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PREDICATION AND EXTENSIONALIZATION

ABSTRACT. In his 2000 book *Logical Properties* Colin McGinn argues that predicates denote properties rather than sets or individuals. I support the thesis, but show that it is vulnerable to a type-incongruity objection, if properties are (modelled as) functions, unless a device for extensionalizing properties is added. Alternatively, properties may be construed as primitive intensional entities, as in George Bealer. However, I object to Bealer's construal of predication as a primitive operation inputting two primitive entities and outputting a third primitive entity. Instead I recommend we follow Pavel Tichý in construing both predication and extensionalization as instances of the primitive operation of functional application.

1. INTRODUCTION

I am in favour of the semantic thesis that predicates denote properties rather than sets or individuals. However, as it stands, the thesis is open to a type-incongruity objection. The objection is that the property and the individual that the thesis correlates in singular predication are an incongruous union, the former being an intensional entity and the latter an extensional entity.

The objection is a knockdown argument of the simple version of the thesis as just stated, but can be dismantled by a reformed version. The latter comes with a device of extensionalization that inputs properties and outputs sets congruent with individuals. My particular device presupposes that sets are identified with their characteristic functions, such that sets are functions defined on individuals. In this paper I propose to embed the semantic thesis within a particular possible-world framework in which functional application is the logic both of extensionalization and predication.

I make no attempt to provide a profound philosophical explanation of the concept of predication. The task I set out to solve is restricted to devising a workable formal semantics that will allow predicates to denote properties.

In sum, the position I am going to defend boils down to this. I agree with the intensionalist semanticist that predicates denote properties, and with the extensionalist semanticist that predication relates sets (not properties) and individuals.[1]

First I present Colin McGinn's recent advocacy of the thesis that predicates denote properties, adding an argument in its favour, and I set out the type-incongruity objection. Then I show how to technically overcome the objection. Finally I consider George Bealer's intensional logic in which properties are predicated of individuals without prior extensionalization. I object that by construing predication as a primitive operation defined over two primitive entities, Bealer's semantics is multiplying primitive operations and primitive entities too freely. I consider it methodologically superior to handle predication, or any other operation, as an instance of a more general, primitive operation, in this case functional application.

2. MCGINN ON PREDICATION

Colin McGinn devotes a chapter of his (2000) to predication.[2] There he espouses the semantic thesis that predicates denote properties. Call this thesis 'the Thesis'. Thus, in McGinn's chosen sentence, "Russell is bald", two entities receive reference, Russell the man and baldness the property. The predicate 'is bald' neither denotes, plurally, each and every bald individual nor, singularly, the set of all bald individuals. His general reason is that the extensions of the properties constitute no additional semantic level alongside the properties themselves: "Extensions will no longer be in the picture" (ibid., p. 63), he says, and continues:

> A predicate refers to a property with many instances; a name refers to an object with many properties: that is all. The meaning of each category of terms stops at its ordinary reference without reaching out further into the non-semantic world of property instantiation. Extensions of both kinds are fixed by the facts of the world, not by the meaning of the terms. They are extra-semantic items. (Ibid., pp. 65–6.)
> Predicates are also rigid designators for me, as they cannot be if taken to designate their extensions, since these vary from world to world [and from time to time, cf. p. 59]. I say that 'red' designates the property of redness in every possible world, as 'Bertrand Russell' designates Bertrand Russell in every possible world. Here again names and predicates are semantically analogous. (Ibid., p. 67, n. 11.)

I am strongly sympathetic to McGinn's claim that predicates denote properties. Unfortunately, McGinn provides little by way of argument in its favour. What he offers amounts to the claims that the Thesis "meshes naturally with speakers' understanding" and that "we know antecedently that names denote objects and predicates denote properties" (ibid., p. 57, pp. 58–9, resp.). I think a cogent argument can be offered in favour of the

Thesis. Pavel Tichý presents, in a number of places, a modal argument designed to show that, to take McGinn's example, only if 'is bald' denotes the property of being bald can the contingency of "Russell is bald" (or rather of the proposition it denotes) be guaranteed.

The argument runs in outline as follows. If it is true that Russell is bald, then it might have been false. If it is false that Russell is bald, then it might have been true. That is, it is a contingent truth or falsehood that Russell is bald. The semantics in terms of which we analyze "Russell is bald" cannot confine itself entirely to set membership, for the following is trivially true and trivially false, respectively: $a \in \{a, ...\}$, $a \notin \{b, c, d\}$, and $a \in \{b, c, d\}$, $a \notin \{a...\}$. Let $\{a, b, c\}$ be the set of all and only those individuals who are actually (and presently) bald. Then consider a world (and a time) at which a is not bald:

> In such a world (as in any world) it is true that a is a member of $\{a, b, c\}$, yet "Russell is bald" is false. Now surely a sentence cannot be false in a state of affairs where what it says is the case. Consequently, what "Russell is bald" says cannot be to the effect that a is a member of $\{a, b, c\}$ (or any other class). (Tichý 1975, p. 83.)[3]

Hence, modal (and arguably also temporal) variability must be built into the semantics. Non-triviality, or contingency, can be restored if 'is bald' does not designate a set, but designates instead a property with *different* sets as its extensions.[4] Tichý follows the prevailing way of doing so by modelling a property as a function from possible worlds (and instants of time) to sets (of individuals) rather than by treating properties as primitive. The truth-value that the sentence (proposition) takes is functionally dependent on the indices at which it is evaluated. The upside is that since properties are sets-in-intension, the predicate 'is bald' does not single out a set, but instead something that 'presents' sets.

