The Institution of Civil Engineers.

1 20 pag.'s

COMMITTEE ON WAVE PRESSURES:

INTERIM REPORT

ON

WAVE-PRESSURE RESEARCH

BY

MAJOR R. A. BAGNOLD.

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ENGINEERING RESEARCH.

THE INSTITUTION RESEARCH COMMITTEE.

Committee on Wave Pressures.

In the November 1935 Journal [1] it was announced that a Committee had been formed to investigate the problems of wave pressures on sea structures, and proposed preliminary investigations were outlined. Those investigations were duly carried out, and it was reported in the December 1936 Journal [2] that they had indicated the value of small-scale model experiments and that a programme of such experiments had accordingly been drawn up in collaboration with the Department of Scientific and Industrial Research. The initial programme provided for 1 year's work, to be carried out by Dr. C. M. White, of the Imperial College of Science and Technology, London, at a cost of about £600, The Institution contributing £300. Major R. A. Bagnold subsequently took over the experimental work, under the supervision of Dr. White, and the programme was extended for a further year (1938–39). Major Bagnold has now prepared a comprehensive report on the experimental work so far carried out, which is printed below as an Interim Report of the Committee.

The Committee hope that it may be possible to arrange for the extension of the research for a further year to deal with some of the points requiring attention that are listed at the conclusion of the Report. The present personnel of the Committee is :—

Sir Leopold Savile, K.C.B. (*Chairman*).
A. L. Anderson, C.B.
Major R. A. Bagnold, M.A.
W. T. Halcrow.
R. E. Stradling, C.B., M.C., Ph.D., D.Sc., Director of the Building Research Station.
Assistant Professor C. M. White, B.Sc., Ph.D., of the Imperial College of Science and Technology.

The Committee wish to record their regret at the loss they have sustained by the deaths in 1938 of their fellow-members, Mr. G. G. Lynde and Mr. H. H. G. Mitchell, whose interest and support in the work of the Committee were greatly valued.

[1] Vol. 1 (1935–36), p. 43.
[2] Vol. 4 (1936–37), p. 310.

Interim Report on Wave-Pressure Research.

By Major RALPH ALGER BAGNOLD, M.A.

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INTRODUCTION.

THE investigation to be described is an attempt to throw light on the nature of the shock pressures exerted on the face of a vertical sea-wall when a wave breaks against it.

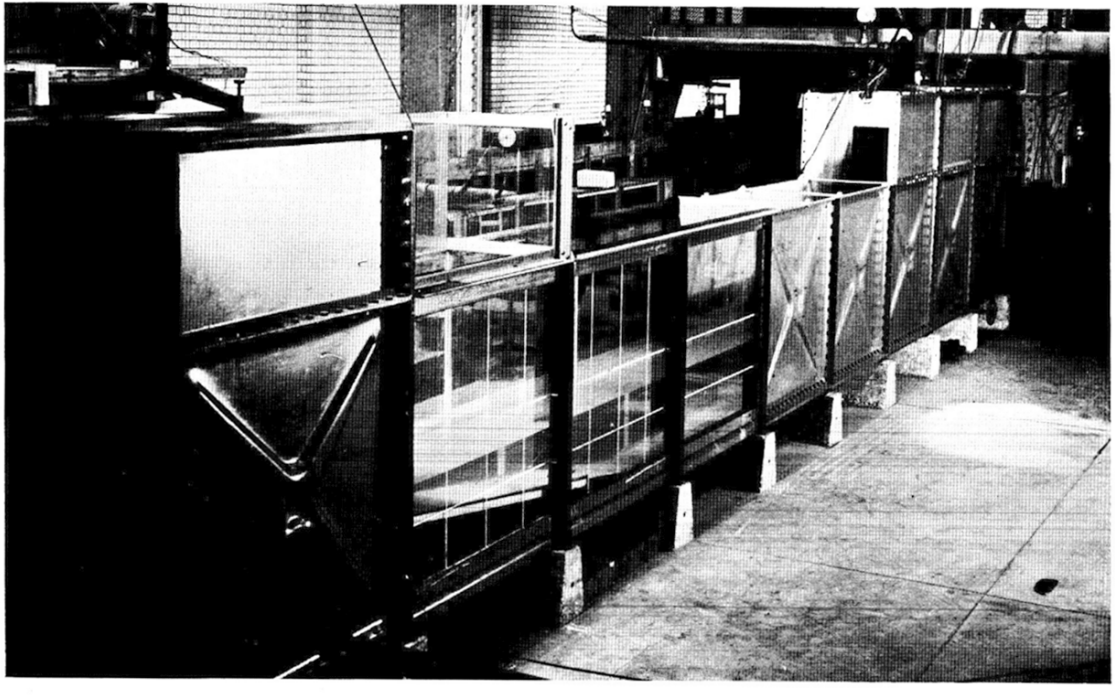
The subject of wave pressures is discussed periodically at the International Navigation Congresses. As a result of the Brussels Congress of 1935 work was initiated in Great Britain by the Research Committee of The Institution of Civil Engineers. It was sponsored jointly by this Committee and by the Building Research Station, and was carried out under the supervision of Assistant Professor C. M. White, Ph.D., B.Sc., in the Civil Engineering Department of the Imperial College of Science and Technology, London. Experiments were started during the summer of 1937. The research, which has been confined to model-experiments, has been to some extent complementary to the elaborate full-scale measurements undertaken by the French Department of Ponts et Chaussées, with whom close touch has been maintained.

The main problem awaiting solution was as follows. It has been known for some years through the work of French and Italian experimenters [1] that two types of pressure are exerted by waves on sea-walls. In deep water the advancing wave does not break, but contact with the wall deflects the water upwards as a *clapoti*. In this case the pressure on the wall at any level is merely the hydrostatic head corresponding to the height of the top of the *clapoti*. The pressures are therefore small, but they last for periods of many seconds while the water is rising and falling. The mathematical theory of the *clapoti* has been worked out by Sainflou [2], and the

[1] Previous work is ably summarized in "Etat actuel des Etudes internationales sur les Efforts dus aux Lames", by A. de Rouville, P. Besson, and P. Pétry. Ann. Ponts et Chaussées, vol. 108 (II) (1938), p. 5.

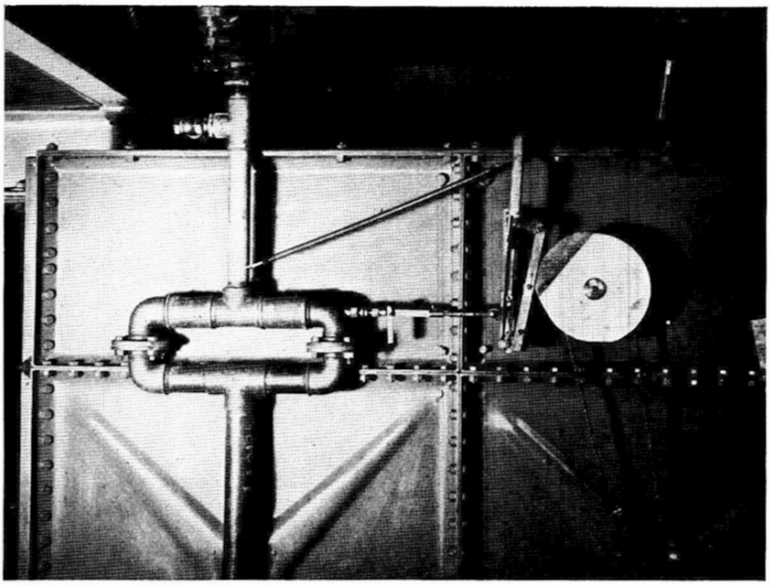
[2] "Essai sur les digues maritimes verticales." Ann. Ponts et Chaussées, vol. 98 (II) (1928), p. 5.

Fig. 1.



