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The R-curve concept, some elementary comments

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Abstract: The present document was primarily written as a kind of an introduction to aspects, which should be recognized if R-curve measurements are carried out. Ample reference is made to the ASTM standard E561-86. It is concluded that R-curves can be useful for characterizing the resistance of sheet material to stable crack tearing under an increasing load. Attention is paid to the question if the R-curve is independent of the initial crack length. It is pointed out that the application of R-curves to the prediction of residual strength of stiffened panels is questionable.

Contents

1 Introduction

Linear elastic fracture mechanics is a most convenient approach to the prediction of residual strength of aircraft structures with a fatigue crack. Unfortunately, it is unable to account for the fracture behaviour in residual strength tests on thin sheet material specimens, e.g. a central cracked sheet specimen. Most high strength materials show plastic deformation before fracture conditions are met, and in the plane stress condition stable tearing often precedes final specimen failure, see Fig.1. The R-curve (or the R-Δa curve) describes the crack growth resistance R as a function of the stable crack extension Δa under an increasing load on a central cracked specimen with an initial crack length $2a_0$.

The R-curve is supposed to be a material characteristic, which is independent of the initial crack length a_0. Because the crack growth resistance can not directly be measured, there is an obvious problem. It is solved by recognizing that the crack driving force, i.e. the strain energy release rate G, is balanced by the crack growth resistance.

In an R-curve test the crack extension Δa (stable tearing) is measured as a function of the applied gross stress. The crack driving force can then be calculated ($G = K^2/E$) as a function of Δa. It thus still employs the K factor. Since R = G during stable crack extension, the R-curve can then be obtained. Crack tip plasticity is accounted for by introducing an effective crack length, which is larger than the physical crack length ($a+\Delta a$). The procedures to be followed in R-curve measurements are described in the ASTM 561 standard [1]. In the present document we will discuss the stable crack growth resistance of a material as described by an R-curve.

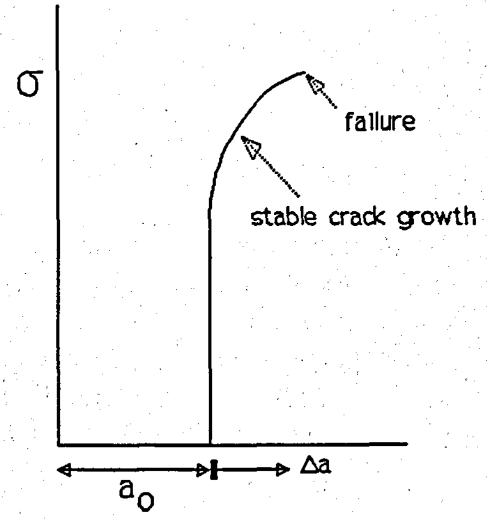

Fig.1 Stable crack growth in a residual strength test.

The energy balance concept is a fundamental aspect of the R-curve approach. It is explained in chapter 2, while some more basic aspects are considered in chapter 3. R-curve measurements procedures are discussed in chapter 4 in relation to the ASTM standard.

2 The energy balance concept

The definition of fracture toughness states:

The fracture toughness of a material is the highest stress intensity that can be supported by a cracked component made of that material.

In principle the toughness can be measured with any kind of a cracked specimen. When the geometry factor (ß) is known for the specimen and the fracture stress is measured, the critical stress intensity factor can be calculated: the result for this critical condition its the fracture toughness.

$$K_c = \beta \, \sigma_c \sqrt{\pi a_o} \tag{1}$$

Because instability in plane stress is often preceded by stable crack growth, a revised definition of toughness is needed. Before this revised definition is given, a relation between the toughness and K or the crack driving force (in the case of R-curves the energy release rate is adopted as the crack driving force) should be explained [2-5]. G is derived by using the energy balance method.

