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Citation (APA)
van der Hulst, M., Gonzalez, R. A., Classens, K., Tacx, P., Dirkx, N., van de Wijdeven, J., & Oomen, T. A. E. (2025). Modal Identification for Multivariable Motion Systems: Applied to a PrototypeWafer-Stage. In R. Carloni, J. Alonso-Mora, J. Dasdemir, & E. Lefeber (Eds.), *Book of Abstracts 44th Benelux Meeting on Systems and Control* (pp. 138-138). Rijksuniversiteit Groningen.

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

44th Benelux Meeting

on

Systems and Control

March 18 − 20, 2025

Egmond aan Zee, The Netherlands

Book of Abstracts

The 44th Benelux Meeting on Systems and Control is sponsored by

Raffaella Carloni, Javier Alonso-Mora, Janset Dasdemir, and Erjen Lefeber (Eds.)
Book of Abstracts - 44th Benelux Meeting on Systems and Control

Modal Identification for Multivariable Motion Systems: Applied to a Prototype Wafer-Stage

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1 Background

Accurate modal identification of multi-input multi-output (MIMO) models is vital for driving the future development of advanced mechatronic motion systems. These modal models offer interpretable, minimal-order representations [1] that enable effective control, provide design validation, and facilitate monitoring and diagnostics of machine dynamics.

2 Problem formulation

Mechanical systems can be modeled in a modal framework [2], where the system consists of rigid-body modes and flexible dynamic modes

$$\mathbf{G}(s,\rho) = \sum_{i=1}^{n_{\text{rbm}}} \frac{\phi_{l,i}\phi_{r,i}^{\top}}{s^2} + \sum_{i=1}^{n_{\text{flex}}} \frac{\phi_{l,i}\phi_{r,i}^{\top}}{s^2+2\zeta_i\omega_i s+\omega_i^2}, \quad (1)$$

where the modal parameters are the resonance frequencies ω_i, the corresponding damping ratios ζ_i, and the left and right mode-shape vectors $\phi_{l,i}$, $\phi_{r,i}$. The modal parameters are jointly stored in the parameter vector ρ. The problem considered is to estimate modal models as described by (1) from data.

3 Approach

A two-stage approach is presented for estimating the modal parameters. First, an additive model is estimated, described by

$$\mathbf{P}(s,\beta) = \frac{\mathbf{M}}{s^2} + \sum_{i=1}^{K} \frac{\mathbf{R}_i}{a_{i,2}s^2+a_{i,1}s+1}, \quad (2)$$

with the parameters stored in the vector β and where the numerator terms $\mathbf{M}$ and $\mathbf{R}_i$ are full rank matrices. This model is obtained by minimizing the weighted least-squares criterion

$$\hat{\beta} = \arg\min_{\beta} \sum_{k=1}^{N} \|\mathbf{G}(\omega_k) - \mathbf{P}(j\omega_k,\beta)\|_{\mathrm{F}}^2, \quad (3)$$

with $\| \cdot \|_{\mathrm{F}}$ the Frobenius norm, for a given dataset of noisy FRF measurements $\mathbf{G}(\omega_k)$. The problem (3) is a non-linear and non-convex optimization problem, which is solved using a novel refined instrumental variable method for additive

systems. In the second stage, a modal model is obtained by optimally reducing the estimated full-rank $\hat{\mathbf{M}}$ and $\hat{\mathbf{R}}_i$ to rank-one matrices, enabling the extraction of the mode-shape vectors. Given a solution $\hat{\beta}$ from stage 1, along with the corresponding covariance $\Sigma_{\hat{\beta}}$, the parameter vector estimate $\hat{\rho}$ of the modal model is found by minimizing

$$\hat{\rho} = \arg\min_{\rho}\|\hat{\beta} - \mathbf{f}(\rho)\|_{\Sigma_{\hat{\beta}}^{-1}}^2, \quad (4)$$

where $\mathbf{f}: \rho \mapsto \beta$ represents the mapping from the modal parameter vector to the additive parameter vector.

4 Results

The developed identification method is experimentally validated on a prototype wafer-stage setup, which features four sensors and thirteen actuators. A 40th-order model is estimated of the 4×13 plant consisting of three rigid-body modes and seventeen proportionally damped flexible modes. A subset of the identified modal model is presented in Figure 1, demonstrating close alignment with the FRF measurement across the considered frequency range.
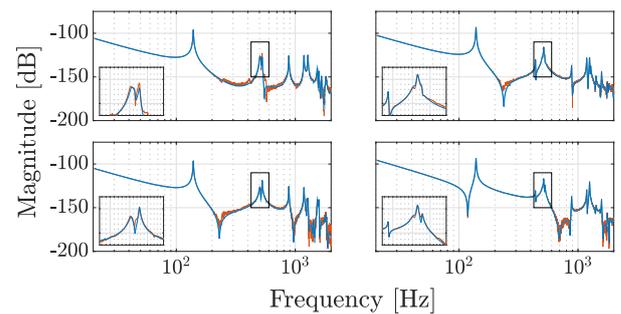

Figure 1: Bode magnitude plot of a subset of the plant with the FRF (——) and identified modal model, described by the eigenfrequencies, damping-ratios, and mode shapes (——).

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