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Bayesian Update of Wave Equation Based Seismic Inversion Using Geological Prior Information and Scenario Testing

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Summary

Reservoir characterisation is a data driven process which involves the integration of different datasets to describe the subsurface. One of the difficulties of integrating geological data with the wave-equation based seismic inversion is that geological information is invariably interpreted as a layer-based model, whereas the wave-equation is defined and solved on a grid. Mapping a layer-based model space onto a grid-based space leads to highly non-Gaussian, multi-modal distribution functions, even when the layer-based properties have simple Gaussian distributions.

In this paper an analytic method is presented that translates the prior layer-model based distributions to grid-based prior distributions. From the unconstrained seismic inversion result a Gaussian likelihood function is constructed and the method to find the maximum a posterior estimate (MAP) and its uncertainty is described. As geological prior information we use well data, a geological concept of the environment of deposition and structural seismic interpretation in the form of some horizons to guide the prior model in between wells.

Given the prior model, a measure for the probability of the data is formulated. When this process is repeated for various prior scenarios, the probability of the scenario, given the data, can be calculated for every location.

Introduction

For reservoir-oriented seismic inversion usually the only constraints used are sparseness constraint on the reflectivities and hard constraints on the properties in the model space. In this paper a new Bayesian framework is presented which allows one to include interpreted layer based prior models in seismic inversion. As geological prior information we use well data, a geological concept of the environment of deposition and structural seismic interpretation in the form of some horizons to guide the prior model in between wells. The prior information is not used directly to steer the inversion process, but is used in a Bayesian way, updating the likelihood function obtained from unconstrained seismic inversion, to a posterior probability density function from which the Maximum a Posteriori (MAP) estimate and its uncertainty can be derived.

Wave-Equation-Based AVP inversion and Likelihood Function

The WEB AVP inversion used in this study is based on the full elastic wave-equation. Like most other AVP inversions, it is based on a locally 1.5D data model, over a target interval comprising the reservoir with top and bottom seals. As a general rule the grid-spacing should be one fifth of the shortest wave-length in the data, including shear waves. The method is based on the integral representation of the elastic wave-equation, which is solved iteratively, with linearised inversions of the data, for the properties, in between updates of the total elastic field in the object. For every linearised inversion a kernel is defined based on the currently best estimate of the total field.

After the last linearised inversion we have the output vector in the gridded model space, the minimum of the objective function and the kernel used in the last inversion. From these ingredients a Gaussian likelihood function can be constructed that represents the solution, with its uncertainty, to the unconstrained inversion problem. The likelihood function is given by:

$$P\left(d\,|\,m\right)=\frac{\exp(-0.5(m-m_0)^T C_m^{-1}(m-m_0))}{\sqrt{\left(2\pi\right)^{Nz}\left|C_m^{-1}\right|}} \quad \text{where} \quad C_m^{-1}=\frac{K^T K}{2E_{\min}} \tag{1}$$

Where m_0 is the output vector of the inversion result, C_m is the covariance matrix in model space, d is the seismic data. We can construct it analytically from the data residual after the inversion and the pseudo hessian (Pratt et. al. 1998), which is the kernel *(K)* of the inversion data equation, multiplied by its adjoint and E_{min} is the minimum of the objective function after the inversion.

Finally, it should be mentioned that the wave-equation based inversion naturally inverts for the properties that define the wave-equation. For the integral representation of the elastic wave-equation these properties are the compressibility (inverse of the bulk modulus), the shear compliance (inverse of the shear modulus) and the density.

