AAS 09-431

DESIGN OF GUIDANCE LAWS FOR LUNAR PINPOINT SOFT LANDING

J. Guo,* and C. Han†

Future lunar missions ask for the capability to perform precise Guidance, Navigation and Control (GNC) to the selected landing sites on the lunar surface. This paper studies the guidance issues for the lunar pinpoint soft landing problem. The primary contribution of this paper is the design of descent guidance law based on the Pontryagin maximum principle. The simulation shows that the proposed polynomial guidance law can achieve precise pinpoint landing. However it is sensitive to the selection of several parameters. Suggestions on lunar pinpoint soft landing strategies are also given according to the simulation results.

INTRODUCTION

As the Earth's closest celestial body, the Moon is recognized as one of the most important destinations for space science and exploration. From the 1950s onwards, various lunar landing missions were carried out by spacecraft such as Surveyor (US), Apollo (US) and Luna (former USSR). Recently, with the rapid growth of the earth's population, energy and resources are becoming increasing scarce. Since the Moon could possibly serve as a new and tremendous supplier of energy and resources for humanity, the exploration of lunar resources has received increasing attentions, and various lunar landing missions have been proposed again by many countries, such as "Return to the moon" (US), the Lunar Logistics Lander (ESA), SELENE-B (Japan), Chang'E-2/-3 (China) and many others. Several organizations such as the X-Prize Foundation have also initiated activities for lunar soft landing. Some of these missions aim to safely reach landing sites containing hazardous terrain features, or to broaden the range of scientific outputs, which ask for the capability to perform precise Guidance, Navigation and Control (GNC) to the selected landing sites on the lunar surface. However, up to this date, lunar landings have only achieved kilometer-level precision. Therefore, the GNC technology for pinpoint soft landing is essential for future lunar missions.

This paper studies the guidance issues for the lunar pinpoint soft landing problem, where pinpoint soft landing aims to guide a lander to a given target on the surface with an error of several meters (in the worst case). The primary contribution of this paper is the design of descent guidance laws based on the Pontryagin maximum principle.

The remains of this paper are organized as follows. Part 1 describes a lunar pinpoint soft landing mission in detail. The preliminary design of a low-cost and robust GNC system is presented

* Researcher, Faculty of Aerospace Engineering, Delft University of Technology, Kluyverweg 1, 2629 HS Delft, The Netherlands.
† PhD Researcher, Surrey Space Center, University of Surrey, Guildford GU2 7XH, United Kingdom.

for a miniaturized lander spacecraft, and the preliminary landing strategy is proposed. Part 2 investigates the "powered descent" pinpoint soft landing guidance problem. A polynomial guidance law is designed, based on the Pontryagin maximum principle. In part 3, the numerical simulations are implemented, and suggestions on improving the proposed lunar pinpoint soft landing strategy are provided according to the simulation results.

MISSION SCENARIO

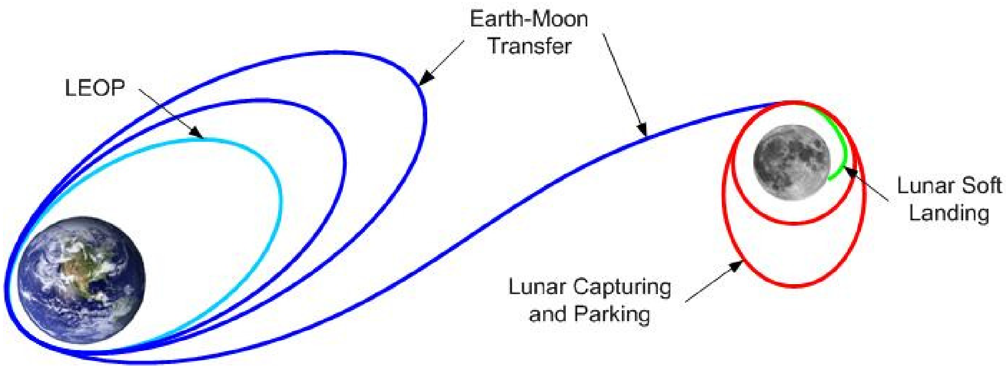

Figure 1. Overall Mission Scenario.

The proposed lunar exploration mission aims to land a miniaturized spacecraft on the pre-selected lunar site for the purposes of technology validation and scientific studies. After the trade-off between costs, launch opportunity, lifetime, mass and many other factors, an overall mission scenario consisting of four stages, i.e. Launch and Early OPerations (LEOP), earth-moon transfer, lunar capturing and parking, and lunar soft landing, is proposed as in Figure 1.

In the LEOP stage, the small spacecraft will be launched into a Geostationary Transfer Orbit (GTO) as piggyback. Then a Phasing Loop Transfer Orbit (PLTO) is selected for earth-moon transfer. The main drivers of using PLTO are from saving propellant and providing higher orbit precision. When the spacecraft approaches the moon, it will be captured and circle around on the moon parking orbit. After the in-orbit test, the lunar lander will be separated from the orbiter and soft land on the expected site.

This paper will only focus on the GNC problem in the soft landing stage, which will be discussed in details below.

The Soft Landing Strategy

One of the most important questions regarding the soft landing is: what is the procedure to land on the expected lunar site?

