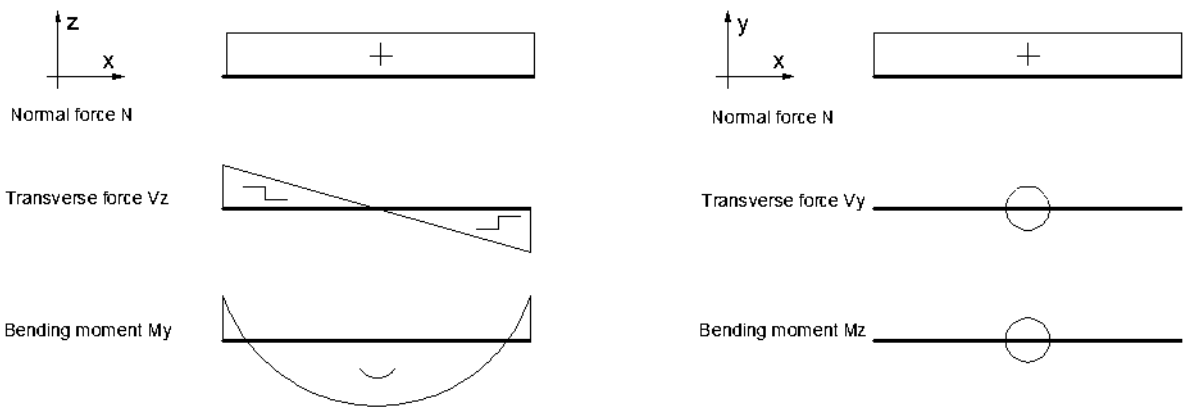


Figure 7.18 Force and moment distribution fixed connection

The fixed connection causes a bending moment in the local x-direction of the ring elements also known as torsion (figure 7.19). To provide enough resistance, the ring elements need to be stiff and strong enough.
The shape of the rings of the reference model is not constant, like a circular shape. Because the shape is not constant, there is no constant curvature which causes a difference in forces and moments in the different spokes. The torsion distribution can therefore be critical in the ring structures.

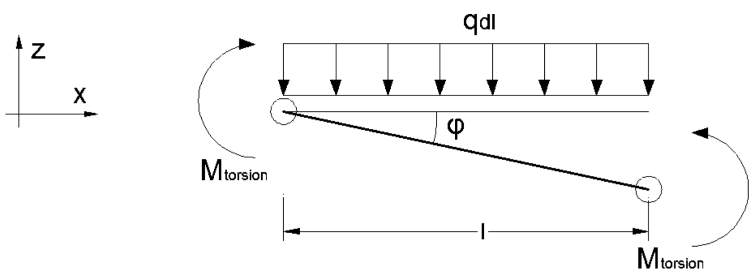


Figure 7.19 Torsion in the rings

Ring elements

Like the connection between the spoke and ring elements, the connection between the ring elements can be either hinged or fixed connected. Both connections will be applied and analysed. Assumed is that the spokes are hinged connected (free rotation around local y- and z-axes) with the inner and outer ring.

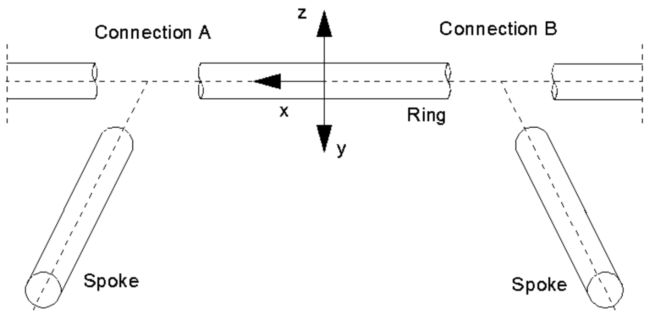


Figure 7.21 Connection A and B between ring elements with global axes

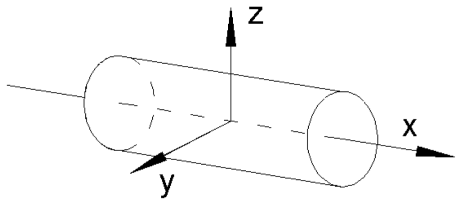


Figure 7.20 Local axes single element

Hinged connection

In the table below the values of the translation and rotation of the connections A and B, displayed in figure 7.21, are presented. The schematization of the hinged connection between the ring elements is illustrated in figure 7.22. The connection between the ring elements is actually a spring connection with free rotations. The ring not only provides transverse support in the global xy-plane, also in the longitudinal direction of the ring itself. Due to the stresses in the ring, the ring elements will also extend along its longitudinal axis. The resistance can be schematized as a spring.

Connection A		**Connection B**	
Translation/rotation	Value	Translation/rotation	Value
Ux	Unknown	Ux	Unknown
Uy	Unknown	Uy	Unknown
Uz	0	Uz	0
Rx	Unknown	Rx	Unknown
Ry	0	Ry	0
Rz	Unknown	Rz	Unknown

Table 7.8 Degrees of freedom hinged connection in the rings

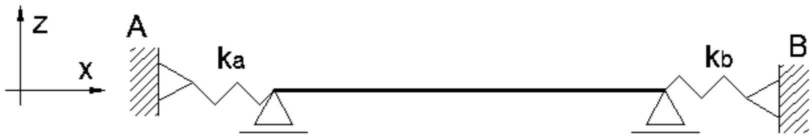


Figure 7.22 Schematization of a single ring element

To use the ring action at full extend, the ring must work as one stiff element. The transverse stiffness however, decreases with the application of the hinged connection. This can be explained using the shape of the reference model.
At the sides of the structure, there is no curvature. Therefore there is no transverse and radial support provided by ring action. The radial load on the ring must be resisted by the ring structure itself. When a hinged connection is applied, the ring will have a very low radial support and will deform at the nodes (figure 7.23). The ring structure cannot take up the radial loads at the sides of the reference model and becomes unstable.

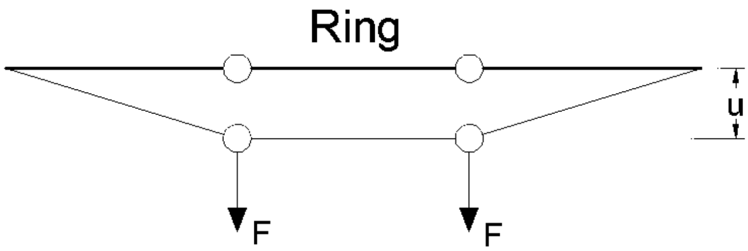


Figure 7.23 Instability of the ring with hinged connection

Fixed connection

In the table below the values of the translation and rotation of the connections A and B (figure 7.20) are presented. The schematization of the fixed connection between the ring elements is illustrated in figure 7.24. The connection between the ring elements is equal to the hinged connection, only with a fixed rotation.

Connection A		**Connection B**	
Translation/rotation	Value	Translation/rotation	Value
Ux	Unknown	Ux	Unknown
Uy	Unknown	Uy	Unknown
Uz	0	Uz	0
Rx	0	Rx	0
Ry	0	Ry	0
Rz	0	Rz	0

Table 7.9 Degrees of freedom fixed connection in the rings

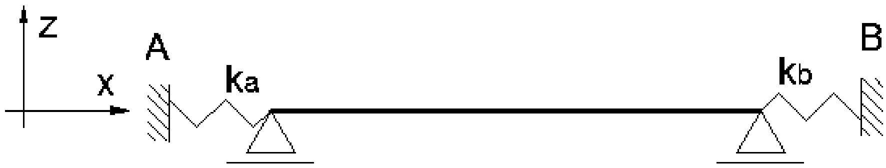


Figure 7.24 Schematization of a single ring element

With a fixed connection between the elements in the rings, the rings will work more as one single element and this will increase the radial stiffness of the ring structure itself. At the points where the ring action is low, due to the lack of curvature, the ring structure remains stable.

Conclusion

A characteristic of the spoke wheel structure is the high transverse/radial stiffness and strength. The reason why this lightweight structure is so strong is due to its ring action and efficient load transfer. The choice of the connections influences the loads (distribution) on the ring structure and therefore indirectly influences the ring action.

First the connection between the spokes and rings has been investigated. It appeared that a hinged as well as a fixed connection can be applied for the spoke-ring connection. However, there are some structural differences between both connections.
Applying a hinged connection less bending moments (Mx, Mz) and transverse forces (Vy) arise in the spokes and the rings. Most of the load is taken up by compression and tension in the spokes. Most of the stiffness will be guaranteed by ring action instead of beam action. Due to the ring action the sag of the structure is limited. This can be explained by looking at the 2nd order effect.
Applying a hinged connection the structure will have an efficient load transfer; most of the loads will be taken up by ring action instead of beam action. The shape of the reference stadium however, provides little ring action and therefore the structure has little overall stiffness.

With a fixed connection a greater part of the stiffness is provided by beam action. The loads are taken up by bending and less by normal forces. The advantage is that the sag of the structure will be less compared to a hinged structure. The disadvantage is the need for stronger and stiffer profiles/elements to withstand the loads. Besides, a fixed connection is more expensive and labour intensive compared to a hinged connection.

For the connection between the spokes and ring a hinged connection will be used. Although greater deformations will arise, a hinged connection will not prevent the structure of using the ring action. This is the essence of a spoke wheel structure and provides a very efficient roof structure. Less forces and moments will arise and therefore smaller elements can be applied, resulting in fewer costs. To decrease the amount of deformations other solutions must be found and applied. The (dis-)advantages of the application of a hinged or fixed connection between the spoke and ring elements are summarized below:

Type of connection	**Advantage**	**Disadvantage**
Hinge	• Loads taken up mainly by ring action • Smaller profiles/elements needed • Easy assembly • Cheaper	• Less ring action provided due to shape of the reference stadium • Larger deformations compared to fixed connection.
Fixed	• Smaller deformations	• Labour intensive • Expensive • Loads mainly taken up by beam action • Stronger/stiffer profiles/elements needed

Table 7.10 (Dis-) advantages of a hinged or fixed spoke-ring connection

The following connection that has been analysed is the connection between the ring elements. The ring structures are mainly radial loaded through the spokes. The spokes are connected at the nodes between the ring elements. To guarantee stability the ring elements need to withstand the radial load themselves. The outer ring structure is supported on a rocker bearing and provides little radial stiffness (assuming the strength and stiffness of the diagonals between the rocker bearings are negligible). The radial stiffness of the ring structure is therefore very important to withstand the loads.
When a hinged connection is used between the ring elements, the ring elements can freely translate in the radial direction. At the straight sides of the structure, there will be no stiffness guaranteed by the ring action ($r = \infty$). The stiffness must be guaranteed by the ring itself. Using a hinged connection the ring structure has a free translation and provides little resistance. The best option is to use a fixed connection between the ring elements. With this connection the ring structure remains stable.

7.1.3 Double inner/outer ring

In chapter 2 the development of the spoke wheel has been described, from which one was the application of a double tension or compression ring for roof structures.
A double tension or compression ring is only the case when the spokes are pre-tensioned. When applying a double inner or outer ring, this will always be a double (outer) compression or (inner) tension ring in case of pre-tensioning.
However, reference projects showed that there are spoke wheel roofs with non pre-tensioned spokes. An example is the Feyenoord stadium in Rotterdam, the Netherlands. In this paragraph the influence of the application of a double ring is investigated for non pre-tensioned as well as pre-tensioned spokes. To use the right terminology, the structures are from now on called double outer or inner ring structure.

The shape study of the reference model in chapter 6 showed that the model has little transverse support in order to withstand the acting loads. Engineers found out that a solution is to increase the transverse support by adding an extra ring. The structural engineer has the choice to apply either a double inner or outer ring.
In this chapter both solutions are applied to the reference model to investigate the results and the structural behaviour of both roof designs.

7.1.3.1 Double inner ring

The main advantage of the application of an extra ring is the major increase of the transverse support of the roof structure. The double inner ring model can be schematized as followed:

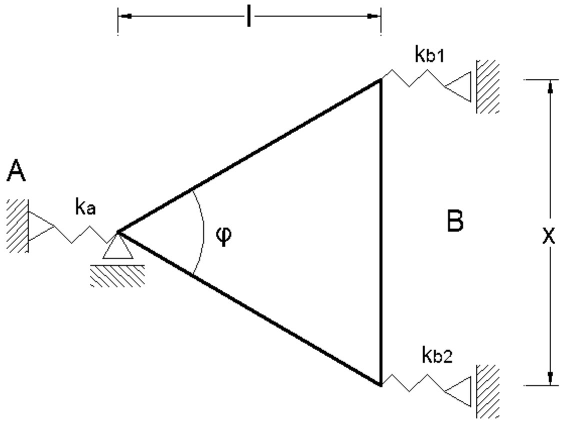


Figure 7.25 Schematization of a double inner ring structure

The reason why this structure has more transverse support is the application of diagonal spokes. In the previous chapter the relation between the ring action and the deformation for a deformed structure has been described. The diagonal spokes show equal characteristics to the deformed structure from figure 7.13 and 7.14. The transverse support of the structure will improve with the application of a double ring. Due to the diagonal spokes vertical force components are introduced. A part of the spring force (i.e. ring action) of the inner and outer ring will now provide transverse support to the roof structure.

The structure in figure 7.25 can be defined as a truss system. There are only forces acting on the nodes of the structure, and the member forces work in the direction of the member itself. The advantage of a truss system is that trusses, compared with massive structures, require less material and therefore have a relatively small dead load. Costs will be saved, when looking at the material savings.
The advantage of using this truss system is that it can take up upward as well as downward loadings. In case of suction by the wind there will be upward loading, or due to wind, dead load or snow downward loading.

The schematization can be seen as a triangular truss with external forces working on the joint; spring forces and reaction forces. The structure has a fixed shape. This can be determined using the formula:

$$s \geq 2k - 3 \qquad (7.5)$$

With s is the amount of bars and k the amount of nodes. The truss system of figure 7.25 consists of 3 nodes and bars and is therefore form fixed and can be seen as a stable truss system.

The influence of pre-tensioned or non pre-tensioned spokes will now be further investigated using the schematization from figure 7.25. Assumed is that the structures are only subjected to a downward load.

Pre-tensioned spokes

When the spokes are pre-tensioned, no compression forces will arise in the spokes. The double inner ring is subjected to tension and the single outer ring to compression. The normal force distribution is illustrated in figure 7.26.

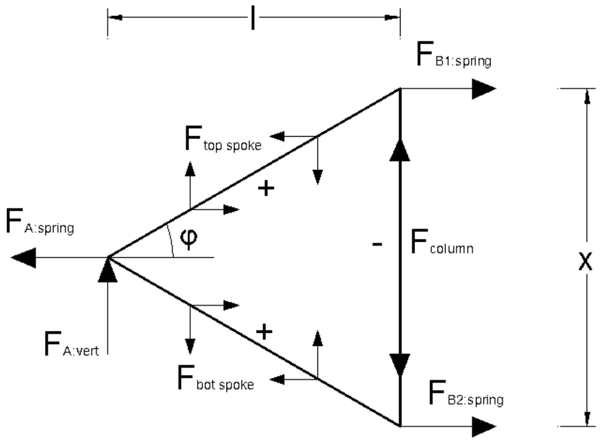


Figure 7.26 Force distribution double inner ring with pre-tensioned spokes

The arising normal force in the spokes can be expressed with the method of joint. The derivation of the equation is presented in appendix A.6. The normal force in the diagonals can be expressed as followed:

$$F_{top\ spoke} = \frac{k_A \cdot u_A}{2\cos(\varphi)} \tag{7.6}$$

$$F_{bot\ spoke} = \frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{\sin(\varphi)} \tag{7.7}$$

$$F_{column} = \frac{k_A \cdot u_A}{2\cos(\varphi)} \cdot \sin(\varphi) \tag{7.8}$$

The relation between the spring force in B, assuming the vertical reaction force in A is zero, is equal to:

$$k_{B1} \cdot u_{B1} = k_{B2} \cdot u_{B2} = (k_A \cdot u_A)/2 \tag{7.9}$$

Equation 7.6 and 7.7 show that the normal forces in the upper and bottom spoke depends from the amount of ring forces and loads (transverse and radial) working on the structure. The load 'activates' the ring action and the structure gains transverse and radial support.

The equations show that the amount of vertical load due to the spring force increases when the angle φ between the spokes increases as well. However, the greater the angle, the smaller the horizontal force component will be.
For the structure, equilibrium has to be found to provide enough support to withstand transverse as well as radial loading. Besides the greater the angle, the more material is needed for the structure, the higher the costs will become. Also at a greater construction height, the structure is more vulnerable for buckling and greater radial loading (wind).

Non pre-tensioned spokes

When the spokes are not pre-tensioned, the normal force distribution depends on the direction of the acting loads on the structure.
As a consequence of the downward loading, compression arises in the top spoke and tension in the bottom. In the upper inner ring compression arises and in the bottom inner ring tension. In the outer ring small compression forces arise. For the force distribution in figure 7.27 the forces in the structure will be expressed. The derivations are presented in appendix A.6.

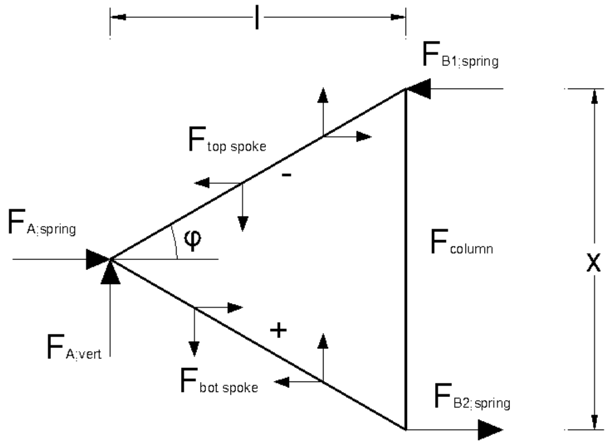


Figure 7.27 Force distribution double inner ring with non pre-tensioned spokes

The expression for the normal force in the top and bottom spoke is as follows:

$$F_{bot\ spoke} = -\frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{2\sin(\varphi)} \qquad (7.10)$$

$$F_{top\ spoke} = \frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{2\sin(\varphi)} \qquad (7.11)$$

In the column no normal forces will arise, because the column is not placed on a support and is subjected to free sag. The structure is stable due to the second order effect. The ring action of the inner ring provides a horizontal force that decreases the sag of the structure until equilibrium has been found. The explanation of the 2nd order effect has been explained in paragraph 7.1.2.2.

The relation between the spring forces in B is, assuming the vertical force in A is zero, equal to:

$$k_{B1} \cdot u_{B1} = (k_A \cdot u_A)/2 \qquad (7.12)$$

$$k_{B2} \cdot u_{B2} = -(k_A \cdot u_A)/2 \qquad (7.13)$$

7.1.3.2 Double outer ring

A double outer ring has the same advantages as a double inner ring. Due to the diagonals, the structure has more transverse support. The equation of 7.6 and 7.7 also holds for this structure, except the ring force in connection A in this structure is twice the ring force in a double inner ring structure. For the double outer ring structure holds that the greater the angle, the higher the vertical spring force component and the smaller the horizontal spring force component. Equilibrium has to be found to withstand both transverse and radial loading.
A disadvantage compared to the double inner ring, is the greater material use for this structure. Due to the application of a double outer ring instead of a double inner ring (smaller than the outer ring) more material is needed. As a consequence the costs of the structure will increase as well.

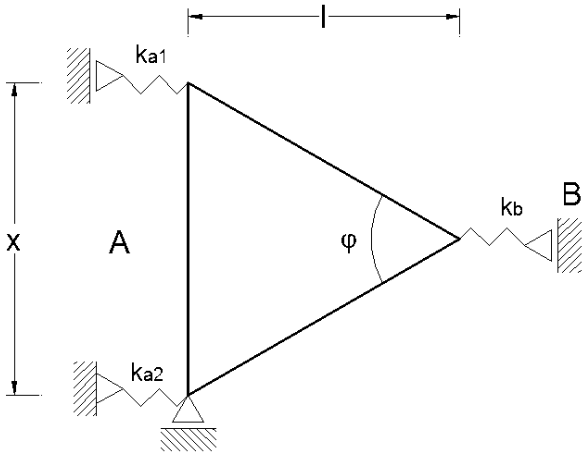


Figure 7.28 Schematization of a double outer ring

Also for this structure two situations are described; the application of pre-tensioned and non pre-tensioned spokes.

Pre-tensioned spokes

The expression for the normal forces in the members is equal to:

$$F_{top\ spoke} = \frac{F_{A;vert}}{\sin(\varphi)} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \tag{7.14}$$

$$F_{bot\ spoke} = \frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)} \tag{7.15}$$

$$F_{column} = F_{A;vert} + \left(\frac{k_A \cdot u_A}{\cos(\varphi)}\right) \cdot \sin(\varphi) \tag{7.16}$$

The relation between the spring force in B is, assuming the vertical reaction force in A is zero, equal to:

$$k_B \cdot u_B = 2(k_{A1} \cdot u_{A1}) = 2(k_{A2} \cdot u_{A2}) \tag{7.17}$$

The figure illustrates that tension arises in both spokes and in the inner ring elements. Compression arises in the column and the outer rings.

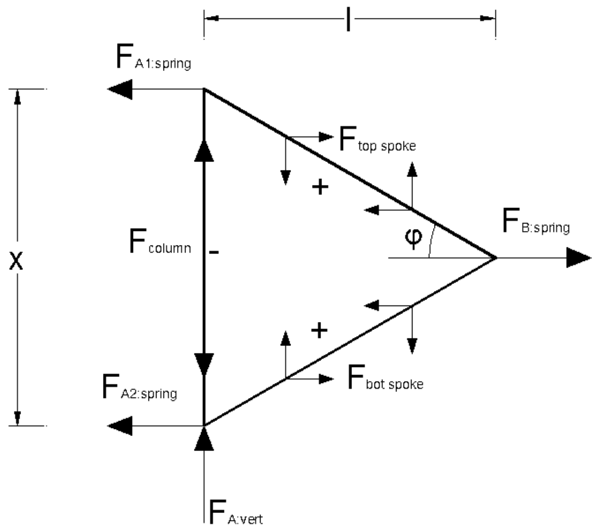


Figure 7.29 Force distribution double outer ring with pre-tensioned spokes

Non pre-tensioned spokes

On the structure is assumed that only a downward load acts on the structure. As a consequence of the loading, compression arises in the bottom spoke and tension in the top. In the upper outer ring compression arises and in the bottom outer ring tension. In the inner ring small tension forces arise. For the force distribution given in figure 7.30 the forces in the structure will be expressed. The derivations are presented in appendix A.6.

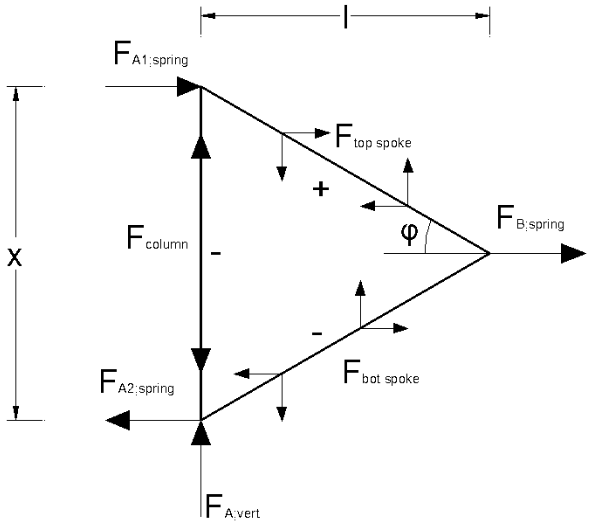


Figure 7.30 Force distribution double outer ring with non pre-tensioned spokes

The expressions of the forces in the members are equal to:

$$F_{top\ spoke} = \frac{F_{A;vert}}{\sin(\varphi)} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \quad (7.18)$$

$$F_{bot\ spoke} = -\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)} \quad (7.19)$$

$$F_{column} = F_{A;vert} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) \quad (7.20)$$

With:

$$k_B \cdot u_B = 2(k_{A;1} \cdot u_{A;1}) = -2(k_{A;2} \cdot u_{A;2}) \quad (7.21)$$

7.1.3.3 Conclusion

The application of a double inner or outer ring provides more transverse support of the structure compared to a single spoke structure. The transverse support increases with increasing of the angle between the spokes. Increasing the angle will however, decreases the amount of radial support. Equilibrium has to be found to withstand all loading.

The structural dilemma is to decide which geometry is the most suitable to come to an efficient spoke wheel roof design. Previous analysis's and investigations of the shape of the reference stadium showed that there is a lack of transverse and radial strength and stiffness due to little curvature of the rings. As a consequence little ring action is provided by the roof structure. To fulfil the structural requirements it is first necessary to provide enough strength and stiffness.
As described in previous chapters, the amount of ring action is heavily dependent from the amount of present curvature. Although the inner ring possesses more curvature compared to the outer ring, a double outer ring will be applied to the structure to increase the transverse support. The amount of ring action is not only dependent from the amount of curvature. Acting loads, extensional rigidity and translation is also of importance.
When a double inner ring structure is used, more loads will act on the outer ring structure. Because the outer ring structure has little stiffness that can be provided by ring action, large radial deformations arise at the sides of the outer ring. As a consequence, the inner ring will further deform radial as well as transverse. To prevent large deformations, the stiffness of the outer ring needs to be increased. By adding another ring to the outer ring, the available curvature will provide more stiffness to the structure. Besides, the extra ring increases the amount of extensional and bending rigidity.

Another advantage is that the outer ring is placed on supports. The structure is able to directly transfer the extra dead load to the underground, which is not possible when a double inner ring is used.

What is left is the determination of the height of the double outer ring structure. In this stage of the design process it is difficult to determine what height would provide the most efficient structural design. Therefore, the height will be determined by looking at reference projects. The Feyenoord stadium in Rotterdam is a similar stadium with almost equal dimensions. The height-span ratio of the roof is around 1:2,8. The span of the global design is already known, it has an average value of 42,50m. The height of the spoke framework would be $\frac{42,50}{2,8} = \pm 15,00m$. This height will be used for the spoke framework.

Looking at the variable loads acting on the roof structure (rain, wind, snow, etc.), the best option to place the roof covering would be at the same level as the bottom spokes. The roof covering will be placed under an average slope of 10 degrees. In this case, rainwater will flow to the outside of the stadium. No extra rain load need to be taken into account, due to water accumulation in the rain gutter at the inner ring.

Furthermore, in this chapter an analysis has been made about the influence of (non-) pre-tensioned spokes for the choice of a double inner or outer ring structure. The consequence of pretensioning the spokes, is that no compression forces arise in the spokes. Smaller profiles can be used for the spokes. The question whether pretensioning the spokes results in a structure that uses less material compared to non-pretensioned spokes cannot be answered at this stage. When pretensioning the spokes, larger forces will act on the rings. The rings need to have sufficient stiffness to withstand these loads. One cannot say of the needed amount of material in the structure is less compared to spoke wheel roof structures that uses non-pretensioned spokes. More research is needed and will be done in a later stage of the thesis.

7.1.4 Conclusion

After the analysis of the design variables regarding the geometry of the structure the fourth sub-question can be (partially) answered. The sub-question is as followed:

'What is the influence of the design variables and the consequence on the structural design?'

Shape inner ring

The requirements and conditions regarding the lines of sight, protection of the elements and quality of the grass determined the design area of the shape of the inner ring. The transverse strength of the roof structure is the leading direction in order to fulfil the strength requirements. The new shape of the inner ring has to be formed within the determined design area.
The more curvature the inner ring has, the more ring action can arise. When the amount of ring action increases, the structure becomes stiffer and the deformations will decrease. In the available design area has been tried to apply the greatest amount of curvature at the inner ring. The maximum radius is present at the centre of the sides. The maximum radius on the short sides is 72.72m and on the long side 2868,06m. The ring action will have the smallest value at these points. With the new shape of the inner ring, the total area of the roof is approximately 22.000m^2.

Support and connections

First the influence of the supports on the structure has been analysed. The choice of support has an influence on the translation of the roof structure, one of the key factors that influence the ring action. Hinged and fixed supports prevent the roof structure to translate in its horizontal plane. When a hinged or fixed support is applied to the outer ring of the roof structure, there will be no ring action. The ring structure will become a simple beam-column system and will resist loads only through beam action.
Besides, when hinged or fixed supports are applied, the stands of the stadium will take up the loads from the spokes. The structural properties of the stands are unknown; therefore it is not possible to apply hinged or fixed supports to the reference stadium. When a whole stadium will be designed from scratch, fixed or hinged supports are an option to apply to the outer ring. However, the amount of forces taken up by ring action will decrease as well as the efficiency of the design. It is not advised to use hinged or fixed support for the spoke wheel roof structure.
What is left is the choice between the use of roll supports or rocker bearings. Rocker bearings have the advantage that it can better take up tensile forces and are cheaper to apply. Besides, the horizontal stiffness of the structure can be adapted by using the right steel profiles for the wall bracings. Due to these reasons, rocker bearings will be applied as supporting system.

After the analysis of the influence of the choice of support, the choice of type of connections has been investigated. The choice of connection has consequences for the way the outer ring is loaded and how the loads will be transferred in the structure.
It appeared that a hinged as well as fixed connection can be applied for the spoke-ring connection. However, there are some structural differences between both connections. Applying a hinged connection, most of the load is taken up by tension or compression instead of bending. The stiffness is mainly provided by ring action instead of beam action. The advantage is that the dimensions of the elements can be minimized compared to a fixed connection. Another advantage of the hinged connection is the assembly of the structure. The assembly of a hinged connection is easier and cheaper compared to a fixed connection. Although the deformation is smaller when a fixed connection is used, the hinged connection will be applied to the structure in order to come to an efficient roof structure.
The following connection that has been analysed is the connection between the ring elements. The ring structures are mainly radial loaded through the spokes. These spokes are connected at the nodes of the ring elements. To guarantee stability the ring elements need to withstand the radial load. The best option is to use a fixed connection. With this connection the ring structure remains stable.

Double inner/outer ring

The application of a double inner or outer ring provides more transverse support of the structure compared to a single spoke structure. The structural dilemma is to decide which geometry is the most suitable to come to an efficient spoke wheel roof design.
Although the inner ring possesses more curvature compared to the outer ring, a double outer ring will be applied to the structure to increase the transverse support. The amount of ring action is not only dependent from the amount of curvature. Acting loads, extensional rigidity and translation is also of importance.
When a double inner ring structure is used, more loads will act on the outer ring structure. Because the outer ring structure has little stiffness that can be provided by ring action, large radial deformations arise at the sides of the outer ring. As a consequence, the inner ring will further deform radial as well as transverse. To prevent large deformations, the stiffness of the outer ring needs to be increased. By adding another ring to the outer ring, the available curvature will provide more stiffness to the structure. Besides, the extra ring increases the amount of extensional and bending rigidity.

The height of the double outer ring structure will be 15,00m. This height has been determined by using the Feyenoord stadium as reference project. Roof covering will be placed at the bottom spoke level.

7.2 Structural materials and elements

When designing a structure research must be done to the available materials and elements that can be used for the structural design. The strength and stiffness of the roof structure and the ring action depends on the properties of the elements. The right material and elements need to be applied in order to fulfil all structural requirements.
The structural design of a tensile-compression ring roof structure can be divided into elements that can be used for the inner and outer rings and the spokes, the so called primary structure. The roof covering, described last, are part of the secondary structure.

7.2.1 General material characteristics

To determine the stresses and deformations of structures, the physical characteristics of the materials should be known. For the design of the stadium roof, only two materials for the main bearing structure will be analysed; steel and concrete.

7.2.1.1 Steel

The tensile – compression behaviour of a material can be expressed by stress-strain diagrams. In these diagrams one can see at what stress value, the material will react. Figure 7.31 gives an example of a stress-strain relation.

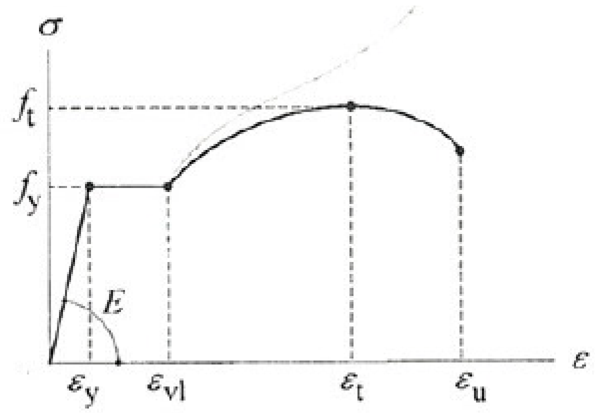


Figure 7.31 Stress strain diagram. Reproduced from [1].

f_y = yield strength
f_t = tensile strength
ε_y = yield strain; the strain when the material starts to yield
ε_{vl} = the strain value at the end of the yield area
ε_t = strain related to the tensile strength
ε_u = strain value at the point of cracking of the material

In the elastic area, there is a linear relation between the stress σ and the strain ε:

$$\sigma = E\varepsilon \quad (7.14)$$

This formula is also called Hooke's law. In the stress-strain diagram of figure 7.31 one can find the modulus of elasticity E back in the slope of the linear-elastic area.

Steel is a tough material that has a relatively high tensile strength f_t compared to the stress value when the material cracks. For steel the compression strength f_c is equal to the tensile strength f_t. In table 7.11 the properties of steel are displayed, with modulus of elasticity value of E = 210 GPA.

Steel	f_y [N/mm²]	f_t [N/mm²]	ε_y [%]	f_c [N/mm²]
S235	235	360	0,112	360
S275	275	430	0,131	430
S355	355	510	0,169	510

Table 7.11 Steel tensile-compression properties

A characteristic property of steel is that the cracking or breaking of the material can be brittle or ductile. This is indicated by the strain value ε_u. The higher the strain value the more ductile steel will be. Ductile cracking appears after yielding of the material.
When a steel structure is subjected to large, varying loads, the crystal grid of the material can be damaged due to the fatigue behaviour of the material. Fatigue will however not been taken into account during this research.
The material can be subjected to thermal expansion as a consequence of temperature changes. The thermal expansion for steel is equal to $12*10^{-6}$ K^{-1} (for temperature T ≤ 100 °C).

7.2.1.2 Concrete

Concrete has different properties compared to steel. Concrete is, in contradiction to steel, a brittle material. The stress-strain diagrams between compression and tension are very different, where the compression strength f_c is higher than the tensile strength. This is clearly visible in figure 7.32. For the calculation of deflections a linear-elastic material behaviour is assumed. The modulus of elasticity for concrete depends on time effecting factors.

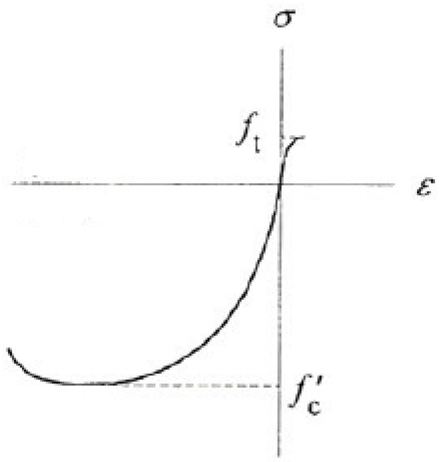


Figure 7.32 Stress-strain diagram concrete. Reproduced from [20].

In table 7.12 the values for the tensile (f_b), the average tensile (f_{bm}) and compression ($f'_{c,k}$) strength of the different concrete grades are visible. The variation of the modulus of elasticity in time can be estimated by:

$$E_{cm}(t) = \left(\frac{f_{cm}(t)}{f_{cm}}\right)^{0,3} E_{cm} \quad (7.15)$$

Where $E_{cm}(t)$ and $f_{cm}(t)$ are values at t days and E_{cm} and f_{cm} the values determined at 28 days. The relation between $f_{cm}(t)$ and f_{cm} is expressed as follows:

$$f_{cm}(t) = \beta_{cc}(t) f_{cm} \quad (7.16)$$

Where $\beta_{cc}(t)$ is a coefficient dependent from time t.

Concrete	E_{cm} [N/mm²]	f_b [N/mm²]	f_{bm} [N/mm²]	$f'_{c,k}$ [N/mm²]	f'_{cm} [N/mm²]
C12/15	27000	1,10	1,60	15	23
C20/25	31000	1,50	2,20	25	33
C28/35	34000	2,20	3,20	45	43
C35/45	36000	2,70	3,80	55	53
C45/55	38000	3,00	4,20	67	63

Table 7.12 Properties of concrete [NEN-EN 1992-1-1]

A characteristic of concrete is that it is subjected to deformations caused by temperature, shrinkage and creep. When designing a concrete structure, these factors must be taken into account.

Temperature

The deformation of concrete due to temperature changes is determined by the extension coefficient. The extension coefficient due to temperature is 10.10^{-6} K^{-1}.

Shrinkage

Shrinkage [20] is a time-depended strain measured in an unloaded and unrestrained specimen at constant temperature.
The shrinkage of concrete is the sum of drying shrinkage and autogenous shrinkage. Drying shrinkage is the reduction in volume caused principally by the loss of water during the drying process. Autogenous shrinkage results from the hardening of the concrete, where the first days after pouring are most important.

The total shrinkage is expressed as follows:

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca} \tag{7.17}$$

Where

ε_{cs} is the total shrinkage extension
ε_{cd} is the drying shrinkage extension
ε_{ca} is the autogenous shrinkage extension

The end value of the drying shrinkage extension ε_{cd} is equal to $k_h \cdot \varepsilon_{cd,0}$. The value of $\varepsilon_{cd,0}$ can be determined using the graphs in the appendix.

The development of the drying shrinkage in time follows from:

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} \tag{7.18}$$

Where k_h is a coefficient dependent from the fictive height h_0 from table A 7.1 and $\varepsilon_{cd,0}$ from table A 7.2 which are presented in the appendix.

$$\beta_{ds}(t, t_s) = \frac{(t-t_s)}{(t-t_s)+0{,}04\sqrt{h_0^3}} \tag{7.19}$$

Where

t is the age in days of the concrete at a certain time.
t_s is the age of the concrete (in days) at the beginning of the drying shrinkage.
h_0 is the fictive thickness (mm) of the cross section; $= \frac{2A_c}{u}$

With A_c is the area of the cross section of the concrete and u the circumference of the part of the cross section that is subjected to drying.

The autogenous shrinkage follows from:

$$\varepsilon_{ca}(t) = \beta_{as}(t)\varepsilon_{ca}(\infty) \tag{7.20}$$

Where

$$\varepsilon_{ca}(\infty) = 2{,}5(f_{ck} - 10)10^6 \tag{7.21}$$

and

$$\beta_{as}(t) = 1 - exp(-0{,}2t^{0{,}5}) \tag{7.22}$$

With t in days.

Creep

Concrete creep [20] is defined as the increase of the deformation of a structure in time under constant load. Basically, long term pressure or stress on concrete can make it change shape. This deformation usually occurs in the direction the force is being applied. Like a concrete ring structure being compressed, due to the acting radial loads from the spokes.

The creep coefficient $\varphi(t, t_0)$ depends from E_c, the tangent modulus, which is equal to $1{,}05E_{cm}$. The creep deformation of the concrete $\varepsilon_{cc}(\infty, t_0)$at time $t = \infty$ for a constant at time t_0 concrete compression stress σ_c is given by

$$\varepsilon_{cc}(\infty, t_0) = \varphi(\infty, t_0) \cdot \left(\frac{\sigma_c}{E_c}\right) \tag{7.23}$$

When the concrete stress compression at time t_0 already has a value of $0{,}45f_{ck}(t_0)$ or more, the creep must be considered non-linear. Then the non-linear creep coefficient is equal to:

$$\varphi_k(\infty, t_0) = \varphi(\infty, t_0)\exp(1{,}5(k_\sigma - 0{,}45) \tag{7.24}$$

With k_σ is the stress-strength ratio $\frac{\sigma_c}{f_{cm}(t_0)}$ where σ_c is the compression stress and $f_{cm}(t_0)$ the average concrete compression strength at the time of loading.

The values of the mentioned factors can be determined using the graphs that are presented in the appendix (A.7).

7.2.2 Primary structural elements

7.2.2.1 Bar elements

Depending from the geometrical design of the spoke wheel roof structure, different type of elements can be used for the spokes and the rings:

H / I profiles

Generally the H-profiles are used for compression loaded columns and for bars loaded by bending were the construction height have to be minimized. H-profiles have a higher moment of inertia value than I-profiles for the same height due to the thicker flanges. H-profiles also have a wider cross section then I-profiles. I-profiles are very often applied for light structures. Both profiles are often used for trusses and are relatively cheap.
For the connection of H/I profiles injection bolts are needed. These are quite expensive in comparison to welded joints [1].

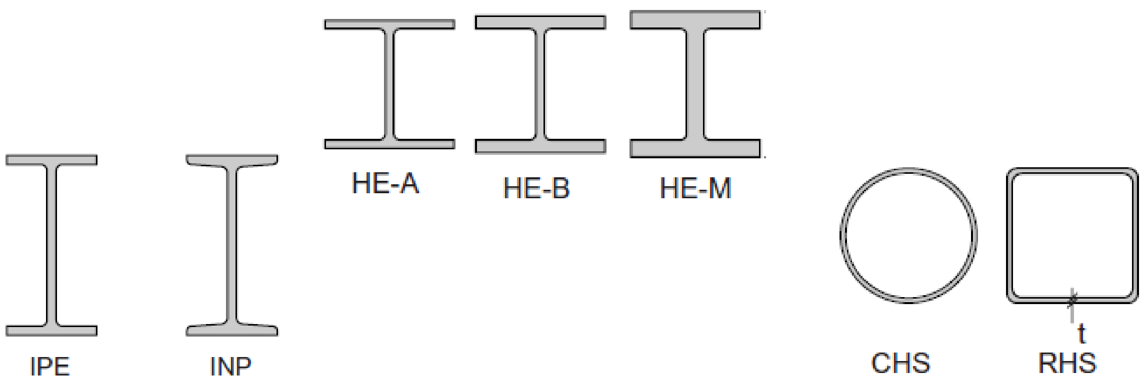


Figure 7.33 H- and I-profiles; CHS and RHS profile. Reproduced from [1].

CHS / RHS profiles

Circular and rectangular hollow sections are often applied for columns and trusses. Due to their high moment of inertia they are very suitable for these applications.
For the connection of these profiles welded or bolted joints are used. The connections are more complex compared to H/I profiles due to the deformation of the cross section. Support plates have to be welded in the profiles [1].
The advantage of a CHS/RHS profile is its high aesthetic value. Architects often prefer these types of profiles compared to HE/I profiles. Besides, CHS/RHS profiles are easier to clean and have lower maintaining costs.

Cables

Cables are used to take up and transfer tensile forces, especially when pre-stressed. The basic element of a cable is the wire. The diameter of a basic hot –formed wire is between 5-13 mm. The hot-formed wire is being cold shaped until it reaches its desired diameter, section and tensile strength. The maximum diameter is about 7mm and the shape of the cross section can be round, z- or s- shaped or conical.

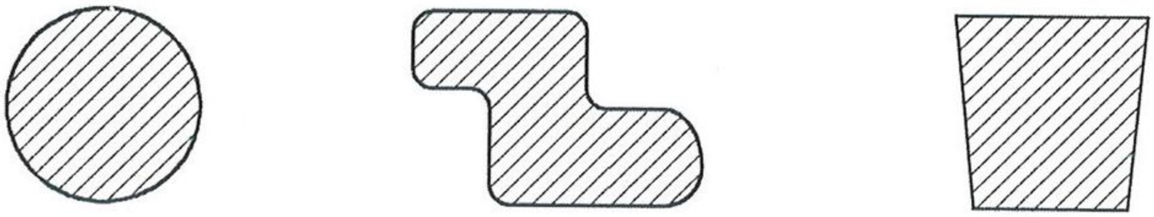


Figure 7.34 Types of wires. Reproduced from [1].

In the building industry cables not only have to fulfil the strength and stiffness requirements, also the requirements with respect to the durability and the corrosion resistance.
To provide the wires more corrosion resistance, the wires are galvanized and have a tensile strength of 1400 – 1800 N/mm^2.
Wires can be collected in spiral or parallel strands. By collecting numerous strands a cable can be made. The strands on its turn can be also be collected spiral- or parallel shaped.

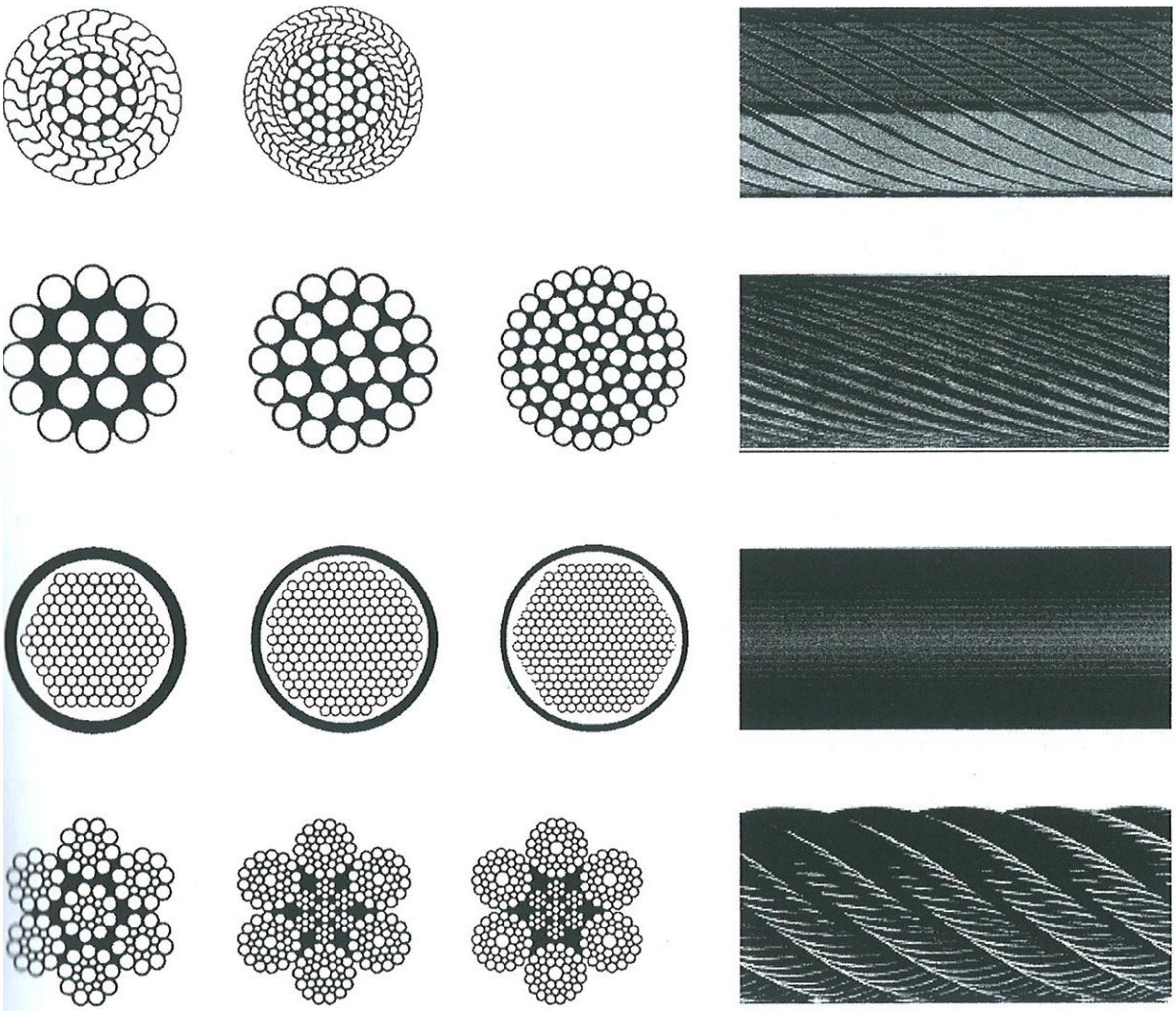


Figure 7.35 From top to bottom: Strand with z-shaped wires; bridge cable; parallel cable; spiral shaped bridge cable. Reproduced from [1].

The bridge cables have a lower E-modulus value then parallel cables and creep can have an influence on these type of cables. Cables are pre-stressed in order to keep the cables under tension. Cables lose their structural strength when they are not loaded. A disadvantage of the application of cables is that they are very expensive in comparison to H/I or CHS/RHS profiles [1].

Concrete elements

Concrete elements can only be used for a compression ring in the spoke wheel roof structure. The disadvantage when using concrete is that dilatation of the ring can only be applied when the structure is imposed on a roller bearing. When using concrete, temperature loading (shrinkage) can be a significant factor [20].

Resume

For all the discussed elements, the advantages and disadvantages are displayed in table 7.13.

Material	Advantage	Disadvantage
H-profiles	• High moment of inertia • Low construction height • Can take up tensile/compression changes • Easy connections	• Protection against corrosion needed • Hard to clean
I-profiles	• Suitable for light structures • Can take up tensile/compression changes • Easy connections	• Protection against corrosion needed • Hard to clean
CHS/RHS profiles	• Very suitable for trusses • Easy to clean • High moment of inertia • High aesthetical value • Can take up tensile/compression changes	• Protection against corrosion needed • Difficult weld connections
Cables	• High tensile strength • Light structure	• Only suitable for taking up tensile forces • Need to be pre-stressed • Very expensive • Resonance response possibility • Protection against corrosion needed
Concrete elements	• High compression strength	• Only suitable for taking up compression forces • Heavy structure • Exposed to shrinkage /creep effects.

Table 7.13 Properties main structural elements

7.2.3 Secondary structural elements

The secondary structural roof elements consist of bar elements (purlins) between the primary structural elements and the roof covering. In the previous paragraph the possible bar elements are already described and in this section only the possible roof covering elements will be analysed.

The covering of a stadium roof need to fulfil some requirements. Materials used for roof covering need to be lightweight, tough, water tight, aesthetically acceptable, cost-effective and durable enough to withstand the effects of the weather, including ultra-violet light. The roof covering also needs to fulfil structural requirements. The roof covering should be strong and stiff enough to span the distance between the primary and secondary elements and supporting different kind of loads; snow loads, imposed loads and wind loads. Besides the roof covering may require additional thermal and acoustic insulation [5].

The materials used for roof covering can be divided into two main categories:

- Opaque covering
- Translucent covering

For each category the materials that can be used for a stadium roof will be briefly described.

7.2.3.1 Opaque covering

Opaque covering as a roof covering can consist of different kind of sheeting's. These properties of these different sheeting's are present in table 7.14 [9]. The physical properties of the opaque covering materials can be found in table 7.15.

Material	Advantage	Disadvantage
Steel sheeting	• Cheap, easy to fix • High E-modulus • High tensile strength • Incombustible	• Protection against corrosion needed • No transparency
Aluminium sheeting	• Light material • High E-modulus • Very high durability • Incombustible • Resistant to atmospheric attack	• Low impact resistance • No transparency
Concrete	• Very high durability • Incombustible	• Heavy material • No transparency • Only suitable for closed roofs

Table 7.14 Material properties opaque covering materials

Material	Steel (S235)	Aluminium	Concrete (C28/35)
Density [kN/m^3]	78,5	38	24
Young's Modulus [GPa]	210	300	31
Tensile strength [MPa]	235	210	1,65
Compression strength [MPa]	235	210	35
Bending strength [MPa]	235	210	

Table 7.15 Physical properties opaque covering

The bending strength of concrete depends on the amount of reinforcement that is applied to the concrete elements. The tensile-, compression and bending strength of steel and aluminium are equal, apart from buckling and torsion.

7.2.3.2 Translucent covering

Materials for translucent covering can be further divided into the categories rigid plastics and non-rigid plastics. For each category the related materials will be described.

Rigid plastics

Rigid plastics have the characteristic that they are waterproof, strong, can withstand reasonable large deformations without damage and have reasonable impact resistance. An example of a rigid translucent roof is the roof of the Munich Olympic stadium from 1972. The following materials are rigid plastics [9]:

Material	Advantage	Disadvantage
Acrylic sheeting	• Durable • High transparency • Scratch resistant	• Lessens moderately in time • Combustible • Medium durability
PVC sheeting	• High transparency • Easy to clean	• Lessens markedly in time • Combustible • Medium durability
Polycarbonate sheeting	• Self-extinguishing • Very high transparency • High durability • Thermal stability	• Low scratch resistance • Lessens moderately in time • Expensive

Table 7.16 Material properties rigid plastics

The physical properties can be found in table 7.17. It must be noticed that the temperature resistance is the temperature at which the material still has its full strength. Also the properties of every material is depended from the producer, the values in table 7.17 and 7.19 are approximations.

Material	Acrylic	PVC	Polycarbonate
Density [kN/m^3]	11,9	14,2	12
Young's Modulus [GPa]	3,0	2,8	2,4
Tensile strength [MPa]	70	52	64
Compression strength [MPa]	120	74	86
Max. elongation [%]	5	20-40	110
Temp. resistance [°C]	75-90	80	130
Translucency [%]	50-75	70-85	80-90
Life Span [Years]	20	15-20	10-20

Table 7.17 Physical properties rigid plastics for translucent covering. Reproduced from [44].

Non-rigid plastics

Non-rigid plastics are widely used for roof covering of lightweight tension structures. These materials are used for fabrics or composite materials. A characteristic of non-rigid plastics is that it has a high tensile strength and is therefore suited for pre-stressing structures like membranes.

Material	Advantage	Disadvantage
Fibreglass	• Incombustible • Maintenance free • High tensile strength • Medium durability • Medium translucency	• Coating needed • Expensive compared to rigid plastics • Long production time
Polyester fibre	• Self-extinguishing • Easy to clean • Medium translucency • Medium durability • High tensile strength	• Coating needed • Expensive compared to rigid plastics • Long production time

Table 7.18 Material properties non-rigid plastics

To protect the materials from environmental degradation, the materials are coated. Examples of coating materials are PTFE (Teflon) and PVC.
Fibreglass coated with PTFE is widely used nowadays. PTFE is corrosion resistant and has low frictional properties so dirt washes of without damaging the coating. The PTFE makes sure the material cleans itself and therefore becomes maintenance free. PTFE is also resistant to abrasion and is highly reflective, absorbing little light as well as heat.
Polyester fibres are often coated with PVC. The PVC coating strengthens the polyester fibres and makes the fabric water proof and easy to clean [30].

Material	PTFE Fibreglass	PVC Polyester fibre
Density [kN/m³]	25	13
Young's Modulus [GPa]	71	30
Tensile strength [MPa]	3450	200-1350
Max. elongation [%]	4,8	4-5
Temp. resistance [°C]	232	240
Translucency [%]	20	15
Life Span [Years]	25-30	15-20

Table 7.19 Physical properties non-rigid plastics for translucent covering. Reproduced from [44].

7.2.4 Conclusion

After the analysis of the different possible materials and elements for the primary and secondary structure a conclusion can be made. Besides, the fourth sub-question can be (partially) answered. The sub-question is as followed:

'What is the influence of the design variables and the consequence on the structural design?'

General material characteristics

Steel material has a high compression and especially tensile strength. The material behaves, depending on the chemical properties, in general ductile. Unlike concrete, this is brittle and breaks in an instant. Concrete is very suitable for taking up very high compression forces. The disadvantage of concrete is that it is subjected to shrinkage and creep. These are factors that must be taken into account when designing with concrete.

Primary structure

The primary structure of the spoke wheel roof can be divided into ring- and spoke elements. The type of profiles/materials that are suitable for both kinds of elements will be described and determined.

Ring elements
The ring elements, especially in the outer ring, are subjected to high forces. Due to the non-circular shape, the structure cannot provide the total stiffness from ring action. A part of the stiffness has to be provided from the bending rigidity of the elements itself. The possible materials are described first.

Steel elements (CHS, RHS, I, HE) are randomly used for stadium roof structures and there are many reasons for. Stadium roofs are often dynamically loaded and therefore large load fluctuations (compression as well as tensile forces) can arise. Besides normal forces, steel profiles can also take up loads through bending (beam action).
An advantage of using steel profiles is that the fabrication of steel elements is relatively cheap and due to prefabrication the building time is short compared to other building materials.

Another option is the use of concrete elements for the ring elements. Concrete elements are only used to take up large compression forces and can therefore only be used in the compression rings. For the application of concrete elements for the spoke wheel roof structure certain conditions need to be met. Due to temperature changes dilation need to be applied. Therefore the spoke wheel roof need be placed on roller bearings. A disadvantage of using concrete is the high building costs and building time.

It can be concluded that steel is the most suitable material for the ring elements. With the use of steel profiles it is possible to take up tensile as well as compression forces. Due to its high modulus of elasticity steel profiles are also able to provide a high amount of extensional and bending rigidity to the structure.
Another reason why steel is more suitable is that a characteristic of a spoke wheel roof structure is its lightweight appearance. Using concrete elements would do no good to this characteristic.

Spoke elements
The spoke elements need to cover a great span and are almost only stressed by normal forces (in an efficient spoke wheel roof). To minimize the use of material and to behold the lightweight character of the spoke wheel roof, steel will be used for the spoke elements. Concrete is not suitable for these elements.

Choice of profiles
The choice of type of profile for the ring and spoke elements depends on the final design variable. The structural engineer has the option to use either pretensioned spokes or not. This choice has a great influence on the use of type of profiles. In this stage of the thesis one cannot say if a pretensioned spoke wheel roof shows better results and benefits compared to a non-pretensioned spoke wheel roof for the reference stadium. More research is needed to answer this question.

When the spokes are pretensioned cables need to be used for the spoke elements. Cables do not have any bending or torsion rigidity. Therefore cables cannot take up loads by beam action. When cables are used the loads will only be taken up by tensile forces. As a consequence a very lightweight roof can be designed. Besides, the building time of a cable net roof structure is very short compared to other materials. The major disadvantage of using cables is the high price. The production and assembly is very expensive.
Because the cables are pretensioned, the ring elements need to be able to withstand the high radial forces. Due to the lack of curvature at some points of the roof, the ring elements need to have sufficient bending rigidity in the radial direction. A good option is to use RHS profiles. With these types of profiles it is possible to provide a high moment of inertia in the radial direction.

A spoke wheel roof structure that does not use pretensioned spokes, needs to find a way to minimize the amount of bending moments in order to provide a structure with a minimum amount of needed material. By using a spatial truss system this is possible. CHS profiles are very suitable for truss structures. The advantages of this type of profile is its high aesthetical value, low maintaining costs, high moment of inertia and low building costs. Due to the geometry of the profile, this type of profile is very suitable for further optimization with for instance parametric modelling.

For the research CHS profiles with a yield strength of 235 N/mm² (S235) will be used and for the cable structure a yield strength of 1770 N/mm². These are the only steel grades that will be used. When an engineer needs to decrease the costs of the design as much as possible, one has to take investigate the influence of the use of different steel grades on the total costs of the design. However, the emphasis of the thesis is merely about the research of the use of tensile compression ring roof structures for stadia and how to use the advantages of this type of structure as much as possible.

Secondary structure

The type of roof covering mainly depends on the choice of the architect. In this thesis the architectural point of view is left out of consideration and the design is based on structural reasons and the requirements set in chapter 5.
For the design only translucent roof covering will be used, because this is a requirement (chapter 5). The choice of using rigid or non-rigid plastics depends on the choice of the primary roof structure. In case of a spoke wheel roof that consists of a spatial truss system one of the rigid plastics have to be used for the roof covering. Looking from a structural and financial perspective acrylic sheeting is most suitable.
Non-rigid plastics are often used for pre-tensioned structures, like tent or cable net structures. Non-rigid plastics are more expensive compared to rigid plastics. It is advised to use PTFE fibreglass for the non-rigid plastics due to its better structural properties compared to polyester fibres. The costs differences between both are minimal.

8. Preliminary design

In the Preliminary Design part the reference shape structure has been analysed. For the occurring problems in the reference structure, design variables have been investigated to see what their influence is on the ring action and efficiency of the structure. With the analysis of all design variables a preliminary design can be made. This preliminary design will be further optimized to a detailed design in the next part of the thesis (part 4 – Detailed Design).

8.1 Structural design

From the investigation of the design variables for the roof structure a preliminary structural design can be determined. However, one design variable has not been determined yet. This holds the choice of using pre-tensioned spokes (cables) or not (CHS profiles). The dilemma is that the costs of a cable element per m^1 are higher compared to CHS profiles per m^1. When cables are pre-stressed, it can take up large loadings and less material is needed compared to CHS profiles. In this phase of the design process it is difficult to say what will provide the most efficient preliminary design that needs the least amount of material.

Although it is unknown whether CHS profiles or cables must be used, it is still possible to determine the geometry of the preliminary design from the conclusion of the other investigated design variables.
The shape of the inner and outer ring remains the same as determined in chapter 7.2. For the design rocker bearings are used for the supports. Next, the use of a double outer ring structure will provide greater transverse support and has smaller deformations compared to a double inner ring structure. The spokes and rings will be hinged connected with each other and the ring elements are fixed.

In the next part of the thesis, investigation is done of the use of a spatial truss structure composed of CHS profiles and a cable structure. By making deeper analysis a better solution can be found in the search for an efficient spoke wheel roof structure.

With the determination of the preliminary design an answer has been given to the following sub-question:

Which design variables together form an efficient structural design?

The preliminary design illustrated in figure 8.1 is a starting point for the structural design part of this thesis where the final design will be determined. The technical drawings of the preliminary design can be found in appendix A.8.

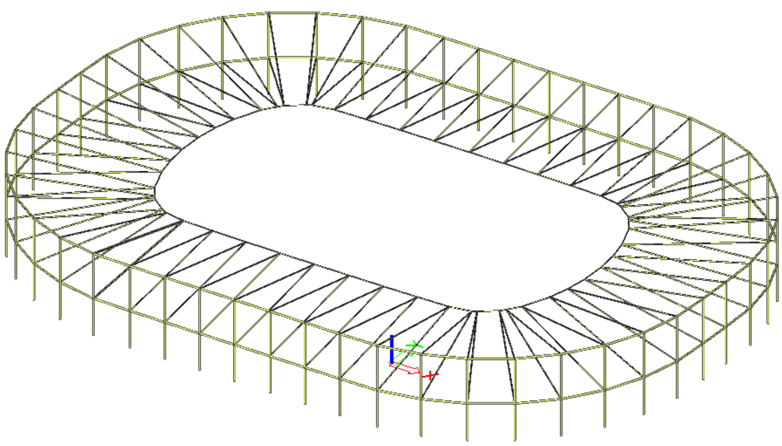


Figure 8.1 Preliminary Design

Category	Sub-category	Design variables			
Geometry main bearing structure	**Supports**	Roll supports	Rocker bearings	Hinged supports	Fixed supports
	Connections	Hinged rings – Hinged spokes	Hinged rings – Fixed spokes	Fixed rings – Hinged spokes	Fixed rings – Fixed spokes
	Ring structure	Double inner ring		Double outer ring	
Structural elements	**Primary structural elements**	Steel profiles (CHS, RHS, HE, I)	Cable elements	Concrete elements	
	Secondary structural elements	Opaque covering	Translucent covering – rigid	Translucent covering non-rigid	

Design 1

Design 2

Table 8.1 Determination of the two preliminary design variants for the reference stadium

8.2 Evaluation design

The preliminary design has been determined after analysing different design variables. The design however, is unstable when second order effects will be taken into account (non-linear unstable). The reason that the structure is non-linear unstable is that the structure still does not provide sufficient stiffness. The structure will deform and cannot find equilibrium. Solutions to stabilize the structure are investigated in chapter 10 and 11.
For now, the preliminary design will be evaluated by looking at the ring action, load transfer, strength, stiffness and roof collapse possibilities.

8.2.1 Ring action

The efficiency of the ring structure can be determined by looking at the amount of ring action, where the ring action depends on the amount curvature of the ring, loads acting on the ring (8.2.2.), extensional rigidity and translation of the ring. The theory of the ring action can be found in chapter 2.

Outer ring

The original shape of the outer ring of the roof has not been subjected to change. The shape is equal to the reference stadium, investigated in chapter 6. The sides of the outer ring have a radius equal to infinite ($r = \infty$), meaning that there will be no ring action. The only ring action is provided in the corners, where the outer ring does have a certain curvature. With the addition of a double outer ring the roof structure is able to benefit from the available ring action in the outer ring.

The structure is supported by rocker bearings. The outer ring is therefore able to translate and will not prevent the roof structure to use the available ring action. Other key factors that influence the ring action are the loads and the extensional rigidity. The loads are discussed in the following paragraph. The extensional rigidity is provided by steel elements. The advantage of this material is that a high extensional and bending rigidity can be provided using the correct profiles.

Inner ring

The preliminary design has, in contradiction to the shape of the inner ring of the reference stadium, some curvature on the sides. The most curvature is present at the short sides, where the largest radius is present at the centre (72,72m). At the long side only little curvature is present. The highest value of the radius is 2868,06m.
The complete inner ring will provide some ring action.

8.2.2 Loads

In a roof structure the leading load component is the transverse load. The radial load acting on the ring is caused by the vertical deformation (up- or downwards) of the inner ring and the spokes as a consequence of the transverse load (figure 6.3). When pre-tension is applied in the spokes, this will cause an extra radial load on the ring structure.

In chapter 6, the consequence of the shape of the reference stadium for the load distribution has been explained. The load transfer from the spokes to the ring structure at the long sides is very inefficient. The spokes are placed almost perpendicular to the ring elements. Using the polygon of forces, almost no radial load will act on the straight sides of the ring structure. Knowing that the amount of curvature at the long side is zero in the outer ring and almost zero in the inner ring, the provided ring action at the long sides of the roof is negligible. The short side of the inner ring has more curvature and a better load transfer compared to the long sides (smaller angle between ring and spokes).
Most curvature is present in the corners of the inner and outer ring. Most of the load is transferred by the spokes in these areas. The high radial loads and curvature causes a high value of the ring action in the corners of the roof structure both in the inner and outer ring.

It can be concluded that the load distribution almost has not changed. Due to the extra little curvature in the inner ring, the structure is able to transfer the loads somewhat more efficiently (polygon of forces). However the improvement is minimal.

8.2.3 Strength

The transverse strength of the roof structure is important in the structural design in order to fulfil all structural requirements. Due to the lack of curvature at the sides of the structure, the structure has little transverse strength. With the addition of an extra outer ring, the roof structure has gained a lot of extra strength in its transverse direction.

The transverse strength can be further increased by pre-tensioning the spokes or using elements with a higher extensional and bending rigidity. A large, hollow cross section gives the ring structure bending and torsion rigidity as well as high resistance to buckling in compression.
The radial strength mainly depends on the strength of the elements themselves. With the addition of an extra outer ring, the radial strength has slightly increased. When the angle between the spokes is too large, this has a negative effect on the radial strength. This is not the case for the chosen angle in the preliminary design.

8.2.4 Stiffness

The stiffness that the preliminary design can provide from ring action heavily depends from the amount of curvature. Due to the lack of curvature at the straight sides, the structure will deform most at these points. With the use of a double outer ring structure, the deformation has decreased. However, the deformation is still critical and the structure is still unstable. In figure 8.2 the deformation of the inner ring of the preliminary design has been compared with a complete circular roof. In a circular roof, the stiffness is equal in the whole roof causing a constant sag. Due to the lack of curvature at the straight sides of the reference stadium, the largest sag arises on the long side of the roof structure.

The ring elements on the straight sides of the structure have to have sufficient stiffness to resists the loads acting on them. Due to the non-circular shape, extra transverse forces and moments arise on the ring elements. Solutions need to be found to create extra stiffness. An option is to use stiffer elements or use pretensioned spokes.
The radial stiffness is primarily influenced by, next to curvature, the thickness of spokes and by the depth of the ring structure. A stiffer ring structure extends the load-affected zone so that more spokes are affected, and it increases the radial and transverse stiffness of the roof.

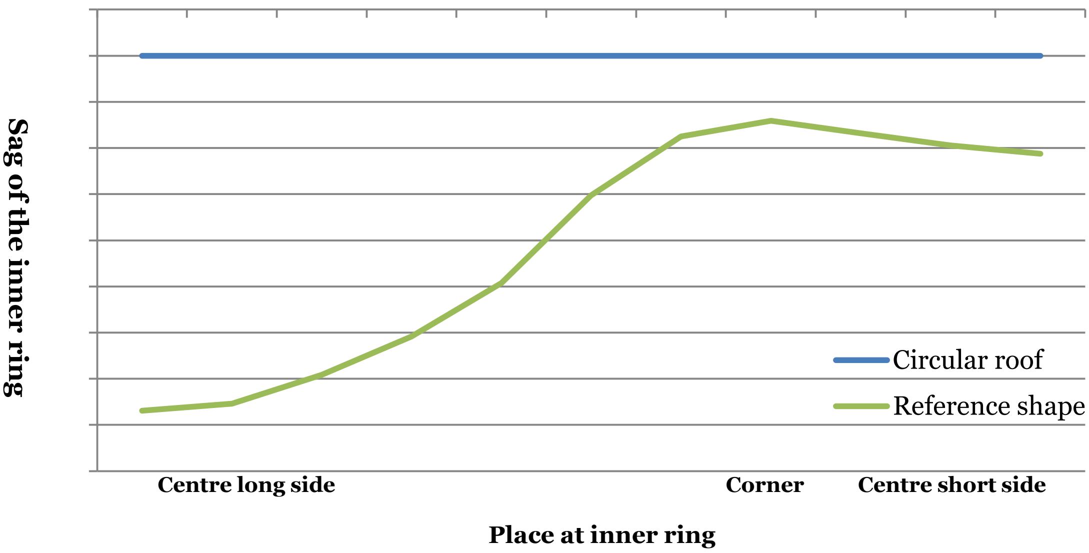


Figure 8.2 Stiffness differences between complete circular shaped roof and shape of reference stadium.

8.2.5 Roof collapse

The most decisive failure will occur at the long sides of the roof structure, where no curvature is present. To prevent the roof from large transverse and radial deformations, the lateral and transverse strength and stiffness need to be increased greatly.
In these points of the structure, the highest force and bending moment's values will occur. The roof will most likely collapse as a consequence of failure of the elements at these points. Especially in the ring elements very large bending moments (Mz and torsion Mx) and transverse forces (Vy) will arise due to the large deformations.
The ring elements need to have sufficient extensional, bending and torsion rigidity to prevent the ring from collapsing (for instance due to buckling).
Other causes of roof collapse can be due to fatigue, buckling or breaking of the spokes or other elements. Fatigue will however not be taken into account. The FEM models are only calculated using static loads instead of dynamic loads. For the wind loads, the results of wind tunnel tests of the Amsterdam ArenA will be used.

8.3 Conclusion

After the investigation of the reference stadium and the design variables a preliminary design has been formed. The problem is that the preliminary design is not yet stable. The addition of extra curvature to the inner ring and the extra outer ring has increased the stiffness and strength. However the stiffness and strength need to be further increased to come to a stable design. The only design variable that is left to investigate is the use of pretensioned spokes, together with the use of the correct profiles for the spoke and ring elements. In the following part the influence of applying either pretensioning the spokes or not will be investigated.
With the investigation of pretensioned spokes a spoke wheel roof with cables will be designed. The other design consists of a spatial truss system that does not use pretensioned spokes.

The question that rises is if both types of structures will provide a stable structure and which design will provide the most efficient structure? Or in other words which design is able to fulfil all structural requirements and use the least amount of material and is most cost beneficial?

When pretensioned spokes (cables) are used for the spoke wheel roof, the stiffness cannot be provided by means of beam action. The question is if it is possible to use the available amount of ring action in the reference stadium to come to a stable and safe structure that fulfils all structural requirements? The next challenge is to investigate whether such a structure is still efficient for roofs similar to the reference stadium.
When the spokes are not pretensioned (regular steel profiles), beam action will play a role in the amount of stiffness in the roof structure. It is interesting to investigate if the available ring action is sufficient to come to a stable design? Is the use of beam action inevitable? If extra beam action is needed, the following question rises to what extend? And is the structural design than still efficient?

For all these questions will be tried to find an answer in the following part of the thesis. By making a design with the two types of structures one will gain insight in the use of spoke wheel roof structures for non-circular shaped roofs.

Conclusion Preliminary Design

The conclusions from the part Preliminary Design are briefly summarized:

Chapter 6 – Shape study

- Straight sides of the outer and inner ring have curvature of $r = \infty$ and provide no ring action.
- Only stiffness due to ring action is provided by the corner ring elements.
- Load can only be transferred by the spokes in the corner of the roof structure. The radial load distribution on the ring structure is therefore not constant.
- Largest transverse deformations arise at the long side of the inner ring.
- Largest radial deformations arise at the short side of the outer ring, due to the larger span of the roof at these points.
- Largest transverse forces (Vy) and bending moments (Mx and Mz) arise in the outer ring at the long sides close to the corners.