The Griffith concept

In order to evaluate the technical and physical meaning of the R-curve some basic aspects of the Griffith energy balance concept are summarized below. A specimen with a central crack is loaded in tension (Fig.2). The work done is stored as potential energy in the specimen. If there is some crack extension, it is also partly stored as surface energy of the newly formed crack surfaces. The energy balance is:

$$F = U + W \qquad (2)$$

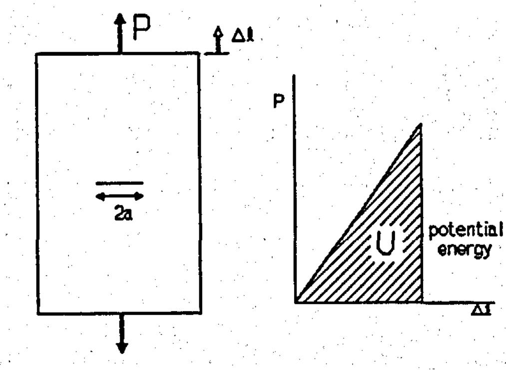

Fig.2 Stored potential energy in a cracked sheet under load.

where F is the work done, U is the potential energy, and W is the crack surface energy. The question now is whether an incremental crack extension da will occur. If it does happen, the energy balance requires:

$$dF = dU + dW \qquad (3)$$

or:

$$d(F - U) = dW \qquad (4)$$

In other words, the incremental work done and the change of the potential energy must provide the energy for an incremental crack extension.

Crack extension results in a lower stiffness of the specimen (an increase of the compliance). If the load remains constant (constant load case, see Fig.3), the specimen will become slightly longer by an amount $d\ell$. The work increment is $dF = P{\cdot}d\ell$. At the same time the potential energy has increased by an amount equal to the triangle ABC in Fig.3, which is $\frac{1}{2}\, dP{\cdot}\ell$. Thus equation (4) then becomes:

$$dW = P{\cdot}d\ell - \tfrac{1}{2}P{\cdot}d\ell = \tfrac{1}{2}P{\cdot}d\ell$$

If the crack extension occurs under fixed grip conditions, then P will slightly decrease, see Fig.3, but no work increment is involved, i.e. $dF = 0$. The potential energy increment is negative and equal to the triangle ABD in Fig.3. For an incremental $d\ell$, ABD = ABC an thus

$dU = -\frac{1}{2}P\cdot d\ell$. Substitution in Eq.(4) leads again to the same result:
$dW = 0 - (-\frac{1}{2}dP\cdot\ell) = \frac{1}{2}dP\cdot\ell$.

As pointed out before in relation to Eq.(4), d(F - U) is the energy available for a crack length increment da. If we consider constant-load conditions, it follows from the above relations that $d(F-U) = \frac{1}{2}P\cdot d\ell = dU$. The potential energy per unit length of crack extension is usually referred to as the *strain energy release* rate G, or:

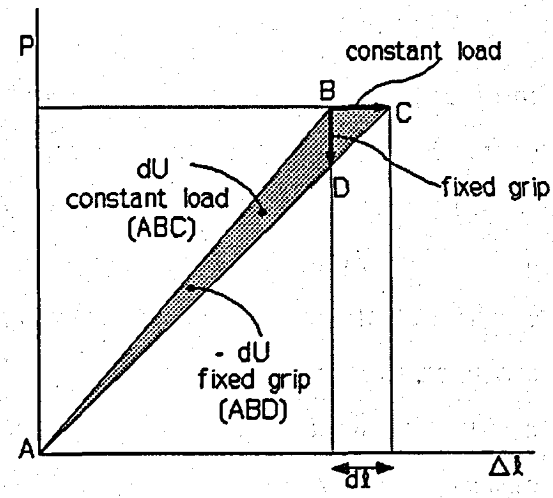

Fig.3 Incremental potential energies during an incremental crack extension.

$$G = \frac{dU}{da} \tag{5}$$

The energy required for crack extension per unit crack length extension is by definition the *crack growth resistance*:

$$R = \frac{dW}{da} \tag{6}$$

As long as $G < R$, crack growth will not occur. Crack extension can occur for $G = R$. As soon as $G > R$ it will occur in an unstable manner.

