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Multi-period railway timetabling to serve time-dependent demand

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ABSTRACT

Passenger railway demand fluctuates daily, peaking at the start and end of the workday due to commuting to school and work. During the off-peak the volumes drop and most people travel for other purposes, like leisure and social visits, which results in different travel destinations. Despite this, many European Railways use fixed line plans and cyclic timetables that remain constant throughout the day. While this approach makes schedules easy to remember and provides ample off-peak travel options, it is primarily designed for peak-hour demand, making it less efficient for the off-peak. Furthermore, due to the different mix of travel purposes, a schedule based on peak-hour demand is not necessarily optimal for off-peak demand. This paper aims to combine the benefits of a cyclic timetable with the flexibility of an acyclic timetable in order to follow the time-dependent demand more closely. We propose a mixed-integer linear programming model that finds a timetable for a day consisting of several periods which each have its own line plan. The resulting timetable is required to be cyclic within each period and provide a good transition between the periods. The model is successfully tested on a case study with changing stopping patterns using data from the Dutch railway network, for which an optimal timetable is found. In this timetable, the transition between cyclic schedules can be done without cancelling trains or shifting trains from the new cyclic times.

1. Introduction

The demand for passenger railway services exhibits significant variation during the day. At the beginning and end of the working day, there is a peak period in which a substantial number of commuters travel to or from their workplaces or educational institutions. Outside these peak periods, the passenger volumes are considerably lower and the travel patterns are different as a larger portion of the travellers are travelling for reasons other than commuting, like leisure activities or social visits. On a regular workday in the Netherlands, there can even be up to 10 distinct homogeneous demand patterns, according to Van der Knaap et al. (2024). Despite this variability in demand, in many European countries a fixed line plan and cyclic railway timetable is used, which is the same for every hour of the day. The primary benefit of a cyclic timetable is its memorability for passengers, which makes it easy to use. Cyclic timetables are often clock faced (i.e., trains always leave a certain number of minutes after the hour) and usually aim to have regular departure intervals between trains on the same line. According to Wardman et al. (2004), these two aspects both improve the memorability of a timetable. Furthermore, a cyclic timetable is also advantageous for people travelling outside the peak hours as there are still multiple travel options available during less busy times. Previous studies have shown that having a cyclic timetable with well-planned connections improves the passenger satisfaction and as a result the demand and revenues for the

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Table 1
Overview of benefits and drawbacks of strictly cyclic and acyclic timetables.

	Benefits	Drawbacks
Cyclic timetables	Easy to remember for passengers No gaps in train service when demand is low Consistent transfers throughout the day	Expensive, especially for low demand Allocation of unused hourly time slots for non-hourly trains (e.g., freight or international trains)
Acyclic timetables	Many trains in periods with high demand Flexible to demand variations Less expensive for low demand	More difficult to remember Less travel opportunities and/or gaps in service for periods with low demand Entire day needs to be considered during planning

railway undertaking (Wardman et al., 2004; Johnson et al., 2006). However, the drawback of a cyclic timetable is its inflexibility to the passenger demand. Cyclic timetables are usually optimised for the peak-hour demand and hence the resulting schedules fail to address the distinct travel patterns of off-peak travellers, resulting in a suboptimal service provision during these times. On the other hand, acyclic timetables provide many opportunities to serve varying demand, but lack ease of use for the passengers. An overview of the benefits and drawbacks of cyclic and acyclic timetables, based on the work of Peeters (2003), is given in Table 1.

In recognition of the benefits of both the cyclic and acyclic timetables, several studies have attempted to create a timetable that has both cyclic and acyclic characteristics. For example, Robenek et al. (2017) investigate three types of cyclicity in railway timetables. They show that out of the three options, a 'hybrid cyclic timetable' in which non-cyclic trains can only be scheduled in hours when also a cyclic train is scheduled, is best able to provide both regularity and flexibility. Other examples are the works of Yin et al. (2019) and Li et al. (2023), where a train is not necessarily scheduled in every hour, but whenever it is scheduled it always departs at the same number of minutes past the hour. However, one potential pitfall of creating a hybrid timetable that has both cyclic and acyclic characteristics, is that passengers do not recognise the regularity in the timetable and hence will not experience the benefits of the cyclicity.

In order to create a timetable that has as much cyclicity as possible while also following the varying demand, this paper proposes a mathematical model for the multi-period timetabling problem. The mixed-integer linear programming (MILP) model takes as input several periods in a day in which the demand is homogeneous and for each of these periods a line plan that can serve this demand. The aim of the multi-period timetabling problem is then to create cyclic timetables for each of the defined periods and a good transition between the different cyclic timetables. Furthermore, regularity should be kept throughout the day wherever possible. Specifically, if lines are present in multiple period-specific line plans, the departures and arrivals of this line should deviate minimally to preserve passenger convenience. By having several periods with cyclic timetables and by keeping the regularity throughout the day wherever possible, we aim to provide both regularity to the passengers while simultaneously serving the demand better through the adapted lines. The proposed MILP model is tested on a case study based on part of the Dutch railway network. We show that the model can produce a multi-period timetable for this case study where the line plan has fixed frequencies but varying stopping patterns throughout the day. Changes in the stopping pattern, including both adding and removing stops, modify the minimal journey times of the trains. This in turn can create timetable conflicts that need to be resolved at some cost, e.g., by driving slower or having irregular intervals between departure. The proposed model aims to minimise these costs when creating the multi-period timetable.

The main contributions of this paper are:

- A new concept of multi-period cyclic timetables to accommodate changing line plans corresponding to time-dependent demand.
- An optimisation model to compute multi-period timetables with smooth transitions between periods.
- A case study analysis that demonstrates the model's capability to generate a conflict-free timetable for a line plan with varying stopping patterns throughout the day.
- A sensitivity analysis of different objectives to illustrate the trade-off between minimising journey time and departure interval irregularity.

The remainder of this paper is organised as follows. An overview of the relevant literature for the multi-period timetabling problem is given in Section 2. Next, in Section 3 the mathematical model is provided. The model is tested on a small case study of which a description and the results can be found in Section 4. A conclusion and suggestions for future research are provided in Section 5.

2. Literature review of timetabling for varying demand

The aim of this paper is to provide a method to create a timetable with multiple cyclic periods that better matches the demand than a strictly cyclic timetable. As a multi-period timetable has both cyclic and acyclic characteristics, we briefly discuss these two types of timetables with a special focus on literature that aims to serve the time-dependent demand in Section 2.1. Next, Section 2.2 provides the literature that aims to add more flexibility to the timetable by allowing lines to have different cycle times. In Section 2.3, the literature that combines cyclic and acyclic timetables is discussed. Section 2.4 provides the literature that combines line planning and timetabling decisions, and the literature gaps are discussed in Section 2.5.

2.1. Cyclic and acyclic timetables

In many European countries, cyclic timetables are prevalent due to the ease-of-use for the passengers. Studies by Wardman et al. (2004) and Johnson et al. (2006) have demonstrated that cyclic timetables with well-coordinated connections enhance passenger satisfaction, thereby increasing demand and revenues for railway companies. The widespread adoption of cyclic timetables can be attributed to these benefits. In a cyclic timetable, the schedule recurs every cycle, usually every hour; for instance, a train departure at 8:05 will also occur at 9:05, 10:05, and so forth. Typically, such timetables are formulated using the Periodic Event Scheduling Problem (PESP) framework, initially developed by Serafini and Ukovich (1989). Given its frequent use in the literature, several overviews on timetabling with PESP exist, including Odijk (1996), Peeters (2003), and Caimi et al. (2017). However, PESP also has some challenges when the aim is to provide a passenger-friendly timetable. For example, a cyclic timetable is usually based on the demand of one cycle, for which often a peak hour is used. Therefore, variations in the demand during the day are not taken into account. Furthermore, Liebchen and Möhring (2007) note that routing passenger flows is beyond the scope of PESP, which means that either assumptions are needed to approximate the passenger travel time or the PESP model needs to be extended.

Acyclic timetables, on the other hand, offer a lot of flexibility to serve demand that varies over time. Niu and Zhou (2013) minimise the number of waiting and denied passengers on an urban rail transit line with time-dependent demand and oversaturated conditions. They use a genetic algorithm to solve the proposed nonlinear mixed-integer program (MIP). Barrena et al. (2014b) create different MILP formulations for the timetabling problem with the objective of minimising the total waiting time and test them on a case study of the Madrid Metropolitan Railway. They show that when demand is fluctuating, significant improvements of the average waiting time can be realised when compared to a regular timetable. In a follow-up study, the same authors propose an Adaptive Large Neighbourhood Search algorithm to solve more realistic instances (Barrena et al., 2014a). Niu et al. (2015) aim to create a timetable on a corridor that minimises the passenger waiting time under time-dependent demand and pre-determined skip-stop patterns. They propose a MIP model with a quadratic objective function and use the General Algebraic Modelling System to implement and solve it. Li et al. (2019) investigate how equity can be incorporated in passenger-oriented railway timetabling with time-dependent demand. They try adding two fairness criteria, min–max fairness and α-fairness, to the objective of a non-linear integer programming model and show that in efficient timetables the waiting time is not always distributed fairly over the passengers. Yin et al. (2021) aim to reduce the crowdedness in (transfer)stations by coordinating the timetable of different lines to better serve the time-dependent demand. They propose two algorithms to solve this MILP model, namely an adaptive large neighbourhood search (ALNS) algorithm and a Decomposition-based ALNS (DALNS).