Chapter 7 – Design Variables

Shape inner ring

- Largest curvature at the short side is 72,72m, at the long side 2868,06m.
- The total area of the roof is around 22.000m^2.

Support and connections

- Hinged and fixed supports are left out of consideration. The reason is because these supports prevent the roof from using ring action to provide stiffness.
- Rocker bearings will be applied to the roof structure. Its advantages compared to roll supports are: lower building costs, higher tensile capacity and the possibility to adapt the horizontal stiffness of the roof structure.
- The rocker bearings will have a height of 12,30m.
- Between the spokes and ring element a hinged connection will be applied. The advantages compared to a fixed connection are: ring action used at full extend, easier assembly, lower building costs.
- The ring elements will be fixed connected in order to provide enough stability and to withstand the high occurring forces and moments in the ring structure.

Double inner / outer ring

- A double outer ring structure will be used for the design of the roof.
- The amount of deformation with a double outer ring structure is lower compared to a double inner ring structure, assuming the same structural profiles are used.
- The height of the double outer ring structure will be 15,00m. The height has been determined using the Feyenoord stadium as reference project.
- The roof covering will be placed at the same level as the bottom spokes of the roof structure.

Structural material and elements

- For the primary structure steel is most suitable for the application for a spoke wheel roof structure.
- For a pretensioned spoke wheel roof, cables are needed for the spokes and RHS profiles for the outer ring elements. RHS profiles have the ability to provide a great amount of bending rigidity in the radial direction of the spoke wheel roof. For the non-pretensioned roof CHS profiles are most suitable.
- The used regular steel profiles (CHS, RHS) have a yield strength of 235 N/mm^2 and the cables a nominal tensile strength of 1770 N/mm^2.
- Rigid plastic (acrylic sheeting) roof covering will be used for trussed roof structures, non-rigid plastic (PTFE fibreglass) roof covering for a cable roof structure.

Chapter 8 – Preliminary Design

- The largest transverse deformation arises at the long side of the inner ring.
- The largest radial deformation arises at the long side of the outer ring.
- The transverse strength and stiffness at the long side of the roof structure is leading in the determination of the structural design.
- The strength and stiffness can be increased by improving the load distribution, increase the extensional, bending and bending/torsion rigidity of the ring elements, use of pre-tension of the spokes.

In the next part of the thesis, the preliminary design will be made into a final detailed design.

Part 4
Detailed Design

Detailed Design

In this detailed design part the preliminary design with a cable and spatial truss system will be further modelled, optimized and investigated. First the loads acting on the roof structure are described, followed by the determination and optimization of the structural design. At the end the construction method and the building costs are presented.

9. Loads

When designing a building the load distribution on the structure is of great importance. The result of the structural design depends on the load acting on the building. The loads that need special attention are the environmental loads with random distributions, durations and magnitude. To achieve a high accuracy in the results from the structural analysis the possible loads on the structure will be investigated.

In the Eurocode NEN-EN 1990 the loads working on a structure are categorized into:

- Permanent load (G)
- Variable load (Q)
- Incidental load (A)

The permanent load consists of the dead load, pre-stressed load and indirect loading due to shrinkage and uneven settlement. Wind, snow and imposed loading are an example of a variable load and incidental loading is loading caused by explosions, earthquakes and collisions from external objects. In this thesis the incidental load will not be taken into account, because the chance of occurrence is extremely small.

In this chapter the load cases will be described first. Also the two main variable loads, wind and snow loading, are explained. Other loads working on a roof structure will be described last.

9.1 Load cases

For every critical load case the values of the load effects must be determined by making load combinations of the load cases which occur at the same time. A load combination will consist of a permanent load and a leading variable load or an incidental load. Because the incidental load is not taken into account a load combination in this thesis will only consist of a permanent and a variable load. For the load combinations the factors (ULS, SLS) from the Eurocode will be applied.

As described in the Eurocode, different limit state cases are taken into account: the Ultimate limit state (ULS) and the Serviceability limit state (SLS). The ULS will be applied when it concerns the safety of people and/or the safety of the structure. The SLS is applied to take care for the comfort of the people, the functioning of the structure under normal conditions and the appearance of the building.
In the National Annex of the Eurocode is described that the instantaneous factor is zero for roofs. Therefore the instantaneous load combinations will not be applied in the ULS and SLS.

9.1.1 Ultimate Limit State

The following ultimate limit states should be checked when relevant:

- EQU: Loss of static equilibrium of the structure or any part considered as rigid body, in which:
  - Minor variations in the value or the spatial distributions of actions from a single source are significant, and;
  - The strengths of construction materials or soil are generally not governing.
- STR: Internal failure or excessive deformation of the structure of structural members, including footings, piles and basement wall, etc. where the strength of construction materials of the structure governs.
- GEO: Failure or excessive deformation of the ground where the strengths of soil or rock are significant in providing resistance.
- FAT: Failure of the structure or structural elements due to fatigue.

The situation of GEO and FAT will be left out of the research.

The load safety factors for the EQU and STR situation can be found in table 9.1. and 9.2.

Permanent actions		**Leading variable action**	**Accompanying variable actions**	
Unfavourable	**Favourable**		**Main (if any)**	**Others**
$\gamma_{gj,sup}\ G_{kj,sup}$	$\gamma_{gj,inf}\ G_{kj,inf}$	$\gamma_{q,1}\ Q_{k,1}$		$\gamma_{q,i}\ \Psi_{0,I}\ Q_{k,i}$

Table 9.1 Load safety factors for EQU. Reproduced from NEN-EN 1990:2002/NB:2007

Where the partial factors for the EQU situation are equal to:

$\gamma_{gj,sup} = 1{,}1$
$\gamma_{gj,inf} = 0{,}9$
$\gamma_{q,1} = 1{,}5$
$\gamma_{q,i} = 1{,}5$

Permanent actions		**Leading variable action**	**Accompanying variable actions**	
Unfavourable	**Favourable**		**Main (if any)**	**Others**
$\gamma_{gj,sup}\ G_{kj,sup}$	$\gamma_{gj,inf}\ G_{kj,inf}$			$\gamma_{q,i}\ \Psi_{0,I}\ Q_{k,i}$
$\varepsilon\gamma_{gj,sup}\ G_{kj,sup}$	$\gamma_{gj,inf}\ G_{kj,inf}$	$\gamma_{q,1}\ Q_{k,1}$		$\gamma_{q,i}\ \Psi_{0,I}\ Q_{k,i}$

Table 9.2 Load safety factors for STR/GEO. Reproduced from NEN-EN 1990:2002/NB:2007

Where the partial factors for the STR/GEO situation are equal to:

$\gamma_{gj,sup} = 1{,}35$
$\gamma_{gj,inf} = 0{,}9$
$\gamma_{q,1} = 1{,}5$
$\gamma_{q,i} = 1{,}5$
$\varepsilon\gamma_{gj,sup} = 1{,}2$

The $\Psi_0\ Q_k$ is the combination value used to check the ultimate limit state and the irreversible service limit state. Because the incidental loading is not taken into account the frequent- (Ψ_1) and quasi-static value (Ψ_2) will be left out of consideration.
In the National Annex of NEN-EN 1990 is written that the factor Ψ_0 is zero for the category roofs, snow- and wind load and loads due to temperature (Table A.1.1. in NEN-EN 1990:2002/NB:2007).

The load in the ULS will be determined using the following formula:

$$F_d = \gamma_f F_{rep} \tag{9.1}$$

$$F_{rep} = \varphi F_{ch} \tag{9.2}$$

Where:
F_d is the value of the load
F_{rep} is the relevant representative value of the load
F_{ch} is the characteristic value of the load
γ_f is the partial factor which takes the possibility of unfavourable distortions of the load values with respect to the representative values in account.
Ψ is 1,00 or Ψ_0, Ψ_1 or Ψ_2.

As mentioned Ψ_0 is zero and Ψ_1 and Ψ_2 are not relevant, because the incidental load is not taken into account. The load in the ULS is as follows:

$$F_d = \gamma_g G_{k,j} + \gamma_q Q_{k,1} \tag{9.3}$$

Failure of the roof structure has major implications regarding the loss of life of people, or very large economic or social impact. Therefore the roof structure is categorized in the highest reliability class, RC3. This means the partial factors can be multiplied by a factor K_{FI}=1,1.

The ULS of STR/GEO is leading and will be taken into account, ULS EQU is left out of consideration. The ULS is calculated with the following equations:

$$F_d = 1{,}35 G_{k,j} \tag{9.4}$$

$$F_d = 1{,}2 G_{k,j} + 1{,}5 Q_{k,1} \tag{9.5}$$

9.1.2 Serviceability Limit State

In the SLS the loading will be checked to guarantee the comfort, the functioning of the structure under normal conditions and the appearance of the building. The load in the SLS is as follows:

$$F_d = \gamma_m G_{k,j} + \gamma_m Q_{k,1} \tag{9.6}$$

Where γ_m is the partial factor for material characteristics and is equal to 1,0.

9.1.3 Structural requirements

In chapter 5 the fundamental and additional requirements are briefly described. One of the fundamental requirements is that the structure has to resist every load and influence that can arise during construction and use. For the ULS and SLS there are structural requirements.

Ultimate Limit State

The unity check for the ULS for the EQU situation is as follows:

$$\frac{E_{d.dst}}{E_{d,stb}} \leq 1 \tag{9.7}$$

Where:
$E_{d.dst}$ is the value of the destabilizing load effect
$E_{d.stb}$ is the value of the stabilizing load effect

The unity check for the ULS for the STR/GEO situation is as follows:

$$\frac{E_d}{R_d} \leq 1 \tag{9.8}$$

Where:
E_d is the value of the load effect
R_d is the value of the corresponding resistance

Serviceability Limit State

The unity check for the SLS situation is as follows:

$$\frac{E_d}{C_d} \leq 1 \tag{9.9}$$

Where:
E_d is the value of the load effect described in the service ability criteria
C_d is the value of the corresponding service ability criteria

As mentioned, the SLS deals with the usability requirements of a structure. A structure cannot function optimal when there are too large deformations and vibrations. With respect to the deformations, requirements are described in the Eurocode. With respect to deformation requirements, the Eurocode refers to the National Annexes (NEN-EN 1990).

Vertical deformation

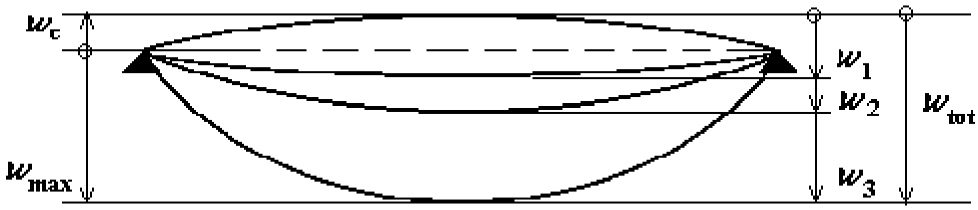


Figure 9.1 Deformation of a beam. Reproduced from NEN-EN 1990.

w_c	sheer of the unloaded structural elements
w_{max}	permanent deformation
w_1	direct deformation due to permanent loads
w_2	long term deformation due to permanent loads
w_3	additional deformation due to variable loads
w_{tot}	total deformation, the sum of $w_1 + w_2 + w_3$

The Eurocode (i.e. National Annexes) describes that the maximum deformation for a roof structure is $0{,}004l$ for a single bar. For a stadium roof there are no definitive deformation requirements. What is most important is that the roof is not subjected to heavy deformation fluctuations. Spectators need to have a safe feeling when the roof is subjected to high variable loads. To prevent the roof from heavy fluctuating deformations, the requirement is set that the largest total span of the roof cannot deform more than 1,00m due to variable loads. This value has been determined by looking at the deformation of other stadia roof structure. For example, the Tunis stadium is a project engineered by Arcadis. The maximum deformation there has been set at 1,00m as a consequence of variable loads.

The preliminary design showed that the weakest point of the roof is the place where the span of the roof is the largest and the ring action is little. The weakest point is therefore present at centre of the inner ring at the long sides. This point is decisive in order to fulfil the deformation requirement.
For the total span deformation of the permanent load there are no fixed deformation requirements. It is important that every roof point at the inner ring has an equal height or distance to the surface. This can be realized during construction of the roof.
For the construction of the roof structure, temporary columns are used to support the roof structure (see chapter 12, Construction Method). On beforehand the deformation due to the permanent load can be determined. By taking into account the deformation of the roof for each point on the inner ring where the temporary supporting columns are placed, the roof can be set at a constant equal height along the whole inner perimeter of the roof after removing the temporary columns.

The maximum deformation of a single bar of a stadium roof depends on the water accumulation and to provide a safe feeling to the spectators. The leading bars that are subjected to large deformation are the ring bars present in the plane of the roof covering, illustrated in figure 9.2.

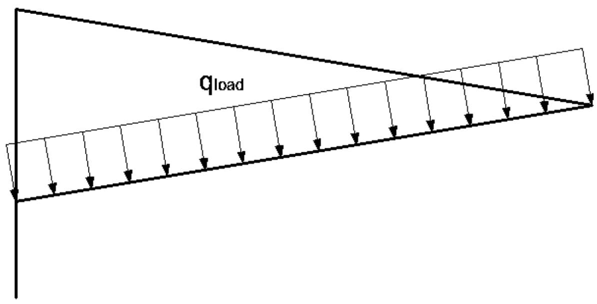


Figure 9.2 Variable loads acting on the roof covering present on the bottom spokes

When water accumulates, large extra variable loads can act on the structural elements. The national annexes of the Eurocode prescribe that a minimum slope of 1,6% together with the deformation requirement is sufficient to prevent water accumulation. This requirement only counts for beams with fixed supports. For roof tops that consist of beams, purlins and plates, as in the case of a stadium roof, a greater slope is needed (figure 9.3).

The roof covering is placed on the bottom spokes and has a slope of around $\frac{7,5}{42,0} = 0,178$ or $tan^{-1}0,178 = 10,12^{\circ}$. The maximum covering distance of the roof covering is 14,00m. Water will accumulate when the roof covering has a local deformation of $\left(\frac{14,00}{2}\right) * \tan 10,12^{\circ} = 1,25m$.
It is assumed that the roof covering has sufficient stiffness capacity to prevent a local sag of 1,25m. Water accumulation is therefore not leading. In order to diminish the amount of deformation of a single bar an adapted stiffness requirement is set for the design. Instead of a maximum deformation of $0,004l$, a single bar of the stadium roof can have a maximum deformation of $0,010l$.

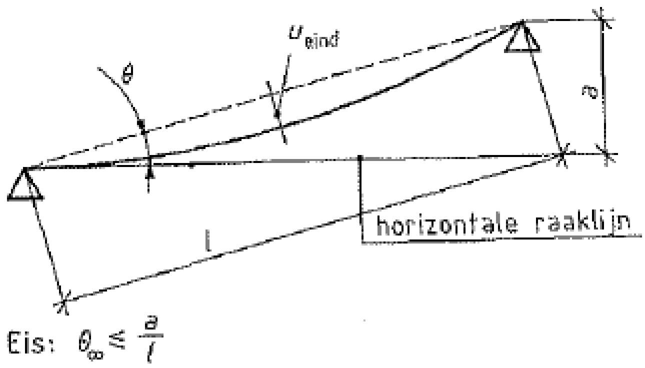


Figure 9.3 Required slope to prevent water accumulation. Reproduced from NEN 6702

Horizontal deformation
The total horizontal deformation of the structure cannot exceed h/500, where h is the height of the structure. The maximum horizontal deformation for a single building story is h/300.

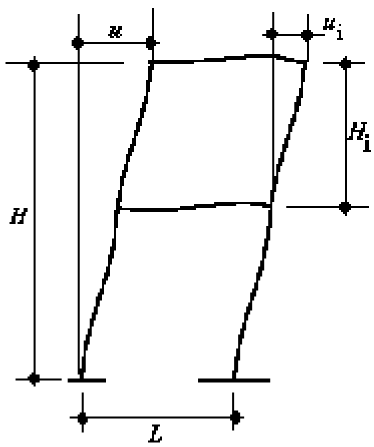


Figure 9.4 Horizontal deformation of a framework

H	total height of the building
H_i	height of a single building story
u	total horizontal deformation over the height H
u_i	horizontal deformation of the height H_i of a single story

9.2 Wind load

Wind loading is, next to seismic loading, the dominant environmental loading for structures. On almost every day of the year a severe windstorm is happening somewhere on earth. With the present knowledge about wind loading, devastating effects due to wind storms can be avoided.
Before the wind load on a stadium structure will be determined, the wind behaviour and its effect will be investigated.

9.2.1 Meteorological aspects

Wind can be described as air movement relative to the earth. The air movement is driven by several different forces. The leading forces are as a consequence of pressure differences in the atmosphere. Due to differential solar heating of different parts of the earth's surfaces pressure differences arise. Another common force that results in air movement is forces generated by the rotation of the earth. Wind depends on different variable forces and is therefore very unpredictable and complex.

Besides wind has a gusty or turbulent character. The reason is that wind is composed of a multitude of eddies of varying sizes and rotational characteristics. The gusty character of the wind hat has a large influence on the structural design when wind load is the decisive load factor.
The gustiness of strong winds in the lower levels of the atmosphere largely arises from interaction with surface features. At great heights above the surface of the earth, frictional forces are negligible. In this free atmosphere there are two important forces acting in this upper air level: the pressure gradients and the Coriolis force.
The pressure gradients are the thermodynamic consequences of variable solar heating of the earth. This force acts from a high pressure region to a low pressure region. The Coriolis force is a force due to the rotation of the earth. It acts to the right of the direction of motion in the northern hemisphere and to the left of the velocity vector in the case of the southern hemisphere. At the equator the Coriolis force is zero [5].

The upper level wind speed in the free atmosphere is known as the gradient wind velocity. Closer to the surface the wind speed is affected by frictional drag of the air stream over the terrain. The layer within which the wind speed varies from almost zero, at the surface, to the gradient wind speed at a certain height is called the boundary layer. The height were the wind velocity is equal to the gradient wind velocity is called the gradient height.
The thickness of the boundary layer, which may vary from 500m to 3000m, depends on the type of terrain. The flow pattern in the boundary layer depends on the conditions in the free atmosphere as well as the roughness of the terrain. The roughness of the earth surface is of great influence on the wind velocity in the turbulent boundary layer. As can be seen in figure 9.5, the gradient height within a large city is higher than it is at open sea, where the roughness is less. It can be concluded that the average wind speed over a time period (which is expressed in the order of ten minutes or more) tends to increase with height, while the gustiness tends to decrease with height [18].

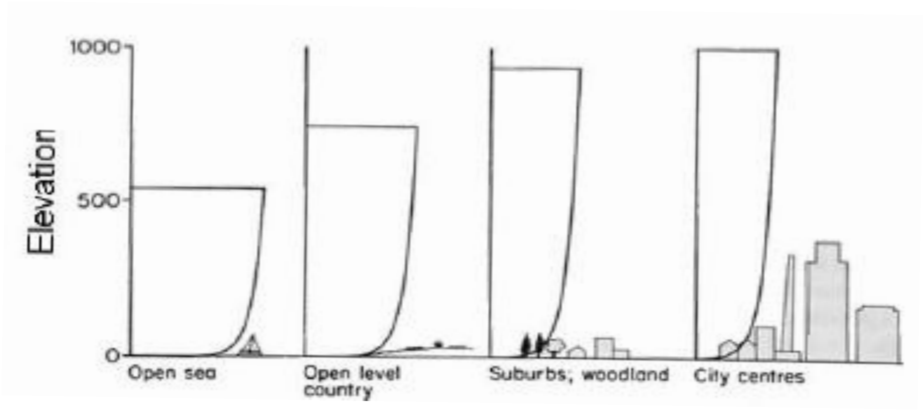


Figure 9.5 Gradient height depending on the terrain roughness. Reproduced from [24].

9.2.2 Wind velocity

The wind velocity in a certain point above the earth surface can be described by means of a wind vector $V(z,t)$ at any time. Due to the gusty and turbulent character of the airflow the magnitude and the direction of $V(z,t)$ can fluctuate from time to time.

The wind vector consists of two components, a mean wind vector $\bar{V}(z)$ (static component) and a dynamic, or turbulence, component $v(z,t)$ [24]. The sum of these components is the wind vector:

$$V(z,t) = \bar{V}(z) + v(z,t) \tag{9.10}$$

9.2.2.1 Static component

Two laws are used to describe the way in which the mean velocity $\bar{V}(z,t)$ varies with the height. The first law is the power law, which has been adopted for many codes and engineers often prefer. The second one is the logarithmic law, which is derived not only from empirical data, also from theoretical considerations. Both laws will be briefly described [7].

Logarithmic law

The logarithmic law is the most accurate mathematical expression in strong wind conditions. The logarithmic law can be derived in a number of different ways. The following derivation is the simplest.

The wind shear or the rate of change of the mean wind speed with height is a function of the following variables:

- The height above the ground, z
- Surface shear stress, τ_0
- Density of air, ρ_a

With the quantities above the wind shear can be formed:

$$\frac{d\bar{V}}{dz} z \sqrt{\frac{\rho_a}{\tau_0}} \tag{9.11}$$

$\sqrt{\frac{\tau_0}{\rho_a}}$ is known as the frictional velocity v_* Which can be derived to:

$\frac{d\bar{V}}{dz}\frac{z}{v_*} = a$ constant, say $\frac{1}{k}$, when integrating follows:

$$\bar{V}(z) = \frac{v_*}{k}(\log_e z - \log_e z_0) = \frac{v_*}{k}\log_e \frac{z}{z_0} \tag{9.12}$$

This equation is known as the usual form of the logarithmic law. z_0 is known as the roughness length and k as the Von Karman's constant and has been found experimentally to have a value of 0,4.

Some problems can occur when using the logarithmic law, one of them is the integration problem. To avoid these kinds of problems, wind engineers often prefer the power law [7].

Power law

The power law has no theoretical basis, but is easily integrated over height. To relate the mean wind speed at any height z, with that at 10m (which can be adjusted if necessary for rougher terrains) the power law can be written as:

$$\bar{V}(z) = \bar{V}_{10}\left(\frac{z}{10}\right)^{\propto} \tag{9.13}$$

The exponent $\propto$ will change with the terrain roughness and also with the height range:

$$\propto = \left(\frac{1}{\log z_{ref}/z_0}\right) \tag{9.14}$$

z_{ref} is the reference height and must be taken as half the maximum height, z_0 is the roughness length (table 9.3).

Terrain type	Roughness length [m]
0 Sea or coast area with wind from the open sea	0,003
I Lakes or open terrain with negligible vegetation and no obstacles	0,01
II Grassland and few trees/buildings with a minimum distance of 20 obstacle heights	0,05
III Area with regular vegetation, buildings with a minimum distance of 20 obstacle heights	0,3
IV Area where at least 15% of the surface is covered with buildings with a minimum height of 15m.	1,0

Table 9.3 Terrain types with roughness length according to NEN-EN 1994-1-4.

In figure 9.6 one can see that the two relationships are very close and that the power law is quite adequate for engineering purposes [7].

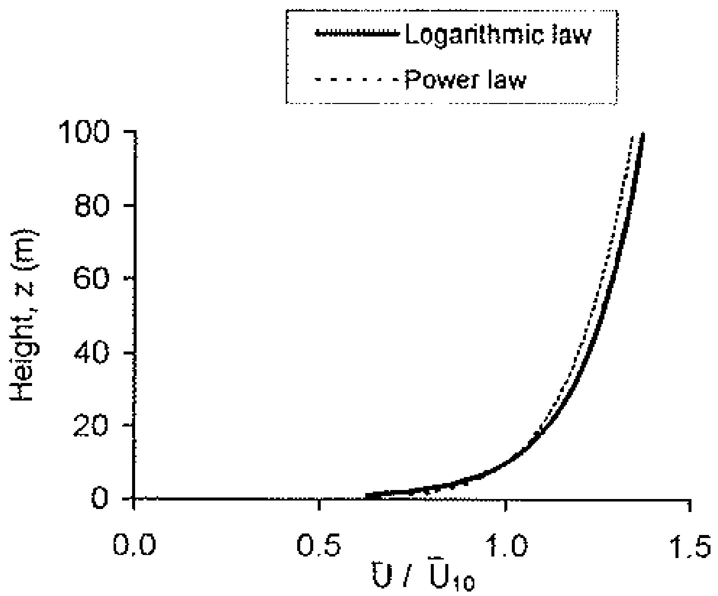


Figure 9.6 Results from logarithmic law and power law. Reproduced from [7]

9.2.2.2 Dynamic component

The wind in the boundary layer is always turbulent due to the roughness of the terrain. This means that the flow is always chaotic, with random periods varying from fraction of seconds to several minutes.

The dynamic component, or the turbulence in the wind speed (figure 9.7), can be measured by its standard deviation, or root-mean-square. First the steady or mean component is subtracted and then the resulting deviations will be quantified.

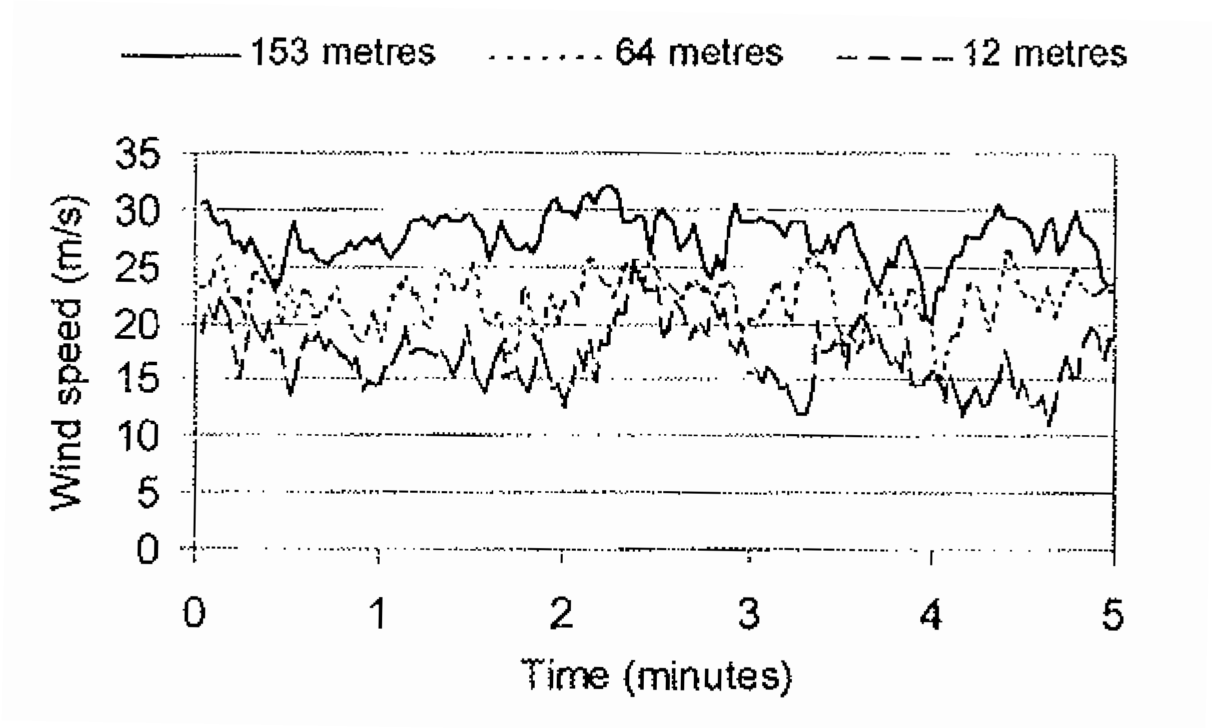


Figure 9.7 Wind speeds at three heights. Reproduced from [7].

The formula for standard deviation can be written as:

$$\sigma_v = \left\{\frac{1}{T}\int_0^T [V(t) - \bar{V}]^2 \, dt\right\}^{\frac{1}{2}} \tag{9.15}$$

Where $V(t)$ is the total velocity component in the direction of the mean wind, equal to $\bar{V} + v(t)$. $v(t)$ is the longitudinal turbulence component.

Turbulence intensity

The ratio of the standard deviation to the mean value is known as the turbulence intensity:

$$I_v = \frac{\sigma_v}{\bar{V}} \tag{9.16}$$

Measurements have found that the standard deviation of longitudinal wind speed, σ_v, is equal a good approximation of $2{,}5v_*$. Then the turbulence intensity is given by:

$$I_v = \frac{\sigma_v}{\bar{V}} = \frac{2{,}5v_*}{(v_*/0{,}4)\log_e(z/z_0)} = \frac{1}{\log_e(z/z_0)} \tag{9.17}$$

The turbulence intensity is simply related to the surface roughness.

This method is not only applicable for the longitudinal component, but also for the lateral horizontal direction and the vertical direction.

Peak gust wind speed

For design purposes the speed used to determine the wind load is the peak gust wind speed. Because the wind has its gusty character, the peak gust wind speed is defined as an expected or average value within a minimal period of 10 minutes. The expected peak gust is given by:

$$\hat{V} = \bar{V} + g\sigma_v \tag{9.18}$$

Where g is a peak factor equal to 3,5.

With the known value for $\hat{V}$ the wind load distribution on a structure can be determined.

9.2.3 Load distribution

The determination of the wind loading is of great importance when making a structural design. As mentioned before wind has a very turbulent or gusty character and so is the wind loading and the distribution on the structure.

The characteristics of wind loading or the pressures on a structure are a function of the characteristics of the approaching wind and the geometry of the structure. The pressures are not steady and more highly fluctuating as a result of the gustiness of the wind. Another cause is the local vortex shedding at the edges of the structures themselves. The fluctuating pressures can result in dynamic excitation / pressure and are not uniformly distributed over the surface of the structure.
These aspects make it very complicated to determine the wind loading on a structure, especially for a stadium. Not only will the external pressure play a role in the load distribution. The internal pressure can also be a dominant load factor.

9.2.3.1 Effective static load distribution

The wind loads on stadia are usually the dominant structural loading and these structures have some significant differences in comparison to smaller low-rise buildings. For these kinds of structures the quasi-steady approach is used in many building codes to determine the wind loading and pressure. The disadvantage is that pressure fluctuations and dynamic response are ignored when using this method. In stadia the resonant dynamic response can be significant and therefore cannot be ignored. To take these aspects into account, the effective static load distribution method is used, which will be described briefly.

The effective static peak loading distributions can be derived for the following three components:

- Mean component
- Background or sub-resonant component
- Resonant component

The background component is derived making use of a formula derived by Kasperski and Neimann (1992). The resonant component comprises of an inertial loading, similar to that used in earthquake loading.
The main advantage of the effective static load distribution approach is that the distributions can be applied to static structural analysis for the use of structural design [7]. Despite the unbalanced and complex loading on football stadia, the effective static load distribution is a good approximation of the wind loading and pressure on stadia structures.

Mean load distributions

The mean wind loading on a stadium structure can be obtained simply by relating the mean local pressure to the mean wind speed:

$$\bar{f}(z) = [0.5\rho_a \bar{V}_h^2]\bar{C}_p b \tag{9.19}$$

Where $\bar{C}_p$ is the mean pressure, that must be determined by wind tunnel testing [7, 24].

Background loading distributions

Background wind loading is produced by fluctuations due to turbulence, but with frequencies too low to excite any resonant response. The background fluctuating load distribution is expressed as followed:

$$f_B(z) = g_B \rho(z) \sigma_p(z) \tag{9.20}$$

$\rho(z)$ denotes the correlation coefficient between the fluctuating load at position z on the structure, and $\sigma_p(z)$ is the root-mean-square fluctuating load at position z [7].

Load distribution for resonant response

Due to the turbulent nature of the wind velocities in storms of all types, the wind loads acting on structures are also highly fluctuating. There is a potential to excite resonant dynamic response for structures, or parts of structures, with natural frequencies less than about 1 Hz.
The resonant response of a structure introduces the complication of a time-history effect, in which the response at any time depends not just on the instantaneous wind gust velocities acting along the structure, but also on the previous time history of wind gusts.

A well-known rule of thumb is that the lowest natural frequency of structure should be below 1 Hz for the resonant response to be significant. When the resonant response is significant i.e. a structure response dynamically the important principle to remember is the mentioned time history effect.

An equivalent load distribution for the resonant response is given by:

$$f_R(z) = g_R m(z)(2\pi n_1)^2\sqrt{\overline{a'^2}}\varphi_1(z) \tag{9.21}$$

Where g_R is the peak factor for resonant response; $m(z)$ is a mass per unit length; n_1 is the first mode natural frequency; $\sqrt{\overline{a'^2}}$ is the root-mean-square modal coordinate and $\varphi_1(z)$ is the mode shape for the first mode of vibration [5].

Combined load distribution

The combined effective load distribution for mean, background and resonant components is obtained as follows [3]:

$$f_c(z) = \bar{f}(z) + \sqrt{[f_B(z)]^2 + [f_R(z)]^2} \tag{9.22}$$

To calculate the combined load distribution on a stadium, wind tunnel testing is needed to determine the pressures on the structure. An assumption will be made for the loading on the reference stadium of this thesis.

9.2.3.2 Wind load assumptions

The determination of an accurate wind load distribution on a stadium is very difficult. As mentioned before, wind tunnel tests are used to calculate the wind loads on these special types of buildings.
For this thesis an assumption of the wind load distribution will be made. The reason is that the dynamic wind load on a stadium can only be calculated accurately with the help of wind tunnel testing. This is unfortunately not possible for this thesis. Therefore wind tunnel results from a reference project will be used.

At Arcadis wind tunnel test results are available from the structural design of the Amsterdam Arena. For this thesis only the static wind load will be used. The reason is that the aim of the thesis is to research the use of tensile-compression ring roof structures for football stadia. To do this, there is no need to use complex dynamic loads.

In the wind tunnel tests from the Amsterdam Arena, different situations where investigated. Because of the retractable roof, they did this for open and closed situations. For this thesis only the results from the open roof situation will be used.

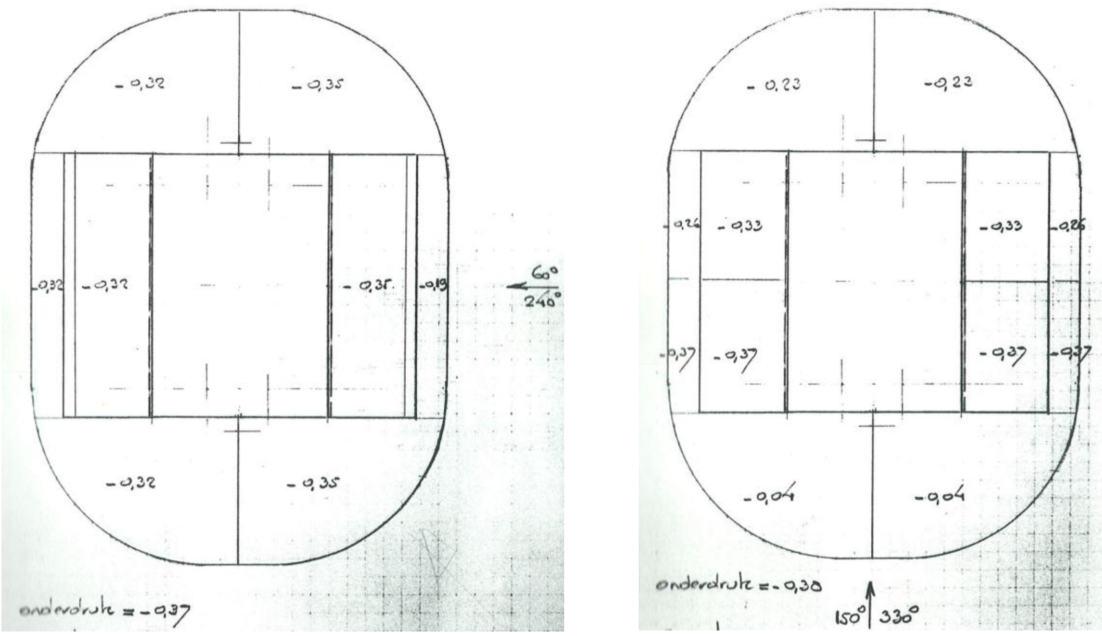


Figure 9.8 Pressure coefficient values from wind tunnel tests. Reproduced from [23].

In figure 9.8 the test results from wind acting on the long side of the stadium (situation 1) and on the short side of the stadium (situation 2) are displayed. The values are pressure coefficients. A pressure coefficient is an average value for a certain area. The results show that there is only external suction on the roof (in figure 9.8 expressed as a negative value; downward load is assumed to be positive), causes by the wind flow over the curved stadium roof. The resulting wind pressures can be calculated by multiplying the pressure coefficients with the extreme wind pressure.
Inside the stadium internal suction is present due to the wind. When wind acts on the long side of the stadium (left model in figure 9.8) the internal suction pressure coefficient inside the stadium is 0,37. The wind working on the short side of the stadium (right model in figure 9.8) activates an internal suction pressure coefficient of 0,38.

For the structural design of the roof of the Amsterdam Arena two normative load cases for each situation were used: a load case with a negative internal suction (load case 1) or a positive external suction (load case 2) on the windward side. On the leeward side only the external suction from the wind tunnel test results are used. In figure 9.9 and 9.10 the normative load cases used for both situations are displayed.

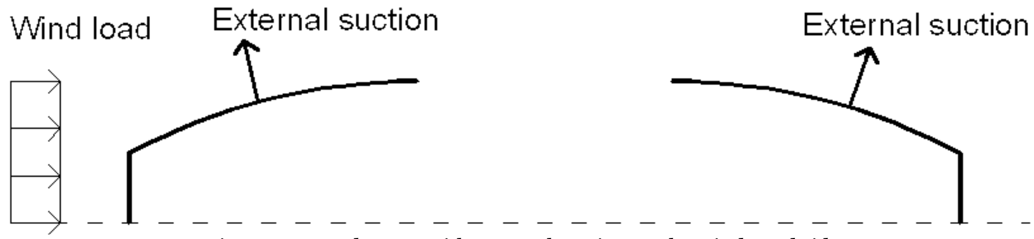


Figure 9.9 Load case 1 with external suction on the windward side

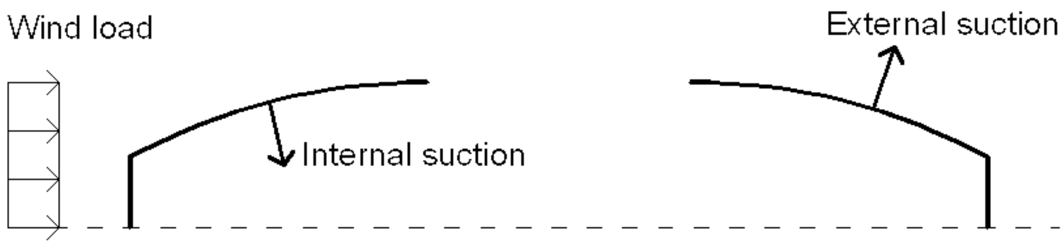


Figure 9.10 Load case 2 with internal suction on the windward side

In the load cases were both the negative internal suction and positive external suction is taken into account (load case 2) a horizontal force due to the wind arises on the roof structure.
The pressure acting on the roof can be determined by, as mentioned, multiply the pressure coefficient with the extreme wind pressure. The extreme wind pressure at area 2 in the Netherlands at height h = 50m (appendix A.9), is 1,38 kN/m^2.
The pressure values of the different load cases are displayed in figure 9.11 and 9.12 and will be used for the structural design of the tensile-compression ring roof (external suction is expressed as a positive value and internal suction as a negative value).

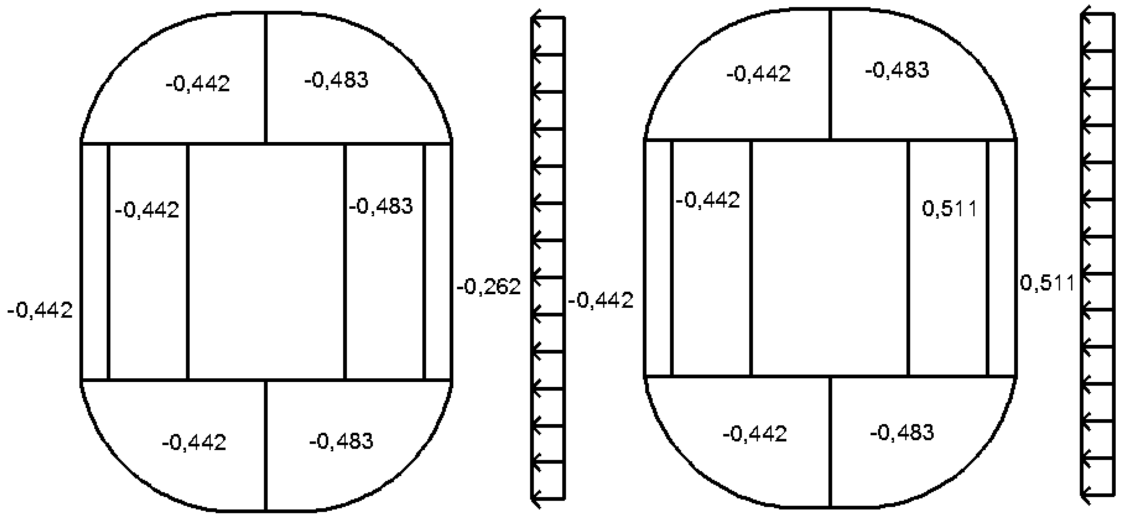


Figure 9.11 Situation 1, with external suction (left, load case 1) and internal suction (coefficient 0,37; pressure 0,511 kN/m²) at windward side (right, load case 2)

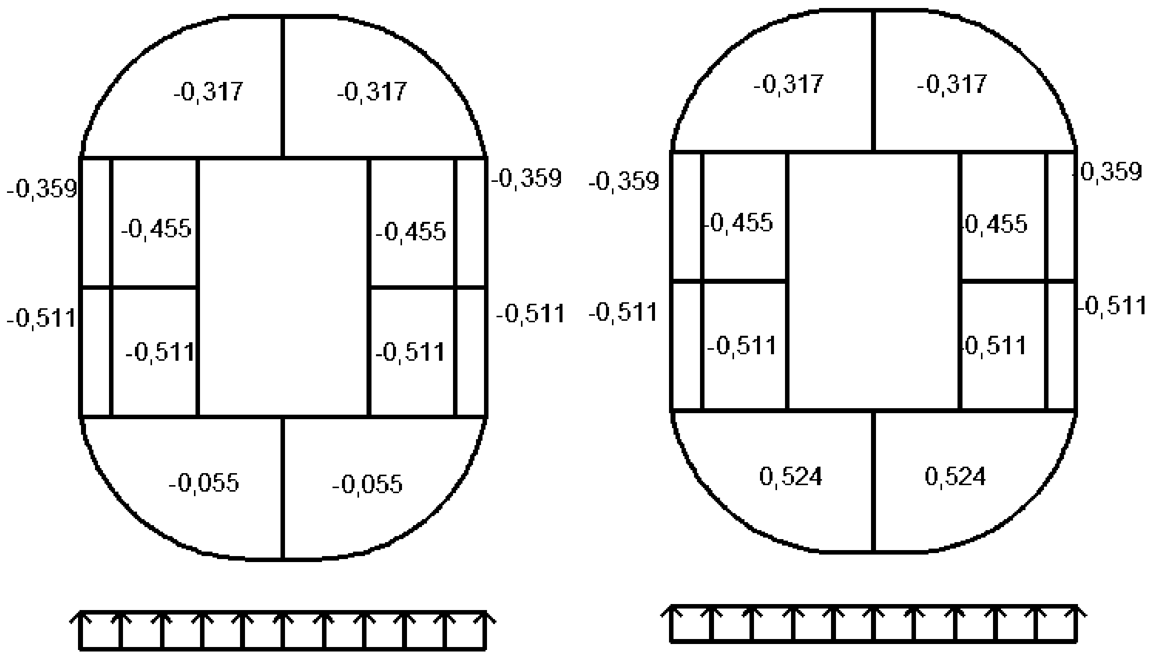


Figure 9.12 Situation 2, with external suction (left, load case 1) and internal suction (coefficient 0,38; pressure 0,524 kN/m²) at windward side (right, load case 2)

Besides a vertical load, the wind pressures cause a horizontal load on the roof of the stadium. The load is acting on the roof covering of the stadium which is placed at the lower spokes (see preliminary design) where the roof covering will be placed. The wind load acts perpendicular (downwards or upwards) to the roof; introducing vertical and horizontal loads (figure 9.13). With the help of the FEM software program Scia Engineer, the loads are automatically taken into account.

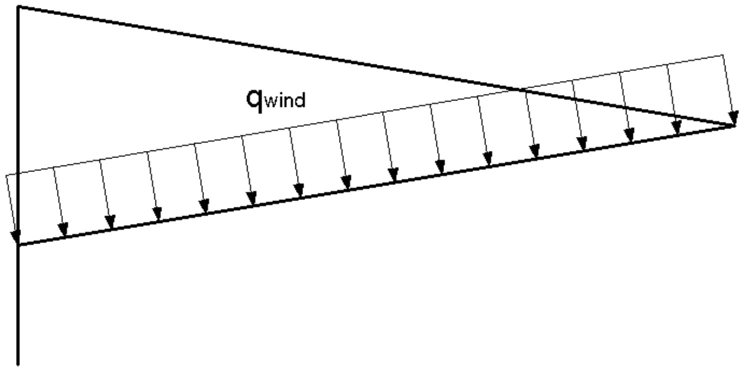


Figure 9.13 Wind load acting on the roof

9.3 Snow load

Snow load plays an important role in the structural engineering and can be the dominant load acting on a structure. In the past many structures collapsed due to the snow loading and therefore it is of great importance to use the correct snow loading data.

9.3.1 Snow load cases

In the Eurocode NEN-EN 1991-1-3 a distinction is made between snow ground load and snow roof load. The basis of snow load computation is the ground load. The roof load is modified by multiplying the ground load by a certain factor. The roof load depends on the following factors:

- The shape of the roof
- The thermal properties of the roof
- The roughness of the surface of the roof
- The amount of height being generated under the roof
- The presence of surrounding buildings
- The surrounding area
- The local climate: wind, temperature fluctuations, probability of precipitation of rain and snow

Snow roof loading is further divided by flat and sloped roofs. For sloped roofs, the flat roof snow load is modified to account for slope and the roughness characteristics of the roof. In the Eurocode several load cases are covered and most of them are traditional shaped, like saddle-, pitched- and cylindrical shaped roofs.

Other aspects that have to be taken into account when determining the snow load on structures is movement of the snow (for instance due to the wind) and drifting. These factors can cause unbalanced load distributions. These aspects have also been taken into account by the Eurocode.

Places where exceptional snowfall or snow blowing is rare (like in the Netherlands), the formulas in the normal conditions from the Eurocode must be used to determine the snow load. The ground snow load [kN/m^2] is determined with the following formula:

$$s_k = 0{,}164Z - 0{,}082 + \frac{H}{966} \tag{9.23}$$

Where Z the zone is number of the place and H is the height [m] above sea level. See Eurocode 1991-1-3 for the zone number. Most parts of the Netherlands have zone number 2. When using this formula a low value will be generated.
In the National Annex of the Eurocode NEN-EN a higher value for the characteristic ground snow load is assumed. The characteristic ground snow load is assumed to be $s_k = 0{,}7 kN/m^2$.

As mentioned the roof snow load depends on certain factors. For roof snow load under normal conditions the following formula must be used:

$$s = \mu_1 \cdot C_e \cdot C_t \cdot s_k \quad [kN/m^2] \tag{9.24}$$

Where μ_1 the snow load shape coefficient is, C_e is the coefficient depending on the exposure to wind and C_t the heat coefficient.

In table 9.4 the values for the coefficient due to wind exposure can be found and the heat coefficient is equal to 1,0 when the heat transmission of the roof is $\leq 1 \ W/m^2 \, K$.

Surroundings	**C_e**
Exposed to wind	0,8
Normal conditions	1,0
Sheltered	1,2

Table 9.4 Recommended values for C_e from Eurocode 1991-1-3.

To determine the snow load several ways are possible, depending on the type structure and material. When for instance tension structures, such as cable and membrane roofs will be used, the problems occurs that these types of structures are not covered by the Eurocode. Due to the flexibility of tension roofs and heavy snow loads sag of flat roof areas can occur. This requires consideration in design and can only be analysed with the aid of wind tunnel or water flume experiments. Water flume experiments are used to determine the snow load in case of movement of the snow due to wind and drifting [12].

Due to the lack of these tests, an assumption will be made for the snow load on the reference stadium of this thesis.

9.3.2 Snow load assumptions

For the snow load, the described load in the National annex of the Eurocode will be used. For every location in the Netherlands the characteristic ground snow load is 0,7 kN/m².

The roof snow load depends not only on the ground snow load, but also from the roof shape coefficient and the coefficient depending on the exposure to wind and the heat transmission.

The roof shape coefficient can be determined using NEN-EN 1991-1-3 5.3.2. The roof covering is placed at the bottom spokes, which has a slope of:

$$tan^{-1}\frac{15,00}{42,10} = 19,61^{\circ}$$

The roof has a slope between 15 and 30 degrees. Therefore the coefficient will be $\mu_1 = 0,8$. The National Annex prescribes that the wind exposure coefficient and the heat coefficient is 1,0 for every location.

When using these factors, the roof snow load will be:

$$s = \mu_1 \cdot C_e \cdot C_t \cdot s_k = 0,8 * 1,0 * 1,0 * 0,7$$
$$s = 0,56\ kN/m^2$$

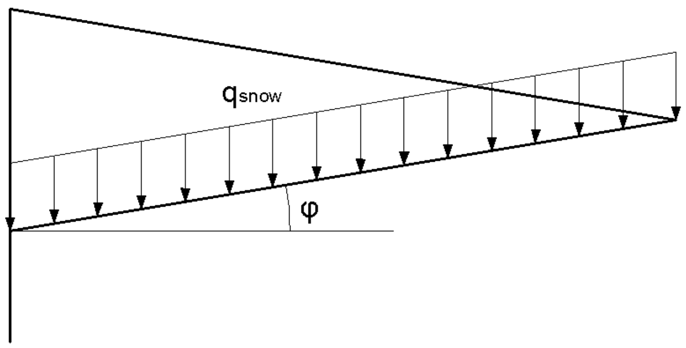


Figure 9.14 Snow load distribution

9.4 Other loads

As mentioned before there are other loads working on a roof structure then just wind and snow. The other loads working on the roof structure will be briefly described.

Imposed load

According to NEN-EN 1990:2002/NB:2007 the imposed load on a roof structure is as follows:

Categories of loaded areas	**q_k [kN/m²]**		**Q_k [kN]**
H (not accessible	Roof slope α	q_k [kN/m²]	1,5
	0 ≤ α ≤ 15 15 ≤ α ≤ 20 α > 20	1,0 4 - 0,2 x {α} 0	

Table 9.5 Imposed load on a roof structure

The load q_k is active on every individual roof element with an area up to 10 m². For the individual roof elements with a larger area, the loaded area shall be equal to 10 m², the largest length shall not be larger than 5 m. Additionally a line load of 2 kN/m active over a length of 1 m and a width of 0,1 m shall be considered. This line load is active on the whole roof area and on every individual roof element.
In case of elements directly under the roof boarding, as purlins, trusses and beams, a point load equal to Q_k = 2 kN shall be considered.

The imposed load will not be taken into account for the determination of the main supporting structure. The imposed load is only used for the determination and calculation of secondary structural elements like purlins.

Pre-stressed load

In tensile-compression ring roof structures, there is the possibility to pre-stress the spokes. Examples are stadia which use cables like the Commerzbank Arena in Frankfurt and the Gottlieb Daimler Stadium in Stuttgart. The pre-stressed load is a permanent load and has to be taken in account.

Rain load

Loads induced by rainwater on flat roofs must be taken into account. Water accumulation on the roof should be avoided. The determination of the rain load depends on the height of the overflow and the shape of the roof. The roof has a positive slope value (figure 9.11). The water will flow to the outside of the stadium. The water will not accumulate at the inner ring of the structure.

Event load

When designing a stadium event loading must also be taken into account. Today stadia have multiple purposes like concerts and football games. For every event, different objects are hung on the roof and this causes extra loading.
In the case of the Amsterdam Arena, engineers used the available bearing capacity of the snow load that almost never occurs on the roof. So no extra loading due to events was taken into account. This is the reason the event load will not be taken into account for the final design of this thesis. Another reason is that most of the event loading is hung above the field on the retractable roof. The final design will not have a retractable/closed roof and therefore there is almost no event loading.

Permanent installations loading

At the roof of a stadium a lot of permanent installations are hung. An example is the lighting and sound installations. Often a stadium has a catwalk at the roof, to provide the accessibility to the installations for control and service. When using a catwalk a significant load acts on the roof and therefore has to be taken into account, as well as other significant installation loadings.
Another well-known permanent installation load is the big TV screen in stadia for the spectators. An example is the two TV screens in the Amsterdam Arena of 65m² which have a weight of a 160kN. This load is however, not taken into account.

9.5 Load combinations

The total load depends on the permanent load and the critical variable load working on the roof structure. The Eurocode prescribes that the total load is the load combination of the acting permanent loads on the structure and/or the critical variable / incidental loads acting on the same time. The incidental loads are however not taken into account.

For the calculations four load combinations have been taken into account. These can be found in appendix A and consist of the following loads:

LC1: Permanent load
LC2: Permanent load + Wind situation 1
LC3: Permanent load + Wind situation 2
LC4: Permanent load + Snow

The permanent load consists of the dead load of the main bearing structure, the roof covering (0,05 - 0,3 kN/m^2, depending on the type of material) and the permanent installations (0,1 kN/m^2).
For load case 2 and 3, where wind is the variable load, only the situation where the internal suction does play a role will be taken into account. Due to internal suction, extra horizontal forces will arise in the structure. The reason why the situations with only external suction is not taken into account, is because the snow load combination (LC4) is leading for downward loading only.
The loads due to events will not been taken into account. It is assumed that the roof has enough extra bearing capacity using the load combinations described above.

The load combinations are summarized below:

LC1 Permanent load

	SLS	ULS
Dead load	1,0	1,35
Permanent installations	1,0	1,35
Roof covering	1,0	1,35

Table 9.6 LC1

LC2 Permanent load + Wind situation 1

	SLS	ULS
Dead load	1,0	1,20
Permanent installations	1,0	1,20
Roof covering	1,0	1,20
Wind situation 1	1,0	1,50

Table 9.7 LC2

LC3 Permanent load + Wind situation 2

	SLS	ULS
Dead load	1,0	1,20
Permanent installations	1,0	1,20
Roof covering	1,0	1,20
Wind situation 2	1,0	1,50

Table 9.8 LC3

LC4 Permanent load + Snow

	SLS	ULS
Dead load	1,0	1,20
Permanent installations	1,0	1,20
Roof covering	1,0	1,20
Snow	1,0	1,50

Table 9.9 LC4

10. Truss structure

At the end of the Preliminary Design part a preliminary design has been determined. To further investigate what is the most efficient spoke wheel design, two type of structures are investigated; a spatial truss structure and a cable structure. In this chapter the use of a spatial truss system for the spoke wheel roof is investigated. By means of making a spoke wheel roof design of a spatial truss structure for the reference stadium, insight will be gained in the use of spoke wheel roof structures for non-circular roofs, like football stadia. The eventual goal is to provide advice and an answer to the research question.
To come to an efficient design, the costs will also be taken into account. The meaning of an efficient design in this thesis is determined to be a design that consists of as little material, where the building costs are low and the construction labour is minimal.

10.1 Design process

At the end of the preliminary design part, the question raised how to come to a stable design of a spoke wheel roof consisting of a spatial truss structure? The following question is if a roof structure with a shape equal to the reference stadium is able to fulfil all requirements and still be efficient using the ring action to the fullest? Or is the available ring action not sufficient and is beam action needed to come to a stable and safe design that also consists of little material? To determine what is the best design in terms of efficiency and fulfilling all structural requirements, multiple designs will be made each with a different ratio ring : beam action.

In this chapter, three different truss designs are presented and analysed, each using the available ring action differently. The reason is that it is not possible to determine the most efficient structural design in an instant. By looking at three different designs, the chance of creating an efficient design is greater.
First the structure of the three designs must become non-linear stable. The preliminary design will be adapted in order to provide enough stability. After the determination of three designs, the designs are further optimized in order to decrease the amount of material. After the optimization the three designs are compared with each other to see what is the best solution to come to an efficient spoke wheel roof for non-circular shaped roofs like football stadia. The designs are optimized using parametric modelling.

Parametric modelling is a design concept where the absolute values of a model, or part of it, are replaced by relative parameters. Examples of parameters are the height of structures, thickness of the profile, etc. When the parameters are defined (fixed and variable parameters), the designer or engineer can adapt certain parameters into new values (variable parameters) and the structural design and its results will change. The engineer can repeat this process (iterative) in order to come to an (cost) efficient design.

The roof structure is an indeterminate, complex model which is loaded by variable loads. Besides, the influence of the second order effect on the structure is not negligible. To determine the right profiles for the different elements, the FEM program Scia Engineer will be used.
At first, the profiles are estimated by looking at reference projects. The reason is that it is not possible making correct estimations by making hand calculations of a complex structure. This first design will be modelled in the FEM program and calculated with the different load combinations (chapter 9) acting on the structure.
After the first calculations the steel check can be used to see if the steel profile is sufficient and if there is room for improvement using parametric modelling. When the steel check has a low value, a smaller, less strong profile can be applied and still fulfil the structural requirements. However, the choice of the profile influences the total roof structure; deformations, forces, moments, etc. will change in every element of the roof structure.
The determination of the right profile is an iterative process. By using fixed (geometry, loads) and variable (profile) parameters multiple calculations must be made until the right profiles are used to come to an (cost) efficient design. This process can be illustrated using the following example.

Iterative step	Steel profile (CHS)	Unity Check
1	355,6*25	0,34
2	244,5*16	0,78
3	219*12,5	0,94

Table 10.1 Example iterative design process

It is important to keep in mind that, next to the unity checks for strength and stability, the requirements regarding the stiffness will be taken into account as well. The requirements and conditions the structure needs to fulfil are explained in the following paragraph.

10.2 Requirements and Conditions

The design has to fulfil certain requirements and conditions to become a safe and stable structure. Besides structural and additional requirements (chapter 5), there are other conditions that need to be met. These conditions are related to ways of construction, building costs , etc. The requirements and conditions important in this stage of the design process are summarized by nature:

Structural

- Structure non-linear stable (stability, strength and stiffness unity check ≤ 1,00)
- Max. additional deformation (variable load) is 1,00m
- The profiles have a cross section class 1-3. Class 4 has been left out to prevent the elements from shear buckling.

Additional

- Continuous beams needs to have the largest diameter, to prevent welding problems.
- Steel elements of equal structural purpose (rocker bearings, spokes, etc.) have equal diameters. To save material, the thicknesses of the steel elements are variable.
- The steel elements consist of CHS profiles. The reason is its aesthetical value, low costs and good structural properties.

Continuous beams need to have a larger diameter compared to other non-continuous beams which are connected to the same node. When this is not the case, problems will arise in the connection (figure 10.1). A gap will arise and during construction this is difficult to repair. Besides, the force transfer from one element to the other will not be efficient.

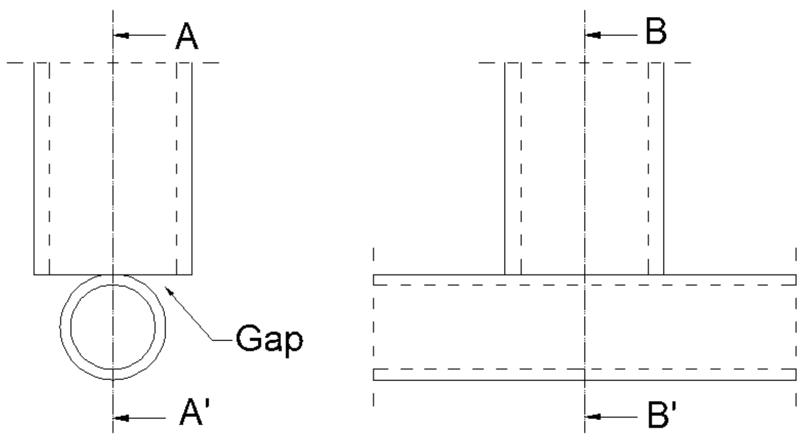


Figure 10.1 Gap at connection with cross section AA' (right) and BB' (left)

The goal is to minimize the amount of material (kg/m²) and fulfil all mentioned requirements and conditions. It is assumed that the costs are directly proportional to the amount of material.

10.3 Structural design

In the previous part of the thesis, the design variables that influence the strength and stiffness of the wheel and are related to the ring action, have been investigated for the use of a roof structure. From these variables, a preliminary design has been made. Investigation showed that when using non-pretensioned (regular steel profiles) spokes a part of the stiffness is provided by beam action. Reference projects, like the Feyenoord stadium, showed that by using a spatial truss system the ring : beam action ratio could be increased. One must take into account that when using regular steel profiles beam action is inevitable in a structure.
In this chapter will be investigated how the available ring action can be used at a greater extend in order to increase the ring : beam action ratio. By increasing the use of ring action compared to beam action the structural design is more able to efficiently transfer the loads to the underground and more material can be saved. However, the question is if the available ring action is sufficient to come to an efficient design?

First the preliminary design has to be adapted in order to provide a stable design. The structure has to provide enough stability to take up vertical as well as horizontal loads. In this chapter the realization of the three different structural designs are presented. By comparing three design variants, one can investigate which design is more efficient and why the design is more efficient.

10.3.1Variant 1

The idea for the first variant is to directly transport the loads to the corners of the roof structure where the deformation is minor and to use as little elements as possible. It is assumed that by directly transporting the loads to the corner areas, the available ring action is used at full extend and it will increase the ratio ring action : beam action. The realization of the variant is described in this paragraph. To provide sufficient non-linear stability, the structural design is determined by looking at the vertical and horizontal loads that need to be resisted.

Vertical loads

The leading design factor is to provide enough stability to take up vertical loads. The vertical loads acting on the structure are taken up by beam and ring action. The spokes applied at the structure are almost all of equal length and profile. It can be assumed that the amount of load that can be taken up by beam action is equal in the whole structure. The stiffness difference will lie in the amount of ring action the structure can provide.
The shape of the preliminary design causes a non-constant ring action capacity in the structure. In the Preliminary Design part is concluded that the amount of ring action heavily depends on the curvature of the rings. The more curvature, the more ring action that can be provided.
The sides of the global roof design are not curved and therefore cannot provide any ring action. Assuming the beam action is equal in the whole structure, it can be concluded that the vertical strength and stiffness at the sides of the structure are lower compared to the corners of the structure. The largest vertical deformations will therefore arise on the sides.

To decrease the amount of vertical deformation, a solution is to transfer the vertical loads to the corners of the structure. The ring action in the corners of the structure can take up a part of the vertical load acting on the sides of the roof structure (figure 10.2). This way the stiffness of the weakest point of the roof is increased by using the ring action from somewhere else. The stiffness will be less provided by beam action and more by ring action, increasing the use of the available ring action in the roof structure.
By placing diagonals from the centre of the side of the inner ring to the outer ring elements in the corners, the loads can be transferred. In order to transfer the loads efficiently, the diagonals need to be placed close to an angle of 45 degrees (figure 10.3).

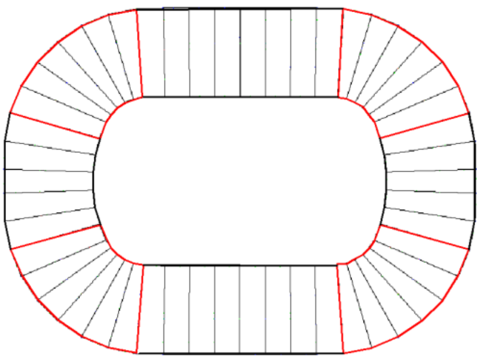


Figure 10.2 Ring action only in the corners (red)

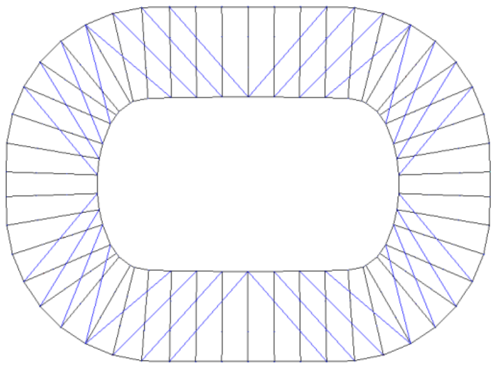


Figure 10.3 Diagonals transfer loads

The distance between the supports is 12,30m, the span of the structure is around 42,10m at the long side and 43,85m at the short side. By letting the diagonals cross three spokes, the diagonals will have an angle closest to 45 degrees. This can be showed as followed:

$\frac{42,10}{12,30} = 3,42$

The diagonals in figure 10.2 are placed in the plane of the upper and lower spokes (figure 10.6). The reason is that when the roof structure is subjected to vertical loads, the roof will slightly deform in vertical direction. In case of downward loads, the roof will sag downwards resulting in a greater slope of the top spokes compared to the bottom spokes (explanation chapter 7.1.3). The diagonals and spokes in the top plane are then more vertical placed and will transfer the spokes more efficiently by tension to the outer ring elements. When the structure is lifted up by upward loads (suction due to wind) the bottom spokes and diagonals are tensioned and will provide the transverse support.

The application of the diagonals influences the load acting on the spokes. When the diagonals are connected to the spokes, a load perpendicular to the spokes is introduced. To prevent the spokes from sideways collapse, bars must be placed between the spokes at the points where the diagonals intersect the spokes. Besides the buckling length will decrease, which has a positive effect on the compression load capacity of the spokes.
Because the diagonals cross three times the spoke distance, the spokes will be divided in three parts. The bars form new 'rings' in the structure. This is illustrated in figure 10.4. The distance between the rings is exactly one-third of the span of the roof.

The final structure can be determined by combining the structures from figure 10.3 and 10.4. It is important that all bars are connected in shared nodes. The diagonals that are illustrated in figure 10.3 are placed between the nodes from figure 10.4. The places of diagonals are therefore slightly different.

In figure 10.3 one can see that the diagonals are not attached at the centre of the short sides. The reason is the even amount of supports. The reference stadium rests on 50 supports. This boundary condition has consequences for the structural design. To take up the vertical load at the short sides as efficient as possible, cross diagonals are applied between the two centre spokes at the short sides. The final design is illustrated in figure 10.5.

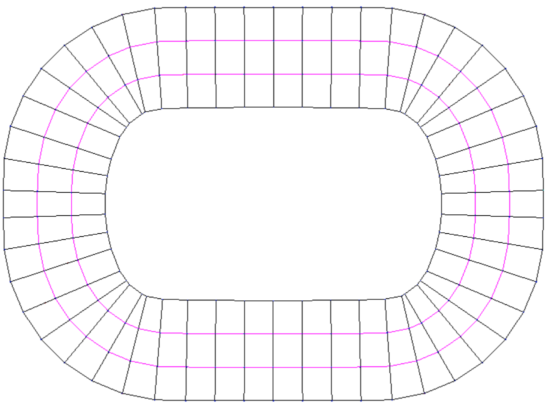


Figure 10.4 Additional bars between spokes

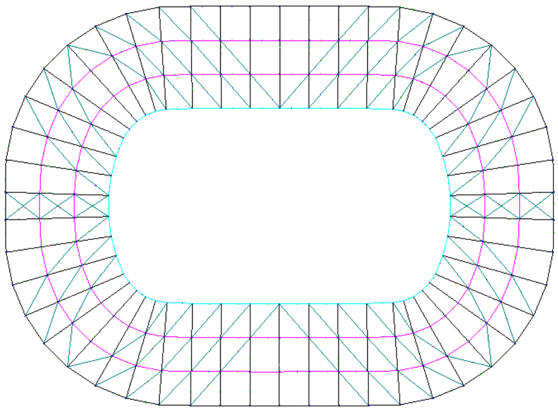


Figure 10.5 Top view design variant 1

Knowing the geometry of the main bearing structure, the spoke framework can now be further optimized. With the application of the bars between the spokes, it is a good option to apply columns between the spokes at the same place where the bars and spokes are connected (figure 10.6). The reason is that when fewer nodes are created where bars come together, less labour is needed. This results in lower costs of the structure.
The advantage of applying columns is that the spokes are supported at smaller distances, which decreases the amount of transverse forces and bending moments acting on the spokes. The buckling length of the compressed spoke will become shorter.

Horizontal loads

The final structural design, illustrated in figure 10.5, is able to take up horizontal loads. The horizontal load must also be resisted by the spoke frameworks (figure 10.6). When a horizontal load (mainly wind) acts on the spoke framework, the columns between the spokes are perpendicular loaded causing bending moments in the columns. To decrease these bending moments, diagonals can be placed between the columns.
The diagonals that are placed between the columns and spokes are illustrated in figure 10.7. Not only will they resist horizontal loads, also vertical loads. In case of downward load acting on the roof structure, tension arises in these diagonals. Extra transverse support is provided to the roof structure.

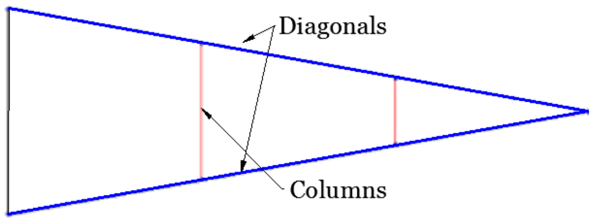


Figure 10.7 Diagonals in spoke framework

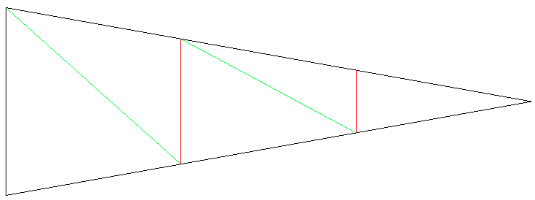


Figure 10.6 Supporting columns between upper and bottom spokes

Choice of profiles

To determine the right profile for every element, the total roof structure will be divided in different groups. Every group consists of elements that have the same purpose. For instance a group of elements that consist of just rocker bearings, one with just inner ring elements, etc. These groups of elements are numbered in figure 10.8 and 10.9.

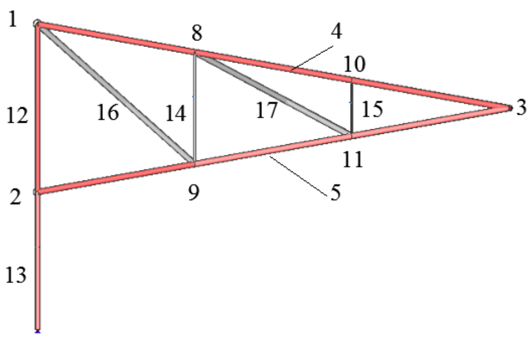


Figure 10.9 Cross section spoke framework variant 1

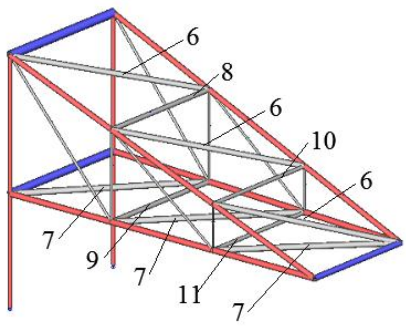


Figure 10.8 Spoke framework variant 1

The choice of the profiles depends on the structural and additional requirements mentioned in paragraph 10.2. One of the requirements is that continuous beams need to have a greater diameter compared to the other connected elements. The reason is to prevent welding problems.

The outer ring (element nr. 1 and 2) and inner ring (3) of the structure will take up most of the loads and are continuous beams (coloured blue in figure 10.9). Besides, the spoke wheel principle showed that the transverse stiffness and strength increases when the outer ring has a large cross sectional area. These elements will therefore have the largest diameter. The spokes (4,5) will have the second largest diameter (red), followed by the other elements number 8-17 (grey). The only exception is that the columns between the outer rings (12) and the rocker bearings (13) only must have a diameter lower than the outer rings (1 and 2). Knowing this, the actual profiles for the elements can be further determined.

The amount of load acting on the profiles is unknown in this phase of the design process. The statically indeterminate structure, acting of variable loads and second order effects makes it difficult to determine the profiles by hand. The profiles will therefore be estimated using reference projects (Feyenoord stadium, Rotterdam).
The applied profiles are presented in table 10.2 using the illustrations in figure 10.8 and 10.9. For the structure, steel grade S235 has been used. In table 10.2 is visualized that element nr. 1-3 must have a greater diameter than element nr. 4-17. Followed by element nr. 4 and 5 which must be greater than element nr. 6-17.