In what follows I shall make two assumptions. One is that McGinn and Tichý are right about predicates denoting properties; i.e., that the Thesis is true. The other is that properties of individuals are (modelled as) functions from worlds and times to sets of individuals. The latter assumption brings out a serious downside of the Thesis: the denotation of a predicate is not directly attributable to the denotation of a singular term denoting an individual. This is serious, because the Thesis would simply render predication impossible. To see why the Thesis, without appropriate theoretical embedding, renders predication impossible, consider the general form of the truth-condition that McGinn assigns to "Russell is bald", or any other sentence in which a property is predicated of an individual. The truth-condition is that individual a has property P, and

McGinn casts it as the ordered pair $<a, P>$ (ibid., p. 63). The problem is: how are a, P to correlate with one another in such a way that P is predicated of a? It would seem that they just cannot. Put syntactically, "the result of juxtaposing an [intensional] abstract and an individual constant does not form a well-formed expression." (Bealer and Mönnich 1984, p. 237.) Analogously, the concatenation of the two English words 'Russell', 'bald' in "Russell bald" does not constitute a sentence of English. Put objectually, the intensionalist Thesis lacks what its foremost extensionalist rival has, namely the compatibility between the two extensional entities a, {. . .} thanks to the relation $\in$ of set membership. Thus, "$a \in$ {. . .}" is a well-formed expression of the syntax of set theory. However, no intensional counterpart of $\in$ is available that would make feasible the predication of P of a, on the assumption made above that P is a function from worlds and times to sets. The intensional entity P cannot take the extensional entity a as an argument. It is probably telling, then, that McGinn in fact sidesteps the issue of how P is to be predicated of a by simply offering the two-membered sequence $<a, P>$ lacking a third member to trigger the predication of P of a. Mere sequences of individuals and properties are incapable of "setting up a great chain of interlocking objects and properties" (ibid., p. 63).

Fortunately, there is a way out of this impasse. What we need to do in order to uphold the Thesis is to embed it within a theory of predication that allows the denotation of a predicate to be *indirectly* attributable to the denotation of a singular term. The particular theory of predication I am going to employ derives from Tichý's 'neo-Fregean' formal semantics called Transparent Intensional Logic.[5]

Below I shall show how to remove the technical obstacle of the missing mediation that blocks the implementation of the Thesis. The obstacle may show up in different semantic frameworks. Here I am going to demonstrate how the obstacle arises in a particular version of possible-world semantics and how it can be overcome within the same framework. The solution consists, not surprisingly, in juxtaposing an individual and a set of individuals for compatibility. However, it is crucial to my solution that the set in question is an extension of the property to be predicated of the individual. That is, the set must be obtained by *extensionalizing* the property.

Let me emphasise the generality of the problem before proceeding. Nothing hinges on the particular *implementation* of extensionalization in a typed possible-world framework. The implementation serves simply to demonstrate one way of accommodating extensionalization. That is, you may reject the particular account I give of predication and extensionalization within my typed possible-world framework, but should you accept

the Thesis I fail to see how you could afford to reject some version or other of extensionalization. For the Thesis presupposes a demarcation between properties and sets (or, between sets-in-intension and sets-in-extension), and the Thesis creates the problem of how to descend from property to set. (As we shall see in the last section, extensionalization may also crop up in connection with stating the truth-condition that such-and-such an individual is an element of an extension of this or that property.) The demarcation between properties and sets must be formally cast at least partly in terms of a difference in their degree of individuation, and properties must be finer than sets. Just how much more finely properties should be individuated than sets is irrelevant to the issue of extensionalization. However, the notion of property I am deploying is, as already mentioned, the one of possible-world semantics, whose intensional entities are extensionally individuated ('f' for 'function'):

$$\forall ff^*(\forall wt(f(w,t) = f^*(w,t)) \rightarrow f = f^*).$$

In that framework properties are also set-theoretic entities. *Qua* function a property is a set of ordered *n*-tuples where the *n*-1 tuples are arguments and the *n*-th members are values. Hence, the contrast between set and property that I am operating with is essentially a contrast between (i) the *actual/present* extension of baldness, say, and (ii) the *possible/past, present and future* extensions of baldness.

In this paper I restrict my treatment to McGinn's chosen sentence, "Russell is bald", which is an instance of singular predication. I do not discuss whether generic predication like "The raven is black" (i.e., "All ravens are black") also requires extensionalization and, if so, which particular form it might take.

3. Simple Type Theory

The framework within which I present my solution comes with a simple type theory of basic and functional types. If you are hostile to type theories, just think of this one as a heuristic device facilitating the account of predication. And if you prefer linguistic types to my objectual types, feel free to reconstrue them accordingly.

DEFINITION (*simple type over the ontological base OB*). Let *OB* be an ontological base, i.e., a collection of pair-wise disjoint, non-empty sets.

1. Each member of *OB* is a *simple type over OB*.
2. If α, $\beta_1,...,\beta_n$ are arbitrary *simple types over OB*, then the set of all (total and properly partial) functions $(\beta_1 \times ... \times \beta_n \rightarrow \alpha)$, denoted $(\alpha\ \beta_1 ... \beta_n)$, is a *simple type over OB*.