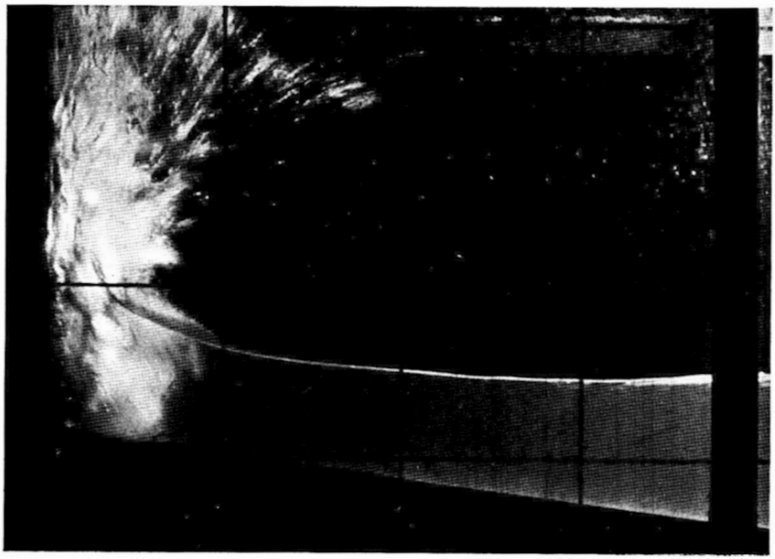
EXPERIMENTAL TANK.

Fig. 2.



WAVE-MAKING CONTROL-APPARATUS.

Fig. 4.



WAVE JUST AFTER IMPACT.

results have been satisfactorily verified both by French investigators at Dieppe, and by Italian investigators at Genoa. In localities where the water-level is constant—for example, in the Mediterranean—a sufficient depth of water is maintained at the wall to ensure that no impact other than that of the *clapoti* type can ever occur. The maximum pressures to be dealt with are therefore known as soon as reliable data have been collected regarding the maximum waves likely to be encountered.

Where the tide varies, however, the depth of water at the wall may be such that at low tide an oncoming wave may strike the wall at the moment of breaking, so that the advancing water face is nearly parallel with that of the wall. In this case, although the ultimate fate of the wave is the same as before—namely, it forms an upward jet which gives rise to a long-period pressure of low intensity—there also occurs, superimposed on the other, a shock pressure of great intensity but of very short duration.

Such shock pressures have been recorded at Dieppe up to a maximum value of 100 lb. per square inch. They occurred fitfully, however, and not more than 2 per cent. of the waves which struck the measuring apparatus gave any pressure at all. Not only has no useful correlation been obtained between the pressure maxima and the characteristics of the waves producing them, but no physical explanation has been forthcoming regarding how these pressures can arise. The problem is made more difficult by the entire absence of any theory or mathematical background for the mechanism of the breaking wave.

It was considered that the most fruitful method of studying such a problem was likely to be the use of a model wave-tank in which the characteristics of the wave causing the pressures could be maintained under close control.

The Model Wave-Tank.

The tank is shown in *Figs. 1, 2,* and *3*. It was made of standard pressed steel tank-plates 4 feet by 4 feet. The observation section, 12 feet long, was made of $\frac{3}{8}$-inch armour-plate glass in panels also 4 feet by 4 feet.

The wave-making apparatus consisted of a double-acting hydraulic ram A (*Figs. 3*, p. 204) working in trunnions and attached to a lever B pivoted at C. This had external and internal slides D and E, to which were attached connecting rods to the top and bottom of a plane steel paddle F whose weight was supported by two rollers G. The travel of the lever B and its speed-variation were controlled by a motor-driven cam shown in *Fig. 2*, which operated the valve gear. The travel of the top and bottom of the paddle could be adjusted independently by varying the heights of the two slides on the lever B.

The experiments were confined to a single wave driven forward along a level floor, made to break, by means of the sloping beach H, against the wall-face J, and reflected back again to meet the next stroke of the paddle, which meanwhile was held stationary. By altering the cam-setting and the

motor-speed two or more colliding waves could be produced, which travelled to and fro between the point or points of collision and the wall and the paddle. Since the water motion of the break (*déferlement*) seemed to be identical in both cases, however, there appeared to be no point in using anything but the one simple wave.

The wall consisted of two separate blocks of concrete of sufficient weight to withstand the maximum pressures by themselves without attachment to the tank sides. In the 4-inch slot between them a ½-inch stiffened steel plate, K, 15 inches high and 8 inches wide, carrying the piezo-electric unit L, could slide up and down. This plate was counterbalanced by the weight M, and was jammed in place by the lever and eccentric pivot N.

Figs. 3.

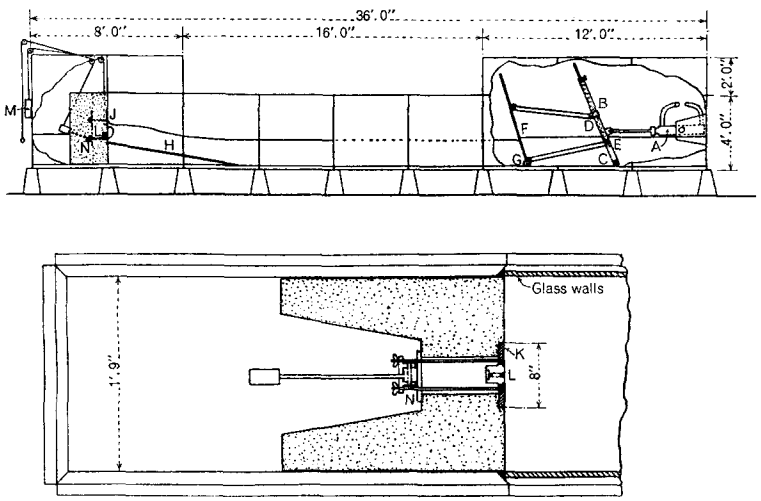


Fig. 4 (facing p. 203) shows a wave immediately after impact. The grid lines imposed on the glass tank-walls are 10 centimetres apart.

Pressure-Measurement.

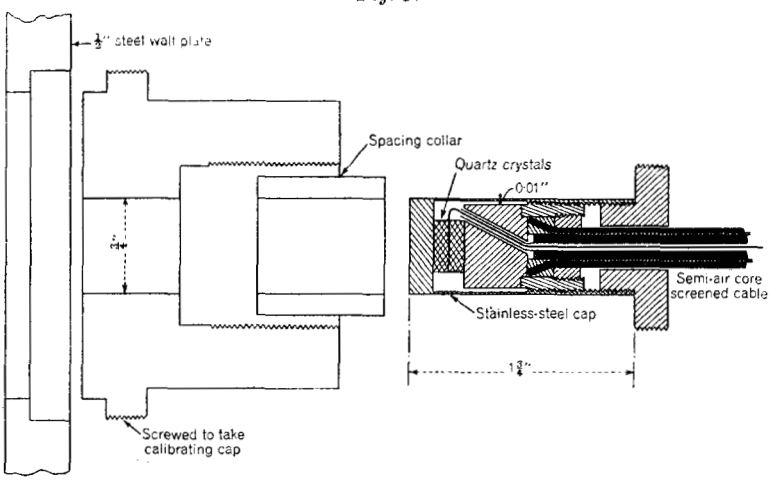
The piezo-electric unit L (*Figs. 3*) consisted of a stainless-steel capsule, shown in detail in *Fig. 5*, containing a pair of quartz crystals. The capsule was mounted in a heavy brass plug. The electric charge produced by the mechanical compression of the crystals was conveyed to a two-valve amplifier through a screened cable of low capacity and high insulation-resistance. The amplified impulses were fed through a second direct-current paraphase amplifier to a cathode-ray oscillograph, equipped with a time-base capable of giving any required spot-frequency. The most convenient spot-frequency was found to be 50 cycles per second. The whole

pressure-measuring apparatus was found to be practically aperiodic, and to be capable of response to shocks—produced by small direct metallic blows on the capsule face—of considerably shorter duration than any that were registered by wave-impacts.

Calibration was effected by screwing a pneumatic cap, connected with a gauge and pump, to the front of the unit. With the 12-centimetre cathode-ray tube used the most convenient amplification gave spot-deflexions of 1 centimetre for each 10 lb. per square inch pressure.