Grid-based prior probability distributions

As mentioned before, geological information derived from wells and a general concept of the environment of deposition will inevitably be interpreted as a layered model, where means and standard deviations are assigned to the layer properties and thicknesses. The standard deviations of the layer properties can easily be determined from the vertical variability observed within the layers in the wells and these quantities are assumed to have Gaussian distributions. The distributions of the layer thicknesses can be described by truncated Gaussians, where the area under the originally Gaussian curve, extending over negative thicknesses, is mapped onto a delta function at zero-thickness. This allows the concept of assigning a finite chance for that layer to be absent, but leaves us with a non-Gaussian distribution. For a single grid point analytically this can be expressed as a weighted sum of Gaussians.

$$P(k(z_j))=\sum_{i=1}^{N} w_{ij}k_i \tag{2}$$

where k_i is the property probability density for the i^{th} layer and w_i is the weight corresponding to the i^{th} layer, j denotes the j^{th} grid node. The w_{ij} are the weights which explains the probability of having a i^{th}

layer at j^{th} grid point. The joint probability distribution for the whole model vector at a specific location, for any property k, is given by:

$$P(k(z)) = \prod_{j=1}^{Nz} P(k(z_j)) \qquad (3)$$

For the prior joint probability density for all depths, at a specific lateral location, we get a product of sums of Gaussians, constituting a highly non-Gaussian, multi-modal distribution. The assumption that all gridpoints are statistically independent may not be strictly true, but it leads to good results and simplifies the analysis considerably, because this very complex function can be described analytically.

Grid-based posterior probability distributions and Scenario Testing

Bayes' Rule, usually written as:

$$P(m/d) = \frac{P(d/m)P(m)}{p(d)} \qquad (4)$$

where $\underline{m}$ is the grid-based model vector and $\underline{d}$ is the data, and normalising factor as the denominator $P(\underline{d})$. The unnormalised grid-based posterior probability density function is the product of the very complex non-Gaussian grid-based prior distribution eq.(2) and the Gaussian seismic likelihood function eq.(1), which is also defined in the grid-based model space.

$$P\left(m \mid d\right) = \frac{\exp(-0.5(m-m_0)^T C_m^{-1}(m-m_0))}{\sqrt{\left(2\pi\right)^{Nz}\left|C_m^{-1}\right|}} \prod_{j=1}^{Nz} P\left(k(z_j)\right) \qquad (5)$$

To find the MAP estimate the posterior distribution needs to be maximised using gradient based method. The procedure adopted in this paper is to use the unconstrained seismic inversion result (i.e. the Maximum Likelihood Estimator) as starting point for a Conjugate Gradient (CG) search for the nearest maximum of the posterior distribution. The gradient can be calculated analytically due to the assumption that all gridpoints are statistically independent. The second derivative at MAP will be a measure for the uncertainty of the solution.

Scenario testing can be implemented by estimating the *P(d)* which is usually considered a normalisation factor to make the posterior a proper probability density function, calculated as:

$$P(d) = \int dm P(d/m)P(m) \qquad (6)$$

Eq. (6) tells us that the probability of the data is a functional of the prior scenario and is calculated as the overlap between the prior and the likelihood function. It is quite expensive to evaluate the probability of data computationally, so we approximate the probability of data as the geometric mean of the mahalanobis distance between prior and likelihood function. If we can calculate this quantity for different scenarios and renormalise the probabilities to add up to one when summed over all scenarios, we can assign probabilities to different scenarios for a given data. This procedure can be carried out for every lateral location, giving a map of scenario probabilities.

Book Cliffs synthetic experiment

The new method was tested on a very detailed model based on a real outcrop and even further downscaled, based on a realistic geological scenario (Feng et al. 2015). In Figure 2, top left, we see part of the true compressibility contrasts against the background of the Book Cliffs model. Clearly visible are the thin coal seams in the section over the first 50 m, which are very soft (high compressibility).

Synthetic data in the tau/p domain was generated over this model with the help of the Kennett method, which is also an exact full elastic method, but very different from the wave-equation based solution in the inversion. The zero-offset section for the synthetic data is shown in Figure 1. Having seen the coal seams in the wells on either side of the section, a seismic interpretation can be attempted, tracking the presence of these seams along the section. The interpreted horizons are horizon 1,2 and 3 as shown in figure 1 with horizon 3 pinching out in between wells .

