Graphic statics in funicular design

calculating force equilibrium through complementary energy

Graduation thesis
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PART I
INTRODUCTION

1. INTRODUCTION

This thesis develops a new methodology for generating thrust diagrams in statically indeterminate structures, and offers an example of how this method can be used to design a form active structure that changes its configurations according to the changing load conditions.

The occurrence of bending moments in constructions causes compressive and tensile stresses in the material, resulting in a deformation by bending. These bending moments usually are determinant in dimensioning the structural elements. This is no surprise, as the most efficient way to transfer loads is through axial forces instead of by bending. It is possible for a construction to have such a configuration that for a given loading, no bending moments occur. When such a structure acts solely in compression or tension, it is called a 'funicular system' [Block].

The term funicular is associated with cords or cables. While this may seem strange, in 1675, English scientist Robert Hooke discovered "the true... ...manner of arches for building," which he summarized with a single phrase: "As hangs the flexible line, so but inverted will stand the rigid arch."
A flexible line, for example a cable or chain, cannot transfer bending moments, nor can it transfer compressive forces. Therefore, all loads can only be transferred by tensile forces; a hanging cable's shape will change according to its loading and will always act solely in tension.

The design of funicular structures, or compression only structures, has been an inspiration for many striking examples of architecture and construction. Because the forces in the structure are mainly axial forces, and there are few bending moments, the structure can be dimensioned much more slender than non-funicular structures of the same size. This type of design, in which form follows force, assumes one major load condition, usually the weight of the structure itself, and builds the shape of the structure in such a way that it is optimised for transferring the load of its own weight. The weight of the structure, i.e. a concrete shell, is large enough to render the effect of external loads such as the wind negligible. The bending moments created by forces other than those for which the shape of the structure was designed, are considered small enough to not have a significant effect on the dimensioning of the structure.

fig. 1.1 funicular concrete shell. Heinz Isler

fig. 1.2 funicular concrete shell. Frei Otto

However this leads to a paradox. On the one hand, the structure is considered heavy enough to nullify the effect of external forces as wind and snow, while at the same time, the fact that the structure has a funicular shape, allows for less material, resulting in a lower weight. Thus, building in a funicular shape allows for a reduction in weight, while weight is needed for the shape to be funicular under all circumstances.

Adaptive structures can be the answer to the paradox. A geometrically adaptive structure can change its configuration in order to adapt to meet certain requirements imposed by its context.

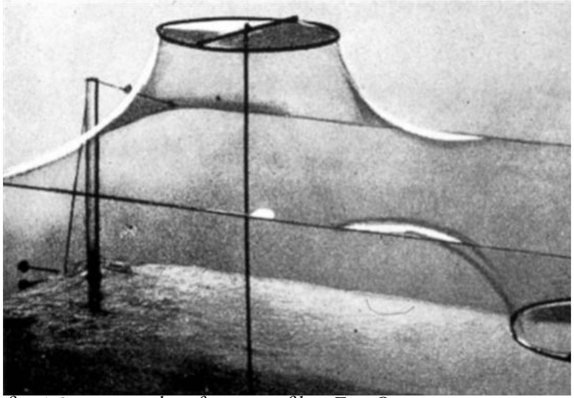


fig. 1.3 minimal surface, soap film. Frei Otto

Where a static structure with a funicular shape is only optimal for one loading, usually its own weight, a geometrically adaptive structure can change its shape continuously in order to always have a funicular shape for its current loading.

These funicular shapes can be generated and analysed through various methods. For a long time, physical models were the primary tool for designing funicular structures. These models were comprised from a multitude of hanging chains, hanging membranes, or soap bubbles, and when finished, their dimensions were measured and used to draw the actual structures. While they present an intuitive way of designing, they are not very flexible and building and measuring these scale models is time consuming. Therefore they might be suitable when designing a static structure, having just one configuration, but for an adaptive structure which constantly changes when load conditions change, they are not.

fig. 1.4 frozen cloth. Heinz Isler

Over the course of history, other methods were developed to generate thrust lines for arches through graphical statics, or mathematical models. Mathematically determining funicular shapes is complicated, and not easy to use as a part of the design process. Graphic statics on the other hand, are much easier to understand and use, but the possibilities for using graphic statics on three dimensional systems are relatively limited.

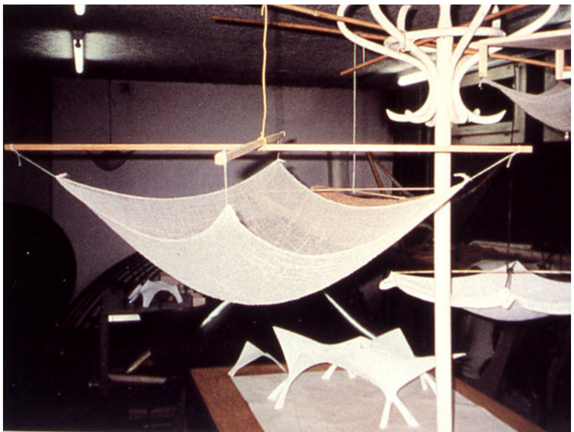


fig. 1.5 hanging cloth models. Heinz Isler

1.1 PROBLEM STATEMENT

From the previous discussion the following problem can be formulated, forming the basis for writing this thesis.

> Existing methods for assessing thrust lines and networks through are either not advanced enough to be used effectively in the design process for funicular formactive structures, or are too complicated or unintuitive to be used as a design tool.

1.2 OBJECTIVE AND APPROACH

This thesis assesses the presumption that by providing a more advanced method to use graphic statics to generate thrust networks, in particular by adding a way to assess indeterminate systems, will provide an easier platform to work with when designing a form active structure that aims to reconfigure itself to the funicular shape. This will be done by developing the thrust network analysis further, and use the output as direct input for describing the configuration of an adaptive structure.

The objective of the thesis can therefore be formulated as: 'Develop a method for assessing thrust networks for both statically determined and indeterminate systems, based on graphic statics, and model it in a parametric design software, where it can be used to describe the configuration of an adaptive structure'

In order to reach the stated objective, a series of secondary objectives is formulated:

a) Define a theoretical framework, stating which methods for generating funicular shapes exist, and use them to improve upon each other to achieve a more complete, intuitive graphical method

b) Use the newly developed method to assess the different funicular shapes that the form-active structure needs to facilitate within a certain range of load conditions

c) Define a reference framework, stating which adaptive systems already exist, and/or which non-adaptive systems can be made adaptive, and how they can be used to facilitate and exploit the configurations that fit the shapes acquired in 'b)'

d) Illustrate the use of the form finding method in the design of a form-active structure

e) Implement the thrust network analysis method and the form-active structure into a parametric design software, and link them such that the output for the method developed in a) provides the input for the configuration of the form-active structure

f) Evaluate performance of the analysis method and the form-finding tool

1.3 THESIS OUTLINE

This thesis presents the proposed method for generating funicular shapes, and gives an example of how the method can be used in an interactive tool for designing a funicular form-active structure.

The thesis is comprised from five parts:

Part I introduces the subject, and defines the motivation and goal of the research.

Part II describes the development of the form finding method. The theoretical framework through a concise explanation of the theory behind the existing methods for generating thrust lines and networks, and the basic framework of the method for generating thrust diagrams. It describes the assumptions, fundamentals and key concepts; outlines the method in an overview of the main steps in the methodology; formulates the problem as a series of systems of linear equations and optimization problems; and explains the solving procedure.

Part III describes the process for the conceptual design of the form-active structure. It explains the categorization of existing systems that are form-active, or can be made form-active; the selection of the system that is used to illustrate the use of the method; and the process of adjusting and improving the system, in order for it to facilitate the required configurations.

Part IV elaborates on the combination of both the script that generates the thrust diagrams, and the script that defines the form-active structure, into one parametric model that changes the configuration of the structure in quasi real-time according to the change of load conditions, which are user defined.

Part V evaluates the form finding method, by discussing its relative strengths and weaknesses; and draws conclusions for the thesis project, defining whether and to what extend the initial goals of the research have been achieved. Additionally, it gives recommendations for future development of the method.

PART II
FUNICULAR FORM FINDING

2 THEORETICAL FRAMEWORK

This chapter reviews the relevant literature. Existing methods for generating thrust diagrams in both 2D and 3D systems and solving undetermined systems are discussed.

The emphasis in this thesis lies on expanding on the graphic statics methods in order to create a relatively simple solution for statically undetermined structures, which is to be implemented in a parametric model that gives instant feedback on a change in load or structural topography.

The selected methods are:

- graphic statics
- thrust network Analysis
- complementary energy method
- force density method

The following paragraphs contain a description of the theoretical framework by explaining the theory behind each method, and an example.

2.1 GRAPHIC STATICS

In graphic statics, the forces acting on a structure are drawn into a force diagram, which is directly linked to their corresponding force polygons through geometrical constraints. By changing either the force diagram, or the force polygons, the other is affected through those constraints.

The external forces on each structure are plotted to a scale of length to force on a load line. Working from the load line, the forces in the members of the structure are determined by scaling the lengths of lines constructed parallel to the members. The diagram of forces that results from this process is called the force polygon. Active Statics implements these graphical methods into an interactive, real-time tool, which allows for an interactive design exploration, including an application for simulating a hanging cable, or arch [Greenwold and Allen, 2001]

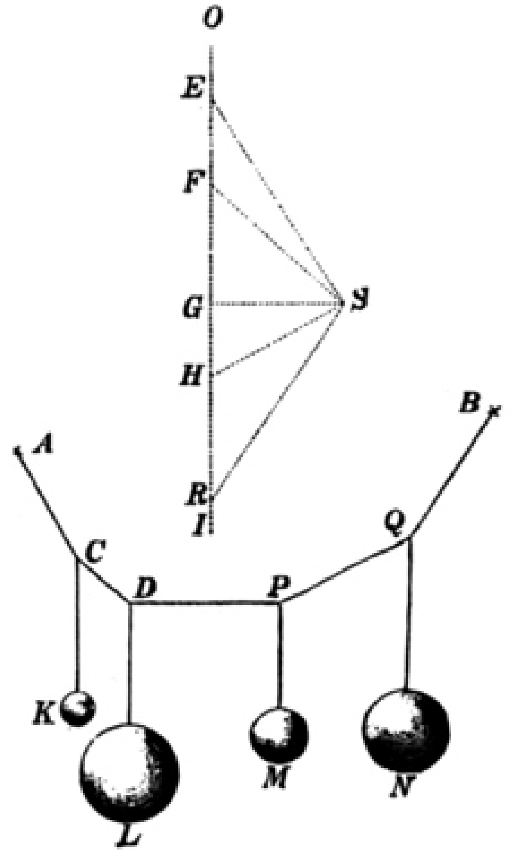


fig. 2.1 graphic statics. pierre varignon

The procedure works as follows:

A load is reduced to a series of forces with their work lines, distributed along the x-axis. The forces are numbered, and drawn as vectors, added together using the kopstaartmethode, in which the vector of the second force starts at the endpoint of the first vector; the vector of the third force starts at the endpoint of the second vector; etc. This results in a polyline, composed of the individual force vectors.

The polar coordinate is drawn. This is a point outside of the force polyline. Lines are drawn between the starting or endpoints from each force vector to the polar coordinate. This process generates a series of triangles, which are the force polygons, describing the force equilibrium in the node of the structure on which each specific force acts.
One side of the force polygon is the external force, drawn in the force polyline. The other two are the forces in the two members connecting the node to its adjacent nodes.

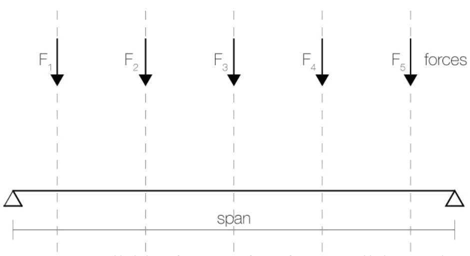


fig. 2.2 parallel loads are reduced to parallel pointloads

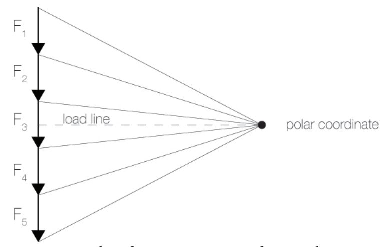


fig. 2.3 pointloads are connected to polar coordinate

As the objective is to construct a structural shape that transfers loads through axial forces only, the direction of the vector corresponds to the direction of the corresponding member.

Since each set of adjacent nodes share one member, they also share the axial force in that member, hence is why the two vectors in the force polygons are of the same length and direction.

The direction of the vectors describing the members of the arch can be used to draw the first member from a starting point to the first intersection with a working line. From that point, the direction of the next vector can be used to draw the second member, and so on until all members are drawn.

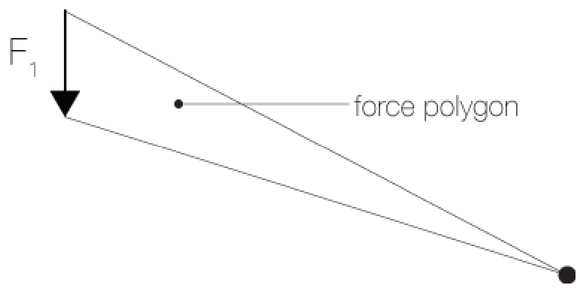


fig. 2.4 three forces form a force polygon

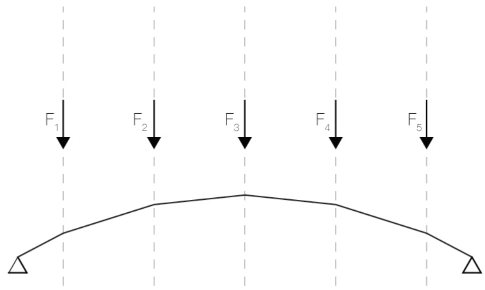


fig. 2.5 the direction of the forces represent the direction of the corresponding parts of the thrust line

Changing the position of the polar coordinate affects the shape and size of the force polygons. Placing the polar coordinate further away from the force polyline increases the magnitude of the forces in the members, and results in a more gradual slope, and a lower height of the arch. Similarly, placing the polar coordinate closer to the force polyline gives smaller forces in the members, and results in a steeper slope, and a higher arch.

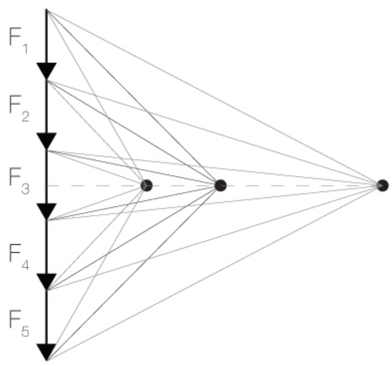


fig. 2.6 placement of the polar coordinate affects the direction of the forces

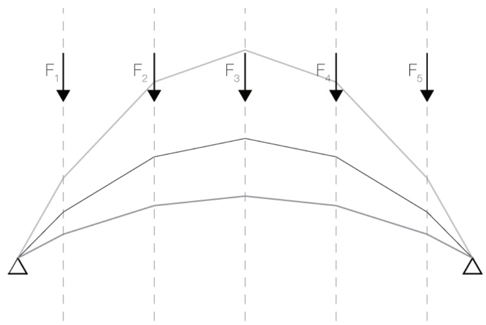


fig. 2.7 placement of the polar coordinate affects the height of the thrustline

The graphic static method for defining hanging cables or standing arches is very intuitive and powerful method for exploring funicular shapes, it is limited to 2 dimensional systems only, and therefore not applicable for 3 dimensional structural designs.

2.2 THRUST NETWORK ANALYSIS

Thrust network is a three-dimensional version of the thrust line analysis using reciprocal diagrams. While primarily intended to assess the equilibrium in masonry vaults, it can also be used more as a design tool for funicular systems.

Like graphic statics, it uses a graphical representation of the forces in a system, using force polygons. Reciprocal figures are introduced to relate the geometry of the three-dimensional systems to their internal forces [Block].

It assumes structure is, or can be reduced to, a discrete network of forces, with discrete loads applied at the vertices, similarly to graphic statics. At the same time, it assumes all loads are vertical.

The method start by making a planar projection of the three-dimensional system, called the primal grid. This grid contains the horizontal components of all the members, and the x and y coordinates for all the nodes.

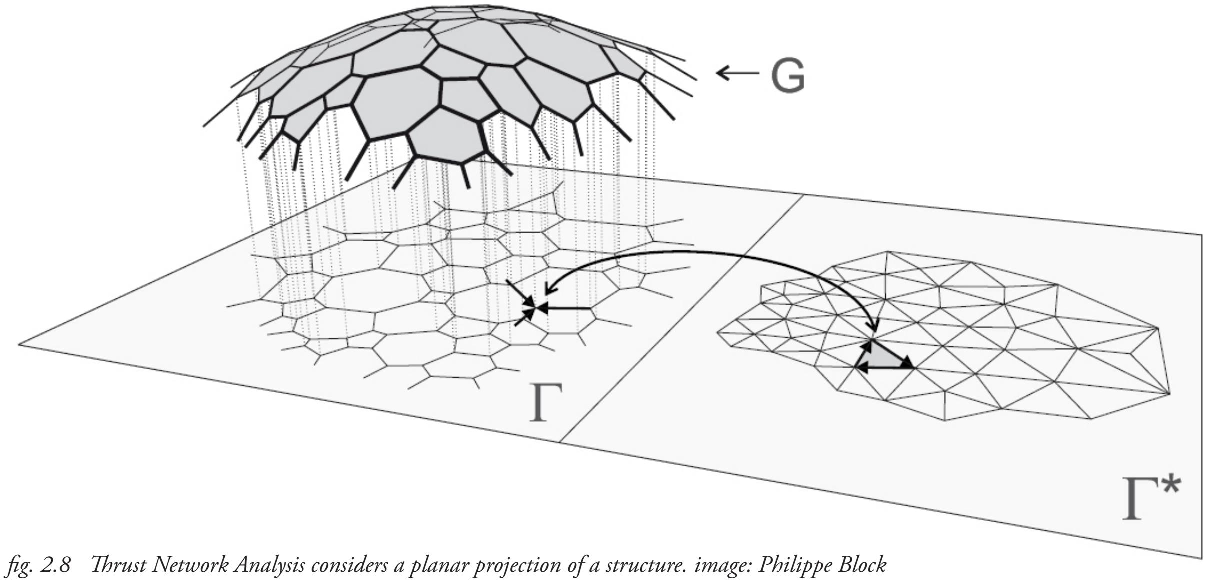


fig. 2.8 Thrust Network Analysis considers a planar projection of a structure. image: Philippe Block

Using the primal grid, the dual grid is created, which is the reciprocal force diagram of the primal grid. Reciprocal figures are geometrically related such that corresponding branches are parallel and branches which come together in a node in one of the networks form a closed polygon in the other and vice versa [Maxwell, 1864].
The force polygon for the forces acting on a node, is the reciprocal figure for the node and the members attached to that node. Consequently, the reciprocal figure for a primal grid, represents the (horizontal) forces in that grid.

The dual grid contains all the information on direction of the forces, and their magnitude relative to each other. Their absolute magnitude is at this point undetermined. Given that the dual grid is a solution for a given set of external vertical loads, the magnitude of the forces in the dual grid determine the slope of the members.