3. Nothing else is a *simple type over OB*.   □

What ought the elements of the ontological base to be? For our present purposes, we need to stretch out a bit, allowing the base to span four different basic types. These are as follows.

o {T, ⊥}
ι Individuals
τ Instants of time
ω Possible worlds

That we need truth-values and individuals goes without saying. Worlds and times are required by intensions, as these are defined in possible-world semantics. Intensional entities such as properties are (modelled as) functions defined over worlds and times in order to simulate the modal and temporal variability pertaining to which extension an intension returns at a given system of indices.[6] A property of individuals is (modelled as) a function of type $(\omega \rightarrow (\tau \rightarrow (o\iota)))$. Abbreviate this notation as '$(o\iota)_{\tau\omega}$'. This is a function from the type of possible worlds to a function from the type of times to a function from the type of individuals to the type of truth-values.[7] A *property* is *empirical* if the function is non-constant. An *empirical predicate* is one that denotes an empirical property. I use De Bruijn's notation 'A : α' to indicate that the object A is of type α (or 'is an α-object', for short).

Notice that when an $(o\iota)_{\tau\omega}$-entity is applied to ω- and τ-entities, the resulting entity is of type (o\iota). Such extensional entities I call *sets*. It is essential to my solution that sets are not treated as primitive, but instead construed as functions, so that we can feed the notion of set into the logic of functions.[8] A set is a function that takes all and only its members to True and non-members to False. Such a construal will be familiar from Frege, whose concepts (*Begriffe*) are also characteristic functions. (See also McGinn, ibid., p. 67.) Functions, on the other hand, figure as primitive entities in my framework.

Let us apply this framework to "Russell is bald". Suppose we assign the types $(o\iota)_{\tau\omega}$ to baldness and ι to Russell. Then the technical problem arises how baldness and Russell must be arranged for baldness to be predicated of Russell.

As already mentioned, what we should *not* do is attempt to apply baldness directly to Russell. For then the incongruity objection wins the day. Instead, baldness demands to be applied to a world and then to a time before it can be predicated of Russell. But then it is not baldness, of

type $(o\iota)_{\tau\omega}$, that gets applied to Russell, but an *extension* of baldness. Only when arriving at an extensional entity of type (oι) have we arrived at the right sort of thing to apply to Russell.

But now, McGinn's whole point is that in attributing baldness to Russell, two entities are involved: Russell and a *property*. Yet here I am talking about a *set*. Isn't my attempt to add a missing piece to McGinn's project in fact taking it into the direction he just objected to? Despite immediate appearances, the answer is No. The reason is this: *empirical properties can be predicated of individuals only relative to worlds and times*. Hence, in formal terms, *predicating* baldness of Russell can be nothing other than applying to Russell the extension that the world- and time-extensionalised property of baldness returns at a given world and time of evaluation.

The proceedings can be explained type-theoretically. After $(o\iota)_{\tau\omega}$ has been applied to w of type ω, $(o\iota)_{\tau}$ demands to be applied to t of type τ. Only then does an entity of the appropriate type emerge, viz., a set of individuals of type (oι). It comes in handy now that sets are functions in our framework. We execute our third functional application by applying the set to an individual. The relevant individual is the referent of '*a*'. The result of the application is an o-object, a truth-value. The truth-value is T, if the individual is a member of the set, and ⊥, if not.

Thus, the solution to the incongruity problem is: extensionalization of P through functional application of P to w and then of this result to t to yield a set, of which a either is or is not an element.

A word on method. I am using a Church-Fregean function/argument logic. The philosophical as well as logical advantage of a logic based on functions is that it can model interlocking logical structures in terms of functional dependencies. Functional dependencies are modelled by how the value of one function becomes the argument of another function, or how a function applicable to some particular argument is handed down by another function. A logic of functions is erected on the idea that one operation typically presupposes that another operation has already been executed so as to provide something to work with. The functional operations are two in number — *application* and *abstraction* — of which the former 'descends' from a function to a value, while the latter 'ascends' to a function from other entities, including other functions. It is key to the logic I am using that the outcome of the execution of an operation may itself be an operation. Otherwise the machinery would grind to a halt far too soon. My functional approach affects also how I think of language. In particular, I adhere to the Fregean tenet that every sentence contains at least one functor, and I construe predicates as functors.[9] Predicates denote functions whose argument(s) must be picked

out by some other expression(s) of the sentence. In the case of "Russell is bald", 'is bald' is the functor and 'Russell' the argument expression.

4. A FUNCTIONAL FRAMEWORK OF PREDICATION

In this section I set out the logic of my theory of predication by showing how to logically combine individuals and properties, worlds and times.

We begin from below. McGinn gives us P, a, and I add w, t, all four of which I arrange thus:

$$(((Pw)t)a) \qquad (1)$$

The occurrences of w, t are free, and so (1) is identical to this or that truth-*value* in accordance with the specific choice of values for w, t. But McGinn wishes, rightly, to state a truth-*condition*. In general, the semantic task of "Russell is bald" cannot be to pick out a mere truth-value, for truth-values are not suitable for asserting, or communicating the truth or falsehood, that Russell is bald. I consider two imaginable solutions: either we bind w, t or we replace them by constants. The latter first. The obvious constants are 'A', denoting the actual world, and 'N', denoting the present moment, *now*. This gives us

$$(((PA)N)a) \qquad (2)$$

However, (2) is flawed for the same reason as (1). It is still a truth-value rather than a truth-condition. The result of applying P to A is a function from times to sets of individuals; the result of applying this function to N is a set of individuals; and the result of applying this set to a is a truth-value.

