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COMPARISON OF STATIONARY AND OSCILLATORY FLOW THROUGH POROUS MEDIA

by

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Chapter 2

Porous Media Flow

2.0 General

Over the past 150 years or so, investigations into porous media flow have yielded a great deal of understanding of the phenomenon. Much of the work has been empirical in nature, similar to the well-known contribution of Darcy (1856). Such investigations have identified the parameters relevant to the phenomenon and have provided some useful relationships between them. The majority of the analytical work has been directed towards deriving such empirical relationships from the equations of motion and continuity (with appropriate simplifications and approximations), thereby isolating the effects of, and the relative importance of, individual terms. The final goal of this research is of course to produce a reliable predictive description of the phenomenon of flow through coarse granular media.

It is therefore useful to review previous theoretical and empirical results in order to better understand the nature of the present investigation. This chapter begins with a general overview of the concepts of porous flow followed by a brief review of Darcy's law, including a description of the parameters comprising this law, its upper limit of validity and the reasons for its failure after this limit. Next, flow laws governing stationary non-Darcy (or post-linear) steady flow are described and, finally, the few previous investigations into unsteady flow are reviewed and the goals of the present study are highlighted.

2.1 Overview of the Phenomenon

A porous medium acts as a resistance to flow; it is the goal of research to describe the form of the resistance coefficient. Fluid particles flowing through the pore spaces pass through expansions and constrictions to the flow and experience other convectional inertia effects caused by the curvilinear flow paths. Given that resistance coefficients for pipe flow through expansions, contractions, etc., are determined by empirical means, the complexity of defining resistance coefficients for a generalized, random porous medium is readily apparent.

A porous medium is generally visualized as a continuum having properties of dimension and porosity (Shih, 1990). The permeability of the porous medium is usually described in terms of directly measurable quantities, most commonly the porosity, and a large body of work has been (and is still being) directed towards relating permeability and porosity. The per.neability is, however, obviously dependent upon other properties including particle size, shape, orientation and surface roughness.

Analytically, the continuum approach requires averaging of the terms in the equations of motion and continuity, as these quantities cannot be used directly owing to the complex boundary conditions of flow through the pore spaces of the medium. Thus, the properties of velocity and pressure must be averaged over a volume which is large enough for the averaging procedure to be valid and yet small enough so as to be considered infinitesimal with respect to the total sample volume (Scheidegger, 1960; Bear, 1972; Le Mehaute, 1976). This requires that the magnitude of the flow be much greater than the pore volume. Therefore, flow through large pores (or past large obstructions), such as waves passing through the armour layer of a breakwater, cannot be validly described by this approach. Gray and O'Neill (1976) described such a technique of "local averaging" to obtain generalized porous flow equations and Le Mehaute (1976) illustrated how such an averaging of the terms in the Navier-Stokes' equations can result in Darcy's law.

2.2 Darcy's Law and Laminar Flow

Darcy's experiments (1856) yielded the results that, over a limited range of flowrates, Q,

$$Q = KA_T \frac{(h_2 - h_1)}{l} \qquad [2.1]$$

where A_T is the cross-sectional flow area, l is the length of the sample, h_1 and h_2 are the piezometric heads at locations 1 and 2 at elevations z (ie $h_i = p_i/\rho g + z_i$), ρ is the density of water, g is the acceleration due to gravity and K is a constant of proportionality which Darcy called the permeability of the material.

Expressing (2.1) in terms of pressure and noting that the average or "bulk" or "superficial" velocity is $q = Q/A_T$, Darcy's law can be written as

$$q = K \cdot \nabla\left(\frac{p}{\rho g} + z\right) = Ki \qquad [2.2]$$

∇ is the gradient operator, i is the slope of the energy grade line (i = dh/dx), commonly termed the hydraulic gradient, ρ is the fluid density, g the acceleration due to gravity and the permeability, K, is a function of the fluid and the porous medium; these two aspects can be separated yielding

$$k = \frac{\mu}{\rho g} K \qquad [2.3]$$

where k is defined as the intrinsic permeability of the material (because it depends only on properties of the material) and has dimensions of $(\text{Length})^2$ and μ is the dynamic viscosity of the fluid.

Hence, Darcy's law states that the energy loss across a porous medium due to friction (μ) is directly proportional to the averaged or "bulk" velocity. However, this law applies only to a limited range of flowrates where effects of inertia are negligible compared to the those due to viscous forces (Wright, 1968; Scheidegger, 1960; Philip, 1970; Dybbs and Edwards, 1982).

For a generalized flow equation, the force balance must include not only the frictional terms but also the effects of inertia and gravity so that, for steady flow (Le Mehaute, 1976)

$$\text{spatial mean value of} \left\{ \begin{matrix}\text{Inertia} \\ \text{force}\end{matrix} + \begin{matrix}\text{Gravity} \\ \text{force}\end{matrix} + \begin{matrix}\text{Pressure} \\ \text{force}\end{matrix} + \begin{matrix}\text{Friction} \\ \text{force}\end{matrix} \right\} = 0 \qquad [2.4]$$

Recently, Gu and Wang (1990) have examined the relative importance of different resistance components due to viscosity, inertia and turbulence for the particular case of gravity waves over porous bottoms. This treatment will be discussed in section 2.3.

The relation (2.4) above can be formulated by the general Navier-Stokes' equation

$$\frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \vec{\nabla})\vec{U} = -\vec{\nabla}\left(\frac{p}{\rho} + \vec{F}\right) + \nu \nabla^2 \vec{U} \qquad [2.5]$$

where $\vec{U}$ is the velocity vector, $\vec{F}$ is an externally applied force (such as gravity), ν is the fluid's kinematic viscosity and t represents time. The two terms on the left hand side represent the temporal and convectional inertia, the first term on the right hand side represents both the pressure and external (ie. gravity) components and the last term is the component due to friction. Therefore, neglecting the effects of inertia, the solution to this equation, averaged over a suitable volume element $\Delta\forall$, becomes (Le Mehaute, 1976)

$$\frac{1}{\Delta\forall}\iiint_{\Delta\forall}(-\nabla(p + \rho g z) + \mu \nabla^2 \vec{U}) d\forall = 0 \qquad [2.6]$$

The integral of the first term becomes

$$-\nabla(\bar{p} + \rho g z) \qquad [2.7]$$

where $\bar{p}$ is the locally averaged pore pressure, and it is seen that the equation becomes identical to Darcy's law if the second term comprising the viscous forces is proportional to the average velocity, $\bar{v}$, as

$$\frac{1}{\Delta\forall}\iiint_{\Delta\forall} \mu \nabla^2 \vec{U} d\forall \sim \bar{v} \qquad [2.8]$$

and then

$$\bar{v} = K\nabla(\bar{p} + \rho g z) = K\nabla H \equiv q \qquad [2.9]$$

where K is the constant of proportionality and H is the total pressure head, $H = \bar{p} + \rho g z$. Darcy's law is therefore seen to be a solution to the Navier - Stokes' equation if inertial effects are neglected and the terms of pressure and velocity are averaged over a suitable elemental volume. This realization has prompted researchers to adopt concepts from traditional fluid mechanics into the field of porous media flow. Such concepts must be modified to account for the nature of a porous medium, which is most commonly defined by its permeability.

2.2.1 Permeability and Porosity

The intrinsic permeability term k in Darcy's law (equation 2.3) is dependent upon the configuration of the granular matrix; the size and shape of the particles, their size distribution and orientation and the porosity of the sample. Owing to the fact that these properties are difficult to measure and control in laboratory and prototype, most effort has been concentrated on the last factor, porosity, as a descriptor of the permeability of the sample. In addition, most studies assume an isotropic porous medium so that k is a constant; for anisotropic media k is a tensor quantity.