Acyclic timetables are more suitable for considering time-dependent demand than strictly cyclic timetables. Since the goal of cyclic timetabling is to create a timetable that is repeated throughout the day, no changes to the timetable can be made to serve time-dependent demand. Instead, variations in demand volumes can be served by varying the rolling stock (e.g., using longer trains in the peak hours than during the off-peak). On the other hand, there have been several studies that consider time-dependent demand in acyclic timetabling in order to minimise the passenger waiting time or reduce crowdedness.

2.2. Timetabling with multiple cycle lengths

Besides the strictly cyclic and strictly acyclic timetable, there are also several studies that have attempted to make timetables that mix cyclic and acyclic characteristics in order to improve the service offered to customers. One direction of research to achieve this is by allowing lines to have different cycle times, e.g., some lines are operated every 2 or 3 h instead of every hour. In that way, certain OD pairs can still have a direct connection even if there is not enough demand to provide this connection every hour. Caimi et al. (2011) present a framework with which a timetable can be created for lines with different cycle times. They create an augmented version of the periodic timetabling problem by projecting all lines on a single period, which can be modelled using e.g., PESP. Solving this augmented version results in slightly longer computation time, but eliminates the need for post-processing in the form of removing lines from certain periods and improves the solution quality of the final timetable. Zhou et al. (2017) propose a model for finding a periodic railway timetable that minimises the total train travel time, in which each train series can have its own period length. They use an algorithm based on Lagrangian relaxation decomposition to solve the proposed model for several case studies based on a small railway network and the high-speed railway between Beijing and Shanghai.

Although considering larger and varying cycle times can help to offer more direct connections, the aim of the aforementioned research is still to create a cyclic timetable that can be repeated throughout the day. However, when the demand is significantly different in different periods (for example in the peak and off-peak hours), simply lengthening the cycle time will not be enough to serve this period-specific demand as different lines could be needed.

2.3. Hybrid timetabling

Other studies have looked beyond varying the cycle times of different lines by including more acyclic characteristics. Robenek et al. (2017) investigate three types of cyclicity in railway timetables. In the 'θ shifted cyclic timetable', each line has cyclic departure time, but trains are allowed to be scheduled with a tolerance of at most θ minutes from this departure time. Alternatively, in the 'ξ partially cyclic timetable', ξ% of the trains in the timetable should be cyclic, while the remainder of the trains have no departure time requirements. Lastly, the 'hybrid cyclic timetable' is investigated, in which non-cyclic trains can only be scheduled in hours in which also a cyclic train is scheduled. Tests on the Israeli Railways network showed that the 'hybrid cyclic timetable' is best able to keep a good level of regularity, while also achieving the flexibility of a non-cyclic timetable. Yin et al. (2019) propose a framework in which the arrival and departure times of trains are generally regular throughout the day, but the distribution of trains over the planning horizon depends on the passenger demand. The proposed MIP model minimises the weighted sum of the total train travel time and the total passenger waiting time and is solved using a 3-step heuristic. Li et al. (2023) propose a multi-objective integer programming model to solve an integration of the railway timetabling problem and the rolling stock circulation problem under time-dependent demand. The model's objective is to minimise the total passenger waiting time at their origin station and the total operating cost of train services. The model is solved exactly using Gurobi and the ϵ-constraint method is used to create a Pareto frontier.

These studies show that hybrid timetabling can create timetables for time-dependent demand that consider both passenger attractiveness and costs for the railway undertaking. However, a key challenge remains in maintaining the recognisability of the timetable's cyclicity, as most benefits of cyclic timetables originate from this recognisability.

2.4. Combining line planning and timetabling

Lastly, there are also several papers that combine line planning and timetabling decisions in order to better serve the time-dependent demand. Kaspi and Raviv (2013) combine line planning, timetabling and passenger routing decisions into one optimisation model. The model's objective is creating a line plan and a cyclic timetable that minimises the operational cost and the total passenger travel time. They use a cross-entropy metaheuristic to solve the model for a case study of Israel Railways. Yan and Goverde (2019) propose an approach to find both an optimal line plan and timetable, such that all OD pairs have at least one direct connection per time period. The suggested approach creates a timetable with both periodic and aperiodic characteristics: the lines' frequencies can take any positive integer value and the departure times of lines are not required to be strictly periodic but can deviate according to some tolerance parameter. The models for line planning and timetabling are solved consecutively, where in case of infeasibility the algorithm returns to the previous stage. Yang et al. (2021) propose a MILP model that uses a short-turning strategy to deal with asymmetric demand. Lagrangian relaxation is used to solve the model for a case study of the Yizhuang Line of Beijing Subway Network, and this shows that the short-turning strategy can effectively reduce the waiting time by almost 10%. Hao et al. (2022) aim to improve both the railway system's efficiency and the level of service to customers by considering the joint optimisation of line planning and timetabling under time-dependent demand. They use simulated annealing to solve the proposed MIP model. Zhang et al. (2021) propose a MILP model to solve the integrated line planning and timetabling problem with time-dependent demand. The model is solved for an instance on the Wuhan-Guangzhou high-speed railway corridor using Gurobi for varying amount of discrete time intervals. These tests show that when smaller time intervals are used, the trains are more equally distributed over the entire period. Qi et al. (2021) integrate train stop planning and timetabling to serve time-dependent demand. Their objective is to minimise the total travel time of the trains, while serving all the demand in their preferred time slot with a direct connection. The resulting model is tested on the Wuhan–Guangzhou high-speed railway line in China, where the model is solved using CPLEX. Xu et al. (2021) combine timetabling for the high-speed railway with stop-skipping, passenger assignment and platform decisions, to improve passenger convenience and better operating efficiency. They formulate the problem as a minimum-cost multicommodity network flow problem and the resulting MILP model is solved using a Lagrangian relaxation-based heuristic and CPLEX. Wu et al. (2021) try to find an equitable stopping pattern in an oversaturated urban rail transit network. The proposed MILP model which minimises the maximum waiting time of passengers in the network, is not only able to find solutions that are more equitable, but also improve other efficiency evaluation indicators such as the average waiting time. Real-life instances can be solved using the proposed Variable Neighbourhood Search algorithm. So when line planning and timetabling are combined to address time-dependent demand, often choices about stopping pattern and less frequently route selection, short-turning and frequency decisions are considered.

2.5. Literature gap

As discussed above, there are various studies on timetabling methods that aim to serve time-dependent demand, focusing primarily on acyclic or hybrid models. While strictly cyclic timetables benefit from their memorability, they fall short when the demand varies a lot throughout the day. Conversely, acyclic timetables offer a lot of flexibility, but lack clarity and ease-of-use for the passengers. While timetables with multiple cycle lengths can offer more direct connections, they still yield a cyclic pattern which assumes consistent demand throughout the day. This method becomes ineffective when the fluctuations in demand throughout the day are large. The works of Yan and Goverde (2019) and Li et al. (2023), which also aim for a cyclic timetable, suffer from similar limitations. Furthermore, minor deviations in scheduled departure times make them harder for passengers to remember, a challenge also present in the 'θ shifted cyclic timetable' by Robenek et al. (2017). Yin et al. (2019) and the 'hybrid cyclic timetable' of Robenek et al. (2017) propose scheduling based on travel demand, which can still result in service gaps in periods with low demand that would reduce the passenger demand during those periods even further. Not investigated before in the literature, are models that generate multiple cyclic timetables throughout the day in order to serve varying demand. Furthermore, we have not found any literature on transitioning from one cyclic timetable to another cyclic timetable. A timetable with multiple cyclic periods is better memorable for passengers than a completely acyclic one, as during several parts of the day, there would be a clock faced timetable. The memorability can be further improved by keeping the departure times on lines that occur in multiple periods the same throughout these periods. Moreover, by also providing a cyclic timetable in periods with less demand, passengers will not experience service gaps, which would reduce demand even further. In the remainder of this paper, we will address these gaps in the literature, by proposing a model that creates a cyclic timetable for each period with homogeneous demand while maintaining regularity for lines that span multiple demand periods and creating smooth transitions between periods.