How about binding w, t by $\exists$ instead? We get:

$$\exists w(\exists t(((Pw)t)a)) \qquad (3)$$

(3) interprets "a is P" as saying that it is *possible* that a should be a P. But the assertoric force of "a is P" is not that a is possibly a P, but that a is actually and presently a P. Thus (3) underdetermines the assertoric force of "a is P". Furthermore, if we are in S5 and if the concession is made that a might (not) have been a P, although a is (not) a P, then (3) states a *logical* truth and not a contingent truth or falsehood. The concession is minimal, if a is Russell and P is baldness. (3) then amounts to nothing other than this logically necessary truth about possibility: possibly, Russell

is bald. (3) also gets the modal profile of "*a* is *P*" wrong by failing to capture its contingency. Thus, ∃-binding *w*, *t* is a non-starter.

But try binding *w*, *t* by λ. The result is:

$$\lambda w \lambda t(((Pw)t)a) \tag{4}$$

(4) is a function from the logical space of possible worlds into a *chronology*, which is a function from instants of time to, in this case, sets of individuals. It is entirely exterior to (4) which particular set might be the actual and present extension of *P*, and whether Russell is a member of it or not. To assert, know or understand that Russell is bald is not to assert, know or understand that Russell is a member of some one particular set {...}. It is instead to assert, know or understand that Russell is a member of whatever set satisfies the condition of being the extension of *P* at the particular world and time at which the assertion is made. The abstraction over worlds and times is a formal way of rendering what competent human language-users know, namely that the property of being a *P*-entity is to be applied to *any* (as opposed to *all*, *some* or the *actual/ present*) *w, t* and that *a* is to be tested for membership of the extension of *P* at *w*, *t*. There is no requirement that we should first identify the actual world and the present moment before checking *a* for *P*-hood.[10]

Thanks to the modal and temporal abstraction, it remains an epistemologically open question which set of individuals is the extension of *P* at the actual world and the present moment, and this explains why the actual and present truth-value of (4) needs to be ascertained by empirical, *a posteriori* means. After all, the extensionalist tenet that predicates correlate sets and individuals is embedded within an intensionalist framework that holds that the relevant sets enter the picture only in their capacity as extensions of properties. This is why the predication of *P* of *a* is not merely a matter of set membership: (*P**a) or *a*∈*P**, *P** : (οι), ∈ : (οι(οι)).

But if (4) disentangles the semantics of "*a* is *P*" from the actual world and the present moment, how is (4) anchored to the actual world and the present moment, which the assertion of "*a* is *P*" is obviously an assertion about? By means of pragmatics rather than semantics. When we *assert* a sentence expressing or denoting (4), our assertion is to the effect that the actual world and the present moment are among the worlds and times at which *a* is a member of the respective extensions of *P*. Once the assertion has been made, it is up to extra-semantic, extra-pragmatic, empirical inquiry to determine whether the assertion was a hit or a miss. This empirical test can be executed in *any* world *w* at *any* time *t*; hence the abstraction over world and time variables.

I wish to put (4) forward as the answer to the question how baldness is to be predicated of Russell. Since (4) is an empirical proposition, its type is $o_{\tau\omega}$: a function from worlds to a function from times to truth-values. Treating intensions as binary functions for expository simplicity, (4) can be viewed as a set of world/time couples, whose membership condition is the following, $\in^* : (o(\tau\omega)(o_{\tau\omega}))$:

$$\langle w, t \rangle \in {}^*\lambda\, w\lambda\, t(((Pw)t)a) \leftrightarrow a \in P\langle w, t\rangle.$$

The truth-condition (4) that a is a P-entity is satisfied by any world/time couple $<w,t>$ at which a is in the extension of P.

The abstraction over worlds and times abstracts from the particular truth-value obtained at the particular values assigned to w, t and yields instead a function from all the values of w, t into $\{T, \perp\}$. Let me tighten up the notation a bit. Rewrite (4) as (5):

$$\lambda w\lambda t(P_{wt}a) \tag{5}$$

Then, more importantly, the use of round brackets '(,)' fails to make it unambiguous that I am using brackets as more than scope indicators. I am also using them to represent the basic operation of functional application.[11] Read '$[P_{wt}\, a]$' as the result of three consecutive executions of functional application, and rewrite (5) as (6), which is the final form:

$$\lambda w\lambda t[P_{wt}a] \tag{6}$$

The truth-condition encapsulated by (6) is verbally in agreement with the truth-condition McGinn states for simple sentences involving singular predication:

The truth-conditions of simple subject-predicate sentences are given [as follows]: a sentence of the form 'Pa' is true if and only if the object referred to by the name has the property referred to by the predicate. (2000, p. 53.)[12]

However, above we saw McGinn offer $<a, P>$ as the logical form of how "the object referred to by the name has the property referred to by the predicate". I offer $\lambda w\lambda t\, [P_{wt}\, a]$ as a rival to $<a, P>$. I already objected that $<a, P>$ fails to indicate how P is to be predicated of a. This lacuna leaves an open flank in McGinn. But, McGinn might argue, once this has been

taken care of, his theory of predication can do without introducing the complicating factor of extensionalization. This he would be in a position to argue, because it is compatible with his thesis that predicates denote properties to construe properties as *primitive* rather than as functions. This option is admittedly a tempting one. For one thing, it renders the w, t indices superfluous and reduces the number of logical steps in one go. No less importantly, the construal takes the wind out of the sails of the mismatch objection, for one simply defines those primitive properties as being directly applicable to individuals. But, as I try to show in the next section, construing properties as primitive sheds little light on the logic of predication, which is why I recommend against the construal.