The magnitude of the dual grid compared to the magnitude of the vertical forces, is called the scale-factor. Assuming a higher scale-factor, results in larger horizontal forces. Since the vertical forces are constant, and the solution results in a funicular shape in which only axial forces exist, the slope of the every member will be smaller, and the force in the member will be larger.

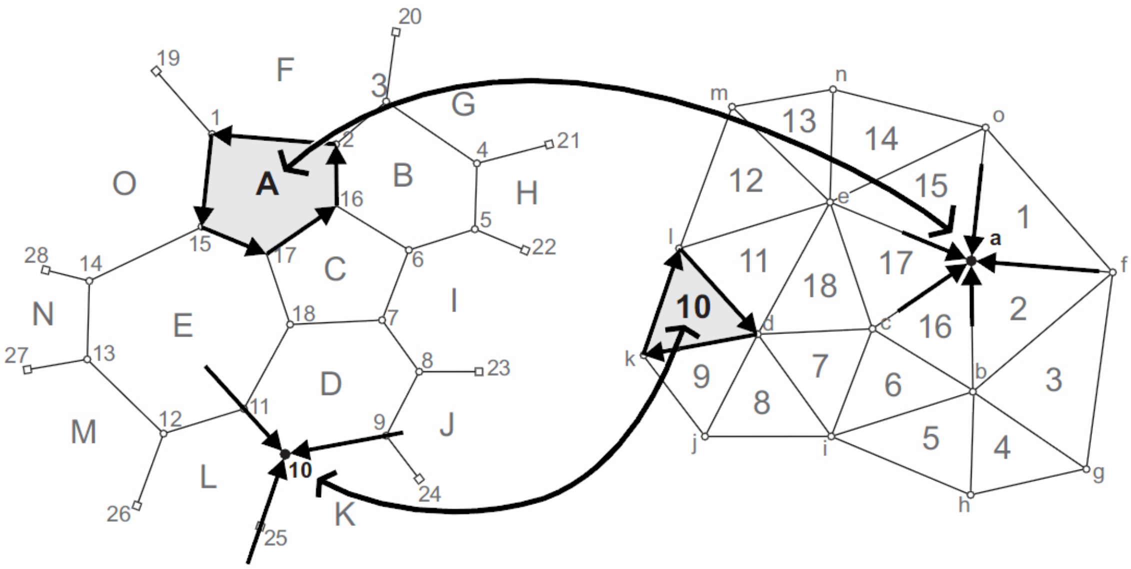


fig. 2.9 reciprocal figures. image: Philippe Block

Assuming a smaller scale-factor, results in smaller horizontal forces, and therefore a steeper slope, and a larger height of the funicular shape.

This process of changing the scale-factor resembles the repositioning of the polar coordinate in graphic statics.

When the scale-factor has been decided, the magnitude of the horizontal forces is known. Since the magnitude of the vertical forces is also known, and the x and y coordinates of the nodes is determined by the primal grid, only the z-coordinates of the nodes are unknown at this point.

Since the x and y coordinates are known, and the horizontal forces are known, the horizontal equilibrium is

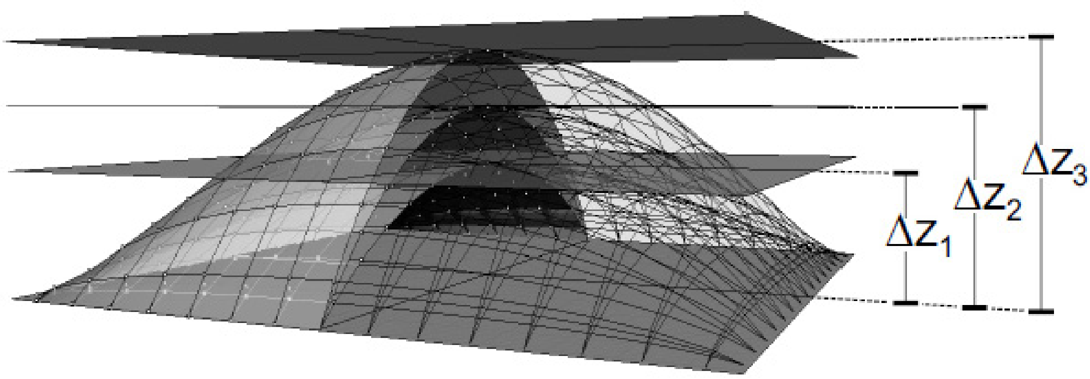


fig. 2.10 effect of changing the scale factor. image: Philippe Block

also known. What is left is the vertical equilibrium, which depends on the z coordinates of the nodes.
For every node, the vertical equilibrium can be described with a linear equation. This results in a number of equations equal to the number of nodes, and an equal number of variables, the z-coordinates.
From that point, the system is solved in a way similar to the force density method [2.1.4], to get the z-coordinate for every node. The three-dimensional shape can then be drawn.

In the case of a network in which all nodes are three-valent, meaning that in every node, three members come together, the reciprocal figure consists of triangles. As three forces act on every node, their force polygon automatically becomes a triangle.
Since all the force polygons are triangles, the size of the force relative to each other is determined; if one force changes in magnitude, the two others will have to grow or shrink proportionally in order to keep the force polygon closed: there is only one solution for equilibrium. A three-valent network is therefore statically determinate.

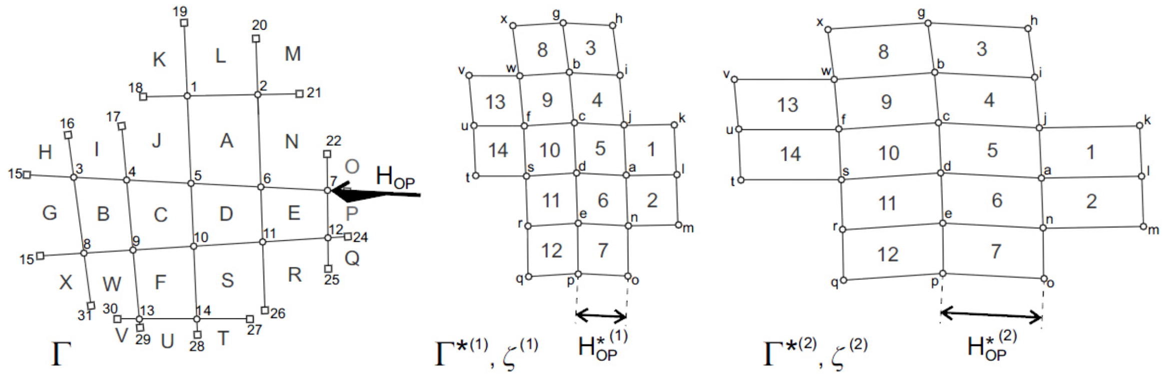


fig. 2.11 for a statically indeterminate system there are multiple solutions to make equilibrium. image: Philippe Block

For a node in which more than three members come together, more than one force polygon can be drawn that makes equilibrium. Consequently, for a primal grid with nodes having a valency higher than three, multiple, different, dual grids can be constructed. The system is then statically indeterminate, meaning that more than one force needs to be known in order to determine all the other forces in the system [Maxwell].

Although infinite possibilities exist for constructing a dual grid for a given statically indeterminate primal grid, all dual grids can be used to give a funicular shape as a result. However, this final shape will be different for different dual grids.

2.3 COMPLEMENTARY ENERGY

When a force acts on a bar, internal stress will occur. As a result, the bar will deform by either elongating or shortening, depending on whether it is under compression or tension. When the material behaves linear elastically, Hooke's law will apply, and the magnitude of the stress will be proportional to the elongation, i.e. when the stress increases, the elongation will increase proportionally.

$$N = EA\varepsilon$$

And since

$$\sigma = \frac{N}{A}$$

$$\sigma = E.\varepsilon$$

Deformation energy can then be described by the formula

$$E_v \int \frac{\sigma^2}{2E} = \int \frac{N^2}{2EA}$$

In figure 2.12, stress and elongation are plotted against each other for a linear elastic material. The red area under the curve is called the deformation energy. The blue area above the curve is called the complementary energy.

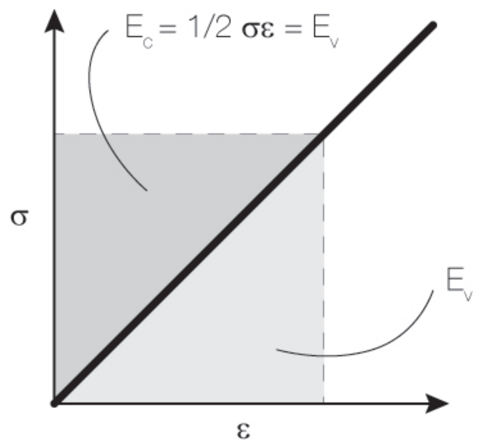


fig. 2.12 elongation as a function of stress

In this case,

$$E_c = \frac{1}{2}\sigma\varepsilon = E_v$$

[Welleman]

When a force acts on a node, the members that come together in that node exert forces on the node, such that the combination of all forces makes equilibrium. The forces acting on the node can be drawn as vectors, and when all vectors are drawn after each other, a polygon has been made, which is the force polygon. When the polygon is closed, the resulting force acting on the node is 0, and the node is in equilibrium.

When the force polygon exists from three forces, there is only one solution for equilibrium, as in a triangle for which all the angles are known, the magnitude of one side determines the magnitude of all the other sides.

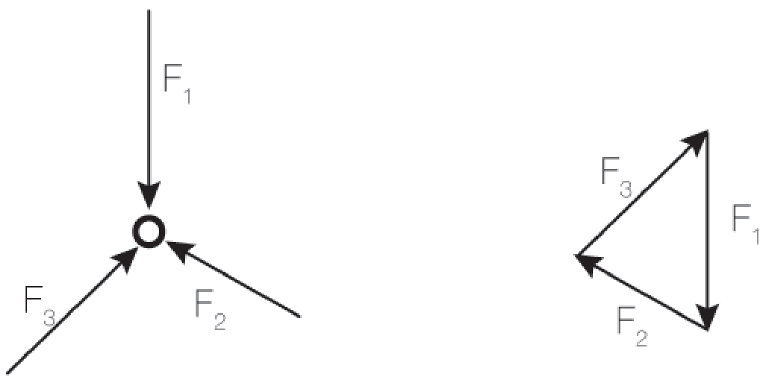


fig. 2.13 a node with three forces acting on it has only one possible force polygon

When however, the force polygon consists of four or more forces, there are more than one polygon that can be drawn, while maintaining equilibrium. The system is then statically indeterminate.

Of course, in reality, there will be only one way in which the forces are distributed through the system. To determine how the forces will be distributed, the complementary energy can be used.

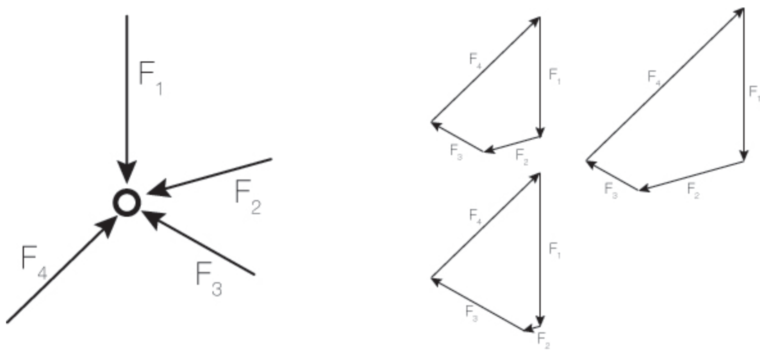


fig. 2.14 a node with more than three forces acting on it has only one possible force polygon

Consider the topology of figure 2.15.
A node is supported by three bars, and a force F acts on the node. The force will be distributed between all three bars, but it is unknown how much force will act in every bar.

fig. 2.15 a force F_1 on a node supported by three bars

To solve this system, the system is regarded as if it is a statically determined structure, by removing the second bar. The force that acts on the second bar is denoted as f and substracted from the force F. The system then consists of a node, supported by two bars, bar1 and bar3, and a force F-f acting on the node. Now, the forces in the two remaining bars can be calculated.

Given the dimensions of the system,
the force in bar1 will be
0,8 (F-ϕ)

The force in bar3 will be
0,6(F-ϕ)

The force in bar2 will be
ϕ

member	N_i	l_i	E_c
1	$0,8(F-\phi)$	3,75l	$1,2\frac{l}{EA}(F-\phi)^2$
2	$0,6(F-\phi)$	5l	$0,9\frac{l}{EA}(F-\phi)^2$
3	ϕ	3l	$1,2\frac{l}{EA}\phi^2$

fig. 2.16 complementary energy expressed in ϕ

For every value of f, the force distribution between the bars will be different. Consequently, the complementary energy for the elongation of the bars will be different as well.

The forces will be distributed in a way which uses the least complementary energy. The system can then be solved by expressing the complementary energy for the entire system as a function of f, by substituting the term N in the formula for the force in the bar, for every bar.

The resulting equation for the complementary energy of the system can then be differentiated, in order find the value for f that results in the minimum energy.

2.4 FORCE DENSITY METHOD

The force density method is currently probably the most used form-finding method for shells and membranes. The force density method calculates the equilibrium state of a predefined initial net structure by transforming a system of non-linear equilibrium equations into a system of linear equilibrium equations by prescribing a constant force-density value. The force density method is based on the mathematical assumption that the ratio between the length and tension within each cable element is a constant value [Lewis 2003].

The net structure consists of a number of points (nodes) which are connected by lines (branches). There are two types of nodes: fixed nodes, and free nodes. For the fixed nodes, the x, y and z coordinates are known. For the free nodes, the x, y and z nodes are unknown. The x, y and z components of the external forces acting on each node are also known.

The method then searches for the equilibrium shape of the tension network. The equilibrium for a node can be described by three equations.

Consider the topology of figure 2.17. In order for the construction to be stable, there must be force equilibrium in each of the free nodes. This force equilibrium can be described for every node, as equilibrium in the x, y and z direction. This presents the following equations:

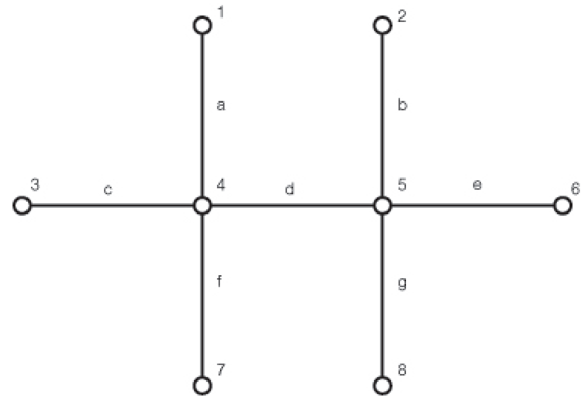


fig. 2.17 planar projection of a system with two free nodes

For node 1
Force equilibrium in the x direction:

$$\sum X;4:\left(x_4-x_1\right)\frac{n_a}{l_a}+\left(x_4-x_5\right)\frac{n_d}{l_d}+\left(x_4-x_7\right)\frac{n_f}{l_f}+\left(x_4-x_3\right)\frac{n_c}{l_c}-F_{x;4}=0$$

Force equilibrium in the y direction:

$$\sum Y;4:\left(y_4-y_1\right)\frac{n_a}{l_a}+\left(y_4-y_5\right)\frac{n_d}{l_d}+\left(y_4-y_7\right)\frac{n_f}{l_f}+\left(y_4-y_3\right)\frac{n_c}{l_c}-F_{y;4}=0$$

Force equilibrium in the z direction:

$$\sum Z;4:\left(z_4-z_1\right)\frac{n_a}{l_a}+\left(z_4-z_5\right)\frac{n_d}{l_d}+\left(z_4-z_7\right)\frac{n_f}{l_f}+\left(z_4-z_3\right)\frac{n_c}{l_c}-F_{z;4}=0$$

For node 2
Force equilibrium in the x direction:

$$\sum X;5:(x_5-x_2)\frac{n_b}{l_b}+(x_5-x_6)\frac{n_e}{l_e}+(x_5-x_8)\frac{n_g}{l_g}+(x_5-x_4)\frac{n_d}{l_d}-F_{x;5}=0$$

Force equilibrium in the y direction:

$$\sum Y;5:(y_5-y_2)\frac{n_b}{l_b}+(y_5-y_6)\frac{n_e}{l_e}+(y_5-y_8)\frac{n_g}{l_g}+(y_5-y_4)\frac{n_d}{l_d}-F_{y;5}=0$$

Force equilibrium in the z direction:

$$\sum Z;5:(z_5-z_2)\frac{n_b}{l_b}+(z_5-z_6)\frac{n_e}{l_e}+(z_5-z_8)\frac{n_g}{l_g}+(z_5-z_4)\frac{n_d}{l_d}-F_{z;5}=0$$

This brings us 6 equations, with a number of 20 unknown variables. Since the number of equations is smaller than the number of variables, the problem cannot be solved.

By specifying a value for the Force Density (Q), which is the ratio between the force acting on a branch, and the length of the branch, the term $\frac{n}{l}$ can be replaced by a known value Q.

The number of unknown variables is thereby reduced to 6. Having 6 unknown variables, and 6 equations, the problem can be solved, providing the x, y and z-coordinates for every free node and the three-dimensional net can be constructed.

If constant force densities are used throughout the network except at the corners and edges where stress concentrations can occur its shape forms a minimal surface between the given boundaries. However, the value chosen for the force density can be completely arbitrary, and changing the value for force density in one or more branches changes the output for the three-dimensional net.

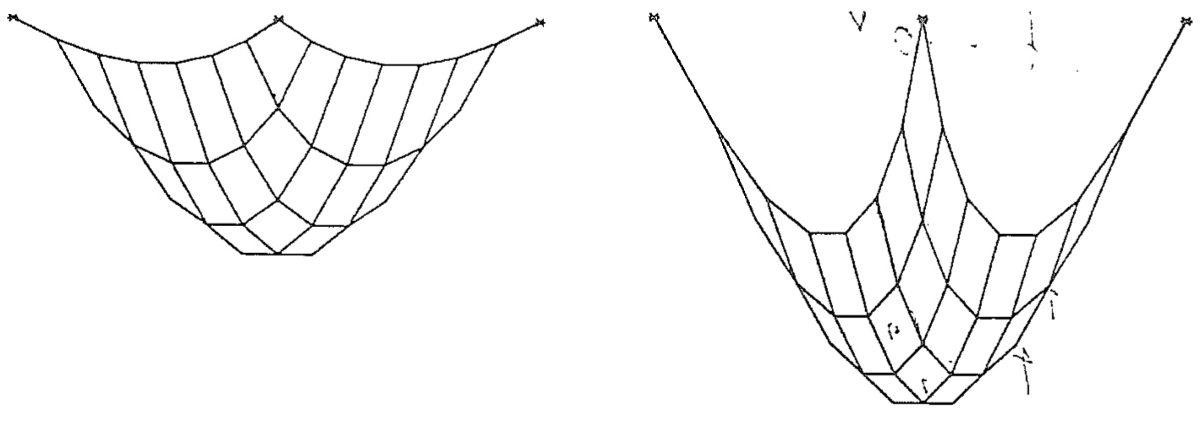


fig. 2.18 different values for force density gives different solutions

2.5 SUMMARY

Graphic statics provides a powerful and intuitive method for assessing thrust lines, where the graphical representation of a force as a vector is directly linked to the force polygon, describing the equilibrium in the system. Changes in either the forces or the force polygons directly influence the other, and thus shows the direct geometrical relation between the force analysis and the load transfer in a structure. However, the method is limited to two-dimensional systems only.