After trials with various early forms of capsule and connexions little difficulty was experienced in handling the extremely minute electric charges involved in the piezo-electric method of pressure-measurement, even under the adverse watery conditions in which the work was carried out.

Fig. 5.



Since no satisfactory method could be found whereby the cathode spot could be released automatically at the moment of wave-impact, photographic records were taken by operating the camera by hand while watching the approaching wave. The only disadvantages of this simple method were (i) the pressure-peaks might occur at any point along the horizontal sweep of the spot, and (ii) the spot made several blank sweeps during the exposure period and so produced a continuous and somewhat heavy trace along the axis of zero pressure.

Experimental Results.

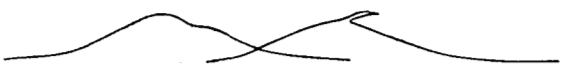
The Breaking Wave.

The following general picture was obtained by means of visual observa-

tion, aided by a slow-motion cinematograph film of the types of in-shore water movement which may occur.

The Partial Break (" white horse ").—A pure water wave of constant velocity and wave-length appears to exist only as a mathematical abstraction. Over a uniform horizontal bottom such a wave would travel unchanged in cross-sectional form. In nature a wave is composed of many components which may vary widely in wave-length and amplitude. It has long been known that when the amplitude of a wave exceeds about one-seventh of its wave-length the wave becomes unstable, and cannot continue without a change of form involving a dissipation of surplus energy. Since the velocity of a water wave depends on its wave-length, phase-changes are continually occurring, and when two waves of large amplitude come into phase the above instability results in a " white horse." Observation shows that the excess energy is ejected outwards very suddenly from the combined summit as a thin jet of water (*Fig. 6*). The jet curls

Fig. 6.



over on one side or the other, and its energy is dissipated in a turbulent disturbance of the surface. If the wind-pressure is negligible, the side on which the jet collapses depends on its initial inclination, and this is always away from the component wave whose amplitude is the greater. The angle of the initial inclination from the vertical appears to vary with the ratio of the two amplitudes. This production of " white horses " is therefore not the direct result of wind-pressure, but is the mechanism whereby the excess of energy produced by the amalgamation of two waves is dissipated.

The curling-over of the jet containing the energy which the wave can no longer retain is called a " break." The " white horse " is a partial break When the whole wave becomes unstable, however, its energy is, if space is available, dissipated altogether in a full break. A partial break may occur by the superposition of any two or more waves, whatever their respective wave-lengths may be, and a small parasitic wavelet may cause a partial break when it is raised to the summit of a larger wave of sufficient amplitude-ratio.

Reflexion from a Wall (*Clapoti*).—The ejection of excess energy when two waves meet occurs whether they are travelling in the same or in opposite directions with regard to the bottom. In the latter case, however, the sense of rotation of the water particles differs, so that the result is not quite the same, unless the two amplitudes are very different. When a wave is reflected from a vertical wall and meets its oncoming fellow, the

opposite sense of rotation of the water particles produces a more symmetrical jet. If the two waves are of nearly equal amplitude the jet rises vertically, and, being broader at the base, collapses on itself instead of curling over. Hence its energy is not dissipated in turbulence.

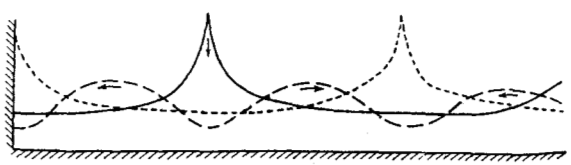
Two distinct wave-motions result, depending on the ratio of the wave-length to the water-depth; that is, on whether the waves are "surface waves" or "long waves" (shallow water). In the case of surface waves, when the water particles have a nearly circular motion, and the effect of the bottom can be neglected, reflexion produces true standing waves or *clapotis* (*Fig. 7*). Here the wave-peak has no horizontal motion, and the centre of curvature of the rest of the surface is always above the surface.

Fig. 7.



In the case of long waves, the collapse of the jet or *clapoti* forms two travelling waves which move away in either direction from the site of the jet, as shown in *Fig. 8*. The subsequent collision of pairs of these travelling

Fig. 8.



waves gives rise to a second set of *clapotis*, and so on. At the wall, therefore, the effect is the same as if a succession of single travelling waves were to advance against it.

Full Break on Beach.—When a travelling surface wave advances sufficiently far over a gently rising bottom, it tends to become a long wave when the water-depth is less than about a quarter of a wave-length. The velocity, which was originally a function of the wave-length only $\left(U=\sqrt{\frac{g\lambda}{2\pi}}\right)$, becomes, when the depth is small, a function of the depth only ($U = \sqrt{gH}$).

The complete expression for the velocity in any depth of water is

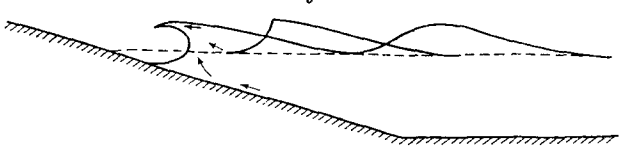
$$U^2 = \frac{g\lambda}{2\pi} \tanh \frac{2\pi H}{\lambda},$$

neglecting the effects of surface-tension. Hence, as the bottom rises, the wave is slowed up, and the wave-length is decreased. The energy of the wave is maintained by a change in the wave-form, the amplitude increasing to offset the reduction in velocity, until the limit of stability is reached at an amplitude/wave-length ratio of about 1/7.

If the slope of the bottom is steep (*Fig. 9*), however, the wave has no time to adjust either its velocity or its amplitude, both of which remain nearly constant (the velocity being taken as that of the summit). The wave-front steepens very rapidly, till it becomes vertical. The water below, which would, but for the bottom, have been pushed on ahead, is now scooped up the wave-front towards the top, as indicated by the arrows in the figure. The wave-energy again tends to be driven out in the form of a jet, but in this case, since there is no second wave, the jet is horizontal. The jet advances at nearly twice the velocity of the wave summit, and falls to the beach where the whole wave finally dissipates itself in a series of vortices.

Defining the position of the break on the beach with regard to the still-water shore-line as that of an ordinate through the point where the wave-

Fig. 9.



front first becomes vertical, this position depends on the beach-angle and on the characteristics of the original wave. Unfortunately the position is also very sensitive indeed to other factors which are hard to define. The chief of these are (i) the amplitudes and phase-relations of the random wavelets which are present on the water-surface before the arrival of the wave; (ii) the strength of the current of water flowing down the beach from the wash of the last wave; and (iii) the surface-roughness and irregularity of the beach.

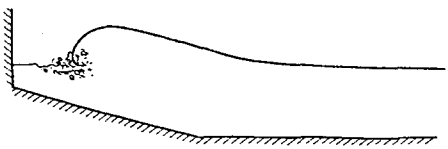
The first and most important of these factors, in addition to affecting the position of the break, also considerably complicates its action, for very often a partial break will occur at the crest just before the main jet should otherwise develop. In view of the fact that no natural sea wave is ever free from these haphazard complications, it was not thought worth while to attempt any quantitative work on the relations between the characteristics of the main wave, the beach-angle, and the break position.

Full Break against a Wall.—If the bottom terminates in a vertical wall whose base is permanently submerged, the advancing wave, whether it

originates far away or results from a reflexion of the type shown in *Fig. 8* (p. 207), has three possible fates :—

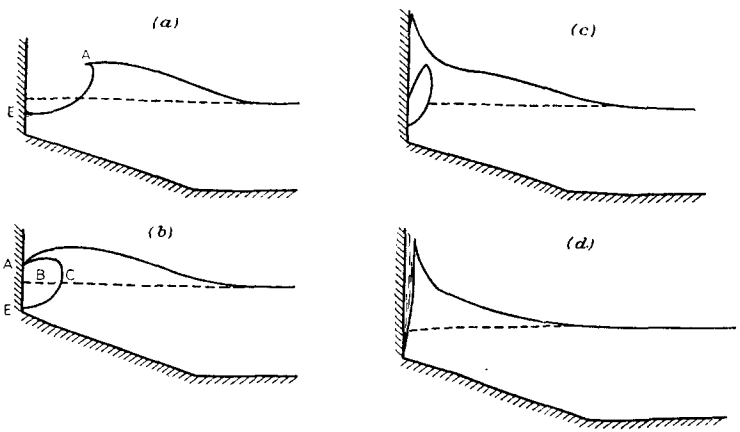
(i) The break may occur so early that the jet has collapsed before reaching the wall (*Fig. 10*).