An alternative to construing properties as primitive would be to construe them as *propositional functions*. If properties are (modelled as) functions from worlds to functions from times to sets of entities, as I suggest, then they are equivalent to, e.g., functions from entities to propositions, where propositions are functions from worlds to functions from times to truth-values. In the present type-theoretic context, propositional functions defined over individuals would be of type $(\iota \rightarrow o_{\tau\omega})$, i.e. $(o_{\tau\omega} \iota)$. Let the property of baldness be the propositional function of being an x such that x is bald. If this propositional function is applied to Russell as argument, the functional value is the proposition that Russell is bald. Modelling properties as propositional functions is not without its attractions. For they do away with the incongruity problem in one fell swoop. No need to groom the property before predicating it of an individual; just apply the propositional function as is to an individual and obtain a proposition in return. Tempting though it might be to embrace propositional functions as properties, there are two reasons for resisting the lure of this construal, one general and the other more specific.

The general reason is the concern to maintain the uniformity of the system of intensions. Properties would no longer be intensional entities defined on possible worlds, unlike all the other intensions. Instead they would be defined on, for instance, individuals. This sort of argument fails, of course, to impress anyone who rejects that intensions are functions from logical space, but ought to strike everyone as being *ad hoc*: why would properties have a wholly different type of argument than all the other intensional entities? Surely, if we are able to maintain a principled, unified, general theory of intensions then we ought *a fortiori* to do so. Linked with this top–down argument is one owing to Bealer, which may be summarised as, "So properties are propositional functions? But then what are propositions?" In particular, are propositions what Bealer calls

coarse-grained (extensionally individuated) or fine-grained (hyperinten-sionally individuated) 0-ary intensions? In his own words,

> How is one to develop a theory of the *other* type of intension? This job will require some new kind of logical machinery, machinery not used in the original propositional-function approach? ... This new logical machinery is likely to be very much like that used in the algebraic approach [Bealer's own] to intensional entities, which is the main competitor to the propositional-function approach. If so, what is gained by not using an algebraic approach to both types of intension [property, proposition] from the start?" (Bealer 1989, p. 10.)

I agree wholeheartedly, except that the 'new logical machinery' is just as likely to be that offered by the rival possible-world approach.

Interestingly, though, the propositional-function approach could, in fact, level an argument from theoretic uniformity against me. A few remarks to set the stage. In general, a property is predicated of something. But something may also be predicated of a property. For instance, the property of being attractive may be predicated of the property of being bald. On the assumption that properties are propositional functions, attractiveness must be a propositional function that takes another propositional function to a proposition. Therefore, if baldness, B, is of type $(o_{\tau\omega}\iota)$ then attractiveness, A, must be of type $(o_{\tau\omega}(o_{\tau\omega}\iota))$. If we retain functional application as the logic of predication, the analyses of "Russell is bald", Russell, R, of type ι, and "Baldness is attractive" turn out as follows:

$$[B\,R]$$

$$[A\,B].$$

B remains unaltered in both cases. Not the form of B (i.e., B as opposed to B_{wt}) but only its position in $[X\,Y]$ determines whether B occurs as subject or object of predication. So the propositional-function approach offers a uniform account of attribution of properties to individuals and properties, whereas a theory such as Transparent Intensional Logic needs to extensionalise B in the case of "Russell is bald". This is admittedly a point in favour of the propositional-function approach. But the simplicity of the logical forms of the two sentences above is detrimental to their ability to capture not only modal but also temporal modalities, as well as the interplay between the two. For an example, consider the non-equivalent sentences, "Frequently, my neighbour is sick" and "My neighbour is frequently sick" as found in (Tichý 1986, pp. 161–63,

propositions L^2 and L^4, respectively). Their respective analyses require explicit (λ-bound) w and t variables, which are nowhere to be found in [B R], [A B].

The specific reason is that it is not entirely obvious how contingency is supposed to be captured. For a concrete example, consider Aczel (1980). Aczel's proposition that the propositional function f is true of a ought not to reduce to a being a member of the *set* $\lambda x f(x)$, as in $a \in \lambda x f(x)$ (ibid., p. 31). For then the proposition $f(a)$ is insufficient for the purposes of modelling contingently satisfied truth-conditions; cf. Tichý's modal argument.[13]

So perhaps we still ought to consider inserting occurrences of w, t; only where? Consider the β-reduced form $f(a)$ of $(\lambda x f(x), a)$. Will $\lambda w \lambda t$ $(f_{wt}(a))$ do? Will $\lambda w \lambda t(f(a))_{wt}$? Neither, in case propositions are of type $o_{\tau\omega}$. The stumbling block in the first case is that f can be extensionalised only after having been applied to a (where I am assuming that a is of a type appropriate for f). Since f_{wt} is ill-typed, $\lambda w \lambda t(f_{wt}(a))$ is no option.

