Particle-driven gravity currents

Part 1: Theory

Master of Science Thesis

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Abstract

Particle-driven gravity currents cause major geological problems. Turbidity currents are highly erosive and can be damaging to structures on the sea bottom such as telecommunication cables. Understanding the mechanisms of sediment transport and deposition is required to predict the erosive powers of turbidity currents (and of the distribution of turbidite deposits) which are fully dependent on the behavior of gravity currents. For this reason, the main question of this thesis was formulated: Which physical parameters of the gravity current are of importance for its behaviour?

The lock-exchange release experiment is a frequently used method to study gravity currents in a laboratory and was also used in this thesis. In order to answer the main question, the following parameters were investigated and their influence specifically on the 4 phases, the run-out length and the PSD: particle size, bed roughness and temperature. The influence of particles size was researched using mono-dispersed vs bi-dispersed experiments. In the bed roughness experiments, sandpaper was attached to the bottom and compared to smooth bed experiments. Finally, to investigate the influence of temperature on the gravity current, experiments with warm water were compared to experiments with colder water.

From these experiments, the most notable results are summarized below.

PSD: For all experiments applies that at low concentration the particles segregate over the run-out length of the gravity current. Smaller particles travelled further than the bigger particles with a higher settling velocity. This does not occur at higher concentrations and the PSD over the entire run-out length is similar.

Four Phases: In all experiments, the four phases could clearly be identified with one exception: the first phase in the rough bed experiments was difficult to distinguish.

Run out length: Some interesting findings were made that were in line with literature: adding fine particles to the mixture of the current cause the run-out length of the current to increase. However, it was also found that if the initial concentration is increased, this effect decreases. Furthermore experiments showed that an increase in temperature can cause the current to travel less far when compared to experiments performed with water with lower temperature.

In the light of this research, the following recommendation are made: Temperature should be taken into account for modelling gravity currents. Otherwise this can lead to an overestimation regarding the run-out length and an underestimation of the deposit density. Furthermore, to get more insight in the effect of the particle sizes in the currents, it would be highly recommended to conduct more experiments with a greater difference between particle sizes. This would allow for a better assessment of the magnitude of the effect of hindered settling

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γ	Light over heavy density ratio ρ_1/ρ_2 [-]
μ	Dynamic viscosity [kg/(m·s)]
ν	Kinematic viscosity [m/s^2]
ϕ	Volume fraction of particles [-]
ϕ_0	Volume fraction of particles in suspension at $t_0[-]$
ρ_a	Density ambient fluid[kg/m^3]
ρ_c	Density of the current[kg/m^3]
σ	Normal stress component[N/m^2]
τ	Shear stress component [N/m^2]
A_0	Surface area of the bottom behind the lock gate [m^2]
C_d	Drag coefficient [-]
d	Particle diameter [m]
d_{50}	Median grain size [m]
F	Force [N]
Fr	Froude number [-]
g	Gravitational acceleration [m/s^2]
g'	Reduced gravity [m/s^2]
H	Height of the surrounding fluid [m]
h	Height of the gravity current[m]
L	Length of the tank [m]
l	Position of the front in the channel [m]
M	Mass of particles added to the suspension [kg]
n_m	Manning friction coefficient [s/m$^{1/3}$]
p	Pressure [kg/(m s^2)]
p_0	Atmospheric pressure [kg/(m s^2)]
Re	Reynolds number [-]
T	Stress deviator tensor [-]
t	Time [s]
u	Velocity in x-direction [m/s]
v_0	Terminal settling velocity of a single particle [m/s]
v_f	Fluid velocity [m/s]
v_p	Particle velocity [m/s]
v_r	Relative velocity [m/s]
v_s	Particle settling velocity [m/s]
W	Width of the channel [m]
x	Horizontal coordinate[m]
x_0	Position of the lock-gate [m]

Chapter 1

Introduction

1.1 Background

A gravity current is a primarily horizontal flow driven by a density difference between two fluids. This density difference can be caused by a temperature difference, chemical composition or suspended materials in one of the two fluids Simpson and Britter (1979). When a density difference is present, the lighter fluid is referred to as the ambient fluid. The structure of a gravity current can be seen in figure 1.1. The front region of the current is referred to as the head and the fluid in the back that follows as the tail. A nose can often be distinguished in the foremost point of the head Simpson and Britter (1979).

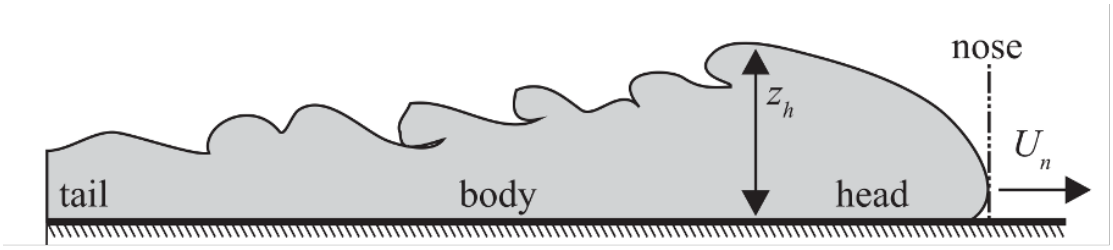

Figure 1.1: Structure of a gravity current.