Based on the stress analysis of Inglis, it was proven by Griffith for an infinite plate under a remote tensile stress S that:

$$G = \frac{\pi a S^2}{E} \tag{7}$$

In Griffith's model the crack growth resistance was associated with creating new crack surfaces and the crack surface energy involved.

$$W = 2\, da\, \gamma \tag{8}$$

where γ is the surface energy per unit area. Eqs.(5) to (8) substituted in $G = R$ then leads to the critical stress:

$$S_c = \sqrt{\frac{2\gamma E}{\pi a}} \tag{9}$$

Griffith's Eq.(7) applies to an infinite plate. It may well be noted that the numerator is equal to K^2 of an infinite plate. The equation can thus be rewritten as:

$$G = \frac{dU}{da} = \frac{K^2}{E} \tag{10}$$

Irwin has shown that this equation is also valid for finite dimensions. For a central crack in a finite width panel it implies:

$$G = \frac{\beta^2 \pi a S^2}{E} \tag{11}$$

where ß is the width geometry factor (e.g. the Feddersen correction).
Equation (9) can be rewritten as:

$$K_c = \sqrt{2\gamma E} \tag{12}$$

Even for low-ductility, high-strength sheet materials, this equation, based on the Griffith concept, underestimates the residual strength considerably. Crack extension will be preceded by some crack tip plastic deformation. The energy needed for plastic deformation is some orders of magnitude larger than the surface energy of the material. The surface energy is negligible as compared to the energy for (limited) plastic deformation along the fracture surface. If the energy for the plastic deformation, indicated by γ_{pl}, would be a constant, Eqs.(8) and (9) can still be used after replacing γ by γ_{pl}. Combining Eqs.(6) and (8) to:

$$R = \frac{dW}{da} = 2\gamma_{pl} \tag{13}$$

it implies that the crack growth resistance is still supposed to be constant. The consequences are considered in Fig.4, where the strain energy release rate G and the crack growth resistance R are plotted vertically as a function of the crack length a. The initial crack length is $2a_0$ and the crack extension is Δa. G(a) is a continuous line (Eq.11), but its position is depending on the stress level S. Three lines for increasing S-values are shown in Fig.4. The crack growth resistance R is supposed to be constant. Loading the specimen will raise the G(a) curve. However, initially G will be lower than R, e.g. in points A and B. The stress must be increased until the G(a) curve passes through point C. Then G = R and incremental crack extension is possible. If the load (stress S) in maintained, it implies that G(a) will increase along the monotonously raising curve through point C, whereas the crack growth resistance will stick to the constant value R. In other words:

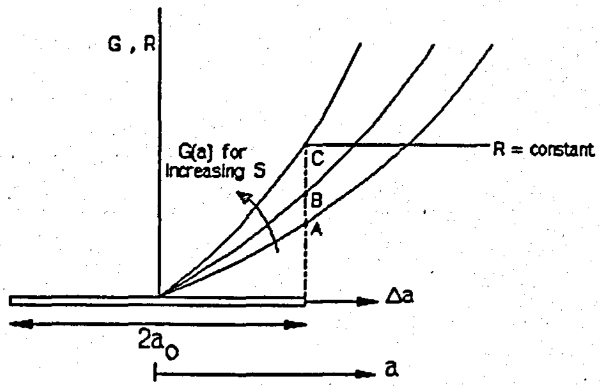

Fig.4 Increasing crack driving force (G) compared to a constant crack growth resistance R.

$$G > R$$

and unstable crack extension will occur immediately. However, in thin sheet material, where plane stress conditions prevail around the crack tip, this does not occur. Some crack extension causes simultaneously a slightly larger plastic zone. It implies that R is not a constant, it increases for increasing Δa. There is a increasing crack growth resistance. The stress must be increased to continue further crack extension, which initially will occur in a stable way. The value of R can not be measured, but it can be obtained from the energy balance:

$$G = R$$

The strain energy release rate G must be calculated. The calculation of G is addressed later. Here we conclude that R is a function of the crack extension Δa. An R-Δa curve is schematically indicated in Fig.5, again with some G(a) curves.