The second suggestion fares ostensibly better. If a is of type ι and f of type $(o_{\omega\tau}\iota)$ then f applied to a yields an $o_{\omega\tau}$-object. Technically, $\lambda w \lambda t(f(a))_{wt}$ works. But as far as philosophical analysis goes, it is somewhat peculiar. The problem is that the addition of w, t is gratuitous: $f(a)$ is already a proposition, so what is the point of $\lambda w \lambda t(f(a))_{wt}$? Well, it might be rejoined, the point is that the latter makes w, t explicit while the former fails to. But this merely goes to show, in my view, that an entity of type $(o_{\omega\tau}\iota)$ such as f is out of place in a framework that comes with explicit w, t variables. Which is to say that propositional functions are, at the very least, at odds with an intensional type theory whose propositions are of type $o_{\tau\omega}$. On the other hand, propositions of this type and properties of type $(o\iota)_{\tau\omega}$ walk hand in hand, as soon as we avail ourselves of a vehicle of extensionalization.

To summarise. The predicate 'is bald' denotes the property of being bald. The predicate picks out an intensional entity that must undergo extensionalization to render it applicable to individuals so that baldness may be predicated of individuals. Extensionalisation takes the form of the logical operation of functional application. The logical form of the sentence "Russell is bald" contains three occurrences of functional application altogether.

[1] The application of Baldness : $(o\iota)_{\tau\omega}$ to w : ω to obtain $(o\iota)_{\tau}$, a chronology which inputs instants of time and outputs the respective sets of bald people at those particular times.

[2] The application of the chronology obtained in [1] to t : τ to obtain a set of individuals : $(o\iota)$.

[3] The application of the set obtained in [2] to Russell : ι to obtain a
truth-value : o.

The truth-value obtained in [3] is relativised to worlds and times by
means of two instances of λ-abstraction to obtain a proposition. The third
functional application, [3], is the predication of Baldness of Russell. The
availability of a set for the operation of predication is functionally
dependent on a property having undergone extensionalization in the two
preceding steps. Steps [1], [2] can be rolled into one step, if we either
eliminate one of the indices or roll worlds and times into pairs, as in
Montague. Conversely, it is also an option to add a third (fourth, ...)
index, which will also require extensionalization. Even a pruned-down
logic of predication must, however, contain two steps: first extensional-
ization, then predication.

5. BEALER ON PREDICATION

Here I set out George Bealer's rival intensionalist conceptions of
predication and extensionalization and compare them with Tichý's. My
outline is based on Bealer (1979), (1993), and Bealer and Mönnich
(1984). First I set out the relevant fragment of his formal semantics, and
then I level a philosophical-methodological objection against it.

Bealer provides his formal analysis of the predication inherent in a
sentence like, "Russell is bald" within the framework of an intensional
algebraic model $M = <D, K, \tau>$. D is the union of denumerably many
disjoint subdomains, such that D_0 is the subdomain of propositions and
D_1 the subdomain of properties (the two kinds of intensional entities we
need here), while D_{-1} is the subdomain of individuals. K is a set of
extensionalization functions.[14] The semantics of the extensionalization
functions $\partial \in K$ is such that they assign the following possible extensions
to individuals, propositions and properties:

- $x \in D_{-1} \rightarrow \partial(x) = x$
- $x \in D_0 \rightarrow \partial(x) = n$ for $n \in \{0, 1\}$
- $x \in D_1 \rightarrow \partial(x) \subseteq D$.

τ is a set of truth-functional connectives and other operations. The set
includes, *inter alia*, the operation pred_s of singular predication, which is,
in the present case, defined as $D_1 \times D_{-1} \rightarrow D_0$. The semantics of pred_s is the
following, if the quantificational range of y is restricted to D_{-1}:

$$\forall x \in D_1 \, \forall y \in D_{-1} \, \forall \partial \in K(\partial(\text{pred}_s \langle x, y \rangle) = 1 \leftrightarrow y \in \partial(x)).$$

Finally, an interpretation function I assigns a value to the individual constant 'a', $I('a') = a \in D_{-1}$, and to the predicate 'P', $I('P') = P \in D_1$.

Singular predication of unary predicates satisfies the following truth-condition:

$$\partial\big(\mathrm{pred}_s\langle P^1, a_1\rangle\big) = 1 \leftrightarrow a_1 \in \partial\big(P^1\big).$$

The extension of the proposition $\mathrm{pred}_s{<}P^1, a_1{>}$ is identical to the truth-value 1 iff the individual a_1 is an element of the extension of the property P^1. That is, the result of predicating a property of an individual is a proposition that is true iff the individual is in the extension of the property.

Assume that the interpretation function I has assigned the unary property Baldness$\in D_1$ to 'B' and Russell$\in D_{-1}$ to 'r'. Then the formal semantics of the predication of baldness of Russell is now straightforward.

$$\partial(\mathrm{pred}_s {<} B, r {>}) = 1 \leftrightarrow r \in \partial(B) \tag{7}$$

That is, the proposition that Russell is bald is true iff Russell is in the extension of the property of Baldness that the semantic interpretation has assigned to 'B'.

One syntactic difference between Bealer's (7) and my (6) immediately stands out. Whereas I write "...P_{wt}..." to indicate the extensionalization of P, Bealer does not write "...$\partial(B)$ ... " to signal an extensionalization of B. Bealer uses property-to-set extensionalization only when stating the set-membership condition and not also prior to predicating B of r. In (7) pred_s is a binary function that inputs an intensional and an extensional entity and outputs a new intensional entity. In fact, pred_s *must* be binary, since the property is not itself an operation: it is a functional argument that is not itself a function.[15] The reason why properties are logically inert is because Bealer construes properties, relations, and propositions—lumped together as so-called PRP's—as *primitive*, irreducibly intensional objects.[16] In general, that an entity e or an operation o is primitive relative to a system s is to say that e or o is not explicated within s. Instead e or o is used to explicate or define other entities or operations within s. The system lays down how e or o behaves technically, but any understanding of what e or o is in the first place must be obtained outside s.