Element nr.	Profile (CHS)	Element nr.	Profile (CHS)
1	864*40	**10**	406,4*20
2	762*36	**11**	406,4*20
3	508*32	**12**	508*25
4	457*20	**13**	508*25
5	457*20	**14**	273*14,2
6	457*30	**15**	219,1*11,1
7	457*36	**16**	244,5*14,2
8	457*22,2	**17**	193,7*10
9	457*22,2		

Table 10.2 List of profiles variant 1

For the determination of the profiles, is assumed that the snow load combination (LC2) is leading. This load combination causes compression in elements 1, 4, 5, 6, 7, 8, 10, 12, 13, 14 and 15. For compression elements it holds that these profiles are preferred to a have a high diameter/thickness ratio compared to tensile elements (this will be further explained in paragraph 10.5). For example, in the bottom (2) and top (1) outer ring elements very high normal forces will arise. The top outer ring element will be compressed. Therefore a profile with a higher diameter/thickness ratio has been applied compared to the bottom outer ring.

The profiles are large compared to reference projects (Feyenoord stadium, Rotterdam) which also use spoke wheel roofs. The reason is that first the structure needs to be non-linear stable . This can be accomplished by minimizing the deformation by increasing the stiffness. When the structure is non-linear stable, the elements can be optimized by using different profiles.

Roof covering

The roof covering is placed on the bottom ring elements (chapter 9). The load acting on the covering (wind, snow, etc.) will be transferred first to the ring elements (2, 3) and the bottom bar elements (9, 11). The load transfer direction of the roof covering is illustrated in figure 10.10.

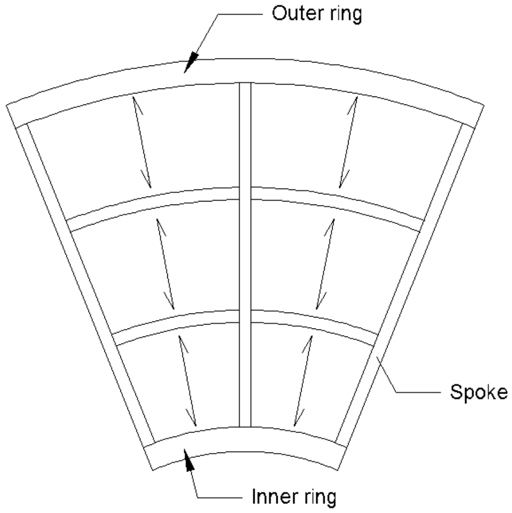


Figure 10.10 Load transfer direction roof covering

Roof design

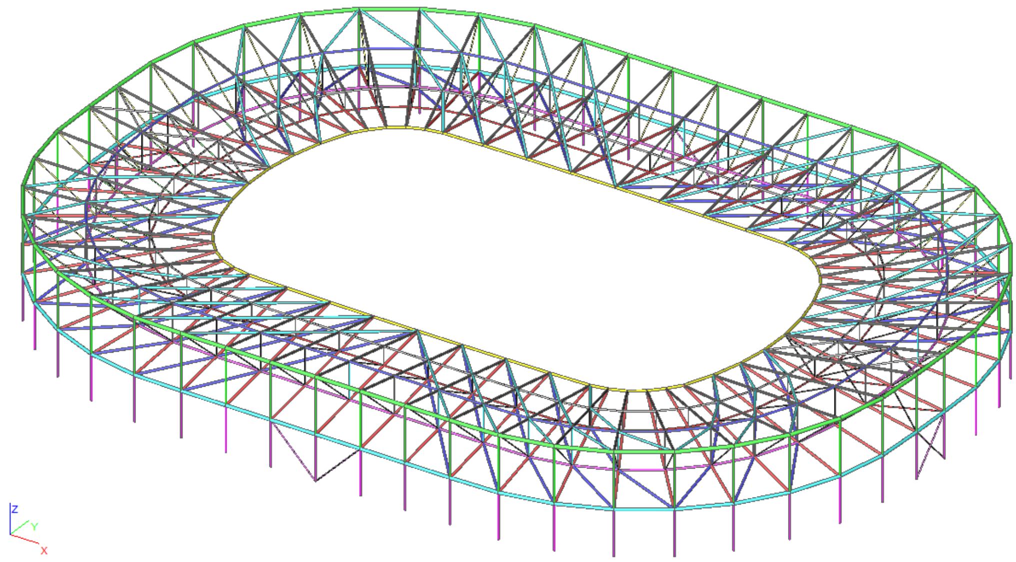


Figure 10.11 3D roof design variant 1

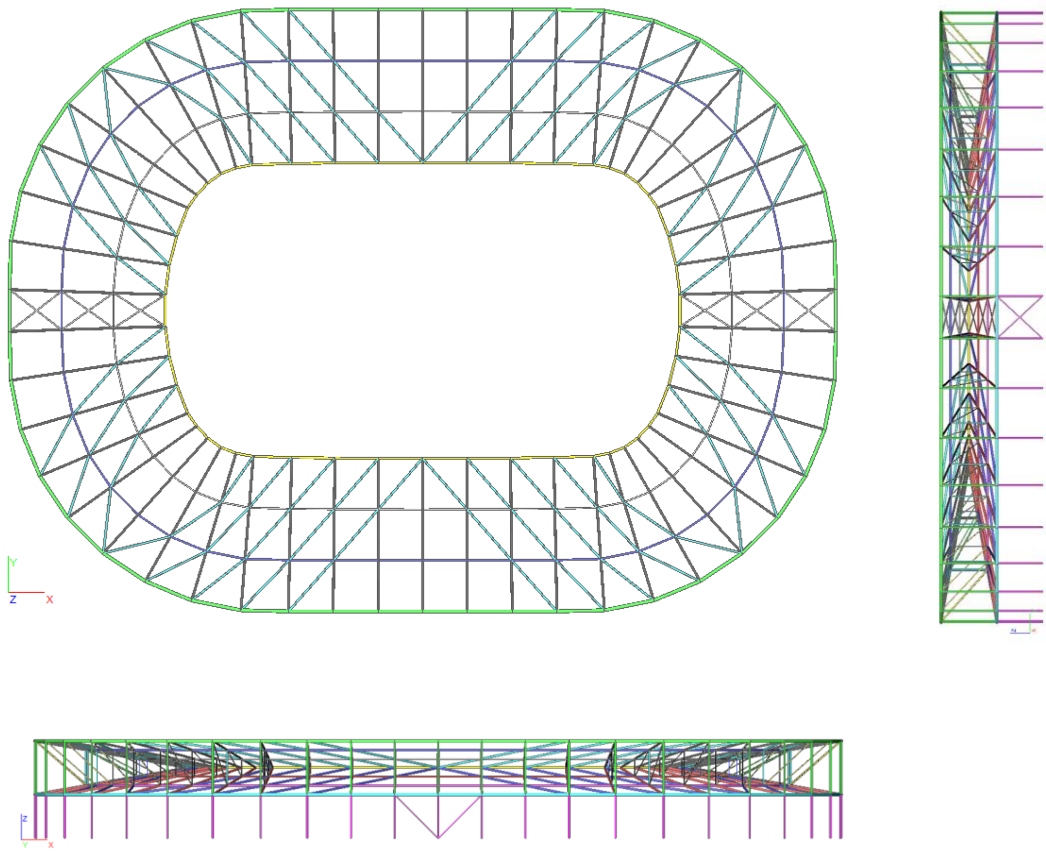


Figure 10.12 View x, y and z-direction variant 1

10.3.2 Variant 2

The idea for the second variant is to use a more regular, symmetric spatial truss almost equal to the Feyenoord stadium. It is expected that the ratio ring : beam action in this structure is lower compared to variant 1, because the loads from the sides of the structure are not directly transported to the corner areas where more stiffness is provided by ring action. By designing this variant insight will be gained in the use of beam action, its advantages and disadvantages compared to using ring action at full extend and to determine whether a more efficient design is possible for non-circular shaped roofs.
For the second variant the ground plan from figure 10.4 is used as a basis for the design. The reason is that the design from figure 10.4 offers the opportunity to apply diagonals under an angle close to 45 degrees.

Vertical loads

A solution to take up vertical loads more efficient is to place the diagonals under a steeper vertical slope. A good option is to apply vertical crosses in the plane of the diagonals (figure 10.13). These cross diagonals are placed at almost 45 degrees and will efficiently transfer the vertical forces to the outer ring. This variant shows equal geometrical characteristics compared to the Feyenoord stadium. Like at the Feyenoord stadium, the cross diagonals for vertical load transfer are placed between the upper and bottom spoke (figure 3.6).

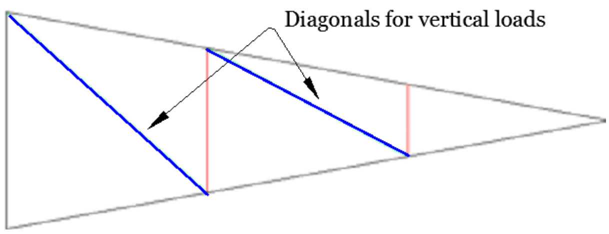


Figure 10.13 *Cross diagonals to take up vertical loads*

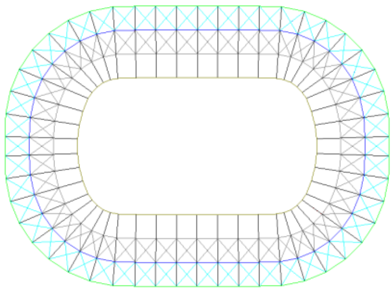


Figure 10.14 Top view vertical cross diagonals

The cross diagonals can take up both up- and downward vertical loads that act on the roof structure. The cross diagonals transfer the loads to the outer ring. A part of the vertical load is taken up by the ring action provided by the ring elements in the corners of the roof structure. Compared to the first variant, the second variant makes more use of beam action.

Horizontal loads

Beside vertical loads, the stadium roof needs to withstand horizontal loads, which are mainly caused by wind. The structure illustrated in figure 10.13 and 10.14 will be further adapted in order to create a structure that is stable in both vertical and horizontal direction.

The direction of the wind is variable. All directions in the horizontal plane of the structure need to be stabilized. This can be accomplished by applying horizontal cross diagonals in the bottom plane along the ring direction.
The spokes in the structure of figure 10.11 is divided in three parts. For the horizontal stability it is an option to apply cross diagonals in the outer ring plane, inner ring plane or in between (figure 10.13). It is important that the horizontal load will be efficiently transferred to the wind bucks that are placed at the outer ring. When the cross diagonals are placed at the inner ring or in-between, extra cross diagonals need to be placed to transfer the horizontal wind loads to the wind bucks.

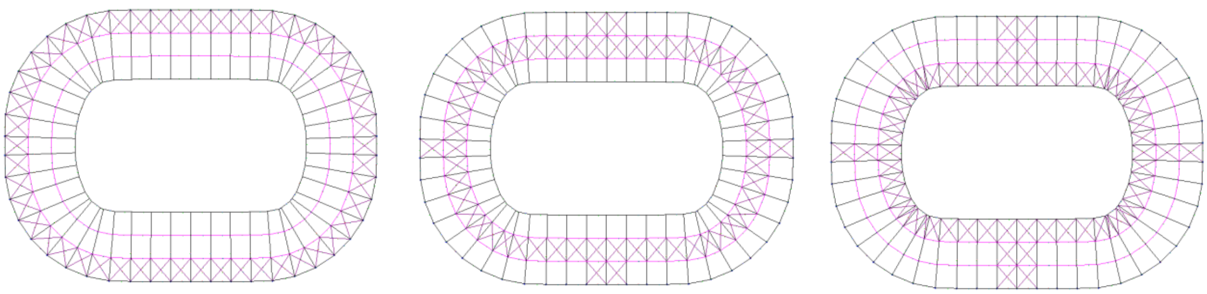


Figure 10.15 Choice of place of horizontal diagonals; along the outer ring, inner ring or in-between

Because there are already cross diagonals needed at the centre of the short sides to carry the vertical loads as explained earlier, this is a good place to apply the wind bucks. To transfer the wind loads from all directions efficiently, wind bucks will also be placed at the centre of the long sides.

The best solution is to apply the cross diagonals along the outer ring. The first reason is that the amount of applied cross diagonals is less than in the case of the other two options. Because the cross diagonals are placed along the outer ring, the forces can be directly transferred to the wind bucks. No extra diagonals are needed to transfer the horizontal loads to these wind bucks, unlike the other two options. This will save labour and costs.
The second reason is the angle of the diagonals. Looking at the cross diagonals in the corners of the structure, the diagonals along the outer ring have an angle closer to 45 degrees compared to when the diagonals are placed along the inner ring (figure 10.15).
When the diagonals have an angle close to 45 degrees, the diagonal are more able to transfer the load efficiently. As a consequence smaller profiles can be used for the diagonals, resulting in lower costs.

With the determination of the structure to provide horizontal stability, the complete structure is determined. The cross section of the total design that provides both horizontal and vertical stability is illustrated in figure 10.16.

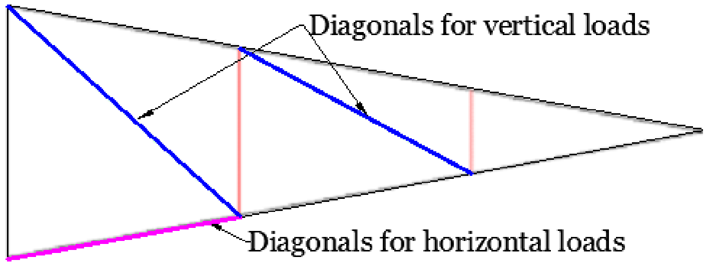


Figure 10.16 Cross section of total design variant

Choice of profiles

To determine the right profiles, the elements will first be divided in groups. The element numbers are presented in figure 10.17 and 10.18.

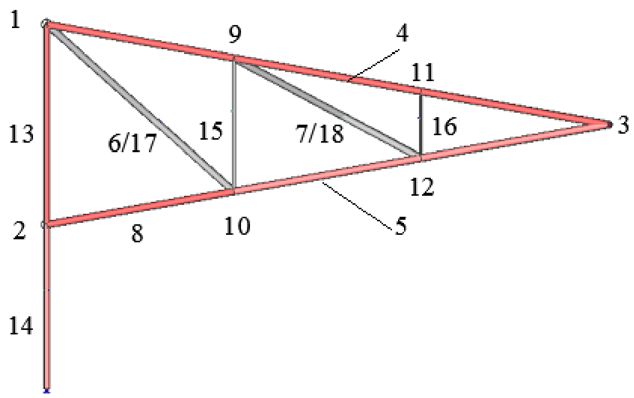


Figure 10.17 Cross section spoke framework variant 2

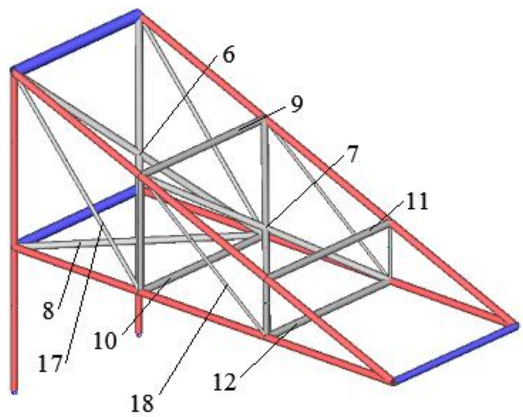


Figure 10.18 Spoke framework variant 2

The profiles for the structure, like variant 1, will be estimated using reference projects. One must keep in mind that continuous beams need to have a greater diameter compared to the other connected elements. The outer (element nr. 1 and 2) and inner ring (3) of the structure will take up most of the loads and are continuous beams (blue). These elements will have the largest diameter. The spokes (4 and 5), the columns between the outer rings (6) and the rocker bearings (7) will have the second largest diameter (red), followed by elements 8-18 (grey). The applied profiles are presented in table 10.3 using the illustrations in figure 10.17 and 10.18. For the structure steel of grade S235 has been used.

Element nr.	Profile (CHS)	Element nr.	Profile (CHS)
1	914*40	**10**	559*25
2	762*36	**11**	559*25
3	559*20	**12**	559*25
4	559*16	**13**	508*20
5	559*16	**14**	508*20
6	559*20	**15**	355,6*16
7	508*20	**16**	219,1*10
8	508*20	**17**	298,5*12,5
9	559*25	**18**	219,1*10

Table 10.3 List of profiles variant 2

The choice of profiles in variant 2 is equal to those of variant 1. The vertical crosses need to take up tensile as well as compression forces. Therefore a large, strong profile has been used (559*20 / 508*20). For the horizontal crosses a CHS 508*20 profile has been applied.

Roof covering

The roof covering is, like variant 1, placed on the bottom ring elements (chapter 9). The load acting on the covering (wind, snow, etc.) will be transferred first to the ring elements (2, 3) and the bottom bar elements (9, 11). The load transfer direction of the roof covering is illustrated in figure 10.10.

Roof design

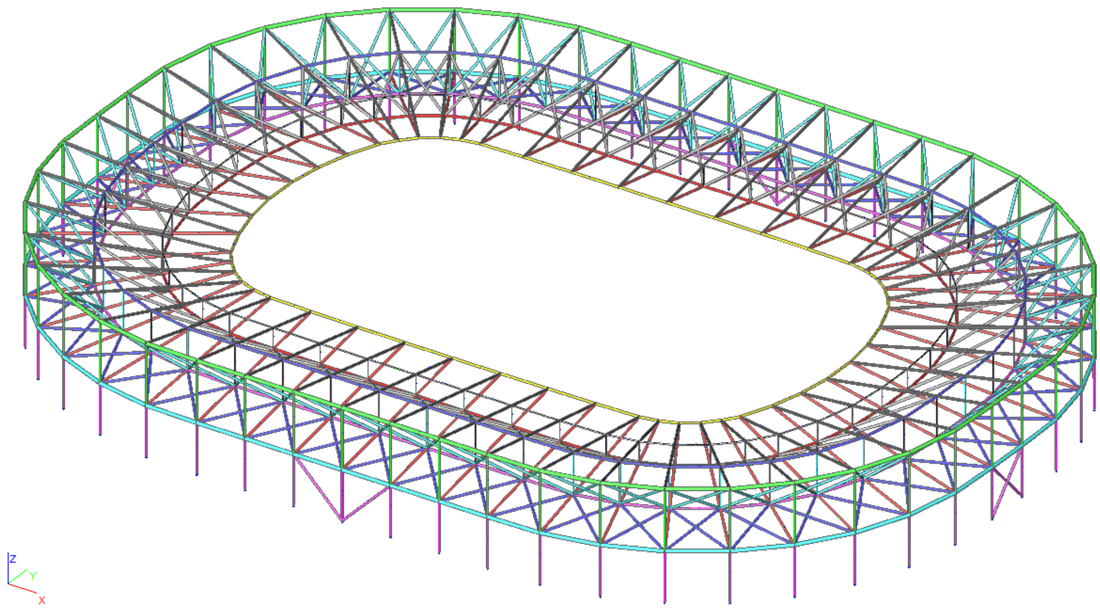


Figure 10.19 3D roof design variant 2

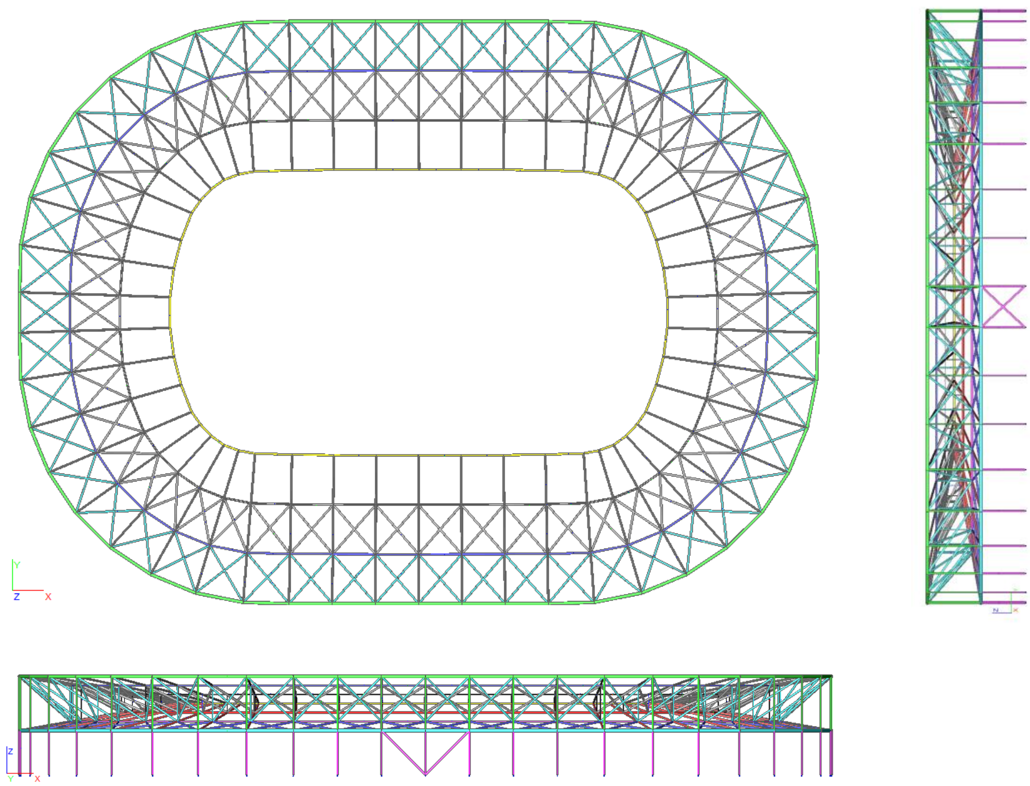


Figure 10.20 View x, y and z –direction variant 2

10.3.3 Variant 3

The idea for the third and final variant is to apply diagonals in the complete plane of the bottom and top plane. The reason is that the loads will flow through the elements that provide the best load transfer. It is expected that this structure will use the available ring action at full extend.
Again the ground plan from figure 10.4 is used as a basis for the design. The reason is that the design from figure 10.3 offers the opportunity to apply diagonals under an angle close to 45 degrees.

Vertical loads

In this third variant crosses are applied in the whole roof in the top and bottom spoke plane. Although more elements need to be applied compared to the two previous variants, smaller and less strong/stiff profiles can be used to carry the same amount of load acting on the roof. Because the ring action is used automatically at full extend, the structure will be very stiff. It is interesting to see how the loads will flow and if material can be saved. The place of the crosses to carry the vertical load is illustrated in figure 10.21 and 10.22.

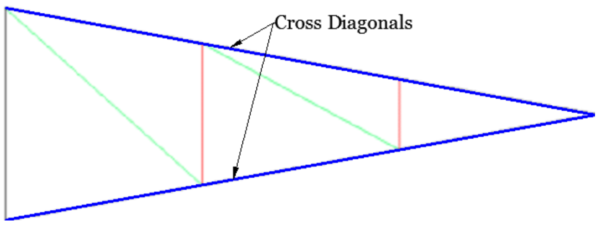


Figure 10.21 Place of cross diagonals

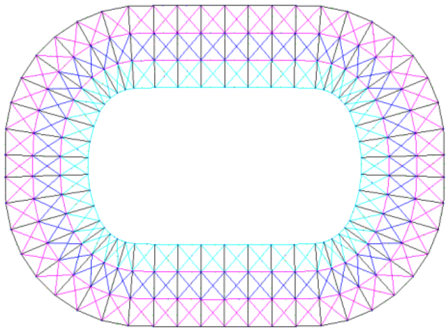


Figure 10.22 Design variant 3

Horizontal loads

The crosses that carry the vertical loads (figure 10.21 and 10.22) will also carry the horizontal loads. There are no extra elements needed to take up the horizontal loads.

Choice of profiles

To determine the right profiles, the elements will first be divided in groups. The element numbers are presented in figure 10.23 and 10.24.

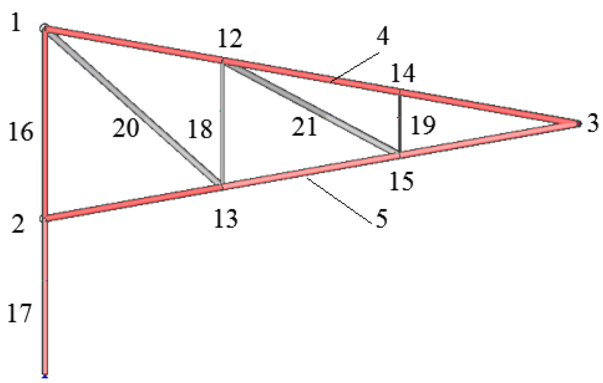


Figure 10.23 Cross section spoke framework variant 3

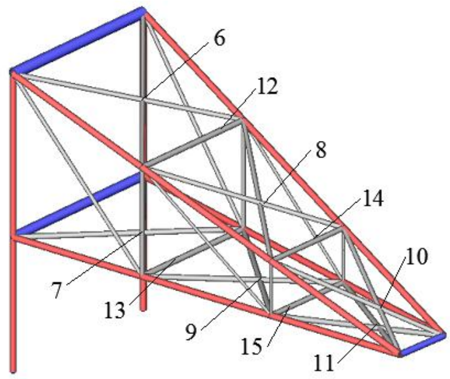


Figure 10.24 Spoke framework variant 3

The profiles for the structure, like variant 1 and 2, will be estimated using reference projects. One must keep in mind that continuous beams need to have a greater diameter compared to the other connected elements. The outer ring (element nr. 1 and 2) and inner ring (3) of the structure will take up most of the loads and are continuous beams (blue). These elements will have the largest diameter. The spokes (4, 5) will have the second largest diameter (red), followed by elements 8-21 (grey). The applied profiles are presented in table 10.4 using the illustrations in figure 10.23 and 10.24. Element numbers 6-11 are the crosses in the plane of the top and bottom spokes. For the structure steel of grade S235 has been used.

Element nr.	Profile (CHS)	Element nr.	Profile (CHS)
1	914*40	**12**	457*22,2
2	762*36	**13**	457*22,2
3	508*32	**14**	355,6*17,5
4	508*20	**15**	355,6*17,5
5	508*20	**16**	508*20
6	508*20	**17**	508*20
7	457*17,5	**18**	273*14,2
8	508*20	**19**	193,7*10
9	457*17,5	**20**	298,5*12,5
10	508*20	**21**	219,1*10
11	457*17,5		

Table 10.4 List of profiles variant 3

The choices of profiles are related to variant 1 and 2. The only differences are the crosses in the bottom and top spoke plane. The top crosses (element 6-8) will have to take up smaller compression forces compared to the bottom crosses (element 9-11). These elements will therefore have a smaller profile.

Roof covering

The roof covering is, like variant 1 and 2, placed on the bottom ring elements (chapter 9). The load acting on the covering (wind, snow, etc.) will be transferred first to the ring elements (2, 3) and the bottom bar elements (9, 11). The load transfer direction of the roof covering is illustrated in figure 10.10.

Roof design

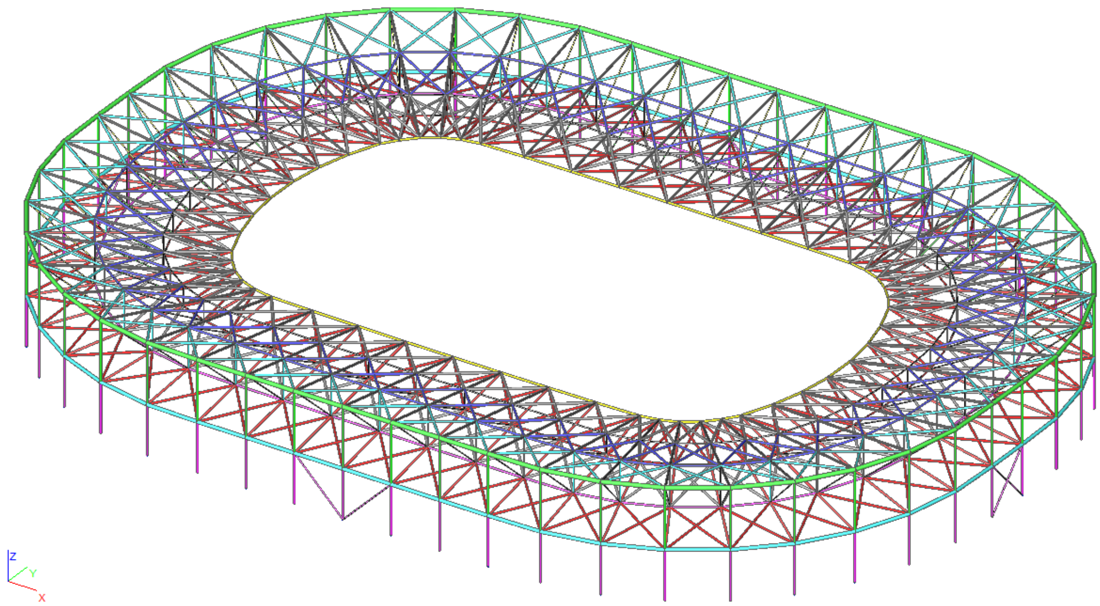


Figure 10.25 3D roof design variant 3

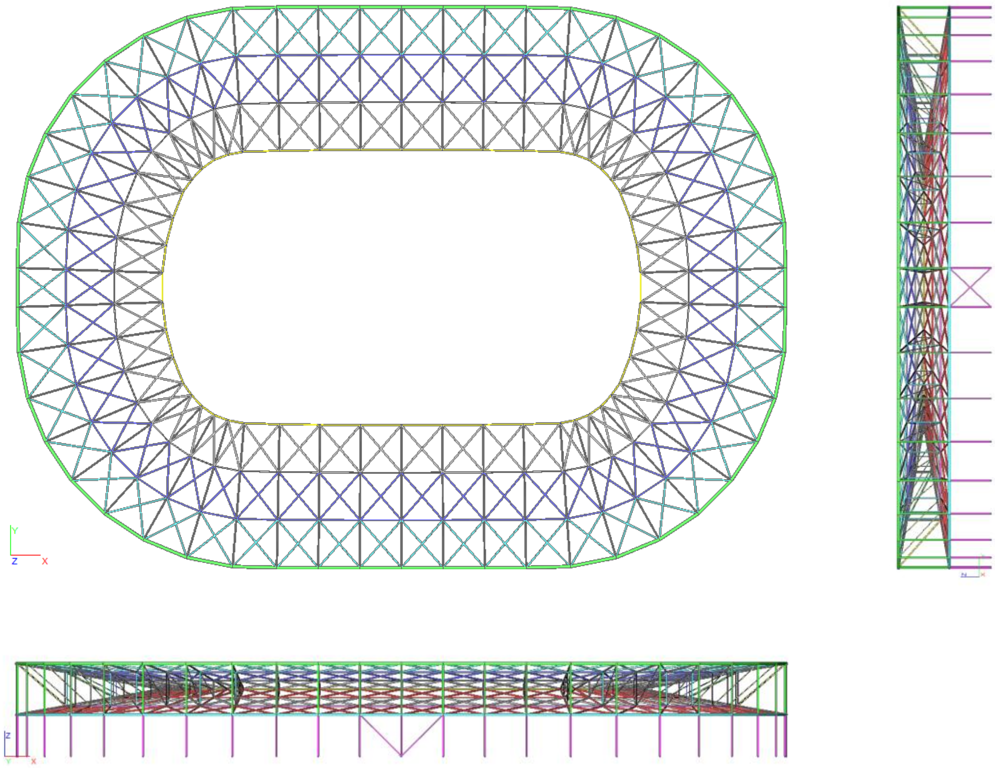


Figure 10.26 View x, y and z-direction variant 3

10.4 Results

The results of the structural design variant 1, 2 and 3 are briefly presented in this paragraph. The results conclude the stiffness, strength and stability performances of the structures. The calculations and results are made with the FEM program Scia Engineer. In appendix A.10 the theory concerning the check of the CHS profiles for stiffness, strength and stability is explained.

10.4.1Variant 1

The results show that the complete structure is non-linear stable. The structure has an average dead load of 190,61 kg/m². The largest deformation arises at mid- span of the long side of the roof structure; 826,40 mm. The unity check values are presented in table 10.5. In the table only the leading UC is presented (stability, stiffness or strength).

Strength
The results show that all elements fulfil the strength requirement and have a stress unity check lower than 1,0. The strength requirement is leading for the elements that are subjected to tension. These are the following element numbers: 2, 3, 9, 11, 16 and 17.

Stiffness
The stiffness requirements only accounts for the ring elements (elements 2, 3, 9 and 11) that are present in the plane of the roof covering. As a consequence of the variable load acting on the roof (snow, wind, etc.) these elements are subjected to the largest additional deformations. All elements fulfil the stiffness requirement.

Stability
The stability requirement is leading for elements subjected to compression forces (elements 1, 4, 5, 6, 7, 8, 10, 12, 13, 14 and 15). For these elements the stability requirements consist of buckling of the bar due to the normal force and the combination normal force/bending moment. CHS profiles are not sensitive for lateral torsion buckling and will therefore not be taken into account.
Table 10.5 shows that element nr. 6 and 7 do not fulfil the stability unity check requirement. A larger and stronger profile (with a higher buckling factor) is needed. Element nr. 6 and 7 however, must have a smaller diameter compared to element numbers 1-5. The reason is that continuous beams must have a greater diameter to prevent welding problems. As a consequence element nr 4 and 5 need to have a greater diameter.

Using the steel checks one can see where the amount of steel can be decreased. The elements are further optimized in chapter 10.5.

	Profile		Results	
Nr.	**(CHS)**	ω_{buc}	**+/-**	**UC**
1	864*40	0,97	-	0,95
2	762*36	0,97	+	0,91
3	508*32	1,00	+	0,88
4	457*20	0,70	-	0,93
5	457*20	0,71	-	0,61
6	457*30	0,47	-	**1,34**
7	457*36	0,44	-	**1,83**
8	457*22,2	0,78	-	0,85
9	457*22,2	0,83	+	0,67
10	406,4*20	0,80	-	0,56
11	406,4*20	0,71	+	0,55
12	508*25	0,71	-	0,24
13	508*25	0,81	-	0,31
14	273*14,2	0,56	-	0,43
15	219,1*11,1	0,84	-	0,17
16	244,5*14,2	0,46	+	0,65
17	193,7*10	0,46	+	0,80

Table 10.5 Results of variant 1; buckling factor, tension/compression and unity check

10.4.2 Variant 2

The results show that the complete structure is non-linear stable. The structure has an average dead load of 220,06 kg/m². The largest deformation arises at mid- span of the long side of the roof structure: 920,60mm. In table 10.6 only the leading UC is presented (stability, stiffness or strength).

Strength
The strength requirement is leading for the elements that are subjected to tension (element nr. 2, 3, 10, 12, 17 and 18). For compressed elements, the stability check is leading.
The results show that not all elements fulfil the strength requirement. Element 2 and 12 have a UC larger than 1,0. A profile with a greater cross sectional area A is needed to fulfil the strength requirement. When a larger diameter is applied, the size of elements 3, 4 and 5 need to be larger as well to fulfil the additional requirement regarding continuous beams.

Stiffness
The stiffness requirements are only leading for the ring elements that are present in the plane of the roof covering. As a consequence of the variable load acting on the roof (snow, wind, etc.) these elements are subjected to the largest additional deformations. All elements fulfil the stiffness requirement.

Stability
The stability requirement is leading for the rest of the elements. Table 10.6 shows that only element 9 does not fulfil the requirement. Buckling due to compression and bending is leading for this element. An element with a greater cross sectional area A and modulus of section W_{pl} is needed. When a larger diameter is applied, the diameter of elements 3, 4 and 5 need to be larger as well to fulfil the additional requirement regarding continuous beams.

The results show that the vertical cross diagonals (element nr. 6 and 8) take up most of the vertical load. As a consequence, the amount of load taken up by the diagonals in the plane of the spoke framework (element nr. 17 and 18) is negligible. The reason is that these diagonals work in the same plane as the vertical cross diagonals. Therefore the diagonals will be deleted from the design.
The results from table 10.6 show that there is room for improvement. In chapter 10.5 the structure will be further optimized.

	Profile	Results		
Nr.	**(CHS)**	ω_{buc}	**+/-**	**UC**
1	914*40	0,97	-	0,97
2	762*36	0,95	+	**1,29**
3	559*20	1,00	+	0,51
4	559*16	0,82	-	0,36
5	559*16	0,80	-	0,66
6	559*20	0,50	-	0,70
7	508*20	0,52	-	0,91
8	508*20	0,52	-	0,91
9	559*25	0,85	-	**1,32**
10	559*25	0,89	+	0,83
11	559*25	0,85	-	0,34
12	559*25	0,85	+	**1,31**
13	508*20	0,72	-	0,40
14	508*20	0,82	-	0,46
15	355,6*16	0,74	-	0,37
16	219,1*10	0,84	-	0,13
17	298,5*12,5	0,23	+	0,29
18	219,1*10	0,17	+	0,33

Table 10.6 Results of variant 2; buckling factor, tension/compression and unity check.

10.4.3 Variant 3

The results show that the complete structure is non-linear stable. The structure has an average dead load of 215,02 kg/m². The largest deformation arises at mid- span of the long side of the roof structure, 1029,50 mm. In table 10.7 only the leading UC is presented (stability, stiffness or strength).

Strength
The results show that all elements fulfil the strength requirement and have a stress unity check lower than 1,0. The strength requirement is leading for the elements that are subjected to tension (element nr. 2, 3, 13, 15, 20 and 21). For compressed elements, the stability check is leading.

Stiffness
The stiffness requirements are only leading for the ring elements that are present in the plane of the roof covering (element nr. 2, 3, 13 and 15). As a consequence of the variable load acting on the roof (snow, wind, etc.) these elements are subjected to the largest additional deformations. All elements fulfil the stiffness requirement.

Stability
The stability requirement is leading for elements subjected to compression forces (elements 1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17, 18 and 19). Table 10.8 shows that element nr. 6 and 11 do not fulfil the stability unity check requirement. A larger and stronger profile (with a higher buckling factor) is needed. Element nr. 6 and 11 however, must have a smaller diameter compared to element numbers 1-5 to fulfil the additional requirement regarding continuous beams.

In chapter 10.5 the structure will be further optimized.

	Profile		Results	
Nr.	**(CHS)**	ω_{buc}	**+/-**	**UC**
1	914*40	0,97	-	0,96
2	762*36	0,97	+	0,97
3	508*32	1,00	+	0,15
4	508*20	0,77	-	0,37
5	508*20	0,79	-	0,68
6	508*20	0,55	-	**1,00**
7	457*17,5	0,46	-	0,69
8	508*20	0,57	-	0,89
9	457*17,5	0,47	-	0,92
10	508*20	0,58	-	0,84
11	457*17,5	0,49	-	**1,33**
12	457*22,2	0,83	-	0,83
13	457*22,2	0,83	+	0,71
14	355,6*17,5	0,86	-	0,51
15	355,6*17,5	0,64	+	0,68
16	508*20	0,72	-	0,35
17	508*20	0,82	-	0,44
18	273*14,2	0,56	-	0,47
19	193,7*10	0,79	-	0,23
20	298,5*12,5	0,61	+	0,64
21	219,1*10	0,50	+	0,70

Table 10.7 Results of variant 3; buckling factor, tension/compression and unity check

10.4.4 Conclusion

The results show that the three designs are non-linear stable. FEM analyses showed that the structures deform and merge to equilibrium. The most important results are presented in table 10.8. Concerning the phase of the design process, it is not possible to come to definitive conclusions. The reason is that the used profiles are roughly estimated using reference projects. The conclusions described here are temporary.

The results show that variant 1 has the lowest dead load value, followed by variant 2 and 3. The reason variant 3 has the highest dead load, is the great amount of elements used in the structure. Variant 1 and 2 almost consist of an equal amount of elements. However, variant 1 has a lower dead load. In the following design step, the right dimensions of every single element will be determined. As a consequence the dead load of the whole structure of variant 1, 2 and 3 will be very different.

Although no definitive conclusions can be made, one can conclude that variant 1 is very efficient. Variant 1 has a lower dead load and a lower additional deformation compared to variant 2. Meaning that the structure can transfer the loads more efficient by using the available ring action better. The following step in the design process will show which variant is most efficient.

Results Structural Design Variants			
Properties	**Variant 1**	**Variant 2**	**Variant 3**
Nodes	400	400	400
Bars	1174	1158	1382
Total dead load	41933,74 kN	47219,15kN	53612,78 kN
Average dead load	190,96 kg/m²	215,02 kg/m²	244,14 kg/m²
Max. deformation	826,40 mm	1029,50 mm	663,20 mm
Max. add. Deformation	195,60 mm	268,40 mm	130,90 mm

Table 10.8 Conclusion results variant 1-3

For the study of the use of tensile-compression ring roof structures for football stadia it is interesting to compare the results from the research with reference projects. The Feyenoord stadium in Rotterdam also has a spoke wheel roof made of a spatial truss system. The average dead loads of the three variants are compared in figure 10.27 with the average dead load of the Feyenoord stadium (90 kg/m²).
One must keep in mind that the advantage of the Feyenoord stadium is that it has more curvature compared to the ground plan of the reference stadium. Besides, the span of the roof of the Feyenoord stadium is smaller compared to the reference stadium (26,00m to 42,50m).
In the following phase the three variants will be further optimized by using parametric modelling.

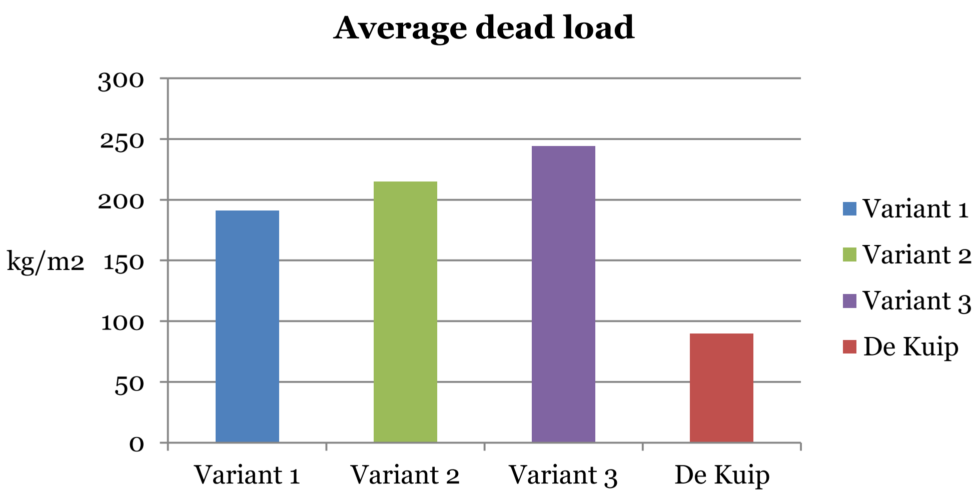


Figure 10.27 Average dead load variant 1-3 after step 1

10.5 Optimization

The results from the design variants show that there is room for improvement. In this paragraph the design will be optimized by using parametric modelling.

Parametric Modelling

The structural design can be further optimized by adapting the profile of the elements. Using parametric modelling it is possible to determine the most suitable profiles to come to an efficient design.

The parametric modelling process consists of multiple (iterative) steps to come to a full optimized design. These steps are briefly described:

- *Check of the original structure*
  The first step is the determination of the steel check of the structure with the original profiles. This step has been done in the previous paragraph. The steel check shows if the chosen profile is sufficient and if there is room for improvement. When the steel check has a low value, a smaller less stiffer/stronger profile can be used. Otherwise, when the steel check is greater than 1,00 a stiffer and stronger profile must be applied.

- *Determination of the diameter*
  The second step is to determine the diameter of the profiles. The structural elements used in the structure, are divided into groups which must have the same diameter. The element in the group that is subjected to the greatest loads, is leading in the determination of the diameter. This element will have the smallest diameter/thickness ratio in that group. The rest of the group element members will have the same diameter, only a smaller thickness.
  The choice of a profile for a group of members has a major impact on the behaviour of the structure. When lightweight profiles are used, the deformation, forces, moments, etc. will become different due to the second order effect of the structure. As a consequence the steel check will become different. The determination of the right profile for a group of elements is an iterative process. The goal of this step is provide a unity check value of around 0,90-1,00 for the leading elements in the different groups.

- *Determination of the thickness*
  After the determination of the diameter of the profile for the elements, the final step is the determination of the thickness of all single elements in the structure. With the help of the FEM program Scia Engineer, an iterative calculation process can be done to determine the right thickness for every single element. The diameter, loads and geometry are fixed parameters in this stage of the optimization process. After an amount of calculation steps the model will find equilibrium and the most possible (cost) efficient structural design is made for a certain design.

For every iteration step a new profile must be chosen based on the unity check. Not only the unity check play a role in the determination of the right profile for a group of elements. The choice of a profile for a group of elements also depends on the following considerations:

- High diameter/thickness ratio is preferred, especially for bars subjected to compression, to provide a high buckling load factor. More material will be saved compared to low diameter/thickness ratio profiles in case of equal loads.
- The profiles need to have a cross section class of 1-3. Class 4 has been left out of consideration to prevent shear buckling. The Eurocode prescribes that the maximum diameter/thickness ratio for steel grade S235 and cross section class 1 is 50, cross section class 2 70 and for cross section class 3 90.
- Requirements prescribe that continuous beams need to have a larger diameter compared to other non-continuous beams that are connected to the same node. This is to prevent welding problems.
- The thickness range of a profile lies between 5 mm and 40 mm. These thicknesses are most common and are financially attractive.

As mentioned, a high diameter/thickness ratio is preferred for compression bars. This will be further explained.
The stability requirements are leading for bars subjected to compression. To increase the stability capacity the bars need to have a, next to sufficient strength $(N_{b,Rd})$, a high buckling factor. A high buckling factor can be achieved, when the relative slenderness has a low value. The formula for the relative slenderness is as follows:

$$\bar{\lambda} = \sqrt{\frac{\mathrm{Af_y}}{\mathrm{N_{cr}}}} \qquad (10.1)$$

Where

$$N_{cr} = \frac{\pi^2 E_d I_y}{l_{buc}^2} \qquad (10.2)$$

The buckling factor depends on the variable parameters in equation 10.1 and 10.2. The variable parameters depending on the profile are the area A and the moment of inertia I.

The expression for the area A and the moment of inertia I of a hollow tube (CHS) are as follows:

$$A = \pi(r_1^2 - r_2^2) \tag{10.3}$$

$$I_{yy} = I_{zz} = \frac{\pi}{4}(r_1^4 - r_2^4) \tag{10.4}$$

Where r_1 is the radius to the outer perimeter of the tube and r_2 the radius until the inner perimeter of the tube.

Equation 10.3 and 10.4 show that with an increasing radius, assuming the thickness ($t = r_1 - r_2$) remains constant, the ratio $I_{yy}\!:A$ increases as well. This results in a higher buckling load. When a profile must be determined for a bar subjected to compression, it is more efficient to choose for a profile with a high diameter/thickness ratio compared to a low diameter/thickness ratio. This will be illustrated with an example below.

Assume two hollow tube profiles with equal cross sectional area and different diameter/thickness ratio in table 10.9. For the buckling length is a length of 12,00m assumed and steel grade S235.

Profile	**CHS 244,5*20**	**CHS 457*10**
Diameter/thickness ratio	1:12,23	1:45,7
Area A	14105 mm^2	14049 mm^2
Moment of Inertia I_{yy}	89,57*10^6 mm^4	351,4*10^6 mm^4
Rel. Slenderness λ_{rel}	1,60	0,01
Buckling factor ω_{buc}	0,27 (curve c)	1,00 (curve c)

Table 10.9 Diameter/thickness ratio

For bars that are subjected to tension instead of compression, the strength requirement will be leading. For the determination of the right profile one only have to take the cross sectional area into consideration (equation 10.3). To decrease the amount of material, a CHS profile with the smallest possible area must be chosen.

Together with the other two mentioned considerations the right profile must be chosen for the groups of elements in order to come to a correct design.

10.5.1 Variant 1

The structure is divided in different groups with elements that have the same structural characteristics. The groups are equal to the element numbers in figure 10.8 and 10.9 and table 10.2. An important requirement is that continuous beams need to have a larger diameter compared to non-continuous beams. This means that the outer and inner rings need to have the largest diameter (element nr. 1, 2 and 3). The spokes in the structure will have the second largest diameter (element nr. 4 and 5) followed by the rest of the elements.

In the first step, the original structure has been calculated and checked. The highest UC values of strength, stiffness or stability for load combination 2, 3 and 4 for the leading element in a group are presented in table 10.10. Using the unity check a new profile (with a possible new diameter) can be determined. After three iterative steps the final diameter of a profile for a group of elements will be determined.

Determination of the diameter

A new profile is determined using the unity check and the considerations mentioned earlier. The goal is to provide a unity check of around 0,90-1,00. A quick way to determine a new profile is explained using the following example:

Original profile : 406,4*28 (A = 33285 mm²), UC = 0,75

Needed area : $\frac{0,75}{0,90} * 33285mm^2 = 27737,5\ mm^2$

New profile : 406,4*25 (A = 29955 mm²), UC = 0,82

When bending moments play a leading role, instead of the value for the area, the moment of section W_{Ed} must be used to determine a new profile for the following iterative step.

In table 10.10 the iterative steps are presented with the chosen profile and its UC values. The diameters used in step 3 are the final diameter sizes that will be used for the referring elements.

	Iterative step 1			Iterative step 2			Iterative step 3		
Nr.	**CHS**	ω_{buc}	**UC**	**CHS**	ω_{buc}	**UC**	**CHS**	ω_{buc}	**UC**
1	864*40	0,97	0,95	864*40	0,97	0,94	864*40	0,97	0,98
2	762*36	0,97	0,91	762*36	0,97	0,91	762*36	0,97	0,93
3	508*32	1,00	0,88	508*32	1,00	0,87	559*28	1,00	0,91
4	457*20	0,70	0,93	508*17,5	0,77	0,85	508*17,5	0,78	0,92
5	457*20	0,71	0,61	508*12,5	0,78	0,72	559*8,8	0,83	0,88
6	457*30	0,47	**1,34**	508*25	0,56	**1,07**	508*32	0,55	0,92
7	457*36	0,44	**1,83**	508*36	0,52	**1,17**	559*40	0,59	0,87
8	457*22,2	0,78	0,85	457*22,2	0,78	0,85	457*20	0,78	0,90
9	457*22,2	0,83	0,67	457*20	0,83	0,71	457*20	0,83	0,73
10	406,4*20	0,80	0,56	406,4*12,5	0,81	0,63	355,6*10	0,74	0,87
11	406,4*20	0,71	0,55	406,4*12,5	0,79	0,75	355,6*17,5	0,64	0,81
12	508*25	0,71	0,24	406,4*10	0,57	0,85	406,4*10	0,57	0,89
13	508*25	0,81	0,31	406,4*12,5	0,71	0,85	406,4*12,5	0,71	0,88
14	273*14,2	0,56	0,43	244,5*8	049	0,92	219,1*11	0,40	0,95
15	219,1*11,1	0,84	0,17	168,3*5	0,72	0,49	127*5	0,51	0,98
16	244,5*14,2	0,46	0,65	244,5*11,1	0,46	0,78	219,1*12,5	0,37	0,82
17	193,7*10	0,46	0,80	193,7*10	0,43	0,77	193,7*10	0,41	0,79

Table 10.10 Highest unity check values step 1 to 3 of variant 1

In figure 10.28 and 10.29 the element numbers in the spoke wheel roof of variant 1 are illustrated again.

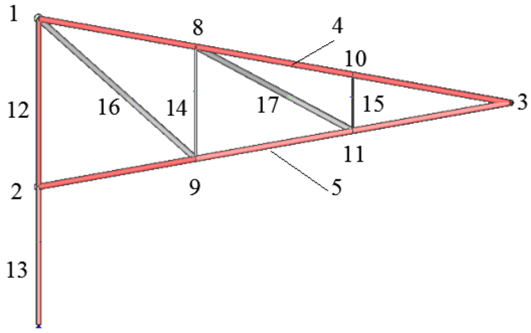


Figure 10.28 Cross section variant 1

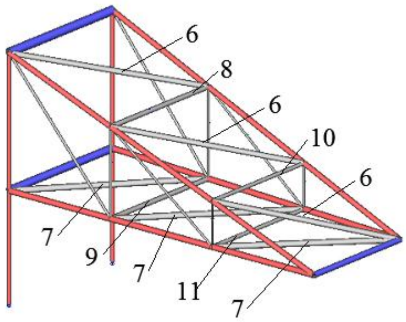


Figure 10.29 Spoke framework variant 1

Determination of the thickness

After the third iteration step the diameter of the elements for every group are known. The following step is to determine the thickness of every single element. This optimization step is performed by the FEM program Scia Engineer.
The leading elements of every group consist of the profiles presented in step 3 of table 10.10. The other elements in the same group will have an equal diameter. The thickness however is variable and can have a lower value than the leading element of the specific group. The calculation results of the final structure of variant 1 are presented in the additional appendix; FEM calculation results. In the results only the unity check value regarding the stability and strength are presented. The stiffness requirements are not leading for the elements.

The final structure has an average dead load of 79,90 kg/m², a total deformation of 1015,1 mm and additional deformation (snow as leading load) of 374,80 mm at the centre of the long side at the inner ring. The structure fulfils all requirements regarding stiffness, strength and stability.
In figure 10.30 the average dead load for each step is presented. The first three steps are the three iterative steps for the determination of the diameter. Step 4 is the final step for the determination of the thickness of every single element with the FEM program Scia Engineer.

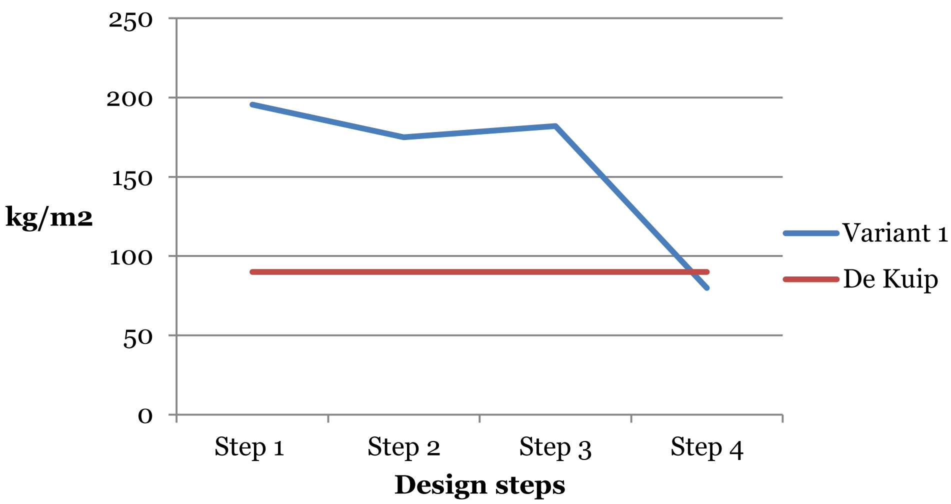


Figure 10.30 Average dead lead variant 1 step 1-4

10.5.2 Variant 2

The optimization process of variant 2 is equal to variant 1. At first the diameter of the elements for every group is determined by three iterative steps. Followed by the determination of the thickness of every element.

Determination of the diameter

In table 10.11 the iterative steps are presented. The chosen profile in step 3 is the final diameter size of the elements.

	Iterative step 1			Iterative step 2			Iterative step 3		
Nr.	**CHS**	ω_{buc}	**UC**	**CHS**	ω_{buc}	**UC**	**CHS**	ω_{buc}	**UC**
1	914*40	0,97	0,97	914*40	0,97	0,76	813*40	0,96	0,86
2	762*36	0,95	**1,29**	864*40	0,95	0,82	813*40	0,96	0,87
3	559*20	1,00	0,51	559*6,3	1,00	0,63	508*6,3	1,00	0,76
4	559*16	0,82	0,36	559*6,3	0,83	0,73	508*6,3	0,84	0,70
5	559*16	0,80	0,66	559*6,3	0,85	**1,04**	508*8,8	0,76	0,82
6	559*20	0,50	0,70	508*16	0,43	0,91	508*17,5	0,43	0,92
7	508*20	0,52	0,91	508*25	0,50	**1,07**	508*30	0,52	0,86
8	508*20	0,52	0,91	508*17,5	0,52	0,75	508*12,5	0,53	0,88
9	559*25	0,85	**1,32**	559*40	0,84	0,83	508*40	0,80	0,95
10	559*25	0,89	0,83	559*17,5	0,83	0,75	508*16	0,82	0,83
11	559*25	0,85	0,34	323,9*10	0,55	0,47	244,5*10	0,34	0,89
12	559*25	0,85	**1,31**	559*36	0,85	0,77	508*22,2	0,82	0,77
13	508*20	0,72	0,40	406,4*12,5	0,56	0,83	406,4*12,5	0,56	0,84
14	508*20	0,82	0,46	406,4*14,2	0,70	0,84	406,4*14,2	0,70	0,85
15	355,6*16	0,74	0,37	298,5*10	0,64	0,61	244,5*10	0,48	0,91
16	219,1*10	0,84	0,13	101,6*5	0,35	0,31	101,6*5	0,35	0,29

Table 10.11 Highest unity check values step 1 to 3 of variant 2

In figure 10.31 and 10.32 the element numbers in the spoke wheel roof of variant 1 are illustrated again.

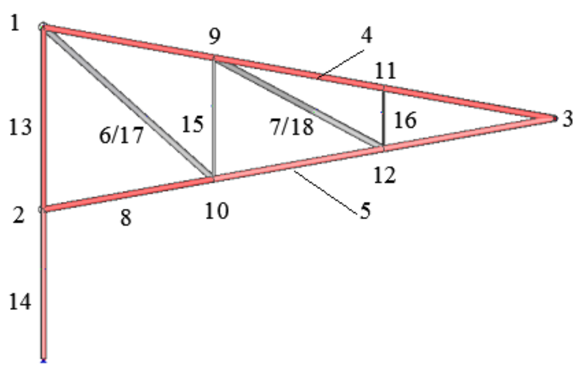


Figure 10.31 Cross section variant 2

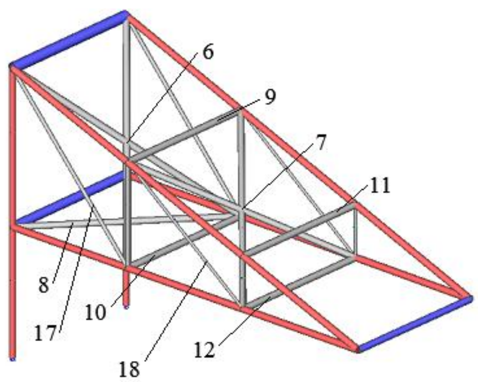


Figure 10.32 Spoke framework variant 2

Determination of the thickness

The calculation results of the final structure of variant 1 are presented in the additional appendix; FEM calculation results. In the results only the unity check value regarding the stability and strength are presented. The stiffness requirements are not leading for the elements.

The final structure has an average dead load of 83,70 kg/m², a total deformation of 1111,00 mm and an additional deformation of 439,60 mm at the centre of the long side at the inner ring. The structure fulfils all requirements regarding stiffness, strength and stability. In figure 10.33 the average dead load for each step is presented.

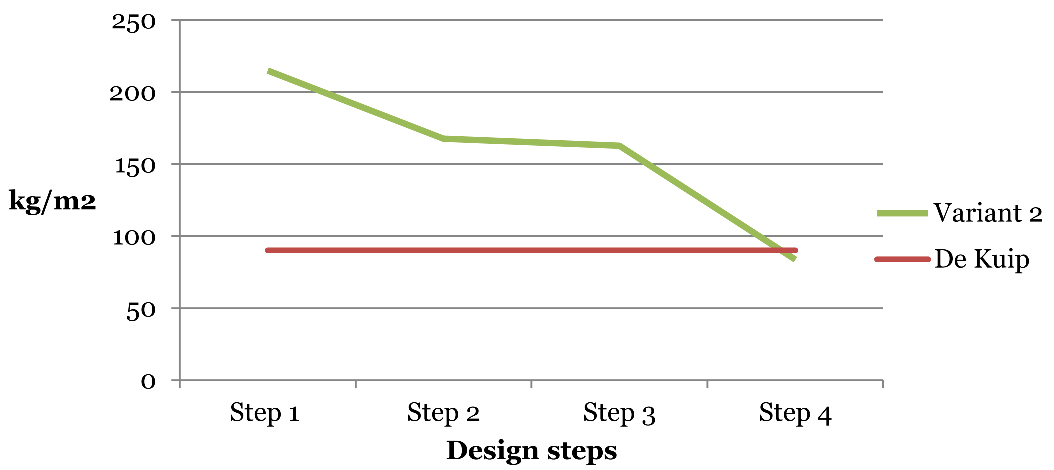


Figure 10.33 Average dead lead variant 2 step 1-4

10.5.3 Variant 3

The optimization process of variant 3 is equal to variant 1 and 2. At first the diameter of the elements for every group is determined by three iterative steps. Followed by the determination of the thickness of every element.

Determination of the diameter

In table 10.12 the iterative steps are presented. The chosen profile in step 3 is the final diameter size of the elements.

	Iterative step 1			Iterative step 2			Iterative step 3		
Nr.	**CHS**	ω_{buc}	**UC**	**CHS**	ω_{buc}	**UC**	**CHS**	ω_{buc}	**UC**
1	914*40	0,97	0,96	914*40	0,97	0,87	914*40	0,97	0,89
2	762*36	0,97	0,97	762*36	0,97	0,90	762*36	0,97	0,92
3	508*32	1,00	0,15	457*10	1,00	0,31	457*10	1,00	0,31
4	508*20	0,77	0,37	457*10	0,74	0,50	457*10	0,74	0,52
5	508*20	0,79	0,68	457*16	0,70	0,86	457*16	0,70	0,87
6	508*20	0,55	**1,00**	457*22,2	0,46	0,93	457*22,2	0,46	0,93
7	457*17,5	0,46	0,69	457*10	0,47	**1,01**	457*12,5	0,47	0,87
8	508*20	0,57	0,89	457*20	0,48	0,97	457*22,2	0,48	0,94
9	457*17,5	0,47	0,92	457*17,5	0,47	0,88	457*17,5	0,47	0,88
10	508*20	0,58	0,84	457*17,5	0,51	**1,10**	457*22,2	0,50	0,94
11	457*17,5	0,49	**1,33**	457*25	0,48	0,94	457*25	0,48	0,96
12	457*22,2	0,83	0,83	406,4*20	0,78	0,90	406,4*20	0,78	0,91
13	457*22,2	0,83	0,71	355,6*17,5	0,61	**1,42**	406,4*17,5	0,70	0,83
14	355,6*17,5	0,86	0,51	273*10	0,56	0,71	244,5*10	0,34	0,84
15	355,6*17,5	0,64	0,68	323,9*16	0,53	0,93	323,9*16	0,53	0,93
16	508*20	0,72	0,35	406,4*10	0,57	0,91	406,4*10	0,57	0,94
17	508*20	0,82	0,44	406,4*12,5	0,71	0,88	406,4*12,5	0,71	0,91
18	273*14,2	0,56	0,47	219,1*11	0,40	0,81	219,1*10	0,40	0,90
19	193,7*10	0,79	0,23	114,3*5	0,43	0,89	127*5	0,43	0,75
20	298,5*12,5	0,61	0,64	193,7*12,5	0,31	0,80	193,7*11	0,31	0,93
21	219,1*10	0,50	0,70	168,3*10	0,32	0,80	168,3*8,8	0,32	0,91

Table 10.12 Highest unity check values step 1 to 3 of variant 3

In figure 10.34 and 10.35 the element numbers in the spoke wheel roof of variant 1 are illustrated again.

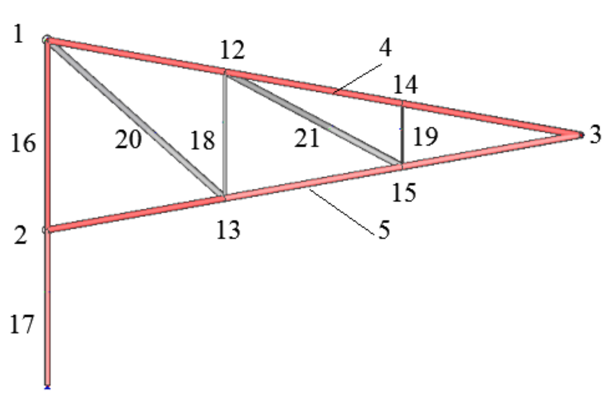


Figure 10.34 Cross section variant 3

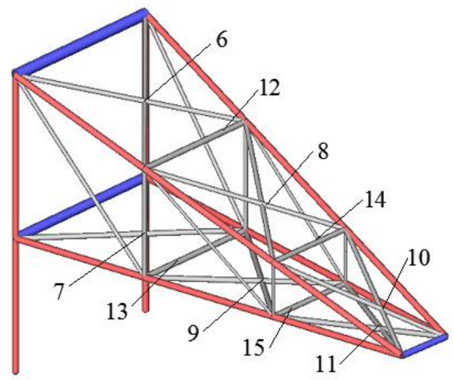


Figure 10.35 Spoke framework variant 3

Determination of the thickness

The final structure has an average dead load of 92,61 kg/m², a total deformation of 749,20 mm and an additional deformation of 260,30 mm at the centre of the long side at the inner ring. The structure fulfils all requirements regarding stiffness, strength and stability. In figure 10.36 the average dead load for each step is presented. The calculation results of the final structure of variant 1 are presented in the additional appendix; FEM calculation results. In the results only the unity check value regarding the stability and strength are presented. The stiffness requirements are not leading for the elements.

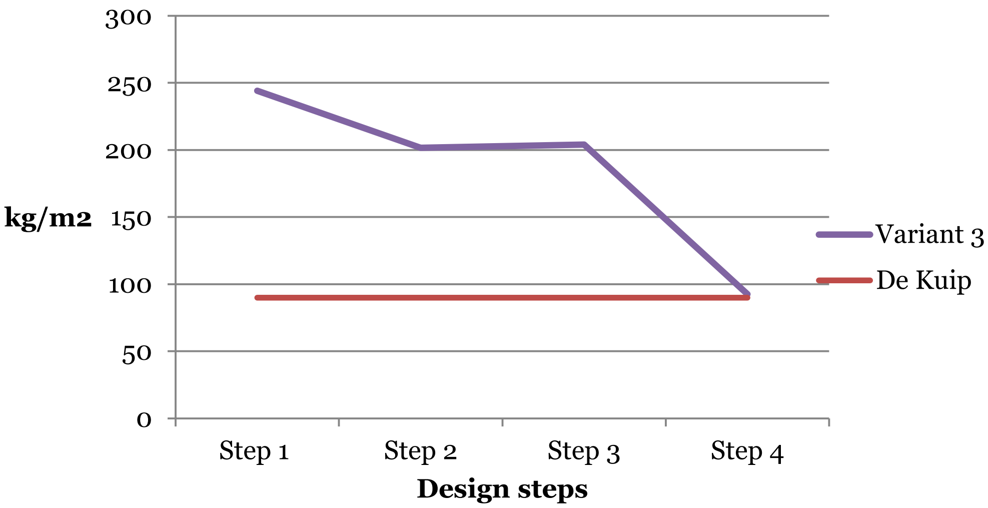


Figure 10.36 Average dead lead variant 3 step 1-4

10.5.4 Conclusion

After the optimization of the three truss variants an answer has been given to sub-question 6 which is as follows:

'Is it possible to further optimize the structural design?'

By adapting the geometry of the three designs and using parametric modelling it was possible to further optimize the structural design. Due to the optimization, the total dead load of the structure have decreased. It can be concluded that truss structure variant 1 uses the least amount of steel to come to a design that fulfils all structural requirements. The average dead load for every step of all variants is summarized in figure 10.37 and 10.38. The designs are compared with the average dead load of the roof of a reference stadium; Feyenoord stadium, also called 'De Kuip'. The roof of the Feyenoord stadium consists of a spatial truss (chapter 3) and is therefore well comparable.
The roof of the Feyenoord stadium has an average dead load of around 90 kg/m². After the optimization can be concluded that variant 1 and 2 has a lower average dead load compared to the Feyenoord stadium and are therefore more efficient. Although there is a span difference between the Feyenoord stadium and the truss structures, (respectively 26,00m and 42,50m) it is possible to come to a more efficient design using the latest engineering possibilities.

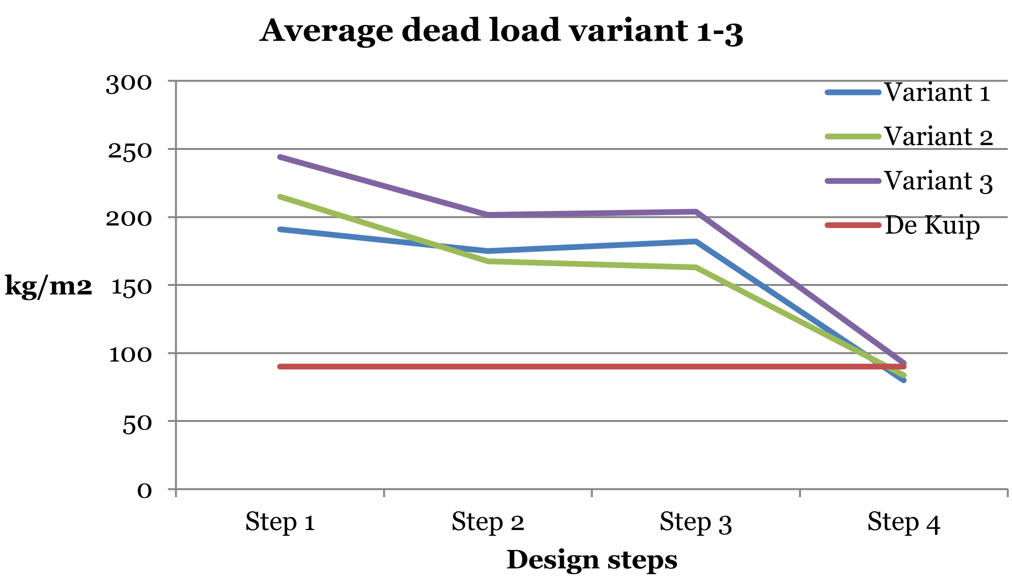


Figure 10.37 Average dead load variant 1-3 for step 1-4

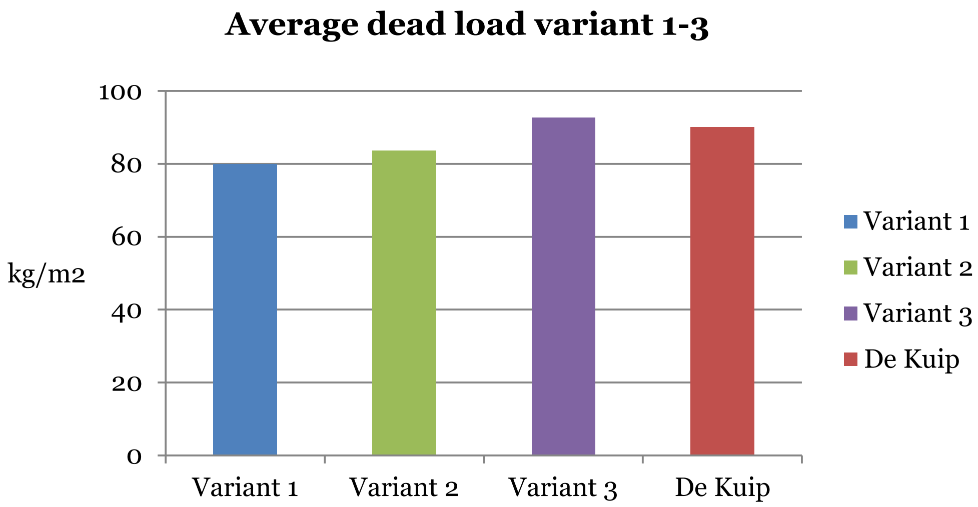


Figure 10.38 Average dead load variants 1-3 after step 4

With the help of Scia engineer it is possible to calculate the right profile for every single element and still fulfil all structural requirements. By using parametric modelling it is therefore possible to design a roof with a larger span and still have a lower average dead load. The disadvantage of using elements that have different thicknesses/diameter is that the construction costs are higher. By using unique elements, the standardization decreases, which results in higher construction costs. When is assumed that the costs are directly proportional to the amount of used material, one can conclude that variant 1 and 2 are more cost effective than the Feyenoord stadium.

From the results has been concluded that variant 1 has the lowest average dead load. Although the difference between the average dead load of variant 1 and 2 is very little, one can conclude that variant 1 is the most efficient structure. The total and additional deformation of variant 1 (1015,10 mm resp. 374,80 mm) compared to variant 2 (1111,00 mm resp. 439,60 mm) also show that the first variant transfer the forces more efficient and therefore needs less material. Variant 3 possess the highest stiffness and strength and therefore deforms less compared to variant 1 and 2. However, variant 3 is not very efficient in terms of material use. It lends its stiffness and strength due to high amount of used elements and material.