Early analytical attempts at defining the permeability led to the development of models that represented the porous medium as a series of capillary tubes, and common pipe flow laws were applied to describe the hydraulics of the system. Scheidegger (1960) gives a good review of these models. Most are based upon the Hagen-Poiseuille equation for laminar flow in straight, circular pipes, (which is also a solution to the Navier-Stokes' equation for these specific boundary conditions) ie.

$$\frac{\partial p}{\partial x} = -32\frac{\mu v_p}{D^2} \qquad [2.10]$$

For porous media flow v_p is replaced by the bulk velocity q and the diameter D is usually replaced by a typical grain diameter, D_{50} for example, because of the difficulty in assigning a typical pore size. This approach is limited because the nature of flow through a porous medium is quite different from that in pipes. At Reynolds' numbers of about 1, convective acceleration terms in the Navier-Stokes equation can no longer be neglected as the fluid must follow curvilinear paths through the granular matrix (Scheidegger 1960; Wright 1968; Dudgeon 1964; Philip 1970). When applied to flow in straight, circular pipes, equation [2.10] is an exact linear equation (non-linear convective terms are identical to zero) which applies to a specific geometry and breaks down suddenly at the onset of turbulence, commencing at Reynolds' numbers of about 2000 (Streeter and Wylie, 1981). In a porous medium, however, the linear flow law (Darcy's law) is conditional upon the non-linear convective terms being "small", ie. at Reynolds' numbers less than about 1 to 10 (Philip 1970; Dybbs and Edwards 1982; Yalin and Franke, 1961). This fundamental difference between Poiseuille's law and Darcy's law is often ignored, with only the coincidental similarity of the hydraulic gradient being proportional to the bulk velocity being considered.

Wright (1968) gave a good qualitative review of the physical processes involved in the failure of Darcy's law. He noted similarities, for the non-linear behaviour of the porous media flow, with flow through pipe coils and in flow past fixed obstructions. In these cases a gradual transition from the linear law is observed as the relative importance of convective inertia increases.

One application of pipe flow analogies to a porous medium is the series of models named Hydraulic Radius theories that make use of the fact that the intrinsic permeability has dimensions of L^2 and is described by a length term, the hydraulic radius, R. The hydraulic radius is defined as the ratio of sample volume to the surface area of the pores (again, difficult to assess for most porous media). The basic form of the permeability relation is

$$k = c\frac{R^2}{\phi(n)} \qquad [2.11]$$

where c is a dimensionless constant and $\phi(n)$ represents some function of the porosity. Of the hydraulic radius theories, that of Kozeny (1923) is the most widely used description of permeability.

Kozeny's theory couples the steady-state Navier-Stokes equation (neglecting inertia terms) with Darcy's law and describes the permeability as

$$k = \frac{cn^3}{S^2} \qquad [2.12]$$

where c is a dimensionless shape factor, analytically derived to be approximately 0.5, and S is the specific surface of the flow tube - a measure of the hydraulic radius. Equation [2.12] is known as the Kozeny equation and has been modified in many ways by different researchers to obtain better fits to experimental results. Sometimes a "Tortuosity" term is introduced so that

$$k = \frac{cn^3}{T_r S^2} \qquad [2.13]$$

where the tortuosity, T_r, accounts for differences in observed and theoretical pressure gradients arising from the curvilinear flow paths. Another common modification to equation [2.12] is that proposed by Carman (1937), resulting in the Kozeny-Carman equation,

$$k = \frac{n^3}{5S_o^2(1-n)^2} \qquad [2.14]$$

where S_o is Carman's specific surface and the empirically determined factor of 1/5 replaces Kozeny's term c=1/2 to give a better fit to the data. The term $(1\text{-}n)^2 / n^3 = \phi(n)$ is a porosity function which differs from that of Kozeny given in equation [2.12]. A list of some of the various porosity functions proposed over the years is given below:

$\phi(n)$	=	$1/n^3$	Kozeny (1927)	
		$(1-n)^2/n^3$	Carman (1937)	
		$(1-n)/n^2$	Rumer and Drinker (1966)	
		$1/n^{3.3}$	Slichter (1897)	[2.15]
		$1/n^5$, $1/n^6$	Mavis & Wilsey (1936)	
		$1/n\phi'^{(n)}$	Rose (1945)	
		$(1-n)^3/n^3$	Chardabellas (1940)	

All of these expressions are extremely sensitive to the porosity value, which is unfortunate for practical usage where the accuracy of permeability estimates are in the order of 5 to 10%. Even in laboratory conditions, Dudgeon (1964) described differences in porosity measurements on a given sample depending on the measurement technique used. Variations of 10% were commonly exhibited. A discussion on how these functions were obtained is included in section 2.3.4.

In a slightly different approach to the analytical description of permeability, Iberall (1950) used the Stokes' flow analogy and, similar to Rumer and Drinker (1966), assumed the total resistance of the porous medium to be the sum of resistances offered by all individual particles. Drag forces were estimated by the Navier-Stokes' equation neglecting inertia terms and any effects of neighbouring particles on flow around the obstructions. Iberall's model schematized the porous medium as a number of cylindrical fibres oriented in 3 orthogonal directions (one of which was parallel to the flow) and used the expressions of drag force on a cylinder in an unbounded fluid, coupled Darcy's law to obtain

$$k = \frac{3}{16}\left(\frac{nD^2}{1-n}\right)\left(\frac{2-\ln(Dq\rho/\mu n)}{4-\ln(Dq\rho/\mu n)}\right) \qquad [2.16]$$

where D is the fibre diameter (constant). Here, it is apparent that permeability is a function of flow rate, q, and is therefore not a constant for a given material. This is contrary to most other theories which describe the permeability as a constant value for a given (isotropic) porous medium.

2.2.2 Comparison between Darcy's Law and Stokes' Law

Similarities between Darcy's law and Stokes' law for flow past a single obstruction have been noted by many researchers (Wright, 1968; Scheidegger, 1961; Philip, 1970). Both are solutions to the Navier-Stokes' equation for an incompressible fluid and negligible inertia. Both Darcy's and Stokes' laws fail when the effects of inertia cease to be small; this occurs when the Reynolds' number ($q \cdot D/\nu$, with D chosen as some characteristic grain diameter) attains values of about 1 and the departure from the linear law is a gradual one. The connection can be illustrated by the use of dimensional analysis as follows:

Figure 2.1(a) and (b) shows, graphically, the flow patterns being compared. Figure 2.1(a) is a homogeneous, isotropic porous medium comprised of equal sized spheres of diameter D (which is small compared to the container dimension H) with porosity n. A uniform steady laminar flow of magnitude v_p is passing through the medium. Figure 2.1(b) shows a single circular cylinder of diameter D in an unbounded laminar flow of magnitude U.