A three-dimensional variant has been developed by Block, thrust network analysis. Thrust network analysis however, does not provide a clear answer to statically indeterminate systems, where multiple solutions for equilibrium force polygons can be drawn. Instead, it defines it as a quality, giving the architect and engineer more freedom to design a funicular shape.
Block does mention that one specific solution can be found using linear elasticity, but does not expand further on how to implement this in the thrust network analysis method.

A statically indeterminate system can indeed be solved using linear elasticity and complementary energy, but the process has to be done by hand, for every knot individually, which is a slow and inconvenient process.

Finally, the force density method can also solve statically indeterminate systems, using force density, the ratio between the length of a branch and the force acting on it. By introducing force density, the force equilibrium in the system can be formulated in a number of linear equations, with an equal amount of unknown variables, meaning that it can be solved. Which values to choose for force density is arbitrary, and different values can give completely different solutions. There doesn't seem to be a straightforward method for deciding which values to use. Also, the force density method is completely computational, lacking the graphical feedback of graphic statics and the thrust network analysis method.

In general, it can be concluded that several very powerful methods exist for assessing funicular shapes. The main aspect that is lacking in the described methods, is the combination between the possibility to solve more complicated systems, and a strong graphical representation of the solution.

3. THE METHOD

3.1 KEY CONCEPTS

3.1.1 Defining the force polygon for a four-valent knot

A four-valent system can be solved by substituting one force for phi and expressing the complementary energy as a function of phi, after which the solution for the minimum energy can be calculated. However, this method is not very convenient as the user has to decide which force is to be substituted, and there is no visual reference to what is being calculated.

To give the user a graphical representation of the calculation, the solutions are drawn as force polygons rather than mathematical formulas.

In a force polygon consisting of three forces, there is only one solution, since all angles are known. One force determines the other two forces. Drawing the force polygon is easy.

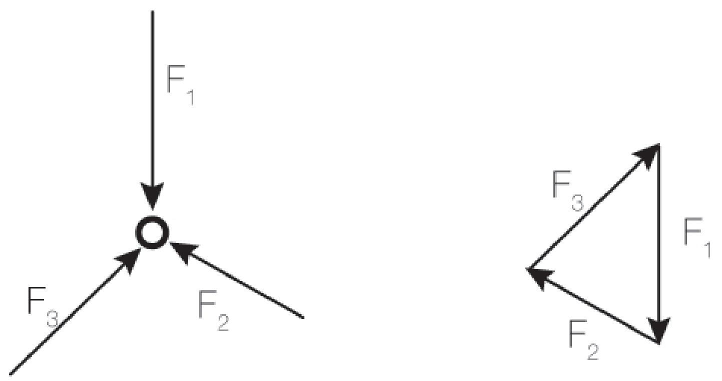


fig. 3.1   a node with three forces acting on it has only one possible force polygon

For a force polygon consisting of four forces, knowing the magnitude of one force is not sufficient for determining the other forces. For any value for F1, there are infinite possible combinations for F2, F3 and F4 to make equilibrium.

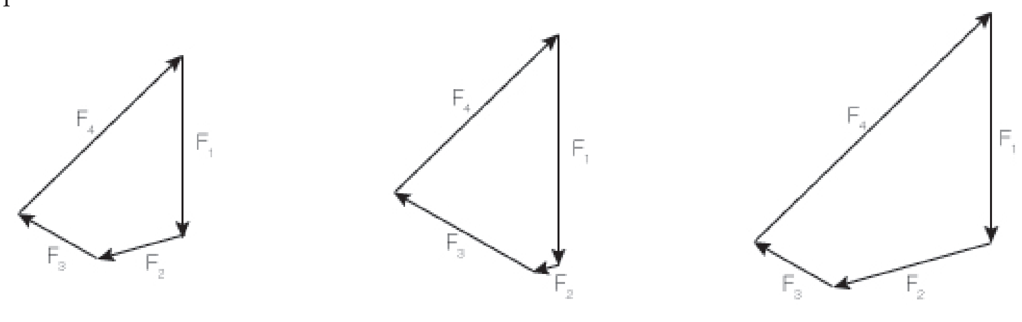


fig. 3.2   for four forces there are multiple ways to make equilibrium

In order to determine all forces, the ratio between two forces needs to be fixed. The two remaining forces can then be calculated, since all angles are known. When F_1 and the ratio F_1:F_2 are determined, F_2 is known. F_3 and F_4 can then be found using the following equations.

$$e = \sqrt{a^2 + b^2 - 2ab\cos\varepsilon}$$

$$\beta_2 = \beta_{12} - \beta_1$$

$$\delta_2 = \delta_{12} - \delta_1$$

$$c = \frac{e}{\sin\gamma}\sin\delta_2$$

$$d = \frac{e}{\sin\gamma}\sin\beta_2$$

So, rather than expressing the solutions as a value for F_1 and the value phi for a certain, user defined member, the solutions are constructed as force polygons, defined by the ratio F_1:F_2. The force polygon contains the magnitude of all the forces, which can then be used to calculate the complementary energy for each solution.

3.1.2 Defining the force density for a four-valent knot

Individual nodes

When several force polygons are constructed for one statically indeterminate system, they are usually not directly comparable. If one solution gives equal forces for all four members in a four-valent system, and a second solution appoints a larger force to one of the members, the forces in the other members will have to become smaller, in order for the polygons to be a solution for the same load.

In figure 3.3, a four-valent knot with a vertical load is considered. Two force polygons are constructed to describe the horizontal components of the axial forces in the members.

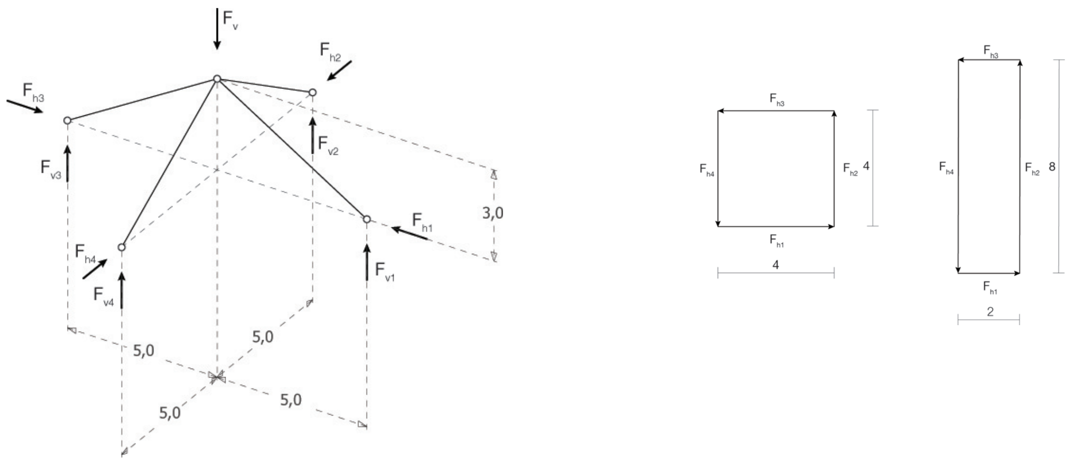
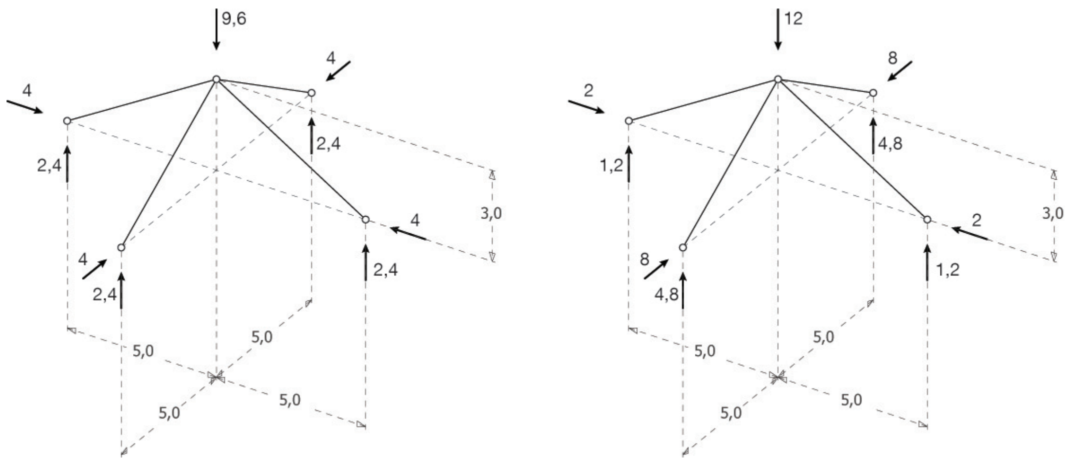


fig. 3.3   an indeterminate system with a single free node and two possible solutions for equilibrium

Since the dimensions of the system are known, using the horizontal forces, the support reactions and the magnitude of the vertical force can be determined.

For the two solutions of horizontal force equilibrium, a different vertical force is achieved. The two force polygons are therefore not solutions for the same problem, and cannot be compared directly.



FD= $\sum \frac{4}{5}+\frac{4}{5}+\frac{4}{5}+\frac{4}{5}=3,2$ FD= $\sum \frac{2}{5}+\frac{8}{5}+\frac{2}{5}+\frac{8}{5}=4$

fig. 3.4   two solutions result in a different force density, and a different vertical force

In figure 3.6, the same 4-valent system with a vertical load is considered. Again, two force polygons are constructed to describe the horizontal force equilibrium. However, in this case, the second force polygon is scaled such that the force density for the complete system is the same for both polygons. Using the horizontal components and the dimensions of the system, the support reactions and vertical load can again be calculated.

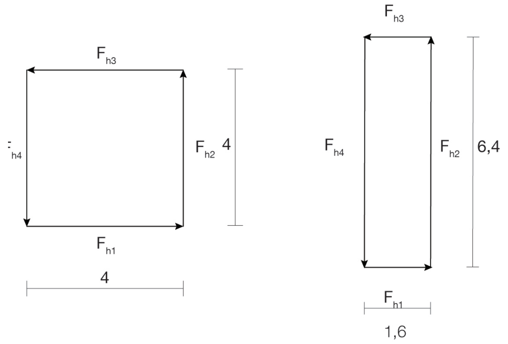
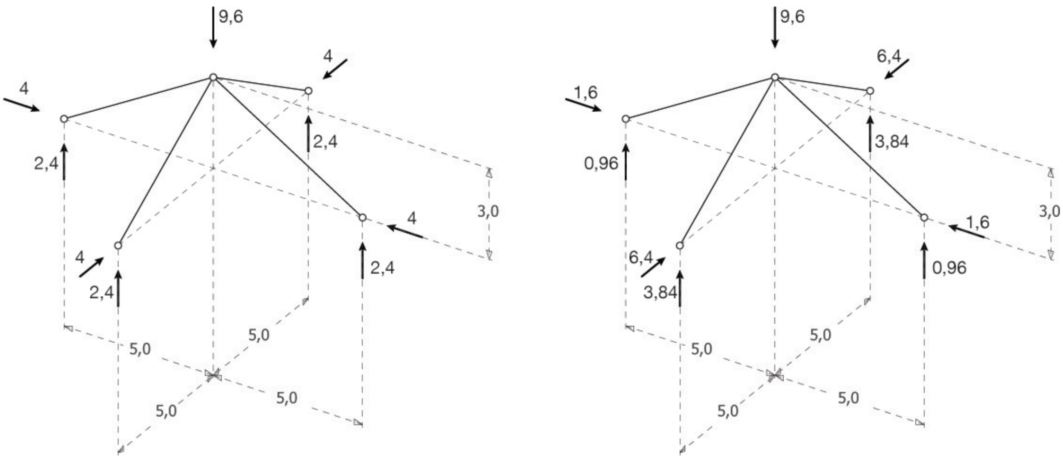


fig. 3.5 the solutions are scaled to match force density



$$FD = \sum \frac{4}{5}+\frac{4}{5}+\frac{4}{5}+\frac{4}{5}=3,2$$

$$FD = \sum \frac{1,6}{5}+\frac{6,4}{5}+\frac{1,6}{5}+\frac{6,4}{5}=3,2$$

fig. 3.6 the two solutions result in the same force density, and the same vertical force

For both solutions for horizontal force equilibrium, the same vertical force is achieved. They are solutions for the same system and load, and can therefore be compared directly.

Systems with multiple nodes

When a system of multiple nodes is considered, multiple force polygons can be pasted together to form a grid. This is the reciprocal grid, as the force polygons are reciprocal figures for the nodes in the original grid. Every node shares a member with the node it is connected to by that member. Therefore, the force acting in that member is also shared by both nodes. The force polygons for those nodes can therefore be pasted together, by overlapping the corresponding forces, which then have to have the same value. Again, infinite solutions for equilibrium are possible, as long as the polygons are closed, and the corresponding forces are of the same magnitude.

To find the optimal solution, all reciprocal grids can again be scaled to a reference force density, and compared for complementary energy. However, for a grid of multiple nodes, the force density is not calculated over all members and forces. Only the support reactions and their corresponding members are to be used. Figure 3.7 shows a system consisting of two, connected nodes, and two possible solutions for equilibrium. When force density is calculated for all members and forces, both solutions have the same result. However, when the vertical components are calculated, it is clear that they are solutions for two different load cases. When both solutions are scaled such that they result in the same force density when it is calculated for the support reactions only, they are solutions for the same load case.

In short: in order to directly compare different force polygons that make equilibrium for a given load on a given system, they have to be scaled in order to match force density in the support reactions. For individual nodes, all forces are support reactions. For a system of multiple nodes, the forces that are shared between two nodes are not to be taken into the calculation, as they are not support reactions.

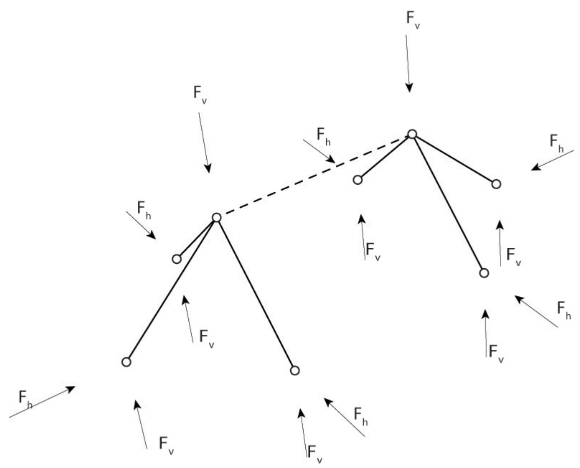


fig. 3.7   the free branch is not to be used in calculating the force density

3.2 PROCEDURE

This chapter describes the main steps in the procedure. For every step, the key steps will be discussed, and the actual actions will be illustrated with an example. The following steps will be described, in order of operation:

- Generating the primal grid
- Extracting relevant information
- Solving the horizontal load distribution
- Adding the vertical loads
- Formulate equilibrium equations
- Constructing the three-dimensional shape

Additionally, some aspects that can still be improved upon are discussed:

- Solving nodes individually
- Adding non-vertical loads
- Solving nodes with a higher valency
- Optimising the scale factor

3.2.1 Generating the primal grid

In the first step of the procedure, the primal grid is defined. The primal grid is considered a planar projection of the funicular shape that will eventually be generated.

This is done by inputting all the fixed nodes and their x, y and z coordinates, all free nodes and their x and y coordinates, and by defining which nodes are connected to which other nodes. Using this information, all the information needed , such as the angles between intersecting branches and their length can be calculated. This information will be needed as input for the following step.

Consider a primal grid consisting of 21 nodes, of which 12 nodes are fixed, and 9 nodes are free. (figure) The free nodes are numbered from left to right, from top to bottom, then the fixed nodes are numbered in clockwise order. The members connecting the nodes are listed clockwise from outside inwards, such that all members containing support reactions are listed first.

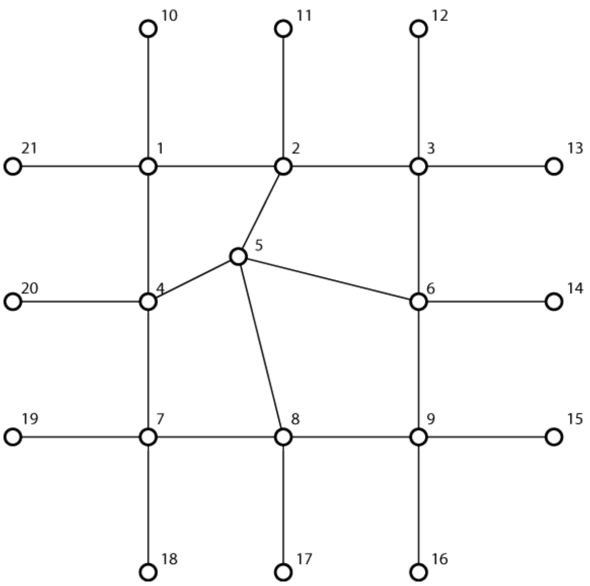


fig. 3.8 the primal grid: nodes

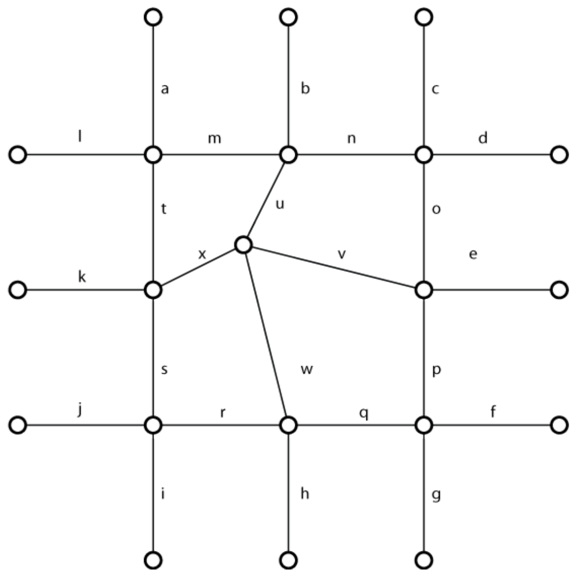


fig. 3.9 the primal grid: branches

3.2.2 Extract relevant information for every knot

A knot is a node and the set of branches that come together in that node. Since the x and y coordinates are known for all nodes, the length of each branch can be calculated using the difference between the x coordinates of two nodes and the difference between the y coordinates. Knowing the coordinates of begin and end points of each branch also means that the direction of the branches are known. Therefore, the angle between two branches can be calculated as well.

