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Machine Learning Surrogate Modeling for Reliability Analysis of Spatially Varying Slopes in 3D

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Abstract: This study investigates the use of machine learning (ML) models as ready-to-use surrogate models for the Random Finite Element Method (RFEM) in a 3D context. The training set comprises 4000 RFEM realizations, with random fields covering multiple spatial correlation lengths as input and their corresponding factors of safety (FoS) as the target. Our results show that a convolutional neural network performs best for predicting the FoS compared to two other ML models – support vector regression and random forest – combined with principal component analysis. The best-performing model in 3D can conduct a stochastic analysis of 4000 simulations within seconds, compared to 83 days for a standard RFEM reliability analysis, highlighting the efficiency of the approach.

Keywords: machine learning; RFEM; slope stability; spatial variability; surrogate modeling; three-dimensional.

1. Introduction

A challenge when analyzing geotechnical structures is addressing uncertainties arising from the spatial variability of soils (Phoon and Kulhawy, 1999). The Random Finite Element Method (RFEM) (Griffiths and Fenton, 2004) accounts for this spatial variability by integrating random field theory with the finite element (FE) method. Through Monte Carlo simulation (MCS), RFEM can compute a factor of safety (FoS) distribution and a probability of failure (p_f). However, MCS requires many realizations, especially for very small values of p_f (e.g., Jiang and Huang, 2016), making it computationally intensive and limiting its adoption in practice.

Surrogate modeling based on machine learning (ML) offers us a potential solution by learning a mathematical function of the system response that can replace the time-consuming physics-based simulations in an MCS (e.g., He et al. 2020; Wang and Goh, 2021). Studies have focused on 'semi-surrogate' modeling, i.e., simulations for a specific case are used to train the ML model, after which it is used to predict the remaining simulations. In practice, this means that time-consuming numerical simulations are still needed. Moreover, for slope stability, a 2D setting is often used for computational efficiency. But, in reality, slope failure mechanisms are three-dimensional.

This paper explores how ML models perform as full-surrogate models for RFEM in 3D slope reliability analyses, with the aim of building pre-trained models that can be directly applied to a broad range of applications.

2. 3D RFEM Slope Model

Our simulations use an idealized 45° slope of height 5 m, crest width 5 m, and length 50 m (Varkey et al., 2019). The base nodes are fixed, while, at the mesh ends, the boundary conditions are set as rollers, allowing only vertical movement (Spencer, 2007). The nodes on the back face of the mesh (i.e., at x=0) are on rollers, allowing movement only in the y-z plane. The slope is discretized into 4000, 20-node, regular hexahedral elements, each of size 1.0×1.0×0.5 m (except for along the sloping face), and each with 2×2×2 Gaussian integration points.

The slope consists of clay modeled by a linear elastic, perfectly plastic Tresca soil model. The elastic component is defined by Young's modulus, E = 100,000 kPa, and Poisson's ratio, ν =0.3. The unit weight of the clay is γ = 20 kN/m^3. The undrained shear strength s_u is a stochastic parameter modeled by random fields generated by Local Average Subdivision (LAS) (Fenton and Vanmarke, 1990). Distinct scales of fluctuation are assumed in the x, y, and z directions (Varkey et al. 2019), and random field cell values are mapped to the FE mesh at the Gauss point level. The slope has been subjected to gravity loading to generate the in situ stresses, and the FoS has been computed using the strength reduction method, as in Hicks and Spencer (2010).

3. Investigated Cases

The five cases used for training and testing (Table 1), labeled T1, T2, V1, V2, and V3, have a consistent vertical scale of fluctuation θ_z of 1 m, as used by Hicks and Spencer (2010). The scales of fluctuation in the horizontal plane, θ_x and θ_y, are set equal to each other for case T1 (Figure 1(a)), which is a common assumption made in 3D modeling. The value is set to 20 m, which promotes discrete three-dimensional failures through horizontal (semi-continuous) weaker zones (Hicks and Spencer, 2010). In case T2 (Figure 1(b)), the scale of fluctuation along the slope length, θ_y, is increased to 100 m. This is based on a study conducted on a Dutch dike, reported by de Gast et al. (2021). It highlighted that the spatial correlation length along the dike's direction can

vary significantly and often exceeds that across the dike. For cases V1, V2, and V3, θ_y is set to a range of values.