The reason why variant 1 and 2 are close is the way both structures transfer the vertical and horizontal loads.
For variant 1 has been tried to use the available amount of ring action optimal, variant 2 takes up the load partly by ring and beam action.
Variant 1 uses the available amount of ring action optimally, by directly transferring the loads to the curved outer ring elements. The weakest point of the stadium is at the centre of the long span. By supporting this point by transferring the loads to the curved outer ring, less material is needed to fulfil the structural requirements. The disadvantage of variant 1 is that the amount of available ring action is limited, especially due to the lack of curvature on the sides of the roof.
Another disadvantage is that the slope of the diagonals and spokes that take up the vertical loads is small ($\ll 45^{\circ}$) compared to the second variant. In variant 2, the vertical cross diagonals are placed at a slope of almost 45° and are therefore more able to take up the (leading) vertical loads directly. Due to these vertical crosses, variant 2 has more beam action capacity. Although the second variant uses less ring action, due to the available beam action the results differences with the first variant is little.

10.6 Structural resilience

When a design is made of a roof structure, the engineer has to prove the structure has sufficient structural resilience. For instance: due to an incidental load (fire, earthquake) or just bad assembly a part of the roof structure can collapse as a consequence of local damage. In case of a stadium roof, the roof structure must possess sufficient strength and stiffness (resilience) to prevent the roof from complete collapse.
For the three design variants some crucial elements will be left out of the structure to check the structural resilience, these are the following: the corner element of the inner tension ring (1), the central element on the long side of the outer tension (2) and compression ring (3).

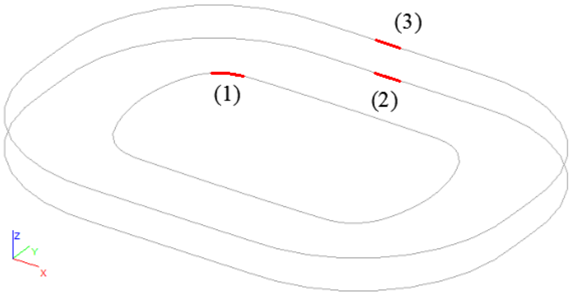


Figure 10.39 Place of local damage

For the calculations different design factors are used compared to regular live or permanent loads. The reason is that the probability of occurrence of an incidental load simultaneously with the occurrence of the maximum value of other live loads is negligible. Therefore, in case of incidental loads the design live load may be reduced [16].
The Eurocode (NEN-EN 1991-1-7) prescribes that for special load combinations, the load factor γ is set to 1,0 and for live loads an instantaneous value is used which is obtained by multiplying with a momentary factor ψ. For roof structures, the momentary factor is zero. For the calculations only the permanent load of the roof structure with a load factor of 1,00 is taken into account.

10.6.1 Variant 1

Variant 1 is a special design that is completely concentrated to use the ring action at full extend. The structure is only symmetrical in one direction. As a consequence the structure is not always able to redistribute all forces in case of local damage.
The largest ring action arises in the corners of the two outer rings. When a corner element of inner tension is damaged, this has little influence on the stiffness of the roof structure. The amount of deformation shows that (table 10.13). On the contrary, a damaged outer tension or compression ring element is crucial for the structure. The outer ring elements provide a large part of the total stiffness and strength of the roof structure. When a local outer ring element on the long side is damaged, the structure becomes unstable. This will be explained using figure 10.40.

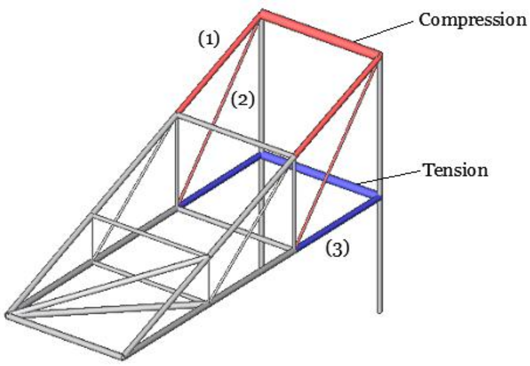


Figure 10.40 Cross section spoke framework centre long side variant 1

In the top outer ring large compression forces arise. Due to the lack of curvature at the sides of the structure, the top outer ring will radially translate to the outside of the structure. When a top outer ring element is damaged, the radial translation will increase greatly. The adjacent elements (number 1 and 2 in figure 10.40) need to prevent this. The adjacent elements need to possess sufficient strength and stiffness to withstand the extreme tensile forces and to transfer the forces to the rest of the structure. These elements however, are not able to do so and the structure becomes unstable.
The same will happen when a part of the bottom outer tensile ring is damaged. The structure wants to radially translate to the inside. The adjacent elements (3) do not have sufficient strength to withstand the high compression forces.

Place of local damage	Highest deformation [mm]	
	Original structure	*Structural resilience*
1	642,90	643,10
2	642,90	unstable
3	642,90	unstable

Table 10.13 Deformation of the inner ring of variant 1 for different local damages

10.6.2 Variant 2

The second variant is, unlike variant 2, double symmetric and so is the distribution of the loads. When there is a local damage, the structure is more able to distribute the force efficiently compared to variant 1.

As mentioned before, the inner ring only provides a small part of the total stiffness and strength of the roof structure. When a key element of the ring is damaged, for instance a corner inner ring element, the structure is able to take up the loads from the inner ring and prevents the roof from collapse.
The outer rings provide a great substantial part of the total stiffness of the roof structure. The weakest point of the roof structure is at the long side. A damaged part at this point of the outer ring is leading for the determination of the structural resilience.
Calculation results (table 10.14) show that the structure remains stable when a part of the outer compression ring is damaged. The adjacent elements (1 and 2) possess sufficient strength and stiffness to withstand the high tensile forces, to distribute the forces to the rest of the structure and to prevent large deformations.
The adjacent elements of the bottom tensile ring (3 and 4) are however not sufficient. The structure becomes unstable. The cross diagonals in the top plane (2) provide more stiffness compared to (3) due to the greater slope the cross diagonals are placed. Therefore these elements are more able to take over the loads from the top ring and redistribute it to the rest of the structure.

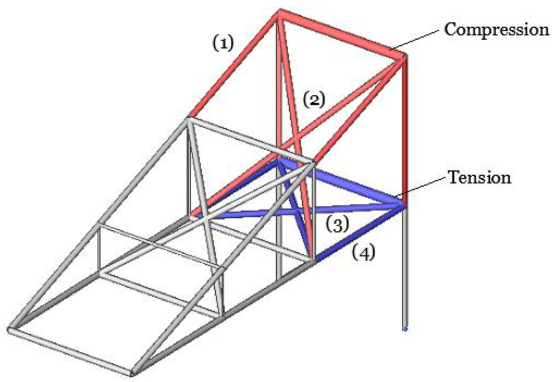


Figure 10.41 Cross section spoke framework centre long side variant 2

Place of local damage	Highest deformation [mm]	
	Original structure	*Structural resilience*
1	679,60	679,70
2	679,60	unstable
3	679,60	1276,30

Table 10.14 Deformation of the inner ring of variant 2 for different local damages

10.6.3 Variant 3

The third and last variant is, like variant 2, also double symmetric. The influence of a damaged inner ring element is again little. Compared to the second variant, the roof structure in the third variant is still stable when an outer tension ring element is damaged. Because diagonal crosses are present in the complete lower spoke plane, the structure is able to redistribute the forces from the outer tension ring through the lower diagonal crosses.
The third variant is however not able to provide stability when an outer compression ring is damaged. The reason is that the top crosses do not provide sufficient strength and stiffness to take over the forces from the outer compression ring. Variant 2 was able to take over these forces, because the adjacent crosses possess more (transverse) stiffness and strength due to the greater slope of the cross diagonals.

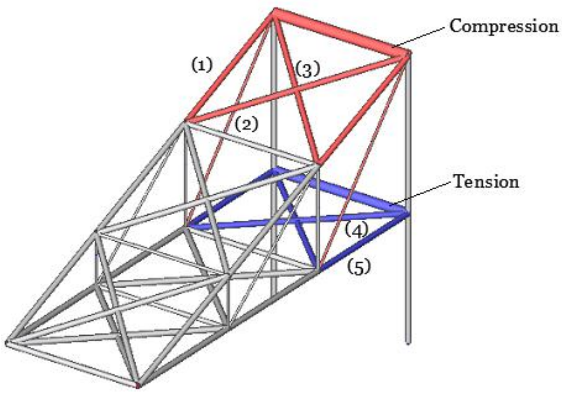


Figure 10.42 Cross section spoke framework centre long side variant 3

Place of local damage	Highest deformation [mm]	
	Original structure	*Structural resilience*
1	490,20	490,20
2	490,20	640,30
3	490,20	unstable

Table 10.15 Deformation of the inner ring of variant 3 for different local damages

10.6.4 Conclusion

The results show that none of the variants is able to remain stable when certain crucial elements are damaged. The inner ring provides only a small part of the total stiffness of the roof structure. When a corner ring element is damaged, the three variants are able to remain stable. The real crucial elements are the central outer ring elements at the long side of the structures. The reason is the lack of curvature at these points. The outer ring elements consist of CHS profile with a large cross sectional area to withstand the high radial forces. When a part is damaged, large radial deformations will arise. The adjacent elements need to take up these loads and are not all stiff and strong enough to stabilize the structure. When the compression ring is damaged, variant 1 and 3 will become unstable; in case of the tensile ring variant 1 and 2 will collapse.

One of the reasons that the elements are not strong enough is because the three variants have been optimized in order to use as little material as possible. The elements in the structures are sufficiently strong and stiff to fulfil all structural requirements. However, the optimization has led to a decrease of structural resilience. When a part of the structure is damaged, the structure still might be able to take up the extra loads. But when a crucial element (on the long sides of the outer ring) is damaged, this has great consequences for the roof structure.

To provide structural resilience there are two options: using stronger elements or adding extra elements.
When is chosen to add more elements, there are some possibilities to guarantee stability in case of damaged critical elements. To prevent an unstable structure in case of a damaged compression outer ring element, extra vertical crosses need to be added (element nr. 2 of variant 2 in figure 10.41). To stabilize the structure when a tensile outer ring element is damaged, extra diagonal crosses need to be applied in the bottom spoke level, see figure 10.30. Using the strengths of the variant 2 and 3 a structure can be suggested that will remain stable in all situations. The suggested structure is illustrated in figure 10.43.
The top outer ring is mainly supported by the vertical crosses (3) and the bottom outer ring by the horizontal crosses (4) in the plane of the bottom spoke level.
For an engineer it is almost impossible to come to an efficient design that has sufficient structural resilience. To prevent a structure from heavy deformations when certain elements are damaged, a lot of extra material is

needed to provide sufficient stiffness or strength. The engineer has the option to use stronger elements or use more elements for extra transfer of loads possibilities.
To come to a resilient and efficient structure more research is needed. The challenge is to provide enough strength and stiffness when a part of the structure is damaged and still use as little material as needed.

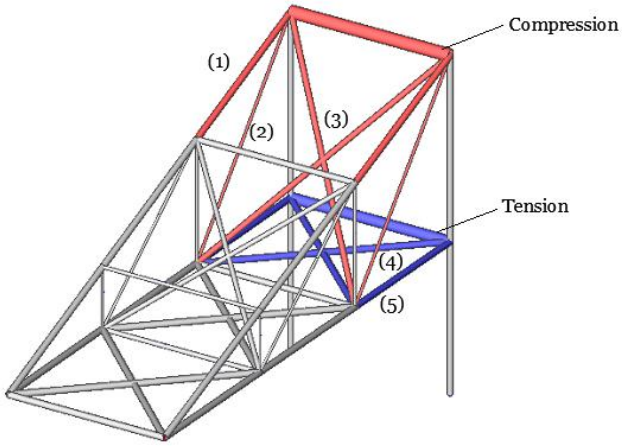


Figure 10.43 Suggested structure to provide sufficient structural resilience

10.7 Connections

For the design of a spatial truss structure, the type of connection is important in the design process. For the choice of the type of node the engineer has to take into account the influence on the way of transport, assembly, prefabrication, finance, aesthetics, etc. The most suitable connection need to be a combination of easy assembly, low costs, easy transport and still have a certain aesthetic level. The possible types of connections that can be applied to the truss design will be briefly discussed.

10.7.1 Welded joints

Welded joints find its application in fixed connections. A good welded joint is stronger than the mother material and therefore well suited as a fixed connection. Welding of CHS profiles require a pre-treatment of the ends of the tubes. For CHS profiles a special shaped tubular end is needed to fit on the following continuous CHS profile. For a T-joint a simple semicircle is sufficient. For other type of joints (K, N, Y), the connections become more complex [5].
Besides the good structural properties of welded connections, it also has its disadvantages. It is not always possible to weld in a work area where the weather conditions do not play a role. Welding on the building site is costly. Sometimes temporary facilities need to be made to weld under safe conditions. Also extra costs can arise for the assembly of temporary bolted joints, inspection and possible reparation.

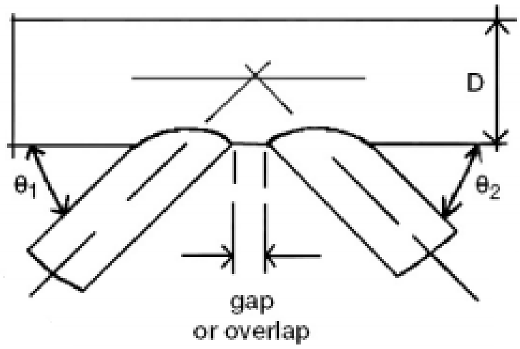


Figure 10.44 Design variables weld connection

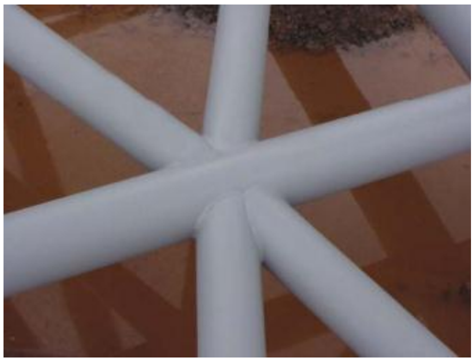


Figure 10.45 Weld connection

10.7.2 Bolted connections

The most common connection between tube components (CHS profile) and node elements is through a bolt connection. The bolt can be directly connected to the tube or by an intermediary node. For three dimensional structures, like the truss design, separate nodes are needed. The roof of a stadium is loaded by dynamic wind forces and therefore tensile as well as compression forces can arise in the connections.

For general buildings 'normal' holes are most commonly used. The holes are wider than the diameter of the bolts. The bolt hole tolerance is about 2 mm. This tolerance has its disadvantages. Due to the tolerance the connection can slip and the bolts can come loose and the connection can rust and become less strong. The dynamic loads will loosen the bolts and therefore the application of normal bolt connections with hole tolerances is not allowed.

For spatial truss systems, like the truss design for the stadium roof, various types of bolt connections are possible. The following companies are specialized in connections for spatial truss systems: Mero (figure 10.41), Merodeck, Montan, Nodus, Octatube (figure 10.40), Tuball, Unibat and Zublin [4].
The tube components and nodes are made in the factory. In the factory the tubes are protected, coated and after transport assembled on the building site. The concept of the bolt connection supports mass production of the components for structures, which leads to standardisation and financial benefits.

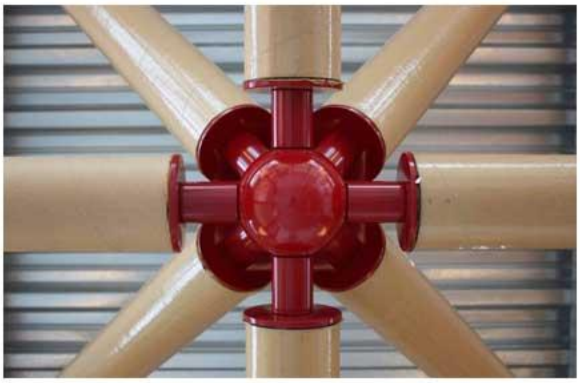


Figure 10.47 Octatube connection

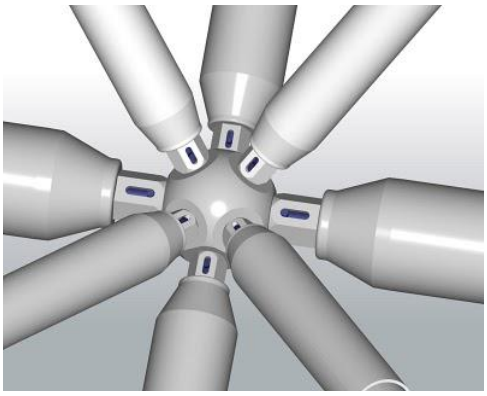


Figure 10.46 Mero connection

10.7.3 Conclusion

The truss design is a large spatial truss system. A bolted connection similar to the Octatube/Mero system is most suitable for the structure. The great advantages and its structural abilities make this type of connection the best solution for the truss design.

A large part of the truss design consists of hinged connections. The only exceptions are the fixed connections between the inner ring elements and outer ring elements. For the hinged connection a bolted connection is needed to provide a certain rotational freedom. A regular bolt-steel plate connection is not suitable for the truss design. Besides its low aesthetical value, this type of connection is not suitable for stadia roofs which are subjected to dynamic loads.
For the hinged connections a bolt connection like the Octatube and Mero system must be applied. Next to its aesthetical and structural advantages, it has great financial and construction advantages. The connections are easy to produce and assemble. Due to the possibility of standardization it has great financial benefits.

The complexity of the nodes has a great influence on the building costs of a design. The more bars that are connected to a single node, the greater the complexity and the higher the costs. Variant 2 has the least amount of complex nodes compared to the other two variants.
Hinged connections using the Octatube or Mero system are very suitable for complex nodes. The assembly is easier compared to complex welded nodes and are cheaper to realize.

Steel structural joints	**Advantages**	**Disadvantages**
Welded joints	• No holes, no loss of strength of the material and structure • Very high aesthetical value • Very strong, stiff connection. • Light connection • Easy to clean	• Extra costs and assembly for temporary bolt connections • Possible extra costs for inspection and reparation • Bad weather conditions can hold up the assembly, temporary facilities needed. • Possibility of shrinkage, occurrence of fatigue behaviour when loaded dynamically.
Bolted connections	• Wide application range (fixed, hinged connections) • Speed of erection • Easy and cheap to realize. • Special bolts can take up dynamic loading. • High aesthetical value	• No bolt hole tolerance allowed, due to dynamic wind loading • Use of pre-stressed, injection or other special types of bolts are more expensive than regular bolts.

Table 10.16 (Dis-)advantages welded and bolted joints

10.8 Conclusion

The basis of the three investigated truss designs is the preliminary design. From chapter 8, preliminary design, has been concluded that the design only has little stiffness and strength. The largest deformations arise at the centre of the long side of the inner ring. These deformations are that substantial that the FEM program cannot find equilibrium. The preliminary design appears to be non-linear unstable. The challenge was to design an efficient and stable structure. A truss structure consists of steel elements that can carry bending moments. Therefore not only ring action plays a role in the amount of stiffness, also beam action plays an important part.

Variant 1

The idea for the first variant is to transport the loads from the sides directly to the curved outer ring elements in the corner areas. It is assumed that in this way the available ring action is used at full extend. The only curvature in the roof structure is present at the corners of the inner and outer ring. The weakest points where the stiffness and strength is very little is present at the sides of roof structure, especially at the long side. The preliminary design showed that the largest deformations arise here.
There has been tried to use the ring action to decrease the deformations and to increase the stiffness and strength at these points. This is been accomplished by directly connecting the centre of the sides of the inner ring with the corner outer ring elements. The structure has become non-linear stable due to these adaptions.
By means of parametric modelling the structure only has an average dead load of 79,90 kg/m² and therefore has the lowest dead weight of all three variants.

Variant 2

The idea for the second variant was to use a more regular, symmetric spatial truss system similar to the Feyenoord stadium. It is expected that the ratio ring : beam action is smaller compared to variant 1. The stiffness and strength has been increased by applying strong vertical crosses at a slope of almost 45 degrees. The vertical crosses will also guide the forces to the corners of the ring elements and use a part of the available ring action as well. After optimization of variant 2, the structure has an average dead load of 83,70 kg/m². The difference in average dead load between variant 1 and 2 is small.

Variant 3

The third variant consists of a roof structure composed of crosses in the whole bottom and top spoke level, with the idea that the force finds the fastest or most efficient way to be distributed. For this variant can be assumed that the force distribution is optimal and that the ring action is used at full extend. The elements need to be less strong and therefore smaller CHS profiles can be used. The disadvantage however, is that many elements have a low unity check value meaning that these elements become unnecessary. Because of the huge amount of elements, the third variant has the highest average dead load: 92,61 kg/m².

Stiffness distribution

In a truss structure beam action plays a significant role in the total stiffness of the roof structure. Assume that the amount of beam action is equal in the whole roof structure, because the span of the roof is constant. The difference in deformation shows the difference in stiffness distribution between the three design variants.
To interpret the stiffness distribution, the deformation due to the permanent load (includes dead load, roof covering and permanent load) of the centre of the long side of the inner ring to the centre of the short side of the inner ring will be investigated (figure 10.48). The deformation distribution of the inner ring is presented in figure 10.43.
The graph (figure 10.49) shows that the stiffness of the roof structures reaches their maximum in the corner of the structure. The present curvature in the corners provides a certain amount of ring action. The graph of the three variants show a minimal value of the deformation in the corner (75 m) of the structure. This proves that at this point, due to the presence of the curvature in the corners, the maximum amount of ring action and stiffness is provided.

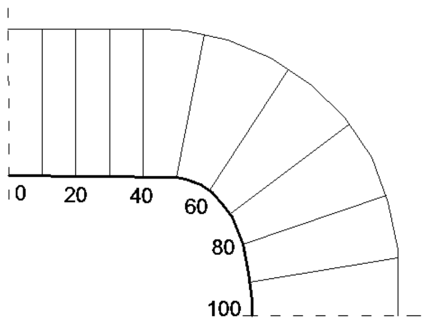


Figure 10.48 Distance from centre long side (0 m) to centre short side (100 m) of the inner ring

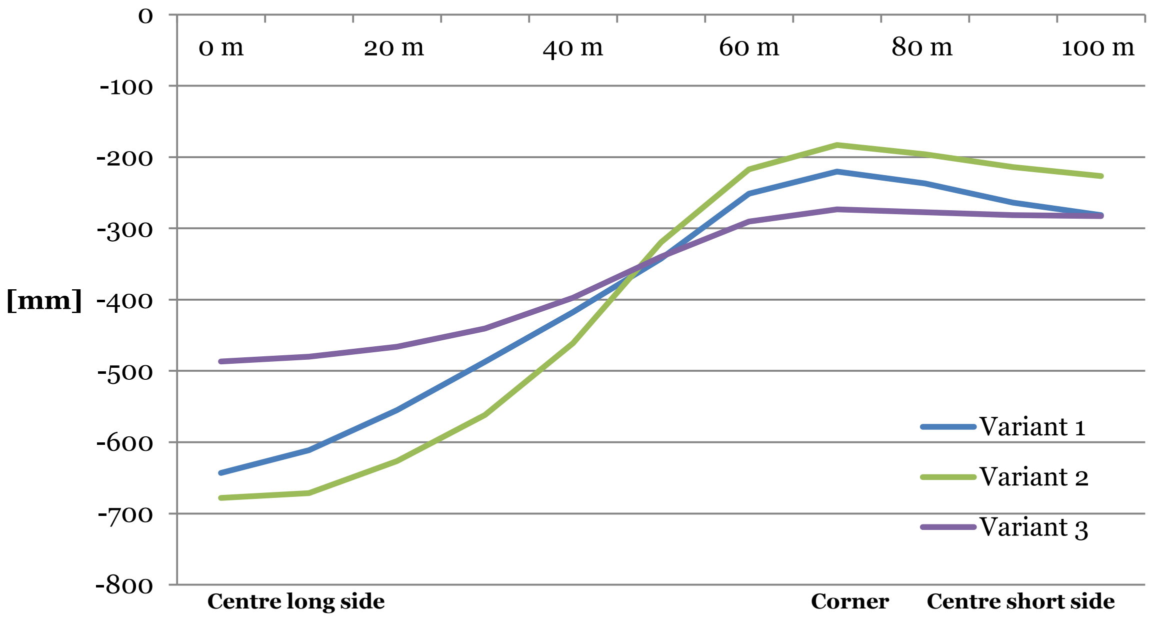


Figure 10.49 Deformation inner ring variant 1-3 due to permanent load

The point where the stiffness is the lowest is, as expected, present on the long side of the structures due to the lack of curvature on the long side. The graph in figure 10.43 further shows that the three variants have almost an equal amount of stiffness at approximately 52m. At this point, the curvature of the rings will become significant ($k \geq 0$) and ring action comes more into play.

The influence of the adaptions that have been made to the different designs can be shown by looking at the deformation due to additional loads, like snow and wind.

Ring action

To show to what extend a design makes use of the ring action, the deformation distribution only due to the snow load will be investigated. The snow load is equally distributed over the whole roof structure. Therefore it is possible to make a fair comparison. The deformation of the inner ring due to the snow load is presented in figure 10.50.

The amount of ring action can be defined by looking at the deformation differences in the inner ring. One must take into account that the available curvature in the rings is equal in all variants and therefore also the amount of available ring action. The span of the roof is almost constant in the whole roof (42,50m) causing a constant amount of beam action. Therefore the difference in deformations can show how the ring action is used in all three variants and which variant shows the greatest use of the available ring action. Table 10.17 shows the deformation difference in percentage.

	Variant 1	**Variant 2**	**Variant 3**
Deformation difference 0 – 75 m	63,53 %	70,72 %	42,61 %

Table 10.17 Deformation differences inner ring variant 1-3.

In a complete circular roof the difference in deformation is zero. When the deformation difference in the roof structure, although the investigated reference stadium is non-circular, is little one conclude that the available ring action is used at great extend assuming the beam action is equal in the whole roof.

In the structural design phase of the three variants the expectation was that variant 1 and 3 will have a high ratio ring : beam action and variant 2 a small ratio ring : beam action. The deformation difference results from table 10.17 shows that these expectations are almost true.
Variant 2 has the lowest ratio ring : beam action and this is illustrated by the deformation differences in graph 10.50. Variant 3 uses the highest amount of the available ring action. By using cross diagonals in the complete plane of the bottom and top spokes, the loads are transferred through the most efficient way.

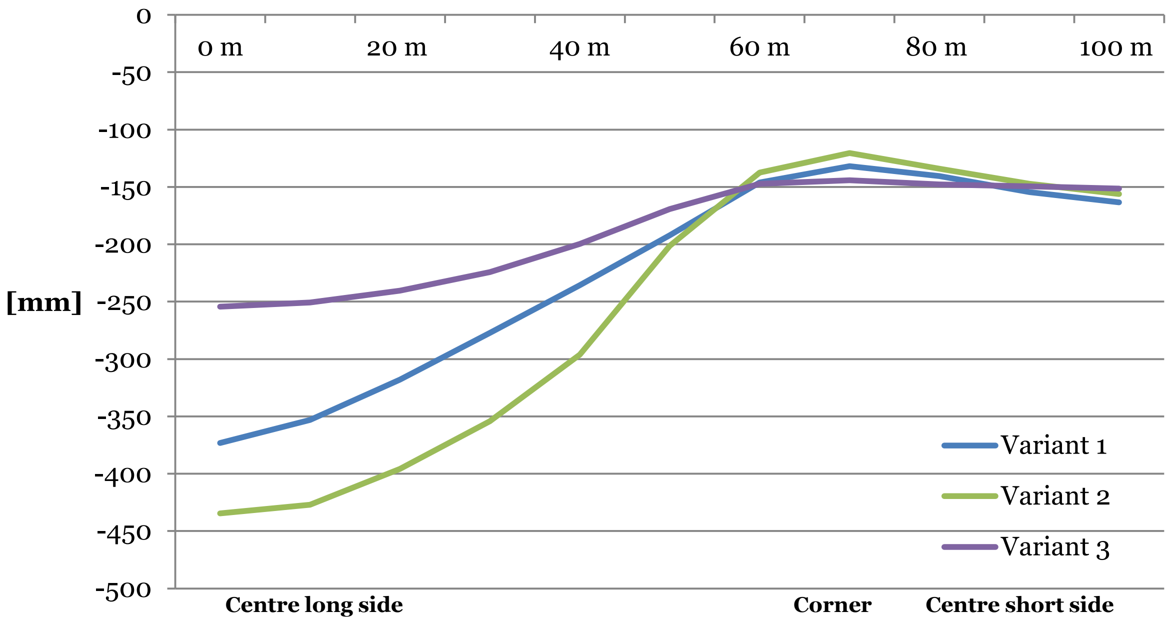


Figure 10.50 Deformation inner ring variant 1-3 due to snow load

The deformation difference shows that variant 1 has a smaller ratio ring : beam action compared to variant 3. The idea of variant 1 was to directly transport the loads to the corner areas and use the ring action at full extend but with as little elements as possible. Because variant 3 consist of many elements, and due to its geometry, the forces will flow though the elements that provide the greatest stiffness to the whole structure causing small deformation differences.
The normal force distribution of variant 3 shows how to efficiently transport the load, or in other words, how to use the available ring action at full extend. Results show that the normal force distribution in variant 1 is almost parallel to variant 3. The largest normal (tensile) forces arise from the centre of the inner ring on the straight sides directly to the curved ring elements (figure 10.52), which provide stiffness due to ring action. Figure 10.51 shows that the diagonals placed in the top spoke level, follow almost the same path as in the case of the normal tensile forces in variant 3.

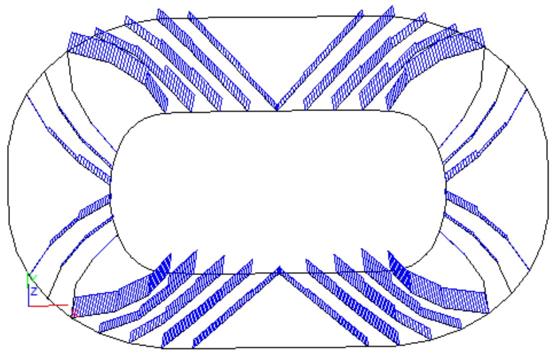


Figure 10.51 Tensile forces in variant 1

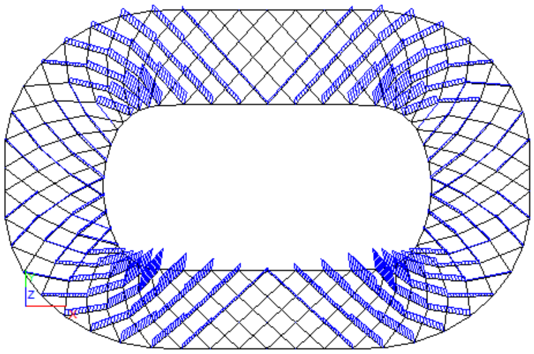


Figure 10.52 Tensile forces in variant 3

The results show that variant 1 transports the loads in a correct way and uses the ring action at full extend. The deformation difference in table 10.17 however shows that variant 3 is more able to use the ring action. Because variant 3 uses a lot of elements and has a high dead load, the question is whether variant 3 is also more efficient compared to variant 1. This will be further determined.

Efficiency

The deformation of the ring has shown that variant 3 uses the amount of available ring action to the fullest, followed by variant 1 and 2. However, the disadvantage of variant 3 is that it is composed of many (unnecessary) elements and therefore has a high average dead load. By taking into account the deformation of the inner ring as well as the average dead load of the structure, the efficiency of the three variants can be determined. The efficiency has been calculated as followed:

$$Efficiency\ [kg/m] = |Deformation\ [m]| * Average\ dead\ load\ [kg/m^2] \quad (10.6)$$

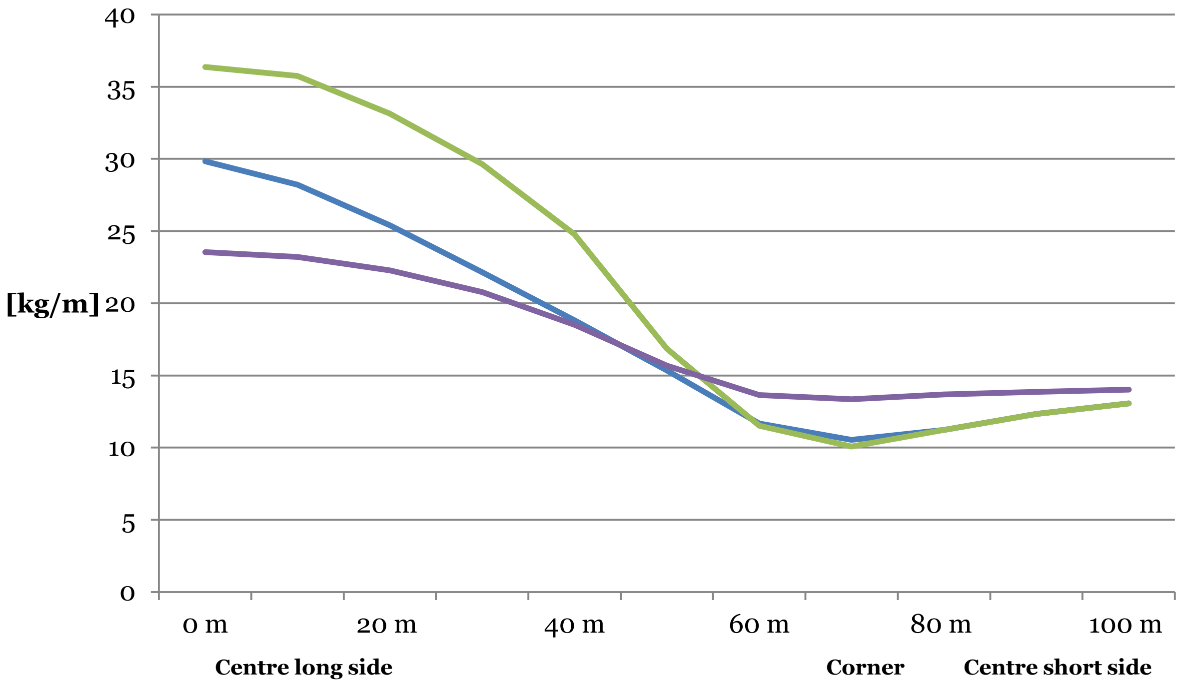


*Figure 10.53 Efficiency value (deformation inner ring [m] * average dead load [kg/m²]) of variant 1-3 due to snow load*

The efficiency of the three variants is illustrated in figure 10.53. The graph shows that variant 3 is the most efficient structure at the centre of the long side of the structure. For 1 m¹ vertical deformation of the inner ring at the centre of the long side (0 m), the structure uses only approximately 24 kg of steel.
Due to the direct connection between the centre of the long side of the inner ring with the outer corner ring elements in variant 1, variant 1 is more efficient at this point than variant 2. Again is shown that the ratio ring : beam action of variant 2 is little.

When looking more to the corner of the structures, the graph shows that variant 1 and 2 are more efficient (both a value of approximately 11 kg/m¹) compared to variant 3 (13,5 kg/m¹). At the centre of the short sides the efficiency value of the three variants comes closer together resulting in a value between 13-14 kg/m¹.
The graph of variants 1-3 in figure 10.53 only shows the efficiency of the structure at one point of the inner ring. To determine the efficiency of the total roof structure, one has to determine the average deformation of the inner ring. By multiplying this value with the total average dead load of the variants, the most efficient design can be determined:

$$Efficiency\ [kg/m] = |Average\ deformation\ [m]| * Average\ dead\ load\ [kg/m^2] \quad (10.7)$$

The results are presented in table 10.18 and show that variant 1 and 3 are the most efficient structure, where variant 3 shows to be slightly more efficient. The results show that using a lot of elements to increase the use of ring action is not always more efficient. Variant 1 and 3 transport the loads in a similar way, but with a different amount of elements and use of geometry. It can be concluded that by directly transporting the loads to the curved ring elements, using the geometry of variant 1, an efficient structure can be made.
Variant 2 appears to be the least efficient structure of the three, because it has the lowest ratio ring : beam action. The results shows that is pays off to use the amount of available ring action at full extend to decrease the amount of material and to come to an efficient design.

	Absolute Av. deformation [m]	**Av. Dead load [kg/m²]**	**Efficiency value [kg/m¹]**
Variant 1	0,248	79,90	**19,63**
Variant 2	0,280	83,70	**23,47**
Variant 3	0,208	92,61	**19,25**

Table 10.18 Determination efficiency value variant 1-3

Structural resilience

Research has showed that none of the variants is able to remain stable when certain crucial elements are damaged. The reason is that the designs have been thoroughly optimized, both the geometry as well as the elements itself, to use as little material as possible. The designs fulfil all structural requirements, but have little reserve in strength and stiffness to provide sufficient structural resilience when crucial elements are damaged.
The leading elements are the centre outer ring elements in the tension ring and compression ring. When an inner tension ring is damaged, the structure will not collapse in any of the three variants.
In order to stabilize the structure, the strengths of variant 2 and 3 need to be used. To take up the loads from the compression ring, vertical sloped crosses need to be used (figure 10.41). For the tensile ring, horizontal crosses in the bottom spoke level (figure 10.42) have to be applied. The suggested structure is illustrated in figure 10.43. Another option is to use stronger elements, which are able to take over the loads.

Overall conclusion

The structural designs of variant 1-3 have shown that by adding the right elements on the right places, a stable efficient design can be accomplished. The sides of the structure have little curvature, only beam action is present. Variant 1 and especially variant 3 uses the ring action at full extend by providing a direct connection between the weakest point of the structure: the centre of the long side at the inner ring, with the strongest points: the corner outer ring elements. The results show that these two structures have the best efficiency values. Although variant 3 uses the available ring action at greatest extend and has the best efficiency value, it is not efficient in terms of material use. Too many elements are applied that carry a minimum amount of loads. Variant 1 has the lowest total dead value, fulfils all structural requirements and therefore this variant is the most attractive one.
It can be concluded that it pays off to use the available ring action at full extend to minimize the amount of needed material. Although little curvature is present, it is advised to use the available ring action at full extend. In the structures beam action plays a significant role to provide sufficient extra stiffness for non-circular shaped roofs to come to a design that fulfils all structural requirements. The results show that although beam action plays a role, it is still possible to come to an efficient design that is an attractive option for stadium roof design and even needs less steel than the Feyenoord stadium. Hereby an answer is given for sub-question 7 which is as follows:

'Is a spoke wheel roof still efficient and attractive to be used for football stadia roofs?'

With the help of parametric modelling is was possible to even further optimize the structural designs. It was possible to lower the amount of steel, by using different profiles for every single element to bring back the unity check values close to 0,90-1,and 00.
The optimization of the structural designs however, have a negative influence on the structural resilience. Calculation results shows that all three designs become unstable when crucial elements are damaged. To remain stable in all situations, another structural design has been suggested in figure 10.43. The disadvantage of this structure is its low efficiency value. For the engineer it is almost impossible to design a roof structure which possess sufficient structural resilience and have an efficiency rate value close to variant 1 and 3.

In the next chapter, it is interesting to see if it is possible to come to a safe, stable and efficient design for the reference stadium that cannot use beam action to provide sufficient stiffness.

11. Cable structure

Engineers found out many decades ago, that a cable system is very suitable for a spoke wheel roof structure. A cable structure resists load only through tension. Cables are not able to provide stiffness through beam action, only by ring action. It is interesting to investigate if it is possible to apply a cable system for a spoke wheel roof structure for football stadia use.
In addition to the absence of bending, cables are light and have a high strength to weight ratio. Small members could therefore be used even in case of a very large span. This property gives cables an advantage compared to other wide-span systems like a truss- or shell system.

In this chapter the use of a cable system as a spoke wheel roof structure for football stadia will be investigated and to see if the cable system is beneficial and efficient as a spoke wheel roof structure. The questions that arose in the conclusion of chapter 8 are tried to be answered.

11.1 Design process

For the preliminary design, presented in chapter 8, a final cable structure will be designed. A cable net roof structure behaves very different from a spatial truss structure. A truss structure has the option to use extra beam action to come to a stable design. The cable structure completely relies on ring action. A basic regular cable roof structure design will be made in order to understand the structural behaviour and the consequences of the use of a cable structure.
When a non-linear stable design has been made, the engineer has the possibility to vary in the used cable elements (profile, size, etc.), in the amount of used pre-tension or even in geometry of the structure. The goal of the structural design is to use as little material and pre-tension as possible to provide a non-linear stable design that fulfils all structural and additional requirements and conditions.

11.2 Requirements and conditions

The design has to fulfil certain requirements and conditions to become a safe and stable structure. Besides structural and additional requirements (chapter 5), there are other conditions that need to be met. These conditions are related to ways of construction, finance, etc. The requirements and conditions important in this stage of the design process are summarized by nature:

Structural
- Structure non-linear stable (stability, strength and stiffness unity check $\leq$ 1,00)
- Max. Additional deformation (variable load) is 1,00m.

Additional
- The cables elements consists of steel with nominal strength 1770 N/mm^2 and regular steel profiles of S235 steel.
- Steel elements of equal structural purpose (rocker bearings, spokes, etc.) have equal profiles.

The goal is to minimize the amount of material (kg/m^2), assuming the costs are directly proportional to the amount of material.

11.3 Structural design

As mentioned, a cable roof structure completely relies on the available ring action in the spoke wheel roof. It is interesting to see if an efficient design is possible for the reference stadium. The most suitable cable system for the spoke wheel roof is the pre-tensioned cable beam system. A cable beam is an improvement of a simply suspended cable system. The reason why has been chosen for a cable beam system, compared to other cable type structures, is described in appendix A.11.

A cable beam structure is formed by adding a second set of cables with a reverse curvature, to the existing suspension cables. This second set of cables can be added in various manners forming three possible combinations: convex, concave and the combination of convex-concave beams. The three combinations are illustrated in figure 11.1.

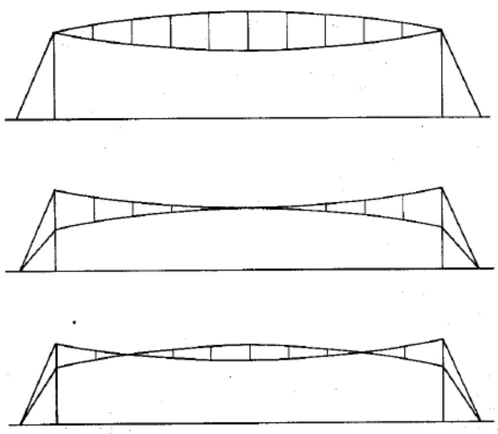


Figure 11.1 From the top: Convex, Concave, Convex-concave cable beam. Reproduced from [3].

Cable beams work efficient as a load bearing system, which results in a light structure. Pre-tensioning of the cables makes the structure stiffer (vertically, radial stiffness is partly provided by the cladding material), and if the cables have the right amount of pre-tension, the cables remain stretched under any combination of applied loading.
A cable beam is, compared to other cable systems, more able to carry vertical loads in both upward and downward direction. For the cable structure a concave cable beam system is very suited regarding the shape of the preliminary design. The way the concave cable beam resists the vertical loads will be briefly described using figure 11.2.

The convex curvature carries the downward force while the concave curvature resists the upward force. When a downward force acts on the structure, the tension force in suspension cable A is increased while the tension in the pre-stressed cable B is reduced. The downward force is thus resisted by suspension cable A. When the structure is subjected to an upward force, tension increases in pre-stressed cable B and reduces the tension force in suspension cable A. Cable B thus resists the upward force.

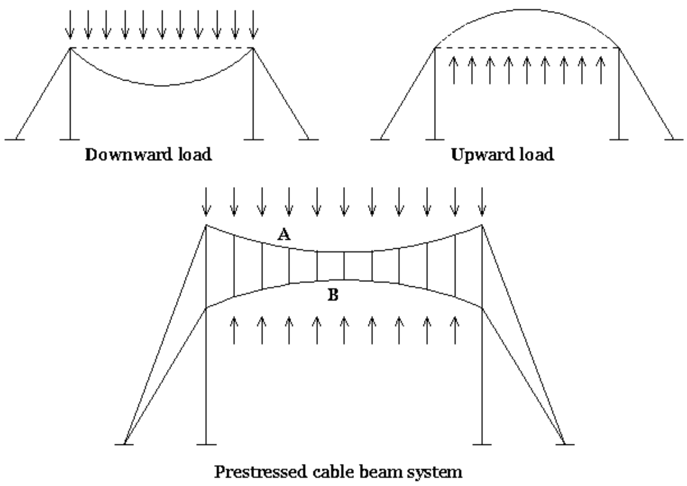


Figure 11.2 Structural behaviour of a concave cable beam.

The suspension cable and pre-stressing cable are connected by struts or ties. The amount of needed struts will be estimated by using reference projects (Gottlieb Daimler Chrysler stadium in Stuttgart illustrated in figure 2.21). The span of the roof will be divided into 6 parts, therefore 5 struts are needed (figure 11.3). For the roof covering non-rigid plastics (PTFE fibreglass) are used to provide a translucent covering. The roof covering is spanned between the smaller rings between the pre-stressing cables, that divide the total span in 6 parts (figure 11.3). The loads are first transported to the rings, than to the cables and further to the ring structure and the underground.

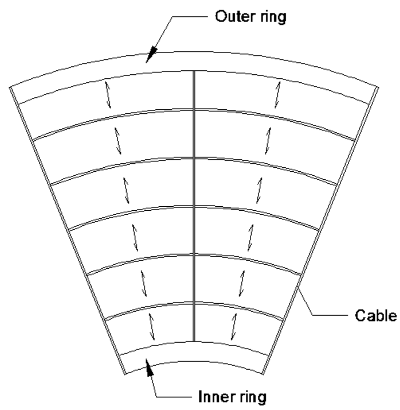


Figure 11.3 Roof covering

To come to a cable roof design, there are certain design variables left that need to investigated. The variables that influence the stiffness of the structure are the design variables that need to be investigated.

The vertical stiffness of the cable beam in vertical direction depends on the following variables:
- Curvature of the cables
- Type of cables
- Dimension of the cables
- Level of pretension
- Stiffness of the supporting structure

Curvature of the cables

The stiffness of the cable beam increases with increasing curvature. In general, the required stiffness will be achieved if the maximum sag of the suspension cable is 5% of the span and for the pre-stressing cable a rise of 3% of the span assuming snow is the dominant load combination for the cable structure [3].

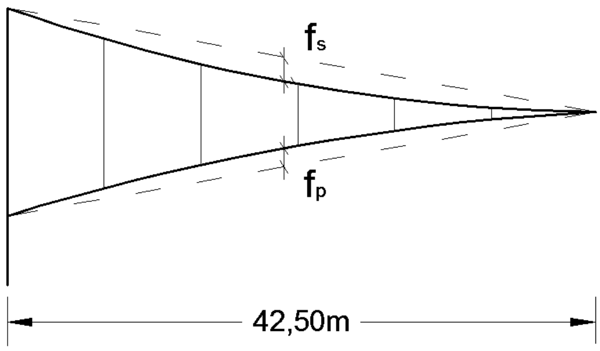


Figure 11.4 Sag of the suspension cable and pre-stressing cable

The maximum span of the roof structure is 42,50 m. The sag of the suspension cable is equal to

$$f_s = \max span * 0{,}05 = 42{,}50m * 0{,}05 = 2{,}125\ m$$

The rise of the pre-stressing cable is equal to

$$f_p = \max span * 0{,}03 = 42{,}50m * 0{,}03 = 1{,}275\ m$$

Type of cables

The choice of type of cable depends on its purpose. Engineers need to make a choice if flexibility or strength is the leading purpose of the cable. In case of the cable structure in this chapter, strength is leading and therefore rigidity is desirable. For this application there is the possibility to use single-strand ropes which are composed of layers of wires built-up around a central wire until the required strength is achieved. Another possibility is a locked coil type of cable. A locked coil consists of one or more layers of interlocking wires. The better fill factor, which is the ratio of steel area to strand area, gives the locked coil construction an advantage regarding the breaking load capacity. For the structural design locked coil ropes are used, because strength is leading and these types of cables have the highest breaking strength capacity.

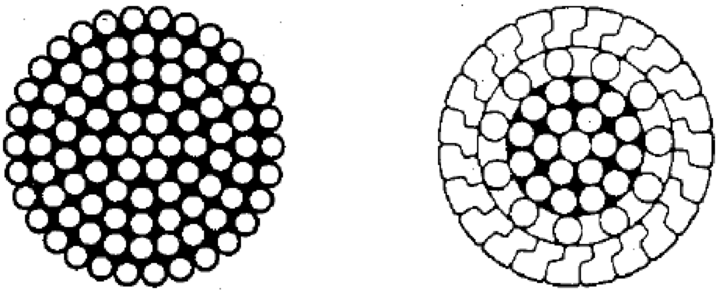


Figure 11.5 Spiral strand and locked coil rope

Dimension of the cables

The larger the size of the cable, the more the stiffness of the structure will increase. The needed cross sectional area of the cables are such that the maximum load is less than or equal to half their breaking strength in order to prevent yielding. The dimension of the cables are estimated using the Gottlieb Daimler stadium in Stuttgart as a reference stadium: a fully locked coil cable with a diameter of 101,5 mm will be used. The cable has a minimum breaking load of 9035,00 kN, a steel area of 6900 mm^2 and a modulus of elasticity value of 158,40 kN/mm^2. The rope consists of more than 3 layers of Z-wires and has a nominal strength of 1770 N/mm^2.

Level of pretension

Also in the case of increasing the level of pretension, the stiffness will increase as well. However, it is more beneficial and less costly to use cables of greater size instead of increasing the pretension force. The stiffening effect of increasing pretension is only marginal. The amount of pretension in the cable should only be sufficient enough to prevent the cables from going slack under any load combination.

Due to the design stage and the lack of information, the following assumptions has been made in order to come to the first calculations:

- The uniformly distributed load acting on the roof structure, can be replaced by concentrated loads acting on the joints (outer ring, struts and inner ring nodes).
- The forces in all the struts are equal in magnitude when subjected to pretension only.
- The tensile forces in the pre-stressing cables are zero when the cables are subjected to the maximum load, since the level of pretension has a marginal effect on the stiffness.

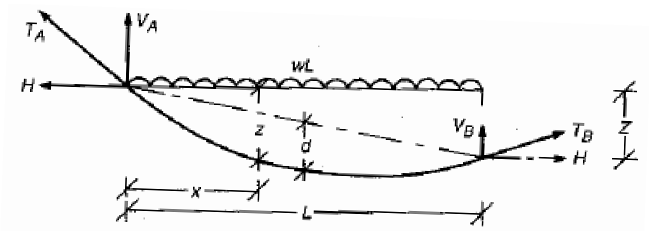


Figure 11.6 Uniformly distributed load on cable. Reproduced from [3].

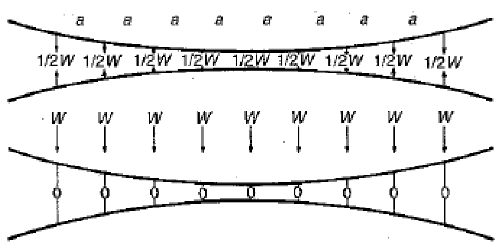


Figure 11.7 Assumed force distribution. Reproduced from [3].

For the pre-stressing and suspension cable the amount of needed pre-tension will be calculated [3]. The theory for the determination of the needed pre-tension holds for complete circular concave cable beam roof structures.

The maximum horizontal component of the cable tensions are:

$$H = \frac{wL^2}{8(d_s + \Delta d)} \tag{11.1}$$

Where d_s is the maximum sag of the suspension cable in the unloaded condition (d_s =2,125 m);
Δd is the deflection of the suspension cable from the pre-tensioned equilibrium condition due to the load $\frac{1}{2}wL$
L is the span (L =42,50m)
w is the total distributed load

For the total distributed load is assumed that the load combination dead load and snow (LC4) is leading:

$$w = 1{,}2\frac{kN}{m^2} * 12{,}30m = 14{,}76\frac{kN}{m} \tag{11.2}$$

The deflection of the suspension cable from the pre-tensioned equilibrium condition due to the load $\frac{1}{2}wL$ is determined as follows:

$$\Delta d = \frac{15L^3}{8(10L^2d - 5Z^2d - 48d^3)}\Delta l \tag{11.3}$$

Where L is the span (L =42,50m)
d is the sag of the suspension cable (d =2,125m)
Z is the height difference (Z=15,00m)

$$\Delta l = \frac{H}{EAL}\left(L^2 + \frac{16}{3}d^2 + Z^2\right) \tag{11.4}$$

Where $H = \frac{\frac{1}{2}wL^2}{8d} = \frac{\frac{1}{2}*14{,}76*42{,}50^2}{8*2{,}125} = 784{,}125\ kN$ (11.5)

It is assumed that a fully locked coil cable with a diameter of 101,5 mm will be used. The cable has a minimum breaking load of 9035 kN, a steel area of 6900 mm² and a modulus of elasticity value of 158,40 kN/mm².

Follows: $\Delta l = \frac{H}{EAL}\left(L^2 + \frac{16}{3}d^2 + Z^2\right) = \Delta l = \frac{784{,}125}{158{,}4*6900*42{,}50}\left(42{,}50^2 + \frac{16}{3}2{,}125^2 + 15^2\right) = 0{,}035m$ (11.6)

$$\Delta d = \frac{15L^3}{8(10L^2d - 5Z^2d - 48d^3)}\Delta l = \frac{15*42{,}5^3}{8(10*42{,}5^2*2{,}125 - 5*15^2*2{,}125 - 48*2{,}125^3)} * 0{,}035 = 0{,}00328m \tag{11.7}$$

As a consequence the horizontal force component is equal to:

$$H = \frac{wL^2}{8(d_s + \Delta d)} = \frac{14{,}76*42{,}50^2}{8*(2{,}125+0{,}00328)} = 1565{,}83\ kN \tag{11.8}$$

The maximum cable tensions are given by:

$$T_{max} = \frac{1}{2}wL\left\{1 + \left(\frac{L^2}{16(d_s+\Delta d)^2}\right)\right\}^{1/2} = \frac{1}{2} * 14{,}76 * 42{,}5\left\{1 + \left(\frac{42{,}50^2}{16(2{,}125+0{,}00328)^2}\right)\right\}^{\frac{1}{2}} = 1596{,}94\ kN \tag{11.9}$$

The maximum cable tension is lower than half of the breaking strength ($1596{,}94\ kN \ll 9035\ kN$) of the cable, therefore the cable will not yield. Assumed is that the cable has sufficient strength to prevent yielding.

The horizontal component of the pretension forces in the suspension and pre-stressing cable are respectively:

$$H_{s0} = \frac{wL^2}{16(d_s+\Delta d)} = \frac{14{,}76*42{,}50^2}{16(2{,}125+0{,}00328)} = 782{,}91\ kN \tag{11.10}$$

$$H_{p0} = \frac{H_{s0}}{d_s/d_p} = \frac{782{,}91}{2{,}125/1{,}275} = 469{,}75\ kN \tag{11.11}$$

As mentioned earlier, the calculated pretension forces are sufficient for complete circular roofs. Ring action plays a significant role in the total deformation. In the reference stadium there is a lack of curvature on the sides of the roof structure. To compensate the lack of curvature, a higher pretension force than the calculated pretension force will be used for the spokes. By looking at the calculated pretension values, a new pretension force for the suspension and prestressing cable are estimated.

For the suspension cable a pretension force of 1000 kN will be used and for the prestressing cable:

$$H_{p0} = \frac{H_{s0}}{d_s/d_p} = \frac{1000}{2{,}125/1{,}275} = 600\ kN \tag{11.12}$$

Stiffness of the supporting structure

The supporting structure need to withstand the cable force which results in high compression forces in the outer ring elements. The rings need to have sufficient buckling resistance to remain a stable structure. The ring elements on the long side are leading in the design of the supporting ring elements. At these points a high radial tensile force acts perpendicular to the ring elements. As a consequence, large horizontal deformations will arise on the sides of the roof structure.
To minimize the horizontal deformations the outer ring elements need to have sufficient stiffness in the plane of the spokes (I_z value). Therefore the most suitable profiles are RHS profiles. Using the Stuttgart Daimler Chrysler stadium as reference project, the dimensions are estimated for the profiles (figure 11.8).

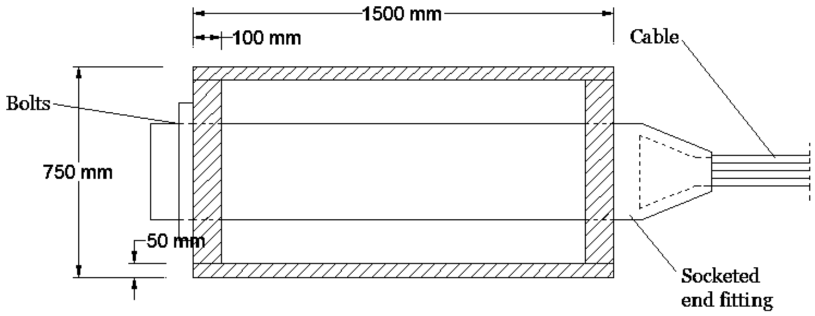


Figure 11.8 RHS profile with connection of the cable

Choice of profiles

To determine the kind of profiles the cable structure is composed of, the structure is divided into groups of elements like in the case of the truss variants. The group numbers are presented in figure 11.9 and 11.10.

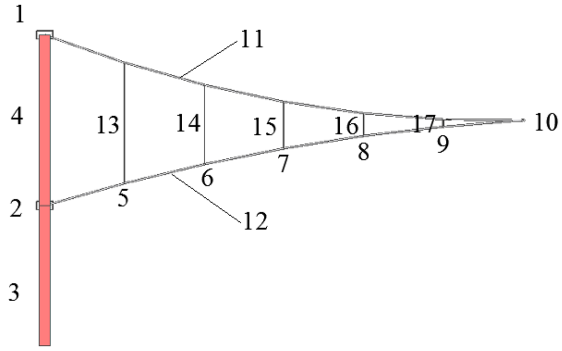


Figure 11.9 Cross section spoke framework cable variant

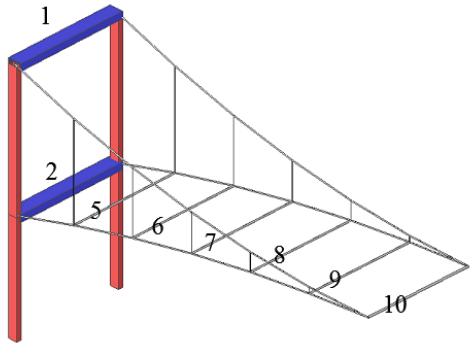


Figure 11.10 Spoke framework cable variant

The profiles for the outer ring elements (element number 1 and 2 with RHS 1500*100*750*50) and for the spokes (element number 11 and 12 with locked coil rope LC101,5) are already known. For the other elements the profiles are yet to be determined by using the Daimler Chrysler stadium as reference project.

For the inner ring (element number 10) a group of 8 locked coil ropes are used each with a diameter of 51mm. The inner ring needs to withstand the high tension forces caused by the pretension in the spokes.
In the outer columns (4) and the rocker bearings (3) high compression forces will arise. For these elements RHS 1000*50*500*50 are used. By using RHS profiles it is easier to connect these columns with the outer ring elements. Besides the columns also need to provide sufficient radial strength and stiffness, RHS profiles are therefore very suitable.
To keep the suspension and prestressing cable in its place, struts (13-17) are placed between these cables. The struts are only subjected to tension and need to have sufficient cross sectional area to prevent the struts from yielding or breaking. For the struts massive steel bars are used with varying circular cross sectional areas.
The last groups of elements are the bars (5-9) connecting the suspension cables. The roof covering is spanned between these bars. For these bars CHS 159*10 profiles are used.

Element nr.	Profile	Element nr.	Profile
1	RHS 1500*100*750*50	**10**	8*LC51
2	RHS 1500*100*750*50	**11**	1*LC101,5
3	RHS 1000*50*500*50	**12**	1*LC101,5
4	RHS 1000*50*500*50	**13**	RD40
5	CHS 159*10	**14**	RD35
6	CHS 159*10	**15**	RD30
7	CHS 159*10	**16**	RD25
8	CHS 159*10	**17**	RD20
9	CHS 159*10		

Table 11.1 Choice of profiles cable variant

Roof design

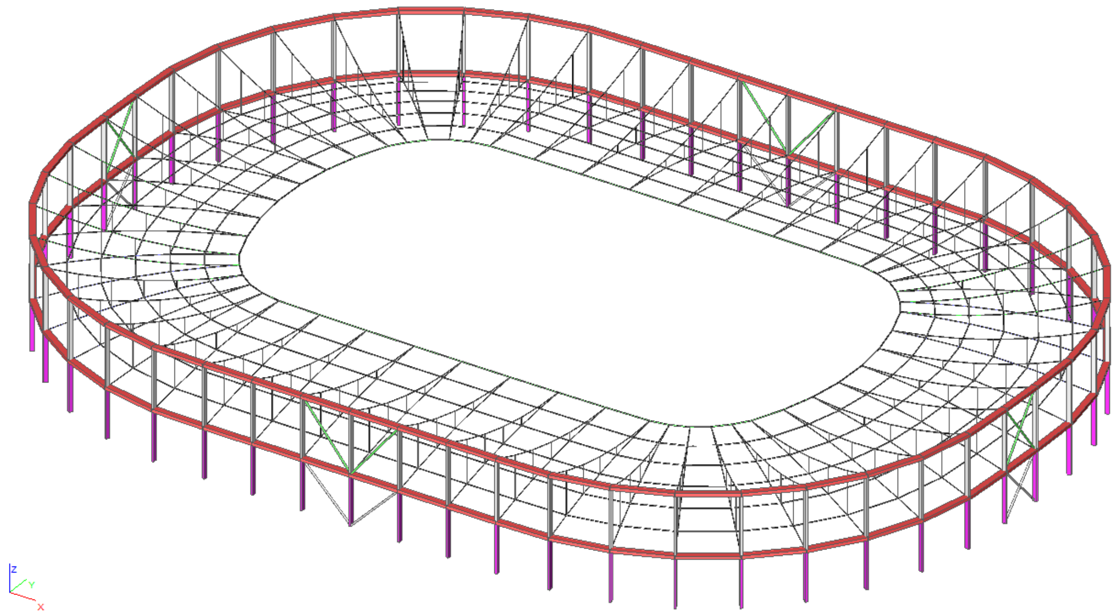


Figure 11.11 3D roof design cable structure

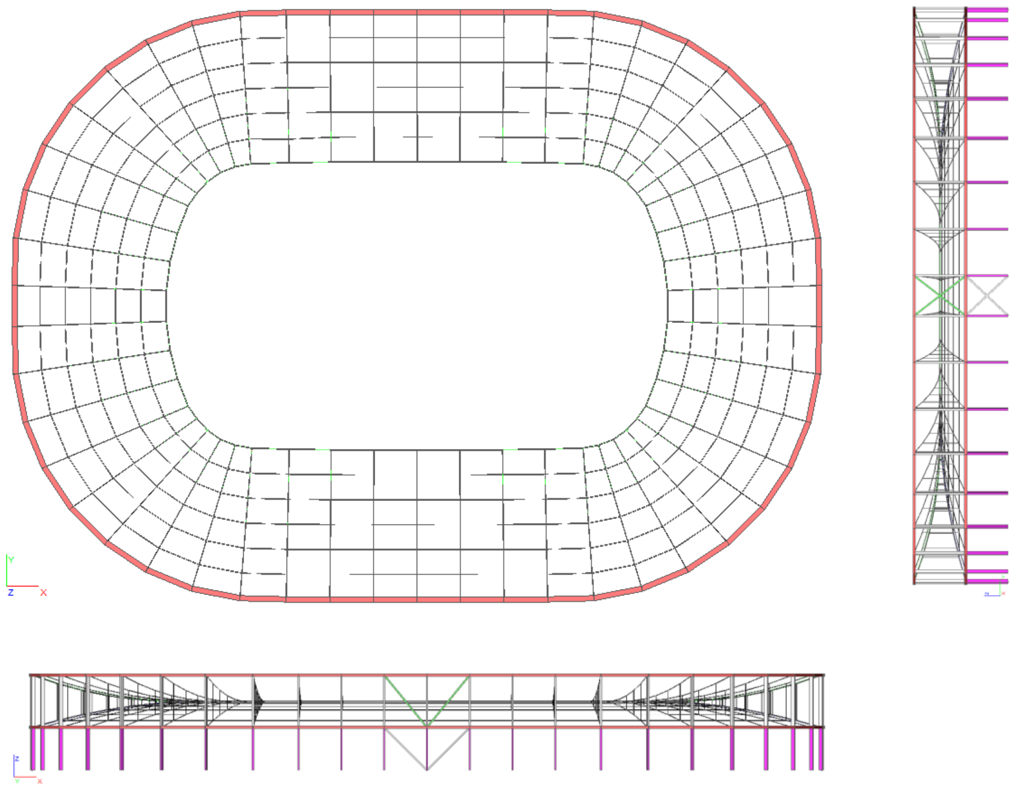


Figure 11.12 View x, y and z-direction cable structure

11.4 Results

The results of the structural design of the cable structure are briefly presented in this paragraph. The results conclude the stiffness, strength and stability performances of the structures. The calculations and results are made with the FEM program Scia Engineer. In appendix A.11 the theory concerning the check of the elements for the structural requirements is explained.

The results show that the complete structure is non-linear stable. The structure has an average dead load of 223,01 kg/m². The largest deformation of the inner ring arises at mid- span of the long side of the roof structure; 2026,20 mm. The unity check values are presented in table 11.2. In the table only the leading UC is presented (stability, stiffness or strength).

Strength
The strength requirement is leading for most of the elements that are subjected to tension: elements 5 – 10 and elements 13 – 17. Table 11.2 shows that the UC values are very high (except for inner ring element number 10) and does not fulfil the strength requirement (≤ 1,00). The reason of the high UC values is due to the large deformations that arise in the structure.

Stiffness
The stiffness requirement is only leading for the suspension and prestressing cable (elements 11 and 12). Calculations show that the UC value for these elements exceed 1,00 for the cables present at the straight sides of the roof structure. The outer ring elements on the long side are not curved and do not provide ring action. As a consequence the outer ring will deform due to the pretensioning of the spokes. The outer ring will deform inwards and the cables will sag further on, which causes a UC value of 1,0.

The large deformations cause extra forces in the ring (5-9) and strut elements (13-17). Due to these high extra forces, these elements cannot fulfil the strength requirements. When the deformations decrease, the amount of forces acting on these elements will decrease as well.

From the results can be concluded that the pretension in the suspension and prestressed cable present on the long sides have very little influence due to the lack of ring action. In figure 11.13 and 11.14 the deformation of the suspension and prestressed cable at the long side and corners of the roof structure are illustrated.
The suspension cable will deform even more than the inner ring. Due to the deformation of the suspension cable, the prestressing cable will undergo the same deformation. Besides, the prestressing cable will deform extra due to its own dead load. This is illustrated by the sags of the individual cable elements in the prestressing cable.
In the corner of the roof structure sufficient ring action is present. Figure 11.14 shows that, with the same amount of pretension, the suspension cable will only slightly deform as well as the prestressing cable.

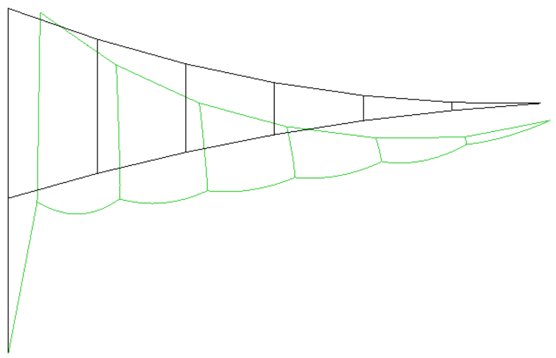


Figure 11.13 Deformation at long side

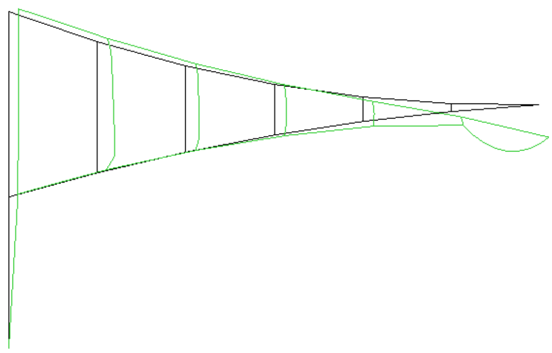


Figure 11.14 Deformation at corners

Stability
The stability requirement is leading for elements subjected to compression forces (element 1-4). For these elements the stability requirements consist of buckling of the bar due to the normal force and the combination normal force/bending moment.
Table 11.2 shows that element nr. 1 and 4 do not fulfil the stability unity check requirement. A larger and stronger profile (with a higher buckling factor) is needed. However, the stiffness results show that due to the lack of ring action at the long side more stiffness is needed at this point. Besides pretension, stiffness can be generated by using much stronger and stiffer elements for the outer ring at the long sides.

To come to a design that will fulfil all structural requirements, the stiffness will be leading in the design process. It is important to increase the amount of stiffness by giving extra attention to the long sides of the roof structure.

The first step is to decrease the amount of deformation by adapting the amount of pretension in the spokes and the amount of stiffness of the outer ring elements. The following step is to determine the right profile for each group of elements which fulfil the strength and stability requirements.

	Profile		Results	
Nr.		ω_{buc}	+/-	**UC**
1	RHS 1500*100*750*50	1,00	-	**1,08**
2	RHS 1500*100*750*50	1,00	-	0,55
3	RHS 1000*50*500*50	1,00	-	0,62
4	RHS 1000*50*500*50	1,00	-	**1,01**
5	CHS 159*10	0,37	+	**3,35**
6	CHS 159*10	0,39	+	**18,55**
7	CHS 159*10	0,67	+	**1233,71**
8	CHS 159*10	0,76	+	**33,40**
9	CHS 159*10	0,82	+	**726,48**
10	8*LC51	0,12	+	0,48
11	1*LC101,5	0,01	+	**2,51**
12	1*LC101,5	0,01	+	**17,65**
13	RD40	0,03	+	**19,60**
14	RD35	0,04	+	**20,72**
15	RD30	0,09	+	**24,10**
16	RD25	0,25	+	**26,85**
17	RD20	0,72	+	**46,07**

Table 11.2 Results of cable variant; buckling factor, tension/compression and unity check

11.5 Optimization

The results have shown that the design needs to be adapted in order to fulfil all structural requirements, the stiffness requirements in particular. In paragraph 11.3 has been described that the design depends on the following design variables:

- Curvature of the cables
- Type of cables
- Dimension of the cables
- Level of pretension
- Stiffness of the supporting structure

The curvature of the cables is a condition that needs to be met in order to gain sufficient stiffness. The curvature is set at 5% for the suspension cable and a 3% rise for the prestressing cable. For the cables fully locked coil ropes with a diameter of 101,50 mm are used. The results showed that the amount of tension in the ropes is little. The ropes have enough capacity to take up more tensile forces. At this stage, the ropes do not need to be adapted. In order to provide more tensile forces in the cables, the amount of pretension and stiffness of the supporting structure need to be adapted.

Determination pretension and stiffness supporting structure

The results showed that there is a great difference in stiffness of the outer ring between the long sides and the corners of the structure. The reason is that the stiffness of the outer ring is only provided by ring action. Beam action does not play a role in this structure. Because the only curvature is present in the corners, the stiffness differences are substantial.

In order to decrease the stiffness differences, the pretension of the cables and the stiffness of the elements in the supporting structure will be adapted. To determine the amount of pretension and extra stiffness, the total roof structure will be divided in multiple areas. Each area will have its own amount of pretension in the cables and will have outer ring elements with a different amount of stiffness. Figure 11.15 illustrates the separation of the roof structure in the different areas.

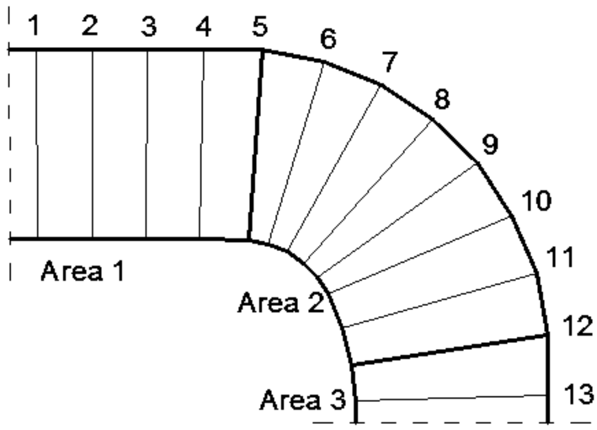


Figure 11.15 Spreading of the areas

The determination of the pretension and the stiffness of the supporting structure is an iterative process. There are no formulas or described methods to come to the right stiffness to pretension ratio between the different areas.
The first step in the iterative process is the determination of the needed stiffness in the outer ring and pretension in the cables of area 2. Most of the curvature is present in area 2, therefore is assumed that the least amount of pretension and stiffness of the supporting structure is needed here due to the greater capacity of ring action compared to area 1 and 3. The goal is to prevent the cables from going slack under any load condition.
When the pretension and the stiffness of the supporting structure have been determined for area 2, area 1 and 3 will follow. These areas probably need more stiffness and pretension and therefore will be determined after area 2.

