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Radar Calibration by Corner Reflectors with Mass-production Errors

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***Abstract*— The paper presents the statistical analysis of trihedral corner reflectors RCS in presence of mass production and installation errors. It is shown that the degradation of RCS from its nominal value can be modeled by Beta distribution. The derived probability density functions (PDF) of corner reflector RCS is further exploited to design an optimal procedure for the radar power calibration technique, taking the aforementioned effect into account. This procedure can be used for real-time estimation of radar sensor healthiness parameter that characterises the sensing quality for awareness of human driver or automated driving system.**

***Keywords*— Radar, calibration, trihedral corner reflector**

I. Introduction

The modern Advanced Driver Assistant Systems (ADAS) consider radar to be the main sensors for the surveillance awareness, together with the lidar and camera. To ensure safety and prevent collisions on road, automotive radars must be fault-proof and have to be tested on reliability and performance, which requires proper diagnostic of well-functioning of the radar so that the car may participate in traffic. One way of the diagnostic of well-functioning of the automotive radar is employing calibration in service stations, e.g. [1]. This, however, does not account for continuous changes in environmental and sensing conditions that can affect the quality of radar measurements and make radar data non-reliable. In contrast to offline calibration in service stations, another way of testing the automotive radar on well-functioning would be through monitoring the state, or in other words the healthiness, of the radar in real-time [2].

One possible solution to monitor the radar state is to use a massive set of calibration targets in road infrastructure. These targets should be cheap in production and maintenance, and thus the passive retro-reflectors, such as trihedral corner reflectors seem the most attractive option. Using a massive set of corner reflectors for calibration leads to the presence of RCS uncertainties due to possible production, installation, and maintenance errors [3], [4].

In this paper, we propose a statistical approach to radar power calibration using non-ideal corner reflectors. In particular, in Section II we demonstrate that in presence of the aforementioned errors, the RCS of the corner reflector can be modeled by four parameters Beta-distribution. The shape parameters of Beta distribution are directly related to the variances of the installation and production errors. This statistical analysis is further exploited to design an efficient self-diagnostics/power calibration technique, which estimates the possible losses of the radar compared to its ideal conditions using corner reflector(s) with mass production errors as calibration target(s). The estimation technique and its performance analysis are presented in Section III. Finally, the conclusions are drawn in Section IV.

II. Trihedral corner reflector RCS distribution in presence of mass-production and installation errors

A. *RCS distribution of a trihedral corner reflector in presence of non-orthogonal sides*

The inter-plate orthogonality is the most important tolerance of the corner reflector because the reflector RCS decreases rapidly as the angle deviates from 90^o [4] (Fig. 1). The effect of surfaces' non-orthogonality on the RCS of the corner reflector has been noticed by Craeye et. al [3]. They claimed that trihedral corner reflector with the angular error ϵ has the loss of RCS compared to the ideal configuration:

$$r_\epsilon = \frac{RCS}{RCS_0} = \mathrm{sinc}^4\left(\frac{2.54 l \epsilon}{\lambda}\right) \tag{1}$$

where $l \gg \lambda$ is the leg of the reflector which is large compared to the wavelength λ, RCS_0 is the RCS of an ideal corner reflector of the same size and ϵ is the same (for three angles) angular deviation from 90^o, given in radians [3]. The model is applicable for the cases when $|\epsilon| < 1$ degree. Different approximations of RCS due to non-orthogonality of surfaces are considered in [4], which for the small angles lead to a similar parabolic approximation of RCS as the function of the angular error, considered below.

If corner reflectors can be produced with some acceptable tolerance in the alignment of the surfaces, then with no extra knowledge about the angular error, it can be assumed Gaussian random variable with zero mean and the variance σ_ϵ:

$$p_1(\epsilon) = \frac{1}{\sqrt{2\pi}\sigma_\epsilon}\exp\left(-\frac{\epsilon^2}{2\sigma_\epsilon^2}\right) \tag{2}$$

The objective is to characterize statistically the RCS of the produced reflectors.

For a small angular error, the function (1) can be approximated with $\mathrm{sinc}(x) \approx 1 - \frac{x^2}{3!}$:

$$r_\epsilon \approx \left(1 - k_\epsilon \epsilon^2\right)^4, \tag{3}$$

where $k_\epsilon = (2.54l)^2 / \left(6\lambda^2\right)$.