There are different types of gravity currents; particle-driven currents and compositional gravity currents. The suspension of particles is the driving force of particle driven gravity currents. For compositional gravity currents, temperature differences or dissolved solutes such as salt are the driving force. Examples of natural particle-driven gravity currents are deep-sea gravity currents also called turbidity currents. These currents are responsible for the dispersion of sediment (from coastal regions) to the deep sea. This type of current can also be found in artificial lakes and can lead to reservoir sedimentation especially during flood season Cesare et al. (2001). Other examples include pyroclastic flows from volcanic eruptions and avalanches. They can also be found in more ordinary situations such as opening a door to a warmer space causing cold air to flow into the less dense warmer air. Simpson and Manga (1998) provides numerous examples of gravity currents both natural and artificial.
Besides occurring frequently in natural situations, gravity currents play an important role in many man-made projects in engineering environments, deliberately or unplanned. These include outflows of wastewater treatment plants and oil spillage which often have a negative impact on the environment. Tomkins et al. (2005).
Thus, they have been a subject of extensive research over the years in order to better understand and predict the behaviour and dynamics of gravity currents and the influence on the environment.

However, some fields are still relatively unexplored; research on the effects of particle sizes, bed roughness and temperature differences is limited in comparison to research into the effects of density differences. An increase in bed roughness seems to decrease the front velocity (Peters and Venart (2000), La Rocca et al. (2008), Adduce et al. (2012)) but the effects of a rough bottom are yet to be completely understood. When the water temperature rises, the viscosity changes which inevitably has effects on the gravity current. Both density and viscosity have an inverse relationship with temperature. This means that at higher temperatures, the viscosity and density decrease. When the density of seawater increases, the salinity increases as well, as these have a positive relationship.

1.2 Experiments - lock exchange release

A frequently used method to study gravity currents in a laboratory is the particle-driven lock-exchange release experiment Huppert and Simpson (1980) , Rottman and Simpson (1983) , Hallworth et al. (1996) , Shin et al. (2004) . A horizontal rectangular-shaped tank is divided in two compartments by placing a vertical barrier in such a way that one of the compartments is smaller than the other. Each compartment is filled with a fluid of a different density. The lock compartment, which is smaller, is filled with a denser fluid and is held back by the barrier. The barrier is then removed as quickly and smoothly as possible releasing the lock fluid into the ambient fluid. The denser fluid will flow on the bottom of the tank while the ambient fluid will flow above it in the opposite direction because of the heterogeneous hydrostatic pressure. Besides the laboratory experiments, numerical modelling is also widely used to study gravity currents (Adduce et al. (2012) , La Rocca et al. (2012))

1.3 Problem description

Particle-driven gravity currents cause major geological problems. Turbidity currents are highly erosive and can be damaging to structures on the sea bottom such as telecommunication cables. Understanding the mechanisms of sediment transport and deposition is required to predict the erosive powers of turbidity currents (and of the distribution of turbidite deposits) which are fully dependent on the behavior of gravity currents.

They also play an important role in engineering. The dredging field is frequently involved with activities that can cause slurry flows which can transport sediment over large distances. These gravity currents can deposit a layer of particles over the ocean floor, burying sea life in the process. The effects on the environment can be detrimental. A good understanding of particle-driven gravity currents, their evolution and factors affecting them, is needed in order to reduce the occurrence of such problems.

Bonnecaze et al. (1993) ,Bonnecaze et al. (1995), Hallworth et al. (1998) and Hallworth and Huppert (1998) have contributed to the literature on particle-driven gravity currents. However, the experiments were performed using a mono-dispersed suspension. Furthermore, the currents that were investigated were either homogeneous gravity currents or particle-driven gravity currents that involved the use of silicon carbide.

A more recent study on particle-driven gravity currents by MPJ Stovers (2016) was published in 2017. The influence of the initial volume concentration on the run out length and the flow velocity was investigated by performing lock-exchange experiments. Sand was used to create a suspension. Furthermore, they determined the particle size distribution (PSD) of settled sediment on the bottom of the tank.
They found that at a higher initial volume concentration, there was no separation of particles sizes in the sediment on the bottom. The experiments with lower initial volume concentrations showed the opposite: the smaller particles traveled further than the bigger particles. Finally, it was shown that the run-out length was proportional to the initial mixture/water level of the experiment. The advice that was given to dredging companies following the results was that offloading should be done at the highest concentration possible as this could be beneficial for

both the environment and the production but more research was needed.

This thesis is a continuation of the research done by MPJ Stovers (2016). The same tank was used to carry out the experiments and the method of performing the experiments was kept the same as far as possible. The analysing techniques were also very similar. Varying concentrations were used in both studies. However, the focus in this thesis is certainly not the same. As mentioned before, the literature on the influence of particle sizes, bed roughness and temperature influence especially is limited which is the reason these subjects are studied in this thesis.

The objective is to firstly identify the influence of the bottom roughness. In the next set of experiments, the influence of the particles on the gravity current by using mono- and bi-dispersed suspensions is investigated. In the final set of experiments, the influence of water temperature will be studied. The results will be used to provide more extensive/detailed advice to dredging companies as well as to improve the numerical model used in this previous thesis.

1.4 Research questions

The topic of this thesis is related the study of physical parameters that influence the behaviour of gravity currents.The following research question is answered in this work:

Which physical parameters of the gravity current are of importance for its behaviour and can the numerical model simulate a gravity current correctly with varying physical parameters?

In order to answer this question, a general understanding of the physical parameters that affect the behaviour of gravity currents is required. In all probability the particle size(s)of the mixture, the bed roughness and temperature could influence the behaviour of a gravity current. Therefore, the sub questions that need to be answered in this thesis are:

1. What is the influence of particle size(s) of the mixture for the behaviour of a gravity current?
2. What is the influence of bed roughness on the behaviour of a gravity current?
3. What is the influence of temperature of the mixture for the behaviour of a gravity current?

