We N106 01

# Utilization of Multiple Scattering - The Next Big Step Forward in Seismic Imaging

A.J. Berkhout* (Delft University of Technology)

## SUMMARY

Until today, seismic imaging is applied to primary reflections. This means that prior to imaging, the multiples need be removed from the measurements ('data linearization'). However, multiples contain valuable information that should not be removed but should be utilized. The message of this presentation is that we should not adapt the measured data to our imaging algorithms but we should adapt our imaging algorithms to the measured data. By doing this, we may expect significantly better images, particularly for deep reservoirs.

## Introduction

A plea is made to stop with investments in multiple removal algorithms. Instead, it is recommended to focus all efforts on the development of new imaging technology that treats multiples as indispensable signal. This paradigm shift involves two disruptive step changes: (1) the signal-to-noise ratio in seismic recordings is significantly raised and (2) the illumination strength, particularly for deep data, is significantly improved. Note from a data acquisition point of view that these advantages are provided free of charge.
The presentation starts with a concise review of the traditional imaging concept, where primaries and multiples are considered as two separate wavefields. This theoretical framework leads to the well-known family of open-loop, two-step algorithms: (1) multiple removal followed by (2) primary migration. Next, I will summarize the full wavefield concept of seismic imaging that treats primaries and multiples as one physical wavefield. This full wavefield approach leads to a closed-loop, single-step algorithm: the full wavefields (primaries + multiples) are simultaneously migrated such that the image is consistent with the input data. It is demonstrated that closed-loop, full wavefield migration is the next big step forward in seismic imaging.

## Data linearization

The feedback model shows that the data at $z_0$ with surface-related multiples $(\mathbf{P}^-)$ can be written as:

$$\mathbf{P}^- = \mathbf{P}_\mathbf{0}^- + \mathbf{X}_\mathbf{0}^\cup \mathbf{R}^\cap \mathbf{P}^- \tag{1a}$$

or

$$\mathbf{P}^- = \mathbf{P}_0^- + \mathbf{P}_0^- \mathbf{A}^\cap \mathbf{P}^- \qquad at\ z_0, \tag{1b}$$

matrix $\mathbf{P}_0^-$ being the data without surface-related multiples ($\mathbf{P}_0^- = \mathbf{X}_0^\cup \mathbf{S}^+$) and matrix $\mathbf{R}^\cap$ being the reflectivity operator at the surface that transforms *upgoing* wavefields $\mathbf{P}^-$ into *downgoing* wavefields $\mathbf{R}^\cap \mathbf{P}^-$. Surface-related operator $\mathbf{A} = (\mathbf{S}^+)^{-1} \mathbf{R}^\cap$ includes compensation for the source matrix in $\mathbf{P}_0^-$.

*Removal of surface-related multiples*

From equation (1b) it follows:

$$\mathbf{P}_0^- = \mathbf{P}^- - \mathbf{P}_0^- \mathbf{A}^\cap \mathbf{P}^- \qquad at\ z_0, \quad (2a)$$

leading to the iterative SRME algorithm:

$$(\mathbf{P}_0^-)^{(i)} = \mathbf{P}^- - (\mathbf{P}_0^-)^{(i-1)} \mathbf{A}^{(i)} \mathbf{P}^- \text{ with } (\mathbf{P}_0^-)^{(0)} = \mathbf{P}^-. \tag{2b}$$

Operator $\mathbf{A}^{(i)}$ is estimated by minimizing the shot records (*matrix columns*). In the first iteration $(\mathbf{P}_0^-)^{(0)}$ is a pre-processed version of $\mathbf{P}^-$ and $\mathbf{A}^{(1)} = A_{jj}^{(1)} \mathbf{I}$ equals a scaled unity matrix.