During increasing of the load on the central cracked panel, the G(a) curve will shift upwards. When passing points A and B there is still a balance between G and R, but contrary to the situation in Fig.4, there is some stable crack extension Δa. The balance can be maintained until the G(a) curve becomes tangent to R(Δa) in point C. After passing this point, G > R and further crack extension will be unstable.

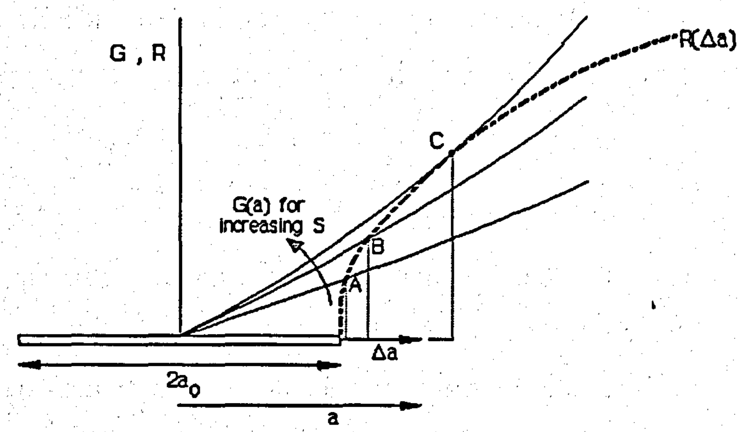

Fig.5 Increasing crack driving force compared to an R-Δa curve. Instability at point C.

In the literature [2,5] it is assumed that the crack growth resistance curve, R(Δa), of an unstiffened sheet is independent of the initial crack length a_0. This assumption is discussed in the next section, but it may already be concluded here that R(Δa) can be used for two purposes:

- To compare the crack growth resistance of different materials with respect to a monotonously increasing load.
- To predict the residual strength by calculating the stress for which G(a) becomes tangent to R(Δa).

3 Some aspects of the R-curve

R-curves are presumed to be a material characteristic independent of the specimen width and the initial crack length. If this is correct, one test should be sufficient to generate the R-curve of a material. The R-curve can then be used to predict residual strength of any specimen of that material for any width and initial crack length.

3.1 Initial crack length

The question whether the R-curve is independent of the initial crack length, is discussed by Broek [2]. He points out, "... there is some experimental evidence that the hypothesis is useful, and some arguments can be given to make it plausible...". Recently, Anderson [5] states that, "... there is no guarantee that an R-curve produced according to this standard (i.e. the ASTM standard) will be a geometry independent material property ...", and "... the in-plane dimensions must be large compared to the plastic zone in order for LEFM to be valid. Also the growing crack must be remote from all external boundaries ...".

Let us now consider a crack in an infinite sheet, see Figure 6. If the first crack extension occurs at a certain K-value with a certain size of the plastic zone. It is reasonable that this initial crack extension is a K-controlled phenomenon, independent of the initial crack length a_0. If the load is increased, the plastic zone will become larger, because of the higher load, and also due to the larger crack length (a_0 + Δa). The plastic zone will also be shifted to a larger x-value, see Fig.6. Again another crack increment will be associated with the migrating plastic zone during the increasing load. This will be continued during further load