For the porous medium, the phenomenon can be completely described by the characteristic parameters (D,n,q) along with the fluid viscosity and density (Yalin and Franke, 1961). Expressing these parameters in terms of dimensionless variables (by using the "Buckingham-Pi Theorem", (Yalin, 1971), and choosing basic quantities D,q and ρ), any given dimensionless property, Y_a, can be described by the relation

$$Y_a = \phi_a\left(\frac{qD}{\nu}, n\right) \qquad [2.17]$$

where ϕ_a represents some function related to the dimensional property "a" and the geometry of the medium under consideration. For the case of filtration flow the velocity heads are neglected as $q^2/2g \rightarrow 0$. Although this is not the case for flow through rockfill structures, the velocity heads remain small compared to other

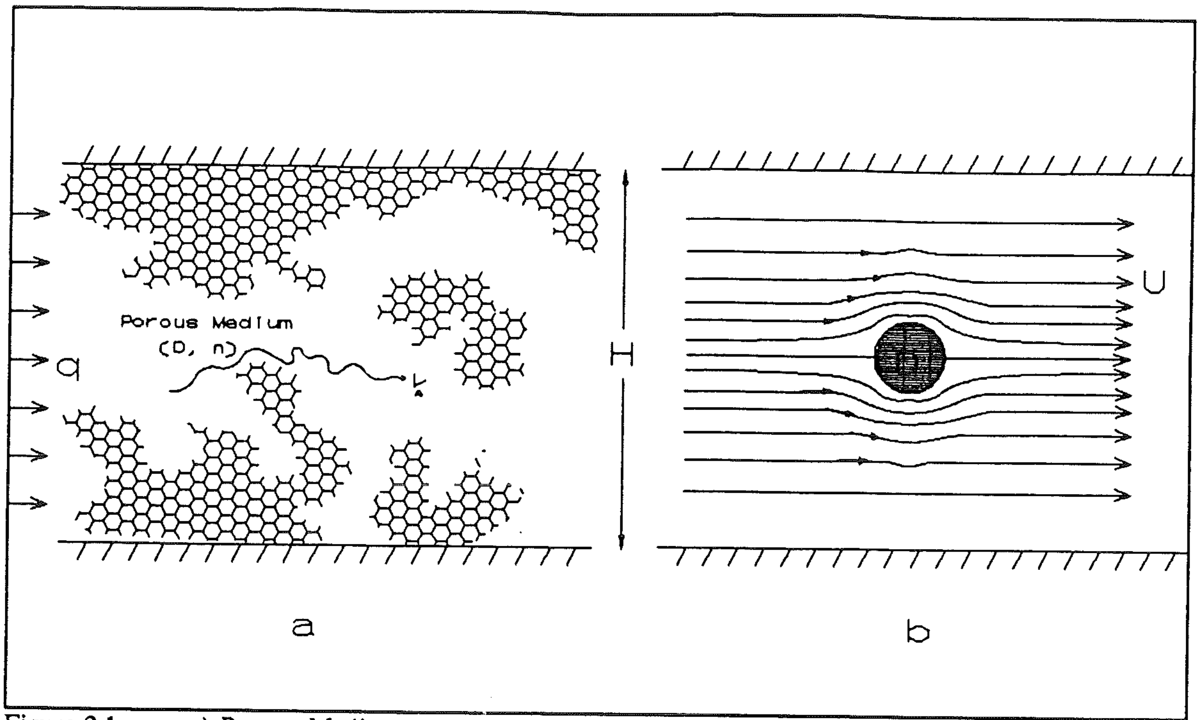


Figure 2.1 a) Porous Medium b) Sphere in Unbounded Fluid

energy losses, even in prototype (Parkin and Trollope, 1966; Hall, 1987). For the case of waves plunging on a breakwater the velocity heads in the armour layers are likely not negligible. However, as explained in section 2.1 that flow situation is not validly described the present system of analysis. In addition, other effects such as air entrainment and fully turbulent flow require a different method of solution to be adopted. Therefore the influence of the velocity heads will not be considered in this analysis of porous media flow.

If the flow is laminar (small $\mathbf{R_e}$) viscous forces dominate over the convective inertial forces, which are represented by ρ. If the inertial forces are not to be considered then ρ (thus $\mathbf{R_e}$) cannot be included in the functional relation but the viscosity, μ, must remain. If the quantity a is the pressure gradient ($dH/dx = \rho gi \equiv I$), then application of dimensional analysis (by choosing basic quantities as D, q, μ) provides the relationship

$$Y_I = \frac{D^2}{\mu q} I = \phi_I(n) \qquad [2.18]$$

Equation [2.18] can be written in terms of the hydraulic gradient, i, as

$$\frac{\rho g D^2}{\mu q} i = \phi_I(n) \qquad [2.19]$$

or, equally,

$$i = \frac{Fr}{Re} \phi_I(n) = \left[\frac{\nu}{g D^2} \phi_I(n) \right] q \qquad [2.20]$$

(where **Fr** is the Froude Number) which is identical to Darcy's law if $1/K = \nu/gD^2 \, \phi_I(n)$. In addition, the quantity $i/\mathbf{F_r}$ is proportional to $1/\mathbf{R_e}$. The term i/F_r is a common expression for the friction factor and will be discussed further in section 2.3.

If the property under investigation is the total force F on the porous medium then, as above, for small Reynolds' numbers,

$$F = \frac{\mu^2}{\rho} \phi_F\left(\frac{\rho q D}{\mu}, n\right) \qquad [2.21]$$

where, for laminar flow, ρ is not a parameter and must vanish as above. To accomplish this F must be linearly proportional to $\mathbf{R_e}$ and therefore

$$F = \mu q D \alpha \cdot \phi(n) \qquad [2.22]$$

where α is the coefficient of proportionality. Equation [2.22] is seen to be identical to Stokes' law for a single particle, that is,

$$F = C_D \mu D \cdot q \qquad [2.23]$$

except that for a porous medium a term for the porosity (which is commonly used to represent the permeability) must be included.

For the flow situation in Figure 2.1(b), the analysis proceeds identically to that given above except that the porosity term is not present. Therefore, any property under investigation becomes a function of the Reynolds' number only (other properties such as surface roughness are incorporated into the functional relationship for all geometrically similar configurations). For the case of the total force, F, on the particle, this becomes

$$F = \frac{\mu^2}{\rho} \phi_F\left(\frac{\rho U D}{\mu}\right) \qquad [2.24]$$

and for laminar flow the density term is neglected, leaving the result (Stokes' law)

$$F = \mu D C_D U \qquad [2.25]$$

where C_D is the proportionality constant commonly called the drag coefficient.

Rumer and Drinker (1966) used this similarity to show that Darcy's law can be derived from a simple force balance if Stokes' law can be assumed to apply to a porous medium (with appropriate modifications). This approach also allowed them to derive a theoretical form of the porosity function ($\phi(n)$) listed in equations [2.11] to [2.15]. They considered a cylindrical element (E), (Figure 2.2) with porosity n, length ds and inclined at an angle θ to the vertical so that $\cos\theta = dz/ds$. The force balance (stationary flow) is then

$$-\frac{\partial H}{\partial s} - \frac{R_F}{\rho\, n\, dA_T\, ds} = 0 \qquad [2.26]$$

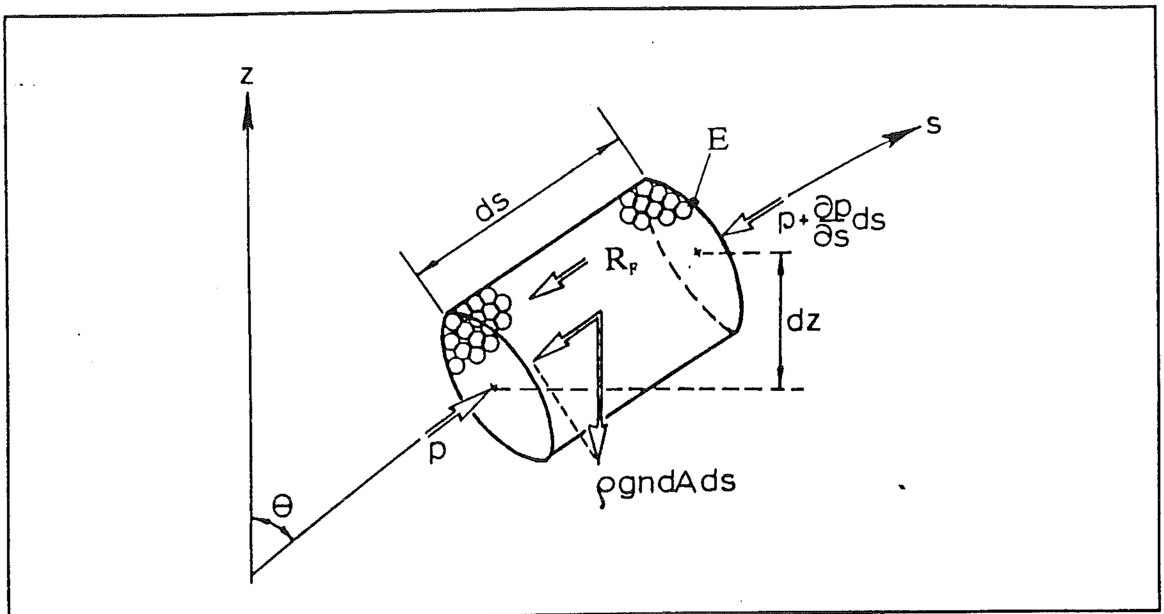


Figure 2.2 Porous Medium Elemental Volume E

where H is the pressure head and R_P is the total resistance force of all the grains within the volume (dA)(ds). For the case of laminar flow all energy is dissipated by friction, as shown above, and the resistance of the solid particles can be described by Stokes' law, modified to account for the effects of many particles, ie.