Many iterative steps have been taken in order to determine the right stiffness and pretension to prevent the cables from going slack. Table 11.3 presents a summary of the taken iterative steps. The presented pretension values are

the amount pretension present in the suspension cable. For the prestressing cable 60% of the pretension force of the suspension cable is used. This ratio is equal to the described theory in equation 11.12.
The greatest deformations arise in the prestressing cable. The presented deformation values are the maximum arising deformations in the prestressing cables in a particular area. The cables remain slack when the maximum deformation has a positive value.

Iterative step	**Properties**						**Max. Deformation [mm]**		
	Area 1		**Area 2**		**Area 3**		**Area 1**	**Area 2**	**Area 3**
	kN	I_z [m4]	kN	I_z [m4]	kN	I_z [m4]			
1	5000	0,01	5000	0,01	5000	0,01	-3662,3	-1420,9	-423,3
2	10.000	0,02	10.000	0,02	10.000	0,02	-5307,7	-1676,3	-323,6
3	20.000	0,02	20.000	0,02	20.000	0,02	-1792,7	10,4	72,5
4	50.000	0,05	20.000	0,02	20.000	0,02	-1863,3	35,4	134,9
5	100.000	0,05	20.000	0,02	20.000	0,02	-3423,9	122,9	281,4
6	100.000	0,05	30.000	0,02	30.000	0,02	-3170,6	104,6	218,6
7	100.000	0,05	50.000	0,02	50.000	0,02	-2644,4	23,8	112,9
8	100.000	2	50.000	1	50.000	1	-937,5	54,6	112,4
9	100.000	5	70.000	2	70.000	2	-251,1	34,1	87,7
10	50.000	10	100.000	2	100.000	2	47,4	28,0	62,0

Table 11.3 Iterative process to determine pretension(kN) and stiffness (m4) supporting structure

The first calculation results show that the original structure, with a pretension value of a 1000 kN and supporting structure with I_z=0,01 m4, was not sufficient to fulfil the stiffness requirements. In the first iterative step, the pretension has been increased to 5000 kN. It appeared that the structure is still not stiff enough to prevent the cables from going slack under any load condition.
The cables in area 2 and 3 remain tight after the third iterative step. A pretension value of 20.000 kN in the suspension cable and a supporting structure with I_z=0,02 m4 is needed to provide sufficient stiffness for area 2 and 3. For the following step, iterative step 4, the pretension and stiffness value for area 2 and 3 will remain fixed to see what the consequences are for the deformation in the three areas.
In step 4 and 5 the pretension in area has been increased to respectively 50.000 kN and 100.000 kN. The results show that the increase of pretension in area 1 has a positive effect on the deformation of the cables in area 2 and 3 and a negative effect on the cables in area 1. The reason is that the higher the amount of pretension in area 1, the more the inner ring in area 1 will radial deform in the direction of the outer ring (figure 11.16). The distance between the inner and outer ring decreases. As a consequence the cables will have a higher sag value. The opposite happens for area 2 and 3; the inner ring will deform from the outer ring and the cables become less slack.

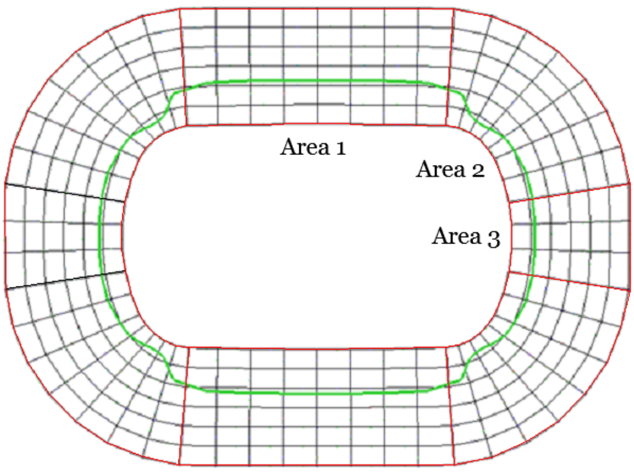


Figure 11.16 Deformation inner ring due to increase pretension straight side

To decrease the sag of the cables in area 1, the pretension value of the cables in area 2 and 3 need to be increased. In step 6 and 7 the pretension in areas 2 and 3 are increased, respectively to a pretension value of 30.000 kN and 50.000 kN. The pretension in these areas causes a decrease of the sag of the cables in area 1.
The sag can be further minimized by increasing the stiffness of the supporting outer ring structure. Due to the pretension, the outer ring will deform inwards due to the high radial pretension force (figure 11.17). The long side of the supporting outer ring structure needs to have a higher bending rigidity to increase their stiffness. The corners of the supporting ring structure need a smaller increase of the bending rigidity, because its stiffness is also provided by the ring action. In step 8 and 9 the moment of inertia of all areas have been increased, resulting in a further improvement.

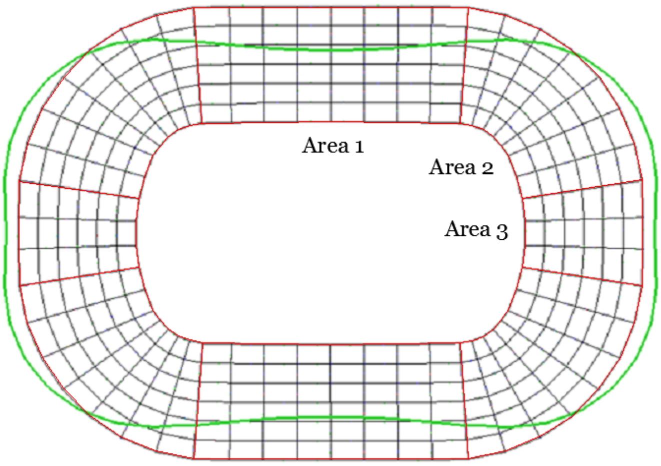


Figure 11.17 Deformation outer ring by constant pretension

From the first nine steps can be concluded that the sag of the cable at the long side can decrease by increasing the stiffness of the supporting structure and by increasing the pretension in the cables of areas 2 and 3. The last possibility is to decrease the amount of pretension in the cables in area 1. The pretension causes an increase of the radial deformation of the inner ring (figure 11.16). By decreasing the pretension in area 1 this can be avoided. In step 10, the pretension in area 1 has a lower value, resulting in a positive effect for the cable. In the final situation none of the cables become slack under load condition.

The conclusion is that the amount of pretension at the straight sides cannot be too high, in order to prevent large radial deformations at the straight sides of the outer and inner ring. These large radial deformations arise because the ring elements at the straight side cannot provide stiffness due to ring action. This can be explained by looking at the polygon of forces. The angle between the spokes and the outer ring at the straight sides is around 90 degrees. This results in a very inefficient load transfer. The supporting structure needs to have sufficient bending rigidity to withstand large radial loads, like pretension, to prevent large radial deformations. The opposite is the case in the corner areas, where the angle between the spokes and rings is smaller than 90 degrees. Both situations are illustrated in figure 11.18.

To prevent large radial deformations, or decrease the amount of deformation at the straight side of the roof, the pretension in the spokes in the corner areas need to be increased. The supporting structure can take up these extra high radial forces by ring action in the corners of the roof. The lack of ring action at the straight sides must be compensated by using stiffer elements for the supporting structure.

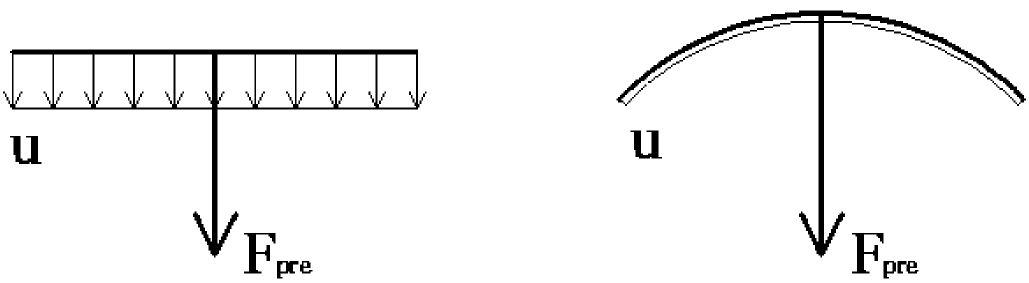


Figure 11.18 Deformation differences between straight ring elements (left) and curved ring elements (right)

The amount of needed pretension in the cables and the amount of needed stiffness of the outer ring structure is much more than has been determined before. The theory described in paragraph 11.3 for the determination of the pretension only holds for perfectly circular structures. The results from table 11.3 show that the available ring action has a great influence on the efficiency of the structure. A lot of pretension and extra stiffness of the supporting structure is needed to prevent the cables from going slack under any load condition.

The unity check values of table 11.2 have changed due to the new applied pretension and stiffness. The new unity check values are presented in table 11.4. Element 1 and 2 represent the top and bottom outer ring. For these profiles there are no regular profiles that are able to possess that great amount of stiffness. Therefore these elements have been left out of the table. Further research is needed to determine the real geometry/shape of the outer ring that can provide such stiffness.

The unity check values presented in table 11.4 are generally smaller compared to table 11.2. By decreasing the amount of deformation, the amount of extra forces in the elements in roof structure has decreased as well. However, the applied elements are still not able to fulfil the structural requirements regarding strength and stability. Stronger elements need to be used to fulfil all structural requirements.
The average dead load of the current design is 1126,38 kg/m². The reason of the extreme high value of the average dead load is the great amount of steel that is needed to provide sufficient stiffness at the straight sides of the roof. Due to the extreme high average dead load, it is not of importance for the thesis research to further increase the strength of the elements that not have a unity check value lower than 1,00 . For the cable design structure further research is needed to come to ideas how to use the amount of available ring action better in order to decrease the needed amount of steel in the supporting structure.

Profile		**Results**		
Nr.		ω_{buc}	**+/-**	**UC**
3	RHS 1000*50*500*50	1,00	-	0,43
4	RHS 1000*50*500*50	1,00	-	0,93
5	CHS 159*10	0,37	+	**4,45**
6	CHS 159*10	0,39	+	**8,72**
7	CHS 159*10	0,67	+	**13,24**
8	CHS 159*10	0,76	+	**18,32**
9	CHS 159*10	0,82	+	**24,57**
10	8*LC51	0,12	+	**3,91**
11	1*LC101,5	0,01	+	**2,47**
12	1*LC101,5	0,01	+	**4,00**
13	RD40	0,03	+	**8,01**
14	RD35	0,04	+	**10,06**
15	RD30	0,09	+	**13,92**
16	RD25	0,25	+	**21,38**
17	RD20	0,72	+	**40,71**

Table 11.4 Unity check values elements after new pretension and stiffness supporting structure

Optimization ideas

To come to an efficient spoke wheel roof for football stadia that makes use of a cable net roof, more research is needed. One can increase its efficiency by using the available ring action at full extend or by increasing the stiffness of the supporting structure. The first option is preferred, because more material can be saved. An idea for further research is the use of cables that are directly spanned from the corner supporting structure to the centre of the inner ring (figure 11.19). This solution is equal to the first variant of the truss structures. By directly connecting the weakest point of the inner ring with the stiff corner ring elements, the amount of deformation can be decreased by make more use of the available ring action. More research is needed to investigate its feasibility and to determine whether this is still an efficient structural design.

The following option is to adapt the supporting structure at the straight side which cannot generate stiffness from ring action. An idea is to apply an additional truss structure (figure 11.20). At the Daimler Chrysler stadium in Stuttgart a similar solution has been used to increase the stiffness of the outer ring. The question is whether this solution is still efficient compared the truss structural designs from chapter 10. A lot of extra steel is needed, causing an increase of the building costs.

After the investigation of the optimization possibilities an answer can be given for sub-question 6 which is:

'Is it possible to further optimize the structural design?'

By increasing the pretension in the spokes connected to the curved ring elements, increase of stiffer supporting structure, use of strong elements and sufficient curvature one can further optimize the design. For non-circular shape roofs more research is needed to come to an efficient and stable design.

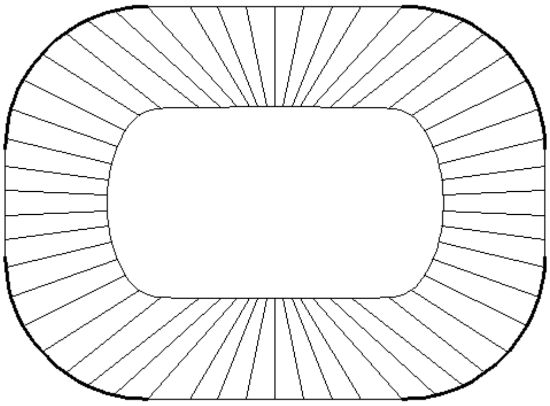


Figure 11.19 Different cable geometry

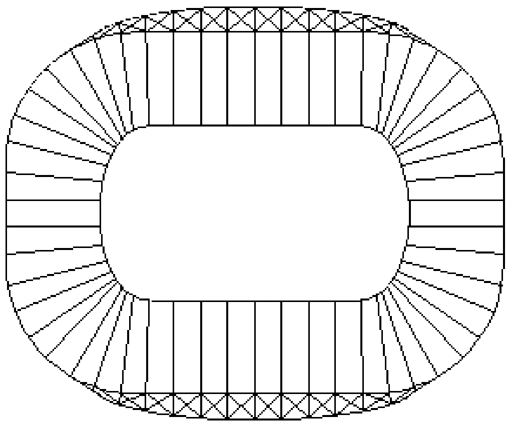


Figure 11.20 Increase stiffness supporting structure

11.6 Connections

For the application of a cable roof structure different type of connections are needed. Cables are pre-stressed and therefore need a special type of connection with the boundary structures. In the design there are three different cable connections. First there are the end fittings of the radial cables connected to the outer ring elements (1). The second type of connection is the connection between the radial cables and the inner cable ring (2). The last one is the connection between the struts and the top and bottom radial cables (3).

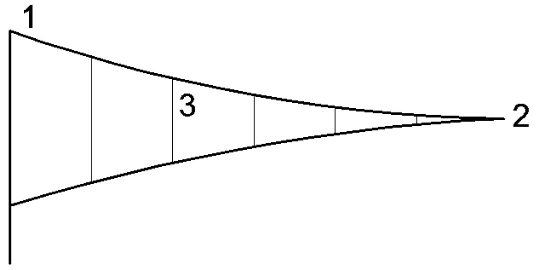


Figure 11.21 Place of cable connections

11.6.1 Connection supporting structure

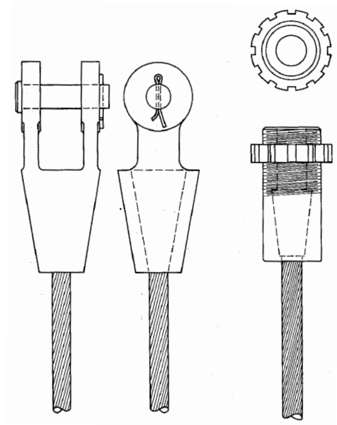
End fittings are used to transmit the cable force to the supporting structure. The end fitting must withstand the full breaking force of the cable without significant yielding; resist the dynamic loading without risk of fatigue failure.
The end fitting depends on the thickness of the cable and the required load range. For different load ranges (different for primary and secondary structures) different types of end fittings can be used.
The end fittings can be mainly categorized into socketed end fittings and swaged end fittings. For both type of end fittings there are a lot of sub end fittings, which have a different load bearing capacity. There are a lot of types of end fittings and differs from every fabricator. Therefore the different kind of (sub-) socketed and swaged end fittings will not be discussed [23,36].

Socketed end fittings

The most reliable and most expensive kind of end fitting is the socketed type. The wires of the end of the cable are sprayed and formed in a conical shaped end fitting. By means of a cast steel shaped cone molten socketing material is poured into the socket. The wires and the molten socketing material form a cone. The used socketing material is either zinc or resin.
Socketed end fittings can be used for all cable sizes, for cables smaller then 38mm often swaged end fittings are used.

Swaged end fittings

Swaged end fittings are cheaper than socketed end fittings and can only guarantee to resist 95% of the minimum breaking load of the cable. Swaged end fittings are more often used for smaller cables which are appropriate for secondary structures.



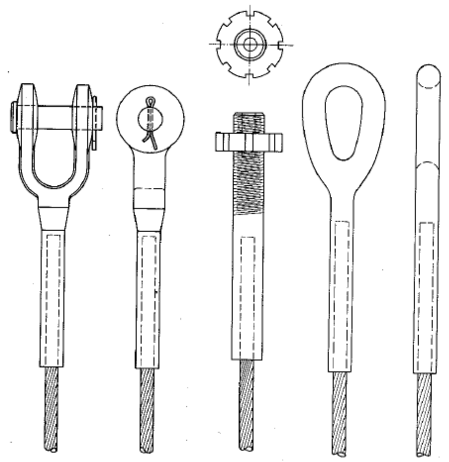


Figure 11.22 Types of socket end fittings (left) and swaged end fittings (right). Reproduced from [3]

For the end fittings in the cable structure socketed end fittings have to be applied. In the structure large cables are used (LC 101,5), therefore socketed end fittings are the only suitable option. In figure 11.8 the connection between the socketed end fitting and the outer supporting ring structure is illustrated. In table 11.5 the advantages and disadvantages of the socketed and swaged end fittings are summarized.

Cable end fittings	Advantages	Disadvantages
Socketed end fittings	• Wide application measurement range • Can resist very high forces	• Very expensive joint
Swaged end fittings	• Can resist 95% of the breaking load of the cable	• Expensive joint • Only applicable for cables smaller then 38mm.

Table 11.5 (Dis-)advantages cable end fittings

11.6.2 Connection inner ring

The second connection is the connection between the radial cable and the inner ring cable. For this connection a socketed end fitting with a pin connector is used. The pin is connected to the steel cast of the inner ring (8*locked coil rope of 51mm) that keeps the cable together. The connection is illustrated below.

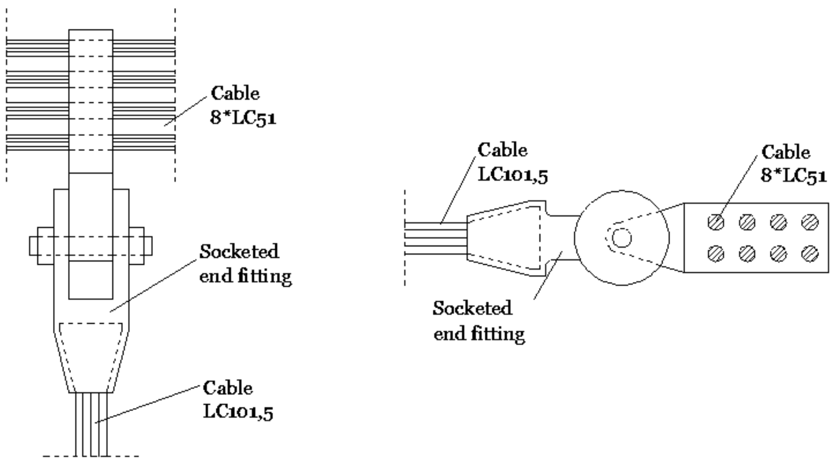


Figure 11.23 Connection radial cable – inner ring

11.6.3 Connection struts

The last connection is the connection between the struts and the radial cable by means of clamps. For the struts regular steel profiles are used. The struts are only subjected to tension. The connection is illustrated below.

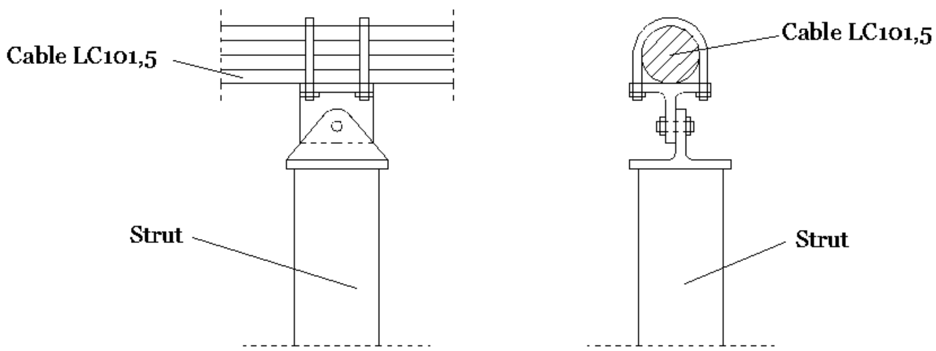


Figure 11.24 Connection strut – radial cable

11.7 Conclusion

The basis of the investigated cable designs is the preliminary design. Unlike the truss structure, the cable structure will not provide stiffness due to beam action. The only additional stiffness can be provided by ring action. Because cables behave different from regular steel profiles, first the behaviour of a cable structure with the shape of the preliminary design has been investigated.

Structural Design

For the structural design a cable beam structure has been used. The design of a cable beam structure depends on the following design variables: curvature of the cables, type of cables, dimension of the cables, level of pretension and stiffness of the supporting structure.
For the structure locked coil ropes with a diameter of 101,5 mm have been used. These cables have the highest breaking strength and therefore have been applied. The suspension cable has a curvature of 5% and the prestressing cable 3%. The amount of pretension differs in both cables. For the suspension cable a pretension value of 1000 kN has been used and for the prestressing cable 600 kN. For the supporting structure RHS profiles are used.
The results show that the stiffness requirements are leading. The stiffness of the structure relies on the amount of ring action. Because no ring action is present at the straight sides, large deformations arise at these points. Due to the great deformations, the unity check values of other elements regarding the strength and stability increases. To provide a structural design that fulfils all requirements, the stiffness of the roof itself needs to be increased.

Optimization

The stiffness of the structure can be increased by increasing the pretension in the spokes and by increasing the stiffness of the supporting structure by using stronger elements.
After a numerous amount of iterative steps of increasing the pretension and stiffness in the supporting structure, some conclusions could be made. Where curvature is present (corners), increasing of the pretension in the cables results in a decrease of the sag of the cables. The reason is that the increase of the pretension of the cables in the corners of the roof structure only has a small effect on the radial deformation of the outer ring. Because the outer rings possess curvature at these points, it can take up the extra loads due to the ring action. When the pretension will be increased at the straight sides, it has an opposite, negative effect resulting in greater deformations.
Because there is no ring action at the straight sides, the supporting structure needs to have a very high stiffness. This can be provided by using very strong ring elements. The calculation results showed that the moment of inertia needs to be extremely high in order to fulfil all structural requirements. As a consequence a lot of steel is needed and the structural design becomes inefficient.
To increase the efficiency of the structural design more research is needed. An option is to change the cable pattern to increase the use of the ring action (for example figure 11.19) or to find a solution to provide very high stiffness in the supporting structure and still use a low amount of material.

Structural Resilience

The structural resilience of the cable structure has not been investigated. The reason is that when a part of a cable is damaged, the pretension in the cable is lost and very large deformations will occur. In a spatial truss design, the steel elements are more able to diminish the deformation. Besides, in a truss structure the loads have more options or ways to be transferred to the rest of the structure.

Overall conclusion

From the results an answer can be given to sub-question 7 which is as follows:

'Is a spoke wheel roof still efficient and attractive to be used for football stadia roofs?'

By using a cable beam structure for a circular spoke wheel roof, a very light weight structure can be designed. However, the stiffness of a cable structure is very dependent from the available ring action. A football stadia roof has less curvature, resulting in a less efficient structure. To come to a design that fulfils all structural requirements, a lot of pretension and steel is needed to provide sufficient stiffness.
Unlike the truss structure, a cable structure cannot use beam action to increase its stiffness. It can be concluded that it is possible to provide a cable design for the reference stadium that fulfils all structural requirements. However, it is not possible to come to an efficient design using only the ring action for the investigated shape.
More research is needed to decrease the amount of needed material and pretension. Possible research options are the use of a truss structure to increase the stiffness of the supporting structure (figure 11.20), or making direct connections of the cables with the curved ring elements (figure 11.19). It is advised to use a cable structure only for shapes that possess curvature in the complete roof.

12. Construction

The construction of a building can be executed in several ways and is depended from the type of structure, the budget, planned time, etc. In order to guarantee the safety of construction workers, the construction of a stadium needs to fulfil the Eurocode requirements. The Eurocode describes that a structure must be designed and constructed in a way that all loadings and influences, which can appear during construction, can be resisted and will stay suitable for its intended use.

12.1 Truss structure

For the final truss structure the construction method is described in this paragraph. The Feyenoord stadium has been used as reference project. The way of constructing the roof structure will be as followed:

Step 1 – Assembly of the components

To construct the roof structure, the roof can be divided in components. The components are preassembled outside the stadium and will be placed on the stands of the stadium in a later stage.
A total of 14 components are made and are almost all of equal size. The components are illustrated in figure 12.1.

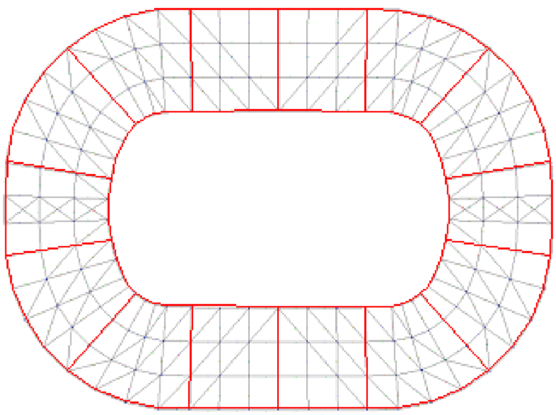


Figure 12.1 Components of the roof structure

Step 2 – Temporary columns/structure

To place the components on the stands, the components need to be stabilized. By using temporary columns or steel structure during construction, the inner ring will be supported (figure 12.2).

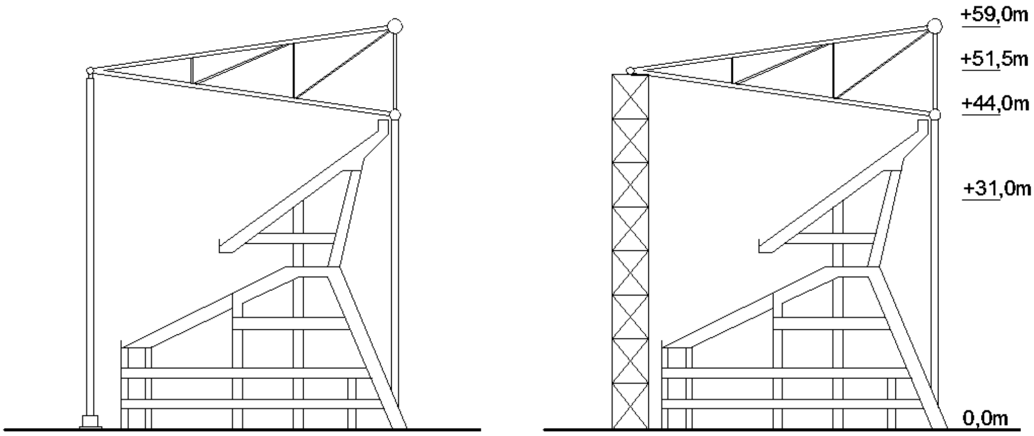


Figure 12.2 Temporary columns or a temporary steel structure (right) supporting the roof structure (left)

After removing the temporary columns/structure, the roof will sag due to its own dead load. By taking into account the deformation of the roof due to its dead load for each point on the inner ring where the temporary supporting columns /structure are placed, the roof can be set at a constant equal height along the whole inner perimeter of the roof after removing the temporary columns/structure.

The roof consists of 50 spokes, which is divided into 14 components (figure 12.1). The components are supported at every corner at the inner ring. Therefore 14 temporary columns/structures are needed (figure 12.3). From the calculation results, the deformation due to the dead load will be used to determine the height of the columns/structure. The total height is equal to:

Total height = Height inner ring + sag due to dead load

The inner ring must be placed at a height of 51,50 m. The deformation due to the dead load of the roof structure is presented in table 12.1. Using figure 12.3 the deformation is given for every node at the inner ring where a temporary supporting structure is needed. The presented values in table 12.1 are the deformation values of truss structure variant 1.

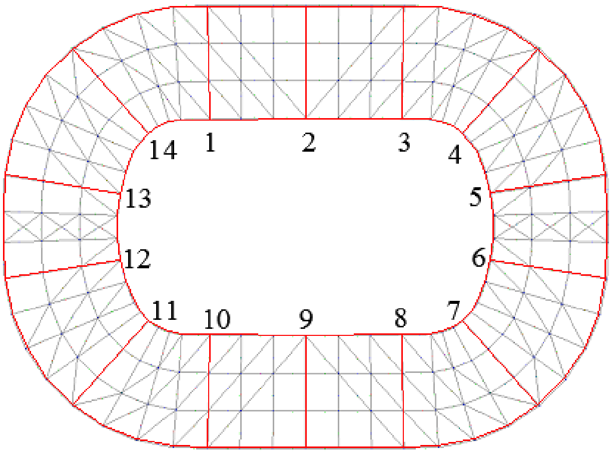


Figure 12.3 Supporting structure numbers

Nr.	Def [mm]
1	269,60
2	374,50
3	270,10
4	146,50
5	143,70
6	144,00
7	146,70
8	270,10
9	374,10
10	269,90
11	144,40
12	142,10
13	144,10
14	146,70

Table 12.1 Deformation inner ring

The columns/structures are subjected to large compression forces due to the dead load of the roof structure. When columns are used, the engineer needs to take the buckling capacity of the columns in to account. The temporary columns must have sufficient strength and stiffness to prevent the columns from buckling.
Due to the great height of the reference stadium, it is advised to use a temporary steel structure. The temporary steel structures have a broader base and possess more strength and stiffness compared to the temporary columns.

Step 3 – Lifting of the components to its final place

The third step is the lifting of the components to its final place. With the help of crane at the outside of the stadium, the components are placed on the rocker bearings and the temporary columns. By using an extra crane at the inside of the stadium, the components can be accurately guided to its final place (figure 12.4).

Figure 12.4 Placement of the units on top of the Feyenoord stadium. Reproduced from [26]

Step 4 – Repeat step 1-3

Step 1-3 must be repeated until all components are placed. An advantage is that step 1 can be executed independent from step 2 and 3. Time can be saved when one component is finished by the assembly of the following component.

The connection between the components is executed by construction workers on site. Specially trained construction workers are needed, to connect the components at these heights. The disadvantage is that these construction workers are expensive.

Step 5 – Closing the roof structure

The last component is the most challenging component to place it on its final position. The final gap is left open. In theory it is possible to place a complete component in the final gap. In practice, the final component is not expected to fit perfectly. The variable space can be either too narrow or too wide. In order to fit the final component, a narrow space is left open on purpose. By adding extra space to the structure, the final structure can be lifted on its final place. The structure is closed by pulling the final elements together, by using for instance ropes. Construction workers can attach the elements by applying bolts (chapter 10.7).

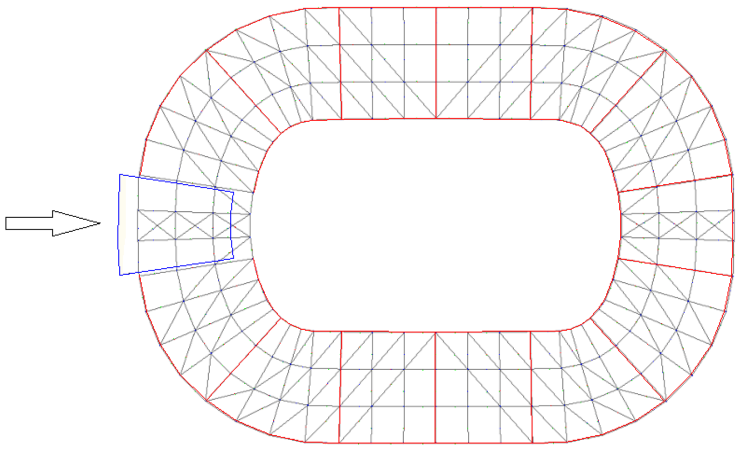


Figure 12.5 Closing of the structure

Step 6 – Removing of the temporary structure

When all components are placed, the temporary structure will be removed. The roof structure is supported by a temporary structure placed on jackets. The support of the whole roof will happen at once. The pressure of every jacket will be lowered simultaneously.
Every jacket need to resist a different load. All jackets need to lower the pressure to zero in the same amount of time in order to release the structure all at once. For example one jacket supports 80 ton and another jacket 50 tons of steel. Both jackets must release the pressure in the same amount of time until the pressure reaches zero. When the roof structure is floating on its own, all temporary supporting structures can be removed.

Step 7 – Assembly of the roof covering

When the mean bearing structure is finished, the roof covering can be placed. For the roof covering steel sheeting is used. The steel sheets are lifted by cranes and are placed on the bottom spoke level (figure 9.14) with the help of construction workers.

12.2 Cable structure

For the cable structure the construction method is described in this paragraph. The Commerzbank Arena stadium has been used as reference project. The way of constructing the roof structure will be as followed:

Step 1 – Construction of the outer ring

The construction of a tensile-compression ring roof starts with the construction of the outer ring. The outer ring, a compression ring, is made of steel or concrete. The outer ring consists of elements that are often prefabricated and are placed by cranes on site (figure 12.1).

Step 2 – Assembly on site

When the outer ring is set in place, the inner ring and the spokes can be constructed to come closer to a final roof structure. Due to the fact that the inner ring 'floats' above the field and the structural interaction with the outer ring, it is a challenge to construct the inner ring.

The cables are often prefabricated and are assembled on site. The cables are first attached to the outer ring using end sockets. The cables are then laid out on the stands and the field. At the centre the cables are connected with each other with cast steel clamps and ring connectors. This way a single cable net is produced.

Step 3 – Lifting of the cable structure

The next step is to lift the cable net, this will happen simultaneously. The radial cables are connected by means of temporary strands to lifting jacks that are fixed to the compression ring. By pulling these temporary strands, the complete cable net can be lift of the floor, to its final position (figure 4.10, Reference projects).

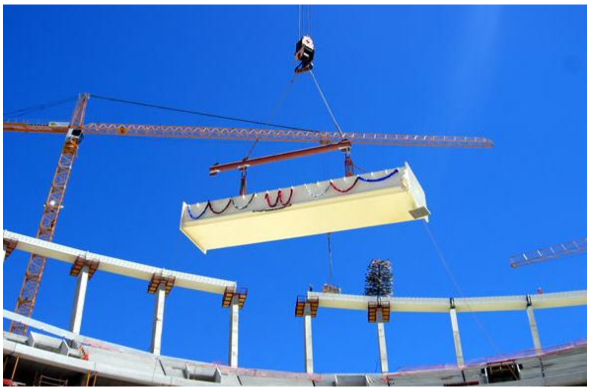



Figure 12.6 Construction concrete compression ring at Greenpoint stadium (left) and construction of cable net structure at the Commerzbank Arena (right)

The complexity in building this system lies in the intricacy in the interaction between the cable net structure, the outer ring and the erection of the structure. During construction, pre-stressing of the cables will cause a considerable deflection and rotation on the rings. This deformation needs to be considered when erecting the cable net, to ensure that the final configuration of the ring would only be obtained after the cables are pre-tensioned. To predict the effect the cable tensioning on the ring, a time history computer analysis program must be used.

Step 4 – Assembly of the roof covering

After completion of the main bearing structure of the roof, the roof covering can be placed. The type of roof covering depends on the structural design of the roof. In chapter 7.2.3 different types of roof covering is described.
The roof covering is assembled bay by bay and special trained construction workers are used for the assembly at great heights.

12.3 Building time

In the building industry, especially when it concerns the construction of a stadium, a lot of money is spend these present days. The building time has a significant influence on the costs. The longer the building time of a structure, the more the costs of a structure increases. To prevent high costs, the planning is very tight. Another reason for the tight planning is that often financial penalties are used when the planned construction time of a structure is exceeded.

A reason for engineers to choose for a tensile-compression ring roof structure is the building time of such a structure. Especially when a cable structure is applied, the building time diminishes rapidly compared to conventional stadium roof structures.
For example, the lifting from the roof structure of the Commerzbank Arena only took two months. The work included the installation of the steel roof structure and the rope supporting framework [26].

When using a cable net structure, the roof is very light. The elements are easier to transport and assemble on the site. The lifting of the roof does not take a lot of time. A tensile –compression ring roof structure composed of a spatial truss system is not as light and easy to handle but still it is possible to decrease the building time. The structural elements can be prefabricated and assembled in the factory and transported to the building site. At the building site the roof structure can be assembled further into units outside the stadium (like at the Feyenoord stadium). The units then can be lifted by cranes and can be placed and assembled on the right spot in the stadium.
The use of units can save time, because major surveying can be avoided. When a stadium roof structure is build element by element, this will cost a lot of time surveying and it is more difficult to assemble.

13. Building costs

The building costs of a project are of major importance in the design phase. In the design process of the tensile-compression ring the costs are one of the parameters that will be taken into account.

The price per unit of the different building materials is fluctuating in time. During the crisis the unit prices of the materials were very high and now have decreased. To determine the unit prices of the different materials assumptions will be made for a recent (July 2011) unit price.

In the following table an assumption of the basic unit prices for different steel grades are given:

Steel grade	**€/kg**
S235 (Truss)	1,20
Y1770 (Cable)	3,00

Table 13.1 Basic unit prices for steel grades

On top of the basic unit price of the material additional costs need to be taken into account. Before steel can be used as a building material it must be fabricated into profiles. The material will be shaped in the needed form (CHS/RHS/Cable) and will be coated and galvanized.

Additional costs	**€/kg**
Galvanizing	0,40
Epoxy coating	1,50
CHS/RHS profiles	0,40
Cables	0,60
Assembly CHS/RHS profile	1,50
Assembly Cables	2,50

Table 13.2 Additional costs on top of the basic unit price

The prices displayed in table 13.1 and 13.2 are prices for every kg steel that will be used for columns or bars. All mentioned prices above are direct building costs. This means the costs without the general building costs, general costs, profit and risk (nl: ABK, AK, W and R). When these prices will be taken into account the direct costs must be increased with 25%.
Further costs like the engineering-fees, unforeseen costs, investment costs, etc. will not be taken into account.

13.1 Truss structure

The total weight of the three truss design variants are presented in table 13.3. These values are used to determine the total costs of the structures.

	Variant 1	**Variant 2**	**Variant 3**
Average dead load [kg/m²]	79,90	83,70	92,61
Total area [m²]	21.960,09	21.960,09	21.960,09
Total dead load [kg]	**1.754.604**	**1.838.052**	**2.033.707**

Table 13.3 General information variant 1-3

The costs of the structures are assumed to be directly proportional to the used amount of steel. The total amount of costs for every variant is presented in table 13.4.

	€/kg	Variant 1	Variant 2	Variant 3
Steel S235	1,20	2.105.524,80	2.205.662,40	2.440.448,40
CHS profile forming	0,40	701.841,60	735.220,80	813.482,80
Galvanizing	0,40	701.841,60	735.220,80	813.482,80
Epoxy Coating	1,50	2.631.906,00	2.757.078,00	3.050.560,50
Assembly	1,50	2.631.906,00	2.757.078,00	3.050.560,50
Total costs/kg	*5,00*	*8.773.020,00*	*9.190.260,00*	*10.168.535,00*
Total costs overall	+25%	**€ 10.966.275,00**	**€ 11.487.825,00**	**12.710.668,75**

Table 13.4 Total costs variant 1-3.

Not every design variable has been taken into account into the determination of the total building costs. The costs due to construction, transportation, etc. are still unknown. A financial disadvantage of the designs is that the structures consist of many different profiles. Due to the optimization of the truss structures, every single element has a different thickness or diameter. It costs extra time and money to place each profile at its correct place. Besides, there is no benefit in standardization of the profiles. It is assumed that saving material by optimization is still the most beneficial solution although there are extra costs due to using unique elements.

Another variable that has not been taken into account is the use of complex nodes. The more bars are connected to a single node, the greater the complexity and the higher the costs. The costs increases because the nodes are more labour intensive. All three variants consist of an equal amount of nodes (400), but from an unequal amount of bars. In this situation can be assumed that the more bars are applied; the more complex nodes are present. Variant 1 and 2 almost consists of an equal amount of bars (respectively 1174 and 1158) and the costs due to the complexity of the nodes are assumed almost equal. Variant 3 consists of the greatest amount of bars (1382) and therefore is most costly.

It can be concluded that variant 1 is still the most beneficial variant, followed by variant 2 and 3 (table 13.1). The ratio between variant 3 and variant 1 and 2 is somewhat different than presented. Due to the greater amount of complex nodes, the total building costs of variant 3 are higher. Costs due to construction and transportation are assumed to be directly proportional to the used amount of material. The costs ratio between the variants will therefore not differ much.

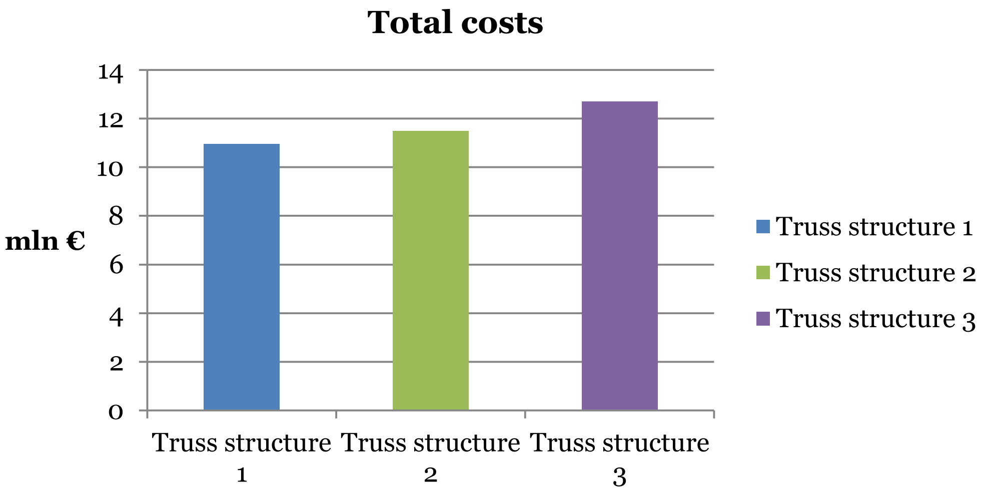


Figure 13.1 Total costs truss structures

13.2 Cable structure

From chapter 11 has been concluded that more research is needed to come to an efficient design that uses a cable roof structure. Because of the little amount of available ring action, the cable structure needs a lot more material compared to the truss structure to fulfil all structural requirements. Therefore the total costs of the cable structure are not taken into account.
Table 13.1 and 13.2 show that the unit prices for cables are substantially higher compared to regular steel profiles. By looking at the costs of the truss structures one can estimate the amount of needed steel in a cable structure to become financially attractive compared to the truss structures. The composition of the unit price of a cable roof structure is presented in table 13.5. The building costs information of cables is reproduced from Spanstaal.

	€/kg
Steel Y1770	6,00
Cable profile forming	1,50
Epoxy Coating	1,50
Assembly	2,50
Total costs/kg	*11,60*
Total costs overall (incl. 25%)	14,38

Table 13.6 Costs of a cable

	€/kg
Steel S235	1,20
RHS profile forming	0,40
Galvanizing	0,40
Epoxy Coating	1,50
Assembly	1,50
Total costs/kg	*5,00*
Total costs overall (incl. 25%)	6,25

Table 13.5 Costs of RHS profiles

For the estimation is assumed that the ratio amount of used steel for RHS profiles (supporting structure) and for cables is 1:1. This ratio depends on the efficiency of the structure. Is the available ring action used at full extend or is the supporting structure stiffened in order to fulfil all requirements? It is very difficult to determine the ratio, because it is dependent from many factors.
The average unit price is (14,38+6,25)/2=10,31 euro/kg. By dividing the total costs of truss variant 1, one can determine the amount of steel:

$$\frac{10966275{,}00}{10{,}31} = 1063396{,}36\ kg \tag{13.1}$$

The average dead load of the structure is therefore:

$$\frac{1063396{,}36\ kg}{21960\ m^2} = 48{,}42\ kg/m^2 \tag{13.2}$$

A cable structure with an average dead load of 48,42 kg/m² is needed to become financially attractive compared to truss structure variant 1. More research is needed to design a cable structure for a spoke wheel roof of a football stadium that uses this small amount of material.

Conclusions Detailed Design

The conclusions from the part Detailed Design are briefly summarized for each chapter.

Chapter 9 – Loads

- The strength, stability and stiffness of the structure are checked according to the Eurocode (NEN-EN 1993-1-1). These are presented in appendix A.10.
- The stiffness requirement is adapted for the use of stadium roof structures. The total additional deformation of the roof is set to a 1000 mm using a reference project (Tunis stadium). The allowed relative deformation of a single beam placed on two supports both ends is 1/100.
- For the wind load, the wind load pressure values of the Amsterdam ArenA are used.
- The influence of the wind load is determined for two situations; wind suction at the long side of the stadium and on the short side. The wind load pressure values are presented in figure 9.11 and 9.12.
- The variable loads act on the roof covering, which is placed at the bottom spoke level. The loads are then transferred to the bottom ring elements and further to the underground (figure 10.9).
- Other loads that are taken into account are the snow load (0,56 kN/m^2), the roof covering (0,05 - 0,30 kN/m^2) and the permanent installations (0,10 kN/m^2).
- Four load combinations are used for the calculations and are presented in table 9.6 -9.9.

Chapter 10 – Truss structure

- From the preliminary design, three non-linear stable structures have been made in order to come to an efficient structural design.
- For the first design the designer has tried to use the available ring action at full extend. The second design is similar to the Feyenoord stadium. The stiffness of the design is more provided by beam action. The third design can be seen as a shell structure. The forces flow through the elements that provide the most efficient way of transfer.
- The three designs are further optimized using parametric modelling. At first the diameter of the leading element of every group is determined. After three iteration steps, the next parametric modelling step follows. Using the FEM software, the thickness of every single element is determined in order to use as little material as needed.
- Truss structure variant 1 uses the least amount of material, followed by variant 2 and 3. The average dead load of variant 1, 2 and 3 are respectively 79,90 kg/m^2, 83,70 kg/m^2 and 92,61 kg/m^2.
- Variant 1 and 3 are most efficient, regarding the efficiency value which is determined by multiplying the average dead load by the average deformation of the inner ring due to a constant load distribution. Variant 2 is the least efficient structure.
- All three variants are not sufficient structural resilient when a leading element is damaged. To become structural resilient a proposition of a structural design has been presented in figure 10.31 or stronger elements need to be used.
- For a spatial truss structure, like variant 1-3, the best option is to use a bolted connection similar to the Octatube, Merodeck, etc. systems. These types of connections have a high aesthetical value, are easy to clean, easy to assemble and are cheap.

Chapter 11 – Cable structure

- The stiffness is leading in the design of cable roof structure.
- The stiffness depends on the following variables: curvature cables, type and dimension cables, stiffness supporting structure and amount of pretension.
- The first design lacked in fulfilling the structural requirements. The amount of pretension and stiffness of the supporting structure need to be adapted. The stiffness of the cable structure is very dependent from the available amount of ring action.
- To prevent the cables from going slack a very high value of pretension is needed in the corners of the structure (100.000 kN) and the outer ring elements at the straight side need a very high moment of inertia (10 m^4). These values are too high to apply for a realistic design.
- More research is needed to provide a very stiff supporting structure which uses less amount of material. More important is to investigate how to use the available ring action better so more material can be saved.

Chapter 12 – Construction

- The way of constructing of the truss structure is as followed: the roof is divided into 14 pre-assembled components, the components are lifted and placed on temporary structures; extra space is left open for the placement of the final component; by means of jackets the total roof structure will be released from the temporary supports; all temporary steel structures are removed; the final step is the assembly of the roof covering.

- The way of constructing of the cable structure is as followed: first the outer ring is constructed; the cable net structure is assembled on site and laid down on the stands and the field; after the assembly of the cable net structure, the cable net is lifted simultaneously to its final position; the final step is the assembly of the roof covering.
- The building time of the truss structure and especially the cable structure is very short compared to conventional stadium roof structures. Because the building time is short, the construction costs can be kept low and penalties can be prevented.

Chapter 13 – Building costs

- For the determination of the total costs of the roof structure is assumed that the costs are directly proportional to the amount of used steel. The costs are described in table 13.1 and 13.2.
- For the determination of the total costs has not been taken into account: construction, transportation, assembly and complexity of the nodes.
- The costs of truss structure variant 1,2 and 3 are respectively € 10,97 mln, € 11,49 mln and € 12,71 mln.
- The total costs of the cable structure cannot be determined. More research is needed to come to an efficient cable roof structure.
- The cable roof structure needs to have an average dead load of 48,42 kg/m^2 , assuming that the cable/RHS material use is equal, to become a financially attractive structural solution compared to truss variant 1.

Part 5
Conclusions & Recommendations

Conclusions & Recommendations

In this final part, Conclusions & Recommendations, the total report will be discussed. What are the conclusions for this thesis and what are the recommendations for this research?

14. Conclusion

In this thesis a study is made for the use of the spoke wheel principle for football stadia. The research question is as followed:

'To what extend is the application of the spoke wheel principle feasible and efficient for football stadia roof structures?'

For the research designs of a spoke wheel roof structure has been made, in order to gain knowledge and insight in the use of the spoke wheel principle for non-circular roofs. The research question will be answered after a summary of the conclusions from the different parts of the thesis.

Analysis

The spoke wheel principle is based on the theory of the ring action. Ring action will arise due to the presence of a curved rim with a certain extensional rigidity and radial force acting on the rim (equation 2.9). The bicycle wheel is a very lightweight structure that uses the ring action at full extend.
When the spoke wheel principle is used for roof structures the conditions will change; transverse loading is leading and the structure is supported at the outer ring instead of the hub in case of a bicycle wheel. The working of the ring action for spoke wheel roof structures depends on the following key factors:

1. Curvature
2. Loads acting on the ring
3. Extension rigidity ring
4. Translation of the ring

The present amount of curvature of a stadium roof is leading for the amount of ring action that can be provided. There will be ring action when the curved ring is subjected to stress. The ring must be able to deform or translate in a radial direction in order to let the ring action work. When the translation is blocked, the forces are not taken up by ring action any more. These factors determine the success of the ring action in a stadium roof structure.

The design of a spoke wheel roof of a football stadium depends on certain design variables that influence the ring action that will work in the roof. These variables are the following and are related to the described key factors:

1. Shape of the (opening of the) roof
2. Double inner / outer ring
3. (Non) pre-tensioning of the spokes
4. Supports / connections
5. Profile / elements

The relation of the design variables to the four mentioned factors that influence the ring action are presented in figure 14.1 and are further explained in the following part.

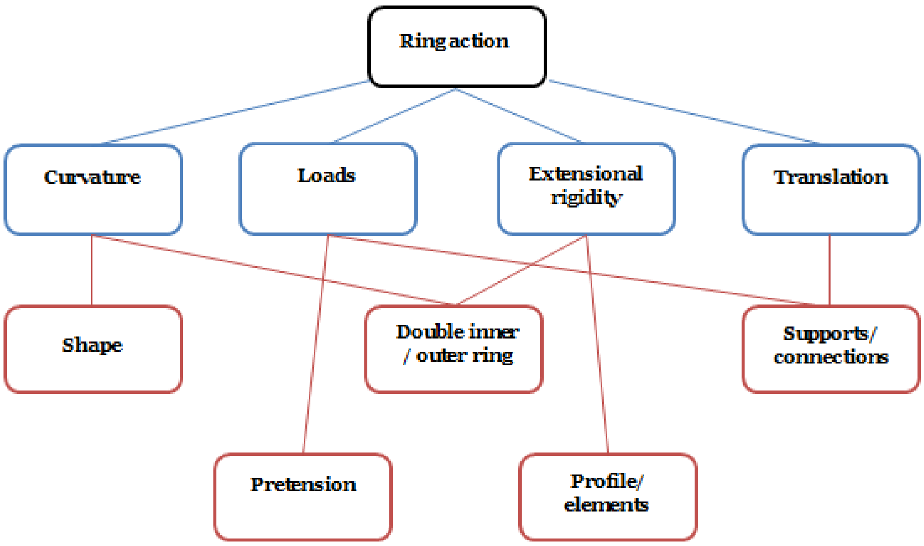


Figure 14.1 Relation between design variables and ring action

Preliminary Design

To gain insight in the behaviour of a non-circular spoke wheel roof a design of a spoke wheel roof is made for a football stadium. No stadium is alike, therefore a ground plan of a reference stadium has been used for the research: the Amsterdam ArenA. The ground plan of the Amsterdam ArenA consists of an oval shaped outer perimeter, with straight sides and an inner rectangular roof opening (figure 6.1). The straight sides will not provide any ring action. To become an efficient design, the available amount of ring action has to be used at full extend. By investigating the consequences of the design variables on the reference stadium, a more efficient design can be made.

The first design variable that has been investigated is the shape of the outer and inner ring. The key aspect of providing as much ring action as possible is to have sufficient curvature in the ring. The shape of the outer ring is fixed. The opening of the roof, or inner ring, has been adapted in order to provide as much curvature as possible. It was possible to apply curvature with a maximum radius of 72,72 m at the short side and 2868,06 m at the long side. The amount of curvature at the long side therefore remains very little.

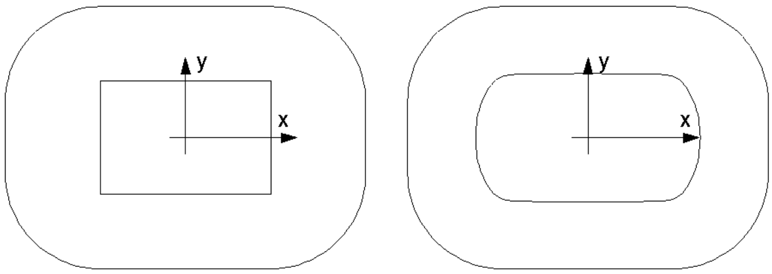


Figure 14.2 Change of the shape of the inner ring

The second design variable is the choice of supports and connections. The outer ring in a spoke wheel roof structure is placed on supports, unlike the bicycle wheel which is only supported in the central hub. In order to let the ring action work, the ring must be able to translate in its radial direction. When the translation is blocked, the loads are not taken up by ring action, only by the strength or stiffness of the elements itself.
The ring needs to be supported by rocker bearings or roll supports. Rocker bearings have the advantage that they are more able to take up high tensile forces and are relatively cheaper than roll supports.

The way of connecting the elements in the roof structure influences the way the ring is subjected to loads. Between the ring and spoke elements it is possible to apply hinged and fixed connections. Although fixed connections show higher stiffness and strength results, hinged connections are preferred to use. The reason is that in case of fixed connections the ring becomes subjected to bending moments. A part of the stiffness is then provided by beam action instead of ring action. The loads are taken up by bending moments instead of normal forces and therefore will work inefficient. Besides, smaller profiles can be used when the amount of bending moments (beam action) are reduced.
The ring elements however, are fixed connected. The sides of the structure have a low stiffness capacity due to the lack of curvature. Large deformations will arise when hinged connections are applied and the structure becomes unstable.

The following design variable is the use of a double inner or outer ring structure. As described in previous chapters, the amount of ring action is heavily dependent from the amount of present curvature. Although the inner ring possesses more curvature compared to the outer ring, a double outer ring will be applied to the structure to increase the transverse support. The amount of ring action is not only dependent from the amount of curvature. Acting loads, extensional rigidity and translation is also of importance.
When a double inner ring structure is used, more loads will act on the single outer ring structure. Because the outer ring structure has little stiffness that can be provided by ring action, large radial deformations arise at the sides of the outer ring. As a consequence, the inner ring will further deform radial as well as transverse. To prevent large deformations, the stiffness of the outer ring needs to be increased. By adding another ring to the outer ring, the available curvature will provide more stiffness to the structure. Besides, the extra ring increases the amount of extensional and bending rigidity.

The fourth design variable is the choice of profile and materials. Steel has the possibility to take up compression as well as tensile forces. Concrete is only preferable in case of high compression forces. Therefore it is preferred to use steel as structural material.
The choice on the kind of profiles depends on the final design variable: pretensioning. To provide stress in the ring, the designer has the option to provide this stress by pretensioning the spokes (bicycle wheel) or only by the dead load of the rest of the structure that automatically provides a radial load on the outer ring.

When pretensioned spokes are applied, cables are very suitable to take up high normal forces. By using RHS profiles for the outer ring, a great amount of stiffness can be provided in the radial direction. However, when cables are used for the spoke wheel roof, the stiffness cannot be provided by means of beam action. The question is if it is possible to use the available amount of ring action in the reference stadium to come to a stable and safe structure that fulfils all structural requirements? The next challenge is to investigate whether such a structure is still efficient for roofs similar to the reference stadium.

For non-pretensioned spoke wheel roofs, it is advised to use a spatial truss system that consists of CHS profiles. The greatest advantage of these profiles is its uniform stiffness and good possibilities for optimization. When the spokes consist of regular steel profiles, beam action will play a role in the amount of stiffness in the roof structure. It is interesting to investigate if the available ring action is sufficient to come to a stable design or if the engineer needs to use the beam action? If extra beam action is needed, the following question rises to what extend? And is the structural design than still efficient?

A design of both types of structures is made in following part to answer these questions.

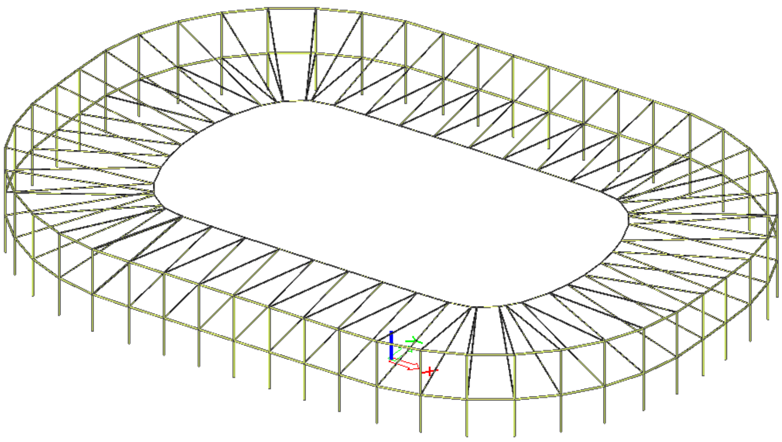


Figure 14.3 Preliminary Design

Detailed Design

The designer has the option to apply either pretensioned spokes (cables) or non-pretensioned spokes (regular steel profiles) in order to provide stress in the rings. The first option is to use non-pretensioned spokes by means of a spatial truss structure.

Truss structure
A truss structure can gain its stiffness, next to ring action, by beam action. By making three different models, each with a different ratio ring : beam action, it was possible to investigate the role of the ring action in the total stiffness of the non-circular structure compared to beam action.
From the deformation results of all three variants can be concluded that the distribution of the stiffness in the roof, due to its shape, is not constant. In all designs beam action does play a role, where each variant uses a different proportion of beam action.

Variant 1 has shown that, although there is little curvature present in the reference stadium, it pays off to use the available ring action at full extend. Although the differences between the average dead load of variant 1 and 2 are little, it becomes substantial when looking at the total amount of dead load (difference of 85 tons of steel).
The normal force distribution in variant 1 and 3 shows that by directly connecting the weakest points of structure (present at the straight sides) with the curved ring elements, the ring action can be used for a greater area of the roof (figure 14.4).
Variant 1 has shown that for the shape of the reference stadium it is still attractive to use a spoke wheel roof structure. However, the use of the spoke wheel principle has its limits. The small difference in average dead load between variant 1 (ring action) and variant 2 (more beam action), shows that when less curvature is present, less can be benefited from the structural strengths of the spoke wheel principle. The role of beam action comes more into play when there is less curvature in the roof structure. At a certain point, when the amount of curvature is minimal, the spoke wheel principle is no more applicable.

The advantage of using a spatial truss system is that it is very suitable for spoke wheel roofs that are not completely circular. Because this type of structure can use beam action to provide extra stiffness, it is easier to come to a safe and stable design. Besides, due to the optimization possibilities for truss structures it is possible to use a minimum amount of material for non-circular shaped spoke wheel roofs.

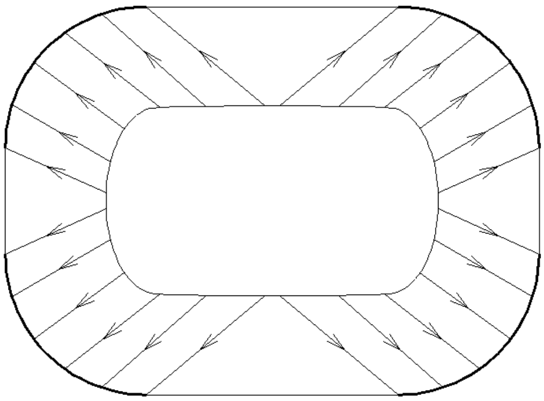


Figure 14.4 Transfer of the loads to the curved ring elements

Cable structure
Unlike the truss structure, the cable structure will not provide stiffness due to beam action. The only additional stiffness can be provided by ring action. The results show that the stiffness requirements are leading. Because no ring action is present at the straight sides, large deformations arise at these points.
The stiffness of the structure can be increased by the following design variables: the curvature in the spokes, the use of stronger cables, increase the pretension in the spokes and by increasing the stiffness of the supporting structure. These design variables all have a certain influence on the total deformation of the roof structure. From the theory regarding cable structures has been concluded that the curvature of the suspension cable is 5% and for the prestressing cable 3% in a concave cable beam. For the cables the strongest possible cable is needed to provide as much possible stiffness.
The amount of pretension needs to be sufficient enough to prevent the cables from going slack, but cannot become too large to prevent large deformations of the ring. Especially the pretension in the spokes connected to straight ring elements must be minimized. The adjacent outer ring elements are not able to transfer the high radial forces efficiently (polygon of forces, figure 11.18), resulting in large radial deformations. The supporting structure (outer ring) at the straight sides have to have a high bending rigidity to provide sufficient radial stiffness. In the corner areas where sufficient curvature is present in the ring elements, large pretension is required. The large pretension in the corner spokes will decrease the amount of deformation in the complete inner ring. The adjacent ring elements are able to efficiently transfer the loads (ring action).

The calculation results showed that the bending rigidity of the supporting structure and the amount of pretension needs to be extremely high in case of the reference stadium in order to fulfil all structural requirements. As a consequence a lot of steel is needed and the structural design becomes inefficient.
The results show that it is not beneficial to use a cable structure for a spoke wheel roof for the reference stadium. It can be concluded that because a cable structure can only provide stiffness by ring action, that such a structure should only be applied for structures where there is sufficient curvature present. The amount of needed curvature to come to an efficient design cannot be determined at this point. More research is needed to answer this question.

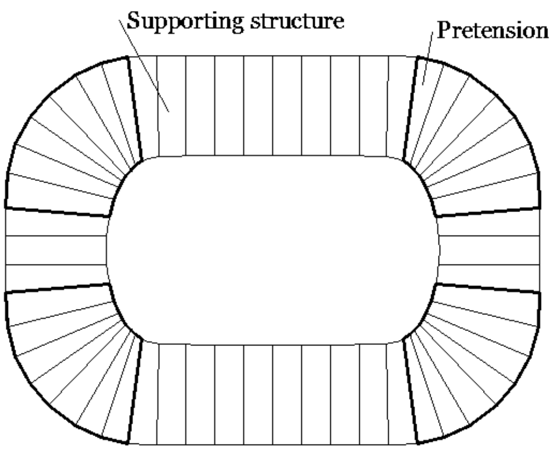


Figure 14.5 Areas that need high pretension (corners) and extra stiffness for supporting structure (sides)

Reference projects have shown that it is possible to apply a cable structure for non-circular spoke wheel roofs. However, all these roofs have in common that extra stiffness has to be gained in order to fulfil all structural requirements. For instance the Gottlieb Daimler Chrysler stadium in Stuttgart is oval shaped and uses a truss structure along the long side of the roof to provide extra stiffness to the supporting structure. The Commerzbank Arena in Frankfurt actually consists of two interlocked spoke wheels to provide sufficient stiffness.
A spoke wheel roof has to have sufficient curvature to come to an efficient design by means of a cable roof structure.

More research is needed to determine for which situation (amount of curvature) an efficient design can be generated using a cable structure. Besides it is interesting to investigate if it is possible to come to an efficient cable roof design, for a non-circular roof, by adapting the structure. For instance by using a different cable pattern to increase the use of the ring action (for example figure 11.6) or to find a solution to provide a very large amount stiffness in the supporting structure (truss structure for example).

Overall Conclusion

After the conclusion from the previous parts an answer can be given to the research question. The application of the spoke wheel principle for football stadia, heavily depends on the presence of curvature. The general form of a football stadium is oval shaped. When straight sides are present, a lot more material is needed to provide a structure that will fulfil all structural requirements.

The feasibility of the spoke wheel principle for stadia roof structures depends on the shape conditions. A spatial truss structure has the ability to use the spoke wheel principle even in case of stadium roofs with straight sides. Due to beam action it is still possible to design an efficient structure.
As mentioned earlier, the use of the spoke wheel principle has its limits. The small difference in average dead load between the use of ring action at full extend (variant 1, 79,90 kg/m²) and the use of more beam action (variant 2, 83,40 kg/m²) for the reference stadium has shown that less curvature results in less benefits from the strength of the spoke wheel principle.

However, the results of the designs for the reference stadium has shown that by means of a spatial truss system the spoke wheel is an attractive type of roof structure for football stadia that possess a certain amount of curvature.
By using the available ring action at full extend and by means of parametric modelling it is possible to decrease the amount of needed material, resulting in a decrease of the building costs. However, the complexity of the structure (nodes), construction, standardization, transport, etc. has not been taken into account. More research is needed to determine the consequences of the complex design of the spoke wheel roof for the building costs.

To show how efficient a spoke wheel roof structure is, it is interesting to compare this type of structure with other types of stadia roof structures. In table 14.1 the average dead load of other type of stadia is presented. Although it is difficult to provide a fair comparison, the results show that the use of the spoke wheel principle is still attractive. The average dead load of truss structure variant 1 is 79,90 kg/ m² for a roof area of 22.000 m².

Stadium	Type of roof structure	Roof area [m²]	Av. dead load [kg/ m²]
Commerzbank Arena	Spoke wheel (cable)	38.000	80
Feyenoord stadium	Spoke wheel (truss)	15.000	90
Radés, Tunis	Stayed	30.000	100
Allianz Arena	Cantilever	38.000	140
Amsterdam Arena	Arch (retractable)	32.500	150
Stade de France	Stayed	60.000	200

Table 14.1 Properties stadia roof structures

A cable structure heavily relies on the amount of the available curvature and can only provide stiffness by means of ring action. A cable structure is still feasible for roofs with straight sides in a structural manner. However, looking from a financial and durability perspective the cable structure is not attractive for owners and is very inefficient. It is advised to use a cable structure only for circular or oval shaped spoke wheel roofs.

In table 14.1 a cable roof structure (Commerzbank Arena) is compared with other type of roof structures. The results show that this cable roof structure uses less material compared to other types of roof structures. What this roof and other cable spoke wheel roofs have in common, is that there is curvature present in the whole roof (circular or oval shaped roof). This way it is possible to still design a cable roof structure that uses a little amount of material. Otherwise measures need to be taken in order to become a stable structure.
As mentioned earlier, more research is needed to determine what amount of curvature is needed to come to an efficient cable roof design. Besides, investigation is needed for the possibility to increase the stiffness of the structures and still become an efficient design. Possibilities are: using different cable patterns, transport the available ring action better or to increase the stiffness of the supporting structure by means of trusses.

15. Recommendations

In this chapter recommendations are given for further analysis. The recommendations are ordered by subject.

Geometry preliminary design

For the determination of the preliminary design some assumptions have been made that require further investigation to come to an even more efficient design. The following assumptions have been made:

- Distance between the outer rings (d)
- Angle of the spokes (α)
- Height of the rocker bearings (h)

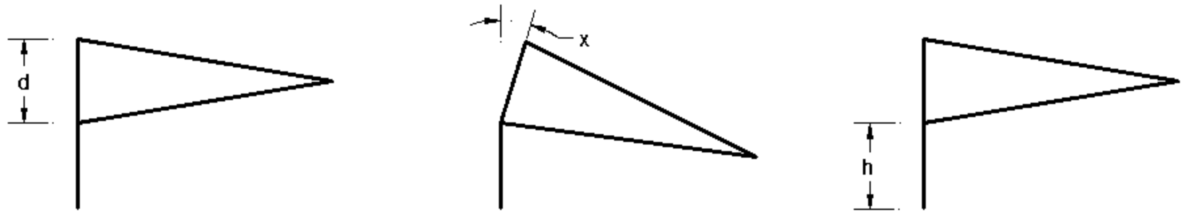


Figure 15.1 Design variables

The dimensions of the described design variables have been estimated using reference projects. When designing a spoke wheel roof, these dimensions need to be determined for the given situation and conditions.

Loads

For the overall design fatigue and dynamic loadings have not been taken into account due to the emphasis of the thesis which is the investigation on how to apply the spoke wheel principle for football stadia roofs. When a real structural design has to be made, the influence of fatigue and dynamic loads need to be taken into account.

Profile/elements

For the design only steel S235 and Y1770 for cables has been used. To determine the most efficient structural solution, different kind of steel grades need to be applied. By using a steel grade with a higher tensile/compression strength, less material is needed. The disadvantage is that the costs of using a higher steel grade are higher per m^2. Investigation is needed to determine whether it is financial beneficial to use a different steel grade.

Additional requirements

The design of the roof does not only have to fulfil structural requirements, additional requirements are important as well. The roof has to protect the spectators from the elements, like wind, rain and snow. To provide enough shelter more research has to be done by means of wind tunnel testing, etc. to find the best way of designing the shape of the roof.
The disadvantage of protection against the elements is that it decreases the quality of the grass. To contain a high quality of grass, grass needs to be subjected to sufficient wind and sunlight. However, with the latest technology it is possible to use artificial sunlight and wind. The consequence is that the exploitation costs of the stadium increases. Equilibrium has to be found in order to provide sufficient comfort to the spectators, the quality of the grass, building costs and a way to still benefit from the strengths of the spoke wheel principle.

Spoke wheel roof

A spoke wheel roof is a special type of structure that, as a roof structure, has great benefits compared to other type of roof structures. Its structural strength will progress with the increase of the amount of present curvature of a roof. When there is little curvature present, it is difficult to design an efficient roof using the spoke wheel principle. When one wants to benefit from the strength of the spoke wheel roof, it is advised to apply the spoke wheel principle to curved shaped roofs.

From the designs that has been made during this graduation project can be concluded that the amount of available curvature (ring action) in the shape of the roof structure, is leading whether a roof structure becomes efficient using the spoke wheel principle. For circular shaped roofs it is advised to apply the spoke wheel roofs, due to its great (structural) advantages.

In case of non-circular shaped roofs measures need to be taken to increase the use of ring action and thus the efficiency of the roof. Possible measures to increase the use of the ring action are the following:

- *Adapt the shape of the roof*
  To increase the efficiency of the spoke wheel roof, the amount of curvature in the whole roof needs to be increased. A solution is to support the roof structure independent from the stands of the stadium. By placing a self-supporting roof over the stands, the roof is able to have a different shape compared to the rest of the stadium. An example is the BayArenA in Leverkusen, Germany (figure 15.3). By placing a complete circular roof over the rectangular shaped stands, the ring action is used at full extend. By placing the roof covering exactly above the shape of the stands, the spectators are still protected from the elements and the pitch remains its quality.

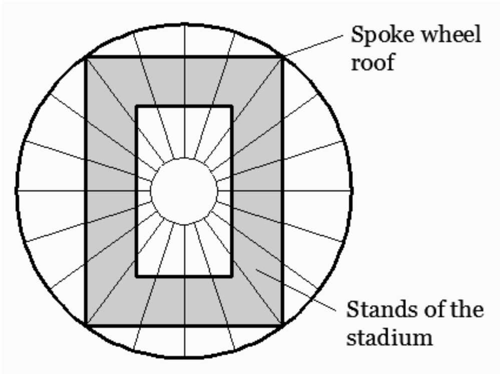


Figure 15.2 Adapt shape of the roof

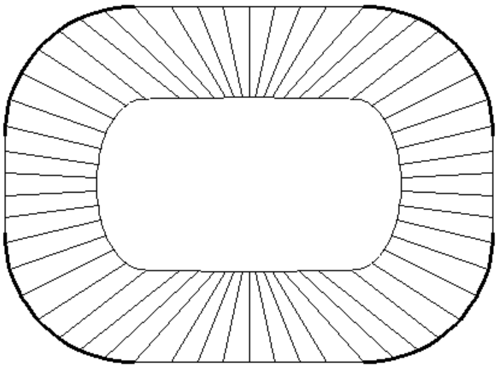


Figure 15.3 Transport the ring action

- *Transport ring action*
  In a non-circular roof there is a lack of curvature at some points of the roof structure. When the structure cannot be adapted, a solution to increase the stiffness is to transport the loads to the areas where there is sufficient curvature in the ring elements, or in other words transporting the ring action. By directly connecting the weakest points of the structure with the curved ring elements, like in case of truss structure variant 1 and 3, it is possible to use the available ring for a greater roof area.