The mean s_u is constant in each case and has been determined to ensure that the p_f derived from 4000 realizations is within the 2.8-3.2% range. This level strikes a balance between the total number of realizations needed to achieve a consistent p_f (i.e., its coefficient of variation, COV[p_f]<0.1) and a practical value. The COV of s_u is 0.25 in all cases, which is within the likely range(Phoon and Kulhawy, 1999). For cases T1 and T2, 2000 additional FE simulations were performed to serve as the training set for the ML models. Note that a single realization typically takes around 30 minutes on the TU Delft computer cluster equipped with Intel XEON E5-6248R 24C 3.0GHz CPUs (DHPC, 2022).

Table 1. The point statistics and scales of fluctuation of s_u for the slope cases considered.

Case	Point statistics			Scale of fluctuation			Number of FE simulations	
	Mean [kPa]	COV	Distribution	θ_z[m]	θ_x[m]	θ_y[m]	Training	Testing
T1	21.96	0.25	Normal	1	20	20	2000	4000
T2	22.04	0.25	Normal	1	20	100	2000	4000
V1	21.19	0.25	Normal	1	20	10	0	4000
V2	22.00	0.25	Normal	1	20	60	0	4000
V3	22.00	0.25	Normal	1	20	120	0	4000

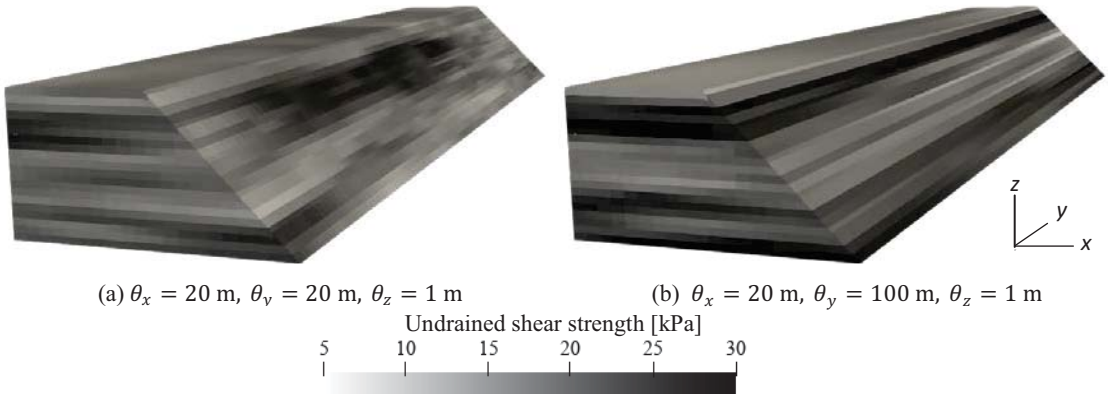


Figure 1. Typical realizations of random fields of the undrained shear strength for cases T1 and T2.

4. Machine learning models and their input procedure

The performance of two ML models – Support Vector Regression (SVR) and Random Forest (RF) – combined with Principal Component Analysis (PCA),and the Convolutional Neural Network (CNN) are investigated for predicting slope stability (see Zhou (2021) and Vermeer (2024) for details of each ML model).

The input for the three models is a random field. For the PCA-SVR and PCA-RF, each node in the random field is standardized to ensure that all features contribute equally to the PCA, preventing features with larger scales from dominating the first components. For the CNN, each random field is transferred into a 3D regular grid using nearest-neighbor interpolation. Zeros are added to the 3D matrix for parts that do not represent any part of the slope.

Hyperparameter tuning for the PCA-SVR uses a grid search with a 5-fold cross-validation on the training set and the mean squared error (MSE). For the PCA-RF, we replace the grid search with a random search of 300 hyperparameter combinations. Vermeer (2024) provides an overview of the hyperparameter ranges tested. The lowest MSE for the PCA-SVR comes from the RBF kernel, C = 0.3, E = 0, and *PCA components* = 130. For the PCA-RF, the best combination is *PCA components* = 6, *Min_samples_split* = 2, *Min_samples_leaf* = 4, and *Max_features* = None.