Chapter 2

Theory

The theoretical background is formed by Shallow Water equations, which have been derived from depth integration of the Navier-Stokes equations and based on certain assumptions. One of those assumptions is the typical gravity current, known to be typically long and thin such that is has horizontal length scale far greater than its vertical length scale. This influences the dynamical properties such that the vertical directions are small and that a hydrostatic momentum balance is established when the current propagates horizontally.
As the name already implies the driving force behind the current is gravity. However, it must be noted that while the current propagates in horizontal direction the gravitational acceleration is in horizontal direction. The horizontal gravitational acceleration occurs when fluids, gasses or a combination of these are oriented in a horizontal direction. This causes hydrostatic pressure that is different in each medium which results in a pressure difference in horizontal direction.
Throughout this report the particle driven gravity currents at the bottom of the ocean are considered. The density difference between the fluids is dealt with through the addition of particles in one fluid. The ambient and interstitial fluid is assumed to be fresh water, whereas sand is considered to be the particle creating the density difference. Between sand and water there is density difference e.g. sands density is 2650 kg/m^3 and that of water 1000 kg/m^3. The concentration of particles in suspension depends on the settling of the particles. However, due to the fact that this behavior is space and time dependent this results in a complicated problem. Eventually, the current will vanish once all particles have settled.

2.1 Navier-Stokes equations

The Navier-Stokes equations, founded and named after Claude-Louis Navier and George Gabriel Stokes. The equations provides a mathematical model of the motion of a fluid and are derived from Newton's second law. The Navier-Stokes equation can be viewed as an application of Newton's second law.

2.1.1 Newton's second law

Newton's second law is defined as:

$$\overrightarrow{F} = m\overrightarrow{a} \tag{2.1}$$

With F being the force on a body, m the mass of the body and a the acceleration of the body. Taking into consideration an in compressible fluid and constant volume, the density for mass can be substituted and using the time-derivative of velocity as accelerations results in the following.

$$\overrightarrow{F} = \rho\frac{d}{dt}\overrightarrow{v}(x, y, z, t) \tag{2.2}$$

Where v is the velocity and the density. Applying the chain rule to the derivative of the velocity equation results in

$$\overrightarrow{F} = \rho\Big(\frac{\partial \overrightarrow{v}}{\partial t} + \frac{\partial \overrightarrow{v}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \overrightarrow{v}}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial \overrightarrow{v}}{\partial z}\frac{\partial z}{\partial t}\Big) \tag{2.3}$$

Which may also be writing as:

$$\overrightarrow{F} = \rho\Big(\frac{\partial \overrightarrow{v}}{\partial t} + \overrightarrow{v}\nabla \cdot \overrightarrow{v}\Big) \tag{2.4}$$

2.1.2 General Navier-Stokes equation

For the body force $\overrightarrow{F}$ in equation it is assumed that it consist of two parts. One part is caused is fluid stresses caused by viscosity(Newtonian fluids) and the other part is formed by external forces defined as $\overrightarrow{f}$, where f is composed of gravitational and friction forces.

$$\overrightarrow{F} = \nabla \cdot \sigma + \overrightarrow{f} \tag{2.5}$$

The stress tensor σ can be divided into two terms. The first term is the volumetric stress term that represent the hydrostatic pressure forces. The second term is responsible for shape changes of the body and is called the stress deviator term. In equation 2.6 the breakdown of the σ in two terms is presented. The stress deviator term is composed of shear stresses as it determines the body deformation. The volumetric stress shows the pressure forces perpendicular on each plane of the volume, whereas the volumetric stress shows the pressure forces perpendicular on each plane of the volume.

$$\sigma = \begin{pmatrix} \sigma'_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma'_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma'_{zz} \end{pmatrix} = -\begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} + \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} \tag{2.6}$$

When defining the stress deviator term as T, the stress tensor σ in the equation above can be reduced to:

$$\sigma = -pI + T \tag{2.7}$$

The general form of the Navier-Stokes equation is obtained by substituting equations 2.5 and 2.7 into equation 2.4.

$$\rho(\frac{\partial \overrightarrow{v}}{\partial t}) + \rho(\overrightarrow{v} \cdot \nabla \overrightarrow{v}) = -\nabla p + \nabla \cdot T + \overrightarrow{f} \tag{2.8}$$

2.1.3 Incompressible Newtonian fluids

For this thesis incompressible Newtonian fluids are considered. In a Newtonian fluid the stress is proportional to the rate of deformation of the fluid. The rate of deformation in x-direction is presented below in equation, with μ representing the viscosity of the fluid.

$$\tau_{ij} = \mu\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big) \tag{2.9}$$

Using the divergence of stress as shown in equation and the rate of deformation as presented in equation the stress term can be calculated and results in the following.

$$\nabla \cdot \sigma = \mu \nabla \cdot \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} = \mu \begin{pmatrix} 2\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & 2\frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} & \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} & 2\frac{\partial w}{\partial z} \end{pmatrix} \tag{2.10}$$

By extending the divergence to the other directions applying the same method, the following is obtained.

$$\nabla \cdot T = \mu \nabla^2 v \tag{2.11}$$

When equation 2.11 is substituted into the general formulation of the Navier-Stokes equation, the Navier-Stokes eqution for an incompressibile Newtonanian fluid in conservative form is obtained. This yields:

$$\Big(\frac{\partial \rho \overrightarrow{v}}{\partial t}\Big) + \nabla \cdot (\rho \overrightarrow{v}\overrightarrow{v}) = -\nabla p + \mu \nabla^2 \overrightarrow{v} + \overrightarrow{f} \tag{2.12}$$

Equation consists of four terms:

1. $\Big(\frac{\partial \rho \overrightarrow{v}}{\partial t}\Big) + \nabla \cdot (\rho \overrightarrow{v}\overrightarrow{v})$, represent the inertial forces of the fluid.
2. $-\nabla p$, represents the pressure forces composed by normal stresses.
3. $\mu \nabla^2 \overrightarrow{v}$, is the stress that causes motion due to horizontal friction and shear stresses.
4. $\overrightarrow{f}$ represent external forces on the fluid, which can include gravitional forces and wall-friction forces.