Bealer's system is his structure M. Since the PRP's are primitive within M, Bealer needs to add to his logic an operation $\mathrm{pred}_s \in \tau$ and

earmark it specifically for singular predication.[17] But also pred_s is primitive by not being an instance of a more general operation that is either defined or explicated within M. Similarly, the extensionalization functions are also introduced into M as primitive operations rather than being instances of a more general operation. The operation of extensionalization is not needed to prevent incongruity, since $x \in D_1$ is immediately compatible with $y \in D_{-1}$. But the operation is primitive relative to M, and must be so, because the framework is designed to lack indices to figure as input for extensionalization. That PRP's are primitive is probably *the* leading philosophical idea informing Bealer's intensional algebra. For without this idea, his intensional logic could not treat PRP's as individuals, and the logic would fail to qualify as first-order. The price exacted for setting up a first-order intensional logic, on the other hand, is an abundance of primitive operations.

Consequently, (7) involves the primitive operations ∂, pred_s and the primitive entities B, r. Bealer's formal semantics of predication comes down to how some primitive operations operate on some primitive entities to generate a new primitive entity. The theory cannot tell us what predication and extensionalization are. Neither can it tell us what a property or an individual is. Nor is M designed to do any of this. It is all something we are supposed to understand pre-theoretically, or intuitively.

Since both extensionalization and predication are primitive, we cannot study the functional dependencies obtaining among these operations and the property, the individual, and the proposition. In particular, Bealer is in no position to say that predication is the application of a property to an individual, the value of which application is a proposition. Predication is instead a matter of applying the operation of predication to <property, individual> to obtain a proposition. But how does a property get predicated directly of an individual? Bealer's semantics tells us *what* pred_s does, by providing a truth-condition for the proposition that emerges from the predication. But to understand its truth-condition we must understand what extensionalization is; otherwise '$r \in \partial(B)$' will be meaningless to us. M tells us (by means of recursion, of which I reproduced a fragment above) what extensionalization does. But again, '$\partial(B)$' simply records the fact *that* B has been extensionalised: it does not tell us *how*. That is, the backtracking stops at ∂ and pred_s. If we do not already understand, pre-theoretically, what extensionalization is, (7) is not an informative analysis of "Russell is bald" or of any other instance of singular predication. And if we do not understand, pre-theoretically, what predication is, we shall not understand the logical operation pred_s.

To see why Bealer's choice of primitives for his formal system may pose a philosophical-methodological problem, consider Bealer's view of the connection between intuition and formal rules:

[O]ur intuitive grasp of these operations [including predication and extensionalization—*BJ*] can be codified by means of appropriate elementary rules. (1993, p. 24.)

However, it would appear to be open to doubt whether we actually have enough of a firm intuitive grasp of such 'techno-logical' notions as predication and extensionalization as to suppose they can be introduced as primitive operations into a system of formal semantics. This is my general philosophical-methodological objection to Bealer's theory of predication. And even if we did understand, pre-theoretically, what predication and extensionalization are, then since Bealer's PRP's are not operations or conditions, pred_s and ∂ need to be added as separate, primitive operations. But then they cannot be subsumed under one overarching primitive operation. However, I believe the following methodological guideline has something to be said for it: the fewer primitive notions (including operations) a theory is furnished with, the better. For then the formal theory is able to presuppose fewer notions (operations) be understood pre-theoretically and can instead elucidate a higher number of notions (operations). This way there is less of a risk of taxing our pre-theoretical intuitions beyond capacity. This is my foremost reason for holding that predication and extensionalization are not two functions, but two cases of functional application. Therefore I also hold that we are better off with properties as functions rather than as functional arguments only.

If, *contra* Bealer, we treat properties and sets as functions, we can study how we form a new entity, e.g., a truth-value or a truth-condition, by means of the operation of functional application. The philosophy and logic of how the descent from intensions to extensions and the ascent (via functional abstraction) from extensions to intensions work then become internal to the framework. But, on pain of infinite regress, at least one operation must be primitive relative to a framework, so that other operations may be explicated or defined in terms of it. I already introduced functional application and abstraction as the two primitive operations of my framework. I am thus able to explain predication and extensionalization as instances of application.

As is seen, I rest my case on two key premises. One is that it is philosophically illuminating and technically rewarding to introduce application as a primitive operation. The other is that we do understand

that by applying the extension of a property to an entity we are *predicating* the property of the entity, rather than executing any other operation that is also reducible to functional application.

It might be objected that by couching the logic of predication in terms of functional application I am simply pushing my account of predication up one level, because we are now supposed instead to understand application pre-theoretically. The point is valid, but let me offer the following by way of justification of my preference for this particular operation as basic. One reason is the methodological one set out already: if two operations can be subsumed under one primitive operation, rather than construing both as primitive, then I suggest we go for the former option. Another is that we arguably do possess a fairly firm grasp of the idea underlying functional application. For the basic idea is simply a matter of inputting something and outputting something else. Elementary arithmetic, for instance, would be beyond us if we did not grasp the simple idea of feeding 7 and 5 into a 'machine of addition' and retrieving 12. Similarly, I suggest, our grasp of predication is such that we do understand that if we take a property and an entity and feed them into a 'machine of predication' enabling us to say something *about* something else we retrieve a Yes or else a No, according as the entity has the property or not. Such an input/output 'machine of predication' is what went into symbols as '$[P_{wt}\,a]$'.