Fig. 10.



(ii) If the break occurs later, the jet A (*Figs. 11* (*a*)) may strike the wall before it falls (*Figs. 11* (*b*)). In doing so it encloses a large cushion of air B between the wall and the lower part C of the wave-front. As this concave front advances, the air is compressed, and finally bursts upwards with a low booming sound and with the formation of much spray. The

Figs. 11.



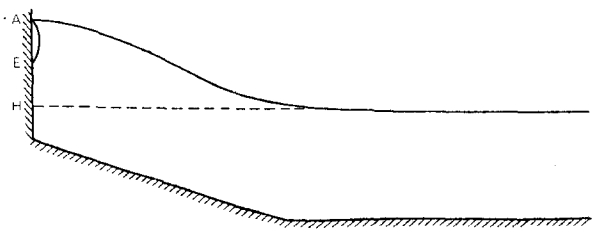
subsequent history of the break is shown in *Figs. 11* (*c*) and (*d*). It should be noticed here that the water-line E is lower than the general level of the surface in front of and behind the wave before it begins to break.

A still later break is shown in *Fig. 12* (p. 210). This is the only case in which the shock pressures appear ever to be produced. The air cushion is much thinner in the horizontal direction, and the water-line E has begun to rise from its lowest position before contact has been made at A. The

height of the cushion is therefore shorter in the vertical direction. The noise of the break is higher and sharper.

The velocity of rise of the water-line E is very great, and if the break happens a fraction of a second too late, E has reached the level of A before contact has been made. There is now no break at all, no air is enclosed,

Fig. 12.



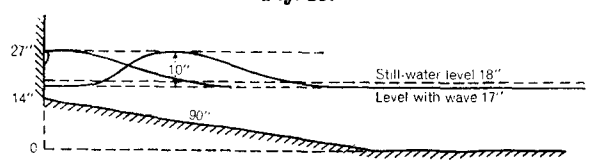
and there is no noise or shock. The water-line E runs steadily up the wall. The wave has now become, in effect, one-half of a *clapoti*.

The condition necessary for the production of shock pressures, that is, a very flat vertical wave-front, enclosing a thin cushion of air between itself and the wall, can only exist for a very short period of time. The condition is therefore extremely critical.

Shock Pressures Observed.

Shock pressures exceeding 10 times the long-period hydrostatic pressures had, as previously stated, been recorded by the French investigators at Dieppe, but attempts to correlate the pressure maxima with the observed characteristics of the waves causing them had failed completely. It was

Fig. 13.



hoped that in the model-tank, with wave-production under control, it would be possible to find this correlation, or at least to repeat at will shock pressures of constant maximum values, so that their distribution over the wall could be accurately explored. Accuracy of repetition being the first essential, attention has been concentrated on that form of wave which was found to give the most consistent results. The arrangement is shown in *Fig. 13.* A single wave was made to travel from the paddle to the wall,

where the break was caused by the slope of the beach. After reflexion the wave travelled back to the paddle, which had remained stationary the while, and which was timed to make another stroke at the right moment to drive the same wave forward once more. The wave-amplitude was 10 inches, the velocity of its summit was 6·8 feet per second, and the velocity of the vertical wave-front just before impact was 8 feet per second.

At the outset hydraulic power to the wave-making paddle was controlled by the speed of a direct-current motor driven off the mains. Although a breaking wave of constant amplitude could then be repeated indefinitely, the mains-voltage fluctuation of some 3 per cent. made it impossible to control the position and timing of the break well enough to maintain a succession of impacts which would all give measurable shock pressures. The shock pressures, as in the French work, occurred fitfully. In 90 per cent. of the impacts there were no shock pressures at all; the majority of the shock pressures when they did occur did not exceed 10 lb. per square inch, but very occasionally maxima as great as 35 lb. per square inch were observed.

A synchronous motor and a " Positive Infinitely Variable " gear for speed-control were then installed, and after a great deal of time had been spent in adjusting the wave-making apparatus, shock pressures could be obtained at every wave-impact. The necessary conditions were so critical that an alteration of but 0·3 per cent. in the paddle timing was sufficient to make the pressures disappear altogether.

The variation in the values of the pressure maxima from impact to impact was, however, still found to be as great as ever. Increasing the probability of their occurrence had only led to increasing the probability that still higher pressures would occasionally be observed. The highest pressure recorded was 80 lb. per square inch (visual observation only).

Typical oscillograms are reproduced in Figs. 14, Plate 1. Cathode-ray pictures of much higher pressure peaks have been observed visually, and examined in greater detail by the use of faster spot-frequencies. Reduced to the same time-scale as the photographs, such curves are typified in Fig. 15, Plate 1. These high pressures happened so seldom that unfortunately, in spite of several hundreds of exposures, no photograph of them has been secured.

From an examination of a large number of plates, and from the impression gained from the visual observation of a far larger number of cathode-ray pictures, when considered together with the simultaneous observation of the breaking wave itself, five conclusions have been drawn :—

(1) The shock pressures occur only when the shape of the advancing wave-front is such as to enclose an air cushion between it and the wall. The pressures are negligible when the thickness of this cushion outwards from the wall exceeds half its height, but they increase in intensity with decreasing thickness of the air cushion.

(2) The great variation in the pressure maxima from impact to impact,

even when to all appearances one wave is indistinguishable from another, must be due to a variation in the mean thickness of the air cushion arising from random irregularities in the relief of the concave water face as it meets the wall. These irregularities are inevitable. They are due to small parasitic wavelets, caused by the disturbances of the last wave-impact, of the paddle, and of partial breaks occurring at the wave crest during its journey from the paddle to the wall.

(3) Although the magnitude of the pressure-peaks varies enormously from impact to impact, the area enclosed by the pressure–time curves tends to approach, and never to exceed, a definite value. Thus a pressure which rises to a high peak value lasts for a shorter time than does a lower pressure.

(4) The horizontal zone along the wall on which the pressures were exerted was explored by sliding the steel wall-plate containing the piezo-capsule up and down the wall. Notwithstanding the difficulty in obtaining satisfactory repetitions of the phenomenon, it is very clear that both the high pressure-peaks and the maximum pressure–time areas only happen over the zone occupied by the air cushion. The height of this zone, between A and E in *Fig. 12* (p. 210), varied from 3 to 4 inches. Above A no shock pressures have been observed, and below E both the pressure maxima and the areas of the curves fell off rapidly with increasing depth. Near the bottom the oscillograms merged into those due to the long-period hydrostatic pressures.

(5) The maximum shock pressures ever observed in the model-tank have not exceeded one-sixth of the theoretical pressures possible if a true " water-hammer " were to occur. In the case of the French full-scale measurements the corresponding fraction is one-fourteenth. The full duration of the pressure in both cases always exceeds by 10 times, and generally by considerably more, the period required by the usual " water-hammer " theory. On this theory the duration of the pressure should be the time taken by a wave of compression to travel with the speed of sound in water (4,000 feet per second) from the seat of the impact to the nearest free surface at which the compression of the water can be relieved. On the other hand, as will appear later, all the observed facts seem to be consistent with the view that the energy of the impact in the case of the breaking wave is stored, not in the compression of the water, but in the compression of the air cushion.

An Explanation of the Observed Shock Pressures.

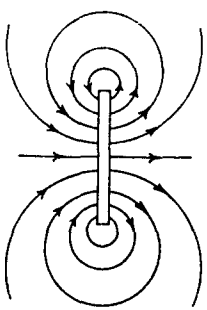
The " Kinetic Mass " of Water.

