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Publication date
2020

Document Version
Accepted author manuscript

Published in
Examples for the Design of Reinforced and Prestressed Concrete Members Under Torsion

Citation (APA)
Granda Valencia, C., & Lantsoght, E. O. L. (2020). Torsion Design Example: Inverted Tee Bent Cap. In E. Lantsoght, G. Greene, & A. Belarbi (Eds.), *Examples for the Design of Reinforced and Prestressed Concrete Members Under Torsion* (Vol. SP-344, pp. 168-182). (ACI Special Publication). American Concrete Institute.
https://www.concrete.org/publications/internationalconcreteabstractsportal.aspx?m=details&id=51728296

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SP-XXXX—XX

Torsion Design Example: Inverted Tee Bent Cap

Camilo Granda Valencia and Eva Lantsoght

Synopsis: This paper provides a practical example of the torsion design of an inverted tee bent cap of a three-span bridge. A full torsional design following the guidelines of the ACI 318-19 building code is carried out and the results are compared with the outcomes from CSA-A23.3-04, AASHTO-LRFD-17, and EN 1992-1-1:2004 codes. Then, a summary of the detailing of the cross-section considering the reinforcement requirements is presented. The objective of this paper is to illustrate the application of ACI 318-19 when designing a structural element subjected to large torsional moments.

Keywords: bridge, codes, concrete, design, inverted tee bent cap, reinforcement, shear, torsion

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INTRODUCTION

An inverted tee bent cap was selected as the structural element to carry out its full reinforcement design, including torsion. ACI 318-19[1] is developed for buildings, not for bridge structures. Nevertheless, this bridge member will be used since its design represents a challenge due to its T-shape cross-section, indeterminacy, and large applied torsional moment. ACI 318-19[1] will be used to illustrate to practicing engineers how to use the torsion provisions of this code, which are applicable to any structural element. Additionally, the final reinforcement layout following the ACI 318-19[1] provisions is compared against the AASHTO-LRFD-17[2] bridge code and other building codes such as CSA-A23.3-04[3] and EN 1992-1-1:2004[4]. Some of the differences between each code, which influenced the final reinforcement layout include: load factors, strength reduction factors, and design philosophy (e.g. space truss analogy or Modified Compression Field Theory).

DESCRIPTION OF DESIGN TASK

Geometry and loads

The inverted tee bent cap is part of the substructure of a three-span bridge. The geometry was taken from the Texas Department of Transportation (TxDOT), LRFD Inverted Tee Bent Cap Design Example[5]. Both end-spans have a length of 54 ft (16.50 m) and the middle-span has a length of 112 ft (34.10 m). The deck has a width of 46 ft (14.00 m) with two external lanes of 15 ft (4.60 m), one middle lane of 14 ft (4.20 m) and two external rails of 1 ft (0.30 m), see Figure 1. The bridge has an overlay of 2 in (0.05 m) and a slab of 8 in (0.20 m). The deck is supported by a set of six beams spaced o.c. @ 8 ft (2.44 m) and each beam weighs 0.851 kip/ft (12.42 kN/m). The rails provide a load of 0.382 kip/ft (5.573 kN/m). The inverted tee bent cap is supported by four 36 in (0.90 m) diameter columns spaced @ 12 ft (3.65 m) each.

The bridge is subjected to the factored dead load of structural and nonstructural components, the factored dead load of the wearing surface, and the factored vehicular live load consisting of the distributed lane load of 0.64 kip/ft (0.868 kN/m) and the design truck specified by the AASHTO-LRFD-17[2] code. The design truck includes the multiple presence and dynamic allowance factor. The way the factored loads are applied on the inverted tee bent cap are shown in Figure 2. The most critical configuration of the loads is sought for the torsion design, which results in placing the live loads only on the longest span. The cross-sectional dimensions of the stem are controlled by the diameter of columns, the distance from the slab to the ledge, the slab thickness, and haunch. The cross-sectional dimensions of the ledge are obtained by knowing the required development length of the reinforcement. The elevation dimensions are governed generally by the girders' spacing and the distance from the centerline of the exterior girder to the end of the cap. The geometry of the inverted tee bent cap can be seen on Figures 3 and 4.