$$f_p = \lambda \mu D v_p \qquad [2.27]$$

where f_p is the drag force per unit volume acting on one particle, D is a characteristic length, diameter for a sphere, v_p is the actual fluid pore velocity, and λ is a factor incorporating effects of the neighbouring particles ($\lambda_{max} = 3\pi$ for a single sphere in an unbounded fluid). Considering N particles within the volume element E, then

$$N = \frac{(1-n)dAds}{\beta D^3} \qquad [2.28]$$

where β is a volumetric shape factor ($\pi/6$ for a sphere). Substituting equations [2.27] and [2.28] into [2.26] and assuming the relationship $v_p = q/n$, then

$$q = -\frac{\beta n^2}{\lambda(1-n)} D^2 \frac{\rho g}{\mu} \frac{\partial H}{\partial s} \qquad [2.29]$$

The term $\beta n^2/[\lambda(1-n)]$ is a function of the pore system only and can thus be replaced by the dimensionless coefficient c. Recalling that the (constant) intrinsic permeability k has dimensions of L^2, as does the constant cD^2, then the product cD^2 can be replaced by the constant k and equation [2.29] becomes identified with Darcy's law, equation [2.2] as

$$q = -k \frac{\rho g}{\mu} \frac{dH}{ds} \qquad [2.30]$$

Note that here the permeability k is a function of the term $(n^2/(1-n))$. This is the form of the porosity function $\phi(n)$ derived on a semi-theoretical basis.

2.2.3 Non-Stationary Laminar Flow

Few studies have addressed the case of accelerated or cyclic flow. In this case an additional external force must be required to accelerate the mass of water (Dean and Dalrymple, 1984; den Adel, 1987). This extra force is $M \cdot dv_p/dt$ where M is mass of accelerated water and v_p is the actual velocity of water in the pore space. Then the total force balance per unit volume (in terms of bulk velocity $q = nv_p$) results in

$$i = \frac{dh}{dx} = \frac{\mu}{\rho g k} q + \frac{1}{g} \frac{dq}{dt} \qquad [2.31]$$

The general form of this equation is

$$i = aq + C' \frac{\partial q}{\partial t} \qquad [2.32]$$

where the coefficient C′ is called the acceleration coefficient and is thought to be a constant for any given media (as is the coefficient a). If the velocity q is described by a sine function with period T, ie.

$$q = q_o \sin(\frac{2\pi t}{T}) \qquad [2.33]$$

then from equation [2.32]

$$i = q_o (a \sin\omega t + C'\omega \cos\omega t) \qquad [2.34]$$

with $\omega = 2\pi/T$ or, if θ is defined by

$$i_o \cos\theta = a q_o \qquad [2.35a]$$

$$i_o \sin\theta = C'\omega q_o \qquad [2.35\ b]$$

where i_o is the magnitude of the gradient, then the hydraulic gradient can be described by

$$i = i_o \sin(\omega t + \theta) \qquad [2.36]$$

where θ is a phase shift induced between the velocity and energy gradient.

den Adel (1987) wrote equation [2.32] in terms of the potential $H = p + \rho g h$, then assumed a sinusoidal applied hydraulic gradient $i = i_o \sin(2\pi t/T)$ so that

$$i = \frac{q}{K} + \frac{1}{ng}\frac{dq}{dt} = i_0 \sin\omega t \qquad [2.37]$$

The analytical solution to equation [2.37] is $q = c_1 \sin\omega t + c_2 \cos\omega t$ with

$$c_1 = \frac{i_o}{K}\frac{1}{(\frac{1}{K})^2 + (\frac{\omega}{ng})^2} \qquad [2.38\ a]$$

and

$$c_2 = -\frac{i_o \omega}{ng} \frac{1}{(\frac{1}{K})^2 + (\frac{\omega}{ng})^2} \qquad [2.38\ b]$$

The velocity, q, may be written as

$$q = q_o \sin(\omega t - \theta) \qquad [2.39]$$

if $q_o = (c_1^2 + c_2^2)^{1/2}$ and $\theta = \tan^{-1}(-c_2/c_1)$. Thus for laminar flow, if either the velocity or gradient is described by a sine function then both will be sine functions with a phase shift between them. This phase shift affects the resistance of the porous medium and will be discussed later in this chapter.

From equations [2.38] and [2.39] den Adel defined a period-dependent cyclic permeability $K_*(T)$ as

$$\left(\frac{1}{K_*(T)}\right)^2 = \frac{i_o^2}{q_o^2} = \left(\frac{2\pi}{ngT}\right)^2 + \left(\frac{1}{K}\right)^2 \qquad [2.40]$$

For applied hydraulic gradients with small periods, the permeability $K_*(T)$ becomes much smaller (ie. increased resistance) than the stationary Darcy permeability, K, (Figure 2.3). It appears that as the period of the applied gradient decreases less pore water can be brought into motion, and the flow becomes less dependent on the applied pressure gradient.

The concept of permeability of a porous medium has yet to be well defined, for it is dependent upon, and sensitive to, many parameters that are difficult to control even in a laboratory environment. To this day the permeability coefficient must still be determined indirectly in laboratory permeameter tests, for no reliable general predictive formulae have been produced.