Applying the transformation of variable (3) to the PDF (2), the distribution $p(r_\epsilon)$ is:

$$p_2(r_\epsilon) = 2p_1\left(g(r_\epsilon)\right)\left|\frac{\partial g(r_\epsilon)}{\partial r_o}\right|, \tag{4}$$

where the factor 2 comes from the even symmetry of the function (3), and

$$g(r_\epsilon) = \left(\frac{1 - r_\epsilon^{1/4}}{k_\epsilon}\right)^{1/2} \tag{5}$$

is the function inverse to (3) with the derivative:

$$\frac{\partial g(r_\epsilon)}{\partial r_\epsilon} = -\frac{r_\epsilon^{-3/4}}{8k_\epsilon\left(\frac{1-r_\epsilon^{1/4}}{k_\epsilon}\right)^{1/2}}. \tag{6}$$

Simplifying the above results, we obtain:

$$p_2(r_\epsilon) = \frac{1}{4\sqrt{2\pi k_\epsilon}\sigma_\epsilon} r_\epsilon^{-3/4}(1 - r_\epsilon^{1/4})^{-1/2} \exp\left(-\frac{1 - r_\epsilon^{1/4}}{2k_\epsilon\sigma_\epsilon^2}\right), \tag{7}$$

with $r_\epsilon \in [0, 1]$.

Next, we show that (7) can be approximated by Beta-distribution for small error. If $\epsilon \approx 0$, then $r_\epsilon \approx 1$, which gives Taylor expansions:

$$\exp\left(-\frac{1 - r_\epsilon^{1/4}}{2k_\epsilon\sigma_\epsilon^2}\right) = \left(e^{1-r_\epsilon^{1/4}}\right)^{-\frac{1}{2k_\epsilon\sigma_\epsilon^2}}\Bigg|_{r_\epsilon\approx 1} \approx r_\epsilon^{\frac{1}{8k_\epsilon\sigma_\epsilon^2}} \tag{8}$$

$$1 - r_\epsilon^{1/4}\Bigg|_{r_\epsilon\approx 1} \approx \frac{1}{4}(1 - r_\epsilon). \tag{9}$$

Using these approximations in (7) gives:

$$p_3(r_\epsilon) \propto \frac{1}{2\sqrt{2\pi k_\epsilon}\sigma_\epsilon} r_\epsilon^{\frac{1}{8k_\epsilon\sigma_\epsilon^2} - \frac{3}{4}}(1 - r_\epsilon)^{-1/2}, \tag{10}$$

which has the shape of Beta-distribution PDF with parameters:

$$r_\epsilon \sim \mathrm{Beta}\left(\frac{1}{8k_\epsilon\sigma_\epsilon^2} + \frac{1}{4}, \frac{1}{2}\right) = \mathrm{Beta}\left(\alpha_\epsilon, \beta_\epsilon\right). \tag{11}$$

We use the sign proportional to $\propto$ to emphasize that the PDF (10) should be normalized such that $\int_0^1 p_3(r_\epsilon) = 1$. Note that the accuracy of angular alignment affects only the α parameter of the distribution, while $\beta = 0.5$ being fixed.

Note that considering (1), we can notice that the PDF of non-ideal corner reflector RCS follows four parameter [5] Beta-distribution:

$$RCS \sim \mathrm{Beta}\left(\frac{1}{8k_\epsilon\sigma_\epsilon^2} + \frac{1}{4}, \frac{1}{2}, 0, RCS_0\right). \tag{12}$$

Moreover, the limiting factor of the proposed transformation lies in posing the error in the main beam of the $sinc$ function, for which (3) can be applied. For larger non-orthogonality of the side surfaces of the corner, the effect of sidelobes occurs, which has to be considered. In particular, we observed the rise of values close to zero in the probability density function $p(r)$.

Interestingly, the application of Beta-distribution for the RCS modeling of simple and complex targets has been previously investigated by Maffett [6]. He demonstrated that it provides better fidelity to model targets RCS compared to widely used Swerling models.