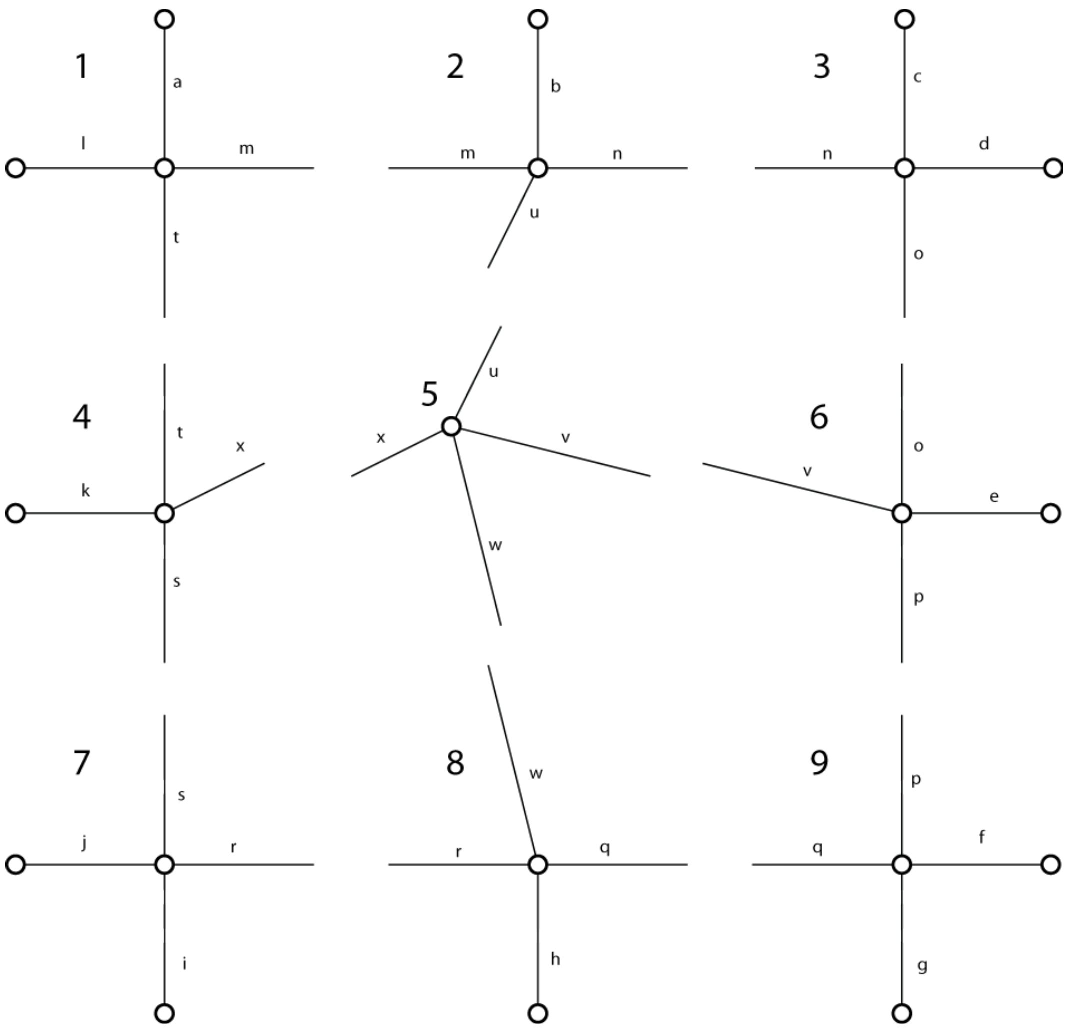


fig. 3.9 the primal grid: extracting information on length, angles and connectivity

3.2.3 Solving the horizontal load distribution

When all angles between members are known, these can be used to draw a reciprocal grid. The reciprocal grid consist of force polygons for every individual node, which are put together by overlapping their corresponding forces. To draw the grid, not all forces need to be determined beforehand. A certain number of forces is needed. In the example, only six forces are needed, for example the forces in a, b ,c ,d, e and f.

When a-f are determined, two of the forces for the force polygon of node 1 are known, and knowing the angles between the forces, the remaining two forces can be constructed. This provides the second force for node 2 and 4, which can then be constructed as well. This then provides additional forces for additional nodes, until the reciprocal grid is completed.

In order to draw a reciprocal grid, an assumption has to be made for (the ratio between) the initial forces. Consequently, different assumptions lead to different reciprocal grids, which are all possible solutions for equilibrium.

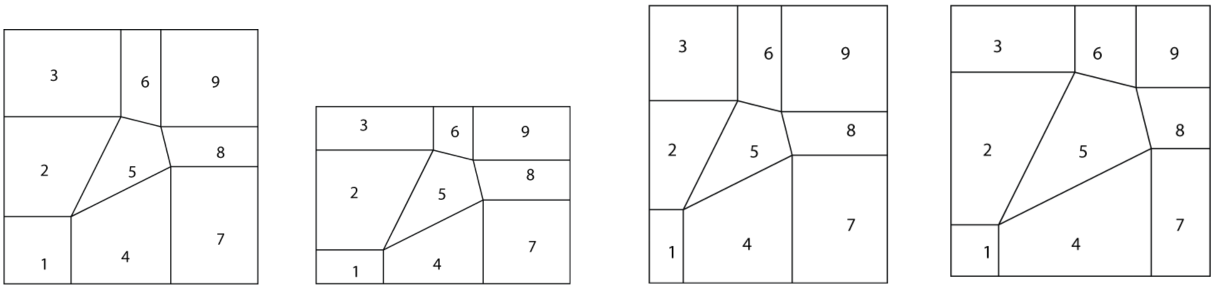


fig. 3.10 there are multiple possible solutions for the reciprocal grid

To determine which reciprocal grid is the optimal solution, they are compared for complementary energy. In order to compare reciprocal grids, they are first scaled to match a reference force density for the support reactions. Then, the complementary energy is calculated for every solution. Whichever solution results in the lowest complementary energy, is the optimal solution.

Since there are infinite possible solutions, and every extra force which is needed to determine the grid enlarges the solution space, this process is better done using an automated solver rather than searching by hand.

3.2.4 Determine the vertical loads

The loads acting on the structure are abstracted to discrete loads acting on the vertices of the grid. Distributed loads are therefore discretized into point loads

3.2.6 Formulate equilibrium equations

For every node, the x and y coordinate are known, as well as the vertical force acting on it. For the fixed nodes, the z coordinates are also known. Now that the exact reciprocal grid has been determined, it can be used to calculate the force density for every member.

Using a similar strategy as in the force density method (and the same as in the thrust network analysis method) the system can be solved by formulating the equations that describe the force equilibrium in the nodes.

The difference between this method and the force density method, is that here, the x and y coordinates for every node are known. Therefore, only the force equilibrium in the z direction has to be formulated. When the equations are formulated, the problem can be solved through a series of matrix operations.

Force equilibrium in the z direction for node1:

$$\sum Z;1:(z_1 - z_{17})\frac{n_a}{l_a} + (z_1 - z_2)\frac{n_f}{l_f} + (z_1 - z_5)\frac{n_j}{l_j} + (z_1 - z_{21})\frac{n_e}{l_e} - F_{z;1} = 0$$

3.2.7 Constructing the three-dimensional shape

When the equilibrium solutions are solved, the z-coordinates for all nodes are known, and the three dimensional force pattern can be constructed.

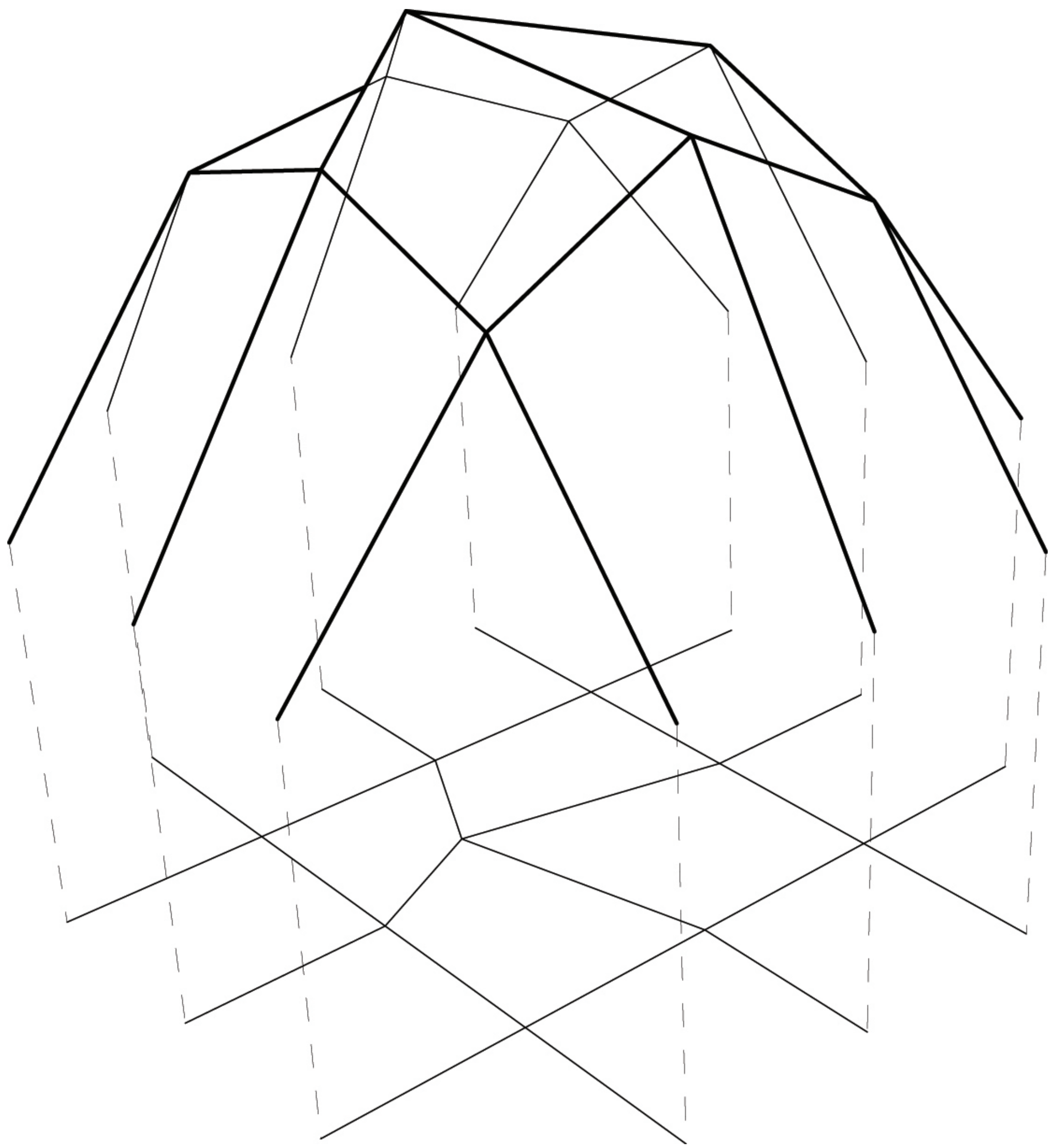


fig. 3.11 using the z-coordinates, the three-dimensional shape can be generated

3.3 FRAMEWORK

Figure 3.12 summarizes the framework

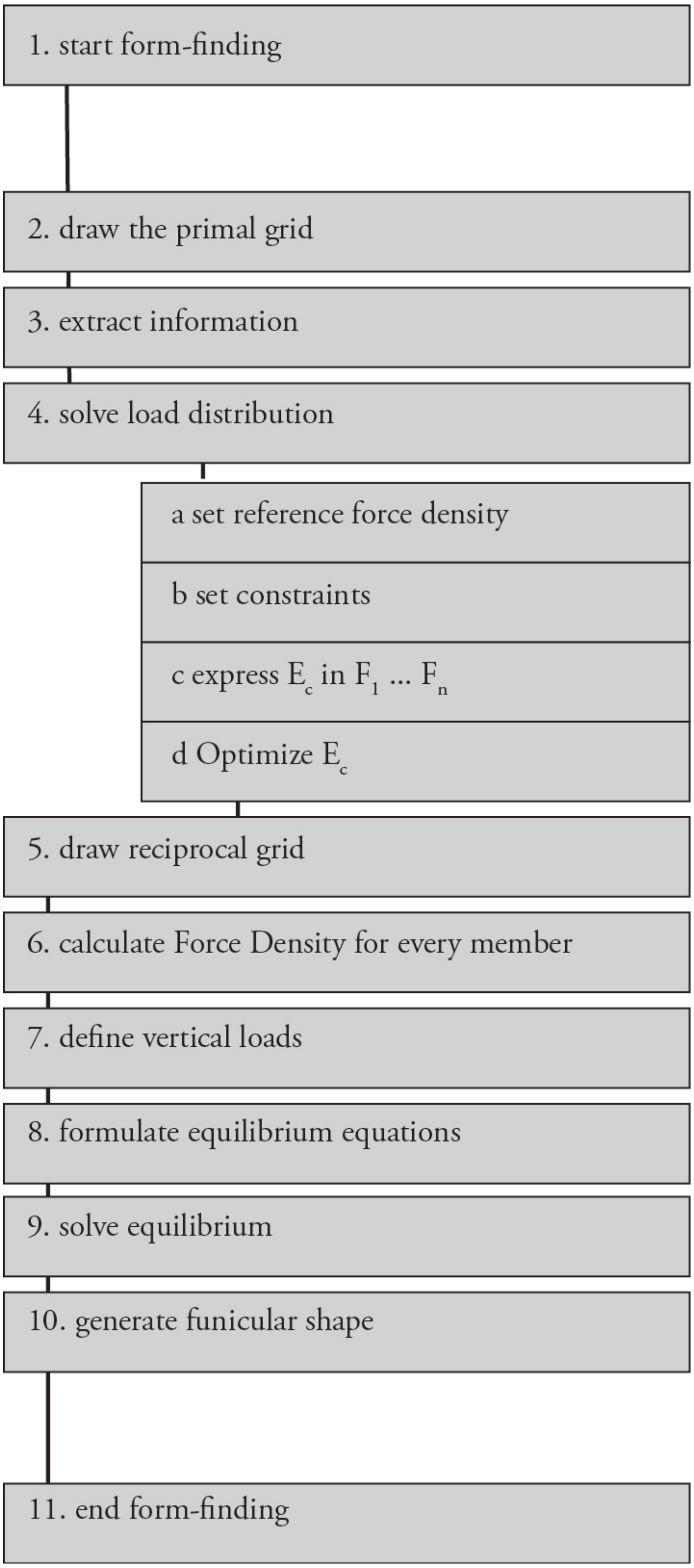


fig. 3.12 step by step flow-chart

3.4 ADDITIONAL CONSIDERATIONS

3.4.1 Scale factor

As the scale factor determines the magnitude of the horizontal forces and the force density in the branches, it also determines the slope of the elements in the three-dimensional structure.

When the scale factor is smaller, the height of the structure will increase, and the axial force in the members will decrease as the horizontal component is smaller. When the scale factor is larger, the structure will be lower, and the axial forces will be higher as the horizontal components increase.

Just as in the graphic statics method, there is an optimal scale factor, providing the optimal height for the structure. This optimal height can be found by calculating the complementary energy for the entire structure over a number of values for the scale factors. The value for the scale factor that results in the lowest complementary energy is the optimal value.

When the structure is meant to facilitate a certain function, there can be a requirement for a minimum or maximum height.

When a specific height is required, the scale factor can be chosen, such that the result has the required height. When the requirement is only a minimum or maximum height, or a range within which the height must be, the scale factor can be optimized for that specific constraint, by again calculating the complementary energy for several scale factors, excluding the one that result in a height that does not meet the minimum or maximum requirements. The optimal result may lay beyond the constraints, but for the given constraints, there can still be found an optimum.

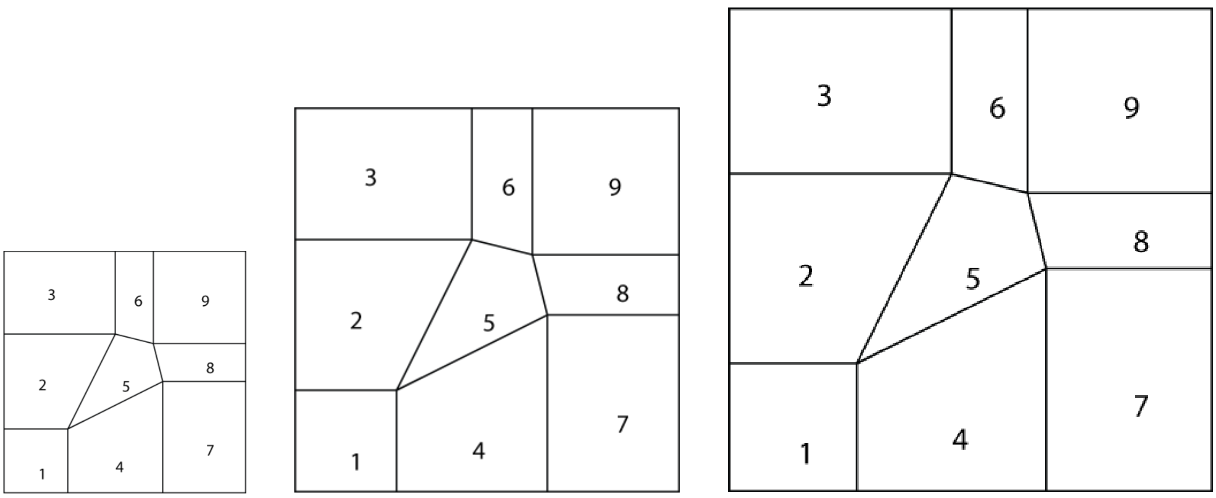


fig. 3.13 the scale factor affects the size of the forces

3.4.2 Non-vertical loads

In reality, loads are rarely exactly vertical. Wind loads occur quite often, and can be significant.

When the vertical and non-vertical loads can be reduced to non-vertical, but parallel, loads, the easiest solution is to, instead of working with non-vertical loads, rotate the entire system, such that the loads become vertical, and the grid is under an angle. A new projection is then made, giving the new x and y coordinates for all nodes, and the z coordinates for the fixed nodes has to be adjusted to match the inclined grid.
The system can then be solved as if it were a regular vertically loaded system. When the z coordinates are extracted, the 3 dimensional shape can be constructed, and the system can be rotated back to its original position.

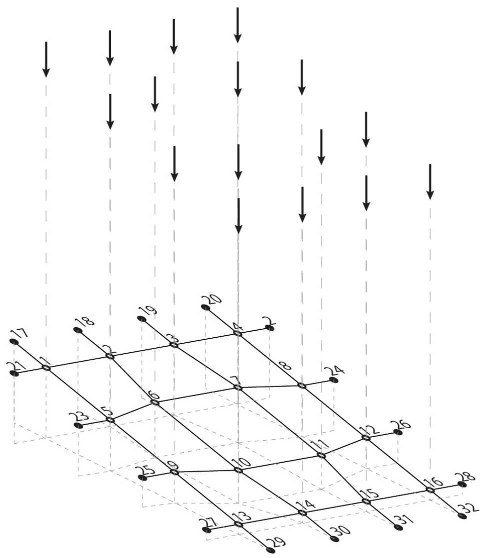


fig. 3.14 when a load acts under an angle, the primal grid is rotated, and a new projcetion is made

When this is not the case, the horizontal component of the non-vertical forces can be drawn in the reciprocal figure, after which the system can again be solved as if it was vertically loaded.
However, before adding the horizontal components of the non-vertical loads to the dual grid, the scale factor will have to be determined first. Searching for the optimal scale factor can then become a time consuming and tedious task, as the extra horizontal components have to be added again, after adjusting the scale factor.