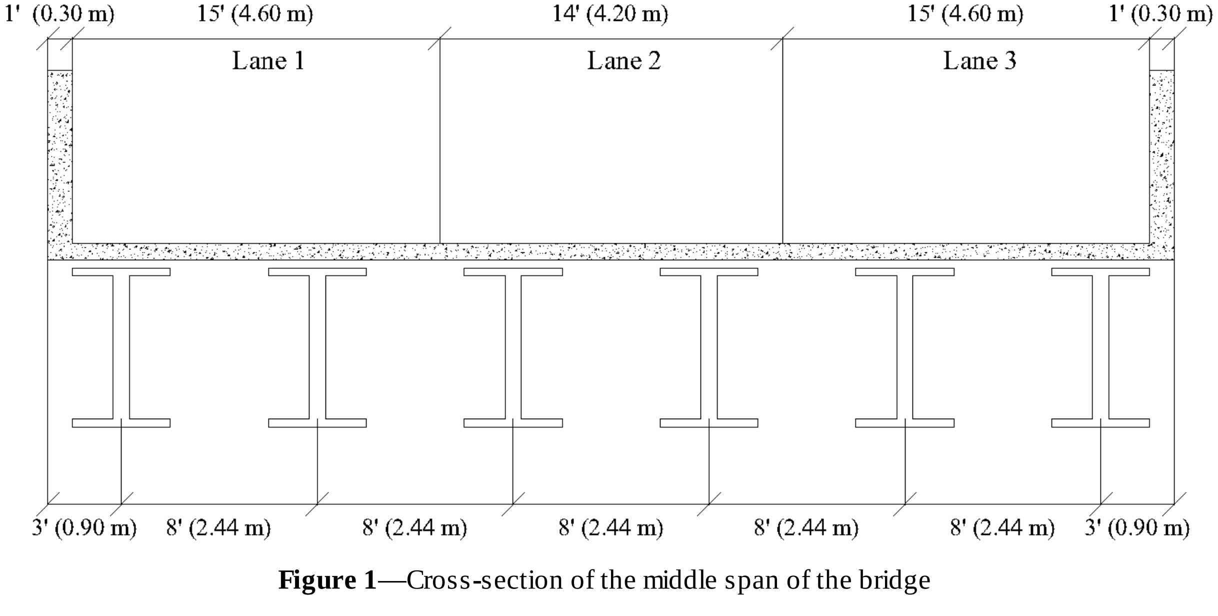

Figure 1—Cross-section of the middle span of the bridge

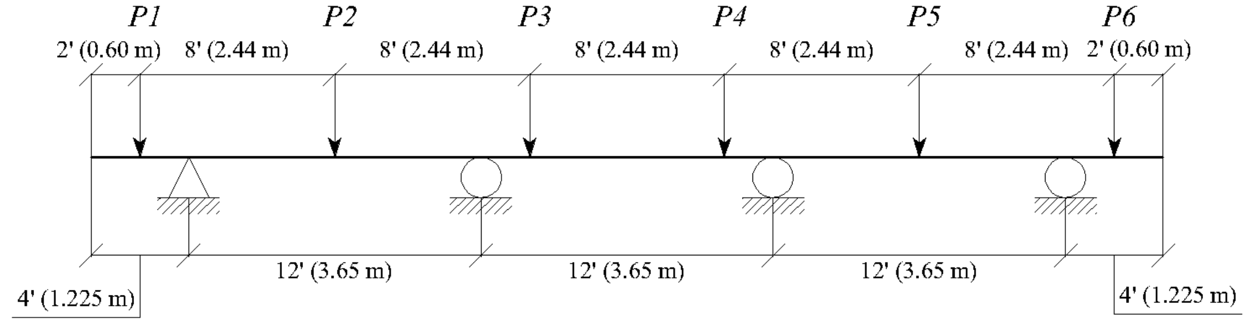

Figure 2—Point loads applied at the ledge of the inverted tee bent cap which produce the torsional moments

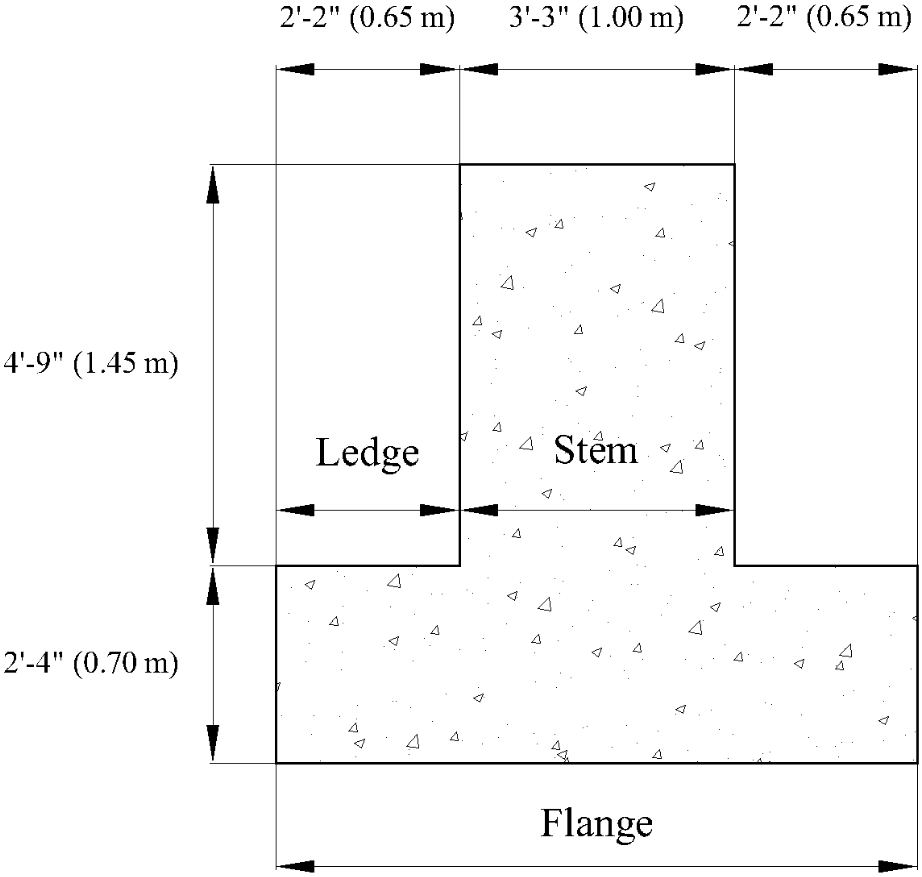

Figure 3—Cross-sectional dimensions of the inverted tee bent cap

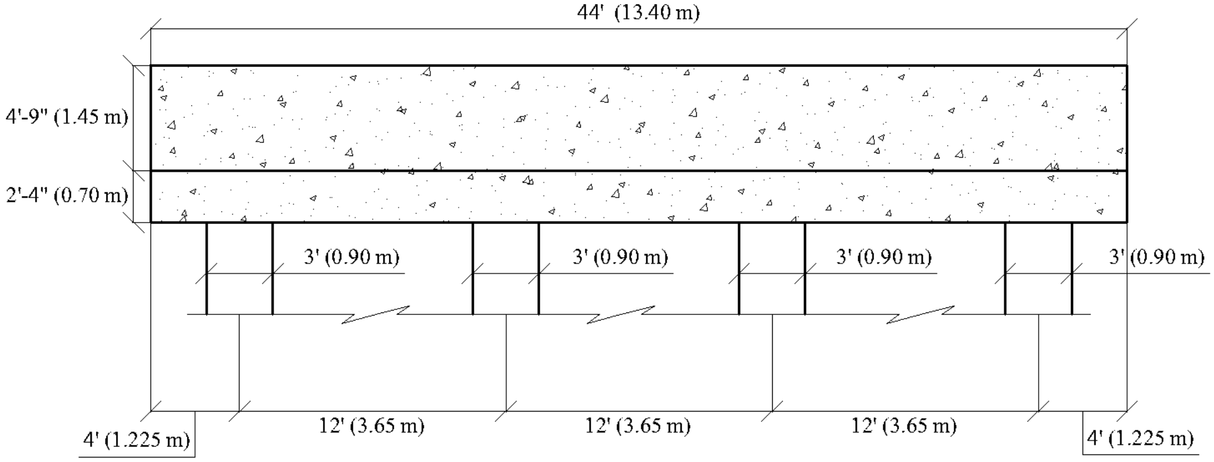

Figure 4—Elevation dimensions of the inverted tee bent cap

Materials

Concrete: $f'_c = 3{,}600$ psi (25 MPa)
$\gamma_c = 150$ pcf (23.56 kN/m^3)

Reinforcement: $f_y = f_{yt} = 60{,}000$ psi (415 MPa)