It must be noted that the Navier-Stokes equation always is solved together with the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \overrightarrow{v}) = 0 \tag{2.13}$$

In essence the contuinity equation represent the conservation of mass, whereas the Navier-Stokes equation represents the conservation of momentum.

2.2 Shallow-Water Equations

These equations describe the evolution of an incompressible fluid in response to gravitational and rotational accelerations. The solutions of shallow water equations represent many types of motion. Due to the fact that these equations have been used for decades the complete derivation of the equations from the Navier-Stokes Equations is often omitted in articles and publications. This section is dedicated to present the physical assumptions and consequences of the shallow water equations on the Navier-Stokes Equations (Vreugdenhil (1994)).

2.2.1 Single Layer

Figure presents a visual representation of a single layer. Since the density differences are minimum, the Boussinesq approximation can be used. This approximation assumes that variations in density only give rise to buoyancy forces and hence have no effect on the flow field. With the Boussinesq approximation the system can be considered as a homogeneous incompressible fluid. With the use of the expansion equation and partial differentiation in all direction, the following is obtained:

$$\rho\Big(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\Big) = -\frac{\partial p}{\partial x} + \mu \nabla^2 u + f_x \tag{2.14}$$

$$\rho\Big(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\Big) = -\frac{\partial p}{\partial y} + \mu \nabla^2 v + f_y \tag{2.15}$$

$$\rho\Big(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\Big) = -\frac{\partial p}{\partial z} + \mu \nabla^2 w + f_z \tag{2.16}$$

Some basic assumptions for this equations are as follows:

1. First of all it is assumed that the depth and the width of the current are very small compared to the length of the current. In essence this means that the flow is essentially one-dimensional and parallel to the walls and bottom of the channel. Resulting in the fact that all terms in equation reduce to zero. The dynamics of the current is mainly driven by the horizontal processes in x-direction, accelerations in the vertical direction are small and a hydrostatic momentum balance is maintained when the current propagates horizontally. All terms in equation including the vertical velocity can be neglected due to the fact that

the depth of the current is assumed to be very small compared to the length. This reduces the hydrostatic balance with gravitational force ρg as the only vertical force acting on the fluid:

$$0 = -\frac{\partial p}{\partial z} - \rho g \tag{2.17}$$

Since all the term in equation that involve the derivative in y or z-direction can be neglected such that equation reduces to:

$$\rho\Big(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\Big) = -\frac{\partial p}{\partial x} + \mu\nabla^2 u + f_x \tag{2.18}$$

2. It is also assumed that the variation of the water height is very small. Since no waves are present and the bottom is flat, the total water height is equal to h. Integration of equation over the height h results in a expression for the pressure p with p_0 the constant atmospheric pressure.

$$p(x,t) = p_0 + \rho g h(x,t) \tag{2.19}$$

Substitution of equation into equation results in

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x} + f_x \tag{2.20}$$

The shallow water equation is obtained by integrating equation over the height of the water column (o,h) and yields

$$\frac{\partial uh}{\partial t} + u\frac{\partial u^2 h}{\partial x} + g\frac{1}{2}\frac{\partial h^2}{\partial x} = f_x \tag{2.21}$$

3. Another assumption is based on the dominate forces which influence the behaviour of the current. These forces can be either inertial or viscous forces, which can be distinguished using the Reynolds number

$$Re = \frac{UL}{\nu} \tag{2.22}$$

In this equation U, L and ν are the typical velocity, length and kinematic viscosity of the current respectively. The height of the current is the typical length scale. In the case that Re is larger than 1 than the current is condered inertial or in viscid, whereas the current is considered viscous when Re is equal or smaller than 1. Typically a current consisting of water is in the interial regime. In the experiments carried out the initial height is large and increasing the height of the current increases the hydrostatic pressure difference which in turn increases the velocity of the current resulting in a even higher Reynolds number. Finally the current will spread and the height and speed will decay at the end of the current and the viscous regime will dominate. Hence, it can be assumed that the current is inviscid. Resulting in the fact that the viscous force term μ can be neglected and reducing equation to

$$\rho\Big(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\Big) = -\frac{\partial p}{\partial x} + f_x \tag{2.23}$$

4. Finally it is assumed that fluids are homogeneous. This mean that the particles in suspension are distributed evenly across height and length before the release of the suspension into the ambient field. The problem can be schematized by the release of a well mixed suspension behind a lock gate. The density difference between the suspension and the ambient fluid will create a current that propagates along the bottom of a rectangular tank of constant volume. The current and ambient fluid are both of constant volume. The density of the ambient fluid is considered constant and the density of the current may vary over length and time as particles will be able to settle out from the current and is given by:

$$\rho_c(\phi) = \phi(x,t)\rho_p + (1 - \phi(x,t))\rho_a \tag{2.24}$$

With ρ_a is the density of the ambient fluid, ρ_p is the density of the particle and ϕ is the volumetric concentration of particles. Since the density difference between the fluids generates hydrostatic pressure difference, the gravitational force term takes form of the reduced gravity g':

$$g'(\phi) = g \cdot \frac{\rho_c(\phi) - \rho_a}{\rho_a} \tag{2.25}$$

Summary of single layer shallow water equations
Applying the reduced gravity results in the following shallow water equations:

Continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = 0 \tag{2.26}$$

The continuity equation consists of the following two terms:

1. the change of height over time, $\frac{\partial h}{\partial t}$
2. the advection of height, $\frac{\partial}{\partial x}(uh)$