3. Mathematical model

In this section, we introduce the mathematical model for the multi-period timetabling problem. The notation and variables are introduced in Section 3.1, the objective is given in Section 3.2, and lastly the constraints of the model are provided in Section 3.3.

3.1. Notation

In this section, the notation and variables of the mathematical model are introduced. A full overview of the notation is provided in Table 2. Similarly to other timetabling problems, the multi-period timetabling problem takes as input a set of lines $\mathcal{L}$, where each line l has a certain route through the network, a stopping pattern, and a frequency f^l denoting how many times per cycle a line should be operated. The cycle time is denoted by C and could for example be 60 min. As lines are not necessarily operated during the entire planning horizon, each line also has a period in which the line should be operated, denoted by the start τ_l^{start} and end τ_l^{end} times (in minutes) of this period. All the trains of this line should depart from its first station within this period. Lastly, lines also have an indicator denoting whether the line is operated in a transition period or not. To explain this transition period, consider the following example. Suppose we consider two different line plans, for which we want to create a multi-period timetable with a cycle time of 60 min. The first line plan should be operated in the period from 7:00 until 9:00 and the second line plan between 9:00 and 15:00. Hence, trains that depart between 7:00 and 9:00 use the first line plan, and trains that depart from 9:00 onwards use the second line plan. However, when we switch between the first and second line plan, there is some time period in which there are trains from both line plans running in the network. We will call this time period the transition period. In this paper we assume that the trains departing during the previous cyclic schedule will keep their cyclic departure and arrival times. This way, we know that only passengers that depart during the transition period will have to deal with an irregular timetable. Trains that depart after 9:00 will use the second line plan. However, to facilitate the transition, we will not enforce the new cyclic schedule for the trains departing in the first cycle(s) of the new schedule. Instead, we try to schedule the trains as close as possible (in time) to the next cyclic schedule, but let the times deviate from the cyclic schedule or cancel the trains if this is necessary to create a feasible timetable. Note that the number of trains for which we should allow extra freedom depends on the case considered. In a case that has long train lines, which take more than C minutes to complete, we need to give freedom to trains in multiple cycles. The number of cycles depends on the length of the longest line in the previous period. Furthermore, it is unlikely that the transition period is an exact multiple of C minutes. However, this is not a problem because trains that depart after the transition period has finished shall be planned according to the new cyclic schedule, as deviations from the cyclic schedule are penalised in the objective. To easily distinguish between trains that start during the transition period (and hence get extra flexibilities), and trains that should be operated in a cyclic manner, two variants of lines are included in the line set. These lines have the same route, stopping pattern, and frequency, but a different period in which they are operated. The transition line is always operated directly before the period of the cyclic line. So returning to the example, we would have the first line plan operated cyclically between 7:00 and 9:00, and the second line plan operated cyclically between 10:00 and 15:00. From 9:00 until 10:00 we have trains departing according to the second line plan, but these trains can have departure and arrival times that deviate from the cyclic schedule. The transition line $l \in \mathcal{L}^{\text{Trans}}$ and the cyclic line $l' \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}$ are included as a pair in the related line set $(l, l') \in \mathcal{P}$. This set is used in the formulation of the constraints to compare the departure times of the transition trains with the departure times of the cyclic trains.

To create a timetable, we need to determine the arrival and departure times for all trains at each of the stations on their routes. We do this by creating departure and arrival events for each train, and for each of these events define an event time variable π_e. A departure event is created for each station $s \in S^l \setminus s^l_{n^l}$, so for each station on the route of a train of line $l \in \mathcal{L}$, except for the last station. However, arrival events are only created for stops at large stations in order to reduce the number of variables needed. For example, at stations where the train does not stop, the arrival and departure times are the same. Hence, if we only have a departure event, and hence departure time at this station, we have all the information we need. Similarly, no arrival events are created for the small stations in the network. At these short stops, the dwell times are usually short as there is relatively low demand and usually no transfer possibilities. By assuming a fixed dwell time for these stations, which will be included in the minimal driving time towards the station, no arrival activity needs to be determined and therefore fewer variables are needed. Hence, arrival events are only created for stops at large stations, as at these stations you might want to dwell longer to facilitate transfers and to be able to fit trains on the infrastructure. The set of all departure and arrival events is denoted by $\mathcal{E}$.

Activities between two events ensure that there is a minimum and/or maximum time between two events. Three types of activities are considered. Drive activities represent a train driving from one station to the next. They are created between the departure event at one station and arrival event at the subsequent station. If the subsequent station is a small station or if the train does not make a stop at that station, then no arrival event is created. In that case, the drive activity is created between the departure events at the two stations. Dwell activities occur when a train stops at a large station. They are created between the arrival and departure events at such stations, indicating the train is dwelling at the station. Note that for stops at smaller stations, no arrival events are created, and thus no dwell activities are generated. Each drive and dwell activity $a \in \mathcal{A}$ has a lower (b_a^{lower}) and upper (b_a^{upper}) bound on the activity time, indicating the minimum and maximum time that the activity can take. Lastly, headway activities are created between the events of two trains utilising the same infrastructure. For trains of the same line, headway activities are only created between successive trains, as the order between trains is known in advance. In contrast, when two lines are involved, the order of events is not known in advance. In that case, headway events are created if trains could be operated around the same time. To determine this, an operating window is assigned to each train, factoring in both the earliest start time and the latest finish time. The earliest start time is set at the beginning of the cycle in which a train is scheduled to depart, under the assumption that, during each cycle,

Table 2
Notation used in the mathematical model.

Sets	
$\mathcal{L}$	Set of lines.
$\mathcal{L}^{\text{Trans}}$	Set of lines that is operated in the transition periods ($\mathcal{L}^{\text{Trans}} \subset \mathcal{L}$).
$\mathcal{P}$	Set of pairs of related lines.
$\mathcal{S}$	Set of stations in the network.
$\mathcal{S}^l$	Stations that line l passes on its route through the network.
$\mathcal{S}^l_{\text{Stop}}$	The set of stations where line l makes a stop.
$\mathcal{E}$	Set of all departure and arrival events. Each event $e \in \mathcal{E}$ corresponds to a line l_e, a specific train on that line t_e, a type u_e which can be departure or arrival, and a station s_e where the event takes place. Hence, event e can be defined by the tuple (l_e, t_e, u_e, s_e).
$\mathcal{A}$	Set of all activities. The types of activities are: drive, dwell, and headway.
$\mathcal{A}^{\text{headway}}$	Set of all headway activities. ($\mathcal{A}^{\text{headway}} \subset \mathcal{A}$).
Parameters	
s^l_i	ith station on line l $\left(s^l_i \in \mathcal{S}^l\right)$.
$s^l_{n^l}$	Last station online l.
τ^{start}_l	Start time (in minutes) of the period during which line l is operated.
τ^{end}_l	End time (in minutes) of the period during which line l is operated.
p^l	The number of cycles during which line l is operated.
f^l	Frequency of line f^l, i.e., how many times per cycle the line is operated.
t^l_i	The ith train scheduled on line l.
b^{lower}_a	The lower bound on an activity time of activity $a \in \mathcal{A}$.
b^{upper}_a	The upper bound on an activity time of activity $a \in \mathcal{A}$.
h	Minimum headway between two events $e, e' \in \mathcal{E}$.
C	Duration of one cycle (in minutes), e.g., 60 min.
Variables	
π_e	Real variable denoting the time at which event $e \in \mathcal{E}$ takes place.
$o_{e,e'}$	Binary variable that takes value 1 if event e occurs before event e', 0 otherwise.
d^l_i	Binary variable which takes value 1 if the ith train of line l is not scheduled, 0 otherwise.
$z^l_{i,s}$	Real variable denoting the absolute deviation from the cyclic departure time of (transition period) train t^l_i at station $s \in \mathcal{S}^l \setminus s^l_{n^l}$.
k^l_s	Real variable denoting the maximum time between two departures of trains of line $l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}$ at station $s \in \mathcal{S}^l_{\text{Stop}} \cup s^l_1$.

f^l trains of line l depart. The latest arrival time is defined as the end time of the cycle in which a train should depart plus the sum of upper bounds on the drive and dwell activities of that train. When the operating windows of two trains overlap and the trains use the same infrastructure, a headway activity is created. Headway activities $a \in \mathcal{A}^{\text{headway}}$ only have a lower bound (b^{lower}_a).