A prior model is built based on a layering observed in the wells and interpolated between the wells with the help of the interpreted horizons. The layer property standard deviations (covariances) for the prior distribution are measured from the variability observed in the wells within the layers. For the standard deviations of the layer thicknesses we test two different scenarios; one where we recognize the coal seams and assign very small standard deviations to the thicknesses, and one where we are not so sure and allow a much wider range of variation in the prior model.

The unconstrained wave-equation based inversion result from this synthetic data, the Maximum Likelihood Estimator, is shown in Figure 2, top right. In Figure 2 middle left, we see the mean realisation of the prior model of Scenario 1, where we assumed small standard deviations for the thicknesses of the layers in the prior model, in particular for the thin coal seams. We should realise that there is more information in the prior model than is observable in the mean realisation.

In Figure 2, middle right, we see the posterior result from the procedure presented in this paper. Clearly the result has improved when compared with the unconstrained maximum likelihood result.

The same exercise was repeated for another prior model scenario, where the prior uncertainty for the thicknesses of the thin layers was much larger, as borne out by the more fuzzy appearance of the mean realisation for this prior in Figure 2, bottom left. Finally, we see the maximum posterior realisation for this scenario in Figure 2, bottom right. Interestingly, the greater uncertainty in the layer thicknesses in Scenario 2 has not greatly reduced the resolution shown by the posterior, but in the covariances of the posterior, not shown here, this would be visible.

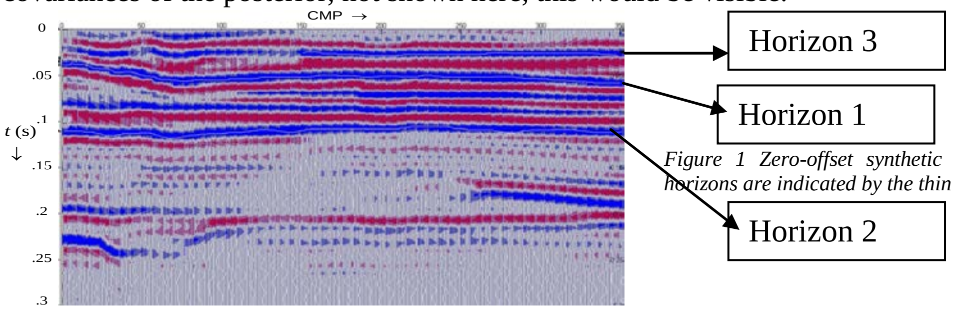

Figure 1 Zero-offset synthetic seismic section. Interpreted horizons are indicated by the thin white lines.

We are now going to analyse the probabilities of these two scenarios, given the data, for every CMP location. As pointed out in the previous section, the probability of the data, given the scenario, defined in Eq. (6), is approximated by the Mahanalobis distance between the means of the likelihood function and the prior. The result is shown in Figure 3.

Since Scenario 1 expresses the prior knowledge that there are thin layers, this scenario is closer to the truth and therefore fits the data, which is based on the truth, better. This applies all along the profile investigated, but, of course, it would be interesting to find that in part of the line, or area, one scenario would be more likely, whereas in another part of the line, or area, another scenario would fit the data better.

Conclusions

The transformation of layer-based prior models to grid-based prior models leads to strong non-linear (non-Gaussian) behaviour. Rather than using this information as a non-linear constraint in an already non-linear inversion, we decided go the Bayesian route and construct a highly non-Gaussian posterior distribution from the highly non-Gaussian prior distribution and the Gaussian likelihood function resulting from unconstrained inversion. We then still face the issue of finding the maximum posterior probability realisation from the posterior. This problem is addressed by electing to use the unconstrained seismic inversion result as a starting point in search for the maximum of the posterior distribution. Although the unconstrained inversion result is already a good result, bringing in prior information based on two wells on either side of the section and guided by some picked seismic horizons, improves the resolution significantly. Two different posteriors were produced, based on different prior model scenarios. Both prior models improved the posterior result, but when the scenario probabilities are estimated it turns out that the data shows a clear preference for the scenario containing coal seams with little thickness variation, in agreement with the true model. Both scenarios increased the resolution of the inversion and removed some of the low wave-number artefacts visible in Figure 2, top right.