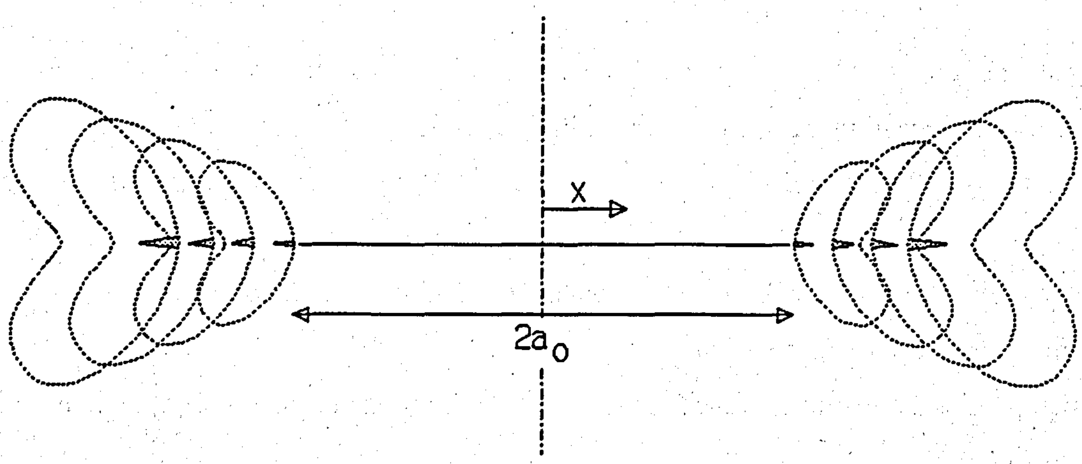

Fig.6 Crack extension in an infinite plate during an increasing load. Migrating plastic zones.

increase. If the crack extension is a plastic deformation controlled mechanism, it is perfectly reasonable that exactly the same crack growth process occurs for any other value of the initial crack length. It is even not obvious that the small-scale yielding condition should be applicable. However, the shape of the plastic zone should not be altered by specimen edge effects. This does not occur in an infinite sheet. However, there is still a possible effect on the shape of the plastic zone for very small initial cracks. A mutual interference of the plastic zones of the two nearby crack tips must be expected. It can no longer be assumed that the same plastic zone shape will occur as for large initial cracks. In a finite width specimen, disturbing edge effects can occur if $2a_0/W$ is large, especially in highly ductile sheet materials. Obviously, it should be recommended to carry out R-curve tests on large specimens, and to avoid too small initial crack lengths. Under such conditions, it appears that R-curves can represent a material characteristic for stable crack growth resistance. The R-curve thus could be relevant to damage tolerance properties of aircraft structures.

3.2 Calculations of the residual strength with an R-curve

If the invariant R-curve is available, the procedure to calculate the residual strength for an arbitrary a_0 and W can easily be explained by referring to Figure 5. For an increasing σ the $G(\Delta a)$ curve moves upward. In order to maintain the energy balance $G = R$, stable crack extension does occur until the G(a) curve becomes tangent to the R curve in C ($dG/da = dR/da$). For a further increase of σ the energy balance can no longer be maintained, because then $G > R$, and unstable failure does occur.

A mathematical solution is possible if the R-curve is represented by a power function as already proposed by Broek [2].

$$R = \alpha\ (\Delta a)^n \tag{14}$$

where α and n ($n < 1$) are obtained by fitting the equation to the experimental R-curve. The energy release rate G is given as:

$$G = \frac{\beta^2\, \sigma^2\, \pi\, a}{E} \tag{15}$$

with $a = a_0 + \Delta a$. The value of the failure stress σ in point C can now be obtained from the two conditions:

$$G = R : \quad \frac{ß^2 \sigma^2 \pi a}{E} = \alpha\ (a - a_o)^n \tag{16}$$

$$\frac{dG}{da} = \frac{dR}{da} : \quad \frac{\pi\, \sigma^2}{E}\ (ß^2 + 2\, a\, ß \frac{dß}{da}) = \alpha\, n\, (a - a_o)^{n-1} \tag{17}$$