Besides the variables denoting the event times, there are four other types of variables. To ensure sufficient time between two events associated with a headway activity, an order variable $o_{e,e'}$ is created for each pair $(e, e') \in \mathcal{A}^{\text{headway}}$. This binary variable $o_{e,e'}$ takes value 1 if event e takes place before event e', and has value 0 otherwise. During the transition periods, we want to schedule the new trains as similar as possible as in the next cyclic schedule. However, to facilitate the transition between two cyclic railway schedules, the departure and arrival times from the transition trains can be shifted from the regular interval times. Variable $z^l_{i,s}$ indicates the absolute deviation from the cyclic departure time at station s by the ith train of line $l \in \mathcal{L}^{\text{Trans}}$. Furthermore, trains of the transition lines can also be cancelled if necessary. Binary variable d^l_i takes value 1 if the ith train of line l is not scheduled, and takes value 0 otherwise. Lastly, we like trains of the same line to be equally spread over the cycle. For example, if we have a line with frequency 4 and a cycle time of 60 min, then ideally the difference between the departures of two subsequent trains is 15 min. Hence, we would like the maximum time between two departures to be as small as possible. Variable k^l_s denotes the maximum time between two departures of trains of line $l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}$ at station s. Note that this variable is not created for transition lines, as we already try to schedule these transition lines as similarly as possible to the cyclic schedule. Hence, if the cyclic lines are scheduled as equally spread as possible, the transition lines will automatically follow.

3.2. Objective

The aim is to create a timetable that is attractive for the passengers, so one that minimises the travel time. However, to calculate the exact travel times, passengers have to be assigned to trains, and this assignment makes the problem significantly more complicated. Therefore, in this work we try to approximate the travel time and the timetable attractiveness with the following components instead. The first component aims to minimise the in-vehicle time by minimising the (avoidable) total journey time:

$$\zeta_1 = \sum_{l \in \mathcal{L}} \sum_{i=1}^{f^l \cdot p^l} \left(\pi_{(l,i,arr,s^l_{n^l})} - \pi_{(l,i,dep,s^l_1)} \right) - \sum_{a \in \mathcal{A} \setminus \mathcal{A}^{\text{headway}}} b^{\text{lower}}_a. \tag{1}$$

The journey time is defined as the difference between the arrival time at the final station and the departure time at the first station. The total journey time is calculated by summing over all trains. If the total journey time is minimised, then a train does not take more time than necessary to travel from A to B and does not dwell at a station longer than necessary. As a result, the in-vehicle time of passengers is minimised, as well as the operating costs since drivers and guards spend less time on the train and less rolling stock compositions are needed. A lower bound for the journey time is the sum of all lower bounds of the drive and dwell activities. By subtracting this lower bound, we obtain the 'avoidable' journey time. This is convenient, as this number is much closer in value to the values of the other components of the objective. Note that the lower bound of the journey time is based on the line plan in each period, so when the stopping pattern is changed the minimum journey time also changes accordingly. Hence, the avoidable journey time denotes the extra running and dwell time used on top of the minimum journey time given the line plan and lower bounds on drive and dwell activities.

The second objective aims to make the timetable more attractive by penalising irregular interval times between trains of the same line. This makes the timetable more attractive, as this provides regular travel opportunities throughout the hour. Regular headways also minimise the deviations from the passengers' desired departure times, when we assume that the desired departure times of the passengers are uniformly distributed over the hour. For example, if a train line has a frequency of 4 per hour, and the departure times of the trains are exactly 15 min apart, then the passengers should depart at most 7.5 min earlier or later than desired (3.75 min on average). Note that this considers a deviation from the desired departure time and not waiting time, since most passengers will check the timetable and arrive accordingly at the station when headways between trains are more than 10 min (Van Oort, 2011; Ingvardson et al., 2018). Furthermore, having equal intervals between trains of the same line increases the memorability of the timetable (Wardman et al., 2004). Hence, in the second part of the objective we penalise unequal spreading of the trains:

$$\zeta_2 = \sum_{l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}}} \sum_{s \in S^l \setminus s^l_{n^l}} \left(k^l_s - \frac{C}{f^l} \right) \cdot p^l. \tag{2}$$

Variable k^l_s denotes the largest difference between the departure times of two trains of line l at station s. If the trains are equally spread over the cycle, then the time between each two departures is the cycle time divided by the frequency. Only if the maximum time between two departures is larger than this optimum, this is penalised in the objective. Not all lines are operated for the same number of cycles. As we want to avoid these deviations as much as possible, we also need to take into account for how many cycles the line is operated. Namely, if an unequal spread is necessary to satisfy the headway constraints, then it is better if this can be restricted to a low number of cycles. Hence, we also multiply the deviation by the number of cycles in which a line l runs (p^l).

To make the schedule more memorable for passengers, we want to stick to the new cyclic pattern as much as possible in the transition periods. Hence, the third part of the objective is the total deviation from the new cyclic schedules, and is given in Eq. (3):

$$\zeta_3 = \sum_{l \in \mathcal{L}^{\text{Trans}}} \sum_{s \in S^l \setminus s^l_{n^l}} \sum_{i=1}^{f^l \cdot p^l} z^l_{i,s}. \tag{3}$$

For each transition train, we calculate how much the departure time deviates from the departure time in the cyclic periods at all its stations. By penalising this absolute deviation, we aim to schedule the trains in the transition period as similar as possible to the new cyclic schedule: if the new cyclic schedule fits immediately, we want to switch instantly to the new cyclic schedule, but this also gives some freedom in case the instant switch is not possible.

Lastly, to provide as much service as possible it would be ideal if all transition trains can be scheduled. Hence, a penalty is added to the objective for not scheduling transition trains, based on the number of trains that are cancelled, which is shown in Eq. (4):

$$\zeta_4 = \sum_{l \in \mathcal{L}^{\text{Trans}}} \sum_{i=1}^{f^l \cdot p^l} d^l_i. \tag{4}$$

The multiple objectives are combined as a weighted sum. However, as all the objectives have different ranges, we first find the optimal (ζ^{ideal}) and nadir (ζ^{nadir}) values for each objective. These values are then used to normalise the objective and the weights are used to set the relative importance of each objective:

$$\text{minimise} \quad w_1 \frac{\zeta_1 - \zeta_1^{\text{ideal}}}{\zeta_1^{\text{nadir}} - \zeta_1^{\text{ideal}}} + w_2 \frac{\zeta_2 - \zeta_2^{\text{ideal}}}{\zeta_2^{\text{nadir}} - \zeta_2^{\text{ideal}}} + w_3 \frac{\zeta_3 - \zeta_3^{\text{ideal}}}{\zeta_3^{\text{nadir}} - \zeta_3^{\text{ideal}}} + w_4 \frac{\zeta_4 - \zeta_4^{\text{ideal}}}{\zeta_4^{\text{nadir}} - \zeta_4^{\text{ideal}}}. \tag{5}$$

Different timetables can be constructed by varying the weights of the different components. For example, by setting large weights for ζ_3 and ζ_4, we can ensure that trains are operated according to the new cyclic schedule as soon as possible, as deviations from the cyclic schedule are minimised. A sensitivity analysis for the weights is provided in Section 4.2.

3.3. Constraints

The following constraints are used in the mathematical model.

$$b^{\text{lower}}_{e,e'} \leq \pi_{e'} - \pi_e \leq b^{\text{upper}}_{e,e'} \qquad \forall (e,e') \in \mathcal{A} \setminus \mathcal{A}^{\text{headway}},\ e,e' \in \mathcal{E} \tag{6}$$

$$\pi_{e'} - \pi_e \geq h - \left(1 - o_{e,e'}\right) M - (d^{l_{e'}}_{t_{e'}} + d^{l_e}_{t_e}) M \qquad \forall (e,e') \in \mathcal{A}^{\text{headway}},\ e,e' \in \mathcal{E} \tag{7}$$

$$\pi_e - \pi_{e'} \geq h - o_{e,e'}M - (d^{l_{e'}}_{t_{e'}} + d^{l_e}_{t_e})M \qquad \forall (e,e') \in \mathcal{A}^{\text{headway}},\ e,e' \in \mathcal{E} \tag{8}$$

$$\tau^{\text{start}}_{l_e} \leq \pi_e \leq \tau^{\text{end}}_{l_e} \qquad \forall e \in \mathcal{E},\ s_e = s^{l_e}_1,\ u_e = \text{dep} \tag{9}$$

$$\pi_e = \pi_{e'} + jC \qquad \forall e,e' \in \mathcal{E},\ l_e = l_{e'} \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}},\ s_e = s_{e'} \in \mathcal{S}^{l_e},\ t_{e'} = 1,\dots,f^{l_e},\ j = 1,\dots,(p^{l_e}-1),\ t_e = t_{e'} + jf^{l_e} \tag{10}$$

$$k^{l_e}_{s_e} \geq \pi_e - \pi_{e'} \qquad \forall e,e' \in \mathcal{E},\ l_e = l_{e'} \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}},\ u_e = u_{e'} = \text{dep},\ s_e = s_{e'} \in \mathcal{S}^{l_e}_{\text{Stop}} \cup s^{l_e}_1,\ t_e = 2,\dots,f^{l_e},\ t_{e'} = t_e - 1 \tag{11}$$