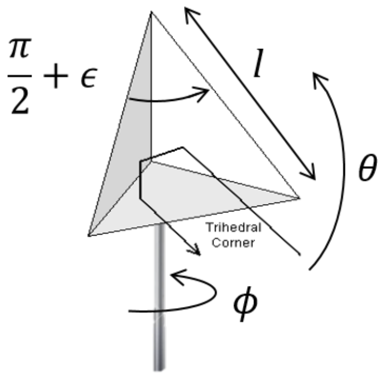


Fig. 1. Geometry of the problem

B. Orientation error

For triangular trihedral corner reflectors, the high-frequency RCS at carrier wavelength λ is defined in [7] as:

$$\begin{aligned} RCS(\theta,\phi) \approx \frac{4\pi}{\lambda^2} l^4 \big(& \cos\theta + \sin\theta\left(\sin\phi + \cos\phi\right) \\ & -2\left(\cos\theta + \sin\theta\left(\sin\phi + \cos\phi\right)\right)^{-1}\big)^2 \end{aligned} \tag{13}$$

where l is the leg length of the reflector, θ, and ϕ are spherical incidence angles in elevation and azimuth planes respectively (Fig. 1). The maximum reflection is obtained at $\theta_0 = 45°$, $\phi_0 = 54.74°$.

If to normalize (13) by the maximum RCS value and expand it in Taylor series around the center of the main beam ($\theta_0 = 45°$, $\phi_0 = 54.74°$), then we can write:

$$r_\theta = \frac{RCS(\theta, \phi_0)}{RCS(\theta_0, \phi_0)} \approx 1 - k_\theta\left(\theta - \theta_0\right)^2; \tag{14}$$

$$r_\phi = \frac{RCS(\theta_0, \phi)}{RCS(\theta_0, \phi_0)} \approx 1 - k_\phi\left(\phi - \phi_0\right)^2, \tag{15}$$

with $k_\theta \approx 5$ and $k_\phi \approx 3.33$ independently on the size of the corner reflector and the wavelength. These expressions have a similar form to (3). Therefore, we can apply the transformation of random variables similar to (4)-(7), and obtain the PDF of loss factor due to the elevation and azimuth orientation errors $\eta \in \{\theta, \rho\}$, $r_\eta \in [0, 1]$:

$$p(r_\eta) = \frac{\sqrt{k_\eta}(1 - r_\eta)^{-1/2}}{\sqrt{2\pi}\sigma_\eta} \exp\left(-\frac{1 - r_\eta}{2k_\eta\sigma_\eta^2}\right). \tag{16}$$

For small angular errors, we can process similarly to (8) and discover that the PDF follows Beta-distribution with parameters:

$$r_\eta \sim \mathrm{Beta}\left(\frac{1}{2k_\eta\sigma_\eta^2}+1, \frac{1}{2}\right) = \mathrm{Beta}\left(\alpha_\eta, \beta_\eta\right). \quad (17)$$

An example of RCS loss distribution is presented in Fig. 2, (a) for the case of azimuths error for a corner reflector with $l = 0.1$ m and azimuth error $\phi \sim \mathcal{N}(0, \sigma_\phi^2)$, $\sigma_\phi = 5^o$. The histogram is evaluated with 10^6 Monte-Carlo simulation of the angular error and the PDF line is plotted according to (17). The cases of the non-orthogonality of side surfaces and the angular elevation errors have the similar shapes of PDF. The applicability of the approximation (8) (applied to the azimuth angle) is validated in Fig. 2, (b) by comparing the predicted values of parameters α of Beta-distribution to that estimated from 10^6 Monte-Carlo realizations of loss factor by means of maximum likelihood estimator (MLE) [8]. Fig. 2, (c) shows Kullback–Leibler divergence between (16) and its approximation by Beta-distribution (17). The results in Fig. 2, (b) and Fig. 2, (c) show the high accuracy of the derived model. The presented results characterize azimuth error; the elevation and non-orthogonality errors show the similar behavior.

C. Total distribution

The total loss factor is obtained by multiplying the loss factors for orientation errors and non-orthogonality:

$$r = r_o \cdot r_\theta \cdot r_\phi. \quad (18)$$

In [9], it is proved that the product of independent Beta distributed variables can be well approximated by another Beta-distribution with parameters:

$$r \sim \mathrm{Beta}\left(\frac{(S-T)\cdot S}{(T-S^2)}, \frac{(S-T)(1-S)}{(T-S^2)}\right), \quad (19)$$

in which

$$\begin{aligned} S &= E(r) = \prod_{\nu\in\xi} E(r_\nu) = \prod_{\nu\in\xi}\frac{\alpha_\nu}{\alpha_\nu+\beta_\nu}, \\ T &= E(r^2) = \prod_{\nu\in\xi} E(r_\nu^2) \\ &= \prod_{\nu\in\xi}\frac{\alpha_\nu(\alpha_\nu+1)}{(\alpha_\nu+\beta_\nu)(\alpha_\nu+\beta_\nu+1)} \end{aligned} \quad (20)$$

and $\xi = \{\epsilon, \theta, \phi\}$. The considered here shape parameters (α, β) were derived in (11), (17).