The physics of the production of short-period shock pressures by a breaking wave is not amenable to direct mathematical approach, because as long as the mathematical problem of the breaking wave itself remains

unsolved the initial conditions of the impact cannot be defined. The general principles which must be involved in the impact can, however, be stated fairly simply; then, by assuming empirical values for the dimensions of a certain real but indefinite volume of water, now to be discussed, useful quantitative results can be obtained.

The motion of a body through a fluid at rest necessitates motion also on the part of that portion of the fluid which surrounds the body. To an observer moving with the body, the movement of the surrounding fluid would appear as sketched in *Fig. 16*. Hence, in order to impart to the body a velocity u, kinetic energy must be given not only to the body, but to that volume of the fluid which is associated with the motion. The fluid velocity varies from point to point within this volume, and the volume is therefore indefinite in extent. It is convenient to imagine an equivalent volume in

Fig. 16.

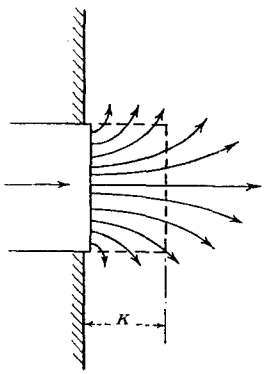


the form of a column of fluid of cross section A equal to that of the body, and of length K in the direction of the motion. This conventionalized fluid column is supposed to be such that if all the particles in it move with the velocity u of the body, the total kinetic energy remains the same as in the case of the real fluid. The associated fluid has, in fact, been replaced by a solid cylinder of the same density. Since this cylinder has a mass ρAK (which may be called the "kinetic mass"), it follows that, if the body be given an acceleration $\frac{du}{dt}$, an additional force $F = AK\frac{du}{dt}$ is required in order to accelerate the mass of the associated fluid. The fictitious length K depends both on the dimensions and on the shape of the body. For a sphere the associated volume of water is known to be half the volume of the sphere. If this volume is replaced by a cylinder of the same diameter d as the sphere, its length K is equal to $\frac{2d}{3}$.

If the body forms part of an otherwise stationary wall, as in *Fig. 17*,

and begins to move forward with an acceleration $\frac{du}{dt}$, there must likewise be an opposing force $\rho AK \frac{du}{dt}$ due to the inertia of the mass ρAK of associated water. Here K is some unknown length, which must, however, be

Fig. 17.



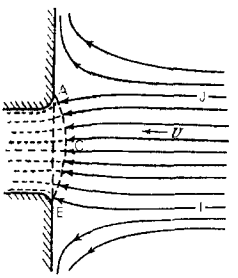
of the same order as the dimension of the accelerated face. It seems likely that K depends only on the boundary conditions, and remains constant for all accelerations as long as the displacement of the face remains inappreciable.

The kinetic mass ρAK, or rather the mass of the real but indefinite volume of water to which it is equivalent, exists as a separate entity, as distinct from the mass of the rest of the fluid, only by virtue of an acceleration of the boundary with which it is in contact. The boundary need not of necessity be the surface of a solid body; acceleration of the boundary, and therefore of the kinetic mass associated with it, can equally well be brought about if the solid body is replaced by a cushion of air at a higher hydrostatic pressure. Moreover, the kinetic mass must also appear in the reverse case when an advancing body of water is retarded.

The system shown in *Fig. 18* is that of steady flow against an infinite plate with a slot in it. It represents an idealized case of a breaking wave a very short time after the jet A of *Fig. 12* (p. 210) has made contact with the wall, and has enclosed a cushion of air whose outer surface is ACE. At the outset this cushion is at atmospheric pressure, and offers no resistance to the advancing water, which therefore behaves as if the slot AE were open. As the air is compressed, however, the force on the surface ACE due to the increasing air-pressure begins to displace the streamlines to the ultimate configuration shown in *Fig. 19*. Thus the retardation of the water is really the rate of displacement of the streamlines, and it is clear that the effect must be felt as a pressure on the wall beyond the limits of the air cushion.

The nature of the fiction of the " kinetic mass " is now apparent. For the extensive and indefinite volume in which the displacement occurs there has been substituted a cylinder of water bounded by the streamlines J and I projecting outwards from the surface ACE for a distance K. This cylinder is supposed to be solid, and to be advancing at the same rate as the mean

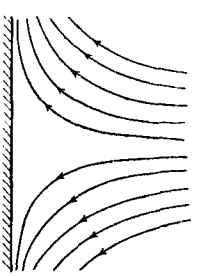
Fig. 18.



surface ACE. At the end of the compression the cylinder is brought to rest, and this is equivalent to the streamlines of the real water having reached their ultimate configuration.

If the length K can be found experimentally as an empirical function of some measurable quantity, the problem of the duration of the compression and the maximum pressure produced becomes, by the above artifice, the

Fig. 19.



straightforward problem of the retardation of a cylinder of water of unit cross section and length K which enters with initial velocity U the mouth of a cup of depth D containing air initially at atmospheric pressure (*Fig. 20*, p. 216).

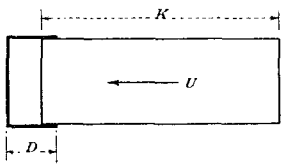
The Shock Momentum and the Length K.

The total forward momentum of the advancing wave is destroyed by the force of the wall acting for the long period during which the water is

rising to the top of the jet. Hence the time-integral of the force on unit length of the whole wall must clearly be equal to the total momentum of unit transverse length of the advancing wave. The time-integral of the force during the fall of the water jet must similarly correspond to the momentum of the reflected wave. Both of these forces are distributed over a great height and over a long period of time, however, and the pressure at any point is consequently small.

In the case of a breaking wave, when an air cushion is enclosed a small portion of the momentum of the wave is destroyed very rapidly owing to the instantaneous application of a large compressed-air force over a small but finite area of the water surface. The value of the momentum so destroyed must be ρUK per unit of superficial area of the air cushion, and this must be equal to the time-integral of the local pressure on the wall. The

Fig. 20.



value of this time-integral can be measured directly from the oscillograms, and the velocity of advance U is also known. Hence the length K can be found very simply:

$$K = \frac{\int p \, . \, dt}{\rho U}.$$

It has already been noted that the areas of the pressure–time curves obtained from the model-waves approach and never exceed a definite limit. This applies also to that portion of the curve between its beginning and its peak; that is, to the portion which, it seems reasonable to suppose, corresponds to the destruction of the original momentum of the kinetic mass of water involved in the impact. In the model-experiments, the area of this part of the curves gave a time-integral which approached a limit of 2·59 lb.-seconds per square foot. Hence, since the velocity U was 8 feet per second, a value of 0·166 foot, or 2 inches, is obtained for K. This is just half the observed height AE of the air cushion (and may be compared with the corresponding figure of two-thirds for a sphere). Since the height AE was found to be 0·4 times the wave-amplitude for those waves which gave appreciable shock pressures, it appears that the length K is empirically one-fifth of the wave-amplitude.

It is interesting at this point to apply the empirical value for K which the above ideas suggest to the case of the real waves whose impulses have

been recorded in the French report referred to on p. 202. Data for seven waves are tabulated in columns 1 to 7 of Table I. The figures for the momentum per unit area in column 7 are obtained by measuring the areas under the compression portions of the pressure–time curves given in the report. They are, therefore, entirely independent of any data other than the pressure actually measured on the sea-wall. The same momentum is arrived at in column 8 from the measured velocity and amplitude of the waves, and from the empirical relation found from the model-work connecting the mean length K of the kinetic mass with the amplitude of the wave.

TABLE I.