Truss structure

In a spatial truss structure, the stiffness is partly provided by beam action. The geometry of the design determines the ratio of ring : beam action. The disadvantage of using beam action, is that relatively more material is needed to provide an equal amount of stiffness generated by ring action.
Research showed that by applying a spatial truss structure to non-circular shaped roofs, it is still possible to use a little amount of material although there is beam action present. From a structural point of view it is advised to use a spatial truss system for non-circular spoke wheel roofs. However, the less curvature is present, the less structurally attractive the spoke wheel roof will be. At some point, when there is a minimum amount of curvature, it is advised to use a different type of roof structure because one cannot benefit anymore from the strengths of the spoke wheel principle.

Cable structure

The research of the cable structure showed that its stiffness depends on the available ring action. In case of complete circular roofs it is advised to use a cable structure for the spoke wheel roof looking from a structural perspective, because only a small amount of material is needed. For non-circular shaped roofs, the efficiency of a cable roof decreases drastically. Due to the lack of curvature, the supporting structure needs to provide a great amount of stiffness in order to prevent large deformations.

Reference projects has (Commerzbank Arena, Daimler Chrysler stadium) shown that it is structurally feasible to use cable structures for non-circular spoke wheel roofs. Measures have been taken to become a stable design, for instance a double spoke wheel or a truss structure to increase the stiffness of the supporting outer ring structure. The average dead load of these structures shows that it is possible to become an efficient design (table 14.1). During this graduation project these measures have not been applied to the design, therefore it is difficult to provide a correct advice. The research of the thesis however showed that roof structures with straight sides, like the reference stadium, are not suitable for the use of a cable net structure.
From the research that has been done an advice that can be given is to use a cable structure for circular or oval shaped roof structures that always have some curvature in the complete roof structure.

More research is needed to investigate the possible solutions to increase the stiffness of non-circular roofs. A possible solution is to adapt the geometry of the cable structure to decrease the amount of deformation. For example by spanning cables from the corner outer ring elements to the centre of the long side of the inner ring (figure 15.3). This solution is similar to the first variant of the truss structure. Another solution is to increase the supporting structure itself by making use of for instance a truss structure.

Structural Resilience

The results from the thesis showed that optimizing a structural design to save material has a negative effect on the structural resilience of a structure. When a design of a structure needs to be made, the engineer has to make a decision whether one wants to provide sufficient resilience or to save as much material as possible. This decision depends on the owner and the structural requirements for the design.

In case of a truss structure it is easier to provide sufficient structural resilience and this type of structure is advised when one needs to fulfil the resilience requirement. Truss structures are more able to take up extra forces by means of bending. Besides the geometry can be adapted in such a way, the loads can be taken up by the adjacent elements.

In case of a cable structure it is hard to provide sufficient structural resilience. As in the case of the truss structure the engineer can adapt the geometry of the structural design, to make sure there is always a way to take up the loads by normal forces. This will however cause a complex structure and the building costs will increase drastically.

Building costs

The building costs depend on many factors. In this thesis the costs are assumed to be directly proportional to the used amount of material. In reality this is not always the case. By using more detailed financial information, the engineer is more capable of designing a financially attractive design. For example, the costs of the way of construction, complexity (for instance nodes), and standardization has not been taken into account. Using all parameters that influence the costs of the structural design, the structural design can become more financial beneficial.

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Software
The following software programs have been used during the thesis:

Scia Engineering 2011
AutoDesk 2010

Building Codes

Eurocode
EN 1990: Basis of structural design (Including the NA)

Eurocode 1
EN 1991: Actions on structures (Including the NA)

Eurocode 3
EN 1990: Design of steel structures

Using the following specifically:
- EN 1993-1-1: General rules and rules for buildings
- EN 1993-1-8: Design of joints
- EN 1993-1-11: Design of structures with tension components made of steel

17. List of symbols

Symbol	Explanation (specific chapter)
Latin capital letters	
A	Area
	Incidental loading (9)
C_e	Wind exposure coefficient
$\bar{C}_p$	Mean pressure coefficient
C_t	Heat coefficient
D	Diameter
E	Modulus of Elasticity
E_{cm}	Modulus of Elasticity of concrete
F	Loading
F_d	Calculation value of the load
F_E	Euler buckling force
F_{rep}	Representative value of the load
F_{ck}	Characteristic value of the load
F_{spring}	Spring force
$F_{pre-tension}$	Pre-tension force
G	Permanent load
G_k	Characteristic value of the permanent load
$G_{k,j}$	Characteristic value of the permanent load j
$G_{kj,sup}$	Upper characteristic value of the permanent load j
$G_{kj,inf}$	Lower characteristic value of the permanent load j
H	Height above sea level (9)
	Horizontal force component (11)
H_{p0}	Horizontal pretension force component in prestressing cable
H_{s0}	Horizontal pretension force component in suspension cable
I	Moment of Inertia
I_c	Turbulence intensity
L	Length
	Span of the cable (11)
M	Moment
M_x	Moment around the x-axes
M_y	Moment around the y-axes
M_z	Moment around the z-axes
N	Axial force
$N_{c,Rd}$	Axial compression force capacity
P	Pre-stressed load
Q	Variable load
Q_k	Characteristic value of a single variable load
$Q_{k,1}$	Characteristic value of the leading variable load 1
$Q_{k,i}$	Characteristic value of the simultaneously acting variable load i
U_x	Translation in x-direction
U_y	Translation in y-direction
U_z	Translation in z-direction
R_x	Rotation in x-direction
R_y	Rotation in y-direction
R_z	Rotation in z-direction
T_{max}	Maximum tension in cable
V	Transverse force
	Wind velocity (9)
V_y	Transverse force in y-direction
V_z	Transverse force in z-direction
$\bar{V}$	Mean wind velocity
$\hat{V}$	Peak gust wind velocity
$\bar{V}_h$	Reference mean wind speed
Z	Zone number for snow load in Europe (9)
	Height difference between suspension and prestressing cable (11)

Latin small letters

a'	Model coordinate for wind resonance
b	Width
d	Depth
	Sag (11)
d_s	Sag of the suspension cable in unloaded condition
Δd	Deflection suspension cable from the pretensioned equilibrium condition
$\bar{f}$	Mean wind load distribution
f_b	Tensile strength concrete
f_{bm}	Average tensile strength concrete
f_B	Background wind load distribution
f_c	Compression strength
f_C	Combined effective wind load distribution
f'_{ck}	Characteristic compression strength concrete
f_{cm}	Average compression strength concrete
f_p	Sag of the prestressed cable
f_R	Resonant response wind load distribution
f_s	Sag of the suspension cable
f_t	Tensile strength
f_y	Yield strength
g	Wind peak factor
g_B	Wind peak factor for background load distributions
g_R	Wind peak factor for resonant response load distributions
k	Spring constant
	Von Karman's constant (9)
k_h	Concrete coefficient dependent from fictive height h_o
l_{buc}	Buckling length
m	Mass
n	Amount of applied spokes
n_1	First mode natural frequency
q	Distributed load
q_k	Characteristic value of the variable load
r	Radius
r_0	Inner radius
s	Roof snow load
s_k	Characteristic value of the ground snow load
t	Radial value
u	Translation in u-direction
	Radial deformation (2)
w	Translation in w-direction
	Total distributed load over a cable (11)
$\mathrm{w_c}$	Sheer of the unloaded structural elements
$\mathrm{w_{max}}$	Permanent deformation
$\mathrm{w_1}$	Direct deformation due to permanent loads
$\mathrm{w_2}$	Long term deformation due to permanent loads
$\mathrm{w_3}$	Additional deformation due to variable loads
$\mathrm{w_{tot}}$	Total deformation, the sum of $w_1 + w_2 + w_3$
x	Distance
	Distance between the spokes (2)
z	Height above the ground
z_0	Roughness length
z_{ref}	Reference height

Greek small letters

α	Angle
β_{cc}	Coefficient dependent from time t for concrete
γ	Partial factor
γ_f	Partial load factor, that takes the possibility of unfavourable deviations of the values of the load with respect to the representative value into account
γ_g	Partial factor for permanent loads, that takes the possibility of unfavourable deviations of the values of the load with respect to the representative value into account

γ_G	Partial factor for permanent loads, that takes the model uncertainties and dimensional deviations into account.
$\gamma_{G,j}$	Partial factor for permanent load j
$\gamma_{Gj,sup}$	Partial factor for the upper permanent load j
$\gamma_{Gj,inf}$	Partial factor for the lower permanent load j
γ_m	Partial material factor
γ_q	Partial factor for variable loads, that takes the possibility of unfavourable deviations of the values of the load with respect to the representative value into account
γ_Q	Partial factor for variable loads, that takes the model uncertainties and dimensional deviations into account.
$\gamma_{Q,1}$	Partial factor for variable load 1
$\gamma_{Q,i}$	Partial factor for variable load i
δ_r	Radial extension ring
δ_r	Total extension ring
ε	Strain
	Factor of reduction (9)
ε_{ca}	Autogeneous shrinkage extension
ε_{cd}	Drying shrinkage extension
$\varepsilon_{cd,0}$	Drying shrinkage extension at time t=0
ε_{cs}	Total shrinkage extension
ε_{vl}	Strain value at the end of the yield area
ε_t	Strain related to the tensile strength
ε_u	Strain value at point of cracking of the material
ε_x	Strain in x-direction
ε_y	Strain in y-direction
	Yield strain (7)
ε_z	Strain in z-direction
κ	Curvature
λ_{rel}	Relative slenderness
μ_1	Snow load shape coefficient
ρ	Density
	Correlation coefficient (9)
ρ_a	Density of air
σ	Stress
σ_v	Standard deviation of the longitudinal wind speed
σ_p	Root-mean-square fluctuating load
σ_x	Stress in x-direction
τ_0	Surface shear stress
φ	Rotation
φ_1	Mode shape for the first mode of vibration
υ	Dynamic wind speed
υ_*	Frictional velocity
Ψ_0	Factor associated with the combination value of a variable load
Ψ_1	Factor associated with the frequent value of a variable load
Ψ_2	Factor associated with the quasi-steady value of a variable load
ω_{buc}	Buckling factor

18. List of figures

19. List of tables

Part 6
Appendices

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Appendix A 1. Design scope factors

To come to a final design of an efficient spoke wheel roof structure, the design scope factors must be taken into account. The meaning of an efficient spoke wheel structure is a spoke wheel structure made of as little material or elements, with low costs and build in a short time (construction). These three factors are all related to each other (figure A 1.1). When little material is used, less time is needed to construct and the costs are kept low. The design of an efficient spoke wheel structure is depended from these three factors. These factors are depended from other parameters which have an indirect influence on the final design and will be described in paragraph A 1.1 as well as their relations in paragraph A 1.2.

One must keep in mind that the emphasis of the research of the thesis will be less on the construction/building costs perspective and more on the research to find a good structural solution for the use of a spoke wheel structure for football stadia. Within a good structural solution the material use and therefore the costs will be kept low.
To get an understanding of the design influence parameters and its relations all parameters must be briefly described.

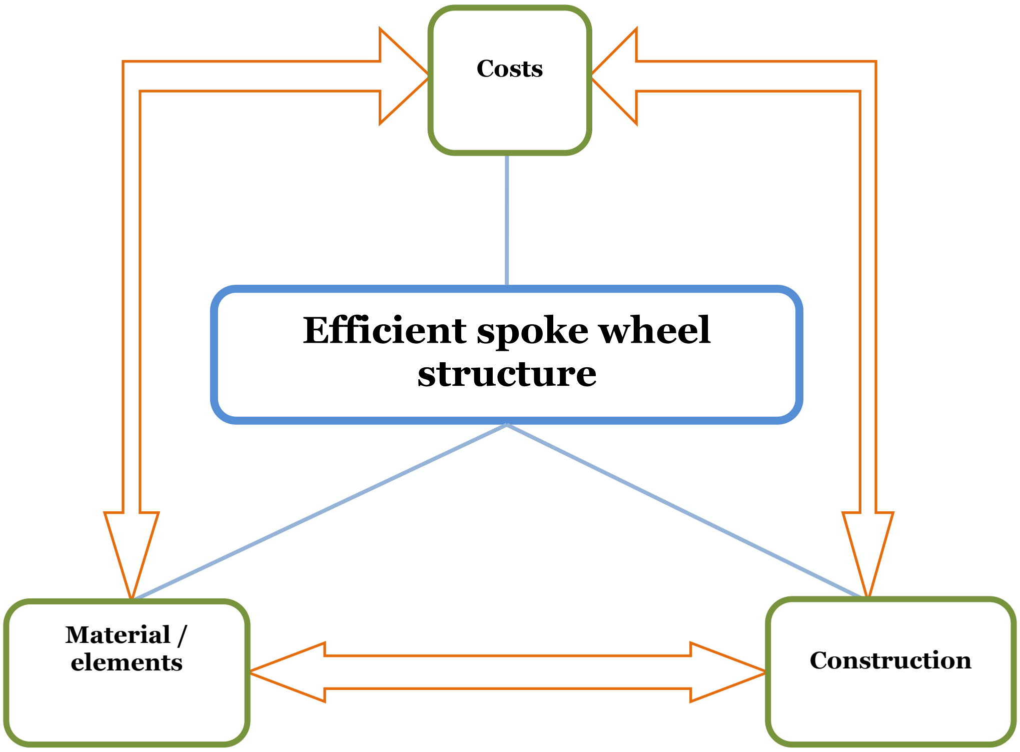


Figure A 1.1 Relations of the design scope factors

Costs
The costs of a structure are probably the most important factor when determining whether a building project will be build or not. The lower the costs, the higher the chance an owner decides to construct a building. The costs of a building depend from the amount and kind of material that is used and the construction and building time. Vice versa, the height of the budget of a project determines the material use and construction.

Materials / elements
Nowadays the choice of the material or element is very important. Materials and elements need to be durable, have a high aesthetical value, low of costs, etc. Especially the influence of the use of the type of material / element on the environment is a big issue these days. The better the quality and durability of the material / element the higher the costs. Next the type of material/element also influences the way of construction and therefore the building time. For special type of elements and materials, extra attention must be paid during construction.

Construction
As described, the construction is depended from the project budget and the type and amount of material. Often the construction planning is very tight and penalties are set up to prevent delay of the project and encourage the

constructor to stay on schedule. The way of constructing, the amount of needed construction capacity and man power has a great influence on the costs.

1.1 Parameters

The previous described factors are not only depended from each other but also from other parameters. These parameters have an indirect influence on the final structural design of the spoke wheel roof structure. The influence parameters on the design factors are the following:

Requirements
The structure of a building has to fulfil the structural requirements (stiffness, stability, strength). These are mainly described in the Eurocode. A structure needs to withstand external influences so the structure is safe in use and during construction. Additional requirements like shading from the sun and shelter from wind and rain have great influence on the design of the roof structure, as well as the aesthetics and appearance of a structure.

Conditions
The structural design needs to meet the conditions that are set to come to an efficient spoke wheel roof structure.

Structural systems
The choice of the structural system is of great influence on the costs, material use and construction. The type of ring structure, shape of the perimeter of the inner and outer rings, choice of elements, the use of a cable net structure or not, are all important choices to be made that have great consequences on the design.

(External) Loads
The amount and type of external loading determines the type of structure/material/construction and eventually the costs of the project. The higher the load, the greater the dimensions and/or the higher needed strength of the material, the more the costs will increase.

Prefabrication / standardization
With prefabrication the elements are produced and assembled in the factory and not on site. On the building site the prefabricated elements or units can be directly placed and a lot of construction time and costs can be saved. Using standardization the assembly time will decrease. When the assembly of the structural elements can be repeated numerous times, this will have a positive effect on the construction time and costs.

Transportation
The transportation depends on the used materials and construction elements. When fragile materials are used, special transportation is needed which is expensive and costs time. This also is the case for the transportation of special construction elements.

Not all parameters that are described above will be thoroughly analysed in this thesis. The reason is that the emphasis of the thesis lies on the research of a structural solution for an efficient spoke wheel roof structure. The parameters that will not be analysed in this thesis are still an important parameter in the building industry but are left out in the thesis due to the described emphasis and the main focus of the thesis.

1.2 Relations design scope

The different parameters not only have an influence on the design, also on each other. They can have a positive as well as a negative effect on each other. To visualize the relations between the parameters a scheme of relations can be found in figure A 1.2. The relations displayed are only the direct relations between the parameters/factors. By following the path one can see the direct and indirect relations between the different parameters and factors.

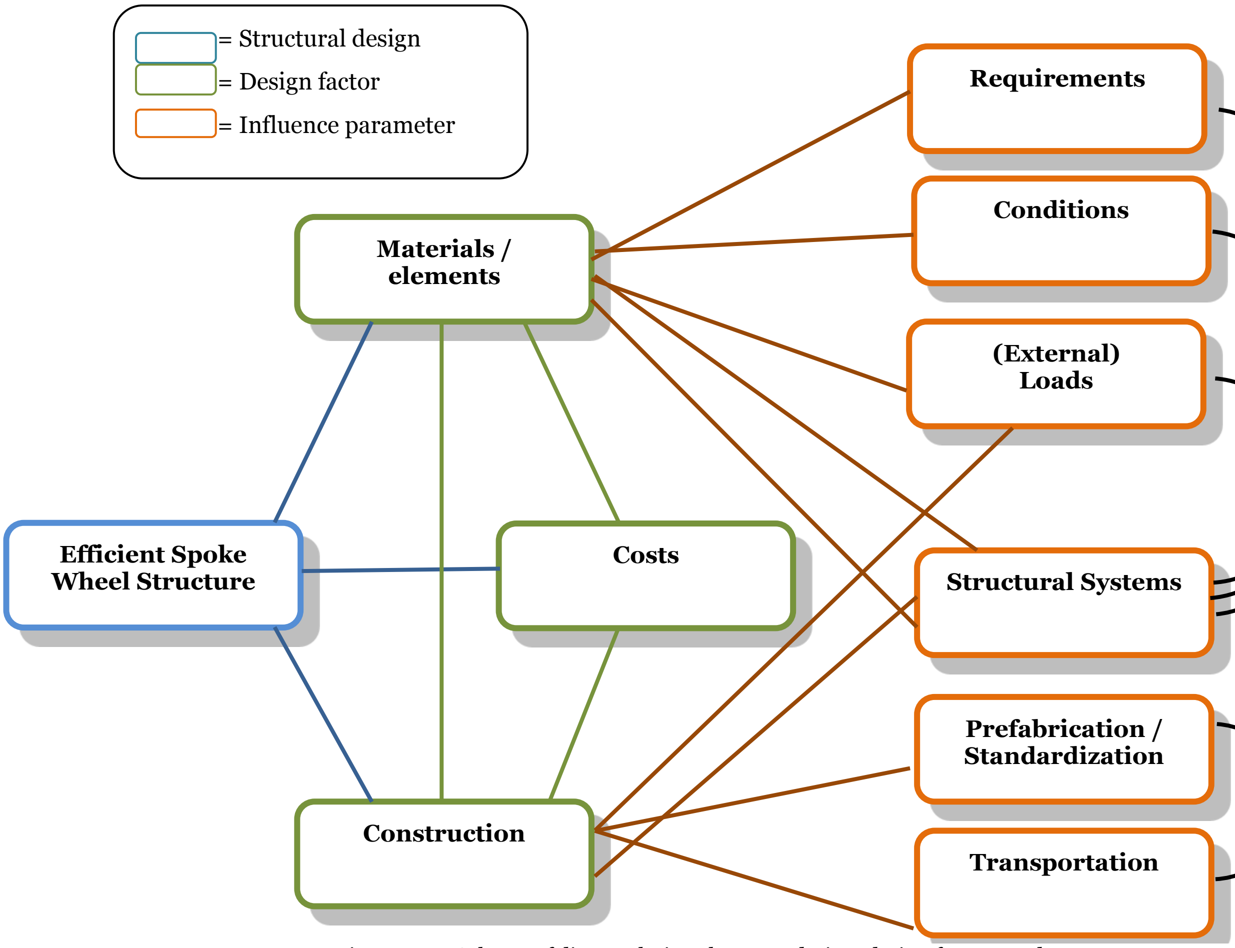
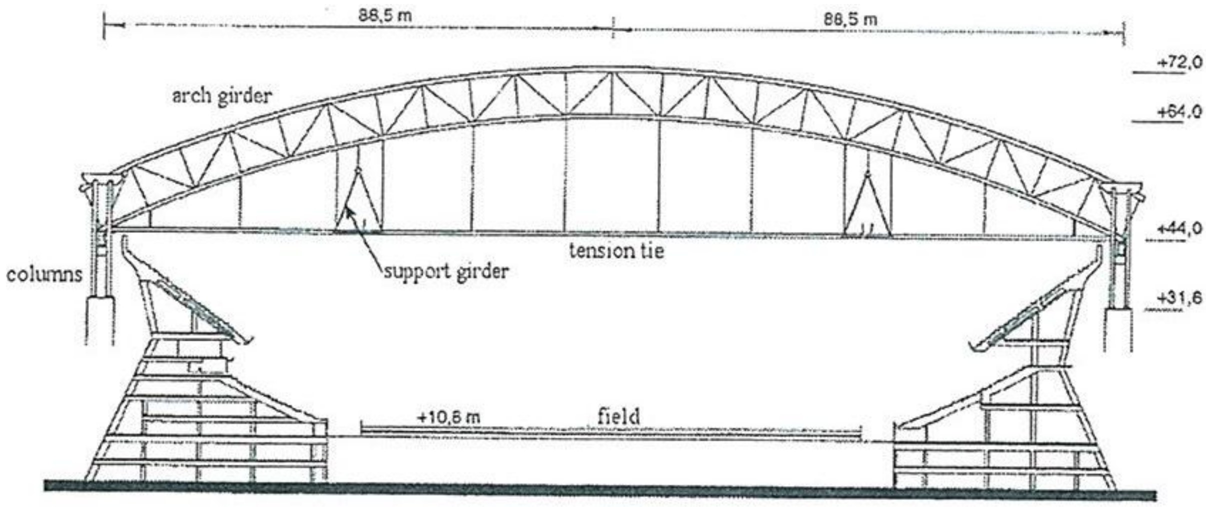


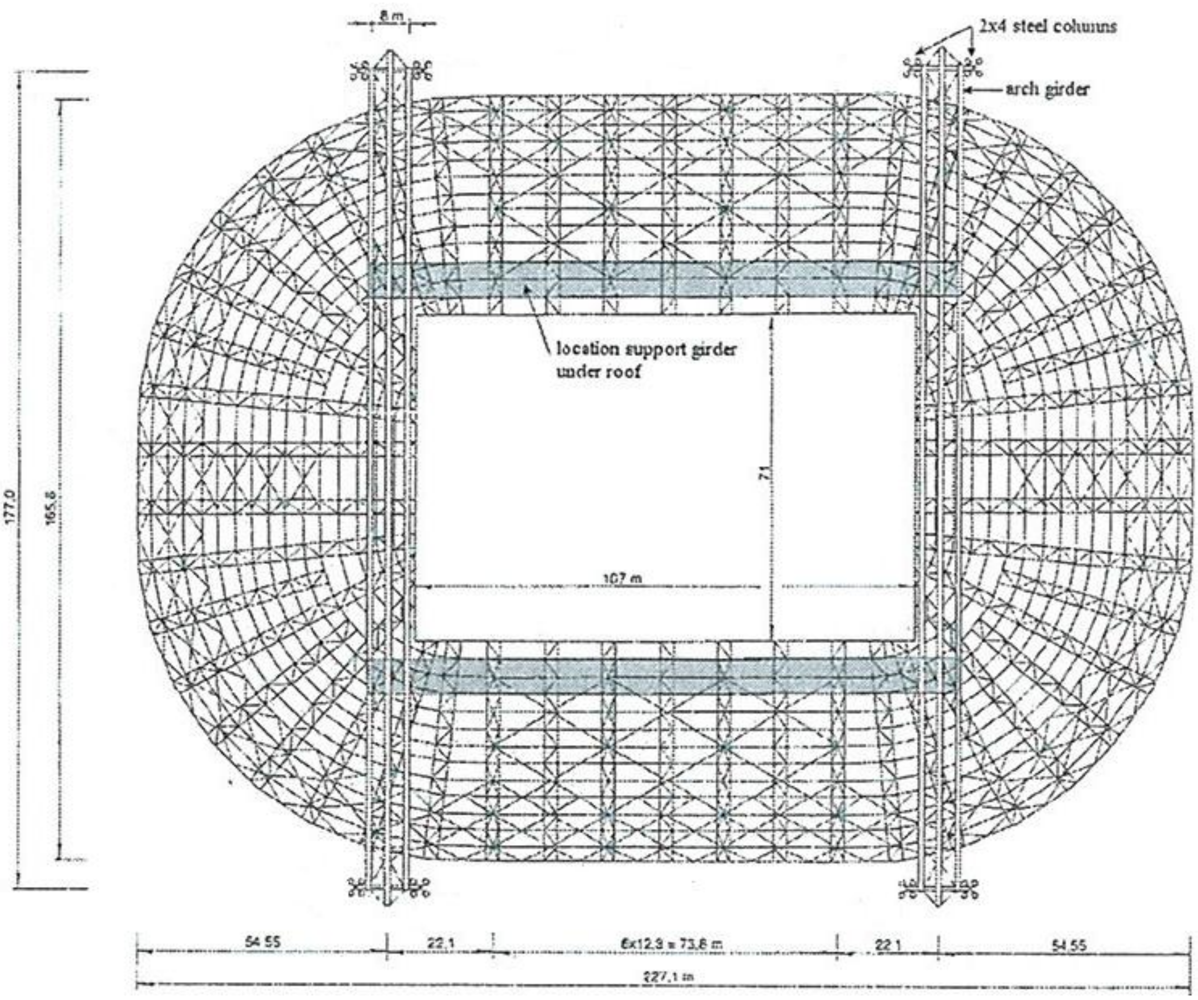
Figure A 1.2 Scheme of direct relations between design, design factors and parameters

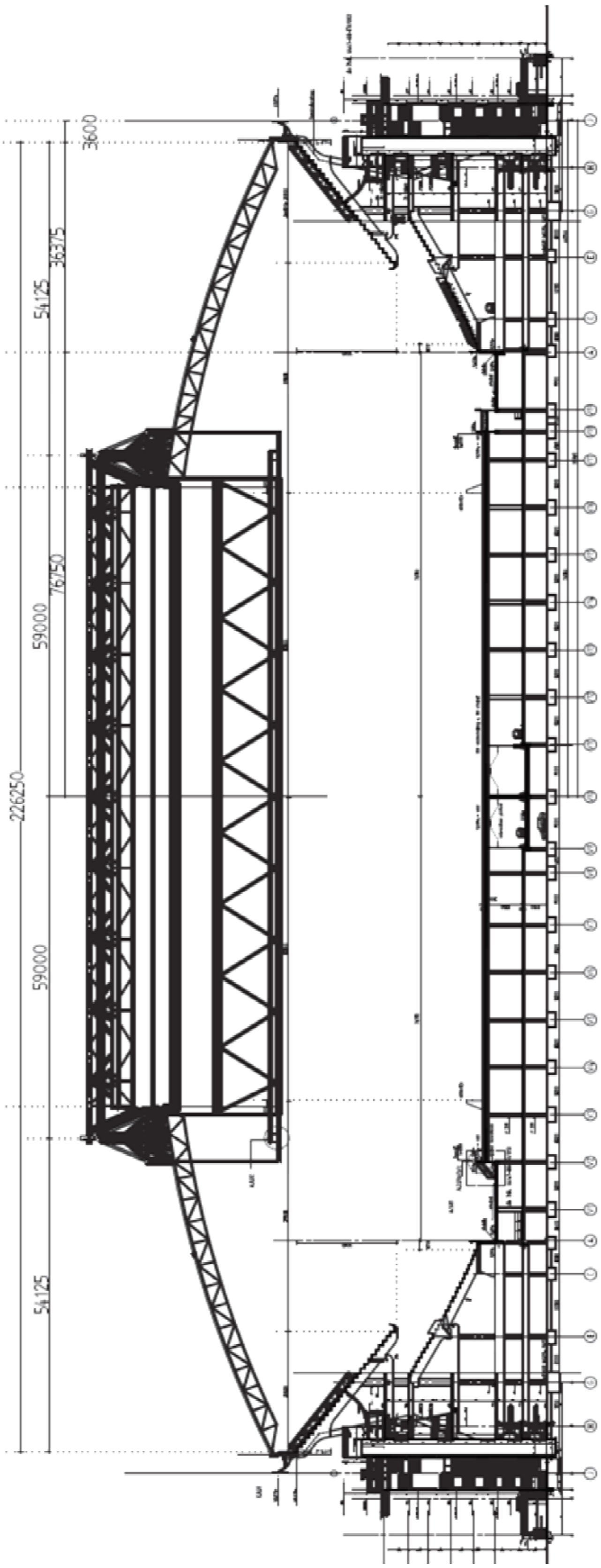
The scheme of relations shows that there are no direct relations between the parameters and the costs. The parameters only have an indirect relation with this design factor. The type and amount of material (or structural elements) and construction determine the costs of a building project.

During the thesis project the most important design factors and parameters, depending on the mentioned emphasis of the thesis, will be further investigated and discussed.

Appendix A 2.Amsterdam ArenA



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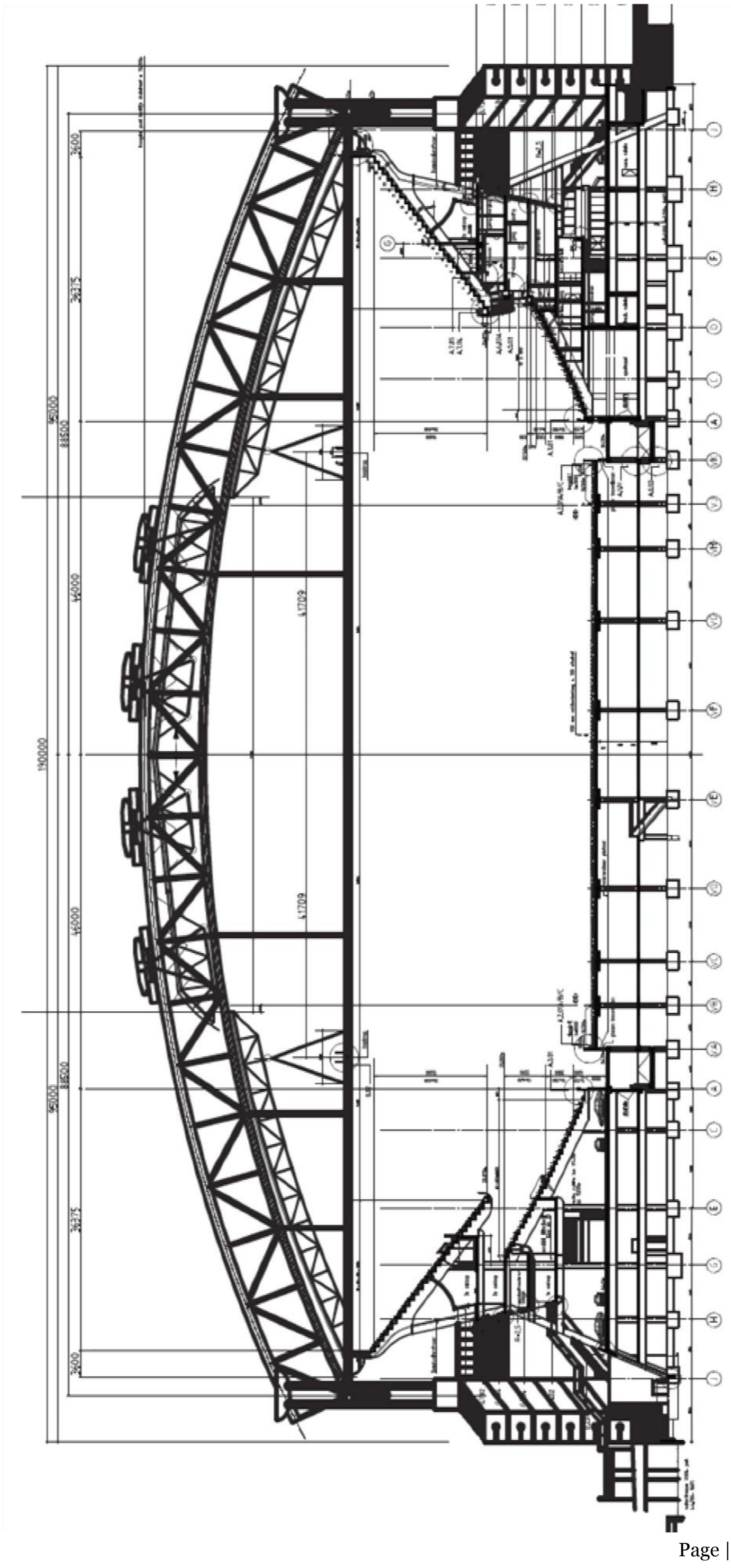
36375
54125
76750
59000
226250
59000
54125


190000
95000
88500
46000
47709
36275
3600

Appendix A 3. Calculation method

To come to a final efficient spoke wheel roof structure, many accurate calculations have to be made to determine the exact values for the deformations and stresses. The spoke wheel structure is a complex, statically indeterminate structure. In order to make precise calculations, computer orientated calculation methods are used, which have a systematic way of calculating. The finite element method founds its origin as a computer program application based on the systematic of the displacement method.

3.1 Finite Element Method

Within the finite element method the structure is split in a number of elements, which are connected by nodes, with each other or with their environment. For a single element first the constitutive equations are determined, which results in an element stiffness matrix. With this matrix, the structural behaviour of a single element is being described. The next step is the description of the structural behaviour of a complete structure as a collection of multiple elements. This is done by a system stiffness matrix.

Example

Assuming a linear elastic structure which stresses and deformation are linear related. The following structure is used for the example:

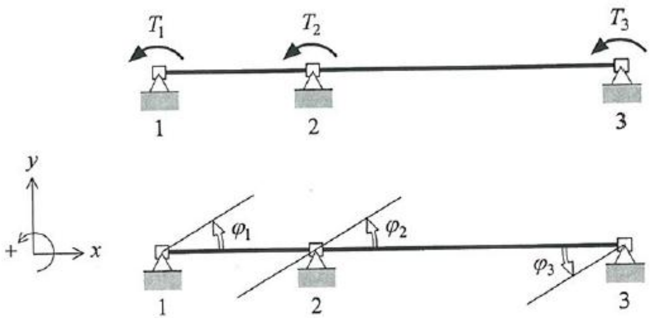


Figure A 3.1 A moment and rotation in every node. Only one single degree of freedom per node

The matrix equation is described as follows:

$$\{f\} = [K]\{u\} \quad \text{(A 3.1)}$$

Where the column matrix $\{f\}$ represent the system force vector:

$$\{f\} = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} \quad \text{(A 3.2)}$$

The same way the degrees of freedom are described in the column matrix $\{u\}$, the system displacement vector:

$$\{u\} = \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{Bmatrix} \quad \text{(A 3.3)}$$

The system stiffness matrix $[K]$ of the structure from figure A 3.1 is expressed as:

$$[K] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \quad \text{(A 3.4)}$$

This systematic approach is used for large complex structures, like in the case of this thesis a stadium roof. A software program based on the finite element method is Scia Engineer. This program will be used during the thesis.

3.2 Computer modelling

To determine the forces, moments, stresses, deformations etc. the finite element calculation method will be used. The structural design will be modelled with the software program Scia Engineer. The major advantage is the ability to calculate large, complex, statically undetermined structural models. Within the program there is a possibility to perform 1st (linear) and 2nd order (non-linear) effect calculations. These methods are briefly described.

1st order effect

When performing 1st order linear calculations the following conditions are needed:

- Loads are conservative, meaning that after the deformation of the structure the loads do not change in size and direction.
- The law of Hooke is valid (flat cross sections remains flat), there is a linear relationship between the stresses and elasticity.
- There is equilibrium in the undistorted geometry.

In reality these conditions can never be fulfilled. This is also the case of the spoke wheel roof structure. The structure is subjected to dynamic loads due to the weather. Although only static loads are applied in this thesis (see chapter 9), the first conditions cannot be met. The second condition can only be met when the load level is relatively low. At a higher load level the cross sections will yield and elements can buckle. The third condition can never be met, as a structure will always deform at a random load.

To approach the 2nd order effect, the 1st order linear calculations always needs to be adapted. A good approach of a 2nd order calculations with a 1st order calculation is well possible.

2nd order effect

When the deformations can be described with sinus functions, the 2nd order linear elastic deformation can be described by multiplying the 1st order linear elastic horizontal deformation with $\frac{n}{n-1}$ with $n = \frac{F_{Euler}}{N_{c,s,d}}$ with F_{Euler} is the Euler buckling force and $N_{c,s,d}$ is the acting compression force.

When the deformations cannot be described by sinus functions, but by parabolic functions, multiplying with $\frac{n}{n-1}$ a good approach can be made of the 2nd order deformation. The 2nd order force distribution can be calculated by multiplying the forces or moments determined with a 1st order linear calculation by $\frac{n}{n-1}$ and sum this up with the 1st order linear calculation

This theory can be used when calculating simple frameworks by hand. This cannot be done in case of a complete spoke wheel roof structure. By using the software program Scia Engineer 2nd order non-linear calculations can be made with the Newton Raphson method.

Newton Raphson
The algorithm is based on Newton-Raphson method for the solution of non-linear problems. It is a method for finding successively better approximations of the roots of a real valued function.

The Newton-Raphson method can be explained as follows:

Given a function $f(x)$, its derivative $f\,'(x)$, and an initial value x_0 for a root of the function. Provided the function is reasonably well-behaved a better approximation x_1 is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad \text{(A 3.5)}$$

Geometrically, x_1 is the intersection with the x-axis of a line tangent to f at $f(x_0)$. The process is repeated until a sufficiently accurate value is reached:

$$x_{n+1} = n - \frac{f(x_n)}{f'(x_n)} \qquad \text{(A 3.6)}$$

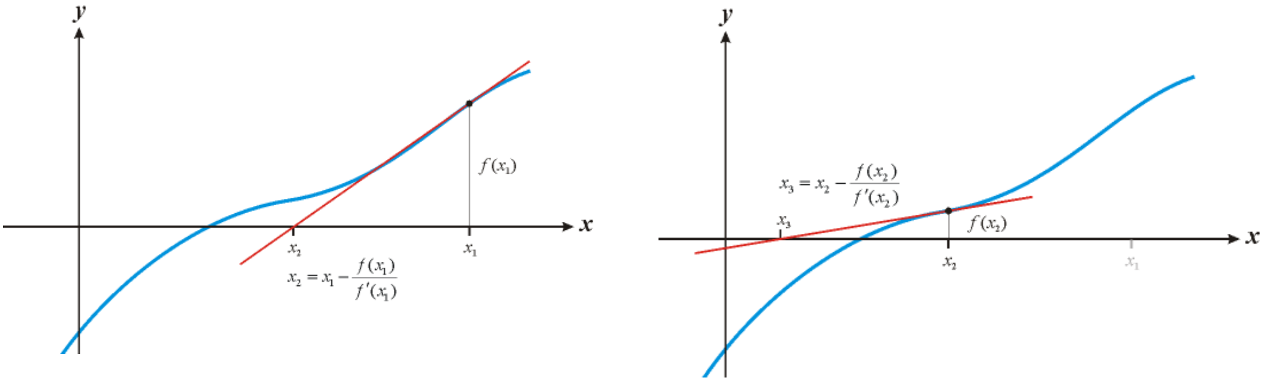


Figure A 3.2 First two steps of the Newton Raphson method

This method can also be used for the calculation of complex structures. In case of the tensile compression ring structure one can calculate the deformations, forces, moments etc. When using this method, the load acting on the structure can be divided into several steps. For example in the first step 20% of the load (the amount of steps and part of the loading is variable). The software program will calculate all the needed values and then will put the next 20% of the load on the structure. The program will again determine the values and the next step follows. Eventually there will be a final calculation with the true data and the values for the deformations, etc.
The calculations that are made with Scia Engineer during this thesis have been done by using 200 iterations with 5 increments.

Scia Engineer model

The different structures will be modelled and loaded in the software program Scia Engineer. For every structure the results (deformations, forces and moments) will be displayed in the paragraph where they are discussed in.
With Scia Engineer it is possible to make 3D models. All results of the models are displayed within the local coordinate system. The interpretation of the local axis for every element is very important to understand the flow of forces in the models. In figure A3.3 the local coordinate system for a single element and an example of a structure is shown. In the results of the different models, the local axes of every model will first be displayed before the results will be shown.

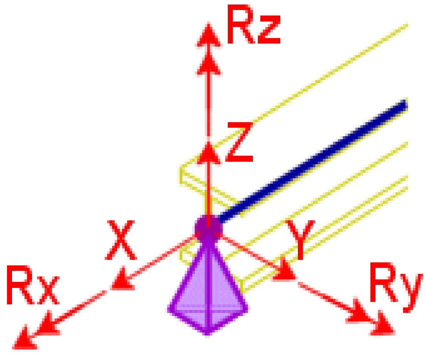


Figure A 3.3 Local axes single element

Appendix A 4. Shape inner ring

4.1 Design Area

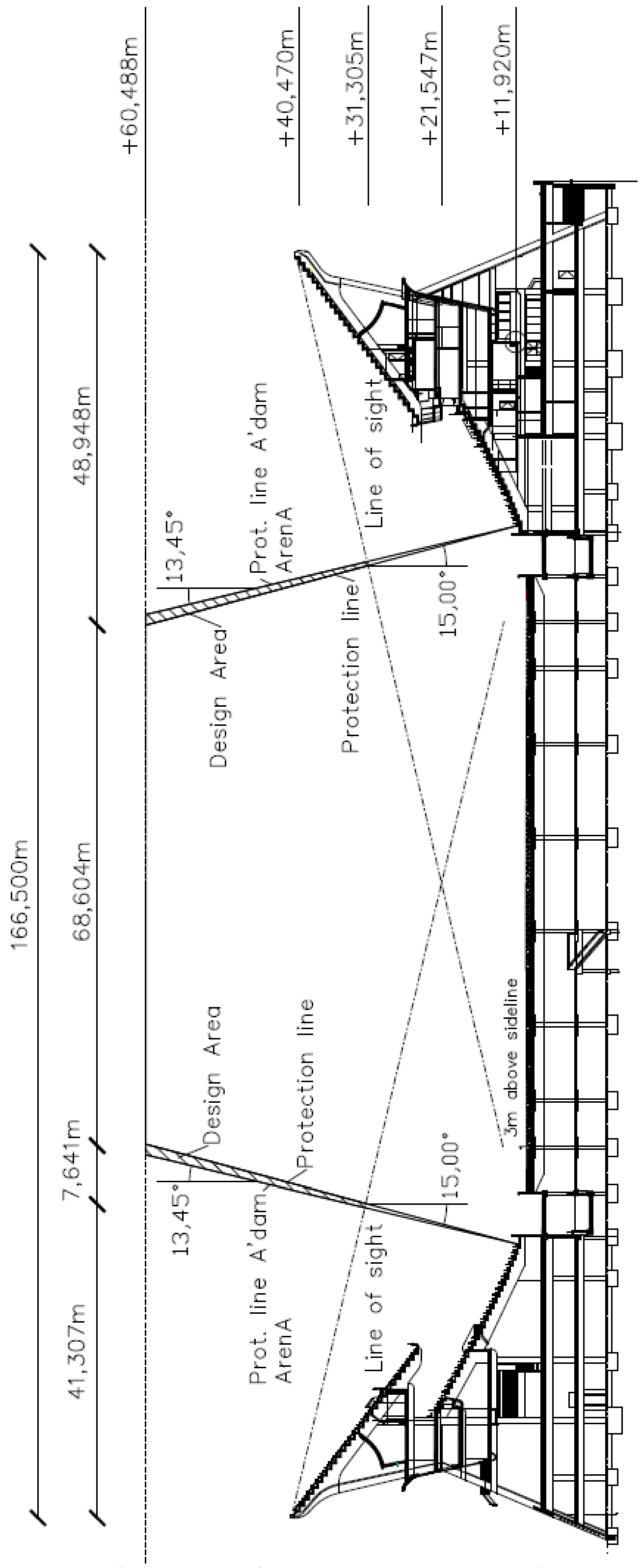


Figure A 4.1 Design area transverse cross section

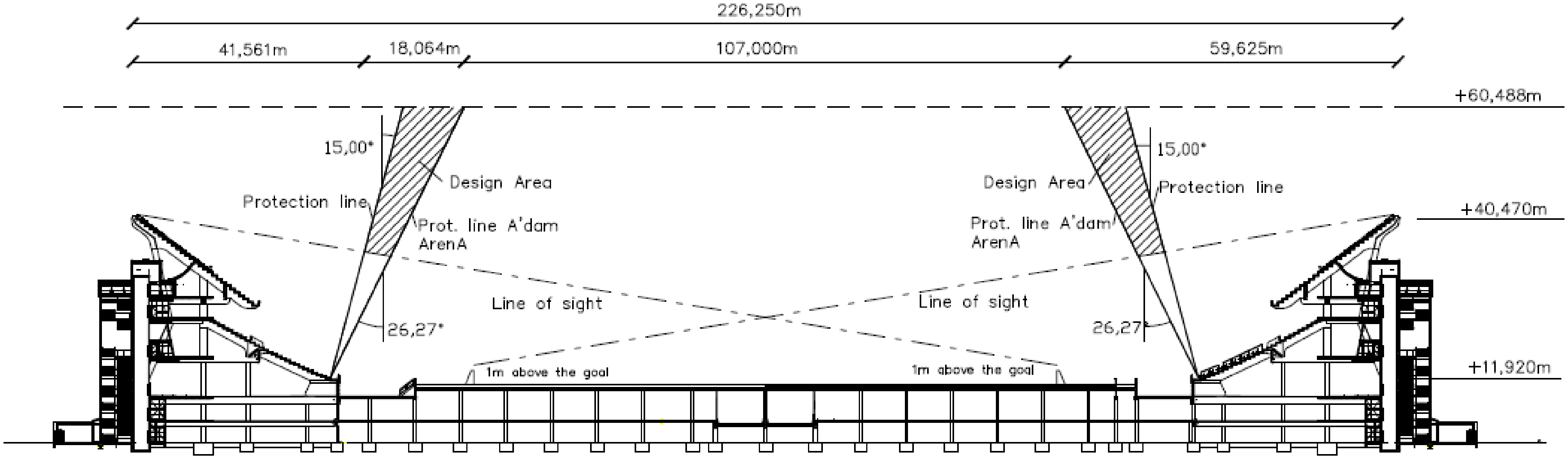


Figure A 4.2 Design area longitudinal cross section

4.2 Shape inner ring

For the global design of the spoke wheel roof structure is assumed that the edge of the roof is in line with the top of the stands. At that height there is some room left for applying curvature at the inner ring. This is illustrated in figure A 4.1 and A 4.2. The total design area at this height of the roof is illustrated in figure 7.2.
Inside the design area it is possible to apply half an ellipse at the short and long side of the inner ring. The ellipses are illustrated in figure A 4.3. The design area at the long side is very small, therefore only a very flat ellipse can be applied (ellipse 1).

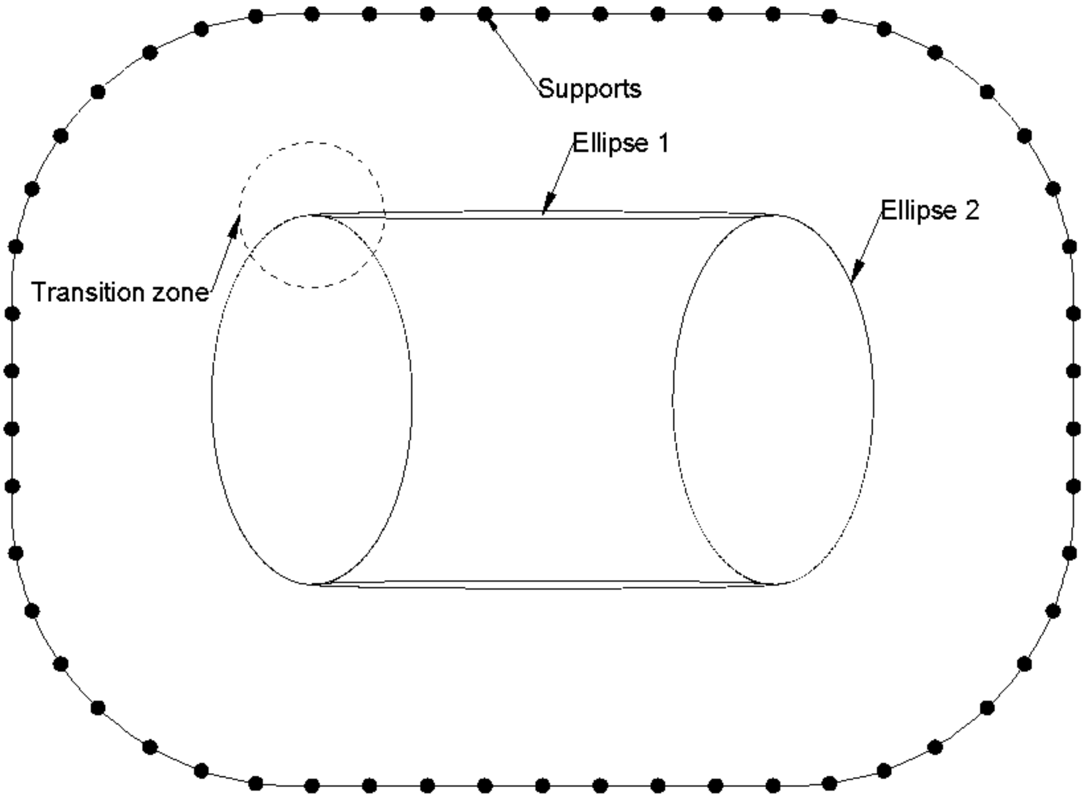


Figure A 4.3 Formation of the definitive shape of the inner ring

An ellipse does not have a constant curvature. The curvature of an ellipse can be described using the following equation:

$$\kappa(t) = \frac{ab}{(b^2 cos^2 t + a^2 sin^2 t)^{3/2}} \quad \text{(A 4.1)}$$

Where a is half the length of the ellipse on the longitudinal axis and b half the length of the ellipse on the perpendicular axis (figure A 4.4). The curvature depends on the radial value $(0 - 2\pi)$.

When the curvature is determined the radius at a certain point $(0 - 2\pi)$ is equal to:

$$r = \frac{1}{\kappa} \quad \text{(A 4.2)}$$

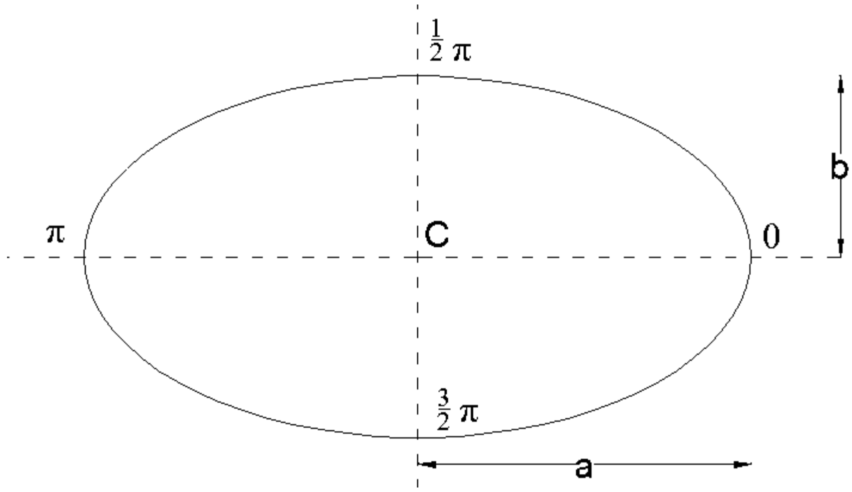


Figure A 4.4 Ellipse

Ellipse 1

For ellipse 1 the value of the radius at point $t = 0$ and $t = \frac{1}{2}\pi$ will be determined.

$a = 49{,}20m \quad ; b = 0{,}844m$

$$\kappa(0) = \frac{(49{,}20*0{,}844)}{\left((0{,}844)^2 cos^2(0) + (49{,}20)^2 sin^2(0)\right)^{\frac{3}{2}}} = 69{,}07m^{-1} \rightarrow \quad r(0) = 0{,}0145m \qquad \text{(A 4.3)}$$

$$\kappa\left(\frac{1}{2}\pi\right) = \frac{(49{,}20*0{,}844)}{\left((0{,}844)^2 cos^2\left(\frac{1}{2}\pi\right) + (49{,}20)^2 sin^2\left(\frac{1}{2}\pi\right)\right)^{\frac{3}{2}}} = 0{,}00348m^{-1} \rightarrow \quad r\left(\frac{1}{2}\pi\right) = 2868{,}06m \qquad \text{(A 4.4)}$$

Ellipse 2

For ellipse 2 the value of the radius at point $t = 0$ and $t = \frac{1}{2}\pi$ will be determined.

$a = 39{,}44m \quad ; b = 21{,}38m$

$$\kappa(0) = \frac{(39{,}44*21{,}383)}{\left((21{,}38)^2 cos^2(0) + (39{,}44)^2 sin^2(0)\right)^{\frac{3}{2}}} = 0{,}087m^{-1} \rightarrow \quad r(0) = 11{,}43m \qquad \text{(A 4.5)}$$

$$\kappa\left(\frac{1}{2}\pi\right) = \frac{(39{,}44*21{,}383)}{\left((21{,}38)^2 cos^2\left(\frac{1}{2}\pi\right) + (39{,}44)^2 sin^2\left(\frac{1}{2}\pi\right)\right)^{\frac{3}{2}}} = 0{,}01375m^{-1} \rightarrow \quad r\left(\frac{1}{2}\pi\right) = 72{,}72m \qquad \text{(A 4.6)}$$

The calculations show that the largest radius is present at $t = \frac{1}{2}\pi$, i.e. in the middle of the ellipse or sides of the inner ring.

The transition from ellipse 1 to ellipse 2 is not very smooth, because the ellipses have different radiuses at $t = 0$ or $t = \pi$. To come to a fluent transition between ellipses, a tangent can be drawn between the ellipse, where the ellipse have the same angle, see figure A 4.5.

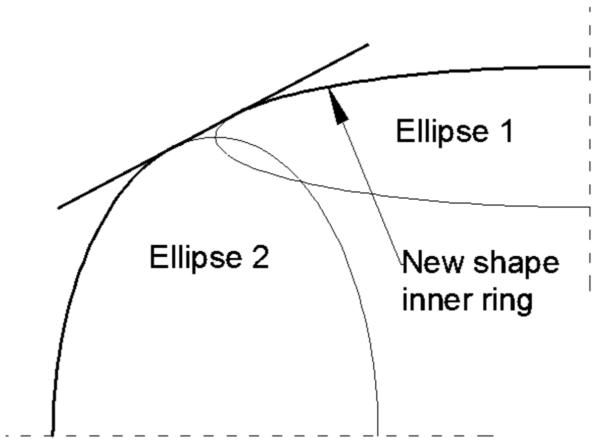


Figure A 4.5 Transition from ellipse 1 to 2

The definitive shape of the inner ring has now been determined and is illustrated in figure A 4.6. The largest radius at the long sides has a value of $r = 2868{,}06m$ and the short side a value of $r = 72{,}72m$.

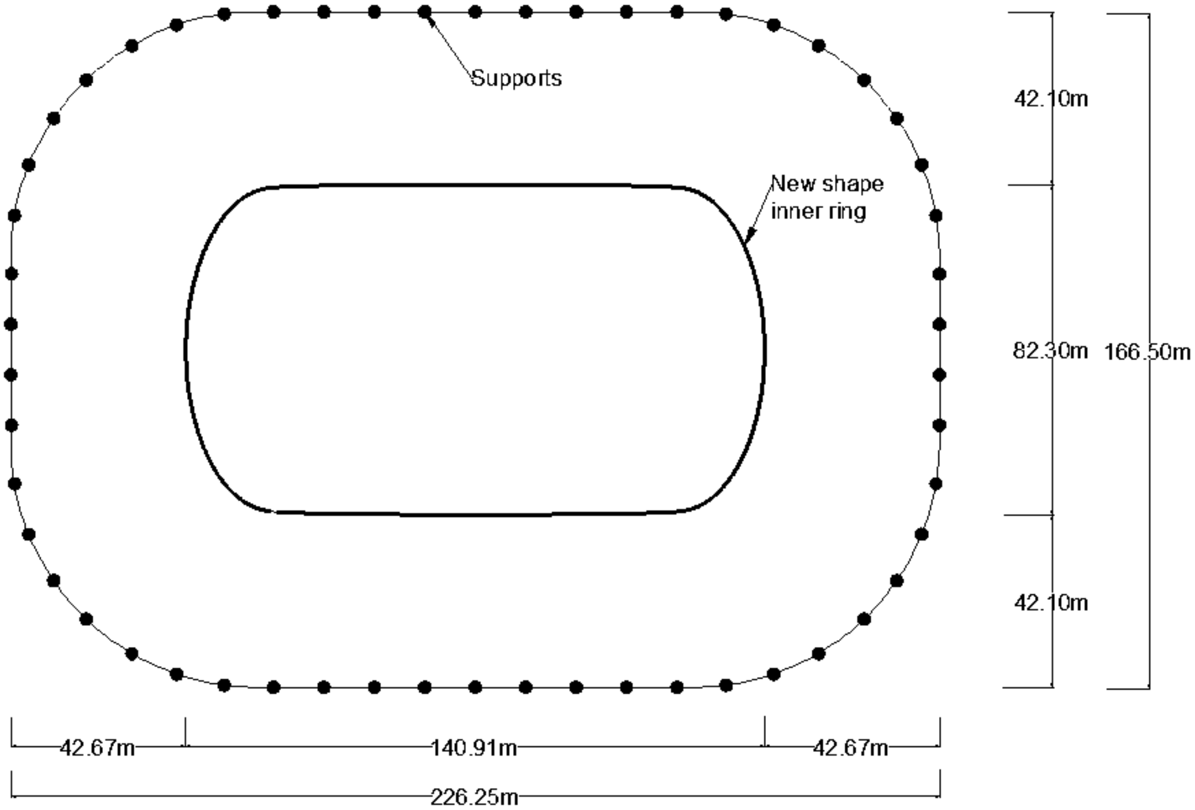


Figure A 4.6 Definitive shape of the inner ring

Area

The total area of the roof can be determined by calculating the total area with the outer ring as boundary minus the area with the inner ring as boundary.

The total area with the outer ring as boundary is equal to the total area of the roof of the Amsterdam ArenA. From sources within Arcadis, the total area of the roof is 32.500 m^2.

The total area between the inner ring can be determined by the summation of the total area of ellipse 1 and 2 and the multiplication of length 2a from ellipse 1 and 2.

Area ellipse 1: $\pi \cdot a \cdot b = \pi \cdot 49{,}20 \cdot 0{,}844 = 130{,}39 m^2$

Area ellipse 2: $\pi \cdot a \cdot b = \pi \cdot 39{,}44 \cdot 21{,}38 = 2647{,}73 m^2$

Rectangular area between ellipses: $\left(2 \cdot a_{ellipse1}\right) \cdot \left(2 \cdot a_{ellipse1}\right) = (2 \cdot 49{,}20) \cdot (2 \cdot 39{,}44) = 7761{,}79 m^2$

The roof area is equal to: $32500 - 130{,}39 - 2647{,}73 - 7761{,}79 = 21960{,}09 m^2$

The extra area due to the transition zone as displayed in figure A4.5 has been assumed negligible. The total area is almost equal to 22.000m^2.

Appendix A 5. Connections

5.1 Hinged connection

Determination of U

The deformation of Ua and Ub can be determined using the displacement method for the structure below. By taking the moment T around node A and B, the deformation can be described. The modulus of elasticity is assumed $E = \infty$. Bending and extension of the spoke do not influence in the deformation u.

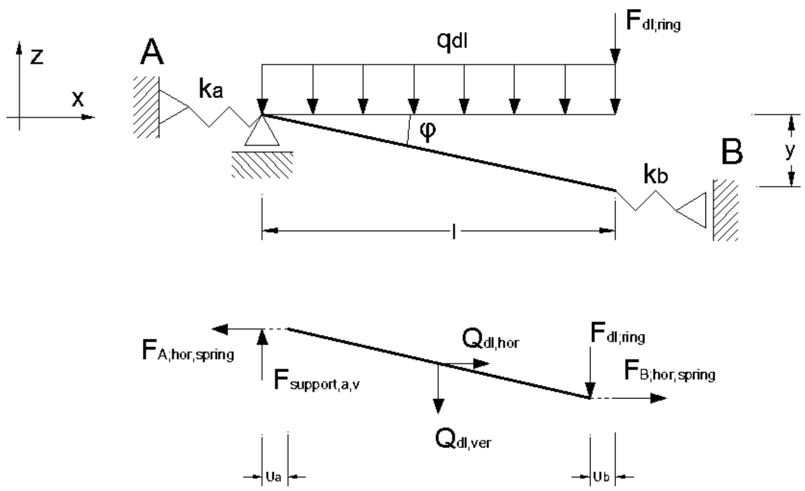


Figure A 5.1 Schematization of the hinged connection

The forces illustrated in figure A 5.1 have the following expression

Dead load:

$$Q_{dl;vert} = q \cdot l \tag{A 5.1}$$

$$Q_{dl;hor} = \frac{ql}{tan(\varphi)} \tag{A 5.2}$$

Spring force

$$F_{A;spring} = k_A \cdot u_A \tag{A 5.3}$$

$$F_{B;spring} = k_B \cdot u_B \tag{A 5.4}$$

Reaction force in node A

The vertical reaction force in node A is determined by the equilibrium of forces in the vertical direction $\sum F_{vert} = 0$

$$F_{A;vert} - Q_{dl;vert} - F_{dl;ring} = 0 \tag{A 5.5}$$

Is equal to:

$$F_{A;vert} - q \cdot l - F_{dl;ring} = 0 \tag{A 5.6}$$

Follows:

$$F_{A;vert} = q \cdot l + F_{dl;ring} \tag{A 5.7}$$

Now the forces in the structure are known, the horizontal deformation in node A and B can be described.

Determination of Ua

$T_B = 0$ (A 5.8)

Follows:

$-F_{A;vert} \cdot (l - u_A) + F_{A;spring} \cdot y + Q_{dl;vert} \cdot \frac{1}{2}(l - u_A) - Q_{dl;hor} \cdot \frac{1}{2} y = 0$ (A 5.9)

When implementing the equation for the forces, follows the equation:

$(k_A \cdot u_A) \cdot y - \left(ql + F_{dl;ring}\right)(l - u_A) + (ql) \cdot \frac{1}{2}(l - u_A) - \left(\frac{ql}{tan(\varphi)}\right) \cdot \frac{1}{2} y = 0$ (A 5.10)

u_A can now be expressed as:

$\frac{ql^2 + F_{dl;ring} l - \frac{1}{2} ql^2 + \frac{qly}{2\tan(\varphi)}}{k_A \cdot y + ql - F_{dl;ring} - \frac{1}{2} ql}$ (A 5.11)

Determination of Ub

$T_A = 0$ (A 5.12)

Follows:
$-Q_{dl;vert} \cdot \frac{1}{2}(l - u_B) + Q_{dl;hor} \cdot \frac{1}{2} y + F_{B;spring,hor} \cdot y - F_{dl;ring} \cdot (l - u_B) = 0$ (A 5.13)

When implementing the equation for the forces, follows the equation:

$-ql \cdot \frac{1}{2}(l - u_B) + \frac{ql}{tan(\varphi)} \cdot \frac{1}{2} y + k_B u_B \cdot y - F_{dl;ring} \cdot (l - u_B) = 0$ (A 5.14)

Is equal to:

$-\frac{1}{2} ql^2 + \frac{1}{2} qlu_B + \frac{ql}{tan(\varphi)} \cdot \frac{1}{2} y + k_B u_B \cdot y - F_{dl;ring} \cdot l + F_{dl;ring} \cdot u_B = 0$ (A 5.15)

u_B can now be expressed as:

$u_B = \frac{\frac{1}{2} ql^2 - \frac{ql}{tan(\varphi)} \cdot \frac{1}{2} y + F_{dl;ring} \cdot l}{\frac{1}{2} ql + F_{dl;ring} + k_B \cdot y}$ (A 5.16)

5.2 Fixed connection

Determination of U

The deformation of Ua and Ub can be determined using the displacement method for the structure below. The fixed connection is replaced by a hinged connection with a certain bending moment in the support. The fixed connections are assumed to be a rotational spring.
By taking the moment T around node A and B, the deformation can be described. The modulus of elasticity is assumed $E = \infty$. Bending and extension of the spoke do not influence in the deformation u.

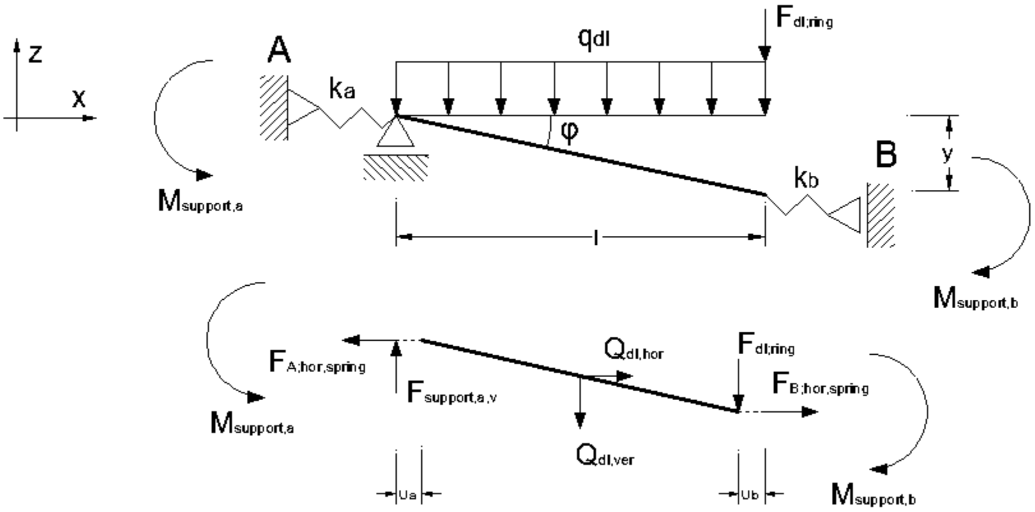


Figure A 5.2 Schematization of the hinged connection with bending moments

The forces illustrated in figure A 5.2 have the following expression:

Dead load:

$$Q_{dl;vert} = q \cdot l \tag{A 5.17}$$

$$Q_{dl;hor} = \frac{ql}{tan(\varphi)} \tag{A 5.18}$$

Spring force

$$F_{A;spring} = k_A \cdot u_A \tag{A 5.19}$$

$$F_{B;spring} = k_B \cdot u_B \tag{A 5.20}$$

Reaction force in node A

The vertical reaction force in node A is determined by the equilibrium of forces in the vertical direction $\sum F_{vert} = 0$

$$F_{A;vert} - Q_{dl;vert} - F_{dl;ring} = 0 \tag{A 5.21}$$

Is equal to:

$$F_{A;vert} - q \cdot l - F_{dl;ring} = 0 \tag{A 5.22}$$

Follows:

$$F_{A;vert} = q \cdot l + F_{dl;ring} \tag{A 5.23}$$

Now the forces in the structure are known, the horizontal deformation in node A and B can be described.

Determination of Ua

$T_B = M_B$ (A 5.24)

Follows:

$-F_{A;vert} \cdot (l - u_A) + F_{A;spring} \cdot y - Q_{dl;hor} \cdot \frac{1}{2}y + Q_{dl;vert} \cdot \frac{1}{2}(l - u_A) = M_B$ (A 5.25)

When implementing the equation for the forces, follows the equation:

$(k_A \cdot u_A) \cdot y - \left(ql + F_{dl;ring}\right)(l - u_A) - \frac{ql}{tan(\varphi)} \cdot \frac{1}{2}y + (ql) \cdot \frac{1}{2}(l - u_A) = M_B$ (A 5.26)

u_A can now be expressed as:

$u_A = \frac{M_B + ql^2 + F_{dl;ring}l + \frac{1}{2}qly(\tan(\varphi))^{-1} - \frac{1}{2}ql^2}{k_A \cdot y + ql - F_{dl;ring} - \frac{1}{2}ql}$ (A 5.27)

Determination of Ub

$T_A = M_A$ (A 5.28)

Follows:

$Q_{dl;hor} \cdot \frac{1}{2}y - Q_{dl;vert} \cdot \frac{1}{2}(l - u_B) + F_{B;spring} \cdot y - F_{dl;ring} \cdot (l - u_B) = M_A$ (A 5.29)

When implementing the equation for the forces, follows the equation:

$\frac{ql}{tan(\varphi)} \cdot \frac{1}{2}y - ql \cdot \frac{1}{2}(l - u_B) + k_B u_B \cdot y - F_{dl;ring} \cdot (l - u_B) = M_A$ (A 5.30)

Is equal to:

$\frac{1}{2}qly(\tan(\varphi))^{-1} - \frac{1}{2}ql^2 + \frac{1}{2}qlu_B + k_B u_B \cdot y - F_{dl;ring} \cdot l + F_{dl;ring} \cdot u_B = M_A$ (A 5.31)

u_B can now be expressed as:

$u_B = \frac{M_A - \frac{1}{2}qly(\tan(\varphi))^{-1} + \frac{1}{2}ql^2 + F_{dl;ring} \cdot l}{\frac{1}{2}ql + F_{dl;ring} + k_B \cdot y}$ (A 5.32)

Appendix A 6. Double inner/outer ring

6.1 Double inner ring

The schematization of a double inner ring is illustrated in figure A6.1. The normal forces are determined using the method of joints. From the static equilibrium, equations can be set up with the assumption that the sum of the forces in the x- and y-direction is equal to zero.

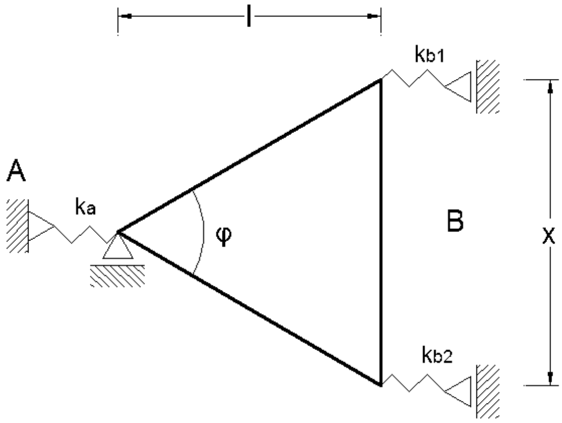


Figure A 6.1 Double inner ring

Using the method of joints, the normal forces in the truss members can be determined. Before the method of joints is applied to a node, the reaction forces have to be determined.

The spring forces are equal to:

$$F_{A;spring} = k_A \cdot u_A \tag{A 6.1}$$

$$F_{B;spring} = k_B \cdot u_B \tag{A 6.2}$$

The vertical reaction force depends on the transverse load acting on the structure. In this design phase the actual transverse loads (dead load, wind, snow etc.) are yet unknown. The normal force will be just expressed as a function of the reaction forces.

Pre-tensioned spokes

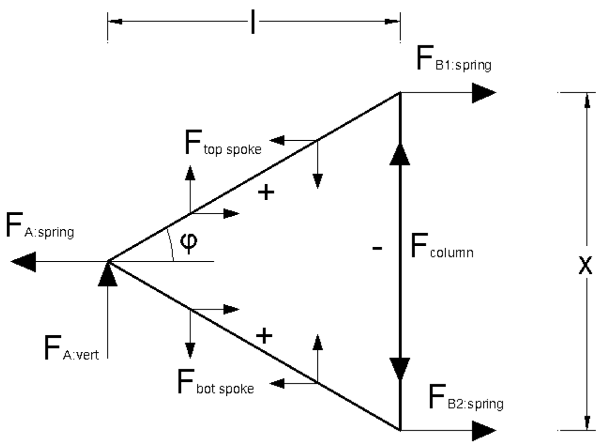


Figure A 6.2 Normal force distribution pre-tensioned double inner ring structure

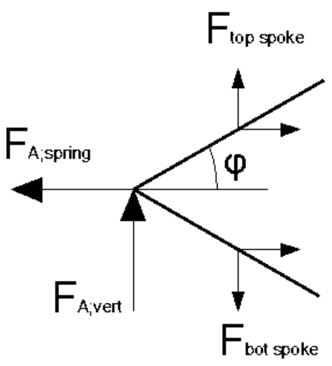


Figure A 6.3 Equilibrium in node at connection A

To determine the normal force in the diagonals, the method of joints is first applied to node A (figure A 6.3).