*Removal of internal multiples*

If we move through the subsurface by wavefield extrapolation (from the surface $z_0$ to the deepest depth level $z_M$), the current depth level $z_m$ plays the role of the surface for which the internal multiple scattering is removed. By applying this concept, we create a *recursive* algorithm for the very complex internal multiple removal process (*m=0, 1, …, M-1*):

$$(\mathbf{P}_0^-)^{(i)} = \mathbf{P}^- - (\mathbf{M}^-)^{(i)} = \mathbf{P}^- - (\mathbf{P}_\mathbf{0}^-)^{(i-1)} \mathbf{A}^{(i)} \mathbf{P}^- \qquad at\ z_m, \tag{3}$$

where the iterative process starts with $(\mathbf{P}_0^-)^{(0)} = \mathbf{P}^-$ and where the output consists of $\mathbf{P}_0^-$ and $\mathbf{A}^\cap$.
When we arrive at the deepest level $z_M$, all multiples have been removed and the 3D *primary* response can be written as:

$$\mathbf{P}_r^-(z_0^+; z_0) = \mathbf{P}^-(z_0^+; z_0) - \alpha(z_0^+) \sum_{m=0}^{M-1} \mathbf{W}^-(z_0^+, z_m^+) \mathbf{M}^-(z_m^+; z_m^+) \mathbf{W}^+(z_m^+, z_0), \tag{4}$$

where $\alpha(z_0^+)$ allows the subtraction to be adaptive (may also be done for every subset of layers). In equation (4) the term m=0 means that the surface-related multiples have been included in the subtraction. Later in this paper we will see how these causal wavefields can be used in a *de-linearization* process.

*Migration of linearized data*

Next, the primary response (equation 4) is input to a primary wavefield migration (PWM) algorithm, such as WEM and RTM. The wavefields that are used in the PWM algorithm are shown in Figure 1a.

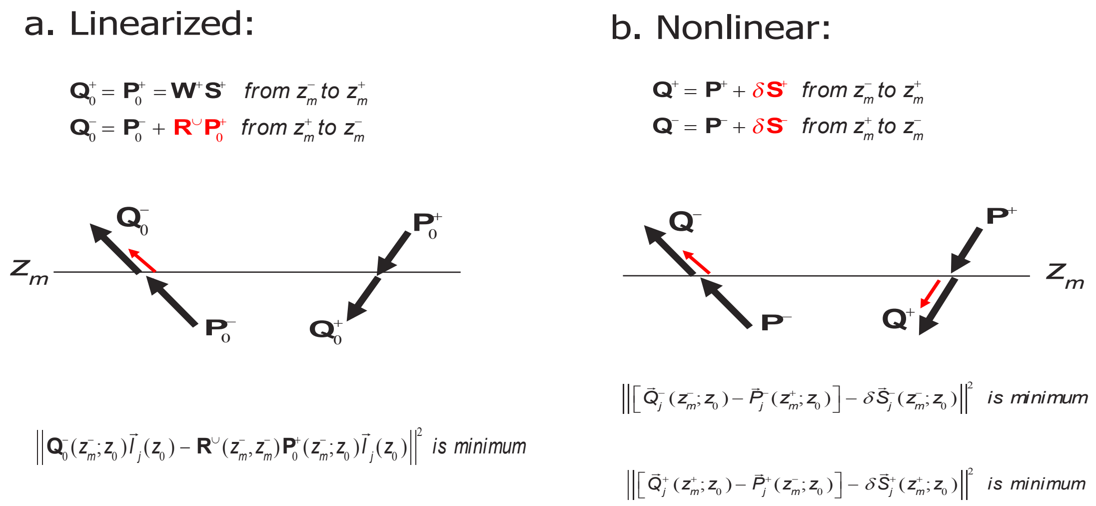


***Figure 1*** *Wavefields and minimization equations at depth level $z_m$ for linearized (left-hand side) migration (PWM) and nonlinear (right-hand side) migration (FWM).*

**Full Wavefield Migration**

In the standard migration practice we have little information about the inconsistency between output and input: mainstream migration has been implemented as an 'open-loop process'. Particularly, if we want to utilize the information in multiple scattering, a simple open-loop approach is not acceptable anymore. By taking the open-loop seismic image as *input* to a suitable forward modeling algorithm (FWMod) we are able to close the loop in migration, meaning that we generate numerically simulated measurements in the feedback path (in this new way of forward modeling the image space is transformed back to the data space). Angle dependency and multiple scattering are an integral part of this process. Next, iterative minimization of the difference between simulated and real measurements allows us to optimize the seismic image.