For an infinite plate $ß = 1$ and then Δa and σ are easily solved from equations 16 and 17. The result for the amount of stable crack extension Δa then is:

$$\delta a = \frac{n\, a_o}{1 - n} \quad \text{with} \quad 0 < n < 1 \tag{18}$$

Actually Broek adopted this equation based on experimental observations, and from the equation he derived the power relation of Equation 14.
The corresponding stress at the moment of failure is:

$$\sigma = \sqrt{\frac{E\, \alpha}{\pi}\, n \left(\frac{n\, a_o}{1 - n}\right)^{n-1}} \tag{19}$$

which is obviously depending on the initial crack length only, as should be expected for an infinite sheet.
For a finite width sheet ($ß \geq 1$) equations 16 and 17 with the Feddersen width correction factor lead to:

$$\frac{a - a_o}{a}\left(1 + \frac{\pi a}{W}\,tg\,\frac{\pi a}{W}\right) = n \tag{20}$$

The crack length a must be solved iteratively from this equation. Substitution in Equation 4 then gives the failure stress:

$$\sigma = \sqrt{\frac{E\alpha}{\pi}\cos\left(\frac{\pi a}{W}\right)\frac{(a - a_o)^n}{a}} \tag{21}$$

4 R-curve determination

4.1 Determination of the effective crack length

The increasing crack growth resistance during stable crack extension, described by the R-curve is closely related to the plastic deformation in the crack tip plastic zone. When the plastic zone becomes larger, the crack starts to behave differently from a crack in a purely elastic material. Plasticity makes a crack behave as if it were longer than its actual physical length, and it thus will show a larger crack opening. The apparently longer crack length is called the *effective crack length*, indicated as a_{eff}. The R-curve is based on this effective crack length. During a residual strength test, the load (P) and the physical crack length (a) are recorded. At the same time the Crack Opening Displacement (COD) at the center line may also be recorded. The physical crack length can be measured visibly by eye, or with a microscope, or by using the electrical potential drop method.

The assessment of the effective crack length is not such a simple problem. The effective crack length has to be defined first. The ASTM E 561 [1] allows two ways to determine the value of a_{eff}, which correspond to two different definitions:

(1) The effective crack length is a fictitious crack length, which gives the same COD at the central line (x=0) as a real crack would do in a fully elastic material.

(2) The effective crack length is a fictitious crack length, obtained from the physical crack length enlarged with a plastic zone size correction.

It is obvious for both definitions that $a_{eff} > a$. The first definition requires a COD value measured in the residual strength tests. The ASTM standard gives two different methods to derive a_{eff} from a COD measurement, which imply to use either a calibration curve or an analytical relation for the a_{eff}-COD relation. The second method is based on a plastic zone correction of a similar type known as the Irwin plastic zone correction, which is used for the determination of K_{Ic}. However, it should be recognized that plane stress conditions prevail in sheet material. Both methods are discussed in more detail below.

4.2 The a_{eff}-COD relation

According to the ASTM standard we have to find a crack length (a_{eff}) of an "elastic" crack, for which the crack opening displacement v at the center of the crack will be the same as for the real crack, if the same stress σ is applied. In other words, the fictitious crack with a length a_{eff} in an elastic sheet should have the same *crack opening compliance* as the real crack with the real length a_{phys} in the elasto-plastic case. The crack length a_{eff} can then be derived from a crack opening compliance measurement, provided that the elastic compliance as a function of crack length is available. This compliance can be obtained in so-called calibration tests, or by the Eftis-Liebowitz equation. In the ASTM standard the normalized compliance $Ev/\sigma W$ is used. It is plotted as a function of 2a/W, see Fig.7. For each value of the measured compliance the effective crack length can then be read from such a graph.