Statement of design problem

Usually, the torsional design of a structural element does not control the final layout of the cross-section. Nevertheless, if the appropriate conditions of loading occur, certain elements like this inverted tee bent cap will experience an important torsional moment. In this example, the lane load plus the design truck were placed on the midspan of the bridge at all lanes. The vertical reaction of all six beams transmitted to the ledge of the inverted tee bent cap produces important torsional moments around this member because the loads are applied out of the axis. There is only one inverted tee bent cap to support torsion, therefore redistribution of torsional moment is not possible. Consequently, the torsion design is needed to maintain the equilibrium of this member. To analyze this, the provisions for torsion given by the ACI 318-19[1] code are used. Although the bridge structures do not fall under the scope of the ACI 318-19[1] building code, the design steps are given here for illustrative purposes. The provisions that cover the bridge structures design are usually given by the AASHTO-LRFD-17[2] code. The AASHTO-LRFD-17[2] required transverse and longitudinal steel for torsion on this example are at the end of this document. ACI 318-19[1] assumes all cross-sections as hollow sections. After cracking, each straight segment of the hollow section will act as a planar truss and the whole member will behave like a space truss. The torsional strength is mainly provided by the transverse and longitudinal reinforcement acting in tension. The compression diagonals will withstand the compression forces. All the steps of the torsion design are listed and explained in order. As a result, the required transverse and longitudinal reinforcement to resist the applied torsional moment is obtained.

DESIGN PROCEDURE

The torsional design is a complement of the moment and shear design i.e. the transverse and longitudinal reinforcement obtained for torsion will be added to the values previously computed to provide flexural and shear resistance. The torsion design consists of the following steps:

Step 1: Determine the factored bending moment, shear force and torsional moment on the inverted tee bent cap
Step 2: Compute the required and provided longitudinal reinforcement for bending moment
Step 3: Compute the required and provided transverse reinforcement for shear
Step 4: Analyze if torsion can be neglected
Step 5: Check if the current dimensions of the cross-section are adequate
Step 6: Limit the maximum spacing of torsion stirrups
Step 7: Determine the required transverse reinforcement for torsion

Step 8: Check the minimum transverse reinforcement for torsion and shear
Step 9: Control of the total transverse reinforcement required for shear and torsion
Step 10: Calculate the required longitudinal reinforcement for torsion
Step 11: Compute the minimum longitudinal reinforcement required for torsion
Step 12: Check the torsional capacity
Step 13: Compute the required hanger reinforcement

DESIGN CALCULATIONS

Step 1: Determine the factored bending moment, shear force and torsional moment on the inverted tee bent cap

The included dead loads are: structural, nonstructural, and wearing surface dead load. The live load consists of the distributed lane load and the design truck including the multiple presence and dynamic allowance (impact factor). The U = 1.2D + 1.6L ACI 318-19[1] load combination was used to compute the factored loads for the ultimate limit state. The shear force, bending moment, and torsional moment were obtained directly from the 3D bridge model developed using CSiBridge. CSiBridge is a software able to model, analyze, and design bridge structures. This software was developed by Computers and Structures Inc[6]. The ultimate torsional moment was computed by obtaining the most critical load combination of T_L, T_{SW}, T_R, and T_W along the length of the inverted tee bent cap.

T_L = 333 kip·ft (452 kN·m)
T_{SW} = 118 kip·ft (160 kN·m)
T_R = -4 kip·ft (6 kN·m)
T_W = 14 kip·ft (19 kN·m)

T_L, T_{SW}, T_R, and T_W are the service torsional moments produced by the HL-93 live truck load, self-weight of the structure, self-weight of nonstructural components, and wearing surface, respectively. It is important to mention that this critical combination occurs from the 26 ft (7.94 m) up to the 28 ft (8.56 m) station from the left tip of the inverted tee ben cap shown in Figure 2. This short length and the proximity to the support will prevent the development of Saint-Venant torsion. In this case, warping torsion will arise. However, the uncertainty of only experiencing warping torsion between the the 26 ft (7.94 m) up to the 28 ft (8.56 m) station is very high. A conservative and safe design following sectional analysis will take this critical torsional moment as the design load. Additionally, there are no provisions in the ACI 318-19[1] building code that follow a sectional analysis design considering warping torsion.

Thus, the ultimate factored torsional moment, T_u, was computed as follows:

$$T_u = 1.2\left(T_R + T_{SW} + T_W\right) + 1.6\left(T_L\right) = 1.2\left(-4 + 118 + 14\right)\text{kip}\cdot\text{ft} + 1.6\left(333\right)\text{kip}\cdot\text{ft} = 687\ \text{kip}\cdot\text{ft}\ \left(931\ \text{kN}\cdot\text{m}\right) \tag{1}$$

The same load factors of Equation (1) were used to calculate the factored bending moments concurrent with the maximum torsional moment = 687 kip·ft (931 kN·m). This occurs from the 26 ft (7.94 m) up to the 28 ft (8.56 m) station. These are not the critical hogging and sagging bending moments throughout the length of the inverted tee bent cap, but the bending moments acting together with the critical torsional moment. Additionally, the factored shear force was also obtained at the station of critical torsional moment. The ultimate state loads for these effects are:

M_u^+ = 738 kip·ft (1002 kN·m)
M_u^- = 531 kip·ft (720 kN·m)
V_u = 461 kip (2051 kN)

With M_u^+ considered as the factored sagging (concave downwards bent) moment, M_u^- is the factored hogging (convex upwards bent) moment, and V_u is the factored shear force.

Step 2: Compute the required and provided longitudinal reinforcement for bending moment

A_s^+ is the required longitudinal reinforcement for sagging moment and is obtained by solving the following quadratic equation:

$$-\frac{f_y}{2\cdot 0.85 f_c' b}\left(A_s^+\right)^2 + dA_s^+ - \frac{M_u^+}{\phi_m f_y} = 0 \rightarrow A_s^+ = 2\text{ in}^2\left(1290\text{ mm}^2\right) \tag{2}$$

In last equation, the effective depth d is equal to 81.87 in (2080 mm). ϕ_m is the reduction factor for the nominal sagging moment capacity, initially taken as 0.9. b is the width of the cross-section under compression. When the factored sagging moment is applied, b is equal to 39 in (1.0 m). When the hogging moment is applied b = 91 in (2.3 m).