Conservation of momentum equation

$$\frac{\partial uh}{\partial t} + \frac{\partial u^2 h}{\partial x} = -g'(\phi)\frac{1}{2}\frac{\partial h^2}{\partial x} \tag{2.27}$$

The Conservation of momentum equation consists of the three terms:

1. the change of velocity over time, $\frac{\partial uh}{\partial t}$
2. the advection of momentum, $\frac{\partial u^2 h}{\partial x}(uh)$
3. the hydrostatic pressure gradient term, $g'(\phi)\frac{1}{2}\frac{\partial h^2}{\partial x}$

Particle conservation
The concentration will vary throughout the current due to the fact the gravity current is particle driven and given that particles settle from the current. In this process the entrainment of water as well as the detrainment of water from particles to the top layer are neglected. Since the velocities just above the bed are insufficient the re-suspension of particles in the current is also neglected.It is also assumed that current speeds are sufficient to keep the flow turbulent such that a vertical uniform particle concentration is sustained through turbulent mixing.Furthermore it is assumed that particles only leave the current at the bottom where viscous forces are greatest and velocities are the smallest. The introduction of particles requires and additional conversation equation, which is presented below.

$$\frac{\partial}{\partial t}(\phi h) + \frac{\partial}{\partial x}(u\phi h) = -v_s\phi \tag{2.28}$$

In which v_s is the term that accounts for the settling velocity of the particles and represent the settling velocity v_s.

2.2.2 Two-layer equations

In this research the effects of the overlying fluid will be taken into account. The reason for this is that the ambient layer is comparable to the height of the intruding gravity current. This implies that the two layer shallow water equations will have to be used instead of the above explained single layer shallow water equations. In the two layer shallow water equations the effects of the upper and in the opposite direction flowing fluid are considered. In order to determine the system of equation for the two-layer situation the approach of will be used.

Similar to the single layer shallow water equations the two-layer shallow water equations also comprises of a conversation of mass equations and a conservation of momentum equations. In the two-layer shallow water equations a conservation of mass and momentum of both layers is required.

Following the same process as earlier result in following shallow water equations for a two layer fluid.

Continuity equation

$$\begin{aligned}\frac{\partial h_c}{\partial t} + \frac{\partial}{\partial x}(u_c h_c) &= 0\\ \frac{\partial h_a}{\partial t} + \frac{\partial}{\partial x}(u_a h_a) &= 0\end{aligned} \tag{2.29}$$

Momentum equation

$$\begin{aligned}\frac{\partial u_c}{\partial t} + u_c \frac{\partial u_c}{\partial x} &= -\frac{1}{\rho_c}\frac{\partial p_0}{\partial x} - g\frac{\partial h_c}{\partial x}\\ \frac{\partial u_a}{\partial t} + u_a \frac{\partial u_a}{\partial x} &= -\frac{1}{\rho_c}\frac{\partial p_0}{\partial x} - g\frac{\partial h_a}{\partial x}\end{aligned} \tag{2.30}$$

In the equations above subscript c represent the current and subscript a represent the ambient fluid. Equations presents the conservation of mass of the two layers and equation the conservation of momentum of both layers in a hydrostratic pressure field.

The momentum equation consists of the following four terms for each layer:

1. the change of velocity over time, $\frac{\partial u}{\partial t}$
2. the advection of momentum, $u\frac{\partial u}{\partial x}$
3. the horizontal pressure difference, $-\frac{1}{\rho}\frac{\partial p_0}{\partial x}$
4. the hydrostatic pressure gradient term, $-g\frac{\partial h}{\partial x}$

The height of the layers are related to each other by a fixed total depth H due to the fact the volume of the tank is fixed and the volume of both layers is constant. This results in the following expression for the total depth H:

$$H = h_c(x,t) + h_a(x,t) \tag{2.31}$$

Because of the constant volume properties and the no flow at the begin wall ($u(0,t) = 0$) condition, the following equation can be deduced:

$$u_c(x,t)h_c(x,t) + u_a(x,t)h_a(x,t) = 0 \tag{2.32}$$

By combining equation 2.30 with equations 2.31 and 2.32, a combined momentum equation can be obtained as a function of the height and velocity of the current only. It is assumed that all velocities and heights will be that of the dense current intruding the ambient fluid.

Combined momentum equation

$$\Big[1 + \frac{\rho_a}{\rho_c}\frac{h}{H-h}\Big]\frac{\partial u}{\partial t} + \Big[1 - \frac{\rho_a}{\rho_c}\frac{h(H+h)}{(H-h)^2}\Big]u\frac{\partial u}{\partial x} + \Big[g_c^{'} - \frac{\rho_a}{\rho_c}\Big(\frac{H}{H-h}\Big)^3\frac{u^2}{H}\Big]\frac{\partial h}{\partial x} = 0 \tag{2.33}$$

Initial and boundary conditions

In order to solve the equations initial conditions need to be defined at $t = 0$ as well as boundary conditions at $x = 0$ and $x = L$. It must be noted that the boundary conditions are reflective boundaries. The initial conditions as presented below in equation are stated such that the height of the suspension behind the lock-gate positioned at x_0 is equal to the total height H and that the height of suspension on the other side of the lock-gate is zero. Velocities are equal to zero at the beginning and end of the tank as can be seen in equation below.

$$\begin{aligned} u(x=0,t) =&0 \\ u(x=L,t) =&0 \end{aligned} \tag{2.34}$$

$$h(x,t=0) = \begin{cases} H, & \text{if } 0 \leq x \leq x_0. \\ 0, & x_0 < x. \end{cases} \tag{2.35}$$