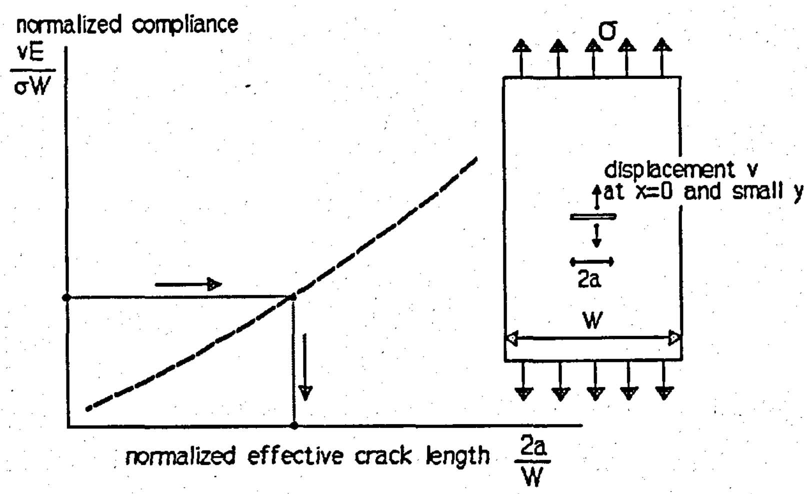

Fig.7 The normalized crack opening compliance, assuming elastic behaviour.

Compliance calibration

The compliance curve of Fig.7 can be obtained by an experimental calibration. It implies that tests have to be performed for different crack lengths. It can be done on one single specimen by increasing the crack length in steps and repeating the compliance measurement. Usually, it is done by introducing a saw cut simulation of the crack. It is easily extended by further sawing. High loads can not be applied, because crack tip plasticity must remain negligible. Obviously, a calibration curve must be obtained for each W-value of interest.

Analytical compliance

A compliance equation proposed by Eftis and Liebowitz [6] is also acceptable according to the ASTM standard. The equation is fairly complex:

$$\frac{v(0,y)\,E}{W\sigma} = \sqrt{\frac{\frac{\pi a}{W}}{\sin\frac{\pi a}{W}}}\left[\frac{2}{\pi}\cosh^{-1}\left(\frac{\cosh\frac{\pi y}{W}}{\cos\frac{\pi a}{W}}\right) - \frac{1+\nu}{W}\,y\left\{1+\left(\frac{\sin\frac{\pi a}{W}}{\sin\frac{\pi y}{W}}\right)^2\right\}^{-1/2} + \nu\,\frac{y}{W}\right] \tag{22}$$

The COD measured is 2v, while y in Eq.(22) corresponds to half the gage length of the COD meter. It should be pointed out that the equation is not an exact solution. An exact solution is not available, but Eq(22) may be supposed to be rather accurate [7]. Obviously, after a compliance is measured, a_{eff} should be solved from equation (22) by an iterative calculation.

Physical crack length plus a plastic zone size correction

Instead of the compliance measurement, a_{eff} can also be obtained from the physically observed crack length, corrected for a plastic zone size effect. This method is based on Irwin's assessment of the plastic zone size r_p with the relation $r_p = 2\,r_y$, where r_y is computed for plane stress conditions. It leads to the well known correction:

$$a_{eff} = a_{phys} + r_y \qquad \text{with} \quad r_y = \frac{1}{2\pi}\left(\frac{K}{\sigma_Y}\right)^2 \tag{23}$$

The ASTM standard states that the formula is most accurate for high strength materials of yield strength to density ratios above 174 kPa/kg.m^3. This criterion is not met by 2024-T3 and barely met by 7075-T6. However, there are several reasons why this a_{eff} procedure should be cautiously considered. The r_y value obtained with Eq.(23) is a rough estimate of the plastic zone size, disregarding the complex shape of the plastic zone, and assuming a simple ideal elasto-plastic material behaviour.

4.3 Evaluation

As discussed before, different methods can be used to obtain Δa_{eff}. They do not necessarily lead to the same Δa_{eff}, which seems to be an unsatisfactory situation. The disadvantage is less important if the same method is used for the determination of the R-curve as well as for the application of this curve.