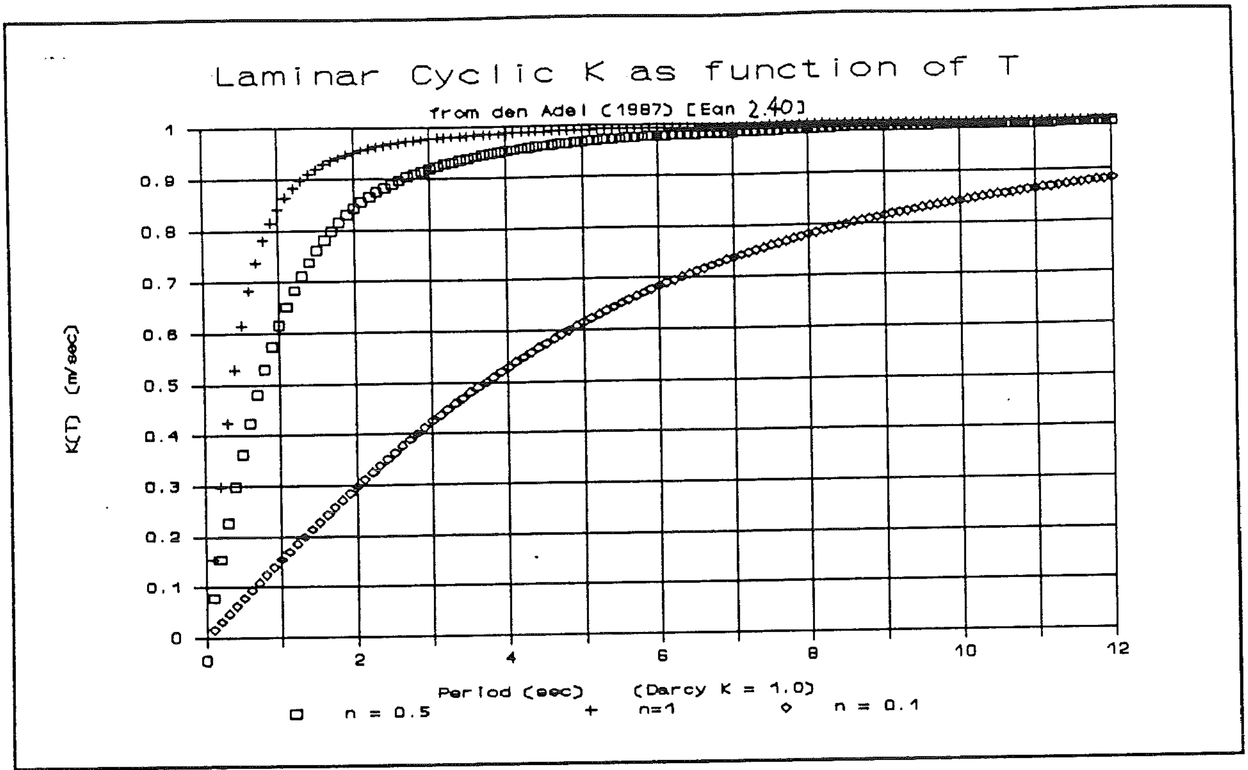


Figure 2.3 Period-dependent Permeability, $K_c(T)$

Thus have the basic concepts of porous media flow been identified for Darcy or laminar flow. For many civil engineering applications flow rates above the laminar regime prevail and must be considered. An overview of the many studies into this field is presented in the following sections.

2.3 Post-Darcy Flow

2.3.1 Steady Flow Experiments

Darcy's law is only valid over a limited range of conditions and is commonly referred to as the Darcy or linear laminar regime of flow. Outside of this range the relation between flowrate and energy loss is not linear. It is the "post-Darcy" regime that is relevant for consideration in this report. Many studies have defined the valid form of the flow law in this range reasonable well but not all aspects of the flow law are accepted as universal and complete.

General agreement of the form of the flow law describing energy losses in the post-Darcy regime has been reached. Two forms are commonly quoted, the series type and the exponential type. The exponential type is of the form

$$i = a' q^{f} \qquad [2.41]$$

where
- i = hydraulic gradient = h/L
- q = bulk velocity = Q/A = nv_p
- a',f = coefficients

Note that if f=1 and a'=$\mu/\rho gk$ then Darcy's law is expressed. This form of flow law is preferred by some (Barends, 1980) because of its similarity to pipe flow equations and thus compatibility with standard measures such as drag coefficient. Muskat (1937), using dimensional analysis, showed that f has an upper limit of 2 for gravity flows, signifying fully developed turbulent flow in all pores in analogy to the pipe flow law. From the result of the dimensional analysis, Muskat has shown that

$$a' \sim \rho D^{f-3} \nu^{2-f} \qquad [2.42]$$

The resistance of the medium must be inversely proportional to the viscosity of the fluid hence a' must be proportional to viscosity. For physically meaningful results f must therefore be less than 2. A drawback to this formulation is that the coefficients a' and f vary continuously over flow regimes and are therefore difficult to parameterize.

Dudgeon (1964) conducted permeability tests on different types of materials covering a wide range of flowrates from the pre-Darcy regime to fully turbulent flow. Figure 2.4 shows results of these tests plotted as hydraulic gradient against velocity on log-log paper. It may be noticed that the slope, f, of the lines does not vary a great deal with porosity while the intercept, a′, is highly sensitive to this factor. The variation of these coefficients with flow regime is also evident.

The same data is presented in figure 2.5 as a friction factor - Reynolds' number plot. As with pipe flow the lines have a slope of 1 in the laminar (Darcy) regime and tend towards horizontal at high flow rates, corresponding to fully turbulent flow. The characteristic length term is taken as D_{50}. The region in between these limits is commonly referred to as the "inertial" regime where $f = \phi_f(R_e)$. Thus three regimes of flow can be identified: laminar, inertial and turbulent. Some researchers (Dybbs and Edwards, 1972; Wright, 1968), however, have subdivided the inertial regime into two components, "steady" and "unsteady", describing relative influence of the convectional inertia, and Dudgeon (1964) interpreted the occurrence of up to five regimes from permeameter test results.

The series form of the (uni-directional) flow law is

$$i = aq + bq^2 \qquad [2.43]$$

where a and b are dimensional coefficients. This formulation is more widely accepted because a and b do not vary significantly with flow regime. For low flow rates $q^2 << q$ and Darcy's law is again expressed if $a = \mu/\rho gk$. For high flow rates $q^2 >> q$, again corresponding to fully turbulent flow. For oscillatory flow the second term must be modified to $b|q|q$ to account for the reversal of friction and an unsteady term, $C \cdot dq/dt$ can be added as described in section 2.2.3 (equation 2.32). For non-laminar flow the coefficient C above may

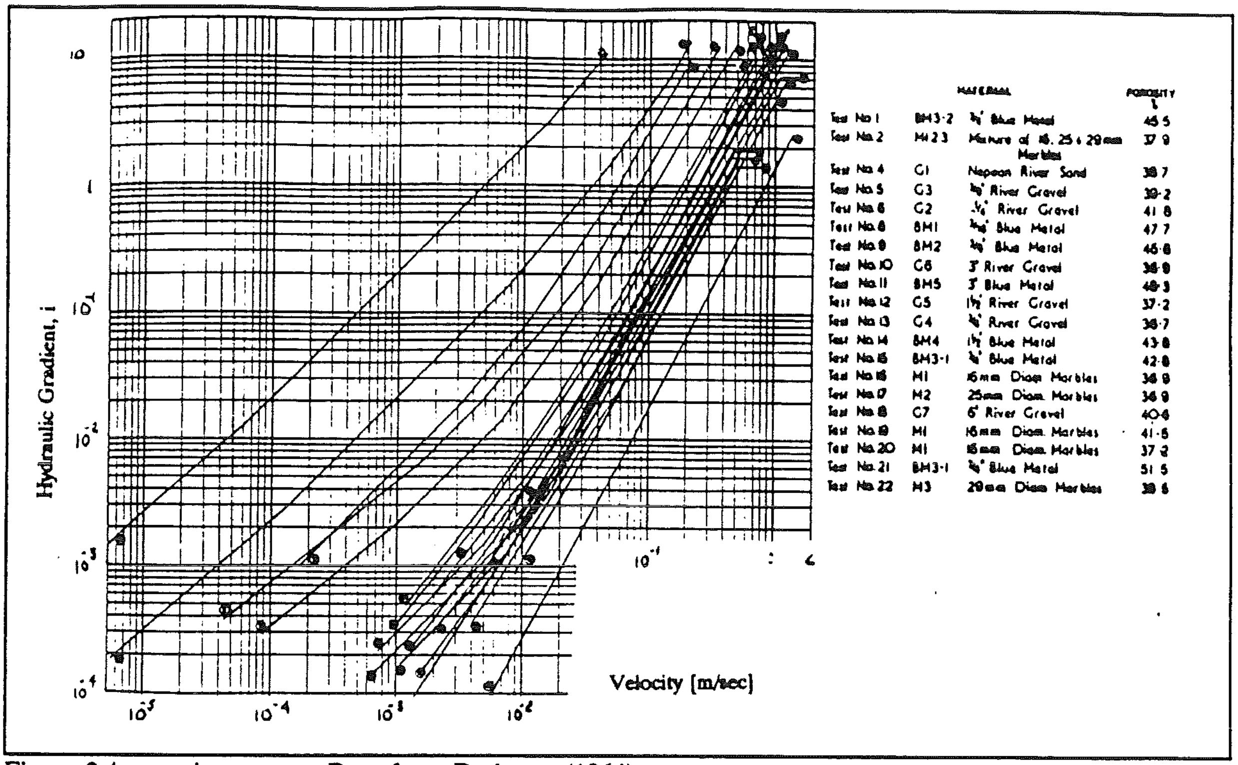


Figure 2.4 i versus q: Data from Dudgeon (1964)