The result obtained in Equation (2) needs to be compared against the minimum longitudinal reinforcement requirement ACI 318-19[1] §9.6.1.2:

$$A_{s,\min}^+ = \max\left(\frac{3\sqrt{f_c'}}{f_y}b_w d, \frac{200}{f_y}b_w d\right) = \frac{200}{60\text{ ksi}}\times 39\text{ in}\times 81.87\text{ in} = 10.64\text{ in}^2\left(6864.5\text{ mm}^2\right) \tag{3}$$

In Equation (3), b_w is the web width. This design example is critical for torsion. For this reason, the minimum longitudinal reinforcement controls the sagging moment design. Finally, it is required to check if the longitudinal reinforcement for sagging moment yields. For this, c, the distance from extreme compression fiber to the neutral axis is needed:

$$c = \frac{A_s^+ f_y}{0.85 f_c' b\beta_1} = \frac{10.64\text{ in}^2\times 60\text{ ksi}}{0.85\times 3600\text{ psi}\times 39\text{ in}\times 0.85} = 6.29\text{ in}\left(159.85\text{ mm}\right) \tag{4}$$

ε_t is the strain at the steel, which needs to be larger than the yield strain of steel $\varepsilon_y = 0.002$ to have a ductile behavior.

$$\varepsilon_t = \frac{0.003}{c}\left(d - c\right) = \frac{0.003}{6.29\text{ in}}\left(81.87 - 6.29\right)\text{in} = 0.036 \tag{5}$$

Since $\varepsilon_t > \varepsilon_y$ the assumption of $\phi_m = 0.9$ is correct. The next step is to compute A_s^-. This is the required longitudinal reinforcement for hogging moment. To obtain it, the procedure from Equation (2) to Equation (5) should be carried out. However, M_u^- should be used instead of M_u^+ in Equation (2).

Finally, the provided longitudinal reinforcement for sagging and hoggin moment, $A_{s,prov}^+$ and $A_{s,prov}^-$, respectively is:

$A_{s,prov}^+$ = 10.8 in^2 (6968 mm^2)
$A_{s,prov}^-$ = 11.0 in^2 (7097 mm^2)

18 #7 bars are used for $A_{s,prov}^+$ and 11 #9 bars for $A_{s,prov}^-$.

Step 3: Compute the required and provided transverse reinforcement for shear

First, the shear strength provided by concrete V_c needs to be computed according to ACI 318-19[1] §22.5.5.1. Normal weight concrete is used; $\lambda = 1$.

$$V_c = \left[2\lambda\sqrt{f_c'} + \frac{N_u}{6A_g}\right]b_w d = \left[2\times 1\sqrt{3600\text{ psi}} + \frac{0}{6\times 4771\text{ in}^2}\right]\times 39\text{ in}\times 81.87\text{ in} = 383\text{ kip }\left(1704\text{ kN}\right) \tag{6}$$

In Equation (6), N_u is the factored axial force and A_g is the gross area of the cross-section. Consequently, the required shear transverse reinforcement to resist factored shear is calculated following ACI 318-19[1] §22.5.8.5:

$$\frac{A_v}{s} = \frac{\left(V_u - \phi V_c\right)}{\phi f_{yt} d} = \frac{461\text{ kip} - 0.75\times 383\text{ kip}}{0.75\times 60\text{ ksi}\times 81.87\text{ in}} = 0.047\ \frac{\text{in}^2}{\text{in}}\left(1.194\ \frac{\text{mm}^2}{\text{mm}}\right) \tag{7}$$

Leaving out the term ϕV_c in Equation (7) yields A_v/s = 0.125. This value is larger than $A_{v,\min}/s$ = 0.119. Consequently, Equation (6) was selected correctly to compute the shear strength provided by concrete according to ACI 318-19[1] §22.5.5.1.

The next step is to compute the spacing of transverse reinforcement when two #5 closed stirrups are provided, see Figure 5.

$$s = \frac{A_v f_{yt} d}{\frac{V_u}{\phi} - V_c} = \frac{\left(4 \times 0.31 \text{ in}^2\right) 60 \text{ ksi} \times 81.87 \text{ in}}{\frac{461 \text{ kip}}{0.75} - 383 \text{ kip}} = 26.3 \text{ in } \left(668 \text{ mm}\right) \tag{8}$$

The spacing computed in Equation (8) needs to be checked against the maximum specified in ACI 318-19[1] §9.7.6.2.2

$$s_{\max} = \min\left(\frac{d}{2}, 24 \text{ in}\right) = 24 \text{ in } \left(610 \text{ mm}\right) \tag{9}$$

The spacing in Equation (8) is larger than (9). Consequently, Equation (9) controls and the spacing of the transverse reinforcement for shear will be 24 in (610 mm).

Step 4: Analyze if torsion can be neglected

To check if torsion can be neglected, the threshold torsion is computed according to ACI 318-19[1] §22.7.4. The equation for a solid non-prestressed cross-section is used $\phi\lambda(f_c')^{0.5}(A_{cp})^2/p_{cp}$ = 0.75×1.0×(3600 psi)$^{0.5}$(4771 in^2)^2/(352 in) = 243 kip·ft (330 kN·m). Torsion can be neglected when the factored threshold torsion exceeds the factored applied torsional moment. ϕ, the reduction factor for the nominal capacity of torsion, is equal to 0.75.
The computed threshold torsion is smaller than factored torsional moment = 687 kip·ft (931 kN·m). Therefore, the torsion analysis is required.

Step 5: Check if the current dimensions of the cross-section are adequate

To prevent crushing of the concrete and excessive cracking, ACI 318-19[1] §22.7.7.1 checks if the dimensions of the cross-section are large enough. The maximum value of the shear and torsion stresses need to be analyzed at their maximum value i.e. where they are added together. If this equation is not fulfilled, the dimensions of the inverted tee bent cap need to be increased and the bending moment and shear design should be repeated. V_c = 383 kip (1704 kN) is the shear strength provided by concrete according to ACI 318-19[1] §22.5.5.1.

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right)$$

$$\sqrt{\left(\frac{472 \text{ kip}}{39 \text{ in} \times 82 \text{ in}}\right)^2 + \left(\frac{687 \text{ kip} \cdot \text{ft} \times 334 \text{ in}}{1.7 \times \;\; 3875 \text{ in}^2 \;\; ^2}\right)^2} \le 0.75\left(\frac{383 \text{ kip}}{39 \text{ in} \times 82 \text{ in}} + 8\sqrt{3600 \text{ psi}}\right)$$

$$0.18 \text{ ksi} \quad 1.26 \text{ MPa} \quad \le 0.45 \text{ ksi} \quad 3.10 \text{ MPa} \tag{10}$$

The last expression is fulfilled, consequently the torsional design can be carried out.