$$k^{l_e}_{s_e} \geq \pi_e + C - \pi_{e'} \qquad \forall e,e' \in \mathcal{E},\ l_e = l_{e'} \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}},\ t_e = 1,\ t_{e'} = f^{l_e},\ u_e = u_{e'} = \text{dep},\ s_e = s_{e'} \in \mathcal{S}^{l_e}_{\text{Stop}} \cup s^{l_e}_1 \tag{12}$$

$$k^l_s \geq \frac{C}{f^l} \qquad \forall l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}},\ s \in \mathcal{S}^l_{\text{Stop}} \cup s^l_1 \tag{13}$$

$$z^{l_e}_{t_e,s_e} \geq \pi_{e'} - \pi_e - (p^{l_e} - j)C - Md^{l_e}_{t_e} \qquad \forall e,e' \in \mathcal{E},\ l_e \in \mathcal{L}^{\text{Trans}},\ (l_e,l_{e'}) \in \mathcal{P},\ u_e = u_{e'} = \text{dep},\ t_{e'} = 1,\dots,f^{l_{e'}},\ j = 0,\dots,(p^{l_{e'}}-1),\ t_e = t_{e'} + jf^{l_e} \tag{14}$$

$$z^{l_e}_{t_e,s_e} \geq -\pi_{e'} + \pi_e + (p^{l_e} - j)C - Md^{l_e}_{t_e} \qquad \forall e,e' \in \mathcal{E},\ l_e \in \mathcal{L}^{\text{Trans}},\ (l_e,l_{e'}) \in \mathcal{P},\ u_e = u_{e'} = \text{dep},\ t_{e'} = 1,\dots,f^{l_{e'}},\ j = 0,\dots,(p^{l_{e'}}-1),\ t_e = t_{e'} + jf^{l_e} \tag{15}$$

$$d^l_i = 0 \qquad \forall l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}},\ i = 1,\dots,f^l \cdot p^l \tag{16}$$

$$\pi_e \in \mathbb{R}_{\geq 0} \qquad \forall e \in \mathcal{E} \tag{17}$$

$$o_{e,e'} \in \{0,1\} \qquad \forall e,e' \in \mathcal{E},\ (e,e') \in \mathcal{A}^{\text{headway}} \tag{18}$$

$$d^l_i \in \{0,1\} \qquad \forall l \in \mathcal{L},\ i = 1,\dots,f^l \cdot p^l \tag{19}$$

$$z^l_{i,s} \in \mathbb{R}_{\geq 0} \qquad \forall l \in \mathcal{L}^{\text{Trans}},\ i = 1,\dots,f^l \cdot p^l,\ s \in \mathcal{S}^l \setminus s^l_{n^l} \tag{20}$$

$$k^l_s \in \mathbb{R}_{\geq 0} \qquad \forall l \in \mathcal{L} \setminus \mathcal{L}^{\text{Trans}},\ s \in \mathcal{S}^l_{\text{Stop}} \cup s^l_1. \tag{21}$$

Constraint (6) ensures that the difference between two event times is between a lower and upper bound if there exists a (non-headway) activity between these two. Constraints (7) and (8) ensure that there is an appropriate minimum headway between two trains that use the same infrastructure. Two constraints are needed to model this, as the order of the trains is not known in advance. Furthermore, the last term of both constraints ensures that if at least one of the trains is cancelled, then the minimum headway constraint is not binding. Note that although we use a fixed minimum headway between two events, this model can easily be extended such that the length of the headway time depends on the order in which the events take place or the types and/or station of the events. Constraint (9) ensures that the trains of each train line depart from the line's first station within the line's time interval. Next, constraint (10) states that the departures and arrivals of non-transition-period trains are exactly the same in each hour in which the line is operated, which ensures the cyclic departure times during the cyclic periods. Constraints (11) and (12) calculate what the maximum interval is between two departures of a line at a station. Constraint (11) does this for trains that depart during the same cycle, and constraint (12) checks the time difference between the last train of the first cycle and the first train of the next cycle. As we know that the maximum difference between two departures is minimised when the trains are equally spread over the hour, we set the cycle length divided by the frequency as the lower bound on the variable k^l_s in (13). Next, constraints (14) and (15) calculate the absolute deviation from the cyclic departure times over all cycles in the transition period. The variable indicates the absolute difference between the departure of a train in the transition period and the cyclic departure time of a train of the corresponding cyclic line. We compare the departure times of the ith train of every cycle in the transition period with the departure times of the ith train in the cyclic period. When the train is cancelled, this constraint should not be binding, so in that case a large value M is subtracted. Constraint (16) states that a train cannot be cancelled if it is not from a transition line. Lastly, constraints (17)–(21) define the signs of the variables in the model.

4. Case study & results

The model presented in the previous section is applied to a small case study. A short description of this case study is provided in Section 4.1. Gurobi is used to solve the model for this case study. Next, in Section 4.2 the results of the case study are presented.

4.1. Description of case study

The case study is based on a small part of the railway network in the Netherlands between the stations Leiden Centraal (Ledn), Den Haag Centraal (Gvc), and Rotterdam Centraal (Rtd). Fig. 1 displays the (simplified) version of the track layout between stations Rtd, Gvc, and Ledn, which is used in this case study. In this figure, the tracks are denoted by lines and the platforms at stations are represented by grey rectangles. We consider a scenario in which the routes and frequencies of the lines stay the same throughout the day, but the stopping pattern of certain lines changes throughout the day to accommodate the varying demand over various

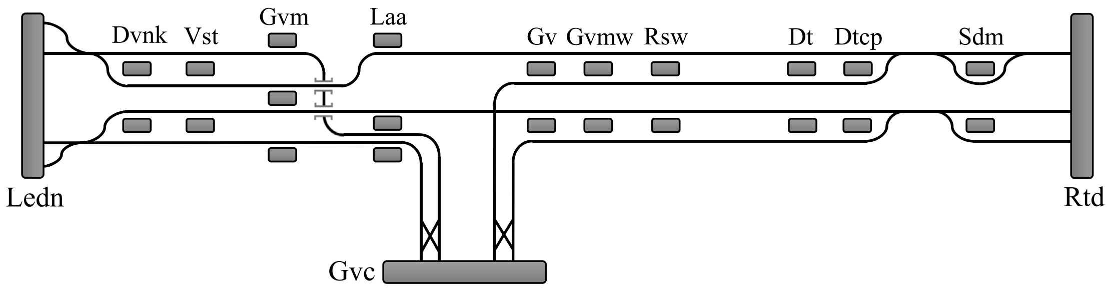

Fig. 1. Simplified representation of the track layout of case study between stations Rotterdam Centraal (Rtd), Den Haag Centraal (Gvc), and Leiden Centraal (Ledn).

Table 3
Line plan of the case study.

Route	Freq.	Transition period	Cyclic period	Stop stations	Min. journey time
Rtd-Sdm-Dtcp-Dt-Rsw-Gvmw-Gv-Laa-Gvm-Dvnk-Ledn	6		07:00–16:59	Sdm, Dt, Gv, Laa	41
Ledn-Dvnk-Vst-Gvm-Laa-Gv-Gvmw-Rsw-Dt-Dtcp-Sdm-Rtd	6		07:00–08:59	Laa, Gv, Rsw, Dt, Sdm	43
	6	09:00–09:59	10:00–16:59	Laa, Gv, Dt, Sdm	41
Rtd-Sdm-Dtcp-Dt-Rsw-Gvmw-Gv-Gvc	4		07:00–08:59	Sdm, Dtcp, Dt	27
	4	09:00–09:59	10:00–14:59	Sdm, Dtcp, Dt, Rsw, Gv	31
	4	15:00–15:59	16:00–16:59	Sdm, Dtcp, Dt, Rsw, Gvmw	31
Gvc-Gv-Gvmw-Rsw-Dt-Dtcp-Sdm-Rtd	4		07:00–08:59	Gvmw, Dt, Dtcp, Sdm	29
	4	09:00–09:59	10:00–14:59	Gv, Gvmw, Rsw, Dt, Dtcp, Sdm	34
	4	15:00–15:59	16:00–16:59	Rsw, Dt, Dtcp, Sdm	30
Gvc-Laa-Gvm-Vst-Dvnk-Ledn	6		07:00–08:59	Laa, Gvm, Vst	21
	6	09:00–09:59	10:00–14:59	Gvm, Dvnk	17
	6	15:00–15:59	16:00–16:59	Dvnk	16
Ledn-Dvnk-Vst-Gvm-Laa-Gvc	6		07:00–08:59	Dvnk	16
	6	09:00–09:59	10:00–14:59	Vst	15
	6	15:00–15:59	16:00–16:59	Vst, Gvm, Laa	20