To answer this question, a preliminary soft landing strategy to minimize the propellant consumption is proposed as shown in Figure 2. This strategy is composed of three phases. The first phase is a "*Hohmann transfer phase*" that decreases the altitude from the lunar parking orbit (around 100km height) by providing a ΔV at the tangential direction. The second phase is a "*powered descent phase*", which decreases the altitude and velocity using continuous burning of the main thruster. At the end of the "*powered descent phase*", ideally both the velocity and the horizontal distance (to the expected landing site) of the lander will approach zero, and the height of the lander is around 1-5m. Then the last phase, i.e. "*free descent phase*" starts by shutting off and main thruster, and the lander descends as a free fall until reach the lunar surface.

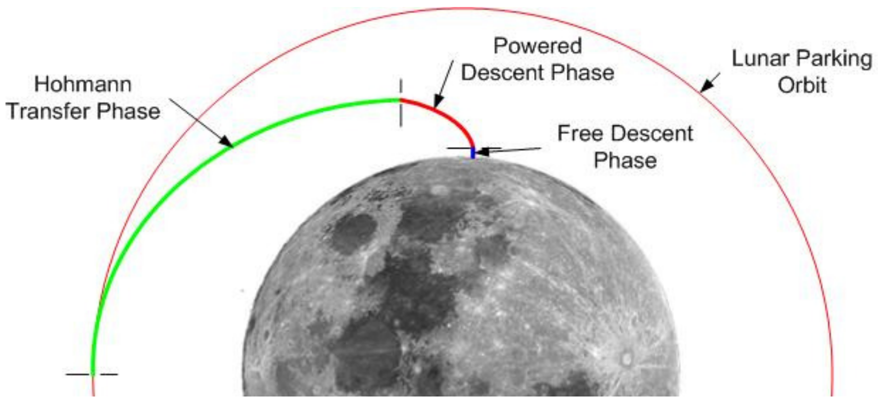

Figure 2. Soft Landing Strategy (not scale).

The GNC System of the Lander

Another important question closely related to lunar landing is how to realize the landing strategy proposed in the previous subsection, or in other words how to define the architecture of the GNC system of the lander.

Due to the limitations on costs, mass and power supply, the following criterions are considered in the design of the GNC system: 1) Low system cost, which implies inexpensive hardware; 2) Low resource consumption, which indicates small size, low mass and low power consumption; 3) High precision, which requires pinpoint accuracy at 10 meters level, with the possibility to achieve 1 meter level; 4) Easy to implement, which stand for low computational workload for OnBoard Computer (OBC); and 5) Robustness, which means insensitive to uncertainties and disturbances.

Table 1. GNC Hardware of the Lander.

Category	Name	Quantity	Manufacturer
Sensor	MIMU	1	TU Delft
	CMOS star tracker	2	ISIS, TU Delft
	Miniaturized sun sensor	2	TNO, Bradford Engineering
	Radar altimeter	1	COTS
Actuator	Control Moment Gyroscopes	4	TU Delft
	Attitude control thruster	12	TNO, Bradford Engineering
	Orbit control thruster	1	Astrium (CHT 400)

Based on the above criterions, the hardware of the GNC system are selected and listed in Table 1.

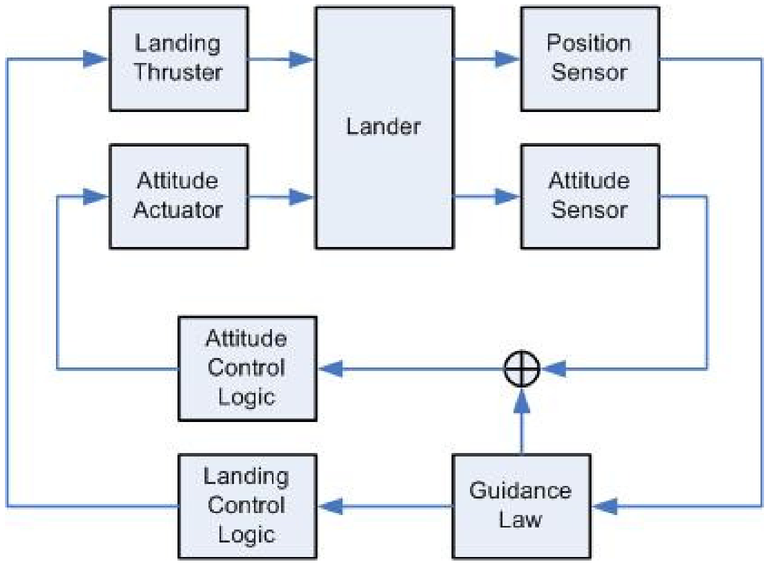

Figure 3. GNC Architecture of the Lander.

Accordingly the architecture of the GNC system is defined as shown in Figure 3. The "*Guidance Law*" block produces guidance commands based on the pre-designed guidance law and the position information provided by "*Position Sensor*". Then the guidance commands are used for creating "*Landing Control Logic*". Since the landing thruster requires appropriate thrust direction, the "*Attitude Control Logic*" should be created by coupling the guidance law with the attitude information provided by "*Attitude Sensor*". In this architecture, "*Position Sensor*" consists of MIMU and radar altimeter, "*Attitude Sensor*" stands for CMOS star trackers and miniaturized sun sensors, and "*Attitude Actuator*" implies CMG and attitude control thrusters.