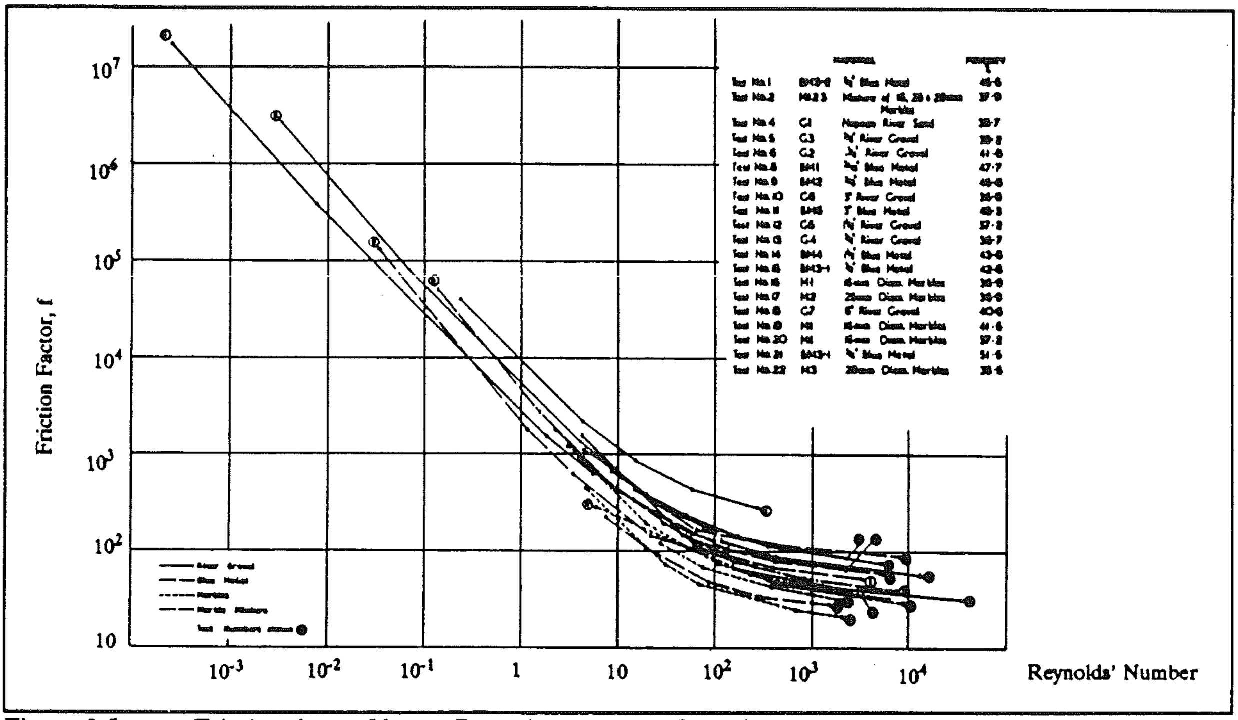


Figure 2.5 Friction factor Versus Reynolds' number: Data from Dudgeon (1964)

be modified to include effects of added mass (Dean and Dalrymple, 1984; Hannoura and McCorquodale, 1978a; Le Mehaute, 1976). This topic is discussed in section 2.3.2.

Both empirical and analytical approaches have been applied to justify each of the flow laws above and to parameterize the coefficients, usually by deriving them from the Navier-Stokes equations or dimensional analysis, etc. Table 2.1 lists some of the common formulations of the coefficients a and b.

Using this form of flow law, Gu and Wang (1990) have examined the relative importance of the three resistance components described above, first introduced by Solitt and Cross (1972), in a non-linear, unsteady porous flow model described by

$$\nabla P = \mu a q + \rho b|q|q + \rho C\frac{\partial q}{\partial t} \qquad [2.44]$$

The expressions adopted for a and b were those of Engelund (1953), listed in Table 2.1. The acceleration coefficient, C, used will discussed in section 2.3.2, which deals exclusively with non-stationary flow in the post-Darcy regime. The three resistance components are therefore

$$i) \ \textit{laminar:} \quad f_l = \frac{\mu}{k}q$$

$$ii) \ \textit{inertial:} \quad f_i = \rho C\frac{\partial q}{\partial t} \qquad [2.45]$$

$$iii) \ \textit{turbulent:} \quad f_t = \rho b|q|q$$

Three dimensionless parameters were determined by considering ratios of these components, namely

a	b	Source
$\frac{180(1-n)^2\nu}{n^3 g D^2}$	$\frac{2.87(1-n)^{1.1}\nu^{0.1}}{n^3 g D^{1.1} q^{0.1}}$	Carmen (1937)
$\frac{150(1-n)^2\nu}{n^3 g D^2}$	$1.75\frac{1-n}{n^3 g D}$	Ergun (1952)
$780\frac{(1-n^3)}{n^2}\frac{\nu}{gD^2}$	$3.6\frac{(1-n)}{n^3}\frac{1}{gD}$	Engelund (1953)
$\frac{\alpha(1-n)^3\mu}{n^2 D^2}$	$\frac{\beta(1-n)\rho}{n^3 D}$	Muskat (1946)
$\frac{\alpha(1-n)^2\mu}{D^2(n_\omega-n)^3}$	$\frac{\beta(1-n)}{gD(n_\omega-n)^3}$	Irmay (1958)
$c_1\frac{\nu T_R^2}{gn}$	$c_2\frac{T_R^3}{gn^2}$	Scheidegger (1960)
$\frac{\nu}{gk}$	$\frac{0.55}{g\sqrt{k}}$	Ward (1964)
$\frac{\nu}{gk}$	$\frac{cD}{gk}$	Sunada (1965)
$\frac{\nu}{gk}$	$\frac{1}{g\sqrt{ck}}$	Ahmed (1967)
$280\frac{\nu(1-n)^2}{gn^3 D^2}$	$0.14\frac{1}{gn^5 D}$	den Adel (1987)
$\left[\alpha_1+\alpha_2(\frac{g}{\nu^2})^{2/3}D_{15}^2\right]\frac{1-n^3}{n^2}\frac{\nu}{g}\frac{1}{D_{15}^2}$	$\left[\beta_1+\beta_2\exp\left(\beta_3(\frac{g}{\nu^2})^{1/3}D_{15}\right)\right]\frac{1-n}{n^3}\frac{1}{gD_{15}}$	Shih (1990)
α, α_1, α_2, β, β_1, β_3, c, c_1, c_2 are all dimensionless constants.		

Table 2.1 Coefficients a and b in Equation [2.43]

$$i)\quad \frac{f_l}{f_l} = \phi_1(R_t)$$

$$ii)\quad \frac{f_t}{f_l} = \phi_2(R_f) \qquad [2.46]$$

$$iii)\quad \frac{f_t}{f_l} = \phi_3(\frac{R_f}{R_t})$$

where the two Reynolds' numbers R_t and R_f are

$$R_f = \frac{|q|D}{\nu} \qquad [2.47\ a]$$

$$R_t = \frac{D^2}{T\nu} = \frac{(D/T)D}{\nu} \qquad [2.47\ b]$$

and signify the relative importance of inertial and viscous forces. However, R_f is a result of convective inertia and R_t a result of local temporal inertial resistance. The ratio of R_f / R_t is a Strouhal number and signifies the different origins of the inertial forces (Le Mehaute, 1976; Gu and Wang, 1990)

From consideration of the relative contributions of these resistance components Gu and Wang produced the following figure (Figure 2.6) which identifies regions where some or all forces have influence on the total resistance. It is assumed here that one force dominates over another if their ratio is greater than 10. This provides a schematic representation of the regions of flow in which each type of resistance plays a role.