Step 6: Limit the maximum spacing of torsion stirrups

The maximum spacing according to ACI 318-19[1] §9.7.6.3.3 is:

$$s_{\max} \le \min\begin{cases} \frac{p_h}{8} \\ 12 \text{ in} \end{cases} = \min\begin{cases} \frac{3334 \text{ in}}{8} \\ 12 \text{ in} \end{cases} = \min\begin{cases} 42 \text{ in} \\ 12 \text{ in} \end{cases} = 12.0 \text{ in} \quad 300 \text{ mm} \tag{11}$$

The spacing of Equation (11) should be compared against the *s* of Equation (13) and (16)

Step 7: Determine the required transverse reinforcement for torsion

According to ACI 318-19[1] §22.7.6.1, θ, the angle between the struts and the tension chord, can be taken as any value between 30 and 60 degrees. ACI 318-19[1] §22.7.6.1.2 states that θ is usually 45° for reinforced concrete members with $A_{ps}f_{se} < 0.4(A_{ps}f_{pu}+A_sf_y)$ and 37.5° for prestressed elements with $A_{ps}f_{se} \geq 0.4(A_{ps}f_{pu}+A_sf_y)$. Because the last expression for non-prestressed reinforced concrete members is satisfied, θ = 45°. The required transverse reinforcement for torsion is:

$$\frac{A_t}{s} \geq \frac{T_u}{1.7\phi A_{oh} f_{yt}} \tan\theta = \frac{687\ \text{kip}\cdot\text{ft}}{1.7\times 0.75\times 3875\ \text{in}^2\times 60\ \text{ksi}} \tan\ 45^\circ\ = 0.0278\ \frac{\text{in}^2}{\text{in}}\ \left(0.706\ \frac{\text{mm}^2}{\text{mm}}\right) \tag{12}$$

The provided transverse reinforcement is two #5 closed stirrups: one for the flange and the other for the stem. However, the number of legs of a stirrup resisting torsion is only one as stated in ACI 318-19[1] §R9.6.4.2, consequently $A_{t,prov}$ = 0.307 in^2 (198 mm^2). Taking the spacing of the torsion stirrups as 10 in (254 mm), the provided transverse reinforcement for torsion is:

$$\frac{A_{t,prov}}{s} = \frac{0.307\ \text{in}^2}{10\ \text{in}} = 0.0307\frac{\text{in}^2}{\text{in}}\ \left(0.780\frac{\text{mm}^2}{\text{mm}}\right) > 0.0278\ \frac{\text{in}^2}{\text{in}}\ \left(0.706\ \frac{\text{mm}^2}{\text{mm}}\right) \tag{13}$$

The provided torsion stirrups spaced @ 10 in (254 mm) provide a larger area per length than the required computed in Equation (12).

Step 8: Check the minimum transverse reinforcement for torsion and shear

For the transverse reinforcement limit, ACI 318-19[1] §9.6.4.2 states that for members under torsion and shear, the stirrups for torsion and shear effects cannot be less than:

$$\frac{A_v+2A_t\ {}_{\min}}{s} = \max\begin{cases} 0.75\sqrt{f'_c}\,\dfrac{b_w}{f_{yt}} \\ 50\dfrac{b_w}{f_{yt}} \end{cases} = \max\begin{cases} 0.75\sqrt{3600\ \text{psi}}\times\dfrac{39\ \text{in}}{60\ \text{ksi}} \\ 50\times\dfrac{39\ \text{in}}{60\ \text{ksi}} \end{cases} = \max\begin{cases} 0.029\ \dfrac{\text{in}^2}{\text{in}} \\ 0.033\ \dfrac{\text{in}^2}{\text{in}} \end{cases} = 0.033\ \frac{\text{in}^2}{\text{in}}\ \left(0.826\frac{\text{mm}^2}{\text{mm}}\right) \tag{14}$$

The required shear A_v/s reinforcement is 0.047 in^2 / in. Therefore, the total transverse required reinforcement is:

$$\frac{A_v}{s}+2\frac{A_t}{s} = 0.047\frac{\text{in}^2}{\text{in}}+2\times 0.0278\frac{\text{in}^2}{\text{in}} = 0.103\ \frac{\text{in}^2}{\text{in}}\left(2.616\frac{\text{mm}^2}{\text{mm}}\right) > 0.033\ \frac{\text{in}^2}{\text{in}}\ \left(0.826\frac{\text{mm}^2}{\text{mm}}\right) \tag{15}$$

The minimum transverse reinforcement for both torsion and shear is less than the required, consequently it does not control the design.

Step 9: Control of the total transverse reinforcement required for shear and torsion

The minimum spacing to control both shear and torsion effects is given by the effective area for shear $A_{v,eff}$ and torsion $A_{t,prov}$ resisting the external loads divided by the required area per unit length computed in Equation (15). $A_{v,eff}$ is the area of the stirrups' legs adjacent to the sides of the beam that are considered to resist torsion. The loads are applied at the ledge of the inverted tee bent cap. Therefore, only two #5 legs = 2×0.307 in^2 = 0.614 in^2 (397 mm^2) will be activated when the load is applied and included into $A_{v,eff}$. These two legs are numbered as leg 1 (blue) and leg 2 (blue) in Figure 5. The inner legs will be ineffective to resist torsion according to ACI 318-19[1] §R9.5.4.2. For the torsion area of stirrups, the effective area resisting the external forces is just the area of one leg of the #5 stirrups provided.

$$s = \frac{A_{v,eff}+2A_{t,prov}}{\dfrac{A_v+2A_t}{s}} = \frac{2\times 0.307\ \text{in}^2\ \ +2\ \ 0.307\ \text{in}^2}{0.103\ \dfrac{\text{in}^2}{\text{in}}} = 11.9\ \text{in (303 mm)} \tag{16}$$

The spacing used Equation (13) is more critical than the one in Equation (16). Consequently, the required spacing to resist shear an torsion stresses is at least 10 in (254 mm).