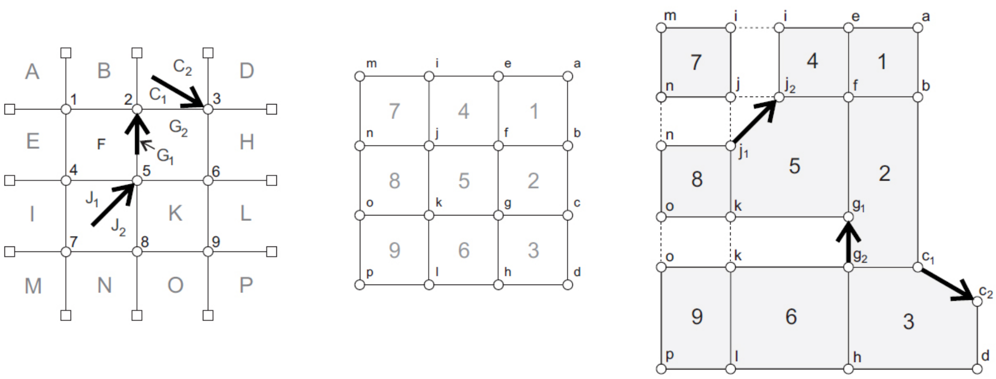


fig. 3.15 non-vertical forces can be drawn into the reciprocal grid. image: Philippe Block

3.4.3 Nodes with a greater valency

When five or more members come together at a node, the problem cannot be solved by the described method for solving four-valent knots. The method can be adjusted to accommodate higher valencies. For a four-valent knot, F1 is set and F2 is determined first through choosing the ratio. They can then be used to determine the two remaining forces F3 and F4.

In the case of a knot with a valency of five, there are three forces that need to be determined before the remaining two can be calculated. This can be done by again setting F1 to 1, assuming a ratio for F1:F2, and then assume a ratio for F3 compared to F1 and F2 combined. The two remaining forces, F4 and F5 can then be determined, and a force polygon can be drawn and scaled.

By keeping the ratio for F1:F2 constant, and changing the ratio for F3, multiple force polygons can be constructed, scaled, and compared in order to find the optimum ratio for F3, for that specific value for F1:F2.

The ratio F1:F2 can then be changed to a different value, after which the optimum ratio for F3 can be sought again. The force polygons that are optimum for the different F1:F2 ratios can be compared again, to find the optimum ratio for F1:F2.

While this method provides a solution, the calculation times increase exponentially when the valency grows. A three-valent system requires only one calculation to generate the triangular force polygon, as there is only one solution. If we assume that the method needs to calculate 50 different values for the F1:F2 ratio in a four-valent system, a five-valent system requires 50 calculations for F3, for 50 values of F1:F2, increasing calculation time again 50 times.

Instead of doing 50^6 calculations for knots with a valency of 9, a smarter approach would be to take the knot apart, and instead model several three-valent nodes close to each other, which should be a close enough approximation.

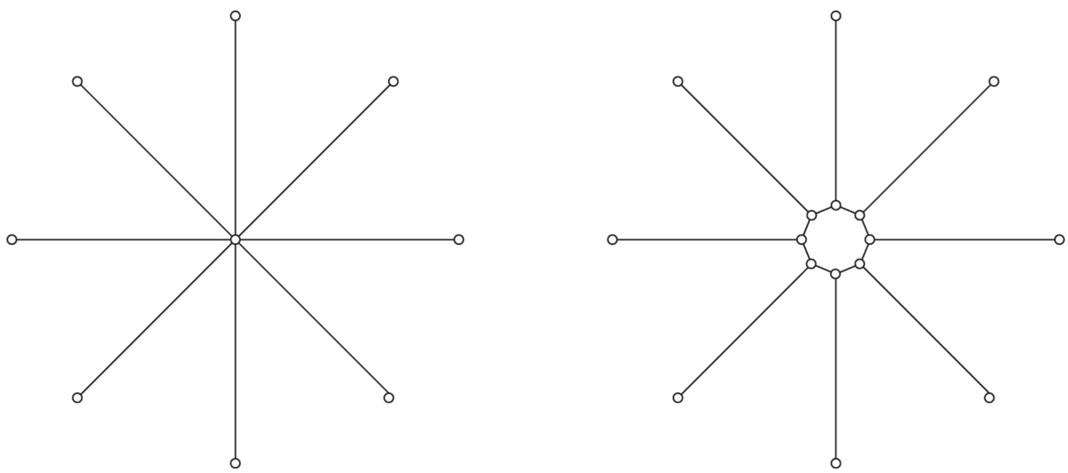


fig. 3.16 nodes of a high valency can be converted to a group of lower valent nodes

3.4.4 Determining vertical loads

Assuming that the structure is loaded by an evenly distributed vertical load, the nodes from the primal grid can be used to make a voronoi diagram to determine which parts of the distributed load act on which nodes. Extra external loads can then be added to the corresponding nodes. All point loads can then be added together to get the final vertical load for each node.

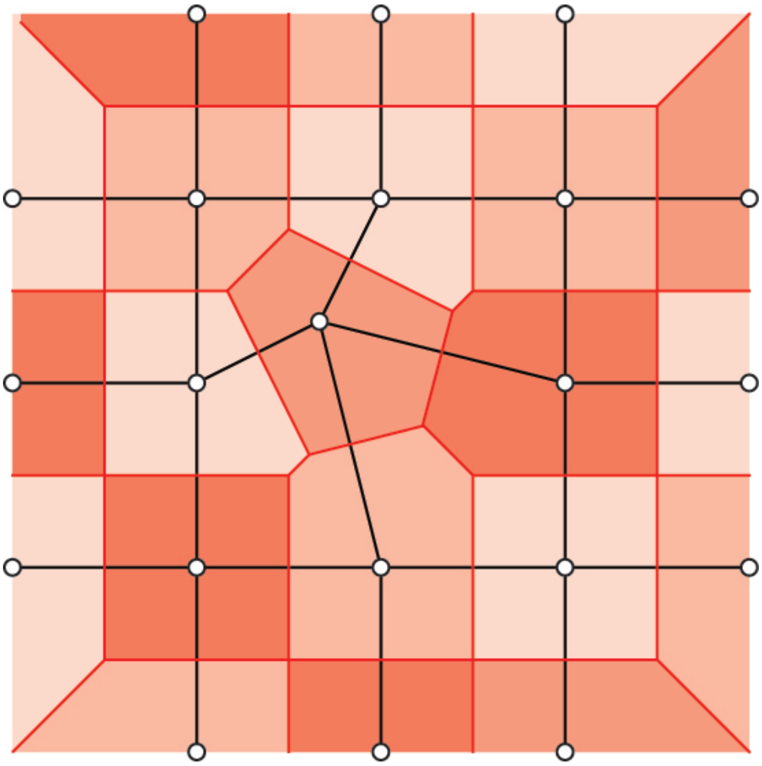
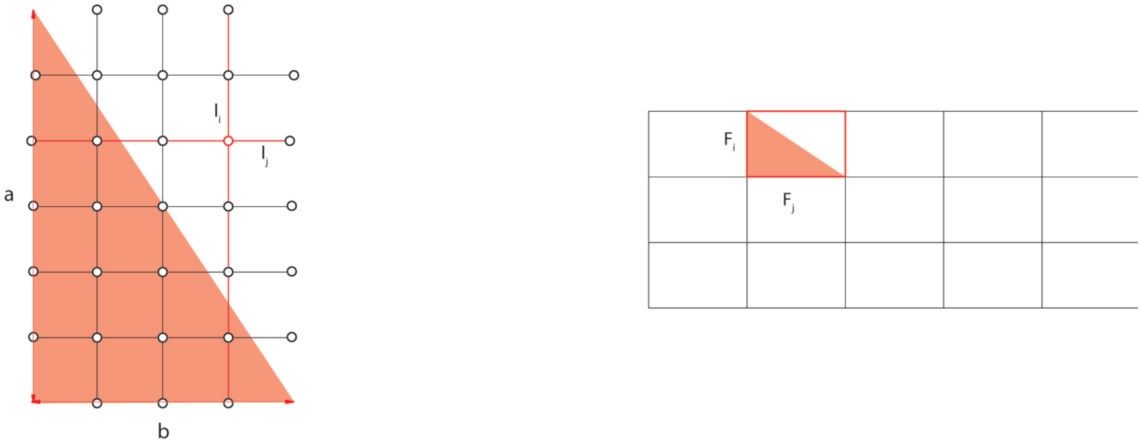


fig. 3.17 determining loads through a venn-diagram

3.4.5 Solving nodes individually

While calculating the entire reciprocal grid at once is a valid approach for solving the horizontal load distribution, its large solution space requires an automated calculation method, using advanced algorithms to efficiently find the solution. It is something which is effectively undoable by hand.
A preferable method would be to find a relation between the geometry of the grid, and the shape of the force polygons, such that no optimization, or at least a simpler one, is needed.

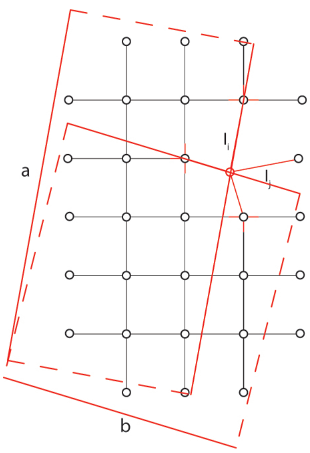
In a regular grid, the magnitude of the forces in the force polygon for a specific node is directly related to the geometry of the grid. To be more precise, the distance from the node to the supports it transfers its load to, the loadpath.



$$a/b = F_j/F_i$$

fig. 3.18 for regular grids there is a direct relation between the length loadpaths and the force polygon

For an irregular node, it is more difficult to find a relation between the geometry of the node, and the shape of the force polygon.



$$a/b = F_j/F_i \ ?$$

fig. 3.19 for irregular grids the relation is not evident

PART III
ADAPTIVITY

4. REFERENCE FRAMEWORK

This chapter will focus on the reference framework for developing a form-active structure, as outlined in objective c.:

"Define a reference framework, stating which adaptive systems already exist, and/or which non-adaptive systems can be made adaptive, and how they can be used to facilitate and exploit the configurations that fit the shapes acquired in 'b)'"

The reference framework acts the starting point for developing the form-active structure, that is specifically tailored to facilitate the configurations that are generated by the thrust diagram tool. A categorization will be made for adaptive structures and structures that may not exist in an adaptive form, but can be adjusted to become adaptive. The categorization will serve as a reference pool from which a suitable system can be taken, depending on the defined criteria, and used directly, or improved upon.

The categorization is not meant to be exhaustive, but does intend to provide the most relevant systems from a broad range, thus giving a representative image of the possible solutions.

4.1 NOMENCLATURE

It has to be noted that over the range of papers regarding this subject, various terms are used to describe the same things, or the same terms for different things, depending on the author and specific context. Therefore, the terms used in this chapter and their intention will be outlined, to prevent misunderstandings.

Adaptive

Adaptive systems or structures, or are generally systems that can adapt. The way in which they adapt can be multitude, and the things they adapt to as well. In this thesis, adaption refers to the ability to change the geometry of the structure. It is used as a synonym to reconfigurable systems.

Form-active

Form-active refers to adaptive structures that do not change their geometry, but do so in an active manner. Instead of reconfiguring as a natural result of the external influences, the form-active structure actively monitors those influences, and decides the proper response. As an example, the form-active structure will reconfigure itself to lean into the wind.

Kinematic

Kinematic structures are structures with a single kinematic degree of freedom (rotational or translational); meaning that the position of one node relative to the others determines the geometry of the structure [Hanaor]. Kinematic systems are not to be confused with reconfigurable systems. A reconfigurable system is any system which can reconfigure its geometry. All kinematic structures are reconfigurable, however not all reconfigurable structures are kinematic.

4.2 CATEGORIZATION

Adaptive systems can be subdivided into two main categories:

discrete systems
continuous systems.

This distinction is based on the deformation of the system, which can be either discrete, or continuous.

In a continuous system, the deformation occurs throughout the material. A rotational deformation is achieved by bending of the element. A translational deformation is achieved by positive or negative elongation of the element. This deformation is continuous, in that it occurs everywhere throughout the system. (figure)

In a discrete system, the deformation occurs at specific points instead of throughout the material. This can be either a rotational connection between members (hinge) or a translational connection. The rotation or translation occurs at those points only, the material itself is considered stiff.

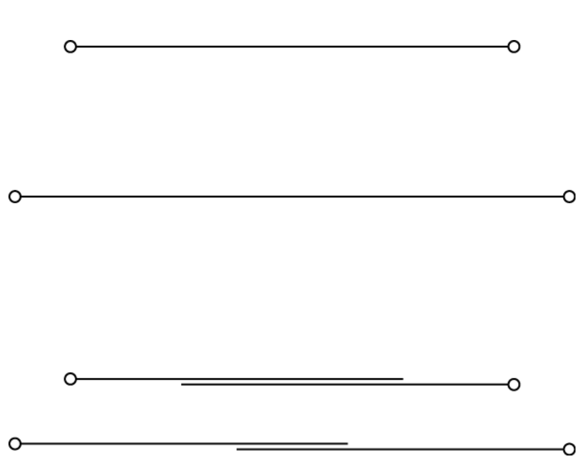


fig. 4.1 continuous elongation versus discrete elongation

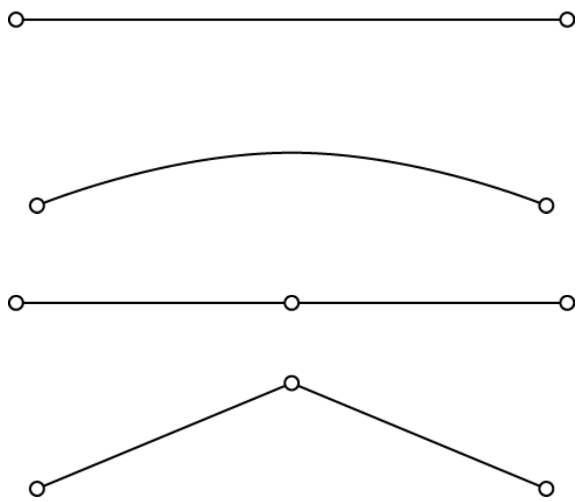


fig. 4.2 continuous curvature versus discrete curvature

The category of discrete systems can be subdivided again, into two smaller groups, based on the elements used in them. The first group is systems which consist of plates and have linear or point connections; the second group is that of systems which consist of bars and have point connections.

The category of continuous systems consists mainly of systems in which a membrane is used.

For each group, several typologies will be described. Since most systems only exist in a static form, the static structure will be described, followed by possible ways of making it (more) deformable.

4.2.1 Discrete systems

In a discrete system, loads are carried through elements which are attached to each other by either rotational and/or translational connections. The deformation occurs at these connections only, rather than throughout the material. The elements can either be lattices or plates.

Lattice structures
Lattice structures consist of skeletal elements which are connected through joints. These structures can be single layered or multiple layered.

Single layered lattice structures are often used to build a polygonal representation of a curved design (fig. 4.3). The lattices or bars are all within the same, curved, surface. Constructional height is the same as the height of each individual bar. Often, a triangular grid is used to create a stable structure

There are two approaches for making an adaptive single layered lattice.
First, instead of a triangular grid, a quadrangular (or more) can be used. This has the result that the structure is instable when all connections are hinges. The shape of the structure can then be altered by actively changing the rotation of the hinges and fixing the joints when in the correct position.

Second, a triangular grid can be used, with members that can actively elongate and shorten to change the geometry of the grid, and thereby changing the shape of the structure. Rotational connections have to be used again, but in this case, the joints do not have to be fixed, as the structure is stable when the length of the members has been fixed.

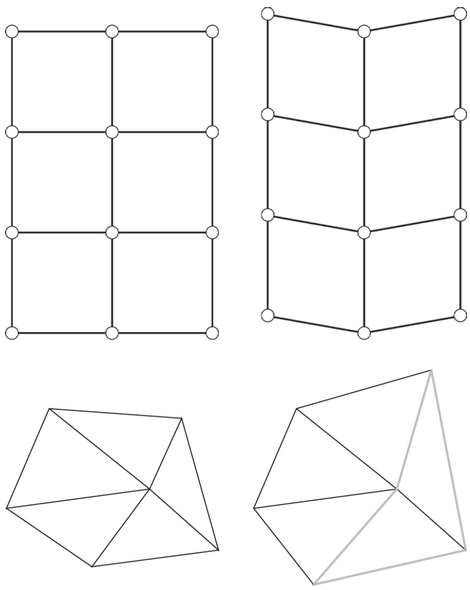


fig. 4.2 adaptivity through hinges and/or elongating bars

fig. 4.3 DZ Bank, Gehry

With multiple layered grids, the construction height is increased by placing several (usually two) layers above each other with some, and connecting them to act as one structure. Common examples of these structures are spaceframes, which are static two layered grids.

Double layered grids can be adaptive when members that can elongate are introduced in one of the layers, or in both layers. When the bars in the bottom layer shorten, while the corresponding bars in the top layer are kept constant, or elongate, the structure will curve inwards.

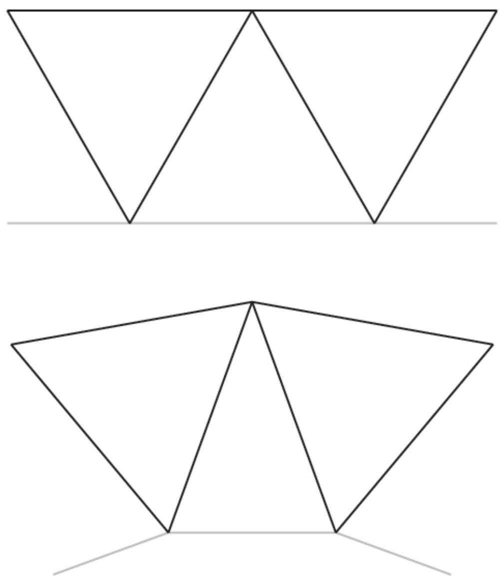


fig. 4.4 adapting a double layered grid by elongating certain members

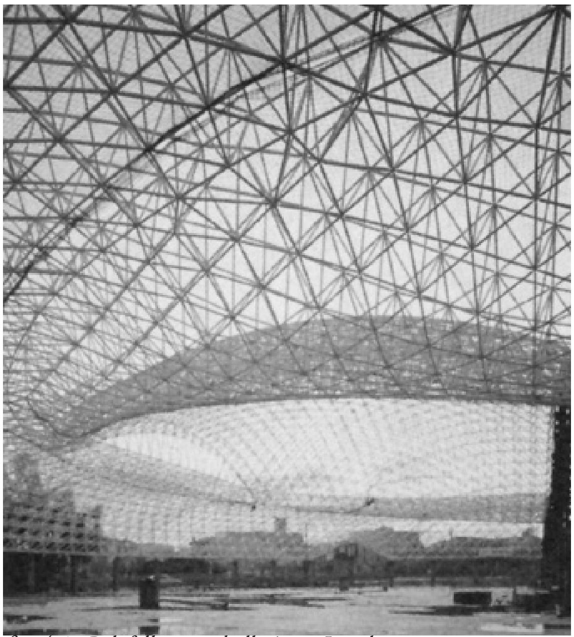


fig. 4.5 Palafolls sports hall. Arata Isozaki

Variable geometry trusses are the linear equivalent of the double layered lattice structure. Variable geometry trusses are trusses in which several elements can elongate, usually telescopic bars. The joints connecting the elements are hinged. Variable geometry trusses can change their configuration by elongation of specific elements. Depending on the geometry and the number and position of elongationable elements, the variable geometry truss can have curvature and rotation. In order for the variable geometry truss to be stable, either the joints will have to be fixed once the desired configuration is achieved, or all elements must form triangles, such that the system becomes stable, and only the telescopic bars need to be fixed to their proper length.

fig. 4.6 variable geometry truss

A variation on the double layered bar grids is cable strut system. In this system, the rigid members that are loaded only with tension, are replaced with cables.