Identification.				Wave Data.		From Model.	Shock momentum per unit area of wall: lbs./ft. sec.	
Col. (1)	Col. (2)		Col. (3)	Col. (4)	Col. (5)	Col. (6)	Col. (7)	Col. (8)
Date.	Time: hr.	min.	Height of instrument: metres above datum.	Amplitude, $2h$: feet.	Velocity, U: feet per second.	$K=2h/5$: feet.	$g\int_0^{\max.} p.dt$	ρUK
							(observed)	(calculated)
2/12/35 .	12	12	5·34	14·6	26·6	2·92	1,740	4,960
	12	15	4·37	14·6	26·6	2·92	2,050	4,960
4/12/35 .	14	08	4·37	9·75	19·5	1·95	3,050	2,440
	14	09	4·37	9·75	19·5	1·95	1,620	2,440
	14	50	4·37	9·75	19·5	1·95	2,150	2,440
18/12/35 .	13	35	4·37	13·0	23·0	2·6	3,400	3,840
23/2/37 .	12	37	4·37	8·12	22·0	1·62	2,420	2,260

In the French work attention was paid to the magnitude of the peak pressures rather than to that of the shock momenta, and the sample curves given in the report were presumably selected with the peak pressure in mind. It is the more satisfactory, therefore, that the shock momenta as derived from the pressure curves should show agreement with those which were derived from the wave-data by means of a relation found from model-experiments on one-twelfth the scale. It should be noted that the figures in column 8 should, according to the model-results, give the maximum impulse that a wave of a given amplitude and velocity can produce, whereas the figures in column 7 may range from zero up to this maximum according to the chance condition of the water surface at the moment of impact. It will be seen that the limit in column 8 has indeed been exceeded in two cases, and it is possible that the maximum value to be allowed for K may have to be made slightly greater than $2h/5$.

The Rise of Pressure. The Compression of the Air Cushion, and the Duration of the Impact.

Having arrived at an empirical method of calculating the length of the

horizontal water column involved in the impact on unit area of the wall, it is now possible to return to the consideration of the rate at which this unit column will compress various thicknesses of enclosed air, and at the same time to calculate the maximum pressure produced when the column is brought to rest.

The water column can be regarded as a solid plunger of length K, entering, at an initial velocity U, a cup of depth D containing air initially at atmospheric pressure p_0. If the distance from the bottom of the cup to the head of the plunger is denoted by x, then for the pressure of the enclosed air

$$p = Ax^{-\gamma}, \text{ where } A = p_0 D^{\gamma},$$

and for the motion of the plunger

$$p = \rho K \frac{d^2x}{dt^2} + p_0.$$

Hence the equation of motion of the plunger is

$$\rho K \frac{d^2x}{dt^2} - Ax^{-\gamma} + p_0 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

On integration this gives

$$\rho K \left(\frac{dx}{dt}\right)^2 + \frac{2A}{\gamma - 1} x^{1-\gamma} + 2p_0 x = c.$$

When $x = D$, $dx/dt = U$. So, substituting the appropriate terms for c,

$$u^2 = \left(\frac{dx}{dt}\right)^2 = U^2 - \frac{2A}{(\gamma - 1)\rho K}\left[\left(\frac{1}{x}\right)^{(\gamma-1)} - \left(\frac{1}{D}\right)^{(\gamma-1)}\right] + \frac{2p_0}{\rho K}(D - x) \quad . \quad . \quad . \quad (2)$$

In the case of isothermal compression, where $\gamma = 1$, the corresponding equation is

$$u^2 = \left(\frac{dx}{dt}\right)^2 = U^2 - \frac{2A}{\rho K} \log \frac{D}{x} + \frac{2p_0}{\rho K}(D - x) \quad . \quad . \quad . \quad . \quad (2a)$$

Assuming that the compression is adiabatic, the time t is given by a second integration,

$$t = \sqrt{\frac{\rho K}{p_0}} \int \frac{dx}{\sqrt{\left\{\frac{\rho K}{p_0} U^2 - 5D^{1\cdot4}\left[\left(\frac{1}{x}\right)^{0\cdot4} - \left(\frac{1}{D}\right)^{0\cdot4}\right] + 2(D - x)\right\}}} \quad . \quad . \quad (3)$$

This second integration for t has not been achieved, but a number of typical pressure–time curves have been computed by graphical integration. Two families of these curves are given in Figs. 21 (a) and (b), Plate 1. In

each set of curves both the velocity of advance U and the length K of the water column have been kept constant, and the thickness D of the air cushion has been varied. In Figs. 21 (a), Plate 1, U and K correspond to the conditions in the model-tank, and in Figs. 21 (b), Plate 1, they correspond to the mean of the full-scale conditions in the French report.

Over the range considered from 2 to 10 atmospheres :—

(*a*) The peak pressure is given within $\pm$ 10 per cent. by

$$(p_{\max} - p_0) = 2{\cdot}7 \frac{\rho U^2 K}{D}$$

in any consistent units.

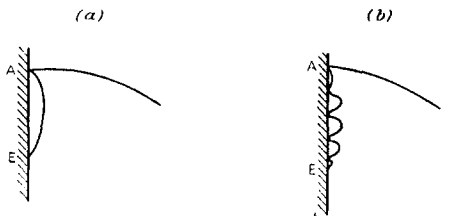
(*b*) The whole duration of the compression approximates, for high peak pressures, to the time taken by the water front to travel the distance D at the initial speed U, but for low pressures the duration is relatively shorter. If $T = a\frac{D}{U}$, a has a value of 1·1 for a peak pressure of 60 lb. per square inch, of 1·7 for a pressure of 20 lb. per square inch, and of 3 for a pressure of 7·5 lb. per square inch. This provides a simple means of estimating the thickness of the air cushion from the pressure curves.

(*c*) The area under the curve is constant as long as the product UK remains constant, no matter what may be the thickness D of the air cushion. This is obvious from the assumed simplified physics of the plunger and cup, for the value of $\int p \,.\, dt$ must be equal to the initial momentum ρUK under any conditions of compression. Under the actual conditions of wave-impact it is probable that the compression-index γ varies during the compression, and that rapid cooling may reduce γ to unity (isothermal compression) towards the end of the stroke. This will affect the shape of the curve, but not the value of $\int p \,.\, dt$.

It is likely, too, that there will be a preliminary small but gradual rise of pressure while the initial shape of the water front becomes modified by the enclosure of the air cushion. The fate of the air cushion during the compression is still not clear, as observation is difficult owing to the very rapid movement of the water surface. Immediately afterwards, however, the air appears in the form of small isolated bubbles. From this it seems likely that the water front becomes unstable during its retardation, and breaks up into little jets, as sketched in *Figs. 22* (p. 220). Since all such jets will certainly not strike the wall at the same instant, the pressure at any one point on the wall is likely to be built up in a series of steps as the pressures propagated from surrounding compressions reinforce that of the compression immediately over the recording instrument.

With these considerations in mind, the calculated pressure curves of Figs. 21 (a) and (b), Plate 1, may be compared with the actual oscillograms obtained from both model- and real waves. Those for the model are given in Figs. 14 and 15, Plate 1, and for the real waves some of the photographs given in the French report have been reproduced in Figs. 23, Plate 1. In each case the calculated curves have been drawn on the same relative scales

Figs. 22.



of pressure and time as the corresponding oscillograms. As far as the compression side is concerned, there is close agreement both as regards the general shape of the curves and the duration of the compression.

The Fall of Pressure.

If the compressive pressure is that required to absorb the momentum ρUK, then the subsequent expansion must recreate a corresponding outward momentum in the water.

It will be seen from the oscillograms that the fall of pressure after the peak has been reached is more irregular and more varied in form than the rise. The general shape of the curve strongly supports the view that an expansion of air follows the initial compression, and that the cycle is repeated as a damped oscillation. That this volumetric oscillation really occurs between the water and the enclosed air, and is not an instrumental effect, is borne out (*a*) by the large variation in its period, even though no part of the apparatus was altered during the series, and (*b*) by the fact that the same general pattern is seen in the French curves, where the time-scale is 10 times as long.

In addition to this phenomenon, however, there are many instances in which a sharp initial rise of pressure is followed by a second longer period during which high pressure is maintained. These cases, it should be noted, only occur when the time-integral for the initial pressure-rise is small compared with the momentum limit set by the dimensions of the wave. It seems reasonable to suppose that in these cases a small local jet of water strikes the recording instrument first, enclosing but a thin air cushion, and producing a short rapid pressure-rise. This is followed by the main com-

pression, which occurs over a much larger area simultaneously, and therefore involves a greater kinetic mass of water. If this main compression encloses a thick air cushion (ratio K/D approaching unity), and if its peak occurs during the expansion of the earlier local shock, the effect is likely to be a pressure picture such as No. 10 of Figs. 14, Plate 1.