In this paper the primary object of study is the guidance problem, which includes the algorithm embedded in the "*Guidance Law*" block, as well as the appropriate conditions for transformations between different landing phases. These topics will be discussed in the following sections.

POLYNOMIAL GUIDANCE LAW FOR PINPOINT SOFT LANDING

Most of the lander's propellant will be consumed in the "*powered descent phase*"; therefore, the guidance law of this phase is the most important for the overall mission. This section investigates the "powered descent" pinpoint soft landing guidance problem, whose purpose is to find a propellant-optimal trajectory that transfers the lander from any given initial state at engine ignition to a desired terminal state without violating propellant limits or any state and control constraint (e.g. actuator saturation) in a gravitational field[1,2]. A polynomial guidance law is designed, based on the Pontryagin maximum principle. This guidance law transforms the pinpoint soft landing problem to a two point boundary value problem and then allows finding the propellant-optimal trajectory using the conventional Nonlinear Programming (NLP) algorithm.

The Motion Equation of the Lander

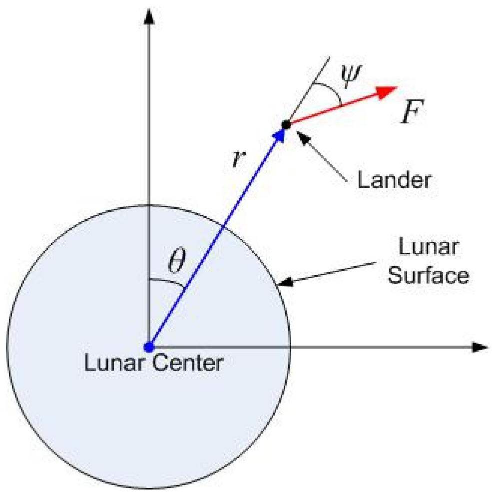

Figure 4. State Variables for Equations of Landing.

Firstly, it is assumed that the lander is a point mass and the descending path is in a vertical plane. Then the state variables of the lander in the lunar polar coordinate system are shown in Figure 4, where r is the radial distance from the lunar center, θ is the azimuth angle, F is the landing thrust force and its vale is constant, and ψ is the attitude angle that is the input variable and defines the angle between the thrust direction and the radial vector.

According to the definitions of the state variables in Figure 4, the motion equation of the lander can be described as:

$$
\begin{aligned}
\dot{r} &= v \\
\dot{\theta} &= \frac{u}{r} \\
\dot{u} &= \frac{F}{m}\sin\psi - 2\frac{u^2}{r} \\
\dot{v} &= \frac{F}{m}\cos\psi - \frac{\mu}{r^2} + \frac{u^2}{r} \\
\dot{m} &= -cF
\end{aligned} \tag{1}
$$

where v is the radial velocity, u is the tangential velocity, m is the mass of the lander, and μ is the gravity constant of the moon. It's also assumed that the specific impulse of the thruster is constant. Therefore, the decreasing rate of mass, cF, is constant.

The Design of the Polynomial Guidance Law

So far the lunar soft landing guidance laws have been validated through the projects of Apollo in US and Luna in Russia and obtained good results. For a pinpoint landing mission, the *powered descent phase* must end not only with a soft landing, but also within a very short distance of the target. Flying to the target under power can require maneuvering several km over the surface, significantly increasing propellant requirements compared to those of non-pinpoint landing. Few results were presented in the context of pinpoint soft landing[3,4]. However, pinpoint landing capability will allow to reach landing sites, which may contain hazardous terrain features (such as escarpments, craters, rocks or slopes), or to land accurately at select landing sites for the purpose of achieving high science values.

Before designing the pinpoint soft landing law, two requirements should be considered: 1) The consumption of propellant must be reduced as low as possible, which means to minimize the period of the descending phase; and 2) Due to the uncertainties on the initial conditions and system parameters, the guidance law should have excellent robustness.

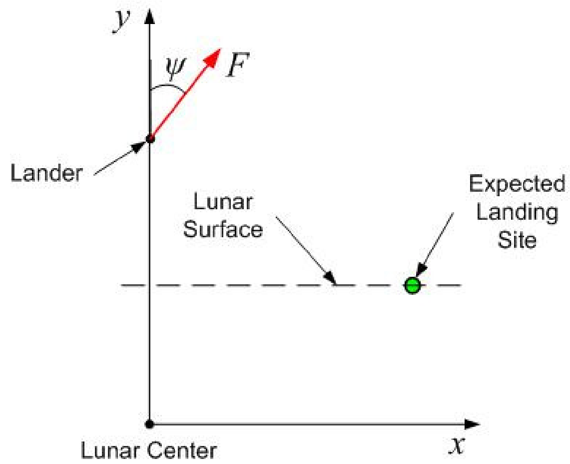

Figure 5. State Variables for Equations of Landing with Flat Surface Assumption.