A special type of cable strut system is the tensegrity structure. In tensegrity structures, no compressive member touches any other compressive member. Everything is held together by cables, and by removing the tension from one or multiple cables, the geometry of the structure can be altered, or the structure can collapse. This can be achieved by actually releasing tension on the cables, or by adjusting the length of the struts, resulting in more or less tension in the cables.

Pantographic structures, also known as scissor structures, "consist of two straight bars connected through a revolute joint, called the intermediate hinge, allowing the bars to pivot about an axis perpendicular to their common plane" [De Temmerman]

They are special in that they can expand and contract, due to their scissor like connections. These structures are deployable; the structure can fold to a relatively small package, or unfold into a much larger structure.

Pantographic structures are reconfigurable, in that they change their configuration from their folded state to their deployed state, and all configurations in between. The freedom for reconfiguration is limited by the property that the position of one node determines the position of all other nodes. Pantographic structures are usually meant to be used as static structures. They can be transported as a relatively small package to their site, where they are deployed once, and then fixed to become static.
A larger variety of possible configurations can be achieved by splitting the structure into multiple groups of pantographic elements, which can fold and unfold independent of each other.

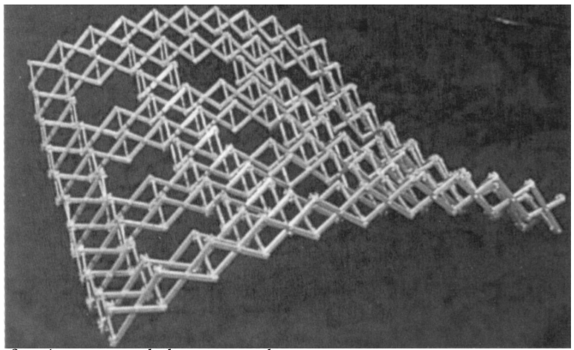


fig. 4.7   expanded pantographic structure

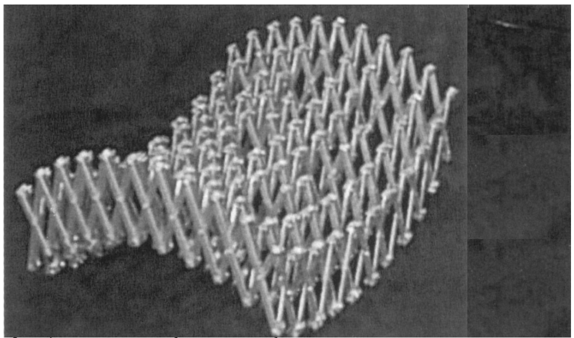


fig. 4.8   contracted pantographic structure

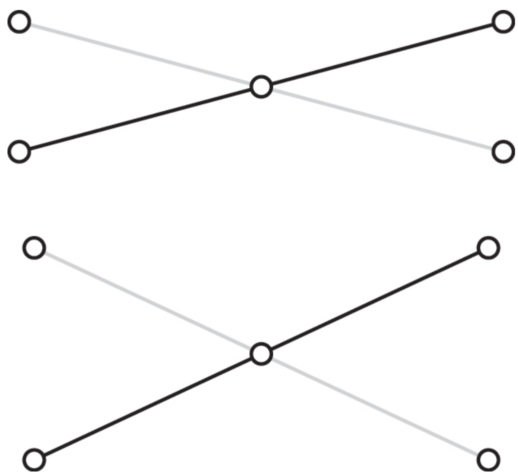


fig. 4.9   pantographic structure principle

Plate structures

Plate structures are structures made of multiple plates, connected to each other along the edges, or at certain points.

Static plate structures are comparable to single layered lattice structures, in which the polygonal shapes, created by the members, are filled with material. The connection between the plates can either be hinged or fixed. When the connections are fixed, the structure will be stable. When hinged, the structure has to be double curved in order to be stable.

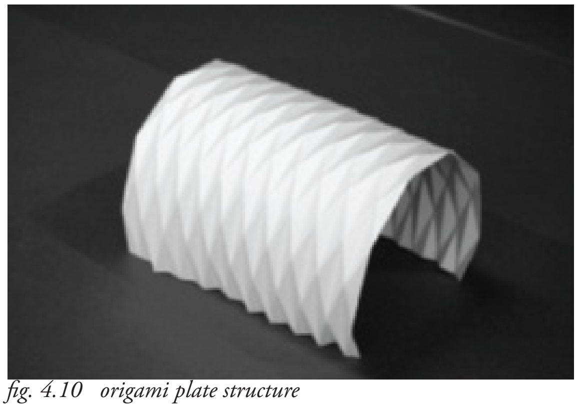


fig. 4.10 origami plate structure

In order to make an adaptive plate structure, the same principles apply as with the single layered lattice structures. By using rotational connections, the structure, when it has the appropriate geometry, can deform by folding, like origami.

Theoretically, a plate structure can also be made deformable by using elements which can elongate and shorten. Where lattices are one dimensional, plates are two dimensional, and depending on the desired configuration, the elements will have to elongate in two directions.

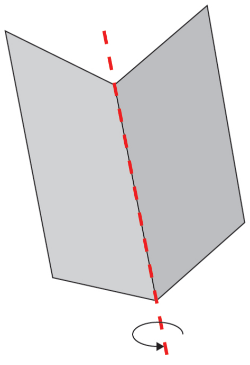


fig. 4.11 linear hinge

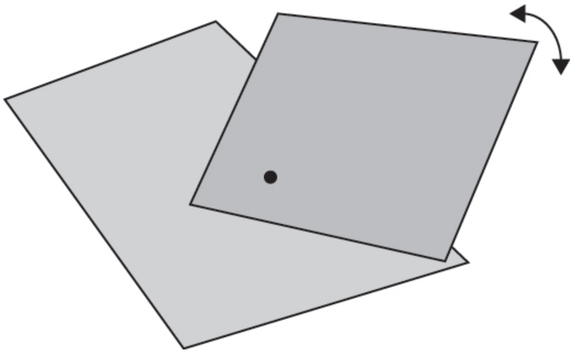


fig. 4.12 pointlike hinge

4.2.2 Ccontinuous systems

In continuous systems, the deformation is continuous, in that it occurs throughout the material. In building practice, continuous systems are always tensioned membrane structures.

No structure can be purely tensional: some compressive elements have to exist to maintain equilibrium [Hanaor]. This compressive element can either be rigid elements, i.e. bars, or compressed air.

Pneumatic structures

In a pneumatic structure, the compressive force is obtained by air pressure. Within pneumatic structures, two types can be distinguished: low pressure structures, and high pressure structures.

In a low pressure structure, the air in the functional space, covered by the membrane, is pressurised. The force created by the pressured air tensions the membrane. The needed pressure is relatively low.

In high pressure pneumatic structures, the structure consists of closed cellular spaces, filled with air under a relatively high pressure. These spaces are not part of the functional space, which is not pressured, though it can be.

Pneumatic structures are generally static, but can be deformed by changing the air pressure inside the cellular spaces (in a high pressure structure), or by manipulating the shape with elements such as cables, struts and bar grids. As the stability and rigidity is obtained by tensioning the membrane through internal air pressure, changes in the external air pressure can influence the shape of the structure. When a wind load is introduced, the asymmetric change in air pressure will cause the structure to change shape, more so than for conventional rigid structures.

fig. 4.13 High pressure pneumatic structureYellow Heart. Haus-Rucker-Co

Tent and ribbed structures
In tent and ribbed structures, the compressive force is obtained by using skeletal elements which interact with the membrane.

The skeletal elements in tent structures are usually masts, which are external to the membrane surface. In ribbed structures, the skeletal elements are usually part of the surface and interact structurally with the membrane.

These structures are also usually static. They can be deformed by altering the geometry of the skeletal elements, so that the forces that tension the membrane change.

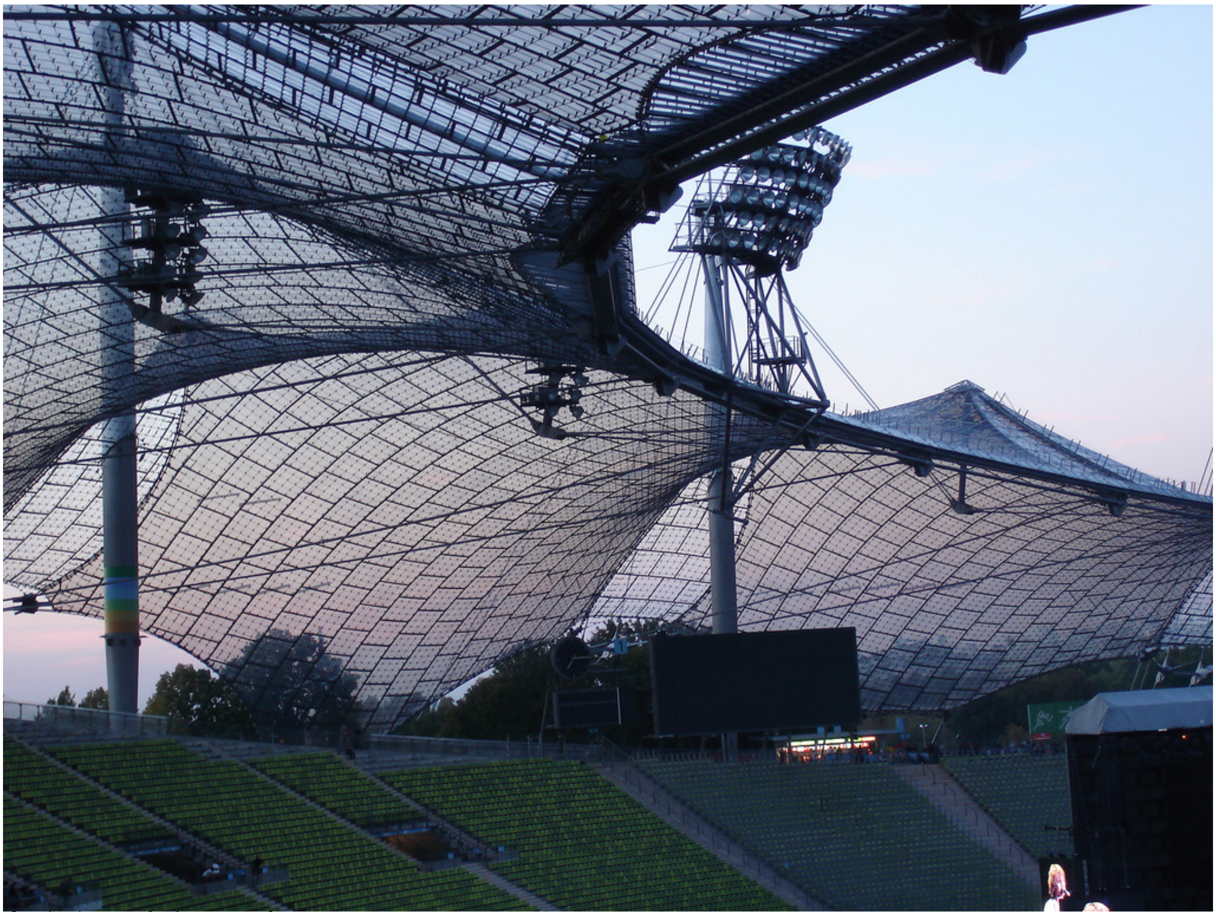


fig. 4.14 Munich olympic stadium, Frei Otto

5. APPLICATION OF A SYSTEM

Using the thrust network tool, a series of funicular shapes can be generated that have to be facilitated by the adaptive structure. From the categorized systems, one system, or a combination of systems can be chosen to develop the form-active structure.
After choosing a system, and knowing the method for reconfiguring it, a thorough analysis can be made of the exact requirements for the structure.
Knowing the requirements, the existing variants of system can be analyzed, and checked if a suitable variant is readily available. When this is not the case, the key concept for operating the system and its limitations are to be analyzed, after which an improved variant can be developed that can be used for the specific task.

5.1 SELECTION CRITERIA

When designing an adaptive structure, the structure will often have a specific function, which sets certain requirements. Using these requirements, a set of selection criteria can be generated. Based on those criteria, some systems may prove to be more suitable than others, and an informed decision can be made.
For this thesis project, the form-active structure has no specified function. The aim is to give an example of one possible way, which may not be the optimal option for such a structure, but at least a possible one. The only criterion therefore is that the system can be adjusted to facilitate the funicular shapes required for optimal load transfer. Theoretically, every system from the categorization can be made suitable.

Therefore, the choice for a specific system has been made based on practical criteria. Since the objective is to develop a descriptive virtual model, a system which can be described more easily is preferred over more complicated systems. The main selection criteria are therefore

1. a system for which the description of the geometry and configuration of the system must be suitable for description in the software package.

2. the number of controls needed to describe and adjust the configuration of the system is preferred to be lower, in order to keep the virtual model manageable.

3. the number of different relations that need to be described is preferred to be lower.

4. the structure must be feasible

Criterion 1 is not so much a property of the system itself, as it is dependent on the ability of the user to describe certain configurations and actions in the software. Criteria 2 and 3 are meant to keep the focus on the linking the system to the script that generates the required configurations, rather than to spend too much time on the problems for describing the system in the first place.

The continuous systems require tensioned membranes. The relation between the tension in the membrane, and the resulting shape is complicated and requires the implementation of a structural analysis to calculate that shape, which is beyond the scope of the thesis. Therefore, using a discrete system is in this case more appropriate.

From the discrete systems, single or double layered grids or trusses are more suitable for the required configurations than plate structures, for which there are few precedents that can be referred to, and no precedents that show the adaptiveness that is required. While pantographic systems are relatively easy to describe and control, the funicular shapes that are required cannot be described a number of configurations in between two extremes.

Variable geometry trusses have been chosen over single and double layered grids, on the basis that while the controls for changing the geometry are similar, the VGT needs a lot less elements and is much easier to describe and control than the double layered grid, and the reconfiguration of a single layered grid seems less feasible due to snap-through behaviour during the active reconfiguration.

Thus, a grid of variable geometry trusses will be used, mainly for practical reasons, while noting that other systems may be just as feasible.

5.2 REQUIREMENTS

As the adaptive system is a discrete system, deformations occur at the connections between elements only. The deformations that may need to be possible are elongation, curvature and twisting.

To analyse the extent to which the system needs to facilitate these deformations and where, the developed method for generating funicular shapes is used to generate several of the extreme shapes.
For a selected group of knots, where two VG trusses intersect, the geometrical configuration has been analysed for every shape. The differences between the different configurations were then measured and documented.

Three aspects were documented. The first is the length of the branches between two nodes. As the structure changes shape, the individual nodes change their position, which changes the distance between the nodes.
The second is the change in curvature in each arch. At every vertex, the truss elements meet under an angle, which is different depending on the configuration. The minimum and maximum angle determine the variance. The elements that connect in the joint have to accommodate the variance.

The third aspect is twist. Each joint has a local coordinate system, defining an x y and z direction. This coordinate system is determined by the geometry of the structure, and changes when the configuration of the structure changes. The truss elements in between the joints also have a local coordinate system. The x-axis lies parallel to the branch connecting the joints. The z-axis for the joints may not have the same direction. The z-axis for the branch is the medium between the z-axes of the two joints. Twist is the differencein rotation around the x-axis between the z-axis of the joint and the branch.

The results show that the curvature difference for one knot has a maximum of 40 degrees. This means that when all knots can facilitate a curvature between 180 and 140 degrees, the VG truss can assume all required curvatures. The required elongation between two knots can be as high as +130%. The maximum twist is less than 15 degrees.
Curvature has to be facilitated at the joints. Twist and elongation can occur throughout the entire branch.

5.3 PRECEDENT ANALYSIS

Currently there are two precedents for variable geometry truss systems.

-prismatic variable geometry truss
-octahedron variable geometry truss

The prismatic VG truss has been applied at the movable monument for the 2005 Expo, and consists of a prismatic triangular truss with extensible actuators, so the monument's shape can be changed variably by controlling the length of each of its extensible actuators [Inoue].
The actuators are located in the longitudinal members of the truss, enabling it to change from a straight line to a three-dimensional curve.

The octahedron VG truss is originally developed as a deployable truss structure, but has great potential as a permanent adaptive structure. It consists of a series of octahedral truss modules, for which the lateral truss members are extensible actuators [Miura].
Varying the length of the actuators, allows the VG truss to follow a three-dimensional curve and change its length.

Current elongable members are limited to a maximum extension of about 30%, before they lose their structural capacities. This is related to the nature of the extension, which is achieved by telescopic bars. After extending a bar, the overlap between the inner and outer bar needs to be large enough in order to preserve the stiffness of the telescopic bar as a whole [turrin].