*Full wavefield forward modeling*

In the theory of Full Wavefield Migration (FWM), each inhomogeneous gridpoint in the subsurface acts as a secondary source $(\delta\vec{S}_j)$ that generates a secondary wavefield down $(\mathbf{W}^+\delta\vec{S}_j^+)$ and up $(\mathbf{W}^-\delta\vec{S}_j^-)$. The properties of each secondary source depend on the incident wavefields $(\vec{P}_j^-, \vec{P}_j^+)$ and the inhomogeneities at that gridpoint. Hence, the measured seismic data (reflections and diffractions, single- and multiple-scattering events) is represented by a *blended* response that consists of a superposition of millions of wavefields that are generated by the primary $(\vec{S}_j)$ and secondary $(\delta\vec{S}_j)$ sources. In FWM the full wavefield extrapolation process is based on the following equations (superscripts +,- meaning down and up in the coordinate system that is chosen):

$$\vec{P}_j^+(z_m^-;z_0) = \mathbf{W}^+(z_m^-,z_0)\,\vec{S}_j^+(z_0) + \sum_{n=0}^{m-1} \mathbf{W}^+(z_m^-,z_n^+)\delta\vec{S}_j^+(z_n^+;z_0) \qquad \text{(FWMod)} \qquad (5a)$$

$$\vec{P}_j^-(z_m^-;z_0) = \mathbf{F}^+(z_m^-;z_0^+)\,\vec{P}_j^-(z_0^+;z_0) - \sum_{n=1}^{m} \mathbf{F}^+(z_m^+,z_n^-)\delta\vec{S}_j^-(z_n^-;z_0), \qquad (\text{FWMod}^{-1}) \qquad (5b)$$

where vectors $\delta\vec{S}_j^{\pm}$ represent the *two-way* secondary sources in the inhomogeneous gridpoints at depth level $z_n$ *(n=0, 1, …, M)*, where matrix $\mathbf{W}^+$ defines the scattering-free forward and reverse propagation operators between two depth levels and where $\mathbf{F}^+ = (\mathbf{W}^+)^*$. Operator $\mathbf{W}^+$ is determined by the propagation velocity model, being piecewise continuous. In equations (5a,b) the secondary source vectors $\delta\vec{S}_j^{\pm}$ are given by a linear combination of the up- and downgoing incident wavefields (converted waves are generated, harmonics are not generated):

$$\delta\vec{S}_j^-(z_n^-;z_0) = \mathbf{R}^{\cup}(z_n^-,z_n^-)\vec{P}_j^+(z_n^-;z_0) + \delta\mathbf{T}^-(z_n^-,z_n^+)\vec{P}_j^-(z_n^+;z_0) \tag{6a}$$

$$\delta\vec{S}_j^+(z_n^+;z_0) = \mathbf{R}^{\cap}(z_n^+,z_n^+)\vec{P}_j^-(z_n^+;z_0) + \delta\mathbf{T}^+(z_n^+,z_n^-)\vec{P}_j^+(z_n^-;z_0), \tag{6b}$$

representing the most general formulation of *elastic* interaction. The column vectors in equations (6a,b) quantify the *dual* scattering process (backward and forward) at the gridpoints of depth level $z_n$ (*n=0,1,…, M*). Note that operators $\mathbf{R}$ *and* $\delta\mathbf{T}$ play the same role in the scattering process.