The proposed polynomial pinpoint soft landing guidance law is based on the optimal control theory. In order to avoid large amount of calculation, another two assumptions are made: 1) The lunar surface close the landing site is flat; and 2) The gravity acceleration is constant. Under these assumptions, the state variables of the lander in the lunar local coordinate system are defined in Figure 5, where the coordinate origin is the expected landing site, x indicates the horizontal distances between the lander and the landing site, and y stands for the vertical distance from the lunar center to the lander. Accordingly, the motion of lander is described as:

$$\begin{aligned} \dot{x} &= u \\ \dot{y} &= v \\ \dot{u} &= \frac{F}{m}\sin\psi \\ \dot{v} &= \frac{F}{m}\cos\psi - \frac{\mu}{R^2} \\ \dot{m} &= -cF \end{aligned} \tag{2}$$

As cF is constant, the acceleration provided by the thrusters can be approximated as a first order function of time t, through Taylor series, i.e.

$$\frac{F}{m}=\frac{F}{-cFt+m_0}\approx a_0(1+\frac{t}{T_A}) \tag{3}$$

where m_0 is the initial mass of the lander before "*powered descent phase*", a_0 and T_A are constants obtained through Taylor series.

In order to obtain the control variable profile, Lagrange multipliers λ_1, λ_2, λ_3, λ_4, also known as co-state variables, are introduced. Based on the principles of Pontryagin[5], the Hamiltonian is formulated as:

$$H=\lambda_1\dot{x}+\lambda_2\dot{y}+\lambda_3\dot{u}+\lambda_4\dot{v} \tag{4}$$

and the variations of the co-state variables are given by

$$\dot{\lambda}_1=-\frac{\partial H}{\partial x}=0,\ \ \dot{\lambda}_2=-\frac{\partial H}{\partial y}=0,\ \ \dot{\lambda}_3=-\frac{\partial H}{\partial u}=-\lambda_3,\ \ \dot{\lambda}_4=-\frac{\partial H}{\partial v}=-\lambda_1 \tag{5}$$

The optimal control angle profile is obtained by minimizing the Hamiltonian at each instant of time with respect to the control variable, i.e.

$$\frac{\partial H}{\partial \psi}=0\Rightarrow \psi=\tan^{-1}\frac{\lambda_3}{\lambda_4} \tag{6}$$

The Pontryagin's principle requires that the terminal value of a co-state variable corresponding to a non-free state variable is to be a constant at the final moment[5]. Therefore, the co-states variables can be expressed as:

$$\lambda_1(t)=\lambda_{10},\ \ \lambda_2(t)=\lambda_{20},\ \ \lambda_3(t)=\lambda_{30}-\lambda_{10}t,\ \ \lambda_4(t)=\lambda_{40}-\lambda_{20}t \tag{7}$$

And the control angle ψ in Eq.(6) can be re-formulated as:

$$\psi=\tan^{-1}\frac{\lambda_{30}-\lambda_{10}t}{\lambda_{40}-\lambda_{20}t}=\tan^{-1}(c_1+\frac{c_2}{\lambda_{40}-\lambda_{20}t}) \tag{8}$$

where c_1 and c_2 are constants.

Keep in mind that thrusters onboard the lander are mainly used for the deceleration of the initial vertical velocity for soft landing, and meanwhile give the lander a time-varying horizontal velocity and make it land on the desired site. Then through Taylor series expansion, Eq.(8) can be transformed to

$$\psi=\overline{\psi}+k_2t-k_1 \tag{9}$$

where $\overline{\psi}$ is a constant angle that contributes for soft landing, (k_2t-k_1) stands for precise pinpoint landing and k_1,k_2 are constants that will be discussed in following paragraphs.

Combining Eqs.(2), (3) and (9), and expanding triangular functions with first order Taylor series, Eq.(2) can be re-formulated as:

$$
\begin{aligned}
&\dot{x} = u \\
&\dot{y} = v \\
&\dot{u} = a_0(1+\frac{t}{T_A})\sin\overline{\psi} + a_0(1+\frac{t}{T_A})\cos\overline{\psi}(k_2 t - k_1) \\
&\dot{v} = a_0(1+\frac{t}{T_A})\cos\overline{\psi} - a_0(1+\frac{t}{T_A})\sin\overline{\psi}(k_2 t - k_1) - \frac{\mu}{R^2}
\end{aligned}
\tag{10}
$$

Then integrate Eq.(10), x, y, u and v can be expressed as functions of time t as:

$$
\begin{aligned}
x(t) - x(0) &= u(0)t + a_0(\sin\overline{\psi} - \cos\overline{\psi}k_1)\frac{t^2}{2} + a_0(\cos\overline{\psi}k_2 + \frac{1}{T_A}\sin\overline{\psi} - k_1\cos\overline{\psi}\frac{1}{T_A})\frac{t^3}{6} \\
&\quad + a_0\frac{t^4}{12T_A}\cos\overline{\psi}k_2 \\
y(t) - y(0) &= v(0)t + [a_0(\cos\overline{\psi} + k_1\sin\overline{\psi}) - \frac{\mu}{R^2}]\frac{t^2}{2} \\
&\quad + a_0(-\sin\overline{\psi}k_2 + \frac{1}{T_A}\cos\overline{\psi} + k_1\frac{1}{T_A}\sin\overline{\psi})\frac{t^3}{6} - a_0\frac{t^4}{12T_A}\sin\overline{\psi}k_2 \\
u(t) - u(0) &= a_0(\sin\overline{\psi} - \cos\overline{\psi}k_1)t + a_0(\cos\overline{\psi}k_2 + \frac{1}{T_A}\sin\overline{\psi} - k_1\cos\overline{\psi}\frac{1}{T_A})\frac{t^2}{2} \\
&\quad + a_0\frac{t^3}{3T_A}\cos\overline{\psi}k_2 \\
v(t) - v(0) &= [a_0(\cos\overline{\psi} + k_1\sin\overline{\psi}) - \frac{\mu}{R^2}]t \\
&\quad + a_0(-\sin\overline{\psi}k_2 + \frac{1}{T_A}\cos\overline{\psi} + k_1\frac{1}{T_A}\sin\overline{\psi})\frac{t^2}{2} - a_0\frac{t^3}{3T_A}\sin\overline{\psi}k_2
\end{aligned}
\tag{11}
$$

Given the boundary conditions, $\overline{\psi}$, k_1, k_2, t can be obtained by solving Eq.(11). However, if a very strict boundary condition, i.e. $x(t)$=0, is applied, it's very possible that there is no solution for Eq.(11). Therefore the problem of solving a set of linear equations in Eq.(11) is transferred to the problem of how to obtain a minimum value for the horizontal distance x. This optimization problem can be described as:

$$
\begin{aligned}
\min. \quad & |x(t)| \\
s.t. \quad & \begin{cases} |u| \le u_{error} \\ |v| \le v_{error} \\ y_{lunar} \le y \le y_{\max} \end{cases}
\end{aligned}
\tag{12}
$$

where u_{error}, v_{error} are maximum tolerated errors, $y_{\max}$ is the maximal accepted vertical distance from the lunar center to the lander at the end of the "*powered descent phase*", and y_{lunar} is the radius of the Moon. Using conventional NLP algorithm, the values of $\overline{\psi}$, k_1, k_2, t can be obtained.

NUMERICAL SIMULATION

In order to investigate the performance of the proposed polynomial guidance law and the conditions for landing phases transformation, it is necessary to implement the simulation for different situations. In this section, focus is on the conditions of transforming from the "*Hohmann transfer phase*" to the "*powered descent phase*". In total four cases are selected, which cover the conditions from high initial velocity to relative low velocity and from long initial distance to short distance. However for all the four cases, identical GNC hardware, as shown in Table 1, is utilized.

Case 1: Long Initial Distance and High Initial Velocity

For the first case, it is assumed that the lander transforms from the "*Hohmann transfer phase*" to the "*powered descent phase*" at a position where the horizontal and the vertical distances to the desired landing site are 100000m and 10000m, respectively. The lander will not do any orbit maneuver during the transforming procedure, i.e. its velocity at the end of the "*Hohmann transfer phase*" and in the beginning of the "*powered descent phase*" will be kept same, assumed to be 1800m/s on the horizontal direction and -100m/s on the vertical direction. It's also expected that at the end of the "*powered descent phase*", the lander's velocity can be reduced to less than 2m/s on both directions. In addition, according to Table 1, the specifications of the Astrium CHT 400 orbit control thruster are: F=400N, I_{sp}=224s, and the gross mass of the lander is m_0=224kg.