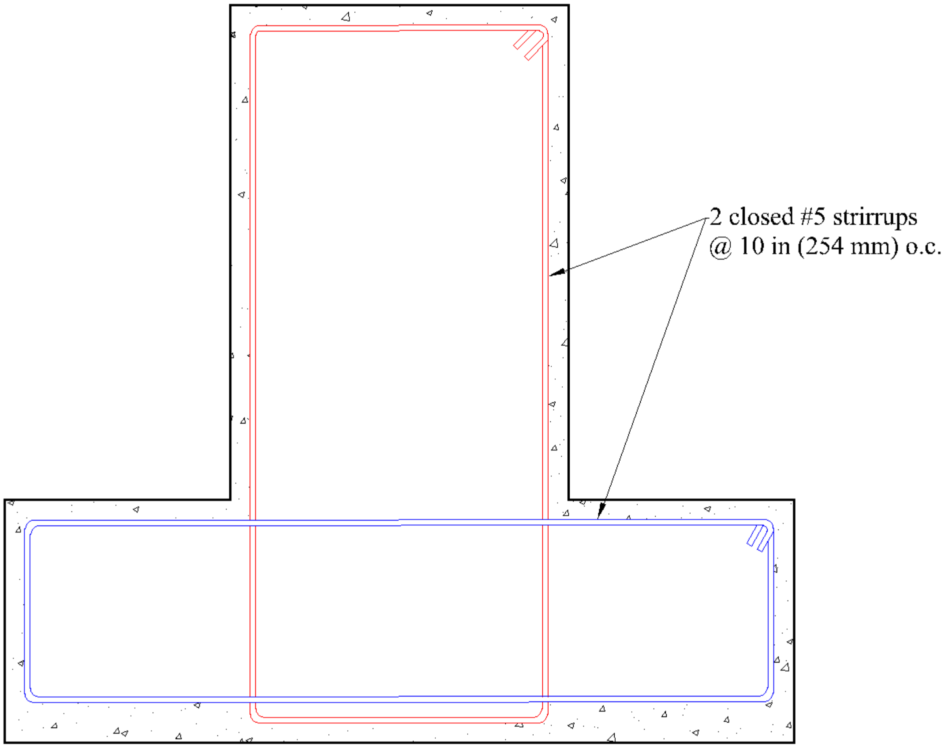

Figure 5— General layout of transverse reinforcement

The spacing computed only for the provided shear reinforcement is 24 in (610 mm), for both shear and torsion is 10 in (254 mm) and the maximum spacing for torsion is 12 in (300 mm). With these values, the spacing that controls the transverse reinforcement is 10 in (254 mm). As shown in Figure 5, two #5 stirrups spaced @ 10 in. (254 mm) o.c. will be provided to resist shear, torsion and their combination of actions.

Step 10: Calculate the required longitudinal reinforcement for torsion

The equation used to compute the longitudinal reinforcement for torsion in terms of the provided transverse reinforcement for torsion is obtained by combining the equations presented in ACI 318-19[1] §22.7.6.1:

$$A_l \geq \frac{A_t}{s}\frac{f_{yt}}{f_y} p_h \cot^2\theta = \frac{0.307\text{ in}^2}{10\text{ in}} \times \frac{60{,}000\text{ psi}}{60{,}000\text{ psi}} \times 334\text{ in} \times \cot^2\ 45^\circ\ = 10.3\text{ in}^2\quad 6615\text{ mm}^2 \qquad (17)$$

The required longitudinal reinforcement for torsion will be compared to the minimum longitudinal reinforcement for torsion and the largest one will govern the design.

Step 11: Compute the minimum longitudinal reinforcement required for torsion

The minimum area of longitudinal steel reinforcement for torsion $A_{l,\min}$ can be calculated with ACI 318-19[1] §9.6.4.3

$$A_{l,\min} = \min\begin{cases} \dfrac{5\sqrt{f'_c}A_{cp}}{f_y} - \left(\dfrac{A_t}{s}\right)p_h\dfrac{f_{yt}}{f_y} \\ \dfrac{5\sqrt{f'_c}A_{cp}}{f_y} - \left(\dfrac{25b_w}{f_{yt}}\right)p_h\dfrac{f_{yt}}{f_y} \end{cases} = \min\begin{cases} \dfrac{5\sqrt{3600\text{ psi}}\times 4771\text{ in}^2}{60{,}000\text{ psi}} - \left(\dfrac{0.307\text{ in}^2}{10\text{ in}}\right)\times 334\text{ in}\times\dfrac{60{,}000\text{ psi}}{60{,}000\text{ psi}} \\ \dfrac{5\sqrt{3600\text{ psi}}\times 4771\text{ in}^2}{60{,}000\text{ psi}} - \left(\dfrac{25\times 39\text{ in}}{60{,}000\text{ psi}}\right)\times 334\text{ in}\times\dfrac{60{,}000\text{ psi}}{60{,}000\text{ psi}} \end{cases} \tag{18}$$

$$= \min\begin{cases} 13.60\text{ in}^2 \\ 18.45\text{ in}^2 \end{cases} = 13.60\text{ in}^2 \ (8775\text{ mm}^2)$$

In this example, the minimum longitudinal reinforcement for torsion is larger than A_l according to Equation (17). Consequently, $A_{l,\min}$ controls and the provided longitudinal reinforcement for torsion should be at least 13.6 in^2 (8775 mm^2). When #6 bars are used, 31 bars are required. However, 38 bars will be used to have a symmetrical layout and to fulfill the spacing requirement of ACI 318-19[1] §25.7.2.3. This provision mentions that the maximum clear spacing between unsupported longitudinal reinforcement is 6 in (152 mm). Out of the 7 extra bars, 4 will be used to hang 4 crossties at the ledge. The provided longitudinal reinforcement for torsion becomes $A_{l,prov}$ = 16.72 in^2 (10,787 mm^2). ACI 318-19[1] §9.7.5.1 states that the longitudinal reinforcement for torsion needs to be distributed around the perimeter and inside the closed stirrups. The spacing between the longitudinal bars for torsion cannot exceed 12 in (300 mm). At least one bar should be placed in each corner of the stirrups.

Step 12: Check the torsional capacity

ACI 318-19[1] §22.7.6.1 gives two equations to analyze the torsional strength T_n. The final torsional capacity of the inverted tee bent cap will be the minimum value of:

$$T_n = \min\begin{cases} \dfrac{1.7A_{oh}A_{t,prov}f_{yt}}{s}\cot\theta \\ \dfrac{1.7A_{oh}A_{l,prov}f_y}{p_h}\tan\theta \end{cases} = \min\begin{cases} \dfrac{1.7\times 3875\text{ in}^2\times 0.307\text{ in}^2\times 60{,}000\text{ psi}}{10\text{ in}}\cot\ 45^\circ \\ \dfrac{1.7\times 3875\text{ in}^2\times 16.72\text{ in}^2\times 60{,}000\text{ psi}}{334\text{ in}}\tan\ 45^\circ \end{cases} \tag{19}$$

$$= \min\begin{cases} 1011\text{ kip}\cdot\text{ft} \\ 1649\text{ kip}\cdot\text{ft} \end{cases} = 1011\text{ kip}\cdot\text{ft}\ (1371\text{ kN}\cdot\text{m})$$

The factored torsional nominal capacity is ϕT_n = 758 kip·ft (1027 kN·m) which is larger than T_u = 687 kip·ft (931 kN·m). Therefore, the presented design fulfills the ACI 318-19[1] code requirements.