The hyperparameters for the CNN architecture are tuned by iteratively adjusting them until the model becomes complex enough to have good performance while maintaining a low number of trainable parameters to reduce training time. Experiments were done using either one or two stacked convolutional layers, either one or two fully connected (FC) layers, and a dropout layer. Training is done on batches of 256 samples with an Adam optimizer and a learning rate of 10^{-4} over a maximum of 5000 epochs. Early stopping halts training when there is no improvement in the validation loss (MSE) after 500 epochs. 30% of the full training data is allocated for validation. Vermeer (2024) provides an overview of the tested hyperparameter settings.

The network that showed the lowest loss on the validation set consists of two convolutional layers, two MaxPooling layers, and two FC layers. The two convolutional layers use a kernel size of 7×7 and 5×5, and the two stacked MaxPooling layers have a filter size of 2×2. The pooled maps are flattened after the second MaxPooling layer, and the two FC layers are included hereafter. Finally, a dropout layer with a dropout rate of 0.4 is added. The output layer is a single neuron. All convolutional and FC layers use the ReLU activation function.

5. Results and discussion

The three trained ML models were used to make FoS predictions for the five slope cases. Cases T1 and T2 have no overlapping FE simulations between the training and the testing sets. Figure 2 shows the coefficient of determination r^2 and Root Mean Squared Error (RMSE) of the predictions on the testing sets. For the CNN model, those metrics are averaged over 10 training and prediction cycles. This repetition was done to account for potential variability due to the random initialization of the trainable parameters and numerical errors.

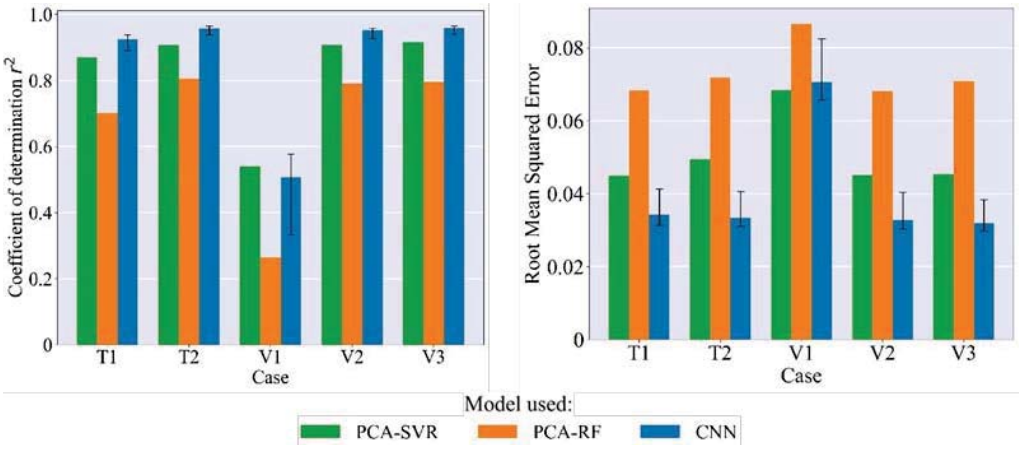


Figure 2. Performance metrics r^2 (left) and RMSE (right) of the surrogate models on the test sets of five different cases.

The PCA-SVR and CNN surrogate models show a strong ability to accurately predict the FoS for slope cases with a θ_y within the range of values they have been trained on, as evidenced by high r^2 (>0.83) and low RMSE (<0.05) values for cases T1, T2, and V2 (Figure 2). The CNN outperforms the other two models overall on both metrics. Additionally, both sets of ML models show high performance when applied to case V3, even though θ_y=120 m and therefore falls outside the range used for training. This supports previous findings that ML surrogate models perform better with larger spatial correlation lengths (Aminpouret al., 2022). Upon examining the slope realizations for this case, it was found that the failure mode of many realizations was similar to what is called failure Mode 3 (Hicks and Spencer, 2010). This is characterized by failure extending along the length of the slope, which mirrors slope failure in 2D. Consequently, slope analysis in 2D would be sufficient for such a case.