III. Power calibration with a non-ideal corner reflector

Assume that we have a corner reflector, produced with a certain tolerance of angles and installed with a certain precision in orientation angles. The objective is to incorporate the possible uncertainty of observed RCS in the power calibration procedure. The measurements of such corner reflector can be described with:

$$\mathbf{y} = Qr + \mathbf{w}, \quad (21)$$

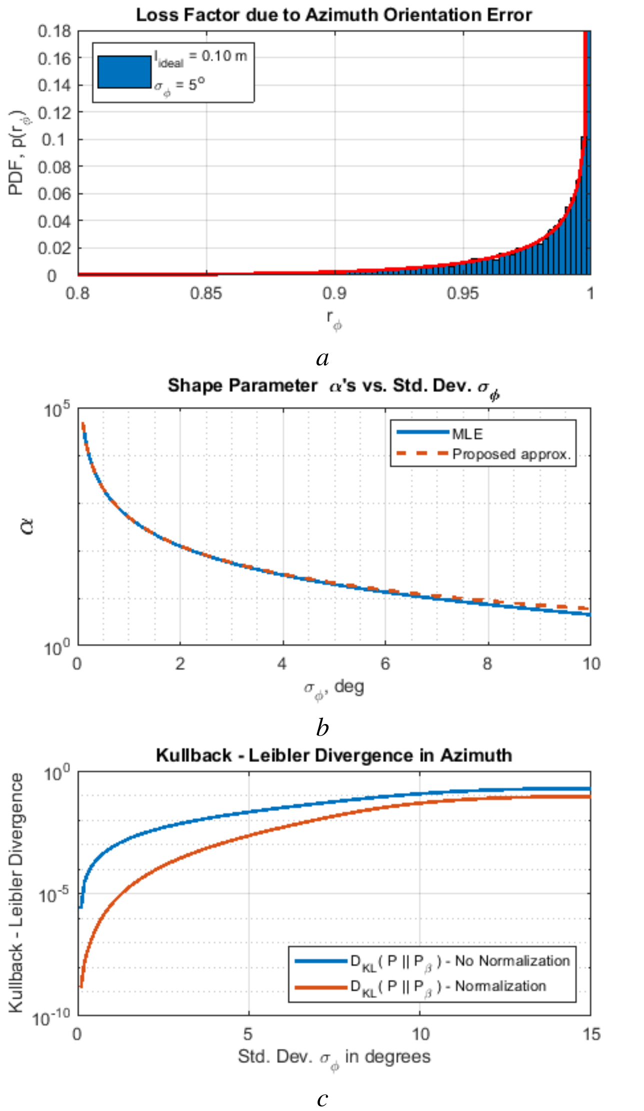
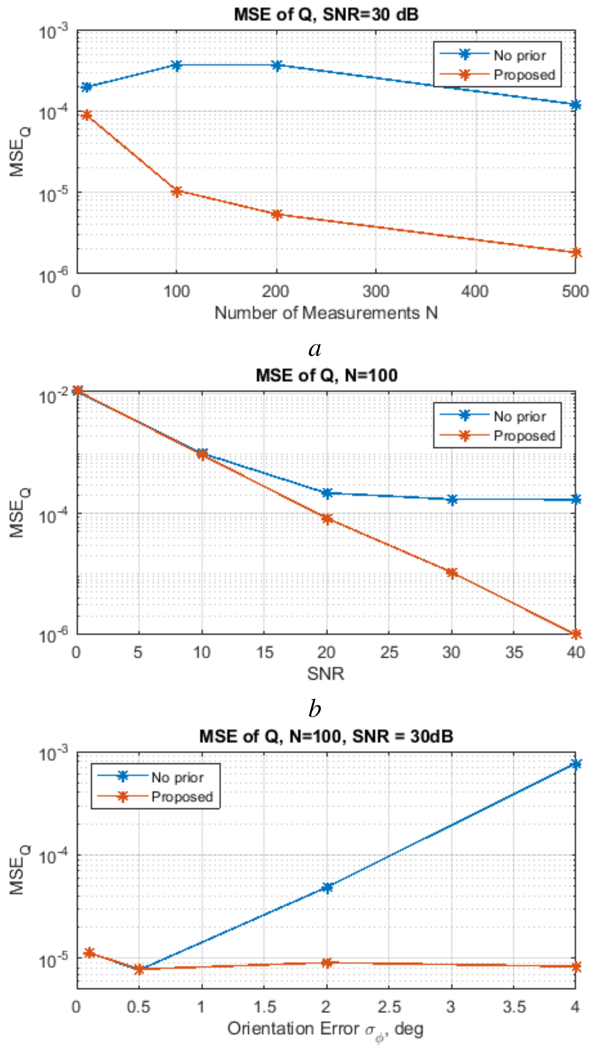