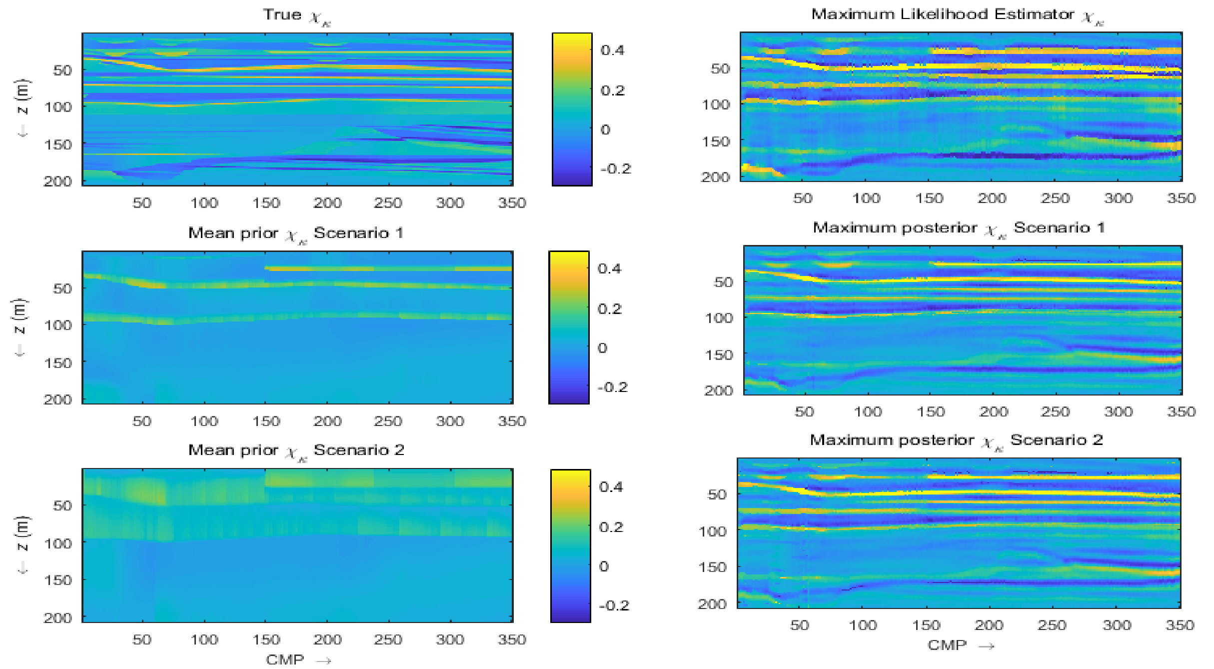

Figure 2: Top left: True compressibility contrasts for part of the Book Cliffs model. Top right: Unconstrained inversion result from synthetic seismic. Middle left: Mean realisation for the prior probability density functions for Scenario1. Middle right: Maximum posterior realisation for Scenario 1. Bottom left: Mean realisation for the prior probability density functions for Scenari21. Bottom right: Maximum posterior realisation for Scenario 2.

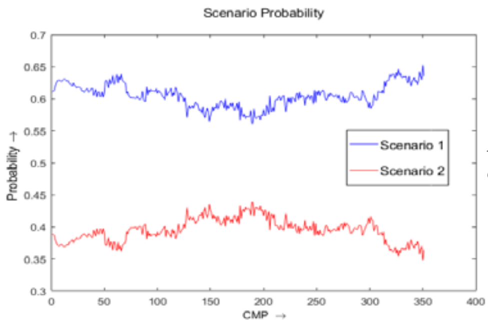

Figure 3:Probabilities of the two scenarios (1: blue, 2: red), given the data.

Acknowledgement

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