Combined momentum equation with friction

In the model no external forces are present that retard the current. It is necessary to include a friction term to the momentum equations to represent all friction experienced by the current such as friction from the bottom and walls. By using the classical Manning formulation which incorporates n_m, the Manning roughness coefficient(Manning et al. (1890)) a friction term can be added to the momentum equation as presented in equation. However, it is very difficult to determine a suitable value for the Manning friction coefficient.

$$\Big[1+\frac{\rho_a}{\rho_c}\frac{h}{H-h}\Big]\frac{\partial u}{\partial t}+\Big[1-\frac{\rho_a}{\rho_c}\frac{h(H+h)}{(H-h)^2}\Big]u\frac{\partial u}{\partial x}+\Big[g_c^{'}-\frac{\rho_a}{\rho_c}\Big(\frac{H}{H-h}\Big)^3\frac{u^2}{H}\Big]\frac{\partial h}{\partial x}=\frac{n_m^2}{h^{7/3}}u|u| \tag{2.36}$$

The number of equations that needs to be solved is 3, which are the continuity equation, the equation of particle conservation and the combined momentum equation with friction.

2.3 Settling velocity of particles

2.3.1 Forces acting on submerged particles

The nature of forces exerted on solid bodies and small particles as they move through fluids will be described in this subsection. Besides the gravitational force, the particles experience forces that find their origin in either particle-fluid interaction or in particle-particle interaction. Buoyancy, drag and lift force are the forces that originate from the particle-fluid interaction. Forces that arise from the particle-particle interaction are transmitted through via physical contact of the particles in the form of inter-particle stress.
Since the ratio between the channel diameter and the particle size D/d_{50} $\approx=$ 2000 the wall friction experienced by a settling particle can be neglected (Di Felice (1996)). In addition, the flow is regarded to be uniform, which means the velocities are constant and the particles will not experience lift force.

The total sum of forces acting on a single particle in steady state is equal to:

$$F_D + F_B - F_g = 0 \tag{2.37}$$

The following three terms can be distinguished

1. The gravitational force, F_g, follows from applying Newton's second law, $F = ma$, and is determined as:

$$F_g = \rho_p \frac{\pi d^3}{6} g, \tag{2.38}$$

   where $\rho_p \pi d^3/6$ is equal to the mass of a spherical particle, in which rho_p is defined as the particle density and d as the particle diameter. The acceleration is defined as the gravitational acceleration g.

2. The buoyancy force, F_B, is an upward force exerted by a fluid that opposes the weight of an immersed object. The submerged weight of a particle is the sum of the gravitational and the in opposite direction acting buoyancy force. The submerged weight of a spherical can be determined with

$$F_B = (\rho_p - \rho_a)\frac{\pi d^3}{6} g \tag{2.39}$$

3. The drag force, F_D, depends on the viscosity of the fluid as well as the size and the shape of the particle. Drag force is experienced when the surrounding fluid flows along the particle and therefore is a function of the relative velocity that the fluid has to the particle: $v_r = v_f - v_p$, where v_f is the fluid velocity and v_p the particle velocity. The drag is described by the following equation:

$$F_D = \frac{1}{8} C_D \pi d^2 v_r |v_r| \rho_a \tag{2.40}$$

The drag coefficient C_D in the above equations depends on the particle Reynolds numbers, which is given by

$$Re_p = \frac{\rho_a |v_r| d}{\mu} \tag{2.41}$$

2.3.2 Settling of a single particle

Various studies and experiments have been carried out to find a correlation between the particle Reynolds number and the drag coefficient. Ferguson and Church (2004) Ferguson and Church have found an equation that can be used to determine the settling velocity of a single particle over a wide range of particle Reynolds numbers Re_p and yields:

$$v_o = \frac{\Delta g d^2}{C_1 \nu + \sqrt{0.75 C_2 \Delta g d^3}}, \tag{2.42}$$

where Δ denotes as the submerged specific gravity and is equal to $\Delta = \frac{(\rho_p - \rho_a)}{\rho_a}$. Furthermore, ν denotes viscosity of the water. The terminal settling velocity of a single particle v_0 is equal to the relative settling velocity v_r and the coefficients for natural sands are $C_1 = 18$ and $C_2 = 1$.

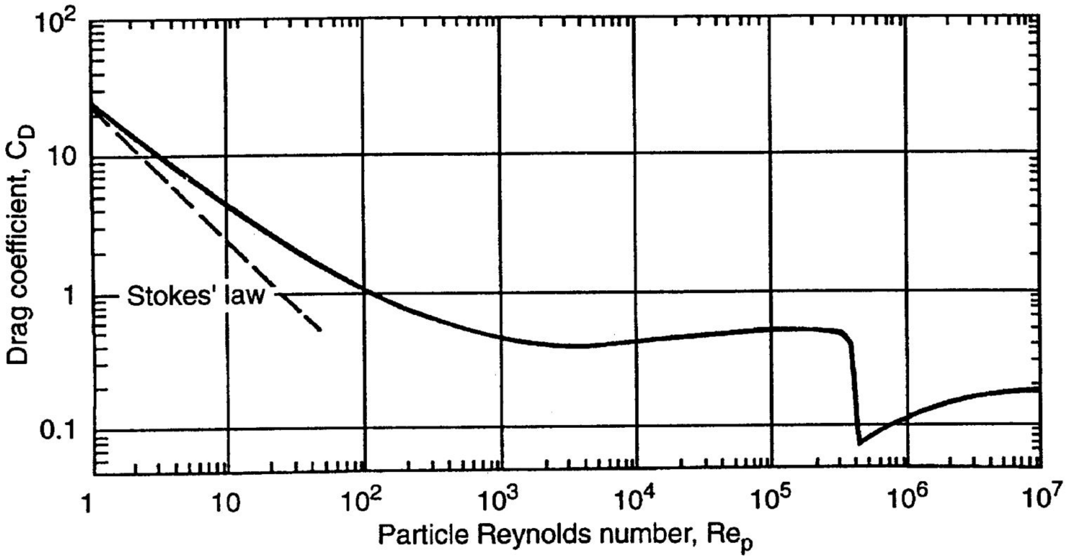

Figure 2.1: Functional relation between particle Reynolds number and drag coefficient