*Full wavefield migration*

In full wavefield migration (FWM) measurements and propagation operators (based on a user-specified velocity model) are given and the unknown source properties and scattering operators are computed (source and operator estimation process). As expected, this process is iterative. It consists of several roundtrips, starting at both the receiver and source locations. In each roundtrip forward (FWMod) and reverse modeling (FWMod$^{-1}$) are *simultaneously* applied. The simulated measurements are compared with the real measurements. After minimizing the difference between the two datasets the next roundtrip starts. The wavefields that are used in the FWM algorithm are shown in Figure 1b.

**Importance of utilizing multiples**

In the foregoing two principally different approaches to seismic imaging have been summarized. In the *linear* approach, being the mainstream in today's industry, the data is linearized first. This is accomplished by recursively removing the multiple scattering events from the measurements. Next, the two linearized wavefields at each depth level $z_m^-$, are used as input to a primary wavefield migration algorithm (see Figure 1a). In the *nonlinear* approach to seismic imaging the four full wavefields are directly used in the full wavefield migration algorithm (see Figure 1b). To characterize the principal difference between the two approaches, I will compare the illuminating wavefields $\left(\mathbf{P}^{\pm}\ vs\ \mathbf{P}_0^{\pm}\right)$ at each depth level:

1. Illumination from above

*Nonlinear*: $$\mathbf{P}^+(z_m^-;z_0) = \mathbf{W}^+(z_m^-,z_0)\mathbf{S}^+(z_0) + \sum_{n=0}^{m-1}\mathbf{W}^+(z_m^-,z_n^+)\delta\mathbf{S}^+\left(z_n^+;z_0\right), \tag{7a}$$

*Linearized*: $$\mathbf{P}_0^+(z_m^-;z_0) = \mathbf{W}^+(z_m^-,z_0)\mathbf{S}^+(z_0). \tag{7b}$$

2. Illumination from below

*Nonlinear*: $$\mathbf{P}^-(z_m^+;z_0) = \mathbf{X}_0^{\cup}(z_m^+,z_m^+)\left[\mathbf{W}^+(z_m^-,z_0)\mathbf{S}^+(z_0) + \sum_{n=0}^{m}\mathbf{W}^+(z_m^-,z_n^+)\delta\mathbf{S}^+\left(z_n^+;z_0\right)\right] \tag{8a}$$

*Linearized*: $$\mathbf{P}_0^-(z_m^+;z_0) = \mathbf{X}_0^{\cup}(z_m^+,z_m^+)\mathbf{W}^+(z_m^+,z_0)\mathbf{S}^+(z_0) \quad \textit{(not used in linearized imaging).} \tag{8b}$$

From equations (7a,b) and (8a,b) we see that in the nonlinear version of migration the illuminating wavefields $(\mathbf{P}^+,\mathbf{P}^-)$ are strengthened by the multiple scattering. From a physics point of view, the extra illuminating wavefields are generated by the downward-reflected full wavefields (role of $\mathbf{R}^{\cap}$). These extra wavefield components $(\mathbf{R}^{\cap}\mathbf{P}^-)$ are explicitly generated in the FWM algorithm at each depth level (backscatter part of $\delta\mathbf{S}^+$). The result yields high-density angle gathers. This theoretical insight confirms the fundamental weakness of the current migration approach: "Removing multiple scattering means diminishing illumination strength". During the presentation this important conclusion will be illustrated by numerical examples.

**From linearization output back to full wavefields**

An important property of the linearization algorithm is that the resulting decomposed wavefields (primaries and multiples are separated at each depth level) obey the *causality* property. This physical property can be used to transform both decomposed causal wavefields (primaries, multiples) at each depth level into *full* causal wavefields, being the wavefields that are needed to create a full wavefield image. I call this wavefield composition process: 'de-linearization'. From a theoretical and practical point of view, it is interesting to note that the Marchenko algorithm can be interpreted as a combined linearization + de-linearization process. This alternative interpretation suggests a close relationship between the Marchenko and the Inverse Scattering Series approach (ISS).