It is therefore difficult to draw the dividing line between compression and expansion; the net result may be that due to pressures rising and falling in different areas at different times. From the practical standpoint it is probably more convenient to consider only the total impulse, as represented by the area under the whole pressure–time curve. It appears from the curves that the maximum value of this total impulse is given by merely doubling the value of ρUK.

Probability of the Occurrence of High Shock Pressures.

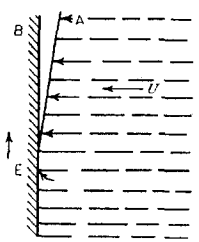
Under the idealized conditions in which a single continuous cushion of air is evenly compressed by a smooth face of advancing water, the probability of the occurrence of a pressure peak exceeding any given value can be estimated at once: it is only necessary to postulate a varying succession of waves such that there are equal chances that the thickness of the air cushion enclosed has any random value between $D = 0$ and $D = K$. Then since for a constant wave-velocity U the value of the pressure peak varies as K/D, the chances of any pressure higher than that corresponding to any particular value of D are simply 1 to K/D. For example, in Figs. 21 (a), Plate 1, the value of K/D which gives a pressure of 36 lb. per square inch is 16·6, so that the chances that a pressure exceeding 36 lb. per square inch will occur should be 1 to 16·6. Actually, however, observation shows that the chances are very much smaller, and the reason is not far to seek: the initial assumption of a continuous smooth advancing water face can rarely, if ever, be fulfilled. The water surface must always contain irregularities and prominences. Suppose that in the above succession of waves the distance by which these prominences project in front of the general surface is denoted by s. Then it is clear that the air cushion can never be continuous if its initial thickness D is less than s. In the above example, for instance, where the mean value of D is $\frac{1}{8}$ inch, instantaneous compression will not occur evenly over the whole area covered by the cushion unless the prominences on the water face are less than $\frac{1}{8}$ inch high. Thus for the very small values of D which alone can give rise to high shock pressures, the cushion must in general be split up into a number of isolated air pockets, the compression of which will very rarely occur simultaneously. Hence the compression of one such pocket must in general be relieved by a sideways movement of the pocket along the wall into a region of lower pressure where the compression has not yet proceeded so far.

This raises the paradox that under atmospheric conditions, if the air

cushion is absent, or if the air in it is allowed to escape, the shock pressure, far from becoming more violent, disappears altogether.

The explanation seems to lie in the fact that, whereas in the case of the compression of the air cushion pressure is applied simultaneously over a finite water surface, in the latter case it is applied successively over different filaments of that surface. *Fig. 24* is an enlarged section of a water–air–solid junction at the edge of a real or potential air pocket. The water surface above the contact-line E is supposed to be advancing unchecked with velocity U, and the water particles below E and close to the wall can have no horizontal velocity. The wedge of air ABE is pushed upwards, supposedly without offering any appreciable back-pressure to the movement of the water. The water surface AE may be imagined to consist of an infinite number of parallel filaments perpendicular to the

Fig. 24.



plane of the paper. As each filament comes successively into contact with the wall at E the momentum of the water in it and behind it is destroyed, and the water particles are deflected upwards. At the moment of the impact, however, there is no pressure above E, neither is there any pressure below E other than a small hydrostatic pressure. Hence at any one instant of time a shock pressure can only occur along the actual contact-line E. Further, since (*a*) the pressure there is in any case limited to the finite value set by the known true water-hammer pressure corresponding to the velocity U, and (*b*) the pressure occurs at any one instant of time only over the infinitesimal width of one filament, the force on the wall at E at any time must itself be infinitesimal.

This appears to hold good provided that the successive time-instants do not overlap; that is, provided that the upward velocity of the water-line E is less than the velocity c_w of the propagation of sound in water. That the upward velocity could ever exceed this value is highly improbable, for even if the presence of the wedge of air is neglected altogether, and vacuum-conditions are assumed, the velocity of E could only exceed c_w if the angle AEB in *Fig. 24* were less than $\tan^{-1} U/c_w$: for a practical value of 20 feet per second for U, the critical angle is found to be only

18 minutes of arc. In a vacuum, therefore, true water-hammer pressures should ensue over a finite area, if the water surface is very nearly parallel to the wall. The presence of atmospheric air must make a considerable difference, however, even though it be free to escape upwards, for the velocity c_a of the propagation of a displacement in air is only a quarter of that in water. Hence, for angles AEB of less than 4 times the above value, the air in the wedge could not get away without checking the advance of the water by the exertion of a back-pressure. It therefore appears that under atmospheric conditions true water-hammer pressures can never occur in a free unrestricted impact between a body of water and a solid, because there must always be an air cushion interposed.

On the other hand, the presence of air may entirely alter the argument that with contact-angles greater than $\tan^{-1}U/c_w$ no finite shock pressure can occur anywhere; for, once the air becomes compressed, either by becoming enclosed or by being unable to get away in time (when the angle AEB is less than $\tan^{-1}U/c_a$), the instantaneous pressure on the wall, instead of being limited to an infinitesimal area at E, is distributed over the whole surface in contact with the air. As a result, there is a sudden destruction of water momentum over a finite and large area, and a consequent large compressive force.

The above ideas are crudely expressed, but a rigid treatment is impossible in the absence of any mathematical background defining the true motion of the water.

There remains the question of why the maximum shock pressures recorded for full-scale waves are relatively so small compared with those observed in the model-experiments. The peak pressures, assuming adiabatic compression, are given approximately by

$$(p_{max} - p_0) = 2{\cdot}7\,\frac{\rho U^2 K}{D}.$$

K appears to be proportional to the wave-amplitude, and U^2 is approximately proportional to the water-depth. Hence, assuming the same probability modulus, it would be expected that if the wave-shape is similar the peak pressures observed should be proportional to the general linear scale of the phenomena. Since the model-pressures reached a maximum value of 80 lb. per square inch and the linear-scale ratio was 12, full-scale pressures of 1,000 lb. per square inch ought to have been encountered, whereas the highest pressure peak recorded by the French is only 100 lb. per square inch.

Whilst much of the discrepancy could no doubt be explained by the smaller number of waves of the required shape and timing observed in the uncontrolled full-scale conditions under which the French were obliged to work, yet the main cause is probably to be found in the very different physical properties of the surfaces of sea-water and fresh water. In sea-

water the permanence of air–water emulsions, and of froths and foam, is far greater than in the case of fresh water, and in the violently-disturbed water near a sea wall the volume of cellular air associated with the breakers is by common experience very large. This air must of itself form an effective cushion, whether or not any additional continuous cushion is also enclosed between the wave front and the wall.

The volume needed to prevent any high pressure peaks from occurring is quite small. With the typical values of U and K taken for the pressure curves of Figs. 21 (b), Plate 1, the value of D corresponding to a pressure peak of 100 lb. per square inch is only 0·36 foot, or 4·3 inches, and it is not unreasonable to suppose that the equivalent thickness of the air enclosed in the internal emulsion and in the external foam is rarely much less than this. Unfortunately, no figures are available.

If this reasoning is correct, the ultimate limit to the intensity of the shock pressures exerted on a sea wall is set by the quantity of air locked in and on the surface of the wave before impact. Since the foam-forming properties of the water are very sensitive to its organic and mineral content, it is likely that the ultimate pressure-limit varies greatly with the geographical locality.

General Conclusions Concerning Shock Pressures.

(1) The shock pressure exerted by a breaking wave is due to the violent simultaneous retardation of a certain limited mass of water which is brought to rest by the action of a thin cushion of air, which in the process becomes compressed by the advancing wave front. The volume of water concerned is approximately equal to that of a horizontal column whose cross section parallel to the wall is that of the frontal aspect of the air cushion, and whose length K is half the vertical width of the cushion. Shock pressures some 10 times greater than the ordinary hydrostatic wave-pressure can be generated in this way.