Step 13: Compute the required hanger reinforcement

To transfer the load effects from the ledge to the stem (main beam) hanger reinforcement is required. ACI 318-19[1] does not provide any recommendation for it. The following procedure[3,7,8] can be applied to compute the required hanger reinforcement. First, the ultimate shear at the left and right ledge, $V_{u,L}$ = 58.6 kip·ft (79.4 kN·m) and $V_{u,R}$ = 358.8 kip·ft (487 kN·m) respectively, is needed. With these loads the following equation can be used to compute the hanger reinforcement:

$$A_h \ge \left(1-\frac{h_b}{h_1}\right)\left(\frac{V_{u,L}+V_{u,R}}{\phi f_{yt}}\right) = \left(1-\frac{57\text{ in}}{85\text{ in}}\right)\left(\frac{58.6\text{ kip}+358.8\text{ kip}}{0.75\times 60\text{ ksi}}\right) = 3.06\text{ in}^2\ \left(1975\text{ mm}^2\right) \tag{20}$$

h_b is the vertical distance from the bottom of the supporting beam to the bottom of the supported beam. h_1 is the overall depth of the supporting beam. Finally, three #5 double-leg stirrups will be provided for A_h, see the green stirrups in Figure 6. These stirrups should be placed within a length of $(b_{w2} + h_2 + 2h_b)/2$ from the station where $V_{u,L}$ and $V_{u,R}$ was computed, on both directions along the length of the inverted tee bent cap. It is recommended to place the hanger reinforcement concurrent with the stirrups for shear and torsion. b_{w2} is the width of the supported beam and h_2 is the is the overall depth of the supported beam.

DESIGN SUMMARY

Final layout and detailing

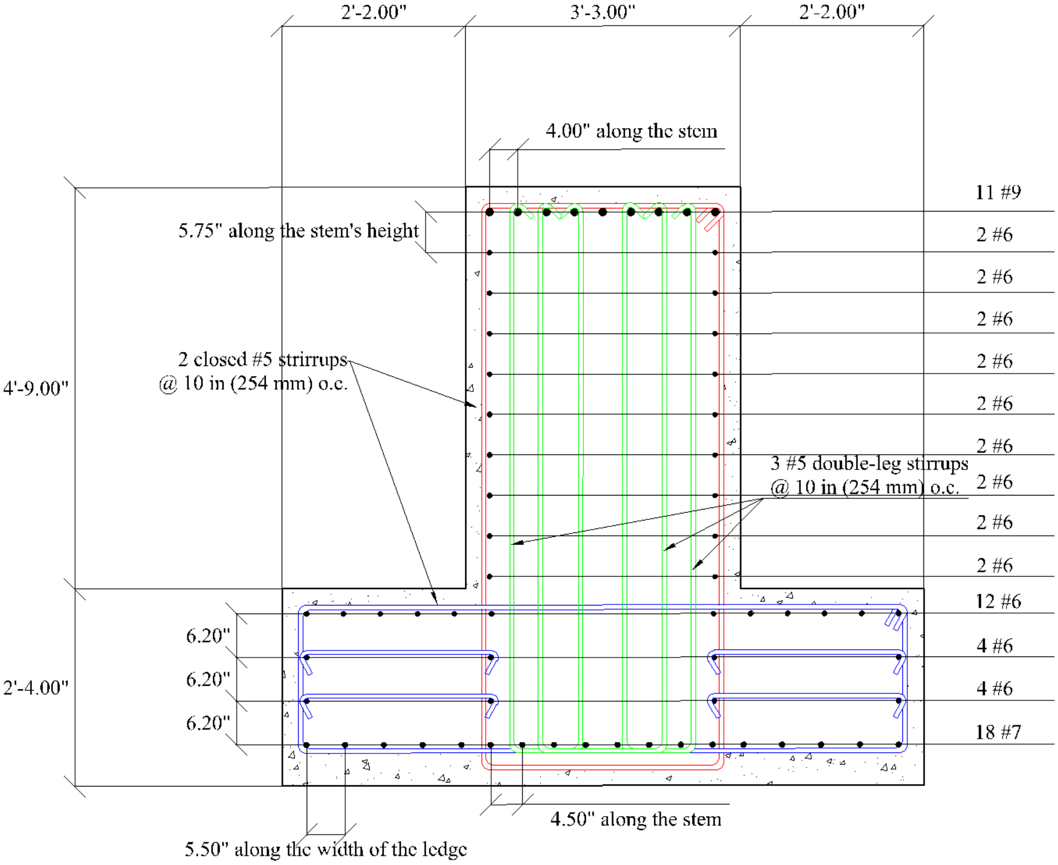

Figure 6—Final layout and detailing of the inverted tee bent cap according to ACI 318-19[1] requirements. All dimensions are center-to center. Conversion: 1ft = 304.8 mm and 1 in = 25.4 mm

Comparison between other provisions for torsion found in other codes

Other codes use different approaches and load combinations to solve the torsion problem. For example, the CSA-A23.3-04[3] code uses the Modified Compression Field Theory (MCFT) and includes the tensile contribution of concrete by considering aggregate interlock. AASHTO-LRFD-17[2] also develops the provisions for torsion from the MCFT. The Eurocode EN 1992-1-1:2004[4] uses a spatial truss model with an equivalent thin-walled tube and wall thickness for the torsion design. Table 1 and 2 provides a comparison of the required longitudinal and transverse, respectively, reinforcement for torsion by each code. ρ_l is the longitudinal reinforcement ratio and considers the required longitudinal reinforcement for torsion and the required longitudinal reinforcement for bending moment that acts together with torsion, in this design example is the longitudinal reinforcement for hogging moment. ρ_w is the transverse reinforcement ratio and considers the required transverse reinforcement for shear and torsion.

Table 1—Comparison of the longitudinal reinforcement required for torsion and hogging moment by each code. Conversion: 1 in^2 = 645.15 mm^2 and 1 in = 25.4 mm

Code	Required longitudinal reinforcement for torsion (in^2)	Required longitudinal reinforcement for hogging moment (in^2)	Number of longitudinal bars provided for torsion	Number of longitudinal bars provided for hogging moment	ρ_l (%)
ACI 318-19	13.6	10.64	38#6	18#7	0.784
CSA-A23.3-04	7.58	7.99	26#5	11#8	0.487
AASHTO-LRFD-17	5.52	3.94	30#4	13#5	0.296
EN 1992-1-1:2004	10.38	5.20	34#5	12#6	0.488

Table 2—Comparison of the transverse reinforcement required for both torsion and shear by each code. Conversion: 1 in^2 = 645.15 mm^2 and 1 in = 25.4 mm

Code	Required transverse reinforcement for torsion and shear (in^2)	Transverse reinforcement provided for both torsion and shear	ρ_w (%)
ACI 318-19	1.16 @ 11 in	2#5 @ 11 in	0.271
CSA-A23.3-04	1.40 @ 13 in	2#5 @ 13 in	0.275
AASHTO-LRFD-17	1.13 @ 7.5 in	2#5 @ 7.5 in	0.386
EN 1992-1-1:2004	0.86 @ 8 in	2#5 @ 8	0.276

A point of discussion is the angle of the compressive field obtained either by the direct (50°) or iterative method (36.4°) using the AASHTO-LRFD-2017[2] code. Either method should give a similar inclination for the compressive stress field, nevertheless, for the presented example, different angles were found. One of the possible causes of this variation is the amount of longitudinal reinforcement for hogging moment. Moreover, the longitudinal reinforcement for hogging moment also causes that the angle of the compressive field found from CSA-A23.3-04[3] guidelines (43.725°) differ from the AASHTO-LRFD-2017[2] code, even though both codes are based on the same theory (MCFT) and follow the same principles for finding the inclination of the compressive field.