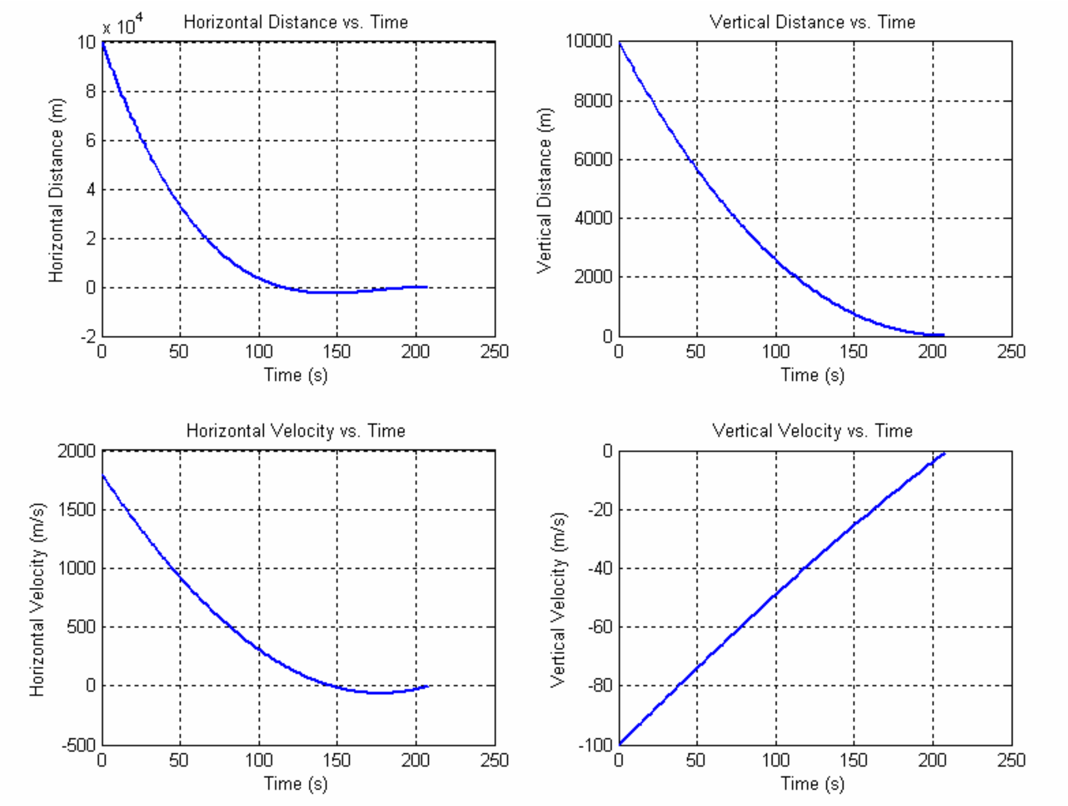

Figure 6. Variations of State Variables (x_0=100000m, v_0=1800m/s).

Based on the above assumptions and conditions, the simulation is implemented and the results are shown in Figure 6. It can be found that after 208 seconds the pre-defined conditions for shutting down the orbit control thruster are satisfied, and the lander's horizontal distance to the expected landing site is less than 3cm. That implies that the proposed polynomial guidance law can achieve precise pinpoint soft landing, with long initial distance and high initial velocity. The propellant consumption is 37.9kg.

Case 2: Long Initial Distance and Low Initial Velocity

In the second case, it is assumed that the lander has a negative velocity impulse at the end of the "*Hohmann transfer phase*" and, therefore, the initial horizontal velocity of the "*powered descent phase*" is reduced to 100m/s. All other assumptions and conditions are remained without any change.

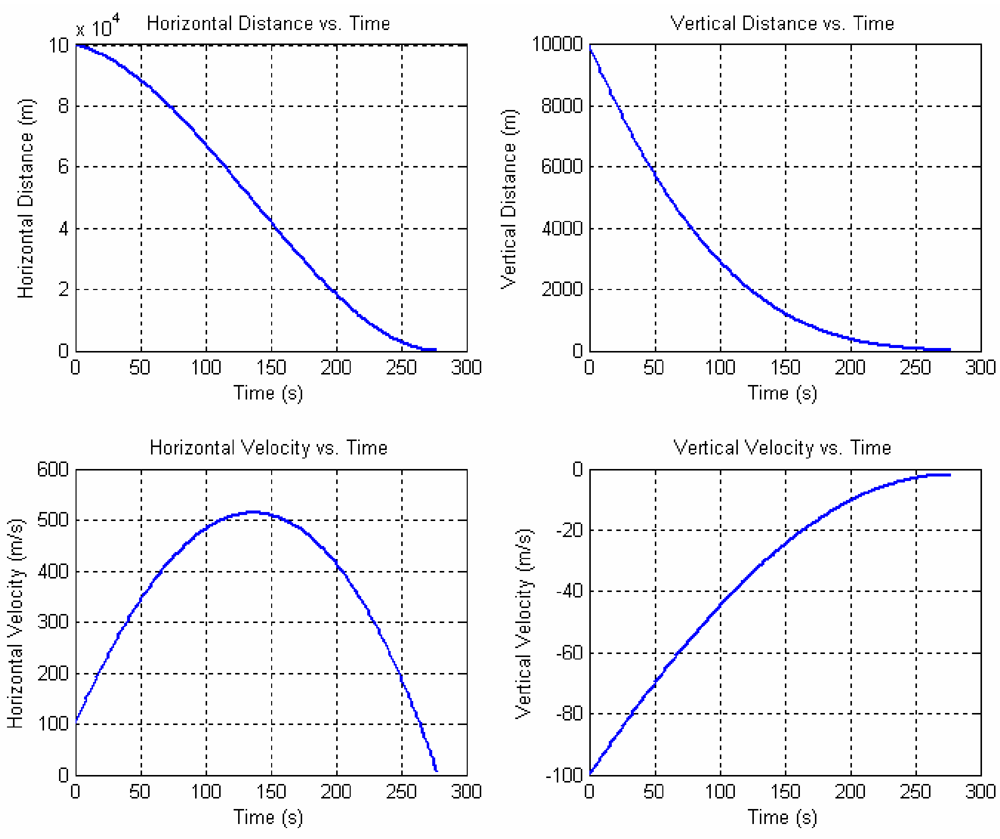

Figure 7. Variations of State Variables (x_0=100000m, v_0=100m/s).

The simulation results for Case 2 are shown in Figure 7, which exhibits similar performance as for Case 1. After 278s powered descending, the lander arrives at the expected landing site with the accuracy of better than 0.7m, and consumes 50.7kg propellant. This means that the proposed guidance law works well for this case, but with higher propellant consumptions. This is easy to be understood if looking at the left bottom plot of Figure 7. The initial horizontal velocity is relatively low; therefore much propellant has to be used on accelerating and then on decelerating.