(2) The pressure set up is determined by the initial velocity of approach U of this water column, by its length K, and by the mean initial thickness D of the air cushion. The pressure at any moment during the impact can readily be calculated from these three quantities by regarding the water column as a heavy free piston which compresses the air cushion adiabatically. The maximum pressure is given approximately by

$$(p_{\max} - p_0) = 2{\cdot}7\rho U^2 \frac{K}{D}.$$

(3) The greatest pressure-maxima therefore occur only when the thickness D of the air cushion is small, and when compression is simultaneous over a large area of this thin cushion. The wave front must thus be approximately plane and parallel to the wall at the moment of impact.

(4) Waves originally of many quite different forms can conform to this condition, but the maximum vertical width of a thin cushion of enclosed air is of the order of 0·4 times the wave-amplitude $2h$. Hence the maximum value of K is, by (1), equal to 0·2 times $2h$, and the maximum pressure-rise can be written

$$(p_{\max} - p_0) = 0{\cdot}54\rho U^2 \frac{2h}{D},$$

and so, for any known wave-amplitude and velocity, varies inversely as the thickness of the air cushion.

(5) This thickness D is indefinite, since it is sensitive to the random irregularities of the surface of the advancing wave front, so that maximum pressures cannot easily be calculated. It seems probable that in the very disturbed water inevitably found near sea walls during storms, enough air is entrained or held on the surface as foam to provide in itself a lower limit to the possible values of D, and that a limit is thereby set to the maximum shock pressure which can be exerted. This, however, could only be established by full-scale measurements.

(6) On the other hand, the impulse I has a definite maximum which is independent of the air-thickness D, and can be predicted from values disclosed by the model. Per unit area of the wall the impulse is simply

$$I = \int p \, . \, dt = 2\rho UK,$$

and this has the maximum value $0{\cdot}8\rho Uh$, where $2h$ denotes the wave-amplitude, and U its velocity. The impulse is thus equal to twice the initial momentum ρK of the " kinetic mass " of water. When maximum impulses are compared, full-scale measurements at Dieppe are consistent with those made on the model.

(7) The main pressure-zone on the wall extends only over the area covered by the air cushion. Below the bottom of the cushion the pressures decrease rapidly. In the model the cushions which gave high shock pressures extended from the wave top, at a height $2h$, to a height $1{\cdot}2h$ above the wave trough. The full-scale results from Dieppe seem to indicate a somewhat lower level for the pressure-zone, but the wall was not vertical, and it was impossible to determine the exact cross section of the waves at impact.

Suggested Lines for Further Research.

Laboratory Work with the Present Model Wave-tank.

(i) Further pressure-measurements on the previous lines but with reflected travelling waves of the type shown in *Fig. 8* (p. 207), and with varying beach conditions, to verify that no larger shock impulses than those already found are likely to occur.

(ii) Experiments on ribbing the outer wall surface to reduce the instantaneous pressure area and the resulting shock momentum.

(iii) The mechanical effect of short-period impulses of high peak pressure in moving concrete blocks which are variously loaded with superstructure.

(iv) The flotation-effects of the long-period hydrostatic pressures due to the rising *clapoti* when such pressures are allowed to penetrate into ill-fitting joints; the degree of close fitting necessary to prevent possible flotation.

(v) Experiments with loose beaches of sand and shingle to ascertain the effects of the bottom-movement of breaking waves in building up and removing the beach material. This might, in addition, provide useful information on coast-erosion.

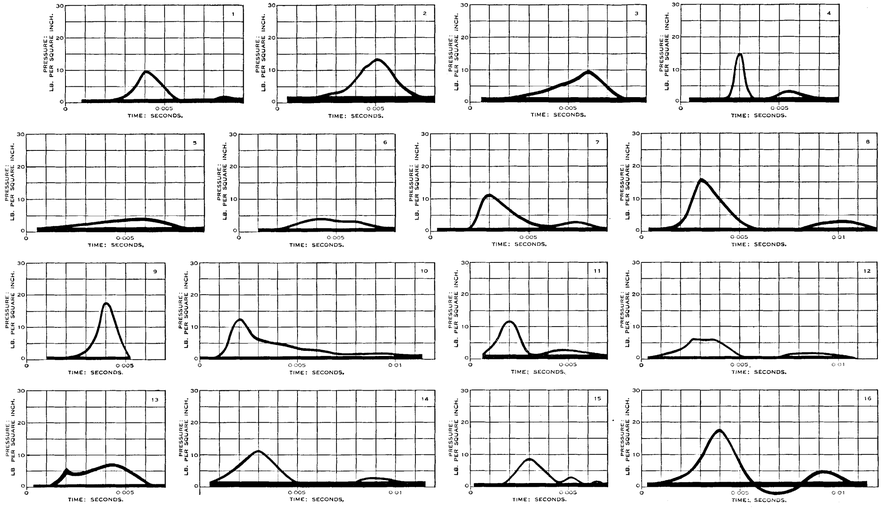
Full-Scale Work.

(i) The collection of information on the minimum air-content of breaking sea waves in different localities; the effect of differences in the organic and mineral make-up of the water on the permanence and volume of the foam and air-emulsion.

(ii) Pressure-measurements on sea walls in confirmation of the French results, with special reference to the position and extent of the pressure-zone.

(iii) Verification of any useful results which may be suggested by the above laboratory work.

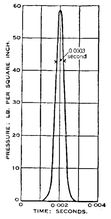
FIGS: 14.



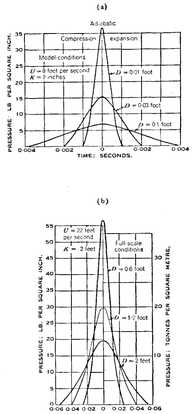
MODEL-WAVE. PRESSURE IMPULSES PHOTOGRAPHICALLY RECORDED.

FIGS. 21.

FIG: 15.



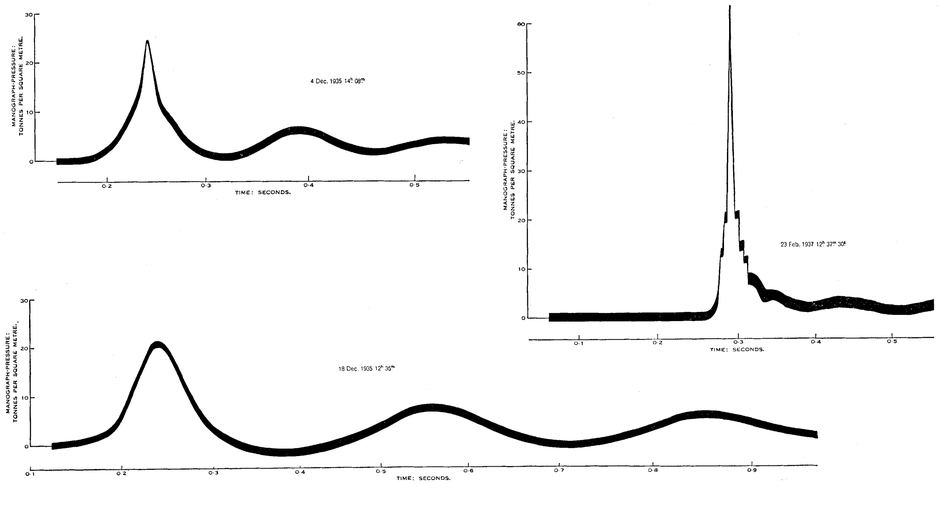
MODEL-WAVE. RARE HIGH-PRESSURE IMPULSES AS OBSERVED VISUALLY.



(a) MODEL-CONDITIONS ITO BE COMPARED WITH OSCILLOGRAMS IN FIGS: 14 & 15?.
(b) MEAN FULL-SCALE CONDITIONS OF WAVES AT DIEPPE ITO BE COMPARED WITH OSCILLOGRAMS IN FIGS: 23?.

PRESSURE IMPULSES AS CALCULATED FROM WAVE DATA, ASSUMING ADIABATIC COMPRESSION OF THE ENCLOSED AIR CUSHION.

FIGS: 23.



FULL-SCALE SEA WAVES. PRESSURE IMPULSES AS RECORDED AT DIEPPE.

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