2.3.3 Hindered Settling

The settling velocity decreases when a large concentration of particles is present. The particles interact with the fluid as well as other particles, which is known as hindered settling. The influence of the concentration on the particle settling velocity has been found by Richardson and Zaki (1954) and yields:

$$v_s = v_0 (1 - \phi)^n \tag{2.43}$$

In which the exponent n depends on the Reynolds particle number and is obtained as follow:

$$n = \frac{4.7 + 0.41 Re_p^0.75}{1 + 0.175 Re_p^0.75} \tag{2.44}$$

See Rowe (1987) for a more detailed explanation about the exponent.

2.3.4 Poly-dispersed mixture of particles

A mixture in which particles of different sizes are present, is called a poly-dispersed mixture of particles. These particles affect each other during settling. The method of (Ferguson and Church (2004);Rowe (1987)) is only valid for mono-dispersed mixtures. In order to determine the effect of a poly-disperse mixture the approach of (Smith (1966)) is used. According to this theory, when a particle settles, the surrounding water will have a velocity in the opposite direction of the particle due to the in compressible nature of the fluid and particle. As a results the water will have to flow in the opposite direction of the particle to fill the void left by the particle in order to fill the volume that was previously occupied by the particle. The settling velocity of a particle is determined as the sum of the water velocity v_w and the relative velocity of the particle to the ambient water v_r:

$$v_s = v_w + v_r \tag{2.45}$$

Since the fluid is in compressible and the densities are constant, mass conversation results in the total volume displacement being equal to zero. Therefore the volume fraction of particles leaving the domain with the settling velocity v_s must be equal to the volume fraction of water entering the domain at the water velocity:

$$\phi v_s = -(1-\phi) v_w \rightarrow v_w = -\frac{\phi}{1-\phi} v_s \tag{2.46}$$

Substitution in equation (2.45) results:

$$v_r = v_s \Big(\frac{1-\phi}{1-\phi} + \frac{\phi}{1-\phi} \Big) = v_s \frac{1}{1-\phi} \tag{2.47}$$

Substitution of equation(2.43) in this expression results in the following expression for the relative velocity of a single fraction:

$$v_r = v_0 (1-\phi)^{n-1} \tag{2.48}$$

The method of Mirza and Richardson (1979) showed that when dealing with a poly-disperse mixture of particles the relative velocity of fraction i can be determined using equation (2.49) with ϕ the total concentration.

$$v_{r,i} = v_{0,i} (1-\phi)^{n_i - 1} \tag{2.49}$$

Using the relative velocity of each fraction as determined by equation (2.49) and the total volume displacement being zero as was found in equation (2.46), a formulation of the settling velocity for each fraction i is obtained and yields:

$$v_{s,i} = \sum_{k=1}^{N} \phi_k v_{r,k} + v_{r,i} \tag{2.50}$$

In this equation ϕ_k is the concentration of fraction k, whereas v_r is the relative velocity which is determined per fraction using equation (2.49). Finally, this equation uses a summation of N different fraction of particles sizes.

Chapter 3

Numerical verification

In this chapter the verification of the numerical model, as developed and implemented by Stovers (2016), is presented. Firstly the numerical method that describes a particle-driven gravity current is discussed in paragraph 3.1. Also the derivation of the numerical model and the discretized numerical model are presented in paragraph 3.1. As the numerical model is usable for all cases regarding particle-driven gravity currents, the verification of the numerical method should be performed by verifying all variables of influence separately.

3.1 Numerical model

A two dimensional gravity current is induced in a tank by the release of a well mixed suspension of density ρ_c into an ambient fluid of a lesser density ρ_a. Both ends on the left and right side of the tank are restricted by vertical walls. Both fluids are of constant volume and it is assumed that both fluids are inviscid and incompressible. The flow of both fluids is assumed to be governed by the balance between buoyancy and inertial forces. Viscous forces and forces due to entrainment are therefore neglected. After removing the lock, the heavier fluid is released into the ambient fluid and the gravity current propagates along the horizontal plane at the bottom of the tank. Subsequently, the length of the current becomes much greater than its height, which leads to very small vertical accelerations. On that basis, a hydrostatic pressure distribution is assumed. Moreover, it is concluded that a gravity current is a very complex physical problem, for which a numerical model is restricted to certain assumptions that only describe the main driving forces behind the current.

In this research the height of the ambient layer is comparable to the height of the intruding gravity current. As elaborated in paragraph 2.2, the effects of the overlying fluid have to be taken into account. This means that the two-layer shallow water equations should be employed instead of the single-layer shallow water equations. The two-layer shallow water equations take the effects of the upper and in the opposite direction flowing fluid into account.

The numerical model that will be used, is presented in 3.1.1. The method that will be used in order to discretize the equations that describe the gravity current are presented in section 3.1.2. Finally, the discretized model is presented in section 3.1.3.

3.1.1 Numerical method

In order to choose a numerical method that is able to simulate the motion of both the mixed suspension and the ambient fluid, the numerical method of choice should satisfy certain requirements. Particularly, the numerical method should be able to handle discontinuities in this system of hyperbolic differential equations and initial conditions, such as shocks and jumps. Furthermore, the accuracy of the numerical representation should be sufficient.