Case 3: Short Initial Distance and High Initial Velocity

Now assume that the lander starts powered descending only when the horizontal distance to the expected landing site is less than 5000m, and other assumptions and conditions are kept same as in Case 1. That means the lander's initial horizontal velocity is 1800m/s.

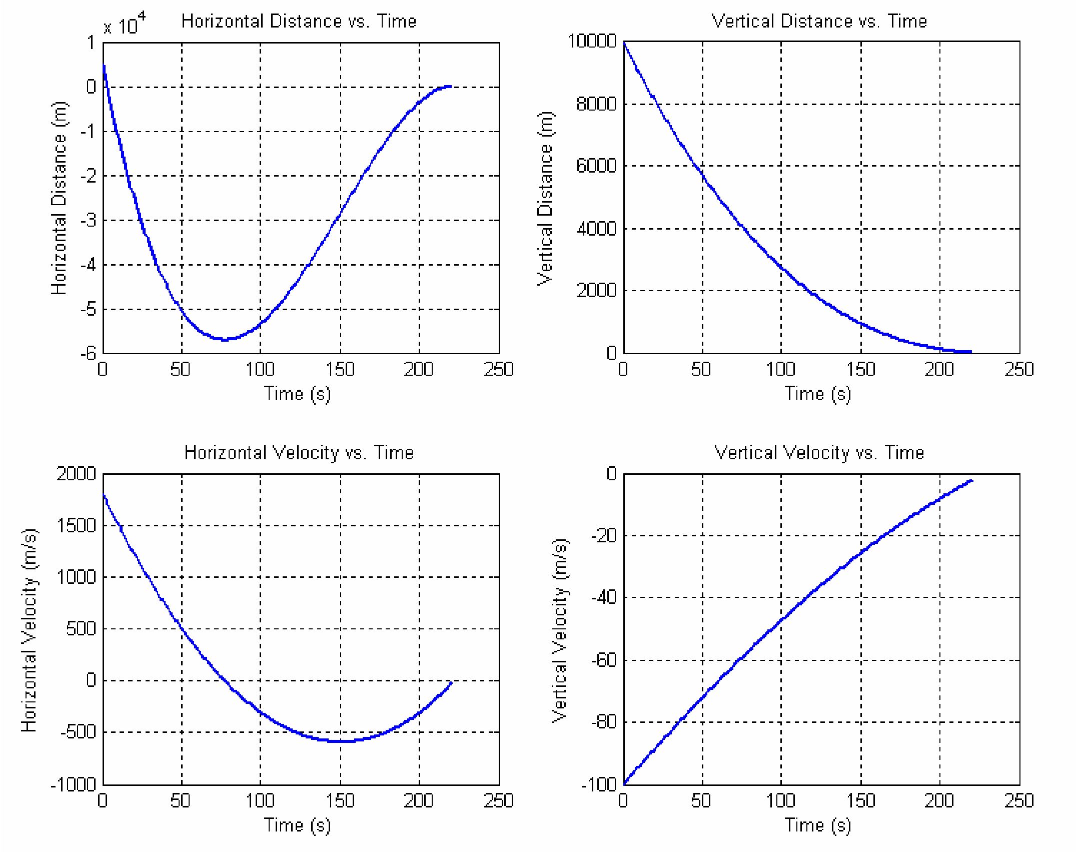

Figure 8. Variations of State Variables (x_0=5000m, v_0=1800m/s).

Figure 8 provides the simulation results for this case. Due to the short distance and high velocity, the lander flies over the landing site shortly after powered descending. However, the orbit control thruster contributes a lot to decelerate the horizontal velocity. After flying over the landing site for almost 57km, the lander flies back and eventually arrives at a location that is 3m away from the expected landing site. This procedure takes 221s and consumes 40.3kg propellant.

Case 4: Short Initial Distance and Low Initial Velocity

At last, the case of short initial distance and low initial velocity is considered. In other words, it's assumed that the lander still starts its "*powered descent phase*" when approaching the expected landing site for less than 5000m, but at the beginning of this phase the lander's horizontal velocity has already been reduced to 100m/s. All other assumptions and conditions are kept same with other cases.