The steady form of equation 2.44 is commonly known as the Forchheimer equation (see equation 2.43),

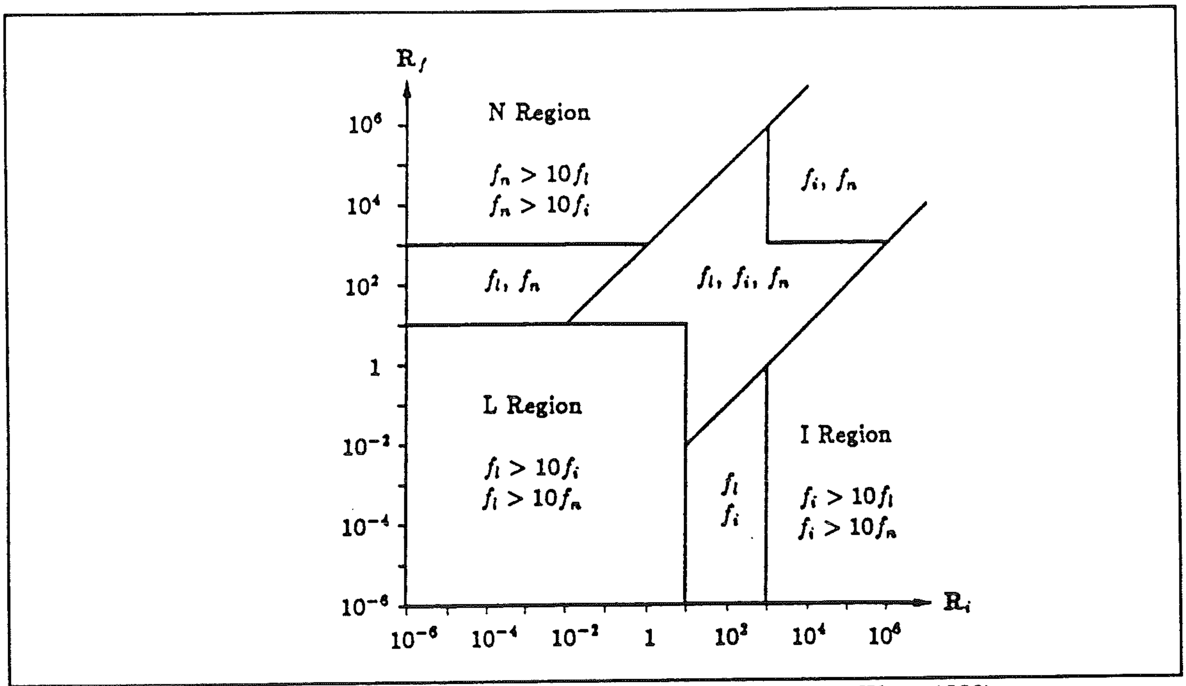


Figure 2.6 Regions of Influence of Resistance Components (Gu and Wang 1990)

associated with development by Forchheimer in 1901, but Ergun (1952) notes that Osborne Reynolds (1900) was the first to describe energy losses through a porous medium as the sum of viscous losses, proportional to flowrate, and kinetic losses, proportional to the flowrate squared. Equation [2.43] can be written as a linear function as

$$\frac{i}{q} = a + b\,|q| \qquad [2.48]$$

and it can be seen (Figure 2.7) that plots of i/q versus q yield a straight line for a given permeability or porosity over a wide range of flowrates. Both a and b are dependent upon permeability but most studies have expressed the relations in terms of porosity. Ergun conducted highly controlled experiments using crushed material and upward gas flows and obtained straight line relationships between a,b and poros factors (Figure 2.8) using the same data as shown on Figure 2.7. These expressions are

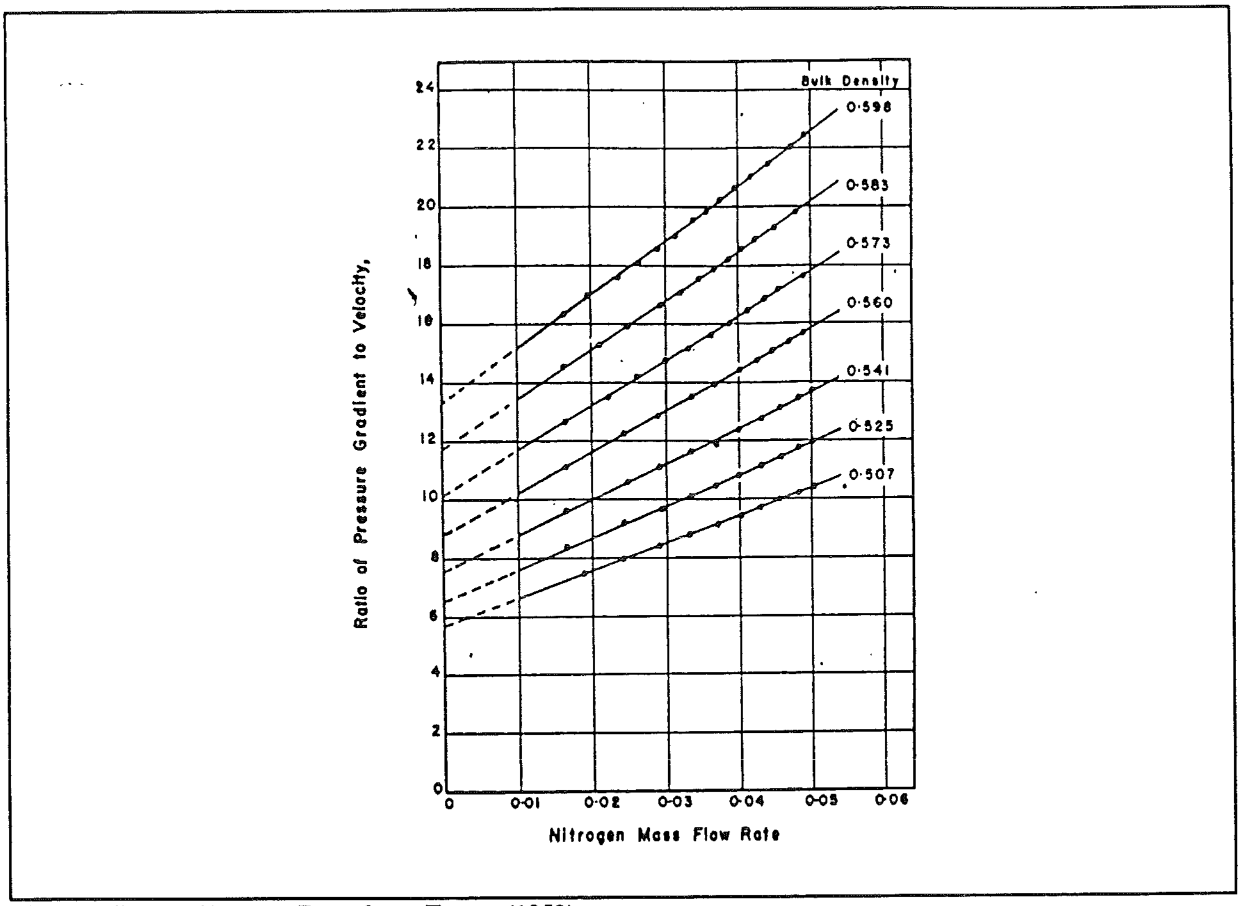


Figure 2.7 i/q vs q Data from Ergun (1952)