Figure 2 also shows that all models have the weakest performance in case V1, which is characterized by the smallest θ_y of 10 m. While the RMSE remains acceptable, the r^2 is significantly low. This is due to the models almost consistently overpredicting FoS for this case (Figure 3): at such a correlation length, it is easier for failure to propagate through semi-continuous weaker zones while avoiding stronger zones, resulting in a lower FoS. Realizations of case V1 were excluded from the training set, which explains the observed overestimation.

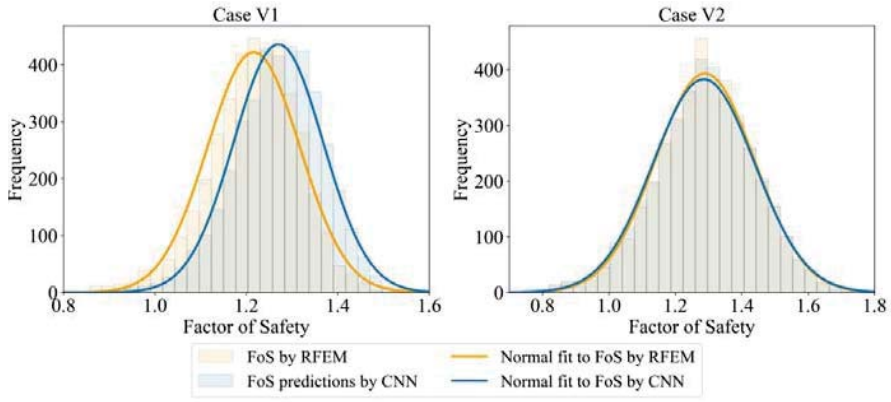


Figure3. Histograms of the CNN-predicted and RFEM realized FoS for cases V1 and V2.

Figure 4 presents the relative error in ML-predicted p_f, calculated based on the ML-predicted FoS datasets and the p_f derived from 4000 FE simulations for all cases. Model predictions are better for cases with larger spatial correlation lengths. For example, the relative error in the p_f prediction by the CNN is as low as 6.6% for case V2 in which θ_y= 60 m. However, more analyses are needed for a definitive explanation of this phenomenon. To further demonstrate the accuracy of the predictions by the CNN model applied to case V2, Figure 3 compares the distributions for the CNN-predicted and RFEM-computed FoS.

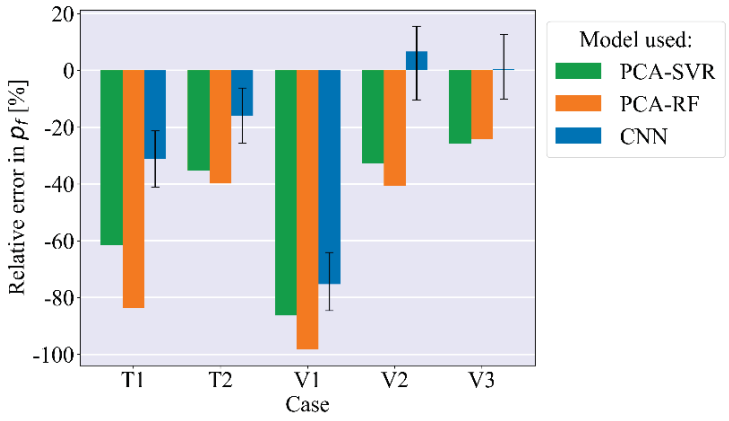


Figure 4. Relative error of the obtained p_f by the ML models to the p_f obtained by RFEM analysis for five different cases.

6. Conclusions

The potential for developing a ready-to-use ML surrogate model for RFEM for 3D slope reliability analysis has been demonstrated. The results indicate that the surrogate models can make FoS predictions that are highly consistent with values from RFEM simulations when θ_y of the testing case is within, or exceeds, the range of values used for the realizations in the training set. Among the models tested, the CNN demonstrated superior performance in terms of both FoS and p_f predictions across all cases. The ML surrogate model performs a stochastic analysis of 4000 realizations within seconds, compared to 83 days of computational time required for RFEM.

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