The forces acting on the nodes follow from the sum of forces in the x and y direction:

$\sum F_x = 0$and $\sum F_y = 0$

The normal force in the top and bottom spoke can be expressed as:

$$F_{top\ spoke;hor} = F_{top\ spoke} \cdot \cos(\varphi) \quad \text{(A 6.3)}$$

$$F_{top\ spoke;vert} = F_{top\ spoke} \cdot \sin(\varphi) \quad \text{(A 6.4)}$$

$$F_{bot\ spoke;hor} = F_{bot\ spoke} \cdot \cos(\varphi) \quad \text{(A 6.5)}$$

$$F_{bot\ spoke;vert} = F_{bot\ spoke} \cdot \sin(\varphi) \quad \text{(A 6.6)}$$

First the sum of forces in the x-direction is determined:

$$F_x: -F_{A;spring} + F_{top\ spoke;hor} + F_{bot\ spoke;hor} = 0 \quad \text{(A 6.7)}$$

$$F_y: F_{A;vert} + F_{top\ spoke;vert} - F_{bot\ spoke;vert} = 0 \quad \text{(A 6.8)}$$

Implementing the values of the spring forces in equations A 6.7 and A 6.8 follows:

$$F_x: -k_A \cdot u_A + F_{top\ spoke} \cdot \cos(\varphi) + F_{bot\ spoke} \cdot \cos(\varphi) = 0 \quad \text{(A 6.9)}$$

$$F_y: F_{A;vert} + F_{top\ spoke} \cdot \sin(\varphi) - F_{bot\ spoke} \cdot \sin(\varphi) = 0 \quad \text{(A 6.10)}$$

The normal force in the top spoke from the equation of F_y (A 6.10) can be expressed as:

$$F_{top\ spoke} = F_{bot\ spoke} - \frac{F_{A;vert}}{\sin(\varphi)} \quad \text{(A 6.11)}$$

Implementing the expression for $F_{top\ spoke}$ in the equation for F_x (A 6.9) follows the expression for $F_{bot\ spoke}$:

$$F_x: -k_A \cdot u_A + \left(F_{bot\ spoke} - \frac{F_{A;vert}}{\sin(\varphi)}\right) \cdot \cos(\varphi) + F_{bot\ spoke} \cdot \cos(\varphi) = 0 \quad \text{(A 6.12)}$$

$$F_{bot\ spoke} = \frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{\sin(\varphi)} \quad \text{(A 6.13)}$$

Now the normal force in the top spoke can be expressed as a function not depended from the bottom spoke from equation A 7.10:

$$F_y: F_{A;vert} + F_{top\ spoke} \cdot \sin(\varphi) - \left(\frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{\sin(\varphi)}\right) \cdot \sin(\varphi) = 0 \quad \text{(A 6.14)}$$

$$F_{top\ spoke} = \frac{k_A \cdot u_A}{2\cos(\varphi)} \tag{A 6.15}$$

This can be controlled by implementing the final expressions of the normal force in the top and bottom spoke (equation A 6.13 and A 6.15) in the equation of the sum of forces in a single direction. This will be done for equation A 6.10:

$$F_y: F_{A;vert} + F_{top\ spoke} \cdot \sin(\varphi) - F_{bot\ spoke} \cdot \sin(\varphi) = 0 \tag{A 6.16}$$

Follows:

$$F_y: F_{A;vert} + \left(\frac{k_A \cdot u_A}{2\cos(\varphi)}\right) \cdot \sin(\varphi) - \left(\frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{\sin(\varphi)}\right) \cdot \sin(\varphi) = 0 \tag{A 6.17}$$

The sum is still equal to zero, it can be concluded that the expressions for $F_{top\ spoke}$ and $F_{bot\ spoke}$ are correct.

Now the normal force in the diagonal is known, the normal force in the column bar of the truss can be determined using the equilibrium of forces in the top node (illustrated in figure A 6.4).

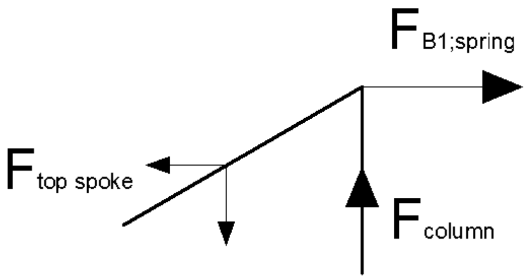


Figure A 6.4 Equilibrium in top node at connection B

To determine the normal force in the column, the equation of the sum of forces in the x-direction is sufficient:

$$F_x: F_{column} - F_{top\ spoke;vert} = 0 \tag{A 6.18}$$

Follows:

$$F_{column} = F_{top\ spoke;vert} \tag{A 6.19}$$

Is equal to:

$$F_{column} = \frac{k_A \cdot u_A}{2\cos(\varphi)} \cdot \sin(\varphi) \tag{A 6.20}$$

All forces in the members are expressed as a function of only the spring force in A. The influence of the spring force in node B1 can be determined as an expression of the spring force in A looking at the last equilibrium in the top node at B1, see again figure A 6.4.

Looking at the force equilibrium in the x-direction:

$$F_x: F_{B1;spring} - F_{top\ spoke;hor} = 0 \tag{A 6.21}$$

Follows:

$$F_{B1;spring} = F_{top\ spoke;hor} \tag{A 6.22}$$

Implementing the values for the forces:

$$k_{B1} \cdot u_{B1} = \frac{k_A \cdot u_A}{2\cos(\varphi)} \cdot \cos(\varphi) \tag{A 6.23}$$

Follows:

$$k_{B1} \cdot u_{B1} = 2(k_A \cdot u_A) \tag{A 6.24}$$

Looking only at the reaction force equilibrium, with $F_{A;vert}$ equal to zero, the equation A 6.24 is correct.

The influence of the spring force in node B2 can be determined as an expression of the spring force in A looking at the last equilibrium in the bottom node at B2, see figure A 6.2.

Looking at the force equilibrium in the x-direction:

$$F_x: F_{B2;spring} - F_{bot;\,spoke;hor} = 0 \tag{A 6.25}$$

Follows:

$$F_{B2;spring} = F_{bot\ spoke;hor} \tag{A 6.26}$$

Implementing the values for the forces:

$$k_{B2} \cdot u_{B2} = \frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{\sin(\varphi)} \tag{A 6.27}$$

Assume the vertical force is equal to zero follows:

$$k_{B2} \cdot u_{B2} = 2(k_A \cdot u_A) \tag{A 6.28}$$

From equation 6.24 and 6.28 follows that the spring force in B1 and B2 are equal.

Non pre-tensioned spokes

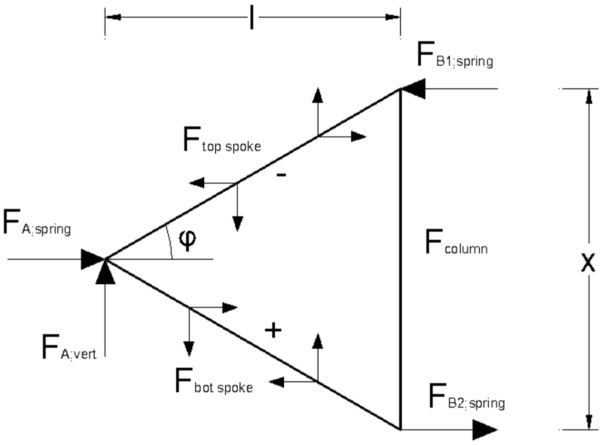


Figure A 6.5 Normal force distribution non pre-tensioned double inner ring structure

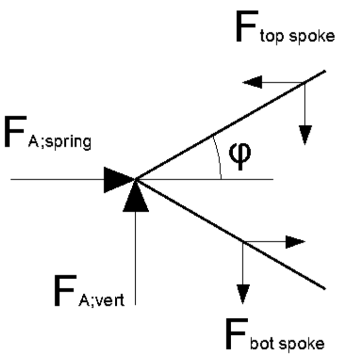


Figure A 6.6 Equilibrium in node at connection A

To determine the normal force in the diagonals, the method of joints is first applied to node A (figure A 6.6).

The forces acting on the nodes follow from the sum of forces in the x and y direction:

$\sum F_x = 0$and $\sum F_y = 0$

The normal force in the top and bottom spoke can be expressed as:

$$F_{top\ spoke;hor} = F_{top\ spoke} \cdot \cos(\varphi) \quad \text{(A 6.29)}$$

$$F_{top\ spoke;vert} = F_{top\ spoke} \cdot \sin(\varphi) \quad \text{(A 6.30)}$$

$$F_{bot\ spoke;hor} = F_{bot\ spoke} \cdot \cos(\varphi) \quad \text{(A 6.31)}$$

$$F_{bot\ spoke;vert} = F_{bot\ spoke} \cdot \sin(\varphi) \quad \text{(A 6.32)}$$

First the sum of forces in the x-direction is determined:

$$F_x: F_{A;spring} - F_{top\ spoke;hor} + F_{bot\ spoke;hor} = 0 \quad \text{(A 6.33)}$$

$$F_y: F_{A;vert} - F_{top\ spoke;vert} - F_{bot\ spoke;vert} = 0 \quad \text{(A 6.34)}$$

Implementing the values of the spring forces in equations A 6.29 and A 6.30 follows:

$$F_x: k_A \cdot u_A - F_{top\ spoke} \cdot \cos(\varphi) + F_{bot\ spoke} \cdot \cos(\varphi) = 0 \quad \text{(A 6.35)}$$

$$F_y: F_{A;vert} - F_{top\ spoke} \cdot \sin(\varphi) - F_{bot\ spoke} \cdot \sin(\varphi) = 0 \quad \text{(A 6.36)}$$

The normal force in the top spoke from the equation of F_y (A 6.32) can be expressed as:

$$F_{top\ spoke} = -F_{bot\ spoke} + \frac{F_{A;vert}}{\sin(\varphi)} \quad \text{(A 6.37)}$$

Implementing the expression for $F_{top\ spoke}$ in the equation for F_x (A 6.35) follows the expression for $F_{bot\ spoke}$:

$$F_x: k_A \cdot u_A - \left(-F_{bot\ spoke} + \frac{F_{A;vert}}{\sin(\varphi)}\right) \cdot \cos(\varphi) + F_{bot\ spoke} \cdot \cos(\varphi) = 0 \quad \text{(A 6.38)}$$

$$F_{bot\ spoke} = -\frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{2\sin(\varphi)} \quad \text{(A 6.39)}$$

Now the normal force in the top spoke can be expressed as a function not depended from the bottom spoke from equation A 6.34:

$$F_y: F_{A;vert} - F_{top\ spoke} \cdot \sin(\varphi) - \left(-\frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{2\sin(\varphi)}\right) \cdot \sin(\varphi) = 0 \quad \text{(A 6.40)}$$

$$F_{top\ spoke} = \frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{2\sin(\varphi)} \quad \text{(A 6.41)}$$

This can be controlled by implementing the final expressions of the normal force in the top and bottom spoke (equation A 6.37 and A 6.39) in the equation of the sum of forces in a single direction. This will be done for equation A 6.34:

$$F_y: F_{A;vert} + F_{top\ spoke} \cdot \sin(\varphi) - F_{bot\ spoke} \cdot \sin(\varphi) = 0 \quad \text{(A 6.42)}$$

Follows:

$$F_y: F_{A;vert} - \left(\frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{2\sin(\varphi)}\right) \cdot \sin(\varphi) - \left(-\frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{2\sin(\varphi)}\right) \cdot \sin(\varphi) = 0 \quad \text{(A 6.43)}$$

The sum is still equal to zero, it can be concluded that the expressions for $F_{top\ spoke}$ and $F_{bot\ spoke}$ are correct.

In the column there will be, in theory no normal forces. The inner ring is not supported during its sag by actual vertical reaction forces. The structure remains stable due to the 2nd order effect. The horizontal ring force causes a 'contra' force, resulting in a decrease of the amount of sag. The description of the 2nd order effect is described in chapter 7.1.2.2.

Assuming the normal force in the column in equal to zero, the bottom spring force in B can be expressed as a function of the spring force in A:

$$F_{bot\ spoke;hor} = F_{B;spring;bottom} = F_{B;spring;2} \quad \text{(A 6.44)}$$

Follows:

$$\left(-\frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{2\sin(\varphi)}\right) \cdot \cos(\varphi) = k_{B;2} \cdot u_{B;2} \quad \text{(A 6.45)}$$

Assuming that the vertical force in the column is zero, $F_{A;vert} = 0$:

$$\left(-\frac{k_A \cdot u_A}{2\cos(\varphi)}\right) \cdot \cos(\varphi) = k_{B;2} \cdot u_{B;2} \quad \text{(A 6.46)}$$

The spring force in the bottom inner ring can be expressed as:

$$-\frac{k_A \cdot u_A}{2} = k_{B;2} \cdot u_{B;2} \quad \text{(A 6.47)}$$

The spring force in the top inner ring B can also be expressed as a function of the spring force in A:

$$F_{top\ spoke;hor} = F_{B;spring;top} = F_{B;spring;1} \tag{A 6.48}$$

Follows:

$$\left(\frac{k_A \cdot u_A}{2\cos(\varphi)} + \frac{F_{A;vert}}{2\sin(\varphi)}\right) \cdot \cos(\varphi) = k_{B;1} \cdot u_{B;1} \tag{A 6.49}$$

Assuming that the vertical force in the column is zero, $F_{A;vert} = 0$:

$$\left(\frac{k_A \cdot u_A}{2\cos(\varphi)}\right) \cdot \cos(\varphi) = k_{B;1} \cdot u_{B;1} \tag{A 6.50}$$

The spring force in the bottom inner ring can be expressed as:

$$\frac{k_A \cdot u_A}{2} = k_{B;1} \cdot u_{B;1} \tag{A 6.51}$$

The spring force relation between B1 and B2 can be expressed as followed using equation A 6.47 and A 6.51:

$$k_{B;1} \cdot u_{B;1} = -k_{B;2} \cdot u_{B;2} \tag{A 6.52}$$

6.2 Double outer ring

The schematization of a double outer ring is illustrated in figure A 6.7. The normal forces are determined using the method of joints. From the static equilibrium, equations can be set up with the assumption that the sum of the forces in the x- and y-direction is equal to zero.

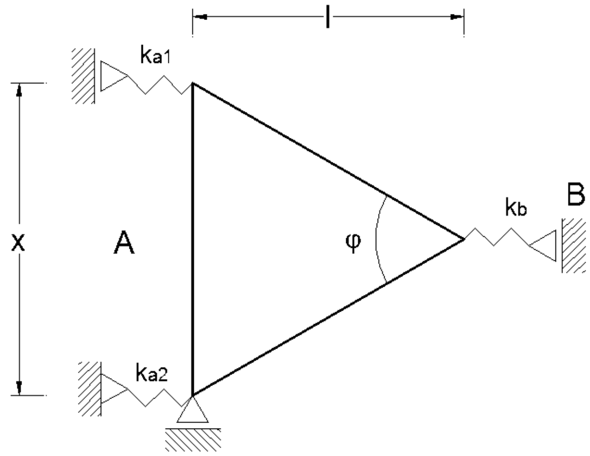


Figure A 6.7 Double outer ring

Following the same method as in the previous paragraph the normal forces in the truss members can be determined for a pre-tensioned and a non pre-tensioned structure

Pre-tensioned spokes

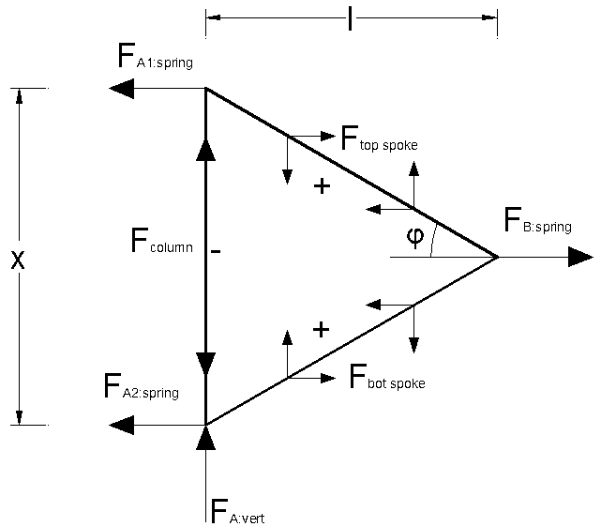


Figure A 6.8 Normal force distribution pre-tensioned double outer ring structure

The method of joints is first applied to node A, see figure A 6.9.

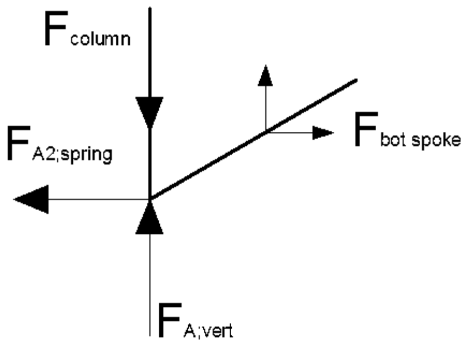


Figure A 6.9 Equilibrium in node at connection A

First the sum of forces in the x-direction is determined:

$$F_x: -F_{A;spring} + F_{bot\ spoke;hor} = 0 \tag{A 6.53}$$

$$F_y: F_{A;vert} + F_{bot\ spoke;vert} - F_{column} = 0 \tag{A 6.54}$$

Implementing the values of the spring forces in equations A 6.53 and A 6.54 follows:

$$F_x: -k_{A;2} \cdot u_{A;2} + F_{bot\ spoke} \cdot \cos(\varphi) = 0 \tag{A 6.55}$$

$$F_y: F_{A;vert} + F_{bot\ spoke} \cdot \sin(\varphi) - F_{column} = 0 \tag{A 6.56}$$

The normal force in the bottom spoke in equation A 6.56 can be expressed as:

$$F_{bot\ spoke} = \frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)} \tag{A 6.57}$$

Implementing the expression for $F_{bot\ spoke}$ in the equation for F_y (A 6.56) follows the expression for F_{column}:

$$F_y: F_{A;vert} + \left(\frac{k_{A;2} \cdot u_{;2A}}{\cos(\varphi)}\right) \cdot \sin(\varphi) - F_{column} = 0 \tag{A 6.58}$$

$$F_{column} = F_{A;vert} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) \tag{A 6.59}$$

The expression for the normal forces in the bottom spoke and column can be controlled by implementing the final expressions of the normal forces (equation A 6.57 and A 6.59) in the equation of the sum of forces in a single direction. This will be done for equation A 6.54:

$$F_y: F_{A;vert} + F_{bot\ spoke} \cdot \sin(\varphi) - F_{column} = 0 \tag{A 6.60}$$

Follows:

$$F_y: F_{A;vert} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) - F_{A;vert} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) = 0 \tag{A 6.61}$$

The sum is still equal to zero, it can be concluded that the expressions for F_{column} and $F_{bot\ spoke}$ are correct.

Now the expression for F_{column} and $F_{bot\ spoke}$ are known, the expression of the normal force in the top spoke can be determined looking at the equilibrium of forces in the top node at connection A:

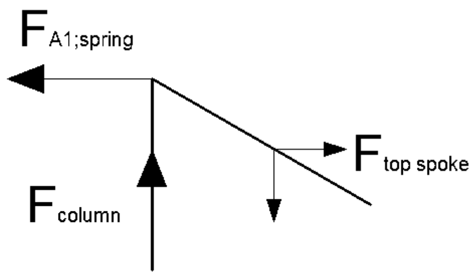


Figure A 6.10 Equilibrium in top node at connection A

To determine the normal force in the top spoke, the equation of the sum of forces in the y-direction is sufficient:

$$F_y: F_{column} - F_{top\ spoke;vert} = 0 \qquad \text{(A 6.62)}$$

Follows:

$$F_{top\ spoke;vert} = F_{column} \qquad \text{(A 6.63)}$$

Is equal to:

$$F_{top\ spoke;vert} = F_{A;vert} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) \qquad \text{(A 6.64)}$$

Follows:

$$F_{top\ spoke} = \frac{\left(F_{A;vert} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi)\right)}{\sin(\varphi)} = \frac{F_{A;vert}}{\sin(\varphi)} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \qquad \text{(A 6.65)}$$

The force in the top and bottom spoke are equal. It can be concluded that the spring force in the top and bottom spoke are equal:

$$k_{A;1} \cdot u_{A;1} = k_{A;2} \cdot u_{A;2}$$

All forces in the members are expressed as a function of only the spring force in A2. The influence of the spring force in node B can be determined as an expression of the spring force in A looking at the last equilibrium in the node at B:

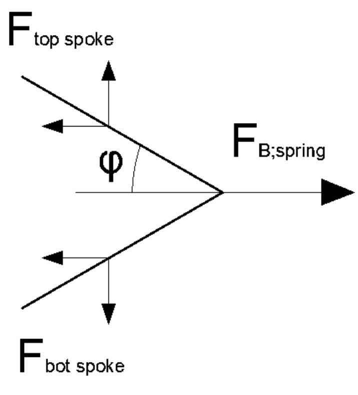


Figure A 6.11 Equilibrium in connection B

Looking at the force equilibrium in the x-direction:

$$F_x: F_{B;spring} - F_{bot\ spoke;hor} - F_{top\ spoke;hor} = 0 \qquad \text{(A 6.66)}$$

Follows:

$$F_{B;spring} = F_{bot\ spoke;hor} + F_{top\ spoke;hor} \qquad \text{(A 6.67)}$$

Implementing the values for the forces:

$$k_B \cdot u_B = \frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)} \cdot \sin(\varphi) + \left(\frac{F_{A;vert}}{\sin(\varphi)} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right)\right) \cdot \sin(\varphi) \qquad \text{(A 6.68)}$$

The vertical reaction force in connection A depends on the dead load of the structure. When leaving out the dead load the following relation can be described between the spring force in A and B:

$$k_B \cdot u_B = 2(k_{A;2} \cdot u_{A;2}) = 2(k_{A;1} \cdot u_{A;1}) \qquad \text{(A 6.69)}$$

Looking only at the reaction force equilibrium, with $F_{A;vert}$ equal to zero, the equation A 6.69 is correct.

Non pre-tensioned spokes

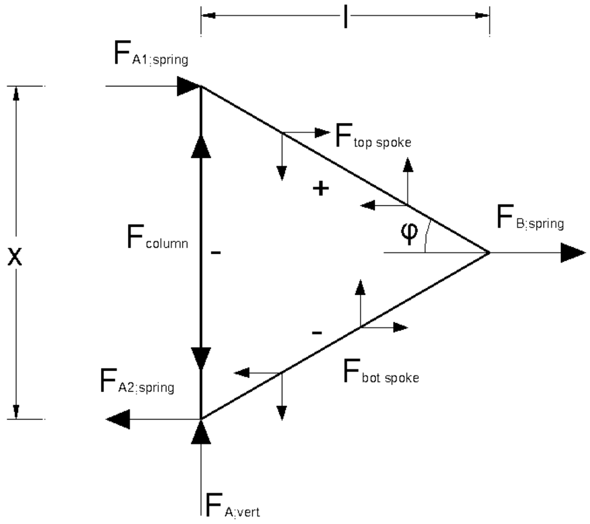


Figure A 6.12 Normal force distribution non pre-tensioned double outer ring structure

The method of joints is first applied to node A, see figure A 6.13.

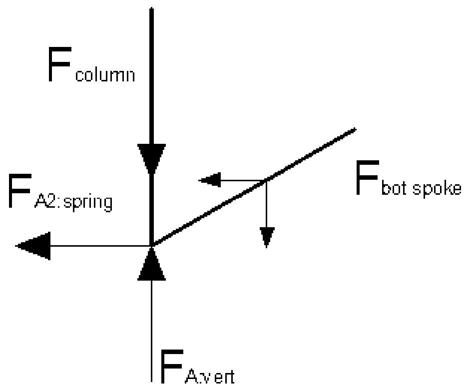


Figure A 6.13 Equilibrium in node at connection A

First the sum of forces in the x-direction is determined:

$$F_x: -F_{A;spring} - F_{bot\ spoke;hor} = 0 \tag{A 6.70}$$

$$F_y: F_{A;vert} - F_{bot\ spoke;vert} - F_{column} = 0 \tag{A 6.71}$$

Implementing the values of the spring forces in equations A 6.70 and A 6.71 follows:

$$F_x: -k_A \cdot u_A - F_{bot\ spoke} \cdot \cos(\varphi) = 0 \tag{A 6.72}$$

$$F_y: F_{A;vert} - F_{bot\ spoke} \cdot \sin(\varphi) - F_{column} = 0 \tag{A 6.73}$$

The normal force in the bottom spoke in equation A 6.73 can be expressed as:

$$F_{bot\ spoke} = -\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)} \tag{A 6.74}$$

Implementing the expression for $F_{bot\ spoke}$ in the equation for F_y (A 6.73) follows the expression for F_{column}:

$$F_y: F_{A;vert} - \left(-\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) - F_{column} = 0 \qquad \text{(A 6.75)}$$

$$F_{column} = F_{A;vert} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) \qquad \text{(A 6.76)}$$

The expression for the normal forces in the bottom spoke and column can be controlled by implementing the final expressions of the normal forces (equation A 6.74 and A 6.76) in the equation of the sum of forces in a single direction. This will be done for equation A 6.73:

$$F_y: F_{A;vert} - F_{bot\ spoke} \cdot \sin(\varphi) - F_{column} = 0 \qquad \text{(A 6.77)}$$

Follows:

$$F_y: F_{A;vert} - \left(-\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) - F_{A;vert} - \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) = 0 \qquad \text{(A 6.78)}$$

The sum is still equal to zero, it can be concluded that the expressions for F_{column} and $F_{bot\ spoke}$ are correct.

Now the expression for F_{column} and $F_{bot\ spoke}$ are known, the expression of the normal force in the top spoke can be determined looking at the equilibrium of forces in the top node at connection A:

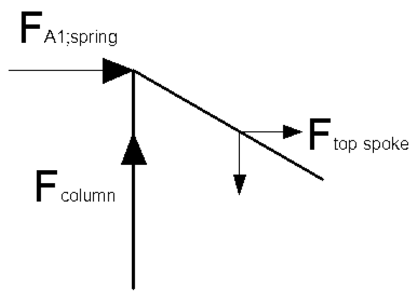


Figure A 6.14 Equilibrium in top node at connection A

To determine the normal force in the top spoke, the equation of the sum of forces in the y-direction is sufficient:

$$F_y: F_{column} - F_{top\ spoke;vert} = 0 \qquad \text{(A 6.79)}$$

Follows:

$$F_{top\ spoke;vert} = F_{column} \qquad \text{(A 6.80)}$$

Is equal to:

$$F_{top\ spoke;vert} = F_{A;vert} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) \qquad \text{(A 6.81)}$$

Follows:

$$F_{top\ spoke} = \frac{\left(F_{A;vert} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi)\right)}{\sin(\varphi)} = \frac{F_{A;vert}}{\sin(\varphi)} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \qquad \text{(A 6.82)}$$

It can be concluded that the relation between the spring force in the bottom and top spokes is, assuming the vertical force is equal to zero, as follows:

$$k_{A;1} \cdot u_{A;1} = -k_{A;2} \cdot u_{A;2} \qquad \text{(A 6.83)}$$

All forces in the members are expressed as a function of only the spring force in A (the top and bottom spring force are equal). The influence of the spring force in node B can be determined as an expression of the spring force in A looking at the last equilibrium in the node at B:

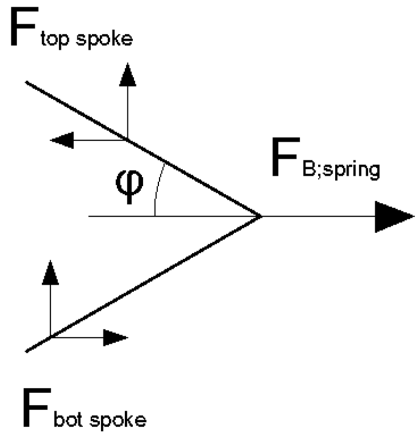


Figure A 6.15 Equilibrium in connection B

Looking at the force equilibrium in the x-direction:

$$F_x: F_{B;spring} + F_{bot\ spoke;hor} - F_{top\ spoke;hor} = 0 \quad \text{(A 6.84)}$$

Follows:

$$F_{B;spring} = -F_{bot\ spoke;hor} + F_{top\ spoke;hor} \quad \text{(A 6.85)}$$

Implementing the values for the forces:

$$k_B \cdot u_B = -\left(-\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right) \cdot \sin(\varphi) + \left(\frac{F_{A;vert}}{\sin(\varphi)} + \left(\frac{k_{A;2} \cdot u_{A;2}}{\cos(\varphi)}\right)\right) \cdot \sin(\varphi) \quad \text{(A 6.86)}$$

The vertical reaction force in connection A depends on the dead load of the structure. When leaving out the dead load the following relation can be described between the spring force in A and B:

$$k_B \cdot u_B = 2(k_{A;2} \cdot u_{A;2}) = -2(k_{A;2} \cdot u_{A;2}) \quad \text{(A 6.87)}$$

Looking only at the reaction force equilibrium, with $F_{A;vert}$ equal to zero, the equation A 6.87 is correct.

Appendix A 7. Structural materials

7.1 Shrinkage

h_0	k_h
100	1,0
200	0,85
300	0,75
≥ 500	0,70

Figure A 7.1 Value of k_h for height h_o

$f_{ck}/f_{ck,cube}$ (MPa)	Relatieve vochtigheid ($^0/_0$)					
	20	40	60	80	90	100
20/25	0.62	0.58	0.49	0.30	0.17	0.00
40/50	0.48	0.46	0.38	0.24	0.13	0.00
60/75	0.38	0.36	0.30	0.19	0.10	0.00
80/95	0.30	0.28	0.24	0.15	0.08	0.00
90/105	0.27	0.25	0.21	0.13	0.07	0.00

Figure A 7.2 Value of $\varepsilon_{cd,0}$

7.2 Creep

The extension coefficient of creep can be calculated by determining the needed factors of equation 7.23 and 7.24. The factors can be determined as displayed in figure A 7.3.

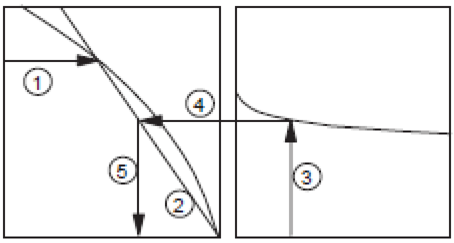


Figure A 7.3 determination of the creep coefficient

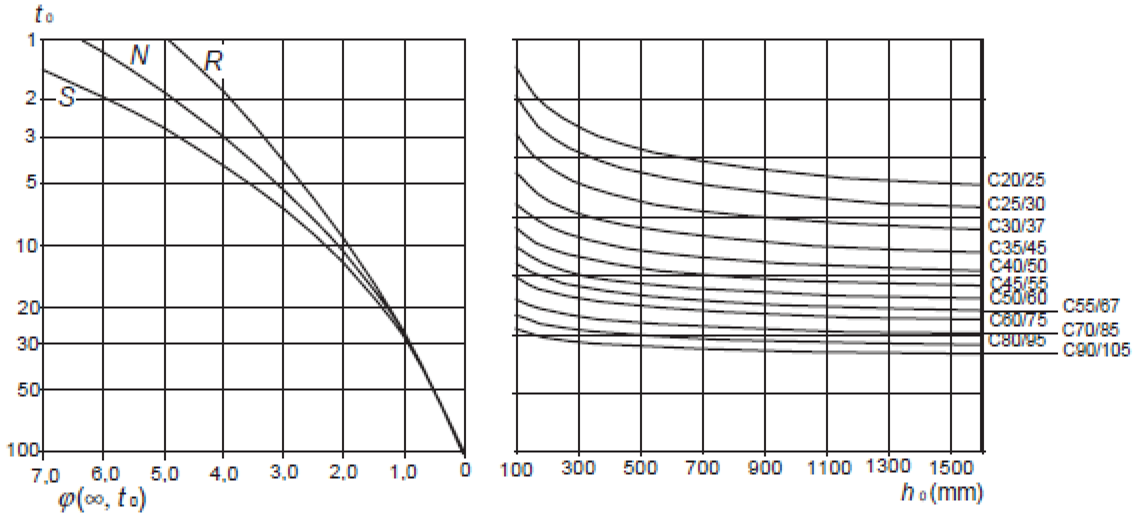


Figure A 7.4 Inside environment – RH = 50%

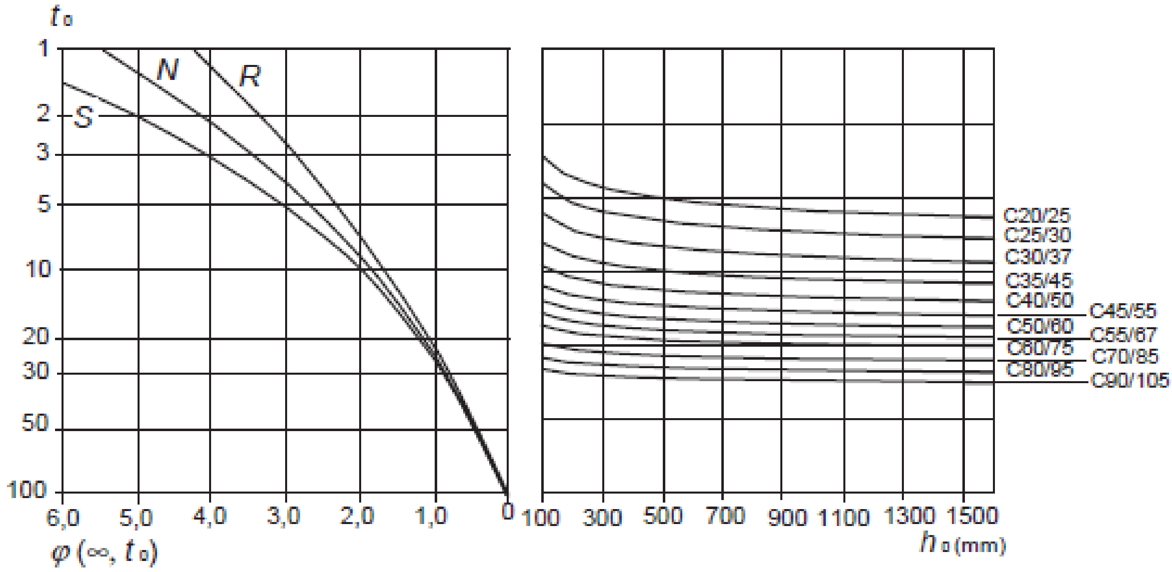


Figure A 7.5 Outside environment – RH = 80%

Appendix A 8. Preliminary Design

During the Design Research part different design factors and aspects has been investigated. From the investigation a global design has been made, that is most efficient in this phase of the design process.
The global design is illustrated in figure A 8.1. This global design will be a starting point for the Detailed Design part of the thesis. In this part a final design will be made for the global design with cable elements and steel profiles. The characteristics of both elements are different and therefore cause a different final structural design.

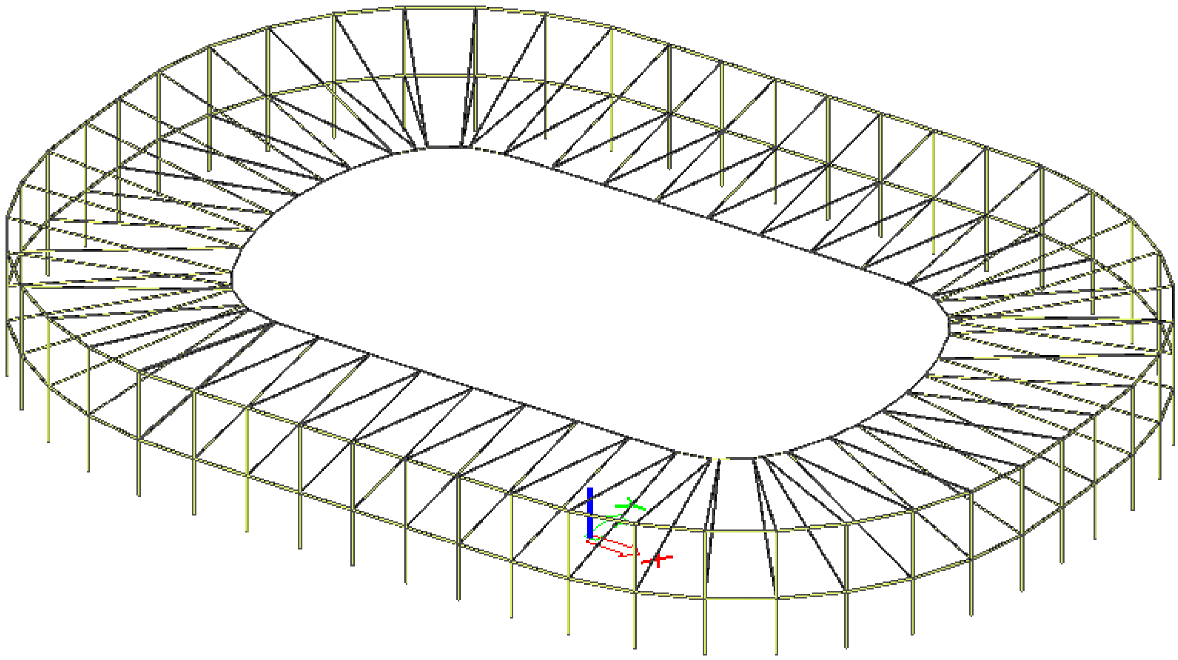


Figure A 8.1 Overall view preliminary design

The dimensions of the global design are presented in figure A 8.2. The global design is based on the dimensions of the Amsterdam ArenA, causing boundary conditions. One of them is the place of the supports. Therefore the place of the spokes is also fixed. The distance between the supports/spokes is 12,30m.
Because the place of the supports is fixed, the shape of the outer perimeter is known. The shape of the inner perimeter has been determined by applying the greatest curvature possible in the given design area (explained in chapter 4). The shape of the inner ring is elliptical and has been determined in chapter 7. The largest curvature is present at the centre of the sides and is presented in figure A 8.2.
What is left are the determination of the height between the two outer ring elements and the height of the rocker bearings. For these two variables an assumption has been made. Looking at reference projects one can estimate the needed dimensions. The Feyenoord stadium in Rotterdam, is a similar stadium with almost equal dimensions. The height-span ratio of the roof is around 1:2,8. The span of the global design is already known, it has an average value of 42,50m. The height of the spoke framework would be $\frac{42,50}{2,8} = \pm 15,00m$. This is height that will be used for the spoke framework.
The only design variable dimension is the height of the rocker bearing. The height of the rocker bearing is set to 12,30m. The reason is that the distance between the spokes is also 12,30m. Between the rocker bearings diagonal wall bracings have to be applied, in order to stabilize the structure. When the height and width of the rocker bearings are equal, the diagonals are placed under 45 degrees. As a consequence the loads can be transferred most efficient.

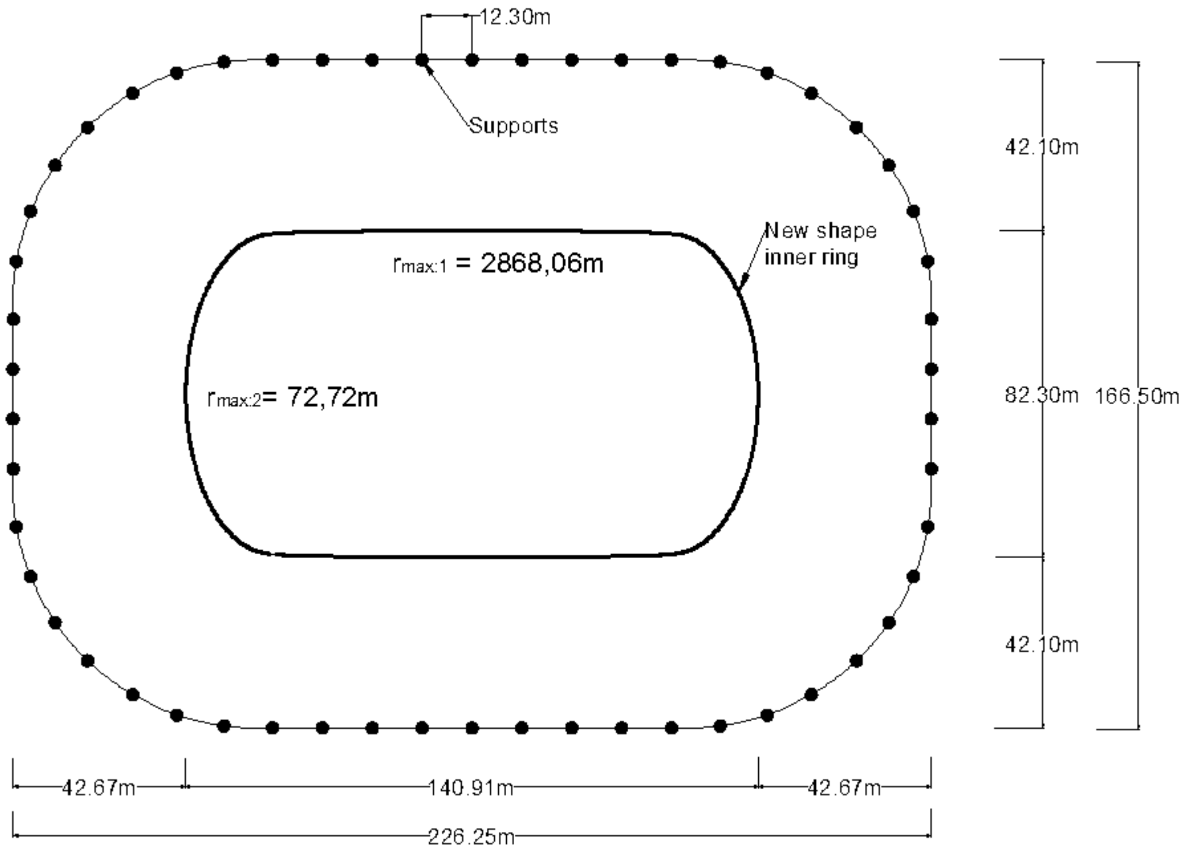


Figure A 8.2 Dimensions preliminary design

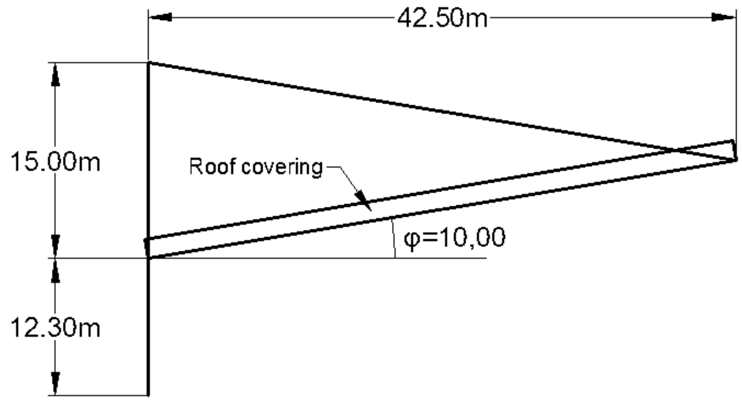


Figure A 8.3 Dimensions spoke framework

Appendix A 9. Load calculations

Wind load

The wind pressure acting on the stadium roof can be calculated using the following formula:

$$w = q_p(z_e) \cdot c_{pr} \qquad \text{(A 9.1)}$$

Where

$q_p(z_e)$ is the extreme trust or wind pressure at height z
c_{pr} is the pressure coefficient depending from the shape of the structure

The values for the pressure coefficient for the stadium follow from wind tunnel tests for the Amsterdam ArenA roof. These pressure are illustrated in figure A 9.1. The extreme wind pressure can be found using figure A 9.2 and table A 9.1. Assumed is wind area 2 for the reference stadium at height 50m. Follows that the extreme wind pressure is 1,38 kN/m².

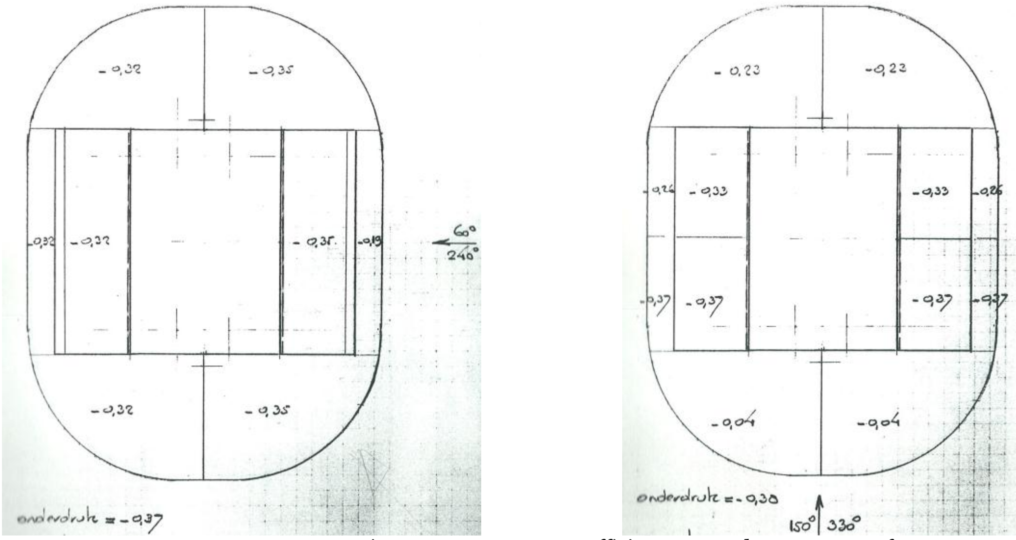


Figure A 9.1 Pressure coefficient Amsterdam ArenA roof

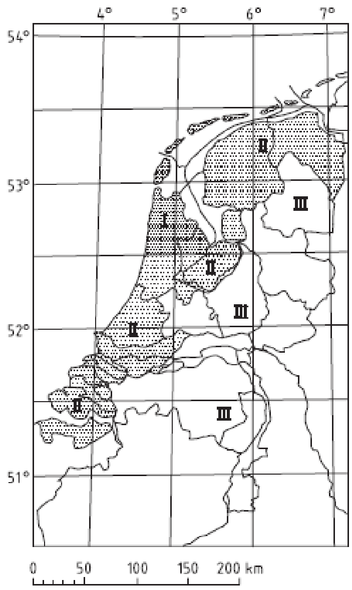


Figure A 9.2 Wind areas in the Netherlands

Hoogte	Gebied I			Gebied II			Gebied III	
		onbe-	be-		onbe-	be-	onbe-	be-
m	kust	bouwd	bouwd	kust	bouwd	bouwd	bouwd	bouwd
1	0,93	0,71	0,69	0,78	0,60	0,58	0,49	0,48
2	1,11	0,71	0,69	0,93	0,60	0,58	0,49	0,48
3	1,22	0,71	0,69	1,02	0,60	0,58	0,49	0,48
4	1,30	0,71	0,69	1,09	0,60	0,58	0,49	0,48
5	1,37	0,78	0,69	1,14	0,66	0,58	0,54	0,48
6	1,42	0,84	0,69	1,19	0,71	0,58	0,58	0,48
7	1,47	0,89	0,69	1,23	0,75	0,58	0,62	0,48
8	1,51	0,94	0,73	1,26	0,79	0,62	0,65	0,51
9	1,55	0,98	0,77	1,29	0,82	0,65	0,68	0,53
10	1,58	1,02	0,81	1,32	0,85	0,68	0,70	0,56
15	1,71	1,16	0,96	1,43	0,98	0,80	0,80	0,66
20	1,80	1,27	1,07	1,51	1,07	0,90	0,88	0,74
25	1,88	1,36	1,16	1,57	1,14	0,97	0,94	0,80
30	1,94	1,43	1,23	1,63	1,20	1,03	0,99	0,85
35	2,00	1,50	1,30	1,67	1,25	1,09	1,03	0,89
40	2,04	1,55	1,35	1,71	1,30	1,13	1,07	0,93
45	2,09	1,60	1,40	1,75	1,34	1,17	1,11	0,97
50	2,12	1,65	1,45	1,78	1,38	1,21	1,14	1,00
55	2,16	1,69	1,49	1,81	1,42	1,25	1,17	1,03
60	2,19	1,73	1,53	1,83	1,45	1,28	1,19	1,05
65	2,22	1,76	1,57	1,86	1,48	1,31	1,22	1,08
70	2,25	1,80	1,60	1,88	1,50	1,34	1,24	1,10
75	2,27	1,83	1,63	1,90	1,53	1,37	1,26	1,13
80	2,30	1,86	1,66	1,92	1,55	1,39	1,28	1,15
85	2,32	1,88	1,69	1,94	1,58	1,42	1,30	1,17
90	2,34	1,91	1,72	1,96	1,60	1,44	1,32	1,18
95	2,36	1,93	1,74	1,98	1,62	1,46	1,33	1,20
100	2,38	1,96	1,77	1,99	1,64	1,48	1,35	1,22
110	2,42	2,00	1,81	2,03	1,68	1,52	1,38	1,25
120	2,45	2,04	1,85	2,05	1,71	1,55	1,41	1,28
130	2,48	2,08	1,89	2,08	1,74	1,59	1,44	1,31
140	2,51	2,12	1,93	2,10	1,77	1,62	1,46	1,33
150	2,54	2,15	1,96	2,13	1,80	1,65	1,48	1,35
160	2,56	2,18	2,00	2,15	1,83	1,67	1,50	1,38
170	2,59	2,21	2,03	2,17	1,85	1,70	1,52	1,40
180	2,61	2,24	2,06	2,19	1,88	1,72	1,54	1,42
190	2,63	2,27	2,08	2,20	1,90	1,75	1,56	1,44
200	2,65	2,29	2,11	2,22	1,92	1,77	1,58	1,46

Table A 9.1 Extreme wind pressure values

Appendix A 10. Truss system

For the truss system three design variants have been investigated. In this chapter the calculation theory and an example of the determination of a CHS profile for a certain element in the design. The calculation results are presented in an additional report; FEM Calculations.

10.1 Calculation theory

The theory regarding the strength, stiffness and stability are according Eurocode NEN-EN 1993-1-1.

10.1.1 General information

Material

For the structural design variants only CHS profiles with a steel grade S235 have been applied and has the following properties:

Yield strength: $f_y = 235\ N/mm^2$

Ultimate tensile strength: $f_u = 360\ N/mm^2$

Modulus of Elasticity: $E = 210.000 N/mm^2$

Shear modulus: $G = \frac{E}{2(1+\upsilon)} \cong 81.000 N/mm^2$

Poisson's ratio: $\upsilon = 0{,}3$

Coeff. of linear thermal expansion: $\alpha = 12 * 10^{-6}\ perK$ (for $T \leq 100°C$)

Method of analysis

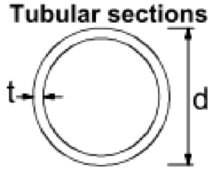
Due to the complex structures, a FEM program is used for the calculations. For the structure second order analysis is made, which takes the influence of the deformation of the structure into account.

Cross section

The profiles are calculated using cross section class 1-3. Class 4 cross sections have been left out of consideration in order to prevent the elements from shear buckling.

The cross section of a bar need to fulfil certain requirements regarding plastic analysis. The cross section of a CHS profile has a limited diameter/thickness ratio for compressed bars. The requirements are presented in table A 10.1.

Tubular sections



Class	Section in bending and/or compression
1	$d/t \leq 50\varepsilon^2$
2	$d/t \leq 70\varepsilon^2$
3	$d/t \leq 90\varepsilon^2$ **NOTE** For $d/t > 90\varepsilon^2$ see EN 1993-1-6.

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71
	ε^2	1,00	0,85	0,66	0,56	0,51

Figure A 10.1 Maximum diameter/thickness ratio for compressed tubular sections. Reproduced from NEN-EN 1993-1-1 table 5.2.

For the structural models steel with a steel grade S235 has been applied. The maximum ratio for the CHS profiles is 90 (cross section class 3).

10.1.2 Ultimate Limit State

The requirements regarding normal forces, transverse forces and bending moments that are used for the calculations are presented in this paragraph.

10.1.2.1 Strength Requirements

The partial factors have the values $\gamma_{m0} = 1{,}10$; $\gamma_{m1} = 1{,}10$ and $\gamma_{m2} = 1{,}25$

Axial tension

The calculation value of tension force N_{Ed} in every cross section must fulfil to:

$$\frac{N_{Ed}}{N_{t,Rd}} \leq 1{,}0 \qquad \text{(A 10.1)}$$

With

$$N_{t,Rd} = N_{pl,Rd} = \frac{Af_y}{\gamma_{M0}} \qquad \text{(A 10.2)}$$

Axial compression

The calculation value of compression force N_{Ed} in every cross section must fulfil to:

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1{,}0 \qquad \text{(A 10.3)}$$

With

$$N_{c,Rd} = \frac{Af_y}{\gamma_{M0}} \qquad \text{(A 10.4)}$$

Bending moment

The calculation value of bending moment M_{Ed} in every cross section must fulfil to:

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1{,}0 \qquad \text{(A 10.5)}$$

With

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl}f_y}{\gamma_{M0}} \qquad \text{(A 10.6)}$$

Transverse force

The calculation value of transverse force V_{Ed} in every cross section must fulfil to:

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1{,}0 \qquad \text{(A 10.7)}$$

The steel CHS profiles are checked with the plastic calculation method. The plastic transverse force value is determined using formula 6.18 from NEN-EN 1993-1-1.

$$V_{pl.Rd} = \frac{A_v\left(f_y/\sqrt{3}\right)}{\gamma_{M0}} \qquad \text{(A 10.8)}$$

With

$$A_v = 2A/\pi \qquad \text{(A 10.9)}$$

Torsion

The calculation value of torsion T_{Ed} in every cross section must fulfil to:

$$\frac{T_{Ed}}{T_{Rd}} \leq 1{,}0 \qquad \text{(A 10.10)}$$

With T_{Rd} is the calculation value of the torsion moment resistance capacity of the cross section. In case of CHS profiles, the effects of cambering of the cross section can be neglected.

Interaction Bending moment and Transverse force

When $V_{Ed} \leq 0{,}50V_{pl,Rd}$ the influence of the transverse force on W_{pl} can be neglected. Except when shear buckling due to transverse force the moment resistance of the cross section decreases.

When $V_{Ed} > 0{,}50V_{pl,Rd}$ a reduced W_{pl} value must be used, calculated with a reduced yield point for the shear area:

$$(1-\rho)f_y \qquad \text{(A 10.13)}$$

Where $\rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1\right)^2$ and with $V_{pl,Rd}$ determined by using equation 10.8.

When torsion arises, $\rho = \left(\frac{2V_{Ed}}{V_{pl,T,Rd}} - 1\right)^2$, with $V_{pl,T,Rd}$ according to equation 10.12. $\rho = 0$ when $V_{Ed} \leq 0{,}50V_{pl,T,Rd}$.

Interaction Bending moment and Normal force

The calculation value of bending moment M_{Ed} must fulfil the following requirement:

$$\frac{M_{Ed}}{M_{N,Rd}} \leq 1{,}0 \qquad \text{(A 10.14)}$$

$M_{N,Rd}$ is the reduced calculation value of the yield moment as a consequence of the normal force N_{Ed}.

$M_{N,y,Rd} = M_{pl,y,Rd}(1-n)/(1-0{,}5a_w)$ with requirement: $M_{N,y,Rd} \leq M_{pl,y,Rd}$ (A 10.15)

$M_{N,z,Rd} = M_{pl,z,Rd}(1-n)/(1-0{,}5a_w)$ with requirement: $M_{N,z,Rd} \leq M_{pl,z,Rd}$ (A 10.16)

Where:

$a_w = (A-2bt)/A$ with requirement $a_w \leq 0{,}5$ for rolled CHS profiles

$a_w = (A-2bt_f)/A$ with requirement $a_w \leq 0{,}5$ for welded CHS profiles

In case of bending moment in two directions (y and z), the following required must be fulfilled:

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^2 + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^2 \leq 1{,}0 \qquad \text{(A 10.17)}$$

Interaction Bending moment, Transverse force and Normal force

When transverse forces and normal forces arise, the influence of these forces on the bending moments must be taken into account.

When $V_{Ed} \leq 0{,}50V_{pl,Rd}$ the influence of the transverse force on equation 10.14 and 10.17 can be neglected. Except when shear buckling due to transverse force the moment resistance of the cross section decreases.

When $V_{Ed} > 0{,}50V_{pl,Rd}$ the interaction between the bending moment and normal force must be determined using a reduced yield point for the shear area:

$$(1-\rho)f_y \qquad \text{(A 10.18)}$$

Where $\rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1\right)^2$ and with $V_{pl,Rd}$ determined by using equation 10.8.

10.1.2.2 Stability Requirements

Buckling stability

The buckling stability of a compressed bar need to be checked as follows:

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1{,}0 \qquad \text{(A 10.19)}$$

$N_{b,Rd}$ is the calculation value of the buckling resistance of the compressed bar and can be calculated as follows:

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \qquad \text{(A 10.20)}$$

Where χ is the reduction factor. The equation for the reduction factor is equal to:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \quad \text{with requirement } \chi \leq 1{,}0 \qquad \text{(A 10.21)}$$

Where

$$\Phi = 0{,}5\left[1 + \alpha\left(\bar{\lambda} - 0{,}2\right) + \bar{\lambda}^2\right] \qquad \text{(A 10.22)}$$

$$\bar{\lambda} = \sqrt{\frac{\mathrm{Af_y}}{\mathrm{N_{cr}}}} \qquad \text{(A 10.23)}$$

$\mathrm{N_{cr}}$ is the critical elastic force for the needed buckling curve based on the properties of the gross cross section.

α is the imperfection factor. The imperfection factor can be determined using table A 10.1. or figure A 10.1. The buckling curve can be determined using table A 10.2

Buckling curve	**a_0**	**a**	**b**	**c**	**d**
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

Table A 10.1 Imperfection factor for buckling curves. Reproduced from NEN-EN 1993-1-1 table 6.1.

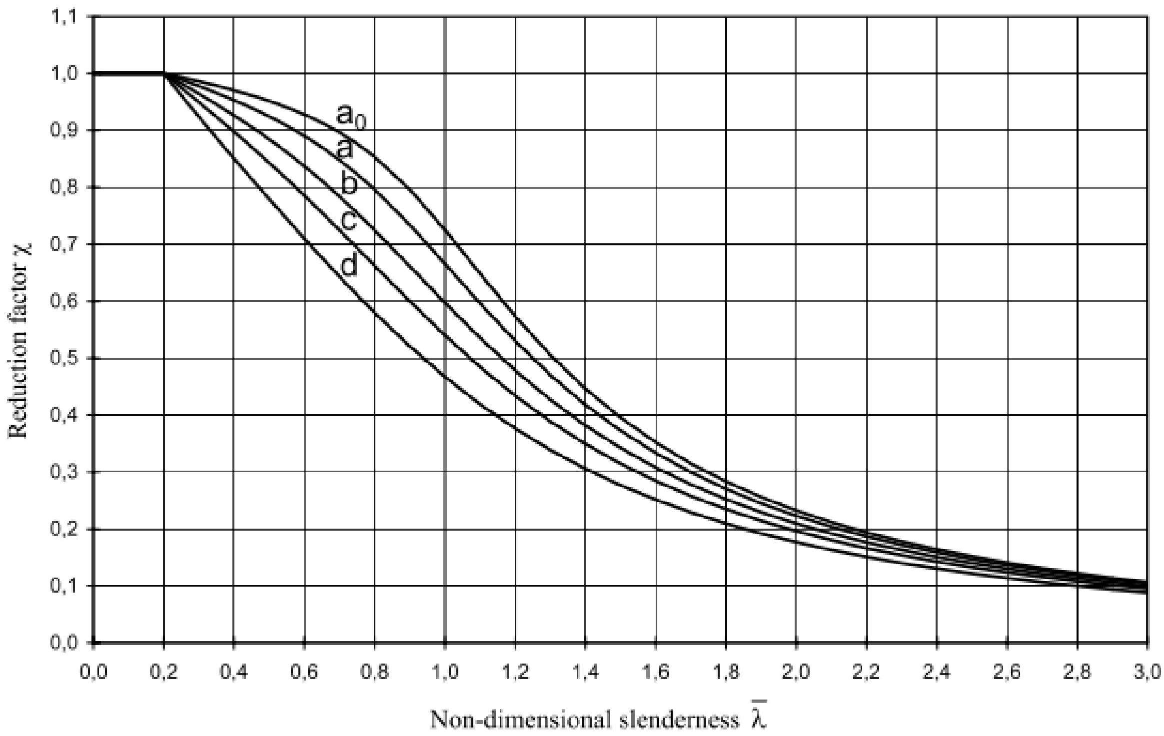


Figure A 10.2 Buckling curves. Reproduced from NEN-EN 1993-1-1 figure 6.4.

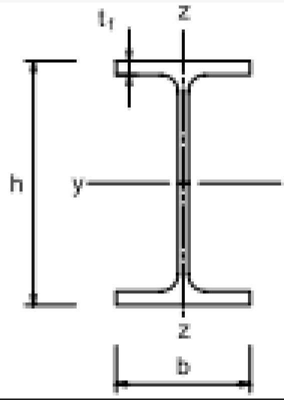
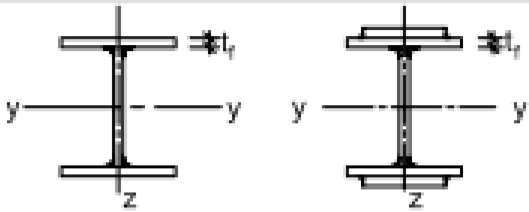
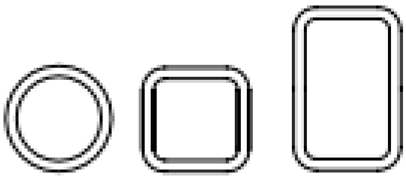
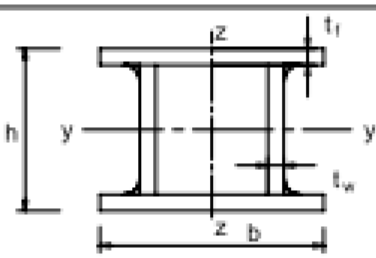
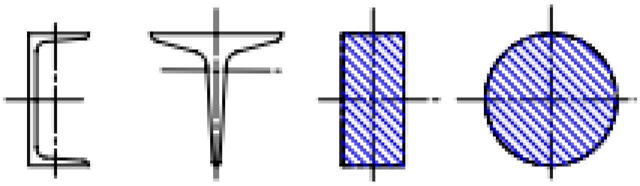
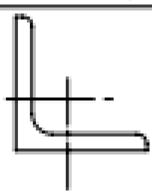
Cross section		Limits		Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	Buckling curve S 460
Rolled sections		$h/b > 1{,}2$	$t_f \le 40$ mm	y – y	a	a_0
				z – z	b	a_0
			40 mm $< t_f \le 100$	y – y	b	a
				z – z	c	a
		$h/b \le 1{,}2$	$t_f \le 100$ mm	y – y	b	a
				z – z	c	a
			$t_f > 100$ mm	y – y	d	c
				z – z	d	c
Welded I-sections		$t_f \le 40$ mm		y – y	b	b
				z – z	c	c
		$t_f > 40$ mm		y – y	c	c
				z – z	d	d
Hollow sections		hot finished		any	a	a_0
		cold formed		any	c	c
Welded box sections		generally (except as below)		any	b	b
		thick welds: $a > 0{,}5t_f$ $b/t_f < 30$ $h/t_w < 30$		any	c	c
U-, T- and solid sections				any	c	c
L-sections				any	b	b

Table A 10.2 Buckling curve. Reproduced from NEN-EN 1993-1-1 table 6.2.

For the CHS profiles used for the structural designs buckling curve a must be used. The imperfection factor is therefore 0,21.

For a relative slenderness of $\bar{\lambda} \leq 0{,}2$, or $\frac{N_{Ed}}{N_{cr}} \leq 0{,}04$ the buckling effects may be neglected.

Torsion buckling

Torsion buckling only has to be taken into account for open (not closed) cross sections. For CHS profiles torsion buckling is not leading and will not be taken into account.

Buckling resistance (bending moments)

CHS profiles are not sensitive for lateral torsion buckling. The buckling curve of CHS profiles are equal to: $\chi_{LT} = 1{,}0$.

Stability of bars subjected to bending moments and compression forces

Bars that are subjected to combined bending and compression must fulfil the following requirement:

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\chi_{LT}\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1{,}0 \qquad \text{(A 10.24)}$$

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\chi_{LT}\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1{,}0 \qquad \text{(A 10.25)}$$

Where

$N_{Ed}, M_{y,Ed}, M_{z,Ed}$ are calculation values of the compression force and the maximum bending moments around the y-y and z-z axes;

$\Delta M_{y,Ed}, \Delta M_{z,Ed}$ are the moments due to change of the point of gravity of the cross section, see table A 10.3;

χ_y, χ_z are the buckling reduction factors according equation A 10.21;

χ_{LT} is the lateral torsion buckling reduction factor and is equal to $\chi_{LT} = 1{,}0$;

$k_{yy}, k_{yz}, k_{zy}, k_{zz}$ are the interaction factors.

Steel class	**1**	**2**	**3**	**4**
A_i	$\boldsymbol{A}$	A	A	A_{eff}
W_y	$\boldsymbol{W_{pl,y}}$	$W_{pl,y}$	$W_{el,y}$	$W_{eff,y}$
W_z	$\boldsymbol{W_{pl,z}}$	$W_{pl,z}$	$W_{el,z}$	$W_{eff,z}$
$\Delta M_{y,Ed}$	**0**	0	0	$e_{N,y}N_{Ed}$
$\Delta M_{z,Ed}$	**0**	0	0	$e_{N,z}N_{Ed}$

Table A 10.3 Values for the steel classes. Reproduced from NEN-EN 1993-1-1 table 6.7.

The CHS profiles are of cross section class 1-3. Knowing that γ_{Mo} and γ_{M1} are equal to 1,1; equations 10.24 and 10.25 can be derived into the following equations:

$$\frac{N_{Ed}}{\chi_y N_{Rk}/1{,}1} + k_{yy}\frac{M_{y,Ed}}{M_{y,Rk}/1{,}1} + k_{yz}\frac{M_{z,Ed}}{M_{z,Rk}/1{,}1} \leq 1{,}0 \qquad \text{(A 10.26)}$$

$$\frac{N_{Ed}}{\chi_y N_{Rk}/1{,}1} + k_{zy}\frac{M_{y,Ed}}{M_{y,Rk}/1{,}1} + k_{zz}\frac{M_{z,Ed}}{M_{z,Rk}/1{,}1} \leq 1{,}0 \qquad \text{(A 10.27)}$$

The interaction factors can be determined using table 10.4-7.

Interaction factors	Design assumptions	
	elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{yy}	$C_{my} C_{mLT} \dfrac{\mu_y}{1 - \dfrac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \dfrac{\mu_y}{1 - \dfrac{N_{Ed}}{N_{cr,y}}} \dfrac{1}{C_{yy}}$
k_{yz}	$C_{mz} \dfrac{\mu_y}{1 - \dfrac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \dfrac{\mu_y}{1 - \dfrac{N_{Ed}}{N_{cr,z}}} \dfrac{1}{C_{yz}} 0{,}6 \sqrt{\dfrac{w_z}{w_y}}$
k_{zy}	$C_{my} C_{mLT} \dfrac{\mu_z}{1 - \dfrac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \dfrac{\mu_z}{1 - \dfrac{N_{Ed}}{N_{cr,y}}} \dfrac{1}{C_{zy}} 0{,}6 \sqrt{\dfrac{w_y}{w_z}}$
k_{zz}	$C_{mz} \dfrac{\mu_z}{1 - \dfrac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \dfrac{\mu_z}{1 - \dfrac{N_{Ed}}{N_{cr,z}}} \dfrac{1}{C_{zz}}$

Table A 10.4 Interaction factors k_{ij}. Reproduced from NEN-EN 1993-1-1 table A.1.

$\mu_y = \dfrac{1 - \dfrac{N_{Ed}}{N_{cr,y}}}{1 - \chi_y \dfrac{N_{Ed}}{N_{cr,y}}}$	$C_{yy} = 1 + (w_y - 1)\left[\left(2 - \dfrac{1{,}6}{w_y} C_{my}^2 \bar{\lambda}_{max} - \dfrac{1{,}6}{w_y} C_{my}^2 \bar{\lambda}_{max}^2\right) n_{pl} - b_{LT}\right] \geq \dfrac{W_{el,y}}{W_{pl,y}}$ with $b_{LT} = 0{,}5\, a_{LT}\, \bar{\lambda}_0^2 \dfrac{M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} \dfrac{M_{z,Ed}}{M_{pl,z,Rd}}$
$\mu_z = \dfrac{1 - \dfrac{N_{Ed}}{N_{cr,z}}}{1 - \chi_z \dfrac{N_{Ed}}{N_{cr,z}}}$	$C_{yz} = 1 + (w_z - 1)\left[\left(2 - 14 \dfrac{C_{mz}^2 \bar{\lambda}_{max}^2}{w_z^5}\right) n_{pl} - c_{LT}\right] \geq 0{,}6 \sqrt{\dfrac{w_z}{w_y}} \dfrac{W_{el,z}}{W_{pl,z}}$ with $c_{LT} = 10\, a_{LT} \dfrac{\bar{\lambda}_0^2}{5 + \bar{\lambda}_z^4} \dfrac{M_{y,Ed}}{C_{my}\, \chi_{LT}\, M_{pl,y,Rd}}$
$w_y = \dfrac{W_{pl,y}}{W_{el,y}} \leq 1{,}5$ $w_z = \dfrac{W_{pl,z}}{W_{el,z}} \leq 1{,}5$	$C_{zy} = 1 + (w_y - 1)\left[\left(2 - 14 \dfrac{C_{my}^2 \bar{\lambda}_{max}^2}{w_y^5}\right) n_{pl} - d_{LT}\right] \geq 0{,}6 \sqrt{\dfrac{w_y}{w_z}} \dfrac{W_{el,y}}{W_{pl,y}}$
$n_{pl} = \dfrac{N_{Ed}}{N_{Rk} / \gamma_{M1}}$ C_{my} see Table A.2	with $d_{LT} = 2\, a_{LT} \dfrac{\bar{\lambda}_0}{0{,}1 + \bar{\lambda}_z^4} \dfrac{M_{y,Ed}}{C_{my}\, \chi_{LT}\, M_{pl,y,Rd}} \dfrac{M_{z,Ed}}{C_{mz}\, M_{pl,z,Rd}}$
$a_{LT} = 1 - \dfrac{I_T}{I_y} \geq 0$	$C_{zz} = 1 + (w_z - 1)\left[\left(2 - \dfrac{1{,}6}{w_z} C_{mz}^2 \bar{\lambda}_{max} - \dfrac{1{,}6}{w_z} C_{mz}^2 \bar{\lambda}_{max}^2\right) n_{pl} - e_{LT}\right] \geq \dfrac{W_{el,z}}{W_{pl,z}}$ with $e_{LT} = 1{,}7\, a_{LT} \dfrac{\bar{\lambda}_0}{0{,}1 + \bar{\lambda}_z^4} \dfrac{M_{y,Ed}}{C_{my}\, \chi_{LT}\, M_{pl,y,Rd}}$

Table A 10.5 Auxiliary terms for the determination of the interaction factors. Reproduced from NEN-EN 1993-1-1 table A.1.

$$\bar{\lambda}_{max} = \max\begin{cases}\bar{\lambda}_y \\ \bar{\lambda}_z\end{cases}$$

$\bar{\lambda}_0$ = non-dimensional slenderness for lateral-torsional buckling due to uniform bending moment, i.e. $\psi_y = 1{,}0$ in Table A.2

$\bar{\lambda}_{LT}$ = non-dimensional slenderness for lateral-torsional buckling

If $\bar{\lambda}_0 \leq 0{,}2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,TF}}\right)}$:

$$C_{my} = C_{my,0}$$
$$C_{mz} = C_{mz,0}$$
$$C_{mLT} = 1{,}0$$

If $\bar{\lambda}_0 > 0{,}2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,TF}}\right)}$:

$$C_{my} = C_{my,0} + \left(1 - C_{my,0}\right)\frac{\sqrt{\varepsilon_y}\,a_{LT}}{1+\sqrt{\varepsilon_y}\,a_{LT}}$$
$$C_{mz} = C_{mz,0}$$
$$C_{mLT} = C_{my}^2 \frac{a_{LT}}{\sqrt{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}} \geq 1$$

$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A}{W_{el,y}}$ for class 1, 2 and 3 cross-sections

$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A_{eff}}{W_{eff,y}}$ for class 4 cross-sections

$N_{cr,y}$ = elastic flexural buckling force about the y-y axis

$N_{cr,z}$ = elastic flexural buckling force about the z-z axis

$N_{cr,T}$ = elastic torsional buckling force

I_T = St. Venant torsional constant

I_y = second moment of area about y-y axis

Table A 10.6 Continue of auxiliary terms of table A 10.5. Reproduced from NEN-EN 1993-1-1 table A.1.