The ratio of the required longitudinal reinforcement in CSA-A23.3-04[3] and AASHTO-LRFD-2017[2] is smaller compared to the ACI 318-19[1] and EN 1992-1-1:2004[4] codes. Both CSA-A23.3-04[3] and AASHTO-LRFD-2017[2] codes consider the compressive torsional and the aggregate interlock contribution to the torsional strength. On the other hand, ACI 318-19[1] and EN 1992-1-1:2004[4] contemplate that the torsional stresses are carried only by the longitudinal and transverse reinforcement. The provisions based on the MCFT (CSA-A23.3-04[3] and AASHTO-LRFD-2017[2]) require more computational time and effort than those based on a 3D-truss and thin-walled tube analogy (ACI 318-19[1] and EN 1992-1-1:2004[4]), but result in a more economic solution.

DISCUSSION

The factored applied torsional moment obtained in this example = 687 kip·ft (931 kN·m) is very similar compared to the computed value in the TxDOT, LRFD Inverted Tee Bent Cap Design Example[5] = 660 kip·ft (895 kN·m). The small difference might be caused by the method used to get the loads at the inverted tee bent cap. In this design example, a full 3D model was developed to obtain the torsional moment, the TxDOT example used live load distribution factors to compute it.

It is important to remark that the sagging and hogging design bending moment computed in this design example are the bending moments acting concurrently with the maximum torsional moment. The maximum sagging and hogging moment occur at a different station. Therefore, the required longitudinal reinforcement needs to be evaluated and designed at several locations, where the torsional moment may not be critical.

As it was mentioned, the critical torsional moment occurs from the 26 ft (7.94 m) up to the 28 ft (8.56 m) station from the left tip of the inverted tee ben cap shown in Figure 2. This short length and the proximity to the support will prevent the development of Saint-Venant torsion. In this case, warping torsion will arise. However, the uncertainty of only experiencing warping torsion between the 26 ft (7.94 m) up to the 28 ft (8.56 m) station is very high. A conservative and safe design following sectional analysis will take this critical torsional moment as the design load. Additionally, there are no provisions in the ACI 318-19[1] building code that follow a sectional analysis design considering warping torsion. Moreover, the point loads are applied at the ledges. This concentrated loads plus the small cross-section of each ledge yield a disturbed region (beam theory is not applicable) at the flange of the inverted tee bent cap. The hanger reinforcement is not intended to contribute to the capacity of the analyzed cross-section. However, it will help to lift up the reinforcement at the disturbed (flange) region. As a side note, there is no provision in the ACI 318-19[1] building code to compute the hanger reinforcement. A strut-and-tie design approach is recommended to correctly assess the behavior at the flange of the inverted tee bent cap.

LIST OF NOTATIONS

b = width of the cross-section under compression,
b_w = web width,
b_{w2} =width of the supported beam,
d = effective depth,
f'_c = specified compressive strength of concrete,
f_{se} = effective stress in prestressing reinforcement, after allowance for all prestress losses,
f_{pu} = specified tensile strength of prestressing reinforcement,
f_y = specified yield strength for non-prestressed longitudinal reinforcement,
f_{yt} = specified yield strength of transverse reinforcement,
h_b = vertical distance from the bottom of the supporting beam to the bottom of the supported beam,
h_1 = overall depth of the supporting beam,
h_2 = overall depth of the supported beam,
p_{cp} = outside perimeter of concrete cross-section,
p_h = perimeter of the centerline of outermost closed transverse torsion reinforcement,
s = center-to-center spacing of stirrups,
A_{cp} = area enclosed by the outside perimeter of concrete cross-section,
A_g = gross area of cross-section,
A_h = hanger transverse reinforcement,
A_l = required area of longitudinal reinforcement to resist torsion,
$A_{l,\min}$ = minimum area of longitudinal reinforcement to resist torsion,
$A_{l,\text{prov}}$ = provided area of longitudinal reinforcement to resist torsion,
A_{ps} = area of prestressed longitudinal tension reinforcement,
A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement, including area holes,
A_s = area of non-prestressed longitudinal tension reinforcement,
$A_{s,prov}^+$ = area of non-prestressed longitudinal reinforcement provided to withstand the factored sagging moment,
$A_{s,prov}^-$ = area of non-prestressed longitudinal reinforcement provided to withstand the factored hogging moment,
A_t = area of one leg of a closed stirrup resisting torsion,
$A_{t,prov}$ = area of one leg of a closed stirrup provided to resist torsion,
A_v = required area of the total legs of a closed stirrup for shear,
$A_{v,eff}$ = total area of the adjacent legs to the sides of the beam of the stirrups provided to resist shear,
$A_{v,prov}$ = provided area of the total legs of a closed stirrup to resist shear,
M_u^+ = factored positive moment considered as the sagging moment,
M_u^- = factored negative moment considered as the hogging moment,
N_u = factored axial force, positive for compression and negative for tension,
T_L = service HL-93 live truck load,
T_n = nominal torsional resistance,
T_R = service nonstructural dead load,
T_{SW} = service structural self-weight dead load,

T_W = service wearing surface dead load,
T_{th} = threshold torsional moment,
T_u = applied factored torsional moment,
V_c = shear strength provided by concrete,
V_u = factored shear force,
$V_{u,L}$ = factored shear force applied at the left ledge,
$V_{u,R}$ = factored shear force applied at the right ledge,
γ_c = unit weight of reinforced concrete,
λ = modification factor which accounts for the properties of lightweight concrete,
ρ_l = longitudinal reinforcement torsion ratio,
ρ_w = transverse reinforcement ratio,
ε_t = strain at longitudinal reinforcement for sagging or hogging moment,
ε_y = yield strain of steel reinforcement,
ϕ = torsional moment and shear force resistance factor,
ϕ_m = bending moment resistance factor,

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