It is little surprise, then, that these two reasons converge in a preference for a functional approach underpinned by the lambda-calculus, which needs functional application and functional abstraction as its only two primitive operations.

6. Conclusion

The task of this paper was to devise a theory of predication that would allow predicates to denote properties, subject to the constraints that neither properties nor the operation of predication be primitive. To this end I have argued the following. An operation of descending to *extensions* is what is required in order to uphold the *intensionalist* thesis that empirical predicates denote properties rather than sets or individuals. Without prior extensionalization any predication of a property of an individual is vulnerable to a category-mistake objection, unless properties are introduced as primitive entities immediately compatible with individuals. The solution I propose uses the operation of functional application to account both for how the extensionalization of a property and the predication of a property of an individual work within a formal semantics that construes properties as functions rather than as functional

arguments only. The construal avoids casting predication as a primitive operation defined over two primitive entities. My objections to this approach are that it introduces too many primitive operations and entities, and that it taxes our intuitive understanding of them.[18]

NOTES

[1] It might seem tempting to cut the Gordian knot of what predicates denote by denying that they denote in the first place. But I would recommend against such a move. Suppose we wish to determine the denotation of a compound expression, in which there is at least one occurrence of a non-denoting predicate. Then the principle of compositionality entails that the compound lacks a denotation, too. However, I for one would have to be convinced that, say, "Mary is happy" fails to denote. Alternatively, one might relinquish compositionality and still have "Mary is happy" denote, while 'is happy' does not. But in my view compositionality is a condition sine quā non for any formal semantics.

[2] Predication is not among the issues discussed in McGinn (2003), which contains his replies to three discussions of other themes raised in McGinn (2000). Nor does the chapter on predication receive attention in MacFarlane (2002).

[3] Example adapted from, "Venus is a planet". See also Tichý (1980), (1988).

[4] The Thesis turns predicates into Kripke-style rigid designators. This is only appropriate, though, since properties are world-invariant: baldness is the same property in all worlds (and at all times), whereas it has different extensions. Both McGinn (ibid., p. 59, p. 67, n. 11), as we saw, and Tichý (1986, p. 255) embrace this consequence.

[5] For background on Transparent Intensional Logic, see Tichý (1988), (2004), Jespersen (2004b). For predication as functional application, see Tichý (1988, p. 174).

[6] I am aware that treating properties as functions is anathema to Bealer. He adduces in several places what I would call *the argument from aroma*: the aroma of coffee is a property, but certainly not a function, hence properties are not functions. The earliest reference I know of is (1982, p. 90). My justification for this treatment is that a function (from worlds, times, or whatnot) *models* the property of being the aroma of coffee without being *identical* to it — though for technical convenience the modelling and what is so modelled may be identified.

[7] Alternatively, intensional entities might be modelled as binary functions defined over ordered pairs of ω- and τ-entities. But for reasons external to this paper, I reject this alternative; see Tichý (1982).

[8] See Jespersen (2004a) for an example of what this construal of sets can do for a procedural conception of set-forming operations.

[9] Which is to say that I adhere to 'the Fregean doctrine that predicates name functions' (Bealer 1982, p. 89).

[10] The technique consisting in λ-binding w, t variables is known as *explicit intensionalisation*. For further details, see Jespersen (2005).

[11] So does Tichý: see his (1988, p. 64). However, I deviate provisionally from Tichý by reading '[...]' as the *result* of the application and not as the very *procedure* of applying a function to an argument. The deviation is an innocuous shortcut in this essay, however. Since we are not ascending into the ramified type hierarchy of the higher-order objects of procedures that Transparent Intensional Logic comes with, we do not need notation for picking out procedures.

[12] 'Fa' was changed into 'Pa'.

[13] In all fairness, though, it is obvious from Aczel (1980) that he is concerned with mathematical propositions only.

[14] One of the extensionalization functions is G, which "tells us the actual extension of the elements of D", (1993, p. 25). G is comparable to Montague's $\vee$ and susceptible to the same confutation that Tichý provides in (1988, pp. 151ff) of Montague's 'downer', that the identification of the function $\vee$ requires empirical omniscience.

[15] Just to amplify the point, on my conception predication is not a relation or any other kind of function at all. If we wanted to, though, we could construe it as one, along the lines of: pred$<P_{wt}, a>$. Pred would be a function inputting sets and individuals and outputting True or False, according as a is an element of P_{wt} or not. But the construal would run counter to the thesis I am advocating, that predication is a case of functional application. So instead of 'pred$<P_{wt}, a>$' I write '$[P_{wt}\ a]$'.

[16] Bealer's M is *intensional*, because some elements of D defy the axiom of extensionality: they are necessarily co-extensional, yet not co-intensional. This is a hyperintensional notion of intensionality exceeding the 'possible-world'-semantic notion of intensionality.

[17] I am leaving out of consideration Bealer's independently motivated introduction of the operation of predication, which has to do with his project of establishing a logic that is both *intensional* and *first-order*.

[18] I am indebted to Marie Duží for valuable comments.

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