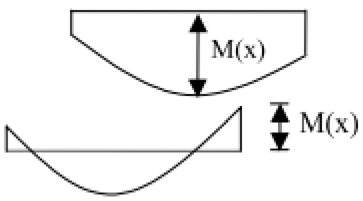
Moment diagram	$C_{mi,0}$
M_1  ψM_1 $-1 \leq \psi \leq 1$	$C_{mi,0} = 0{,}79 + 0{,}21\psi_i + 0{,}36(\psi_i - 0{,}33)\frac{N_{Ed}}{N_{cr,i}}$
	$C_{mi,0} = 1 + \left(\frac{\pi^2 EI_i\lvert\delta_x\rvert}{L^2\lvert M_{i,Ed}(x)\rvert} - 1\right)\frac{N_{Ed}}{N_{cr,i}}$ $M_{i,Ed}(x)$ is the maximum moment $M_{y,Ed}$ or $M_{z,Ed}$ $\lvert\delta_x\rvert$ is the maximum member displacement along the member
 	$C_{mi,0} = 1 - 0{,}18\frac{N_{Ed}}{N_{cr,i}}$ $C_{mi,0} = 1 + 0{,}03\frac{N_{Ed}}{N_{cr,i}}$

Table A 10.7 Equivalent uniform moment factors $C_{mi,0}$. Reproduced from NEN-EN 1993-1-1 table A.2.

10.1.3 Serviceability Limit State

Vertical deformation
The national annexes of the Eurocode NEN-EN 1990 prescribe that the maximum total vertical deformation for roof bars between two fixed supports is 0,004l and is equal to the maximum additional vertical deformation. For the designs the requirement has been adapted: $w \leq 0{,}010l$. See chapter 9 for explanation.

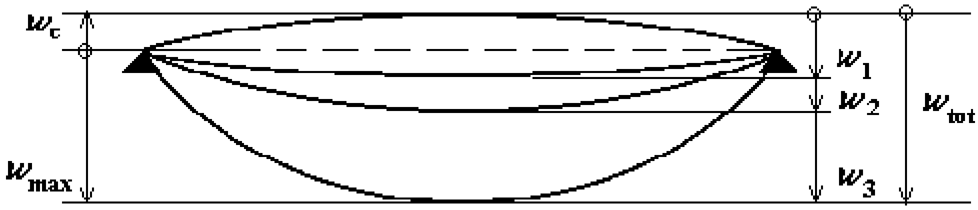


Figure A 10.3 Deformation of a beam. Reproduced from NEN-EN 1990.

w_c	sheer of the unloaded structural elements
w_{max}	permanent deformation
w_1	direct deformation due to permanent loads
w_2	long term deformation due to permanent loads
w_3	additional deformation due to variable loads
w_{tot}	total deformation, the sum of $w_1 + w_2 + w_3$

Horizontal deformation
The total horizontal deformation of the structure cannot exceed h/500, where h is the height of the structure. The maximum horizontal deformation for a single building story is h/300.

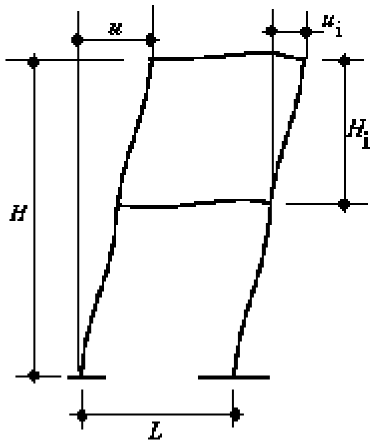


Figure A 10.4 Horizontal deformation of a framework. Reproduced from NEN-EN 1990.

H	total height of the building
H_i	height of a single building story
u	total horizontal deformation over the height H
u_i	horizontal deformation of the height H_i of a single story

10.2 Example Calculation

To come to a final design, the structural designs needs to fulfil all structural requirements regarding the Eurocode. In this chapter an example will be given of the determination of an element in a truss design that eventually will fulfil the requirements.
The following process has been done for all elements in all three the design variants using the FEM program Scia Engineer. The calculation results for all three variants are presented in an additional report: FEM Calculations.

For the example a critical element of variant 1 has been used. Element nr. S2039 is illustrated below.

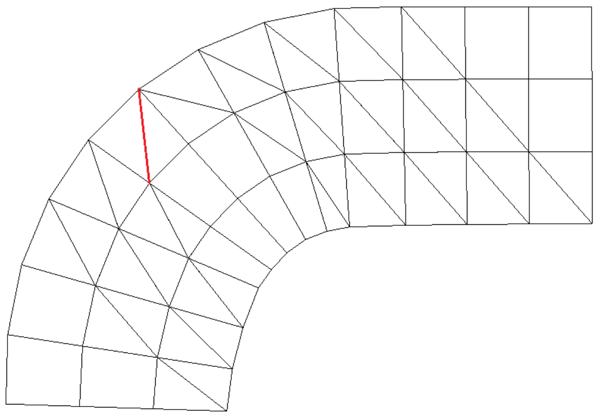


Figure A 10.5 Element S2039

10.2.1General information

For this element is first estimated that a CHS profile with a diameter of 508mm and a thickness of 28mm is needed. The element is composed of steel with a steel grade of S235 and has the following properties:

Yield strength:	$f_y = 235\ N/mm^2$
Ultimate tensile strength:	$f_u = 360\ N/mm^2$
Modulus of Elasticity:	$E = 210.000\ N/mm^2$
Shear modulus:	$G = \frac{E}{2(1+\upsilon)} \cong 81.000\ N/mm^2$
Poisson's ratio:	$\upsilon = 0{,}3$
Coeff. of linear thermal expansion:	$\alpha = 12 * 10^{-6}\ perK$ (for $T \leq 100°C$)
Width/thickness ratio:	18,14
Cross sectional class:	1
Length:	18,46 m
Area:	42223,01 mm^2
Modulus of section:	$5398{,}93 * 10^3\ mm^3$
Moment of Inertia:	$1220{,}16 * 10^6\ mm^4$

10.2.2 Loads

The loads that effect element S2039 are illustrated in figure A 10.6.

At 9,23m (at midspan of the bar) the control of the loads is critical. The loads have the following values:

Internal forces	Value
N_{Ed}	-3344,24 kN
$V_{y,Ed}$	0,02 kN
$V_{z,Ed}$	-0,01 kN
$M_{x,Ed}$	20,93 kNm
$M_{y,Ed}$	295,08 kNm
$M_{z,Ed}$	1,46 kNm

Table A 10.8 Internal forces element S2039

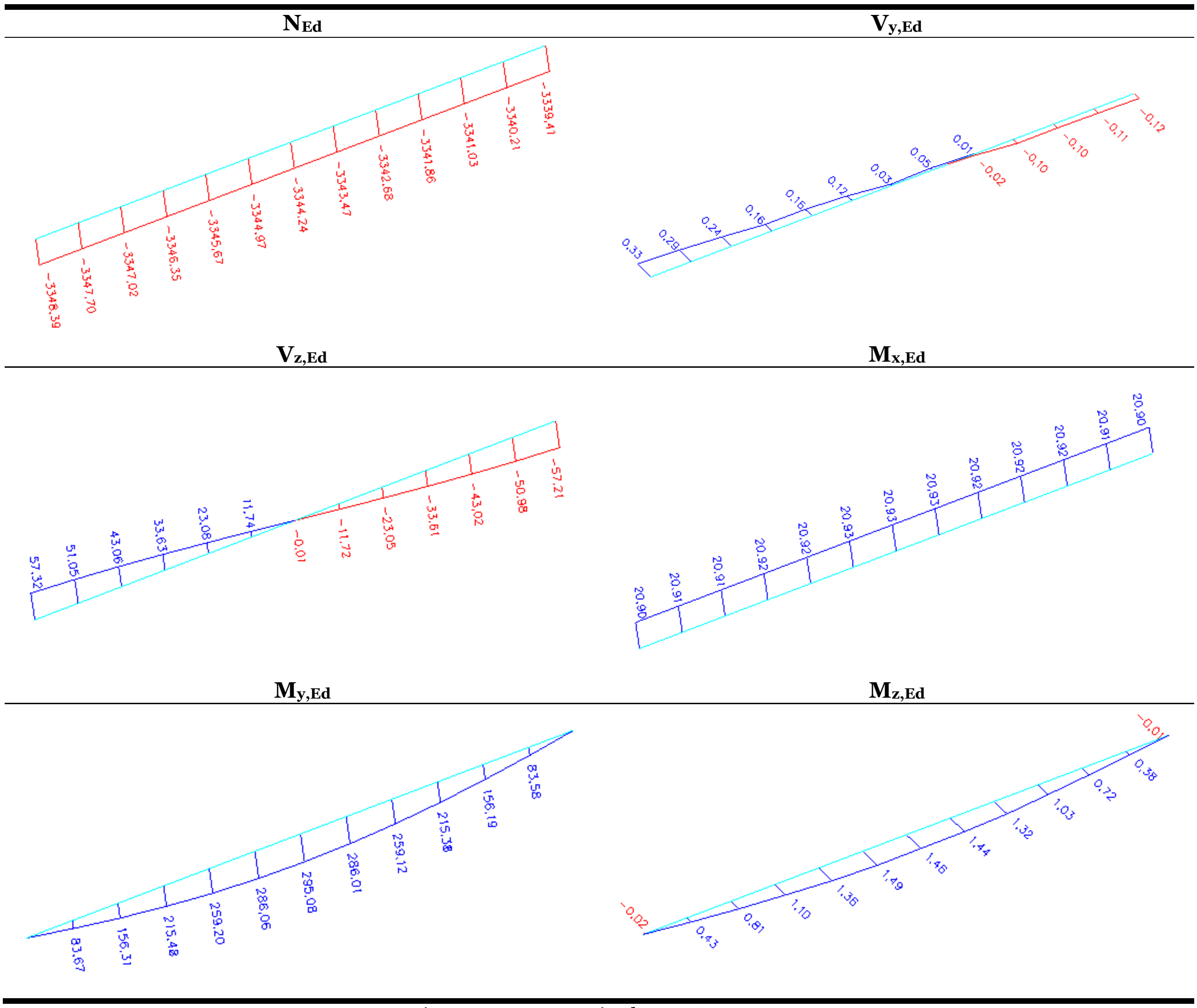


Figure A 10.6 Forces in element S2039

10.2.3 Ultimate Limit State

The requirements regarding normal forces, transverse forces and bending moments are presented in this paragraph.

10.2.3.1 Strength Requirements

The partial factors have the values $\gamma_{m0} = 1{,}10$; $\gamma_{m1} = 1{,}10$ and $\gamma_{m2} = 1{,}25$

Axial compression

The calculation value of compression force N_{Ed}:

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1{,}0 \tag{A 10.28}$$

With

$$N_{c,Rd} = \frac{Af_y}{\gamma_{M0}} = \frac{42223{,}01\ mm^2 * 235\ N/mm^2}{1{,}10} = 9020{,}37kN \tag{A 10.29}$$

Follows:

$$\frac{3344{,}24kN}{9020{,}37kN} = 0{,}37 \leq 1{,}00 \tag{A 10.30}$$

Bending moment

The calculation value of bending moment M_{Ed}:

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1{,}0 \tag{A 10.31}$$

With

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl}f_y}{\gamma_{M0}} = \frac{5398939{,}85\ mm^3 * 235N/mm^2}{1{,}1} = 1153{,}41kNm \tag{A 10.32}$$

Follows:

$$\frac{298{,}05kNm}{1153{,}41kNm} = 0{,}26 \leq 1{,}0 \tag{A 10.33}$$

Transverse force

The calculation value of transverse force V_{Ed}:

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1{,}0 \tag{A 10.34}$$

The steel CHS profiles are checked with the plastic calculation method. The plastic transverse force value is determined using formula 6.18 from NEN-EN 1993-1-1.

$$V_{pl.Rd} = \frac{A_v(f_y/\sqrt{3})}{\gamma_{M0}} \tag{A 10.35}$$

With

$$A_v = 2A/\pi = 2 * \frac{42223{,}01mm^2}{\pi} = 26880{,}00\ \text{mm}^2 \tag{A 10.36}$$

Follows:

$$V_{pl.Rd} = \frac{26880mm^2(235N/mm^2/\sqrt{3})}{1{,}10} = 3315{,}46kN \tag{A 10.37}$$

Follows for V_y

$$\frac{0{,}02kN}{3315{,}46kN} = 0{,}0 \leq 1{,}0 \tag{A 10.38}$$

And for V_z:

$$\frac{0{,}01kN}{3315{,}46kN} = 0{,}0 \leq 1{,}0 \tag{A 10.39}$$

Interaction Bending moment, Transverse force and Normal force

Transverse forces and normal forces arise, therefore the influence of these forces on the bending moments must be taken into account.

However $V_{Ed} \leq 0{,}50V_{pl,Rd}$. The influence of the transverse force on equation 10.14 and 10.17 can be neglected.

There is bending in two directions, the following requirement must be fulfilled:

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^2 + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^2 \leq 1{,}0 \tag{A 10.40}$$

Where:

$M_{N,y,Rd} = M_{pl,y,Rd}(1-n)/(1-0{,}5a_w)$ with requirement: $M_{N,y,Rd} \leq M_{pl,y,Rd}$ (A 10.41)

$M_{N,z,Rd} = M_{pl,z,Rd}(1-n)/(1-0{,}5a_w)$ with requirement: $M_{N,z,Rd} \leq M_{pl,z,Rd}$ (A 10.42)

Where:

$a_w = (A-2bt)/A$ with requirement $a_w \leq 0{,}5$ (A 10.43)
n is equal to equation A 10.30

Follows:

$$a_w = \frac{(42223{,}01mm^2 - 2*508mm*28mm)}{42223{,}01mm^2} = 0{,}326 \leq 0{,}5 \tag{A 10.44}$$

The value for the bending moment due to normal force is:

$$M_{N,y,Rd} = M_{N,z,Rd} = \frac{1153{,}41kNm(1-0{,}37)}{(1-0{,}5(0{,}326)} = 868{,}17kNm \tag{A 10.45}$$

Follows:

$$\left[\frac{298{,}08kNm}{868{,}17kNm}\right]^2 + \left[\frac{1{,}46kNm}{868{,}17kNm}\right]^2 = 0{,}12 \leq 1{,}0 \tag{A 10.46}$$

10.2.3.2 Stability Requirements

Buckling parameters

In the table below the buckling parameters for element S2039 (CHS508*28) are presented.

Parameters	**yy**	**zz**
Buckling curve	a	a
Imperfection factor	0,21	0,21
Lef/Lsys	1,00	1,00
Buckling length	18,46m	18,46m

Table A 10.9 Buckling parameters

Buckling stability

The buckling stability of a compressed bar need to be checked as follows:

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1{,}0 \tag{A 10.47}$$

$N_{b,Rd}$ is the calculation value of the buckling resistance of the compressed bar and can be calculated as follows:

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \tag{A 10.48}$$

Where χ is the reduction factor. The equation for the reduction factor is equal to:

$$\chi = \frac{1}{\Phi+\sqrt{\Phi^2-\bar{\lambda}^2}} \quad \text{with requirement } \chi \leq 1{,}0 \tag{A 10.49}$$

Where

$$\Phi = 0{,}5\left[1 + \alpha\left(\bar{\lambda} - 0{,}2\right) + \bar{\lambda}^2\right] \tag{A 10.50}$$

$$\bar{\lambda} = \sqrt{\frac{\mathrm{Af_y}}{\mathrm{N_{cr}}}} \tag{A 10.51}$$

$\mathrm{N_{cr}}$ is the critical elastic force for the needed buckling curve based on the properties of the gross cross section.

α is the imperfection factor. The imperfection factor can be determined using table A 10.1. or figure A 10.1. The buckling curve can be determined using table A 10.2. The imperfection factor is equal to 0,21.

The critical elastic force is equal to:

$$\mathrm{N_{cr}} = \frac{\pi^2 \mathrm{EI}}{\mathrm{l_{eff}^2}} = \frac{\pi^2 * 2{,}1 * 10^5 \mathrm{N/mm^2} * 1220{,}16 * 10^6 \mathrm{mm^4}}{(18460 \mathrm{mm^2})^2} = 7421{,}17\mathrm{kN} \tag{A 10.52}$$

The slenderness is equal to:

$$\bar{\lambda} = \sqrt{\frac{42223{,}01\ \mathrm{mm^2} 235\ \mathrm{N/mm^2}}{7{,}42 * 10^6 \mathrm{N}}} = 1{,}156 \tag{A 10.53}$$

$$\Phi = 0{,}5[1 + 0{,}21(1{,}156 - 0{,}2) + 1{,}156^2] = 1{,}27 \tag{A 10.54}$$

The reduction factor is equal to:

$$\chi = \frac{1}{1{,}27+\sqrt{1{,}27^2-1{,}156^2}} = 0{,}59 \leq 1{,}0 \tag{A 10.55}$$

The calculation value of the buckling resistance of the compressed bar is:

$$N_{b,Rd} = \frac{0{,}59 * 42223{,}01 mm^2 * 235\ N/mm^2}{1{,}10} = 5372{,}53 kN \tag{A 10.56}$$

The buckling stability of the compressed can now be checked:

$$\frac{3344{,}24\ kN}{5372{,}53\ kN} = 0{,}62 \leq 1{,}0 \tag{A 10.57}$$

Torsion buckling

Torsion buckling only has to be taken into account for open (not closed) cross sections. For CHS profiles torsion buckling is not leading and will not be taken into account.

Buckling resistance (bending moments)

CHS profiles are not sensitive for lateral torsion buckling. The buckling curve of CHS profiles are equal to:

$$\chi_{LT} = 1{,}0 \tag{A 10.58}$$

Stability of bars subjected to bending moments and compression forces

The element is subjected to combined bending and compression and therefore must fulfil the following requirement:

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz} \frac{M_{z,Ed}+\Delta M_{z,Ed}}{\chi_{LT}\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1,0 \qquad \text{(A 10.59)}$$

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed}+\Delta M_{z,Ed}}{\chi_{LT}\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1,0 \qquad \text{(A 10.60)}$$

Where:

$N_{Ed} = 3344{,}24\ kN$

$M_{y,Ed} = 295{,}08\ kNm$

$M_{z,Ed} = 1{,}46\ kNm$

$\Delta M_{y,Ed} = \Delta M_{z,Ed} = 0\ kNm$ (table A 10.3)

$\chi_y = \chi_z = 0{,}59$

$\chi_{LT} = 1{,}0$ (equation A 10.58)

$k_{yy} = k_{yz} = k_{zy} = k_{zz} = 1{,}50$ (table A 10.4-10.7)

The CHS profiles are of cross section class 1-3. Knowing that γ_{Mo} and γ_{M1} are equal to 1,1; equations 10.59 and 10.60 can determined:

$$\frac{N_{Ed}}{\chi_y N_{Rk}} + k_{yy} \frac{M_{y,Ed}}{M_{y,Rk}} + k_{yz} \frac{M_{z,Ed}}{M_{z,Rk}} = \frac{3344{,}24\ kN}{0{,}59*9020{,}37\ kN} + 1{,}50 \frac{298{,}08\ kNm}{1153{,}41\ kNm} + 1{,}50 \frac{1{,}46\ kNm}{1153{,}41\ kNm} = 1{,}02 > 1{,}0 \qquad \text{(A 10.61)}$$

$$\frac{N_{Ed}}{\chi_y N_{Rk}} + k_{zy} \frac{M_{y,Ed}}{M_{y,Rk}} + k_{zz} \frac{M_{z,Ed}}{M_{z,Rk}} = \frac{3344{,}24\ kN}{0{,}59*9020{,}37\ kN} + 1{,}50 \frac{298{,}08\ kNm}{1153{,}41\ kNm} + 1{,}50 \frac{1{,}46\ kNm}{1153{,}41\ kNm} = 1{,}02 > 1{,}0 \qquad \text{(A 10.62)}$$

The element does not fulfil the requirement regarding the stability of the element subjected to compression and bending. A stronger profile, with a higher bending moment and compression capacity, is needed to fulfil the last requirement.

For element S2039 a CHS 508*30 will now be used. Assuming that the same loads act on the element, S2039 will have the following unity checks:

Control	Unity Check
Axial compression	0,31
Transverse force V_y	0,00
Transverse force V_z	0,00
Interaction bending moment, transverse force, axial force	0,05
Buckling stability	0,56
Combined buckling stability bending moment, transverse force, axial force	0,86

*Table A 10.10 Unity check values CHS 508*30 for element S2039*

The results in table A 10.10 show that the chosen profile fulfils all structural requirements. This process has been done for elements by a FEM program. After a long iteration process, a profile has been determined for every single element to eventually come design which used a minimum amount of material.

10.2.3.3 Serviceability Limit State

The requirements regarding the stiffness are not leading for the determination of the profile. The requirement is that the deformation of a single bar can have a maximum value of 1/100 of its length due to variable loadings. The deformation unity check of element S2039 is 0,15

Appendix A 11. Cable structure

11.1 Choice of cable system

Engineers found out many decades ago, that a cable system is very suitable for the spoke wheel roof. A cable structure resists load only through tension. In addition to the absence of bending, cables are light and have a high strength to weight ratio. Small members could therefore be used even in case of a very large span. This property gives cables an advantage compared to other wide-span systems like a truss- or shell system.

Due to the lightweight appearance, the cable system is an attractive choice for architectural reasons. Next cable structures have also proven to be an economical proposition for large structures. With the continuing rise on the cost of steel, cables are becoming an even more attractive economical alternative to conventional forms of wide-span systems.

In the world of cable structures there are five different categories. The categories are based on whether the roof cladding is supported by:

1. Simply suspended cables
2. Pre-tensioned cable beams
3. Pre-tensioned cable nets
4. Tensioned straight cables
5. Tensegric shells

All categories will be briefly described in order to understand its application and properties and to analyse which cable structures are suitable for a spoke wheel roof for football stadia.

Simply suspended cables

Simply suspended cable structures refer to roofs whose cladding is supported by a single layer of non pre-tensioned cables. It is a simple load transferring mechanism where force acting on the roof is first carried by the roof cladding, and then transferred to the cables. This kind of cable structures behave very similarly to the beam column system, where the 'beam' supporting loads through tension instead of bending.

An important characteristic which set the simply suspended cable system from the other cable systems is that the cables in this system are not pre-tensioned. This makes the use of the system cheaper and simpler. However, since the cables are not pre-stressed, they do not offer any stiffness. This results in large deflections under loading and drainage of such roofs has to be thought of during the design process.
Due to the lack of stiffness of the cables also results in much movements of structure under wind loading. Stiffness of the system must thus be supplemented through the stiffness of the cladding material. Next the structure is very vulnerable for dynamic vibrations. In a football stadium wind loading is the decisive load in the engineering process. The continuous various wind loads that act on the roof, downward as well as upward, makes this cable system not suitable for a spoke wheel roof compared to other pre-tensioned cable systems.

Pre-tensioned cable beams

A cable beam is an improvement of a simply suspended cable system. It is formed by adding a second set of cables with a reverse curvature, to the existing suspension cables. This second set of cables can be added in various manners forming three possible combinations: convex, concave and the combination of convex-concave beams. One can see these three combinations in figure A 11.1.

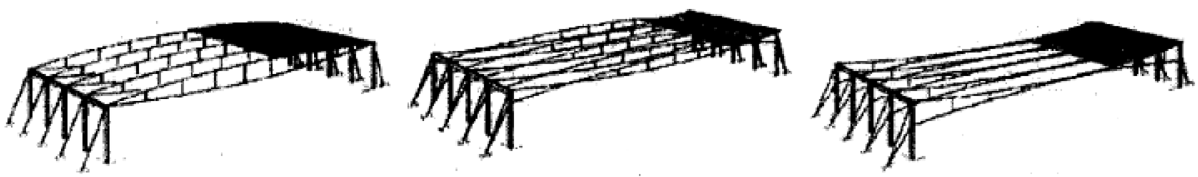


Figure A 11.1 Possible cable combination in cable beams

Cable beams work more efficient as a load bearing system, which results in a lighter structure. Pre-tensioning of the cables also makes the structure stiffer (vertically, lateral stiffness is still provided by the cladding material), and if the cables have the right amount of pre-tension, the cables remain stretched under any combination of applied loading.

A cable beam is more able to carry vertical load in both upward and downward direction. The convex curvature carries the downward force while that with the concave curvature resists the upward force, see figure A 11.2. When a downward force acts on the structure, the pre-tensioned force in cable A is increased while the tension in cable B is reduced. The downward force is thus resisted by cable A. When the structure is subjected to an upward force, tension increases in cable B and reduces the pre-stressing force in cable A. Cable B thus resists the upward force.

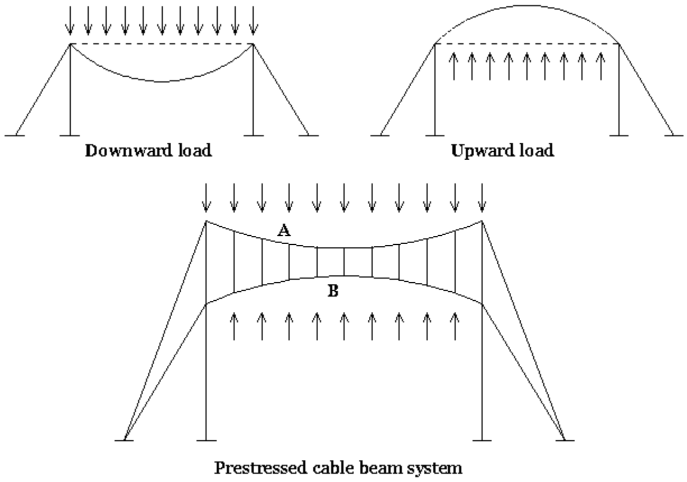


Figure A 11.2 Structural behaviour of a pre-stressed cable beam

Cable beams are most popular among the different cable systems, because they are efficient, easy to erect and can be employed to cover plans of different shapes. This kind of cable system could be very suitable for the use as a spoke wheel roof structure for football stadia.

Pre-tensioned Cable Nets

A cable net structure is obtained when the suspension and pre-tension cables of the cable beams are laid in space to form a single surface. The behaviour of a cable net is thus similar to that of a cable beam, with one set of a curved cable resisting the downward force and another set of oppositely curving cable resisting the upward lift. Since the surface must incorporate cables of opposing curvature, it follows that the net must be anticlastic or saddle shaped.

The net must not accommodate the crossing of cables of the same curvature because in such areas, a local 'basket' will be formed and the cables will not be properly tensioned. As such, the roof in the region will be soft and is subjected to large movements which will damage the cladding. In large roofs, like a football stadium, flat areas should also be avoided because such parts are not very stiff. Due to this structural requirement, boundaries are set to the architectural design of cable net structures. Applying a cable net structure at a spoke wheel roof structure is therefore very difficult. Because the cable net needs to be anticlastic or saddle shaped. In case of the spoke wheel roof structure this is not possible, also due to the large opening in the spoke wheel roof structure.

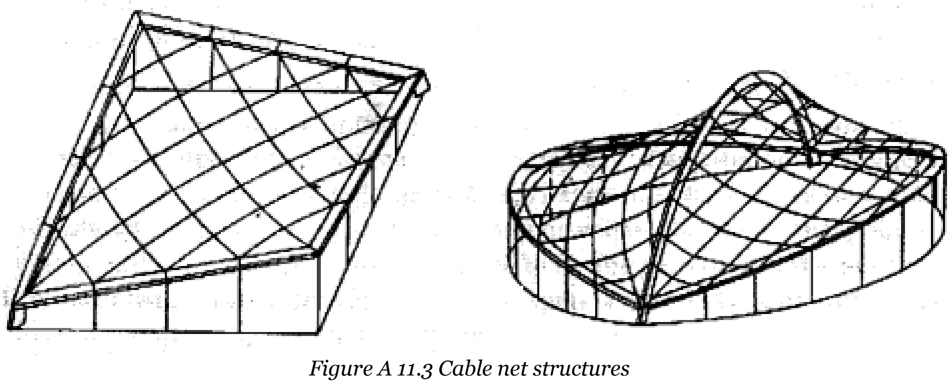


Figure A 11.3 Cable net structures

Tensioned Straight Cables

Buro Happold suggested the use of straight cables with flat fabrics as a new net system for wide-span structures in 1994. They pointed out that there are considerable advantages on using straight tensioned cables which can carry both the downward force and upward lift in one set of cables. Straight cables have the following advantages:

- The elimination of one set of cables brings with it the removal of cross clamps and some anchorages.
- Whether the load is acting downwards or upwards, the cable is tensioned in the same direction, which can be a great advantage if this tension is taken by a funicular arch or ring beam.
- With just one set of cable, connection between the fabric cladding and cable can be greatly simplified. This results in easier roof installation and reduction of the costs.

Straight cables work by having a very high pre-tensioning on the cables to stiffen it sufficiently against any deflection. The fabric which acts as cladding material also needs to be pre-tensioned in the same direction of the cables. Despite the high cost of pre-tensioning, Buro Happold claimed that with the benefit from removing one set of cables, this system would still result in a more economical large span roof structures (as compared to traditional anticlastic cable roof).
However, this cable system also has a downside. The pre-stressed fabric in between the cable elements does not carry concentrated loads as well as the cables. Under local loading, large deflection may be developed in the fabric, which may in turn cause the failure of the structure.
When straight tensioned cables are applied to the spoke wheel roof, where the system needs to cover a large span, the value of the pre-tension needs to be extremely high to avoid large local deflections.

Tensegric Shells

A tensegric shell is a hybrid shell which is made up of a cable net and compression bars. The structure of the tensegric net is derived from the tensegrity system, which was first discovered by Buckminister Fuller in 1962. This system is defined as a state when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space.
In a tensegric system, the structure is self-equilibrating, with the pull in the compressive structures balancing the push from the tensile elements. Although the system is stable on its own, to support loads, the members of a tensegric shell needs to be pre-stressed against each other.
In tensegric shells, the bare are arranged such that they join the cables at the nodes and that they are not connected to each other, see figure 12.4 for an example. The exact placement of the bars can be generated through computer form finding methods.

A tensegric shell is a very effective system for wide-span structures. The system has a high stiffness, which decreases the amount of deformations. Design and construction of a tensegric shell is however more complicated than typical cable systems. The construction of the shell is very difficult because the compression bars need to be fitted into the system individually. The connection of the bar to the cables also needs to be detailed with care.
Due to the difficult design and construction the costs of this cable system is substantial compared to the other discussed cable systems. Therefore this system will not be applied to the spoke wheel roof.

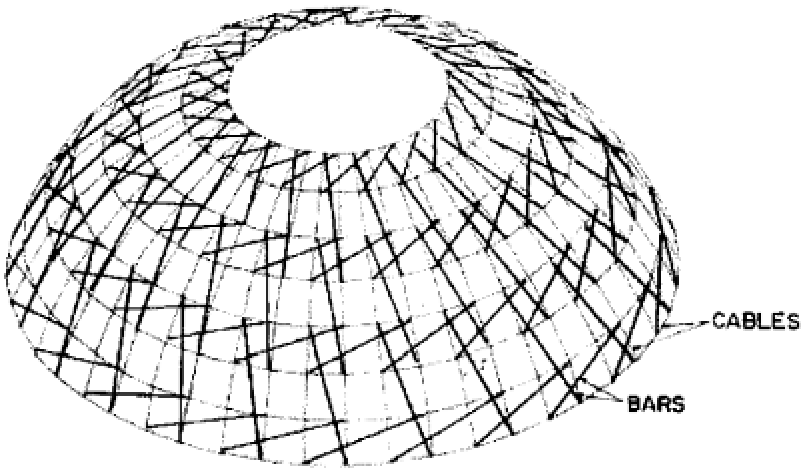


Figure A 11.4 A tensegric shell

Conclusion

To decide which cable system is most suitable to be applied for the spoke wheel roof structure, different aspects need to be taken into account, such as: physical form, span, costs and labour.

Each cable system carries with it a characteristics profile. In general, the choice of the cable system is based on architectural reasons. In this thesis the role of the architect is minimal and the choice is based on structural reasons.
The shape of the area to be enclosed may also affect the choice of the system. A football stadium is more oval shaped. For this shape cable beams, simply suspended beams and cable nets are appropriate.

All of the described cable systems are structurally efficient for long span. However, there is a limit to the maximum span that each system can support effectively. In general, cable nets are able to span greater width than single cable systems. With stiffness acting in both directions (as opposed to one in single cable systems), cable nets form a stiffer structure and thus are able to span to a greater width with relatively smaller deflection.
Between the two single cable systems, cable beams are able to span a greater width because pre-tensioned cables provide more stiffness and can better resist upward and downward loading.

When looking at the construction single cable beams are relatively easy to erect. Cable net systems however, should only be considered if skilled workers experienced in erecting cable structures can be employed.

Most of the cost of building a cable structure goes into the design and the construction. The material costs are not significant. Thus, the simpler the cable system, the cheaper it is. In order this would be simply suspended cables, followed by cable beams and nets.

It can be concluded that cable beams are most suitable for the application for the spoke wheel roof structure. The system is easy to implement the system, can cover the span, easy to construct and is relatively cheap.
Using a suspended cable or straight cables would cause great deflections in case of using it as a stadium roof. Cable nets and tensegric shells however are very suited for stadium roofs. The downside of these systems is that the design and construction is quiet difficult. Next the costs of these systems are significantly higher than cable beam systems.

11.2 Theory of cable beam

Cables are line elements which have a small, neglect able bending resistance. A complete flexible cable cannot transfer bending moments and transverse forces. Only normal forces can be transferred. Cables do not have a fixed shape; the shape of the cable adapts itself to the loading. When a cable is pre-stressed, the cable will have a very small sag value.

In the design of a cable structure it is necessary to know the relation between the sag, tension and span. To describe the relation a piece of a cable element is used (figure A 11.5). It is assumed that the body is in equilibrium, the resistance to bending is negligible. This implies that the force in the cable is always in the direction of the cable.
The following derivations are defined for a cable in its loaded or deformed configuration and therefore geometrically non-linear.

The kinematic relation of the transverse deflection $y(x)$ can be derived by simple geometry to

$$\frac{dy}{dx} = \tan\alpha \qquad \text{(A 11.1)}$$

The constitutive relation can be determined:

$$V = H\tan\alpha \qquad \text{(A 11.2)}$$

The cable can be described by a parabolic curve, assuming that a uniformly distributed load q acts on the cable. When looking at figure A 11.5 the equilibrium in the vertical direction yields

$$\frac{dV}{dx} = -q \qquad \text{(A 11.3)}$$

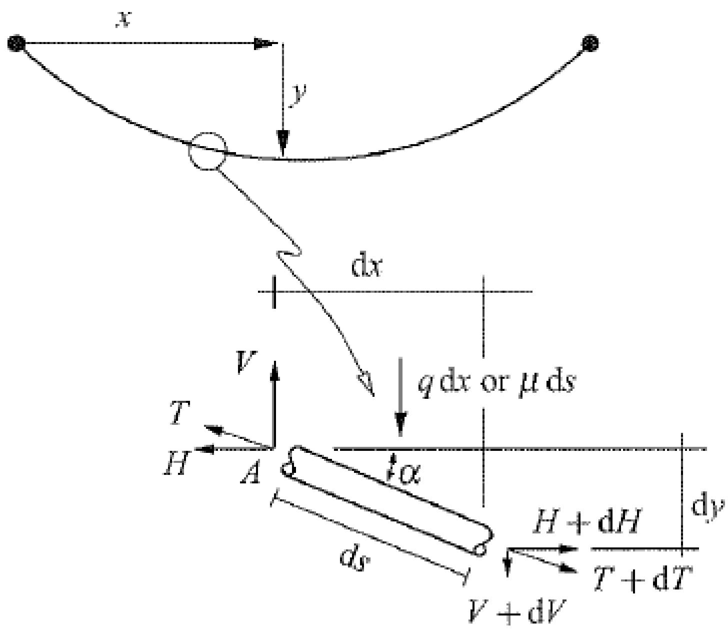


Figure A 11.5 Deflection of a cable element

While the rotational equilibrium around point A, neglecting second order terms of the type $(dx)^2$, gives

$$V = H\frac{dy}{dx} \qquad \text{(A 11.4)}$$

The relation between the deflection y and the applied load q can be expressed as

$$-H\frac{d^2y}{dx^2} = q \qquad \text{(A 11.5)}$$

When integrating gives

$$y = -\frac{q}{H}\frac{x^2}{2} + C_1 x + C_2 \tag{A 11.6}$$

The integration constants C_1 and C_2 can be determined by assuming the boundary conditions $y(0) = 0$ and $y(l) = 0$, where l is the cable span. Due to these boundary conditions the constants are

$$C_1 = \frac{q_0 l}{2H}, C_2 = 0$$

Implementing these constants in equation A 11.6 gives

$$y = -\frac{q_0}{2H} x(l - x) \tag{A 11.7}$$

And

$$V = q_0 \left(\frac{l}{2} - x\right) \tag{A 11.8}$$

The deflection at mid-span of the cable is the cable sag f and is equal to

$$f = \frac{ql^2}{8H} \tag{A 11.9}$$

When a certain value of pre-stressing is applied to the cable, H increases. As a consequence the sag of the cable decreases, assuming the uniformly distributed load remains equal.

11.3 Properties cables

Diameter (mm)	Minimum breaking load (kN)	Elastic weight (kN/100 m)	Steel area (mm^2)	Modulus (kN/mm^2)
25·0	557·2	2·99	374	169·7
30·5	795·6	4·54	546	169·7
33·1	910·0	5·48	678	169·7
42·0	1500·9	8·48	1033	169·7
45·9	1805·0	10·15	1240	169·7
66·0	3747·4	21·11	2570	157·9
75·0	4846·1	27·35	3330	157·9
86·4	6131·3	35·89	4330	147·2
102·0	8524·9	49·97	6020	147·2
116·0	10 486·9	66·01	7862	147·2
127·0	13 371·0	78·33	9450	147·2
137·0	15 068·2	89·10	11 015	147·2

Figure A 11.6 Properties spiral strand cable

Diameter (mm)	Minimum breaking load (kN)	Elastic weight (kN/100 m)	Steel area (mm^2)	modulus (kN/mm^2)
24·0	509·0	3·06	373	158·4
35·8	1106·8	6·58	802	158·4
38·8	1314·5	7·92	966	158·4
48·0	2001·3	12·57	1530	158·4
51·5	2452·5	15·08	1840	158·4
63·2	3492·4	21·42	2610	158·4
73·6	4708·8	29·69	3617	158·4
87·2	6553·1	41·14	5012	158·4
99·0	8721·1	51·89	6320	158·4
101·5	9035·0	56·20	6900	158·4
116·0	12 390·0	75·96	9251	158·4

Figure A 11.7 Properties locked coil cables

11.4 Calculation Theory

The theory regarding the strength, stiffness and stability are according Eurocode NEN-EN 1993-1-11.
For the outer ring elements CHS profiles have been used. The theory regarding these profiles is explained in the previous chapter, chapter 10 – Truss structure.

11.4.1 General information

Tension component

For cable structures the following type of cables are available, see table A 11.1.

Group	Main tension element	Component
A	rod (bar)	tension rod (bar) system, prestressing bar
B	circular wire	spiral strand rope
	circular and Z-wires	fully locked coil rope
	circular wire and stranded wire	strand rope
C	circular wire	parallel wire strand (PWS)
	circular wire	bundle of parallel wires
	seven wire (prestressing) strand	bundle of parallel strands

Table A 11.1 Groups of tension components. Reproduced from NEN-EN 1993-1-11, table 1.1.

For the cable structure fully locked coil ropes (group B) have been applied. Group B products are composed of wires which are anchored in sockets or other end terminations. The products are fabricated primarily in the diameter range of 5 mm to 180mm.
There has been chosen to use fully locked coil ropes because of their high breaking force capacity compared to other type of tension components. Fully locked coil ropes are mainly used as stay cables, suspension cables and stabilizing cables.

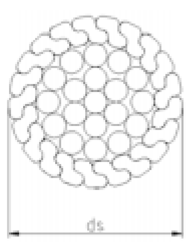
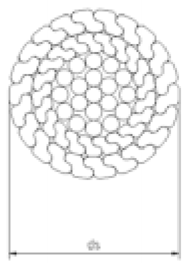
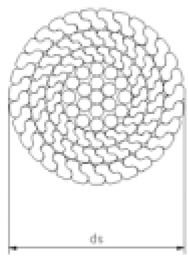
Fully locked coil rope			
			
Construction	1 layer Z-wires	2 layer Z-wires	≥ 3 layer Z-wires
Diameter d_s [mm]	20 to 40	25 to 50	40 to 180
Tolerance for d_s	+5%	+5%	+5%
Nominal metallic area factor C	0,636	0,660	0,700
Breaking force factor K	0,585	0,607	0,643
NOTE: Nominal metallic area factor and breaking force factor acc. EN 12385-2.			

Table A 11.2 Fully locked coil rope. Reproduced from NEN-EN 1993-1-11 annex C.

Terminations

The types of terminations dealt with group B components are:
- Metal and resin sockets
- Sockets with cement grout
- Ferrules and ferrule securing
- Swaged sockets and swaged fitting
- U-bolt wire rope grips
- Anchoring for bundles with wedges, cold formed button heads for wires and nuts for bars.

For the fully locked coil ropes swaged sockets are used for the terminations. In this category different type of sockets are available, see table A 11.3.

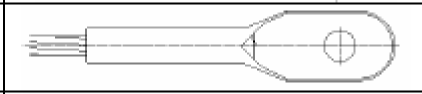
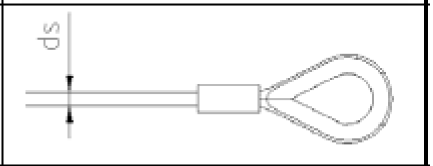
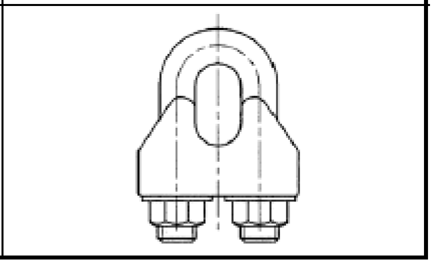
Wire rope end connectors swaged	
Open swaged socket	
Closed swaged socket	
Swaged fitting with thread	
Thimble with swaged aluminium ferrule acc. EN 13411-3	
U-bolt grip acc. EN 13411-5	

Table A 11.3 Types of swaged sockets. Reproduced from NEN-EN 1993-1-11 annex C.

Durability

The tension components (fully locked coil ropes) are subjected to fatigue due to the variable loads acting on the roof structure, both axially and laterally. Next the cables are subjected to corrosion. Therefore exposure class 5 is leading.

Fatigue action	Corrosion action	
	not exposed externally	exposed externally
no significant fatigue action	class 1	class 2
mainly axial fatigue action	class 3	class 4
axial and lateral fatigue actions (wind & rain)	–	class 5

Table A 11.4 Exposure classes. Reproduced from NEN-EN 1993-1-11, table 2.1.

Group B components with exposure class 5 the corrosion protection system should be as follows:
- Individual wires should be protected against corrosion with either zinc or zinc alloy compound;
- The rope interior should be protected to stop the ingress of moisture by filling all interior voids with an active or passive inner filling that should not be displaced by water, heat or vibration;
- The outer surface should be protected against corrosion. After construction additional corrosion protection measures should be applied to compensate for any damage incurred and for the loss of zinc.

At clamps and anchorages additional corrosion protection should be applied to prevent water penetration.

Material

Strength steel:	Nominal tensile strength:	1770 N/mm²
	Yield strength:	1860 N/mm²
Modulus of Elasticity:	E=158,4 kN/mm²	

The modulus of elasticity is dependent on the stress level and whether the cable has been pre-stretched and cyclically loaded and unloaded. There are no tests results available, therefore the modulus of elasticity follows from table A 11.5. The fully locked coil ropes have a value of 160 kN/mm² with a variable value of 10 kN/mm². For the calculations a value of 158,4 kN/mm² has been used [3].

	High strength tension component	E_Q [kN/mm²]	
		steel wires	stainless steel wires
1	Spiral strand ropes	150 ± 10	130 ± 10
2	Fully locked coil ropes	160 ± 10	–
3	Strand wire ropes with CWR	100 ± 10	90 ± 10
4	Strand wire ropes with CF	80 ± 10	–
5	Bundle of parallel wires	205 ± 5	–
6	Bundle of parallel strands	195 ± 5	–

Table A 11.5 Modulus of elasticity E_Q corresponding to variable loads Q. Reproduced from NEN-EN 1993-1-11, table 3.1.

Coefficient of thermal expansion: $\propto_T = 12 * 10^{-6}\ perK$

Friction coefficient: 0,00 (cables are mechanically keyed to the saddles, clamps, etc.)

11.4.2 Ultimate Limit State

The requirements regarding axial loads that are used for the calculations are presented in this paragraph.

The partial factors have the values $\gamma_{m0} = 1{,}10$; $\gamma_{m1} = 1{,}10$ and $\gamma_{m2} = 1{,}25$

11.4.2.1 Group B Component

For the ULS it shall be verified that:

$$\frac{F_{Ed}}{F_{Rd}} \leq 1{,}0 \qquad \text{(A 11.10)}$$

F_{Ed} is the design value of the axial rope force, F_{Rd} is the design value of the tension resistance which is determined as follows:

$$F_{Rd} = \min\left[\frac{F_{uk}}{1{,}5\gamma_r};\frac{F_k}{\gamma_r}\right] \qquad \text{(A 11.11)}$$

Where: F_{uk} is the characteristic value of the breaking strength
F_k is the characteristic value of the proof strength of the tension component regarding table A 11.6.
γ_r is the partial factor regarding table A 11.8

Group	Relevant standard	Proof strength F_k
A	EN 10138-1	$F_{0,1k}$ *)
B	EN 10264	$F_{0,2k}$
C	EN 10138-1	$F_{0,1k}$
*) For prestressing bars see EN 1993-1-1 and EN 1993-1-4		

Table A 11.6 Proof strength of tension components. Reproduced from NEN-EN 1993-1-11 table 6.1.

For a group B component the proof strength is equal to $F_{0,2k}$ from EN 10264. The nominal tensile strength is 1770 MPa. The characteristic value of the proof strength is the nominal tensile strength multiplied by the area:

$$F_k = A_m f_{uk} \qquad \text{(A 11.12)}$$

Where

f_{uk} is equal to the nominal tensile strength
A_m is the metallic cross section. A_m may be determined from:

$$A_m = \frac{\pi d^2}{4} f \tag{A 11.13}$$

Where d is the external diameter of the rope in mm, including any sheathing for corrosion protection
f is the fill factor, see table A 11.7.

		Fill factor f							unit weight $w \times 10^{-7}$ $\left[\frac{N}{mm^3}\right]$
		Core wires + 1 layer z-wires	Core wires + 2 layer z-wires	Core wires + >2 layer z-wires	Number of wire layers around core wire				
					1	2	3-6	>6	
1	Spiral strand ropes				0,77	0,76	0,75	0,73	830
2	Fully locked coil ropes	0,81	0,84	0,88					830
3	Circular wire strand ropes				0,56				930

Table A 11.7 Unit weight w and fill factors f. Reproduced from NEN-EN 1993-1-11, table 2.2.

The fill factor is equal to 0,88, the unit weight factor is 830 $*10^{-7}$ N/mm^3.

Measures to minimise bending stresses at the anchorage	γ_R
Yes	0,90
No	1,00

Table A 11.8 partial factor. Reproduced from NEN-EN 1993-1-11 table 6.2.

$$F_{uk} = F_{min} k_e \tag{A 11.14}$$

Where:

$$F_{min} = \frac{K d^2 R_r}{1000} \tag{A 11.15}$$

Where: K is the minimum breaking force factor taking into account the spinning loss;
d is the nominal diameter of the rope in mm;
R_r is the rope grade in N/mm^2;
k_e is the loss factor given in table A 11.9.

Type of termination	**Loss factor k_e**
Metal filled socket	1,0
Resin filled socket	1,0
Ferrule-secured eye	0,9
Swaged socket	0,9
U-bolt grip	0,8 *)
*) For U-bolt grip a reduction of preload is possible.	

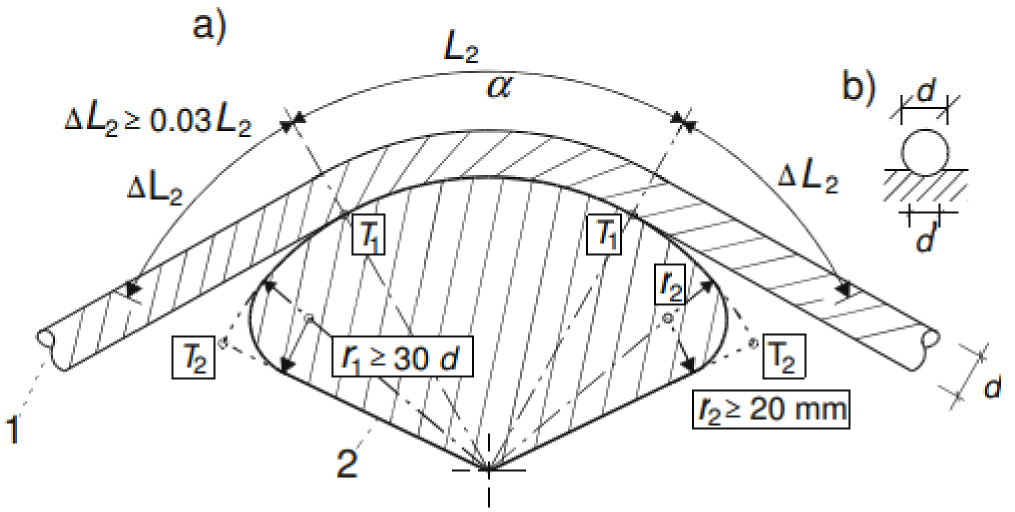
Table A 11.9 loss factor k_e. Reproduced from NEN-EN 1993-1-11 table 6.3.

11.4.2.2 Saddle

The radius r_1 of the saddle should not be less than the greater of $30d$ or $r_1 \geq 400\emptyset$ where,

$\emptyset$ is the diameter of the wire;
d is the diameter of the cable;
d' is the contact width.

The value of r_1 may be reduced to $20d$ when the bedding of the rope on at least 60% of the diameter is coated with soft metal or zinc spray with a minimum of thickness of 1mm.



1 strand/rope
2 saddle
L_2 length of strand/rope between the two theoretical tangent points T_1 under the most unfavourable characteristic combination of loads and the catenary effects
ΔL_2 additional length of wrap

Figure A 11.8 Bedding of a strand/rope over a saddle. Reproduced from NEN-EN 1993-1-11 figure 6.1.

Slipping of the saddle

To prevent slippages the following condition need to be met:

$$max\left\{\frac{F_{Ed,1}}{F_{Ed,2}}\right\} \leq e^{\left\{\frac{\mu\alpha}{\gamma_{m,fr}}\right\}} \quad \text{(A 11.16)}$$

Where:

$F_{Ed,1}; F_{Ed,2}$ are the design values of the maximum and minimum force respectively on either side of the cable;
μ is the coefficient of friction between cable and saddle;
α is the angle in radians, of the cable passing over the saddle;
$\gamma_{m,fr}$ is the partial factor for friction (a value of 1,65 is recommended).

If the condition above is not satisfied, clamps should be provided to impart an additional radial clamping force F_r such that

$$\frac{F_{Ed,1} - \frac{kF_r\mu}{\gamma_{m,fr}}}{F_{Ed,2}} \leq e^{\left\{\frac{\mu\alpha}{\gamma_{m,fr}}\right\}} \quad \text{(A 11.17)}$$

Where

k is normally 2,0 where there is full friction, or else k=1,0.

$\gamma_{m,fr}$ is the partial factor for friction resistance.

Transverse pressure

The transverse pressure q_{Ed} due to the radial clamping force F_r shall be limited to:

$$\frac{q_{Ed}}{q_{Rd}} \leq 1{,}0 \qquad \text{(A 11.18)}$$

Where:

$q_{Ed} = \frac{F_r}{d' L_2}$ and $0{,}60d \leq d' \leq d$. For d' see figure A 11.8 .

$$q_{Rd} = \frac{q_{Rk}}{\gamma_{m,bed}} \qquad \text{(A 11.19)}$$

q_{Rk} is the limiting value of the transverse pressure which is obtained from table A 11.10.
$\gamma_{m,bed}$ is the partial factor.

Type of cable	q_{Rk} [N/mm²]	
	Steel clamps and saddles	Cushioned clamps and saddles
Fully locked coil rope	40	100
Spiral strand rope	25	60

Table A 11.10 Limiting values q_{Rk}. Reproduced from NEN-EN 1993-1-11 table 6.4

For the cable structure steel clamps and saddles are used. The limiting value of the transverse pressure is equal to 40 N/mm².

Design of saddles

Saddles should be designed for a cable force of k times the characteristic breaking strength F_{uk} of the cables.

A value of $k = 1{,}10$ is recommended.

11.4.2.3 Clamps

Slipping of clamps

Where clamps transmit longitudinal forces to a cable and the parts (figure A 11.9.) are not mechanically keyed together, slipping shall be prevented by verifying

$$F_{Ed_{//}} \leq \frac{\left(F_{Ed_{perpendicular}} + F_r\right)\mu}{\gamma_{m,fr}} \qquad \text{(A 11.20)}$$

Where:

$F_{Ed_{//}}$ is the component of external design load parallel to the cable;
$F_{Ed_{perpendicular}}$ is the component of the external design load perpendicular to the cable;
F_r is the radial clamping force;
μ is the coefficient of friction;
$\gamma_{m,fr}$ is the partial factor for friction (a value of 1,65 is recommended)

Transverse pressure

The transverse pressure should meet the requirement of equation A 11.18

Design of clamps

Clamps and their fittings connecting components such as hangers to a main cable should be designed for a notional force equal to 1,15 time the proof force F_k of the secondary components clamped.

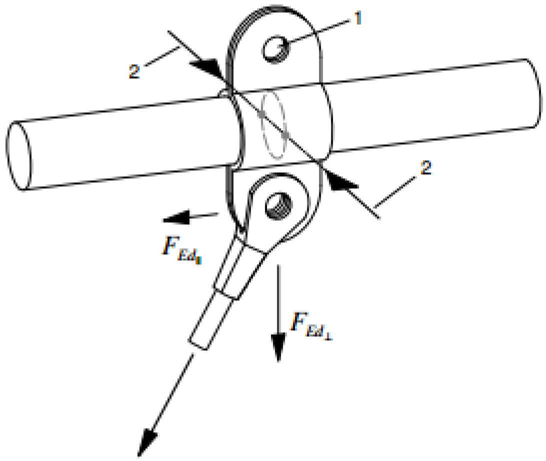


Figure A 11.9 Clamp. Reproduced from NEN-EN 1993-1-11 figure 6.2

11.4.3 Serviceability Limit State

The following serviceability criteria should be considered:
1. Deformations or vibrations;
2. Elastic service conditions

The maximum amount of deformation is set to be 1000 mm due to additional loads. Limits to retain elastic behaviour and durability are related to maximum and minimum values of stresses for serviceability load combinations.

Limiting stress may be specified for the characteristic load combination for the following purposes:
- to keep stresses in the elastic range for the relevant design situations during construction and in the service phase;
- to limit strains such that corrosion control measures are not affected, i.e. cracking of sheaths, hard fillers, opening of joints etc., and also to cater for uncertainty in the fatigue design;
- ULS verifications for linear and sub-linear structural response to actions.

Stress limits should be related to the breaking strength as follows:

$$\sigma_{uk} = \frac{F_{uk}}{A_m} \qquad \text{(A 11.21)}$$

Equation A 11.21 is equal to the ULS requirement of equation A 11.11 where the stress limit has already been taken into account.
For the stress limit there are limits set by the Eurocode for the construction phase and for service conditions. The requirements are as follows:

$$f_{constr} = \frac{0{,}66\sigma_{uk}}{\gamma_r * \gamma_F} \qquad \text{(A 11.22)}$$

Where

$\gamma_r * \gamma_F = 1{,}10$ for short term conditions;
$\gamma_r * \gamma_F = 1{,}20$ for long term conditions.

For the calculations is assumed that the construction is for long term conditions.

Follows that the stress limit is equal to: $f_{constr} = 0{,}55\sigma_{uk}$

$$f_{SLS} = \frac{0{,}66\sigma_{uk}}{\gamma_r * \gamma_F} \qquad \text{(A 11.23)}$$

$\gamma_r * \gamma_F = 1{,}33$ with bending stresses;
$\gamma_r * \gamma_F = 1{,}48$ without bending stresses.

Follows that the stress limit is equal to: $f_{SLS} = 0{,}45\sigma_{uk}$

For the thesis the requirements regarding fatigue and vibration have been left out of the scope of the thesis. The research emphasis lies on the investigation of the use of the spoke wheel principle for football stadia roofs. When a real design has to be made, vibration and fatigue has to be taken into account.

There are some measures to limit vibrations:

- The structure should be monitored for excessive wind and rain induced vibrations.
- Modification of the cable surface (aerodynamic shape)
- Damping devices
- Stabilizing cables.

11.5 Example Calculation

To come to a final design, the structural design needs to fulfil all structural requirements regarding the Eurocode. After research has been concluded that more investigation is needed for the use of cable beams for spoke wheel roof structures that have to be applied to football stadia. The final cable model does not fulfil all structural requirements.

In this chapter an example of the determination of the check values is given the way the FEM program checks the elements. The structural design is not entirely composed of cable elements. In the structural design also RHS profiles are applied. The check process of the structural requirements of the RHS profiles is equal to the process described for the CHS profiles in the previous chapter. In this chapter only an example calculation will be given for cable elements.

The following process has been done for all cable elements by using the FEM program Scia Engineer. The calculation results are presented in an additional report: FEM Calculations. For the example critical element S449 has been used. This element is leading for the determination of the correct kind of cable that needs to be applied.

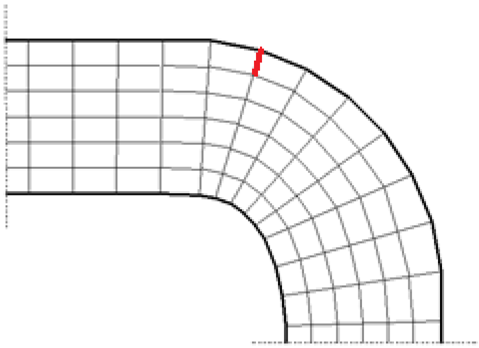


Figure A 11.10 Element S449

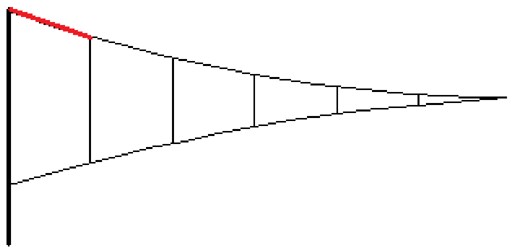


Figure A 11.11 Element S449

11.5.1 General information

For this element is first estimated that a fully locked coil rope with a diameter of 101,5mm. The element has the following properties:

Ultimate tensile strength: $f_u = 1770\ N/mm^2$

Modulus of Elasticity: $E = 158.400\ N/mm^2$

Coeff. of linear thermal expansion: $\alpha = 12 * 10^{-6}\ perK$ (for $T \leq 100°C$)

Breaking force factor: $K = 0,643$

11.5.2 Loads

The loads that effect element S449 are illustrated in figure 11.12

At 0,00 m the control of the loads is critical. The loads have the following values:

Internal forces	Value
N_{Ed}	33241,16 kN
$V_{y,Ed}$	0,00 kN
$V_{z,Ed}$	0,00 kN
$M_{x,Ed}$	0,00 kNm
$M_{y,Ed}$	0,00 kNm
$M_{z,Ed}$	0,00 kNm

Table A 11.11 Internal forces element S449

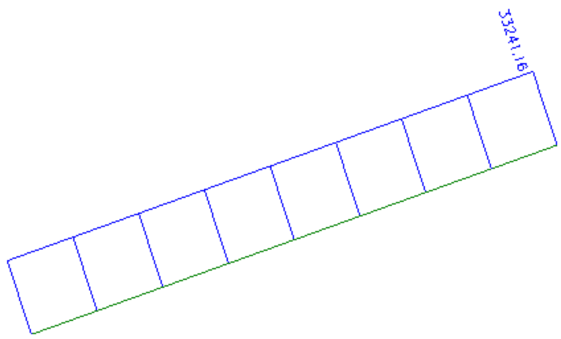


Figure A 11.12 Normal forces in element S449

11.5.3 Ultimate Limit State

The requirements regarding normal forces, transverse forces and bending moments are presented in this paragraph.

11.5.3.1 Group B Component

For the ULS it shall be verified that:

$$\frac{F_{Ed}}{F_{Rd}} \leq 1{,}0 \qquad \text{(A 11.24)}$$

F_{Ed} is the design value of the axial rope force, F_{Rd} is the design value of the tension resistance which is determined as follows:

$$F_{Rd} = \min\left[\frac{F_{uk}}{1{,}5\gamma_r};\frac{F_k}{\gamma_r}\right] \qquad \text{(A 11.25)}$$

Where $F_{uk} = F_{min}k_e$
$F_k = F_{0,2k}$
$\gamma_r = 1{,}00$ (table A 11.6)

For the characteristic value of the breaking strength follows:

$$F_{min} = \frac{Kd^2R_r}{1000} \qquad \text{(A 11.26)}$$

With: $K = 0{,}643$
$d = 101{,}5mm$
$R_r = 1770\ N/mm^2$
$k_e = 0{,}9$ (table A 11.2)

Follows:

$$F_{min} = \frac{0{,}643*101{,}5^2 1770}{1000} = 11725{,}09\ kN \qquad \text{(A 11.27)}$$

$$F_{uk} = 11725{,}09*0{,}9 = 10552{,}58\ kN \qquad \text{(A 11.28)}$$

For a group B component the proof strength is equal to $F_{0,2k}$ from EN 10264. The nominal tensile strength is 1770 MPa. The characteristic value of the proof strength is the nominal tensile strength multiplied by the area:

$$F_k = A_m f_{uk} \qquad \text{(A 11.29)}$$

With: $d = 101{,}5mm$
$f = 0{,}88$ (table A 11.7)

Follows:

$$A_m = \frac{\pi * 101{,}5^2}{4} * 0{,}88 = 7120{,}40\ mm^2 \quad \text{(A 11.30)}$$

$$F_k = A_m f_{uk} = 7120{,}40 mm^2 * 1770 N/mm^2 = 12603.108 kN \quad \text{(A 11.31)}$$

The design value of the tension resistance is the minimal value of equation 11.22:

$$F_{Rd} = \min\left[\frac{10552{,}58\ kN}{1{,}5}; \frac{12602{,}108\ kN}{1{,}0}\right] = 7035{,}05\ kN \quad \text{(A 11.32)}$$

The ULS check will be:

$$\frac{F_{Ed}}{F_{Rd}} = \frac{33241{,}16\ kN}{7035{,}05\ kN} = 4{,}73 > 1{,}0 \quad \text{(A 11.33)}$$

The used cable is not sufficient to fulfil the strength requirement. A stronger cable is needed to withstand the high normal forces in the cable.

11.5.3.2 Saddle

For the check of the saddle, the saddle at the corner inner ring element is used as example. Due to the pretensioning, high normal forces arise in the corner cable elements. The leading element is shown in figure A 11.13. The radial force is equal to 32432,81 kN.

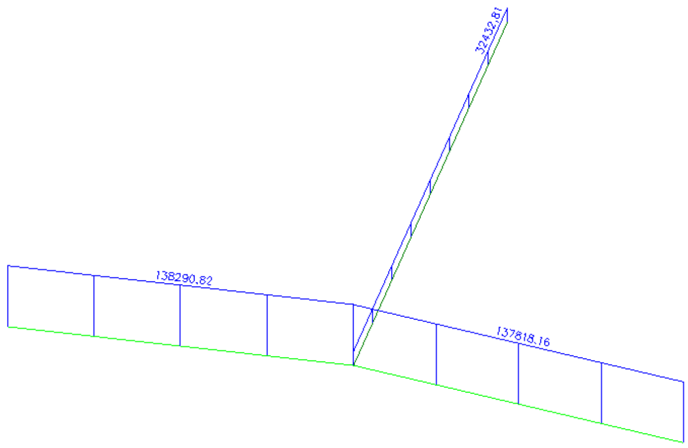


Figure A 11.13 Saddle connection inner ring – corner cable

Slipping of the saddle

To prevent slippages the following condition need to be met:

$$max\left\{\frac{F_{Ed,1}}{F_{Ed,2}}\right\} \le e^{\left\{\frac{\mu\alpha}{\gamma_{m,fr}}\right\}} \quad \text{(A 11.34)}$$

The cables are mechanically keyed together to the saddle. Slipping will not occur and therefore the check of equation A 11.34 does not have to be done.

Transverse pressure

The transverse pressure q_{Ed} due to the radial clamping force F_r shall be limited to:

$$\frac{q_{Ed}}{q_{Rd}} \le 1{,}0 \quad \text{(A 11.35)}$$

Where:

$q_{Ed} = \frac{F_r}{d\prime L_2}$ and $0{,}60d \leq d' \leq d$. For d' see figure A 11.8.

The diameter of the cable is 101,5mm. Assumed is that $d' = 0{,}80d = 0{,}80 * 101{,}5mm = 81{,}2\ mm$

Further:

$F_r = 32432{,}81\ kN$
$L_2 = 0{,}3m$

The above value for the saddle has been estimated. The radial force is presented in figure A 11.13.

$$q_{Ed} = \frac{F_r}{d\prime L_2} = \frac{32432{,}81\ kN}{81{,}2\ mm * 300mm} = 1331{,}39\ N/mm^2 \qquad \text{(A 11.36)}$$

$$q_{Rd} = \frac{q_{Rk}}{\gamma_{m,bed}} \qquad \text{(A 11.37)}$$

$q_{Rk} = 40N/mm^2$ (table A 11.10)
$\gamma_{m,bed} = 1{,}00$ (due to the lack of tests)

Follows:

$$\frac{q_{Ed}}{q_{Rd}} = \frac{1331{,}39}{40} = 33{,}28 > 1{,}0 \qquad \text{(A 11.38)}$$

A regular steel saddle is not sufficient to take up the high force that is a consequence of the amount of pretension in the cables. Special type of saddles needs to be applied. An example of a type of saddle that can resist such high force is illustrated in figure 11.16. For the Stuttgart stadium an equal type of connections have been applied.

Design of saddles

Saddles should be designed for a cable force of k times the characteristic breaking strength F_{uk} of the cables.

A value of $k = 1{,}10$ is recommended. The saddles have to resist:

$$1{,}10 * F_{uk} = 1{,}10 * 10552{,}58\ kN = 11607{,}84\ kN \qquad \text{(A 11.39)}$$

11.5.3.3 Clamps

For the check of the clamps, the clamp at the corner suspension cable is used as example. Due to the pretensioning, high normal forces arise in the corner cable elements. The leading element is shown in figure A 11.14. The radial force is equal to 1502,37 kN.

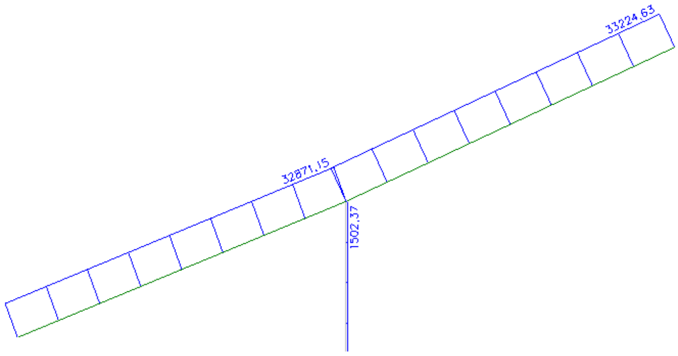


Figure A 11.14 Clamp connection suspension cable - strut

Slipping of clamps

Where clamps transmit longitudinal forces to a cable and the parts (figure A 11.2.) are not mechanically keyed together, slipping shall be prevented by verifying

$$F_{Ed_{//}} \leq \frac{\left(F_{Ed_{perpendicular}} + F_r\right)\mu}{\gamma_{m,fr}} \qquad \text{(A 11.40)}$$

The cables are mechanically keyed together to the clamps. Slipping will not occur and therefore the check of equation A 11.40 does not have to be done.

Transverse pressure

The transverse pressure q_{Ed} due to the radial clamping force F_r shall be limited to:

$$\frac{q_{Ed}}{q_{Rd}} \leq 1{,}0 \qquad \text{(A 11.41)}$$

With:

$q_{Ed} = \frac{F_r}{d\prime L_2}$ and $0{,}60d \leq d' \leq d$. For d' see figure A 11.8.

The diameter of the cable is 101,5mm. Assumed is that $d' = 0{,}80d = 0{,}80 * 101{,}5mm = 81{,}2\ mm$

Further:

$F_r = 1502.37\ kN$
$L_2 = 0{,}1m$

The above value for the clamp has been estimated. The radial force is presented in figure A 11.14.

$$q_{Ed} = \frac{F_r}{d\prime L_2} = \frac{1502{,}37\ kN}{81{,}2\ mm * 100mm} = 185{,}02\ N/mm^2 \qquad \text{(A 11.42)}$$

$$q_{Rd} = \frac{q_{Rk}}{\gamma_{m,bed}} \qquad \text{(A 11.43)}$$

$q_{Rk} = 40N/mm^2$ (table A 11.10)
$\gamma_{m,bed} = 1{,}00$

Follows:

$$\frac{q_{Ed}}{q_{Rd}} = \frac{185,02}{40} = 4,63 > 1,0 \tag{A 11.44}$$

The clamps are not sufficient and do not fulfil the structural requirements.

Design of the clamps

Clamps should be designed for a force of k times the proof force F_k of the secondary component

A value of $k = 1,15$ is recommended.

11.5.4 Serviceability Limit State

Deformations

The maximum additional deformation due to the leading variable load can have a maximum deformation of a 1000 mm. The maximum additional deformation in the structure is …. Mm and therefore fulfil the deformation requirement.
For the cable structure there is no requirement set for the deformation of the single cable elements. For regular steel profiles the maximum deformation can be 1/100 of the length of the bar (see truss design). For the cable elements however, the only requirement is that the cable cannot become slack under any load condition. For element S449 (and all other elements as well) this is not the case and therefore fulfils the stiffness requirement.

Elastic service conditions

For the stress limit there are limits set by the Eurocode for the construction phase and for service conditions. The requirements are as follows:

$$f_{constr} = \frac{0,66\sigma_{uk}}{\gamma_r * \gamma_F} \tag{A 11.45}$$

Where

$\gamma_r * \gamma_F = 1,10$ for short term conditions;
$\gamma_r * \gamma_F = 1,20$ for long term conditions.

For the calculations is assumed that the construction is for long term conditions.

Follows that the stress limit is equal to:

$$f_{constr} = 0,55 * 1770\ N/mm^2 = 973,5 N/mm^2 \tag{A 11.46}$$

For the service conditions holds that:

$$f_{SLS} = \frac{0,66\sigma_{uk}}{\gamma_r * \gamma_F} \tag{A 11.47}$$

Where:

$\gamma_r * \gamma_F = 1,33$ with bending stresses;
$\gamma_r * \gamma_F = 1,48$ without bending stresses.

Follows that the stress limit is equal to:

$$f_{SLS} = 0,45 * 1770\ N/mm^2 = 796,5\ N/mm^2 \tag{A 11.48}$$

The maximum stress (figure A 11.15) arising in element S449 is 3499,6 N/mm² and therefore does not fulfil the requirement.

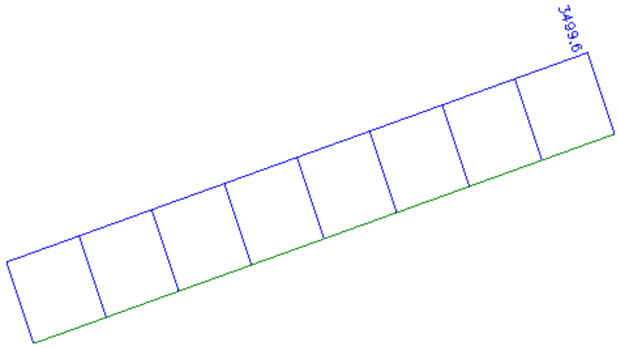


Figure A 11.15 Stress in element S449