There is a practical argument to prefer the compliance approach. The plastic zone correction method requires that $a_{physical}$ is measured during stable crack growth. Unfortunately, it can be difficult to make accurate observations on the location of the moving crack tip. This problem is circumvented by the COD-measurements, which can be made continuously during stable crack growth. If this is done, a choice to be made is to use either a calibration curve or the Eftis/Liebowitz equation. The calibration curve must be determined experimentally, whereas the Eftis/Liebowitz equation is available. An obvious question is whether they give the same result. Comparisons were collected in [7] as found in three references [8-10]. A summary drawn from [7] is given in the table below.

Reference	Material	W (mm)	t (mm)	E (MPa)	v	COD gage length (mm)
[8]	7075-T6	228.6	3.2	71800	1/3	50.8
[9]	probably Al-alloy	300	2.0	68670 (?)	?	6.5
[10]	2024-T3 2091-T84 8090-T81	500	1.6 1.6 1.6	72000 78000 82000	0.33 0.33 0.33	8.0

Table 4: Tests series with a comparison between the empirical and calculated compliances.

The following text comes from [7]:
"The measured compliances of Ref.[8] in the range of 2a/W from 0.2 to 0.6 were about 5% higher than the value calculated with the Eftis/Liebowitz equation. In Ref.[9] the experimental values were also slightly above the calculated results. A percentage can not realistically be derived from the graph in [9]. However, the E-modulus of 68670 MPa (probably measured in a tensile test) can not be an accurate value for 2024-T3 or 7075-T6, for which 72000 MPa (5% higher) is more realistic.

A fairly extensive check on the Eftis/Liebowitz equation was made by t'Hart and Schra [10] on three Al-alloys, including two Al-Li alloys. They performed tests on specimens with saw cuts, as well as specimens with cracks extended by stable crack growth. They found that the empirical compliance on the average was 11% higher than the calculated value. They did not find differences between compliances of specimens with saw cuts and specimens with real cracks.

The practical solution of the above discrepancy can be found in using a COD-a curve obtained by calibration tests." Another conclusion from [7]: "For the time being it appears that an experimental compliance calibration should be carried out in each test program on R-curves."

4.4 Crack edge buckling

In a residual strength test on a sheet specimen with a central crack, crack edge buckling can be observed as a consequence of the compression stresses along the crack edges. According to the ASTM standard E561, crack edge buckling should be restrained, which leads to a higher residual strength as compared to the results of non-crack edge buckling restraint experiments. The effect on the R-curve is investigated in an extensive investigation on sheet specimens of Al-alloys and fiber-metal laminates, which will be published shortly [11]. It will not be discussed any further here. However, it may be stressed that residual strength of realistic aircraft structure is much more complex phenomenon than in an unstiffened panel.

5. Discussion and some conclusions

1. The discussion in the previous chapters has shown that the determination of the R-curve is affected by some elementary problems. This occurs as a consequence of using concepts of Linear Elastic Fracture Mechanics. As should be expected, the relevance of these concepts becomes marginal for residual strength predictions for sheet materials, where significant plasticity does occur. In spite of this conclusion, it appears that the R-Δa_{eff} curve can still be considered as being characteristic for the material resistance to stable crack extension under a monotonously increasing load. The curve can thus be relevant for considerations on the potential of a material with respect to damage tolerance of an aircraft structure. It implies that R-curves can be useful for material

comparison and selection.

2. The basis for assuming that the R-curve is independent of the initial crack length has often been ignored in the literature. The present discussion based on Fig.6 indicates that the R-curve could be independent of the initial crack length, provided that large specimens are used to avoid edge effects. Moreover, small initial cracks should not be used, because of possible interference between the plastic zones of the two nearby crack tips.

3. The application of the R-curve to the prediction of the residual strength of a stiffened panel is a questionable approach for fundamental arguments. Stable crack extension in an R-curve test is a consequence of crack tip plastic deformation. It leads to a moving crack tip plastic zone during the stable crack extension. The development of the moving plastic zones is not constrained by stiffening elements on the sheet. Because such a constraint is present in a stiffened panels, the similarity between the failure processes in an unstiffened and a stiffened panel is no longer valid.

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