periods. Furthermore, we consider asymmetric lines, where e.g., the stopping pattern of a line from Gvc to Ledn does not have to be the same as the stopping pattern from Ledn to Gvc. The line plan is given in Table 3. From left to right, the columns of the table denote the route through the network, the frequency of the lines, the transition time period in which we start operating this line and the extra flexibilities (shifting and cancelling) are allowed, the period during which the line is operated in a cyclic manner, the stopping pattern during these periods, and the minimum journey time of trains operating the lines under different stopping patterns. As can be concluded from the table, a planning time horizon between 7:00 and 17:00 is considered. Furthermore, in our case a route can have up to three different stopping patterns, as can be seen in e.g., the line between Ledn and Gvc (last group in Table 3). Between 07:00 and 08:59, this line only stops at station De Vink (Dvnk), between 09:00 and 14:59 it stops at Voorschoten (Vst), and between 15:00 and 16:59 the line stops at Vst, Den Haag Moerwijk (Gvm) and Den Haag Laan van NOI (Laa). Note that the first hour in which the new stopping pattern is used is indicated in the column called transition period (09:00–09:59 for the second stopping pattern, and 15:00–15:59 for the third stopping pattern). Within these hours, the trains that depart on this line can be scheduled differently from the new cyclic schedule and can also be cancelled if necessary. In the remainder of the periods (10:00–14:59 and 16:00–16:59, respectively) the arrival and departures should be strictly cyclic. Hence, these are denoted in Table 3 under ‘Cyclic period’.

The following parameter values are used in the case study. The duration of a cycle is $C = 60$ minutes, so during the cyclic periods the timetable should be the same in each hour. The lower and upper bounds for the different activities are provided in Table 4. For the drive activities, the minimal driving time is calculated based on the length of the segments between two stations and the maximum speed between these two stations. Furthermore, if the train makes a stop at the station at the beginning and/or end of the track segment, an acceleration correction of 0.85 min and/or a deceleration correction of 0.7 min is added to the calculated value. To derive the lower bound for the drive activities, a 5% running time supplement is added, and then the value is rounded up to the nearest minute. For the upper bound of a drive activity, a 200% running time supplement is added before rounding up. A dwell time of a cyclic train at a large station should be between 1 and 3 min, while a transition train can dwell at a station longer: between 1 and 5 min. A dwell at a small station is set to be 0.5 min and this time is added to the calculated lower and upper bounds for the drive activities that arrive at the small station for a stop. The large stations in the network are Ledn, Laa, Gvc, Den Haag HS

Table 4
Lower and upper bounds for different types of activities (in minutes).

Activity type	Lower bound $\left(b_a^{\text{lower}}\right)$	Upper bound $\left(b_a^{\text{upper}}\right)$
Drive	$\lceil$(Minimal driving time) $*$ 1.05$\rceil$	$\lceil$(Minimal driving time) $*$ 3$\rceil$
Dwell (cyclic train)	1	3
Dwell (transition train)	1	5
Headway	3	–

Table 5
Objective values of the four different objectives under different orders of optimisation.

Order of optimisation	Objective values ζ_1: Avoidable journey time	ζ_2: Unequal spread	ζ_3: Shift transition	ζ_4: Cancel transition
ζ_1 before ζ_2	0	79.5	0	0
ζ_1 last	226	10	0	0
ζ_2 before ζ_1, ζ_4 last	198	10	0	10
ζ_2 before ζ_1, ζ_3 last	198	10	80	0
Weighted sum	100	34.5	0	0

(Gv), Delft (Dt), Schiedam Centrum (Sdm), and Rtd, the other stations are considered small. By summing up the lower bound on all drive and dwell activities, we can calculate the minimum journey time for each line-stopping pattern combination. The last column of Table 3 reports these minimum journey times. This column shows that the absolute difference in journey time with changing stopping patterns lies between 1 and 5 min. The largest percentage change in journey time occurs at line Ledn-Gvc, which increases from 15 min during the off-peak to 20 min in the afternoon peak, an increase of 33.3%. Lastly, the headway between two trains that use the same infrastructure (h) is set to be 3 min.

4.2. Case study results

The objective of the proposed mathematical model, as shown in (5), is a weighted sum of four distinct sub-objectives. These sub-objectives are avoidable journey time, the deviation from the regular departure distribution of cyclic trains, departure time deviations of transition trains from the cyclic plan, and the number of cancelled transition trains. Given the differing units of these objectives, the first step is to determine their best (ideal) and worst case (nadir) values. These values serve as the basis for normalising the objectives, allowing us to set weights according to their relative importance. To derive the ideal (ζ^{ideal}) and nadir (ζ^{nadir}) values, we solve the model for each single objective lexicographically. Specifically, we first solve the model with one objective, then impose a constraint to fix its optimal value, and subsequently optimise the model with another sub-objective. This iterative process continues until the optimal value of each sub-objective is found, taking into account the previously optimised objectives. Given that we have four sub-objectives, there are 24 possible sequences in which the objectives can be optimised. Hence, the complete process needs to be repeated 24 times. Once the ideal and nadir values are obtained, each objective is normalised as shown in Eq. (5). We apply equal weights $w_1 = w_2 = w_3 = w_4 = 0.25$ to find a solution that equally optimises each objective. The model was solved to optimality in 27 min and 35 s on a laptop with Intel® Core™ i7-1185G7 @ 3.00 GHz and 16 GB RAM, using Gurobi Optimizer version 11.0.3.

Although there are 24 sequences in which the objectives can be optimised, only four distinct timetables are generated, as each timetable is optimal for multiple sequences.

- When ζ_1 is optimised before ζ_2, the trains in the timetable have the highest deviation from being equally spread over the hour.
- When ζ_1 is optimised last, the timetable with the highest avoidable journey time is created.
- The plan with the highest number of cancelled trains is created when ζ_2 is optimised before ζ_1 and ζ_4 is optimised last.
- Lastly, the plan we obtain when ζ_2 is optimised before ζ_1 and ζ_3 is optimised last contains the highest shift from cyclic departure times in the transition periods.

Table 5 displays the optimal values of the different objectives for these optimisation orders. Furthermore, the last row of Table 5 shows the objective values for the result obtained using the equal weights in the objective function (5). In this table, the first column denotes the order of optimisation, and the subsequent columns show the four different objective values for each timetable.

Due to the definitions of the four objectives, we know that each objective has to be non-negative. From Table 5 it is clear that this minimum value of 0 can be achieved for all objectives except for ζ_2, which measures the train distribution irregularity. The minimum value for ζ_2 is 10, which is achieved when ζ_2 is optimised before ζ_1. For the different timetables that were obtained, Table 6 shows for which lines and periods the unequal distribution takes place, the follow-up times between consecutive trains, and at which station this unequal departure distribution is planned. Only the follow-up times of the first half hour are shown, as they remain consistent throughout the hour. When ζ_2 is prioritised over ζ_1, train departures for the line between Rtd and Ledn are spaced 9-10.5-10.5 min at stations Rtd and Sdm. Given there is only one track between Rtd & Sdm and Sdm & Delft Campus (Dtcp) that can be used for travelling towards Dtcp (see Fig. 1), lines from Rtd to Ledn and Rtd to Gvc must share tracks. As Rtd-Ledn has a frequency of 6, Rtd-Gvc has a frequency of 4, and there is a headway requirement of 3 min, equal distribution across both

Table 6
Unequal distribution of trains over the half hour.

Order of optimisation	Line	Period	Follow-up times	Stations
ζ_2 before ζ_1, ζ_4 last; ζ_2 before ζ_1, ζ_3 last; ζ_1 last	Rtd-Ledn	07:00–16:59	9-10.5-10.5	Rtd, Sdm
ζ_1 before ζ_2	Rtd-Ledn	07:00–16:59	8-11-11	Rtd, Sdm, Dt, Gv, Laa
	Ledn-Rtd	07:00–08:59	10.5-9-10.5	Ledn, Laa, Gv, Rsw, Dt, Sdm
		10:00–16:59	10.5-9-10.5	Ledn, Laa, Gv, Dt, Sdm
	Rtd-Gvc	16:00–16:59	16-14	Rtd, Sdm, Dtcp, Dt, Rsw, Gvmw
Weighted sum	Rtd-Ledn	07:00–16:59	9-10.5-10.5	Rtd, Sdm
	Ledn-Rtd	07:00–08:59	10.5-9-10.5	Sdm
		10:00–16:59	10.5-9-10.5	Ledn, Laa, Gv, Dt, Sdm
	Rtd-Gvc	16:00–16:59	16-14	Rtd, Sdm, Dtcp, Dt, Rsw, Gvmw

lines is unattainable. By spacing departures of trains on Rtd-Led 9-10.5-10.5 at stations Rtd and Sdm, the unequal distribution is minimised. As the maximum follow-up time is 10.5 at two stations (Rtd and Sdm) during 10 h, this gives the minimum objective value for ζ_2 of $(10.5 - 10) \cdot 2 \cdot 10 = 10$.