Naturally, there are multiple numerical methods can be chosen to model a system of the hyperbolic differential equations of a two-layer shallow water problem. However in this paragraph only two numerical methods, namely the Lax-Friedrich's and the Lax-Wendroff schemes, will be discussed, as these are traditionally used for modelling gravity currents.

- Lax-Friedrich's scheme:
  The Lax-Friedrich's scheme is a first order method, which is a transformation of the unconditionally unstable forward-time central-space scheme (FTCS) that uses spatial averaging to obtain a conditionally stable scheme.

  Advantages:

  - The spatial averaging serves as a numerical dissipation that represents viscosity.

  Disadvantages:

  - The numerical dissipation causes the solution to be very smeared and lose amplitude. Additionally, the loss of amplitude is reinforced by the discontinuous nature of the initial conditions.
  - With a refined grid the accuracy of the scheme is proved to be more than sufficient.

  See Wu et al. (2011) for more information about the generalised Lax-Friedrichs method.

- Lax-Wendroff scheme:
  The Lax-Wendroff scheme is an explicit second order method, wherefore the spatial and temporal discretization is combined in order to globally achieve second order. This results in a method that is stable and classically used for problems that require shock-capturing.

  Advantages:

  - Increased accuracy. More efficient

  Disadvantages:

  - Due to the discontinuous character of the problem under consideration oscillations are bound to occur in the solution. Predominantly, the oscillations will arise around the discontinuity. Due to the oscillations the concentration and height of the mixed suspension become negative which is physically impossible.

  See LeVeque et al. (2002) for more information about the Lax-Wendroff scheme.

The advantages and disadvantages of both the Lax-Friedrich's and Lax-Wendroff schemes are considered in the choice of the numerical method that will be used for the numerical model. Due to the physical impossibility of a negative concentration and height, it is chosen to use the generalised Lax-Friedrich's scheme rather than Lax-Wendroff scheme.

3.1.2 Discretization method

As stated in section 3.1.1, the Lax-Friedrich's method is a modification of the unconditionally unstable forward-time central-space scheme. The forward-time central-space scheme has a second order accuracy in space and a first order accuracy in time. The scheme uses cells for which the cell center's are located at $x = i$ and the cell boundaries are located at $x = i - \frac{1}{2}$ and $x = i + \frac{1}{2}$. Furthermore, the forward-time central-space scheme is an explicit method, which means that the solution at time step $t = n + 1$ is only dependent on the solution at the current time step $t = n$.

3.1.3 Discretized model

The two-layer shallow water equations are derived in paragraph 2.2.2. In this subsection the discretized model is presented. The discretized model consists out of the discretized two-layer shallow water equations, the boundary conditions and the initial conditions. Since this thesis is a continuation of the thesis of Stovers (2016), the same discretization model is used. The elaboration of the discretization model can be found in the thesis of Stovers (2016).

Differential equations:
The following three constants are used in the differential equations describing the continuity and momentum of the fluid and the particle conservation.

$$\begin{aligned} A =& 0.1 \\ B =& 0.1 \\ C =& 1 \end{aligned} \tag{3.1}$$

Continuity equation:

$$h_i^{n+1} = \frac{Ah_{i-1} + Bh_i + Ch_{i+1}}{A+B+C} - \frac{\Delta t}{2\Delta x}(h_{i+1}u_{i+1} - h_{i-1}u_{i_1}) \tag{3.2}$$

Momentum equation:

$$\begin{aligned} D_i =& \left[1 + \frac{\rho_a}{\rho_{c,i}} \frac{h_i}{H - h_i}\right] \\ E_i =& \left[1 - \frac{\rho_a}{\rho_{c,i}} \frac{h_i\left(H + h_i\right)}{\left(H - h_i\right)^2}\right] \\ F_i =& \frac{n^2}{h_i^{7/3}} u_i|u_i| \\ u_i^{n+1} =& \frac{Au_{i-1} + Bu_i + Cu_{i+1}}{(A+B+C)} - \frac{\Delta t}{2\Delta x}\frac{E_i}{D_i} u_i\left(u_{i+1} - u_{i-1}\right) \\ &- \frac{\Delta t}{2\Delta 2}\frac{G_i}{D_i}\left(h_{i+1} - h_{i-1}\right) - \frac{\Delta t}{D_i} F_i \end{aligned} \tag{3.3}$$

Particle conservation equation:

$$[\Phi h]_i^{n+1} = \frac{A\left[\Phi h\right]_{i-1} + B\left[\Phi h\right]_i + C\left[\Phi h\right]_{i+1}}{(A+B+C)} - \frac{\Delta t}{2\Delta x}\left(\left[u\Phi h\right]_{i+1} - \left[u\Phi h\right]_{i-1}\right) - \Delta t v_{s,i}\Phi_i \tag{3.4}$$

Boundary conditions:

$$\begin{aligned} h_0^n =& h_1^n \\ h_{L+1}^n =& h_L^n \end{aligned} \tag{3.5}$$

$$\begin{aligned} u_{i-\frac{1}{2}}^n =& \frac{1}{2}\left(u_{i-1}^n + u_i^n\right) \\ u_{i+\frac{1}{2}}^n =& \frac{1}{2}\left(u_i^n + u_{i+1}^n\right) \end{aligned} \tag{3.6}$$

$$\begin{aligned} \Phi_0^n =& \Phi_1^n \\ \Phi_{L+1}^n =& \Phi_L^n \end{aligned} \tag{3.7}$$

Initial conditions:
The height and concentration is dependent on the experiment on hand. The particles are assumed to be homogeneously mixed in the water. Furthermore, the velocity of the particles is assumed to be zero at $t = 0$.

$$u_i^0 = 0 \tag{3.8}$$

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