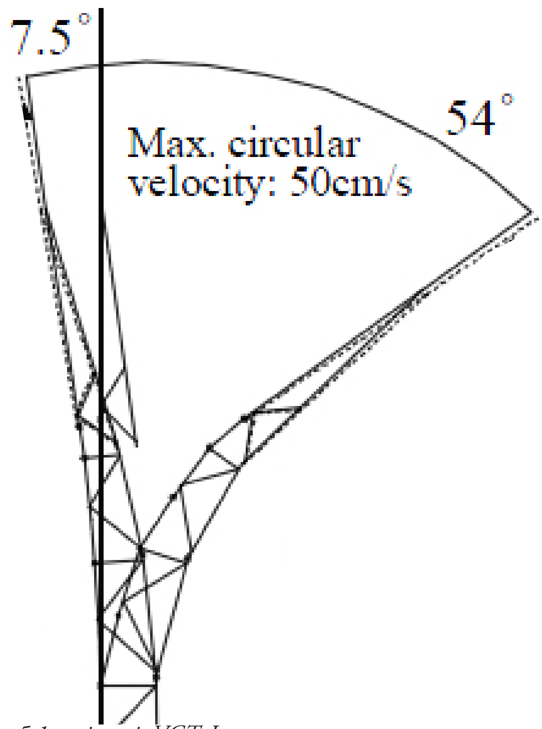


fig. 5.1 prismatic VGT. Inoue

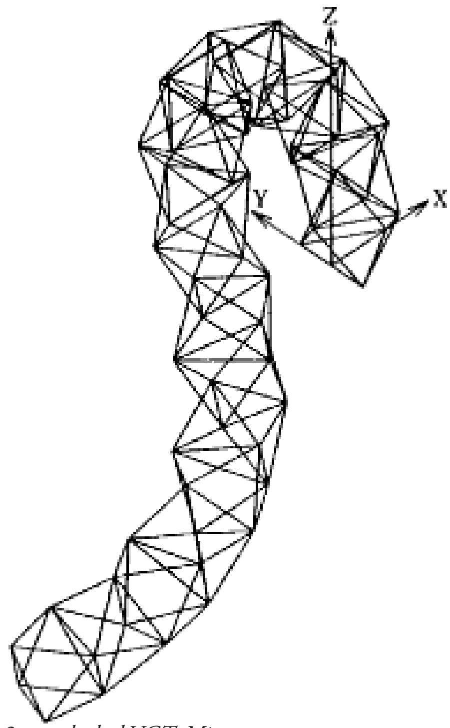


fig. 5.2 octahedral VGT. Miura

5.3.1 Prismatic VG Truss module

The prismatic vg truss module can be regarded as two equilateral triangles, for which every corner point is connected to two other corner points. The members connecting the corner points are supplied with an extensional actuator. By elongating specific members, the truss module can change its curvature, length or twist.

Considered is a module with side faces of 0,5 by 0,5 meters, for which the longitudinal and diagonal members can elongate.

Twisting is possible by elongating the longitudinal members, while keeping the diagonals' length constant, or vice versa. The total length of the module will then grow or diminish. This is not convenient. Therefore the module will be regarded as if the elongation will be half of its maximum. The minimum length of the longitudinal members will then be 0,435m; the maximum will be 0,565m. The maximum twist in one direction is limited by the geometry, which will intersect with each other at a rotation of 60 degrees.

The maximum curvature change is about 16,7 degrees degrees. This is achieved by the maximum length of two longitudinal members, and the minimum length of the third.

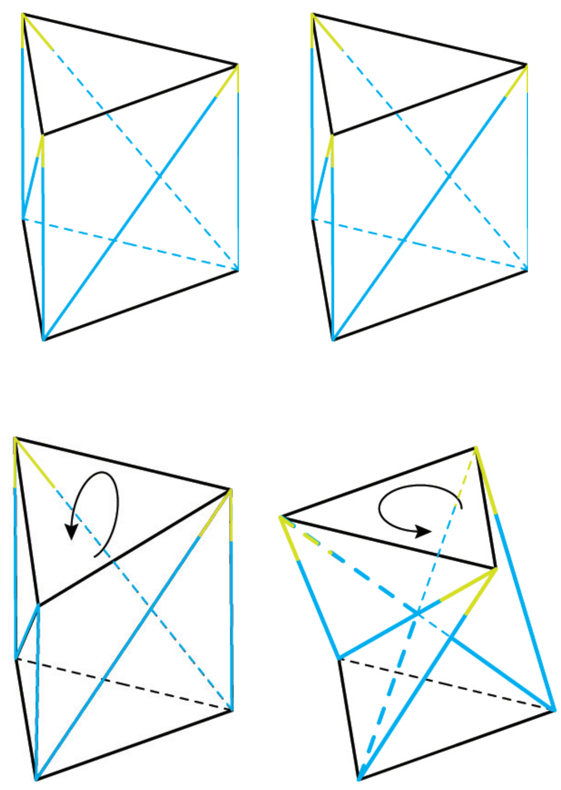


fig. 5.3 curvature and twist

5.3.2 Octahedral VG Truss module

The octahedral truss module is composed of eight equilateral triangles, for which the lateral members are supplied with an extensional actuator. By elongating specific members, the truss module can change its configuration.

Considered is a module built from members with a 0,5 meter length, for which the members that compose the top and bottom triangle can elongate.
By elongating one member, the corners that the member attaches to drop, slightly, providing a change of curvature. Additionally, the opposite result can be achieved by elongating the two other members. The corner point that they both connect to, will drop.
The module can increase its length, when three members forming either the top or the bottom triangle, shorten. The length of the module can be decreased by elongating the members forming either the top or bottom triangle.

If we consider the basic configuration of the module, and assume that the top and bottom triangle members are extended halfway their maximum length, the maximum change in curvature will be around 10 degrees. The maximum length of the module will be 0,43m, while the minimum length will be 0,38m. The octahedral truss module does not allow twisting.

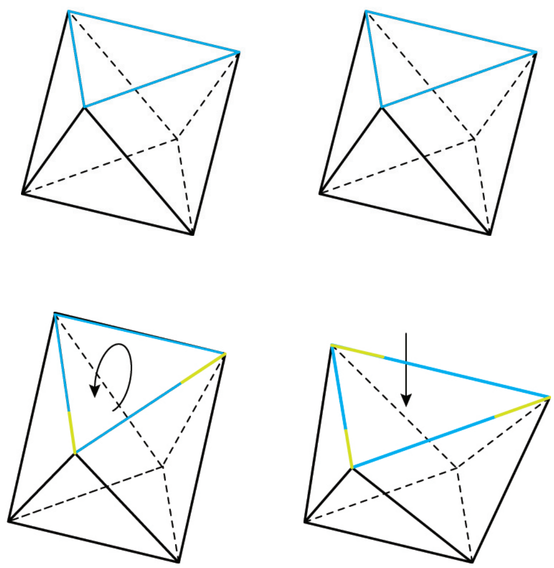


fig. 5.4 curvature and elongation

5.3.3 Summary

As the maximum differences for curvature, twist, and elongation are compared to the requirements, it is clear that the existing VG truss systems are not suitable for the form-active structure. Some adjustments will have to be made in order for them to become suitable, if at all possible.

	requirement	Octahedral truss module	Prismatic truss module
Twist	15 °	Not possible	60 °
Curvature	20 °	10 °	16,7°
elongation	+130% / -76%	+13% / -12%	+30%

The octahedral truss module appears to be very limited. However, for the referenced projects in which it is used, it performs much better in all aspects. This is explained by the fact that the limit of 30% elongation is much lower than the elongation that was possible in the reference projects [Miura]. No specific information has been found on the exact system and limitations. Therefore, the limit of 30% will be kept, severely limiting the potential of the octahedral truss module.

5.4 ADJUSTMENT AND IMPROVEMENT

As neither of the two modules is sufficient, both will be examined for possible improvements. The modules that are used in the joints have to facilitate curvature and twist, while the modules in the trusses have to facilitate elongation. Therefore, for each module, an improved version will be made, focusing on either goal.

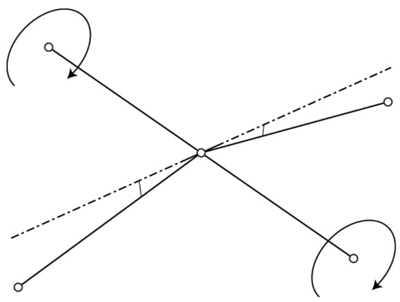


fig. 5.5 joints facilitate curvature and twist

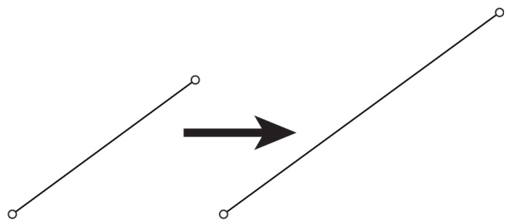


fig. 5.6 trusses facilitate elongation

5.4.1 Truss module

The truss module has to facilitate an extension of +130%. The modules from the precedent project are not sufficient in their current form. The prismatic module achieves the elongation by elongating the longitudinal members. This elongation is limited by the 30% limit, and cannot be improved easily.

However, the octahedral module elongates by shortening the lateral members, in order to change the ratio between the lateral and longitudinal members. The greater the difference, the greater the elongation. By not only shortening the lateral members, but also extending the longitudinal members, the difference in ratio, and therefore the elongation, can be increased.

As the elongation of every member is limited, the initial ratio between longitudinal and lateral members determines the maximum elongation. The shorter the longitudinal members are relative to the lateral, the greater the maximum elongation can be. Thus, shorter elements allow greater elongation.
When longitudinal members start at a length, double that of the lateral members, the element can elongate up to four times its original length.

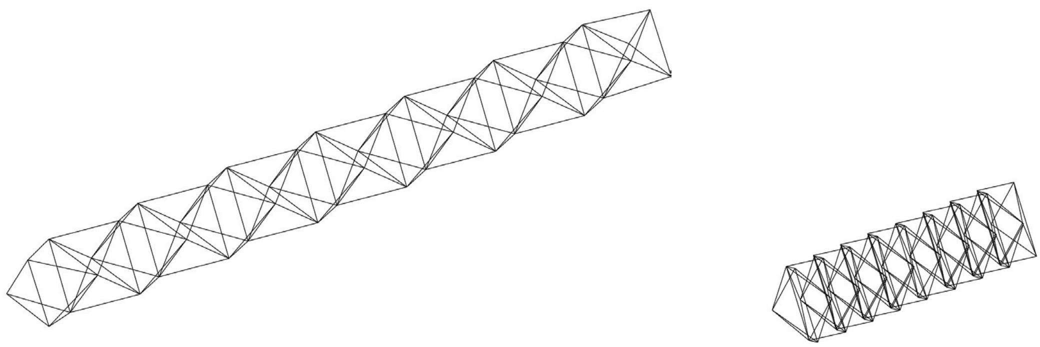


fig. 5.7 elongation of the variable geometry trusses

5.4.2 Joint

The joint elements have to facilitate both twist and curvature differences. Additionally, they have to rotate around the z-axis of the joint. The elements will therefore converge from the triangular cross section of the truss, to a linear hinge to which four elements are attached in each joint.

Curvature difference can be achieved by rotating the triangular face around a virtual point on the hinge. The members connecting the hinge to the face will elongate or shorten in order to achieve this reconfiguration. The ratio between the length of the hinge and the height of the triangular face influences the needed (local) elongation. A relatively shorter hinge allows for a greater angle for the same elongation. A second factor is the distance between the hinge and the face, which is the radius of the circle along which the face rotates. A greater radius requires less elongation.

Since the structure requires an angle of just 20 degrees per joint element, the minimum size of the element is limited, not by the elongation of the members, but the rotation about the hinge. The elements must have freedom to rotate without touching each other.

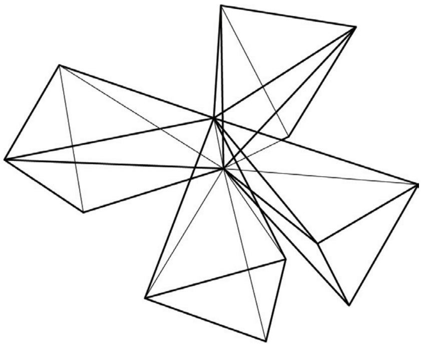


fig. 5.8 joint geometry

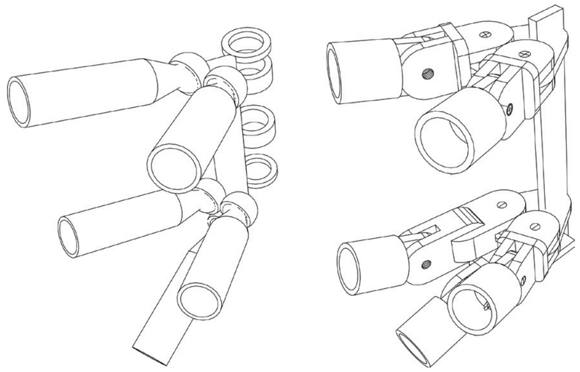


fig. 5.9 possible hinge detail

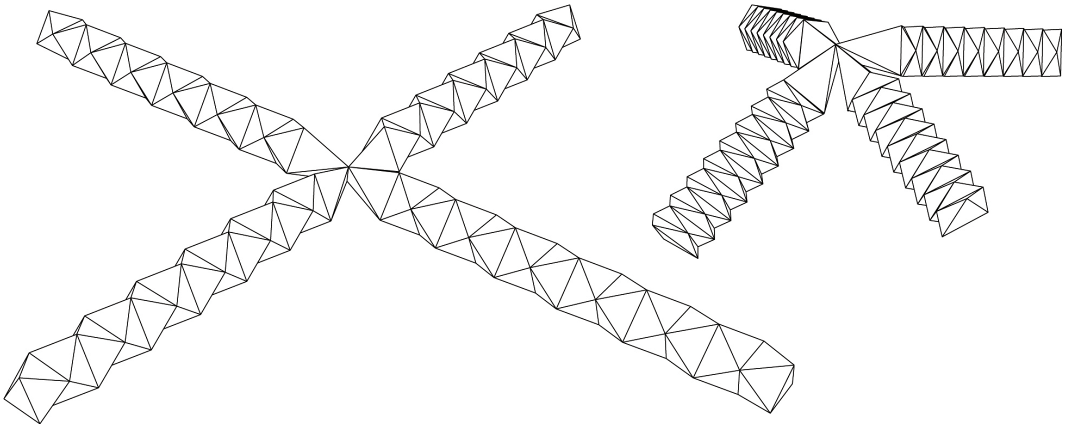


fig. 5.10 joint and four VG trusses in two different configurations

5.5 COMPLETING THE STRUCTURE

The structure consists now of truss elements that can elongate and shorten, and joint elements that can provide curvature and twist.
As the joint module cannot elongate, the elongation of the entire truss, including the joint elements, has to be arranged by the truss elements. It is therefore preferred to keep the joint elements as small as possible. The truss elements however are preferred to be as long as possible, in order to keep the amount of members lower. However, a longer element can't elongate as much. Therefore, in order to use a smaller amount of truss elements while still being able to elongate the required amount, the cross-section of the element also has to increase. This leads to a greater construction depth, which is also not preferred.
A compromise between these aspects, construction height and number of elements, has to be sought.

Assuming the distance between two nodes is l.

The length of the joint elements is a.

The truss elements fill the remaining length (l-2a).

To increase the entire length of l to 2.5 l, the truss elements have to elongate (l-2a) to (2.5l-2a)

The length:depth ratio of a truss element is defined by the elongation that is needed. The size of the joint element (a) defines the size of the trusselement.

When a increases, the number of elements decreases. However, the needed elongation increases, resulting in smaller truss elements, and therefore more elements.

If b = (l-2a)/(2.5l-2a)*4a

Then the optimal number of truss elements is 8, with a being 1/8th of the maximum distance.

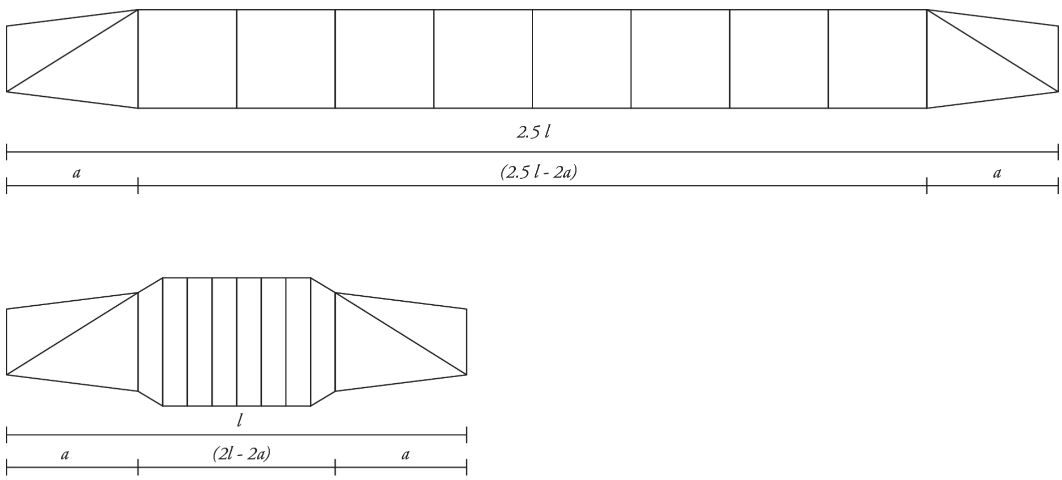


fig. 5.11 VGT division

5.6 SUMMARY

The required funicular shapes have been analysed to find the maximum differences in configuration for the form-active structure. These differences have to be facilitated by the variable geometry truss modules. Two existing types have been tested to those requirements, and were not suitable for direct implementation.

Using the two existing types as a basis, a joint was designed, combining four modules of a new variant, to facilitate both curvature and twisting of the vg trusses. A different variant was developed to facilitate elongation in the trusses, between the joints.

PART IV
IMPLEMENTATION

6. CONNECTING SHAPE AND STRUCTURE

This chapter will describe the how the method for generating funicular shapes can be implemented in the software package, how it generates its output, how this output is used as input for the form-active structure, and finally how the structure itself is generated.

6.1 LAYOUT OF THE PARAMETRIC MODEL

The parametric model is set up using the Grasshopper plug-in for Rhinoceros 4. The grasshopper package allows for graphical scripting of 3D geometry with the Rhino software, and also allows Visual Basic scripting. It therefore can communicate with other softwares too.

6.2.1 Input

The user starts by defining the grid of the structure through modelling points in Rhino, and assigns them in the correct order to the Grasshopper point collections. These points are used to generate branches to and from all nodes.
Reading the points from the 3D model instead of defining them within the script, allows the user to change the geometry of the grid by moving individual points, rather than re-inputting the coordinates. When the points are assigned to the point groups in the script, any change in coordinates is automatically recognised and updated.

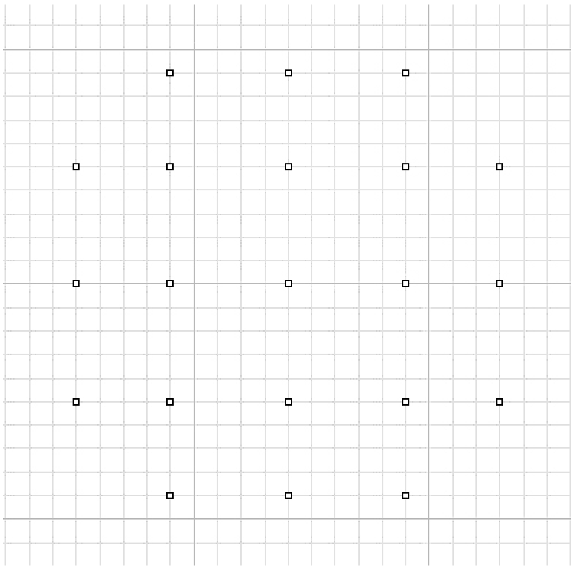


fig. 6.1 points in rhino are the input for the primal grid

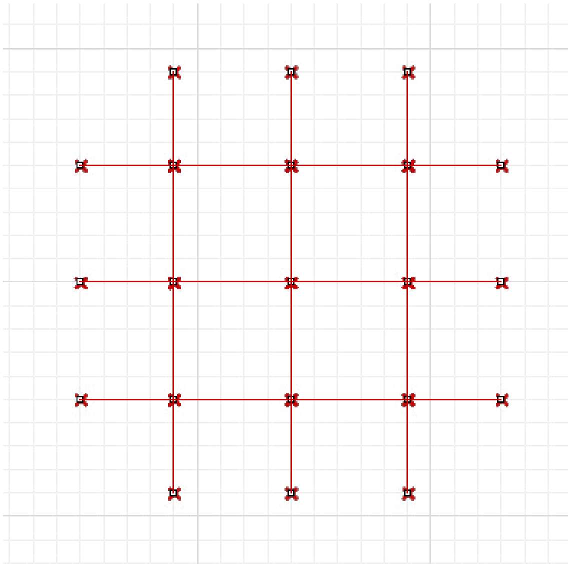


fig. 6.2 the primal grid is generated by connecting the points

6.2.2 Load direction

After inputting the grid vertices, the user can alter the direction of the loads. Since the calculations assumes vertical loads, the direction is altered by rotating the grid, rather than the loads. The user has two sliders to control load direction. One controls the angle of the loads relative to the ground plane, ranging from a completely vertical force, 90 degrees, and combination of an equal vertical and horizontal force, 45 degrees. The other slider controls from which direction the force acts on the structure, as a rotation around the z-axis. The entire grid is rotated according to the slider-values, and a new projection is made on the ground plane. From this projection, the new lengths and angles are determined, and outputted to the calculation software.