Conversely, if ζ_1 is prioritised before ζ_2, the timetable has the most unequal departure distribution and no avoidable journey time. As shown in Table 6, in this timetable there are 3 lines in 4 periods for which the departures are unequally distributed. The line from Rtd to Ledn has the largest deviation from the equal distribution, where the first two departures of the half hour are only 8 min apart, while 10 min is ideal. The variations for lines from Ledn to Rtd and Rtd to Gvc are only 1 min from the ideal separation. Note that for the affected lines, the unequal departures occur at all the stops on the line. When the avoidable journey time (ζ_1) is minimised, trains drive as fast as allowed and do not stop at stations longer than necessary. However, this also means that if somewhere in the network an unequal distribution is needed due to headway constraints, trains should use this unequal distribution during their entire route. By maintaining the unequal distribution on the entire route, the value of ζ_2 is increased to its maximum of 79.5. Instead, if we allow for some longer drive times and/or dwell times, the unequal distribution can be remedied after the headway constraints are met. For example, if the line from Rtd to Ledn has the following departure times from Sdm, following the 9-10.5-10.5 distribution: 3, 12, 22.5, 33, 42, and 52.5. Suppose the minimum drive time to Dt is 5 min and a minimum dwell time is 1 min, then the trains could be ready to depart at 9, 18, 28.5, 39, 48, and 58.5. However, if we would let the second and fifth train dwell an extra minute, and the third and sixth train dwell an extra 0.5 min, then all the trains would depart from Dt exactly 10 min apart at 9, 19, 29, 39, 49, and 59. Hence, there is a clear trade-off between minimising the avoidable journey time and minimising the unequal distribution of trains.

When we optimise ζ_2 before ζ_1 and either ζ_4 or ζ_3 last, we obtain a timetable in which deviations are made in the transition periods from the cyclic schedule. If ζ_4 is optimised last, 10 trains face cancellations during the transitions: all six trains between Ledn and Rtd from 09:00–09:59 and the four between Rtd and Gvc from 15:00–15:59. While this approach reduces the avoidable journey time by 28 min, this reduction mainly stems from the fact that the cancelled trains no longer have avoidable journey time (as they are not running). Meanwhile, the cancellations would make the timetable very unattractive for passengers of the affected lines during the transition periods.

In the timetable that is obtained when ζ_2 is optimised before ζ_1 and ζ_3 is optimised last, the avoidable journey time is reduced by shifting part of the transition trains. The unequal spread between departures is only calculated and penalised for cyclic trains. Since we want to schedule the transition trains as close as possible to the cyclic schedule, the equal spread is automatically also a goal for the transition trains. However, as the transition trains are not penalised for having an unequal distribution, we can reduce the avoidable journey time (ζ_1) for those trains without affecting the unequal spread penalty (ζ_2). Instead, we deviate from the cyclic departure times, which means an increase in ζ_3.

The timetable obtained by the weighted sum appears to blend elements from the timetables obtained when optimising ζ_1 before ζ_2 and when ζ_1 is optimised last. Table 6 shows that in this timetable the same lines have an unequal distribution as in the case where ζ_1 is prioritised over ζ_2. However, there are two notable improvements in the distribution by adding journey time. Firstly, the unequal spread is now confined to fewer stations. The lines from Rtd to Ledn (07:00–16:59) and from Ledn to Rtd (07:00–8:59) only have an unequal departure distribution at one or two stations, as opposed to all stations. Secondly, the departure distribution for the line from Rtd to Ledn has shifted from 8-11-11 to a more balanced 9-10.5-10.5, benefiting many passengers given its all-day operation. Table 5 shows that during the transition periods, the trains immediately depart according to the new cyclic schedule, as both objectives ζ_3 and ζ_4 have value 0. The time-distance diagram in Fig. 2 shows the three cyclic schedules and the transitions between them. The horizontal axis depicts station distances and track layout, while the vertical axis represents time. The figure is divided into four parts: the top section displays the timetable from 8:00 to 10:30 and the bottom section from 14:30 to 16:30. To improve the readability of the figure, we exclude periods 07:00–08:00, 10:30–14:30, and 16:30–17:00 from the figure. Due to the cyclicity between 07:00–09:00, 10:00–15:00, and 15:30–17:00 the timetables from the missing periods can still be viewed by looking at periods 08:00–09:00 for the morning peak, 10:00–10:30 and 14:00–14:30 for the midday off-peak, and 15:30–16:30 for

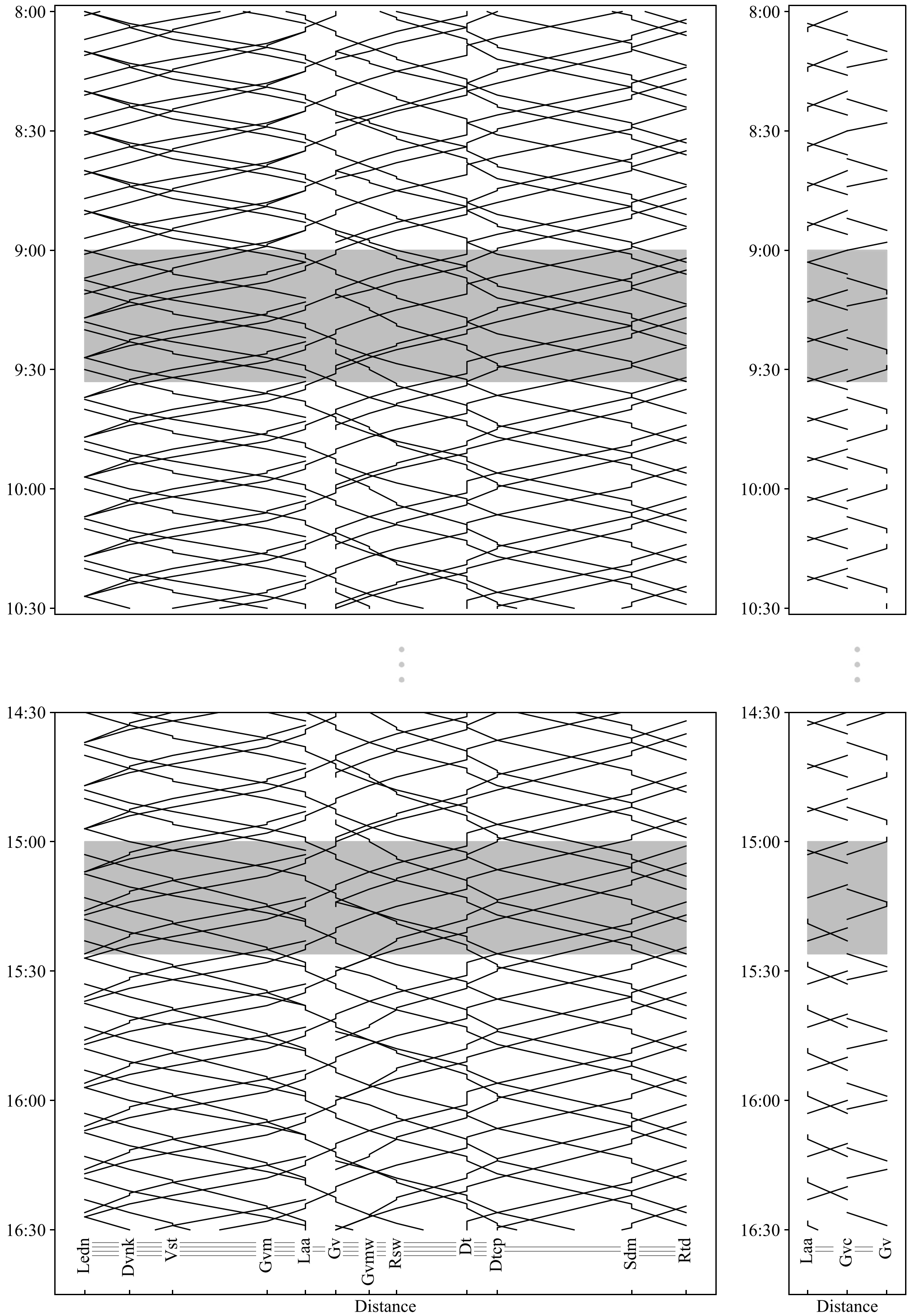

Fig. 2. Time-distance diagram of timetable obtained with the equally weighted sum method. Transition periods are denoted in grey. The corridor between Ledn and Rtd is shown on the left, while the two corridors between Laa and Gvc and Gv and Gvc are shown on the right.