Fig. 2. (a) An example of loss factor distribution in presence of azimuth error; (b) predicted vs maximum likelihood estimation of the shape parameter α; (c) the Kullback–Leibler divergence between (16) and its approximation by Beta distribution (17)

where vector $\mathbf{y} = [y_1, \ldots, y_N]$ denotes the sequence of N independent measurements of the same corner reflector, $\mathbf{w}$ stands for the corresponding vector of noise, $r \sim \mathrm{Beta}(\alpha, \beta)$ is the (normalized) random RCS of current calibration target with known distribution of RCS losses, and Q is the unknown power calibration term. The factor Q can be considered as the measure of radar quality (current unexpected extra losses due to the internal radar state degradation or to the radar signals external propagation factors) when measured in the operational mode – it describes how the received power agrees with the predicted value for an ideally functioning sensor. Herein we assume that the data was normalized by RCS_0.

The likelihood function of the measurements is:

$$p(\mathbf{y}|Q,r) = \frac{1}{(2\pi\sigma_n^2)^{\frac{N}{2}}}\exp\left(-\frac{1}{2\sigma_n^2}\sum_{n=1}^{N}(y_n - Qr)^2\right) \quad (22)$$



a

b

c

Fig. 3. Performance of the proposed power calibration technique:
a - as a function of number of measurements for SNR=30 dB, $\sigma_\phi = 3^o$;
b - as a function of SNR, N=100, $\sigma_\phi = 3^o$;
c - as a function of azumuth orientation error σ_ϕ, N=100, SNR = 30 dB.

where

$$p(r) = \frac{r^{\alpha-1}(1-r)^{\beta-1}}{B(\alpha,\beta)}, \qquad r \in [0,1]. \tag{23}$$

Considering $p(\mathbf{y}|Q) = \int p(\mathbf{y}|Q,r)p(r)dr$, the maximum likelihood estimation of Q is found via:

$$\hat{Q} = \mathrm{argmax}_Q \int_0^1 e^{-\frac{1}{2\sigma_n^2}\sum_{n=1}^{N}(y_n - Qr)^2} r^{\alpha-1}(1-r)^{\beta-1}dr, \tag{24}$$

which is solved numerically by approximating the integral by a finite sum and applying Newton's method to it.

This approach can be similarly generalized for the case of sequential observing a few $M > 1$ different corner reflectors, each characterized by its loss factor r_m. However, the solution, in this case, requires multi-dimensional numerical integration over $r_1, \ldots, r_M$, which can be computationally heavy.

Simulation results of the proposed techniques are demonstrated in Fig. 3 and compared to the results of power calibration assuming no degradation of corner reflector RCS. The variance of the estimated quality metric $\hat{Q}$ decreases with the increase of independent measurements of the target N (Fig. 3, *a*) and for higher SNR of the measurements (Fig. 3, *b*). For moderate number of observations and high SNR, the assumption on no RCS variation leads to large errors. For a single target observation, the shape parameter of the distribution (controlled here via the azimuth error σ_ϕ) does not affect the estimation performance (Fig. 3, *c*) of the proposed technique, while larger RSC uncertainty leads to larger MSE of standard calibration (no proir on RCS). For the proposed method, the uncertainty of the radar installation and production errors are known a priori and they are used to define PDF $p(r)$.

IV. Conclusion

In this paper, we derived the probability distribution of the trihedral corner reflector RCS if it was produced with a certain tolerance of the orthogonality of its sides, or installed with certain orientation errors. We demonstrated that the RCS of a corner reflector with mass production errors can be modeled via four parameters Beta distribution, with the shape parameters determined by the variances of the aforementioned errors. Furthermore, an optimal procedure for radar power calibration technique, accounting for the corner reflector RCS uncertainty due to mass production, has been proposed. Such procedure can be used for real-time estimation of radar sensor healthiness parameter that characterises the sensing quality for awareness of human driver or automated driving system.

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