6.2.2 Force network

When the grid has been generated, all information on length of branches and their respective angles is analysed, and outputted to an external software (Microsoft Excel) containing the optimisation algorithm. The reciprocal grid can be defined knowing the respective angles between branches, and the magnitude of half of the support reactions (left and top side). All other forces can be expressed as a function of the angles and the support reactions.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	Branch
2,121	2,121	2,121	3	3	3	2,1213	2,121	2,121	3	3	3	3	3	2,121	2,121	3	3	2,12132	2,121	2,121	3,000	2,121	3			Length
0,522	0,522	0,522	0,261	0,261	0,261	0,522	0,522	0,522	0,261	0,261	0,261	0,261	0,261	0,522	0,522	0,261	0,261	0,522	0,522	0,522	0,261	0,522	0,261			Force
0,246	0,246	0,246	0,087	0,087	0,087	0,2463	0,246	0,246	0,087	0,087	0,087	0,087	0,087	0,246	0,246	0,087	0,087	0,24627	0,246	0,246	0,087	0,246	0,087			Force Density
0,579	0,579	0,579	0,205	0,205	0,205	0,5789	0,579	0,579	0,205	0,205	0,205	0,205	0,205	0,579	0,579	0,205	0,205	0,57893	0,579	0,579	0,205	0,579	0,205			Ec
																									2	FD support
																									2	FDr
																									9,403	Ec
																									0,01	

1	2	3	4	5	6	7	8	9	node									
1,571	1,571	1,571	1,571	1,571	1,571	1,5708	1,571	1,571	alfa	90	90	90	90	90	90	90	90	90
1,571	1,571	1,571	1,571	1,571	1,571	1,5708	1,571	1,571	beta	90	90	90	90	90	90	90	90	90
1,571	1,571	1,571	1,571	1,571	1,571	1,5708	1,571	1,571	gamma	90	90	90	90	90	90	90	90	90
1,571	1,571	1,571	1,571	1,571	1,571	1,5708	1,571	1,571	delta	90	90	90	90	90	90	90	90	90
0,584	0,584	0,584	0,584	0,584	0,584	0,5841	0,584	0,584	e									
0,464	0,464	0,464	0,464	0,464	0,464	0,4636	0,464	0,464	gamma1	26,57	26,57	26,57	26,57	26,57	26,57	26,57	26,5651	26,57
1,107	1,107	1,107	1,107	1,107	1,107	1,1071	1,107	1,107	gamma2	63,43	63,43	63,43	63,43	63,43	63,43	63,43	63,4349	63,43
1,107	1,107	1,107	1,107	1,107	1,107	1,1071	1,107	1,107	alfa1	63,43	63,43	63,43	63,43	63,43	63,43	63,43	63,4349	63,43
0,464	0,464	0,464	0,464	0,464	0,464	0,4636	0,464	0,464	alfa2	26,57	26,57	26,57	26,57	26,57	26,57	26,57	26,5651	26,57

fig. 6.3 worksheet in Excel optimises the force polygons for the reciprocal grid

The reciprocal grid is then optimised by exploring different values for the support reactions, while complying with two constraints, being all forces must be equal to or larger than 0, and the force density in the supports must be equal to the reference force density, which is predefined.
By ensuring all forces are equal to or greater than 0, tensile forces will be excluded as options for a solution. Keeping the force density for the supports constant, makes sure that all solutions are comparable.
The algorithm intelligently compares different solutions to narrow down the solution space, and finds the solution for minimum Energy.

The optimisation routine is repeated every time a specific value in the worksheet is changed. This happens every time grasshopper outputs variables. So every time the user adjusts the primal grid by moving one or more points, the calculation is updated automatically.

The grasshopper script then retrieves the values for every force from Excel, and uses them to generate the reciprocal grid. This step can be omitted for the calculation, but the reciprocal grid acts as the visual representation of the force distribution within the structure, and is therefore needed for the user to see and understand the solution. Additionally, the user can check the solution for equilibrium. In some cases, the grid has no funicular solution, and tensile forces are required to make equilibrium. The algorithm will then stop calculating, and output the last values it used. Grasshopper retrieves them, while they are not the solution, and generate a reciprocal grid. The grid will not be a closed polygon. The user can identify this easily due to the graphical representation.

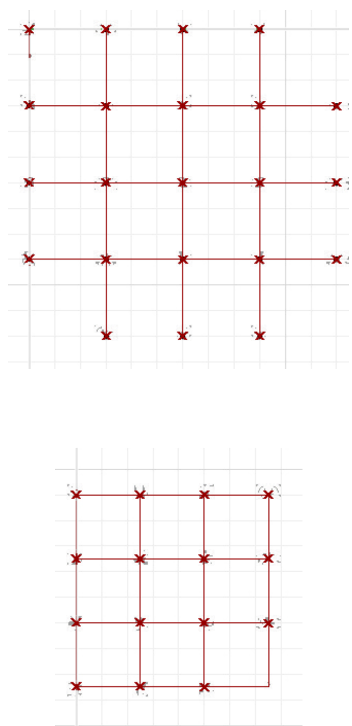


fig. 6.4 the reciprocal grid is generated

6.2.3 Force density and equilibrium equations

Now that the magnitude of every force has been determined, and the length of each branch is known, the force density is calculated for every branch. These values are then used to fill in the matrix containing the equilibrium equations in the z-direction of every node.
The matrix is pre-scripted to match the connectivity of the nodes in the grid, and only needs values for the force density in every branch.

A second matrix contains the 'answers' to the equations, being the magnitude of the forces and the z coordinates of the fixed nodes. The magnitude of the forces can be altered by the user.
To solve the equilibrium equations, the matrix is inverted first; then multiplied with the second matrix to get the z-coordinates for every node.

z1	z2	z3	z4	z5	z6	z7	z8	z9	z10	z11	z12	z13	z14	z15	z16	z17	z18	z19	z20	z21	Fz
0,667	-0,17	0	-0,17	0	0	0	0	0	-0,17	0	0	0	0	0	0	0	0	0	0	-0,17	0,9
-0,17	0,667	-0,17	0	-0,17	0	0	0	0	0	-0,17	0	0	0	0	0	0	0	0	0	0	0,9
0	-0,17	0,667	0	0	-0,17	0	0	0	0	0	-0,17	-0,17	0	0	0	0	0	0	0	0	0,9
-0,17	0	0	0,667	-0,17	0	-0,167	0	0	0	0	0	0	0	0	0	0	0	0	-0,17	0	0,9
0	-0,17	0	-0,17	0,667	-0,17	0	-0,17	0	0	0	0	0	0	0	0	0	0	0	0	0	0,9
0	0	-0,17	0	-0,17	0,667	0	0	-0,17	0	0	0	0	-0,17	0	0	0	0	0	0	0	0,9
0	0	0	-0,17	0	0	0,6667	-0,17	0	0	0	0	0	0	0	0	0	-0,17	-0,16667	0	0	0,9
0	0	0	0	-0,17	0	-0,167	0,667	-0,17	0	0	0	0	0	0	0	-0,17	0	0	0	0	0,9
0	0	0	0	0	-0,17	0	-0,17	0,667	0	0	0	0	0	-0,17	-0,17	0	0	0	0	0	0,9
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

fig. 6.5 the equilibrium equations are formulated in a matrix

1,795	0,573	0,179	0,606	0,375	0,158	0,1962	0,164	0,08	0,31	0,099	0,031	0,029	0,025	0,013	0,014	0,028	0,034	0,03155	0,097	0,289	9,814	z1
0,573	1,974	0,573	0,375	0,763	0,375	0,1636	0,276	0,164	0,099	0,34	0,099	0,092	0,06	0,026	0,028	0,048	0,028	0,02632	0,06	0,092	12,26	z2
0,179	0,573	1,795	0,158	0,375	0,606	0,0802	0,164	0,196	0,031	0,099	0,31	0,289	0,097	0,032	0,034	0,028	0,014	0,01291	0,025	0,029	12,71	z3
0,606	0,375	0,158	1,991	0,737	0,259	0,6056	0,375	0,158	0,104	0,065	0,027	0,025	0,042	0,025	0,027	0,065	0,104	0,09741	0,32	0,097	9,286	z4
0,375	0,763	0,375	0,737	2,25	0,737	0,3748	0,763	0,375	0,065	0,132	0,065	0,06	0,119	0,06	0,065	0,132	0,065	0,06028	0,119	0,06	12,07	z5
0,158	0,375	0,606	0,259	0,737	1,991	0,1575	0,375	0,606	0,027	0,065	0,104	0,097	0,32	0,097	0,104	0,065	0,027	0,02534	0,042	0,025	12,18	z6
0,196	0,164	0,08	0,606	0,375	0,158	1,795	0,573	0,179	0,034	0,028	0,014	0,013	0,025	0,029	0,031	0,099	0,31	0,28873	0,097	0,032	6,711	z7
0,164	0,276	0,164	0,375	0,763	0,375	0,5733	1,974	0,573	0,028	0,048	0,028	0,026	0,06	0,092	0,099	0,34	0,099	0,09221	0,06	0,026	9,158	z8
0,08	0,164	0,196	0,158	0,375	0,606	0,1791	0,573	1,795	0,014	0,028	0,034	0,032	0,097	0,289	0,31	0,099	0,031	0,0288	0,025	0,013	9,604	z9
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	7,652	z10
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	9,099	z11
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	10,55	z12
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	10,44	z13
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	8,89	z14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	7,338	z15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	4,34	z16
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	2,893	z17
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1,447	z18
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1,551	z19
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	3,103	z20
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	4,654	z21

fig. 6.6 the matrix is solved, presenting the z-coordinates for the nodes

6.2.4 Drawing the funicular shape

The z-coordinates are retrieved, and combined with the x and y-coordinates, to draw the points of the 3D funicular shape. All points are again connected as was the primal grid, and the shape is completed. The geometry is the rotated back to a level position.

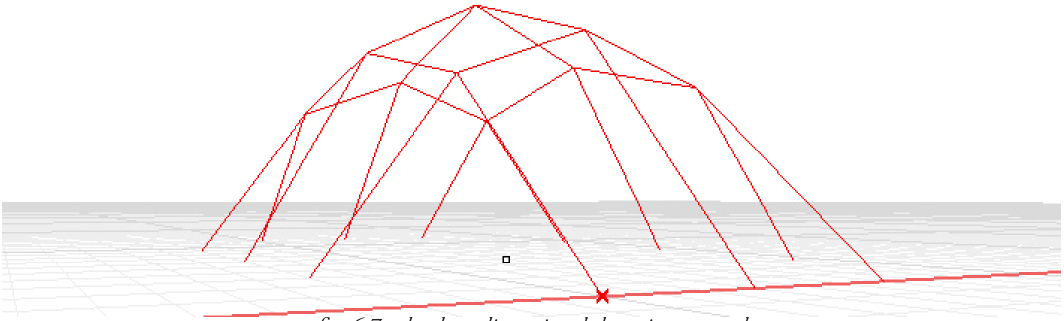


fig. 6.7 the three dimensional shape is generated

6.2.5 Drawing the trusses

At the nodes, a line along the z axis is drawn, and both endpoints are connected. Halfway the branch, a line is drawn perpendicular to the branch, to the line connecting the z-axes of the knots. This line represents the z-axis of the branch

The construction depth and length of the joints can be defined by the user using a slider. For every branch, the number of truss elements is predefined. The length of the branch, minus the joints, is divided into the number of elements. At every division point, a triangle is generated, of which every second triangle is rotated 180 degrees, and the corners of the triangles are connected to the corresponding corners of the other triangles.

6.2.6 Drawing the joints

Joints are drawn by generating a line along the z-axis of the joint, representing the hinge. The start and end points

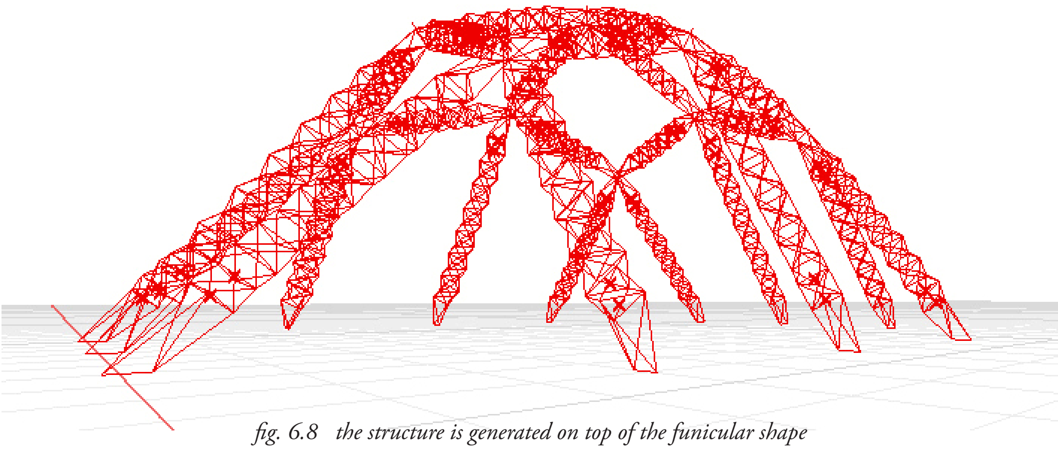


fig. 6.8 the structure is generated on top of the funicular shape

6.2 LIMITATIONS

Although the model is fully functional and provides a certain degree of interactivity, as a design tool it still has limitations:

- The number of rows and columns (of nodes) is predefined. It is therefore not possible to calculate more refined grids or more rectangular layouts without adjusting the script itself.
- Only 4 valent nodes are possible. While the method would work for nodes of a lower or higher valency as well, the script can only handle 4 valent nodes.
- It is not possible to enter radial grids.
- An interaction with Microsoft Excel is needed. This is required to solve the reciprocal grid, and to solve the equilibrium solutions.

These restrictions can be solved by rewriting the script in order to provide options for a more broad user input, regarding the number of nodes and their connectivity. The user would have to provide more information in order for the script to work with, but can be limited to node coordinates and their connectivity. The model would have to be more intelligent in assessing the reciprocal grid. When the valency of a node can be different from 4, the amount of forces, and which ones, needed to determine the reciprocal grid changes from case to case. The need for an external calculation software could be eliminated. Solving matrices, for equilibrium equations, can be done within the grasshopper package through the use of VB scripting and Mapack [Oosterhuis]. Solving the force distribution is done through an algorithm, which is implemented in Excel, but the algorithm, or a similar one, could be scripted in VB script within Grasshopper as well. When the method is improved such that the grid can be examined on a node by node base, the need for an algorithm is eliminated entirely.

Regarding the limitations of the current model, it should not be regarded as a finished form-finding tool, but rather as a proof of concept.

PART V
RESULTS

7. RESULTS AND DISCUSSION

7.1 PERFORMANCE OF THE FORM FINDING METHOD

Regarding the performance of the method, compared to the already existing methods, the following can be stated:

- The method maintains the strong graphical representation of the force distribution in a system as the thrust network analysis method and graphic statics, while adding the functionality for solving statically undetermined structures by calculating the minimal complementary energy.

- In its current form, the method is limited to four-valent systems or less, which did not present a problem for the Thrust Network Analysis method. A valid approach to solve knots with a higher valency has been suggested.

- The method can optimize the scale factor using the complementary energy again, resulting in the most optimal solution.

- The method is limited to parallel loads. A workaround is suggested to cope with parallel non-vertical loads. The suggestions by Block for implementing the horizontal components into the reciprocal grid are still viable.

One alternative approach to solve non-vertical loads might be to keep the x and y coordinates of the free nodes fixed during the creation of the reciprocal grid, but release them during the calculation of the z coordinates. The value for the force density can then still be based on the primal and the dual grid, but non-vertical loads can be entered into the equilibrium equations, and solved. However, there is no guarantee that the values for force density are still correct. This has to be validated.

7.2 IMPLEMENTATION FOR A FORM-ACTIVE STRUCTURE

While the method still has some limitations, it proved useful for the preliminary design of the form-active structure. This is based on the assumption that the governing loads will be the static weight of the building, roughly equal to an evenly distributed load, and the wind load. As long as all loads can be reduced to parallel forces acting on the vertices of the grid, the system can be solved. When this is the case, the output from the script can be used as direct input for the configuration of the form-active structure.

When the design for the form-active structure is developed further, a method will be needed which can cope with different non-vertical loads more easily. This could be the force density method, with the addition of fixed x and y coordinates. However, the force density method lacks the graphical representation, and is therefore not very transparent to the user.

The fact that the Grasshopper package runs within the Rhinoceros environment meant that the results of the structural analysis and the 3D model could be solved within one software package. No interaction with other programs was needed, providing a quasi-real-time feedback.

8. RECOMMENDATIONS

With respect to the development of the method for analyzing thrust networks, the following recommendations can be made:

The solution for four-valent knots can be made more elegantly by formulating the problem as a linear equation, which can then be differentiated to find the minimum, instead of narrowing down the range in which the solution lies. A similar approach might prove useful for knots with a higher valency

Instead of scaling the individual force polygons during the creation of the reciprocal grid, a relation can be sought to determine a value for the reference force density, such that all polygons are calculated at the same scale

The parametric model for the form-active structure may be combined with the kangaroo package for structural analysis, which also runs in the Rhinoceros environment, to enable direct feedback on the structural performance

9. FINAL REFLECTIONS

Reflecting on the main objective for this thesis it can be said that it was successfully achieved within this thesis project: 'Develop a method for assessing thrust networks for both statically determined and undetermined systems, based on graphic statics, and model it in a parametric design software, where it can be used to describe the configuration of an adaptive structure.'

The developed method, while it still has limitations, has been used effectively to generate the optimal shapes regarding load transfer, which made it possible to form a conceptual design for a form-active structure that can reconfigure its geometry to achieve optimal structural performance.

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