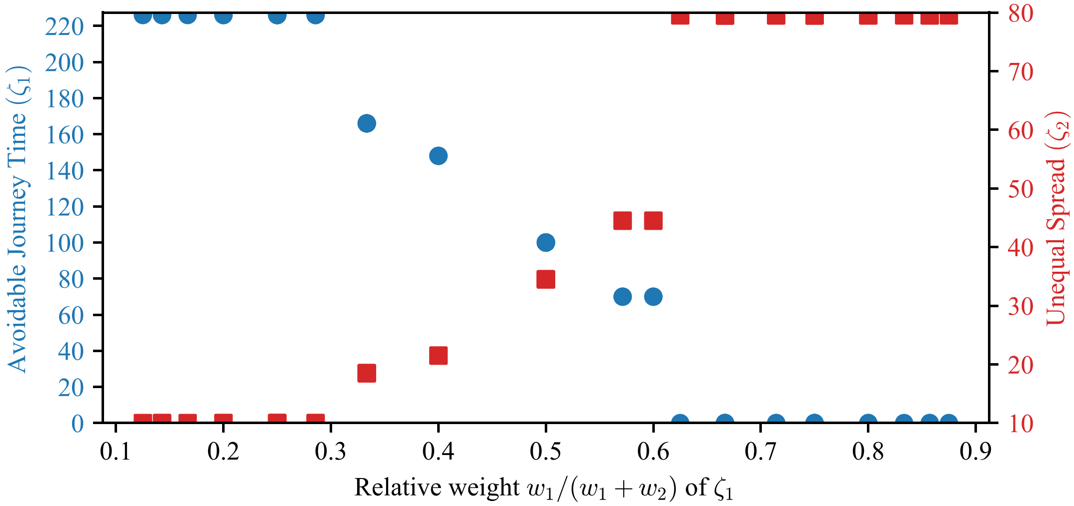

Fig. 3. Sensitivity analysis of w_1 and w_2 when $\zeta_3 = \zeta_4 = 0$.

the afternoon peak. Furthermore, the railway corridor between Ledn and Rtd is shown in the left subfigures, while the tracks between Gvc and Laa and between Gvc and Gv are shown in the right subfigures. The transition periods, during which lines from both the old and the new schedule are operated, are shaded in grey. Fig. 2 confirms that there are three distinct timetables throughout the day: if we compare the time-distance diagrams in the three unshaded parts, we see three different patterns. The transitions between the cyclic schedules take around 30 min, where the second transition is faster than the first transition. This is because during the second transition, the longest lines (between Ledn and Rtd and vice versa) do not change their stopping patterns and hence keep their cyclic times. The diagram also verifies that the timetable is conflict-free, with no overtaking and adequate headways on double-track sections.

As we have several weights in the objective function, we perform a sensitivity analysis on these weights. We do this by varying the weights between 0.1 and 0.7 with increments of 0.1, and under the condition that the sum of the 4 weights is equal to 1. The results confirm that there is mainly a trade-off between ζ_1 and ζ_2, as in most of the scenarios, ζ_3 and ζ_4 attain their minimum values. However, in some combinations where $w_3 = 0.1$ and/or $w_4 = 0.1$, either some trains are shifted from the cyclic departure times (up to 7 min) or one of the transition trains is cancelled. To further investigate the trade-off between ζ_1 and ζ_2, Fig. 3 shows the objective values of ζ_1 (in blue circles) and ζ_2 (in red squares) under the different results that were found (with $\zeta_3 = \zeta_4 = 0$). The horizontal axis displays the relative weight of ζ_1 compared to the weight of ζ_2, so the value of $w_1/(w_1 + w_2)$. The plot shows that the trade-off between the two objectives occurs if the relative weight of ζ_1 is greater than 0.286 and smaller than 0.625. By varying the weights w_1 and w_2 such that the relative weight falls within this range, several timetables could be created based on the preferences of the railway undertaking. Note that in the case of equal weights, the relative weight falls within this range at 0.5.

5. Conclusion

The demand for passenger railway services varies significantly throughout the day in terms of volume and travel purposes, which influences travel destinations. However, in many European countries, a cyclic timetable is used that remains constant throughout the day. While such timetables offer ease-of-use to passengers, they are often optimised for peak-hour demand, leading to inefficiencies during off-peak times. On the other hand, acyclic timetables can more easily be adjusted to the fluctuating demand, but are more difficult to remember.

This study aimed to create a timetable that integrates both cyclic and acyclic elements to better serve the fluctuating demand. The proposed mixed-integer linear programming model creates multiple cyclic timetables within the day and transitions between them. To facilitate the transition between two cyclic timetables, departure times can be shifted and trains can be cancelled during the transition periods. To create a timetable that is optimal for the passengers, the model minimises the avoidable journey time, deviations from the ideal departure time distributions, and the shift from cyclic departure times and train cancellations during the transition periods.

The proposed model was successfully tested on a case study based on a small part of the Dutch railway network. In this case study, we assumed that there are three periods in which the demand is homogeneous: the morning peak, midday off-peak, and afternoon peak. For each of these periods, a line plan is provided that best serves the period-specific demand. For the case study, five different timetables were created. Four timetables were created by solving the model for each single objective lexicographically and the fifth timetable was created by using a weighted sum of the four objectives. Interestingly, in three of the five created timetables, no extra flexibilities were used for the trains that depart during the transition periods. The avoidable journey time and departure distribution are not largely impacted by directly using the cyclic event times and not cancelling trains in the transition periods. Furthermore, when these flexibilities are not used, the resulting timetable is more recognisable for the passengers during the transition periods. Hence, it seems to be favourable to immediately start using the new cyclic event times during the transition periods and not cancel any trains. Moreover, putting insufficient emphasis on avoiding train cancellations can lead to very poor quality timetables during the transition periods, while only minimal reductions of the avoidable journey time can be realised. Furthermore, we see a trade-off between minimising the avoidable journey time (which minimises the passenger in-vehicle time) and achieving an equal departure distribution (which minimises the waiting time at the station). When two trains with incompatible frequencies want to use the same infrastructure, some unequal departure distribution is needed. However, if a lot of weight is put on reducing the avoidable journey time, this unequal distribution is used at all the stations on the train's route. On the other hand, if more weight is put on having equal departure distributions, extra dwell and drive time is used to create equal departure distributions at the stations where this is possible.

We see several avenues for future research. Although the current case study provides some interesting insights, the differences between the line plans is relatively small as only the stopping pattern is adjusted. As a common challenge in practice is reducing or increasing the frequencies of a line or inserting new lines, future work should incorporate these variabilities as well. Furthermore, it will be interesting to apply the proposed model to larger case studies to test its scalability. If needed, one way to potentially improve the scalability of the model is by looking more closely between which trains a headway constraint is needed. For each pair of trains that need a headway constraint, also a binary variable is needed, which makes the problem harder to solve. If we can limit the number of headway activities, and therefore the number of binary variables needed, we could possibly speed up the solving process. Two potentially interesting extensions of the model include adding rolling stock turnaround times and adding transfer times. Considering the rolling stock turnaround times can reduce the cost of the timetable for the railway undertaking. If a timetable is designed such that there are short turnaround times, less rolling stock is needed to operate the timetable. As rolling stock is very costly, considering the turnaround times could greatly reduce the costs for the railway undertaking. Moreover, although the model aims to provide an optimal timetable for passengers, transfers between lines are not considered. In the considered case study, around 4000 transfers are required to transport all passengers from their origin to their destination. The service for these passengers could be improved by also considering the transfer times within the optimisation. Furthermore, while the two objectives of minimising the avoidable train journey time and the unequal spread of trains are used to create an attractive timetable, these do not take into account how many passengers are affected by extra journey time or an unequal distribution. Hence, in order to create a timetable that is favourable for the majority of the passengers, future research could investigate how the amount of affected passengers could be incorporated in the objectives. Another interesting research direction would be to investigate the robustness of the multi-period timetable. As both the timetable and demand are changing throughout the day, it is likely that the robustness of the schedule also varies. For example, when also changes in the frequency are considered, there are periods with more trains per hour, which could have a higher risk of delay propagation. Additional model extensions could be needed to create a schedule that is robust to disturbances in all periods. Lastly, additional information about the benefits and drawbacks would be needed before we can make a recommendation to a railway undertaking about implementing the multi-period timetable. Of course the demand for railway services always depends on the service offered. Therefore, it would be wise to estimate the effect of the new service on the demand and ticket revenues. For example, this could be done by comparing the travel times in the (current) fixed timetable with the travel times in the multi-period one and use travel time elasticities to estimate the effect on demand. When the new demand is determined, the effect on ticket revenues can also be calculated. Furthermore, having a multi-period timetable also adds complexity to creating the timetable, rolling stock plan, crew schedule, and to rescheduling the service after a disruption. Only after all these costs and expected benefits are approximated can we make a definitive recommendation to the railway undertaking.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Co-author is Editor in Chief of the Journal of Rail Transport Planning and Management (JRTPM) — R.M.P. Goverde If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

The authors do not have permission to share data.

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