**Final example**

Let us consider the seismic experiment in Figure 2, where only four sources (at x=600, 900, 4500, 4800m) are used to illuminate the subsurface ($z > z_0$) and where receivers are positioned along the complete surface ($z_0$). Open-loop, primaries-only migration provides the image in Figure 2a. As expected, the middle area is not properly illuminated by the primaries. Next, the multiples are used in the migration process. Figure 2b shows the result of closed loop, all-multiple imaging. Note that the shadow zone of the sparse sources cannot be recognized anymore. Also note the improvement in image of the diffractors. The example confirms what the full wavefield theory predicts, and demonstrates that imaging with multiples improves the illumination significantly: illumination gaps due to sparse source sampling are filled up by the illumination with the multiple scattering. Hence, multiples should not be removed, but they should be utilized! Last but not least, imaging with multiples will also have an important influence on future acquisition geometries.

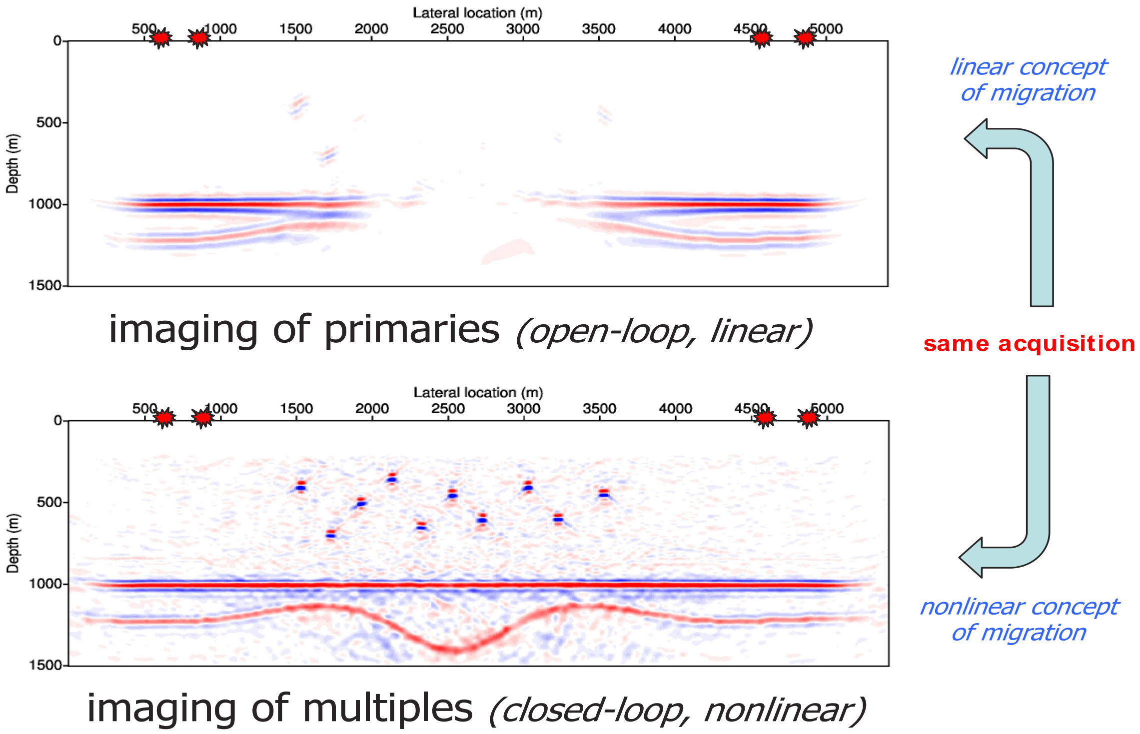


***Figure 2*** *Example to illustrate that multiples provide a significant contribution to illumination.*

**Acknowledgement**

The author would like to thank the Delphi sponsors for the stimulating discussions on industry needs during the Delphi meetings and for their continuing financial support.