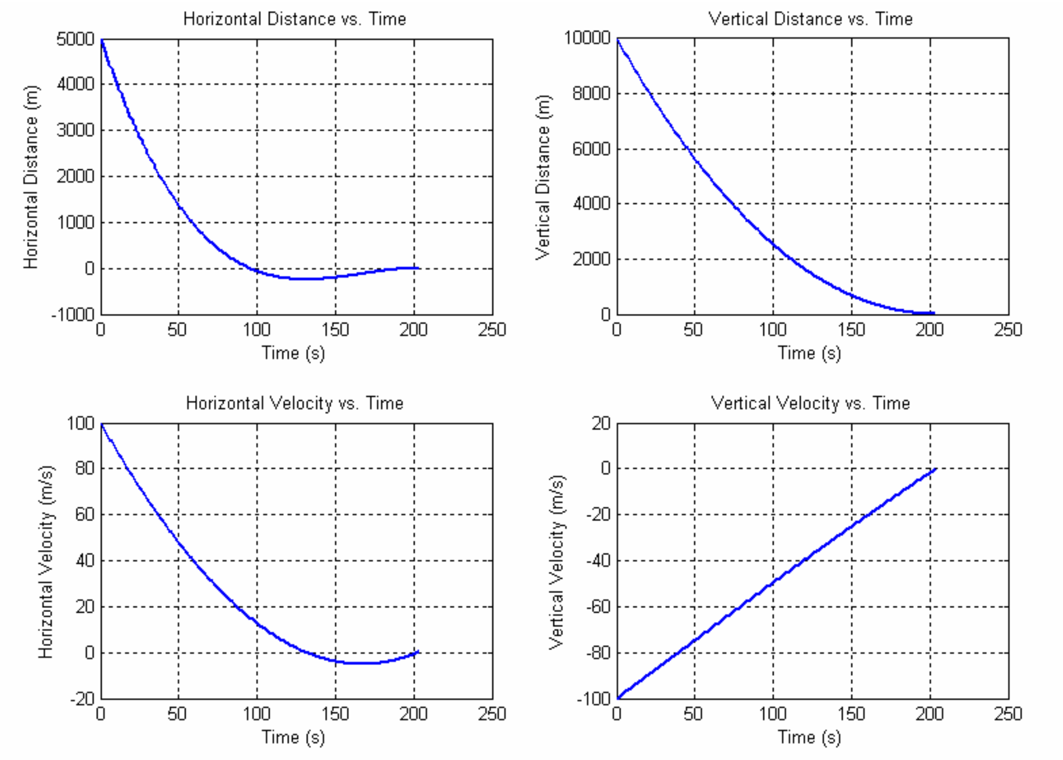

Figure 9. Variations of State Variables (x_0=5000m, v_0=100m/s).

Figure 9 shows the variations of the lander's distance to the landing site and the velocity, on both the horizontal and the vertical directions. Compared with Figure 8, it can be found that although the lander flies over the landing site again, the excess is only 200m, and eventually the lander arrives at the landing site with the accuracy of 0.2m after 204s. As less energy is used on flying back, only 37.2kg propellant is consumed.

CONCLUSIONS

This paper studied the GNC issues, especially the guidance aspect, of the lunar pinpoint soft landing problem. The preliminary design of a low-cost and robust GNC system is presented for a miniaturized lander spacecraft, and the preliminary landing strategy is proposed. The "powered descent" pinpoint soft landing guidance problem is investigated. A polynomial guidance law is designed, based on the Pontryagin maximum principle. The simulations in this paper proved that the proposed polynomial guidance law can help to realize precise lunar pinpoint soft landing with low cost. Furthermore, the results provided useful information to investigate the details of the soft landing strategy.

For a better understanding, the simulation results of the four cases are summarized in Table 2. It can be clearly observed that the proposed polynomial guidance law works well for all the four cases. It also indicates that in order to achieve precise pinpoint soft landing, the better options are either 1) start powered descending without deceleration at the beginning when the lander is still far away from the expected landing site; or 2) if a short initial distance is desired then decelerate the lander's horizontal velocity to a relatively small value before the "*powered descent phase*".

Considering about the propellant that is used for deceleration before powered descending, the first option, i.e. Case 1, is the most appropriate strategy.

Table 2. Summary of Simulation Results of Four Cases.

Case No.	Initial Conditions		Results		
	Horizontal Distance (m)	Horizontal Velocity (m/s)	Time Used (s)	Landing Accuracy (m)	Propellant Consumed (kg)
1	100000	1800	208	<0.03	37.9
2	100000	100	278	<0.7	50.7
3	5000	1800	221	<3	40.3
4	5000	100	204	<0.2	37.2

During the simulation, it's also observed that the proposed guidance law is sensitive to the initial values of the optimization. This is due to the NLP algorithm used for optimization, which always finds the local optimum instead of the global one. Therefore in order to improve the robustness of the proposed guidance law, an appropriate global optimization algorithm is necessary. A possible candidate is interval analysis, and the relevant investigation is undergoing.

REFERENCES

[1] S.R. Ploen, A.B. Acikmese, and A. Wolf, "A Comparison of Powered Descent Guidance Laws for Mars Pinpoint Landing." Proceedings of *AIAA/AAS Astrodynamics Specialist Conference and Exhibit* . AIAA2006-6676, 21-24 August 2006, Keystone, CO, USA.

[2] U. Topcu, J. Casoliva, and K.D. Mease, "Minimum-Fuel Powered Descent for Mars Pinpoint Landing." *Journal of Spacecraft and Rocket*. Vol. 44, No. 2, p324-331.

[3] X. Huang, and D. Wang, "Autonomous Navigation and Guidance for Pinpoint Lunar Soft Landing." Proceedings of *the 2007 IEEE International Conference on Robotics and Biomimetics*. 15-18 December 2007, Sanya, China.

[4] D. Wang, X. Huang, and Y. Guan, "GNC System Scheme for Lunar Soft Landing Spacecraft." *Advances in Space Research*. Vol. 42, No. 2, p379-385.

[5] R.V. Ramanan, and L. Madan, " Analysis of Optimal Strategies for Soft Landing on the Moon from Lunar Parking Orbits", *Journal of Earth System Science*, Vol. 114, No. 6, p807-813.