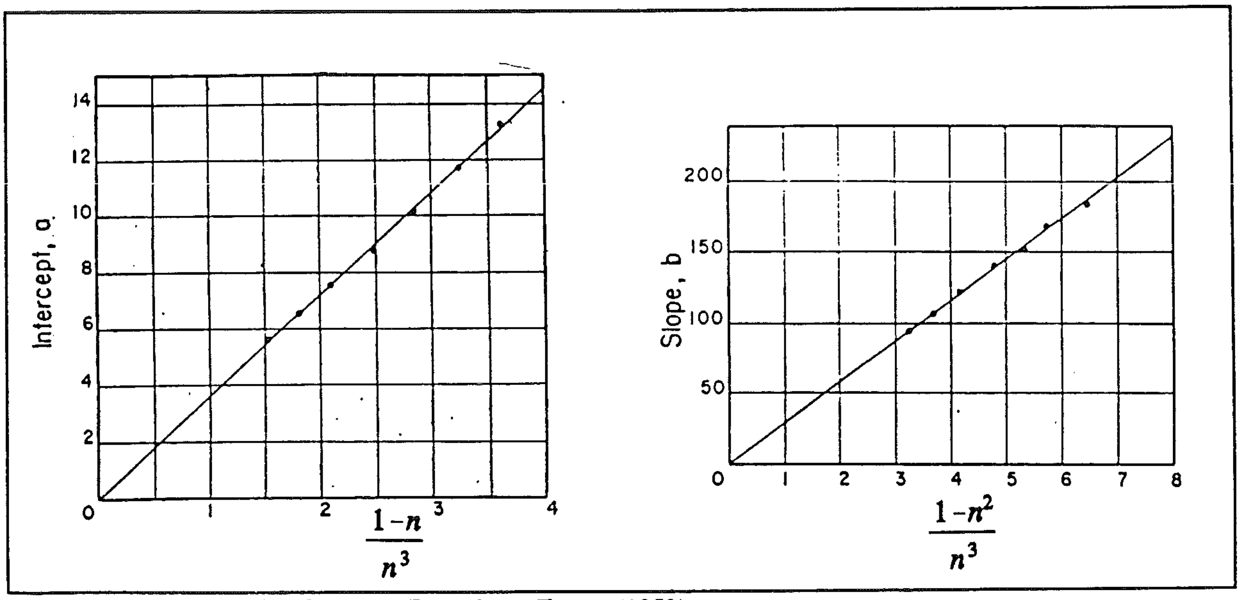


Figure 2.8 Porosity Factors - Data from Ergun (1952)

$$a \sim \frac{(1-n)^2}{n^3}$$
$$b \sim \frac{1-n}{n^3} \qquad [2.49]$$

Difficulties in accurate measurements of porosity for fine materials required Ergun to first calculate n knowing the total mass and volume of the sample and then apply correction factors to account for variations in particle density. Ergun and Orning (1949) also described effects of particle size and shape, incorporating these into equation [2.43] as follows:

$$\frac{g\Delta P}{L} = \left[2\alpha\mu S_v^2 \frac{(1-n)^2}{n^3}\right] q + \left[\left(\frac{\beta}{8}\right)\rho S_v \frac{(1-n)}{n^3}\right] q^2 \qquad [2.50]$$

where α,β are statistical constants and S_v is the specific surface of solids, ie the surface area of solids over the volume of solids. Theoretical development of this equation is based upon the notion of mean hydraulic radius, the drawbacks of which were discussed in section 2.2.1. Ergun plotted data from different experimenters onto a friction factor - Reynolds' number graph (Figure 2.9) where all data falls onto a single curve bounded by the Kozeny-Carman equation (equation 2.14) in the linear laminar regime and an equation by Burke and Plummer (1952) in the fully turbulent regime. Ergun's final equation for all types of flow through porous media is

$$\frac{\rho\Delta P}{L} = 150\frac{(1-n)^2}{n^3}\frac{\mu q}{D^2} + 1.75\frac{1-n}{n^3}\frac{\rho q^2}{D} \qquad [2.51]$$

Other experimental results have not exhibited such a good correlation between the Forchheimer coefficients and porosity factors, probably because such a high degree of control over packing is difficult to obtain especially when working with water flow and mixed media such as rocks. Dudgeon (1968) re-analyzed data from earlier (1964) experiments and found no such relations as those described by Ergun.

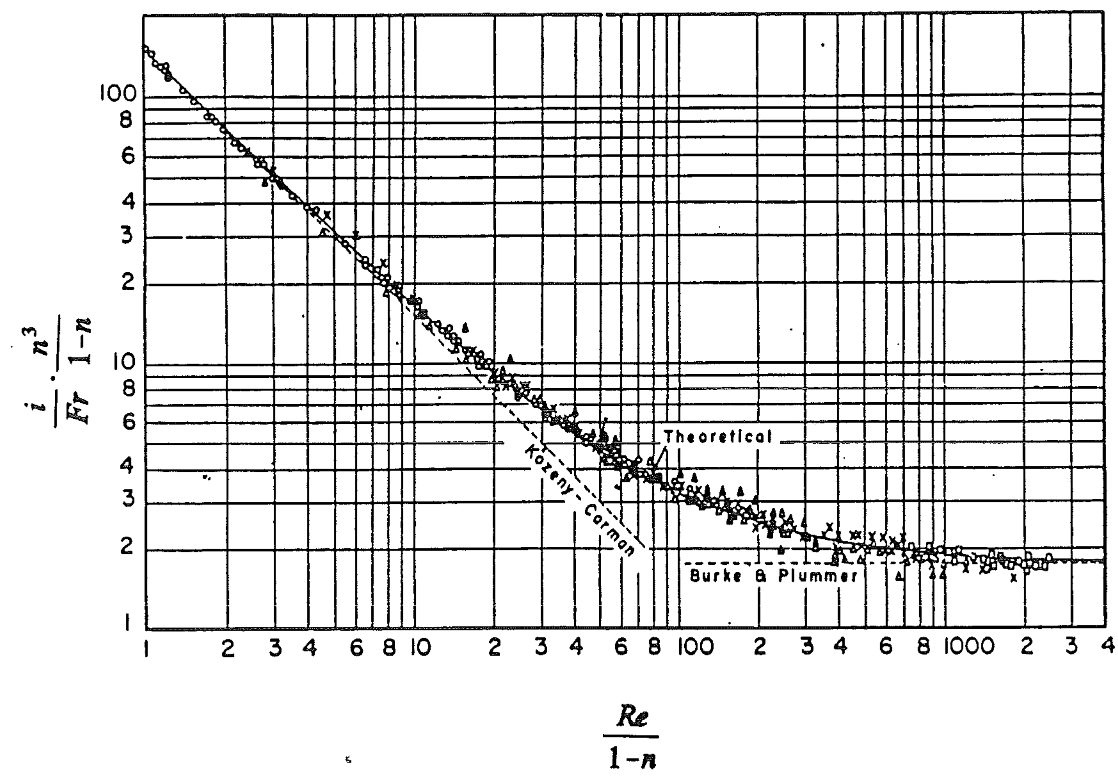


Figure 2.9 Friction Factor Versus Reynolds' Number: Data from Ergun, 1952

Dudgeon's analysis was based upon the exponential form of flow law and since the coefficients, a′ and f, vary continuously over flow regime he equated

$$a'q^f = aq + bq^2 \qquad [2.52]$$
$$\therefore \quad a' = q^{1-n}(a + bq)$$

and he plotted a′ against some porosity functions, two of which are shown on Figure 2.10. He also noted the difficulties associated with porosity measurements, coupled with the dependence of the porosity factor upon n^2 and n^3 as a large source of error. Dudgeon concluded that no universal relation exists linking permeability and porosity for porous materials.

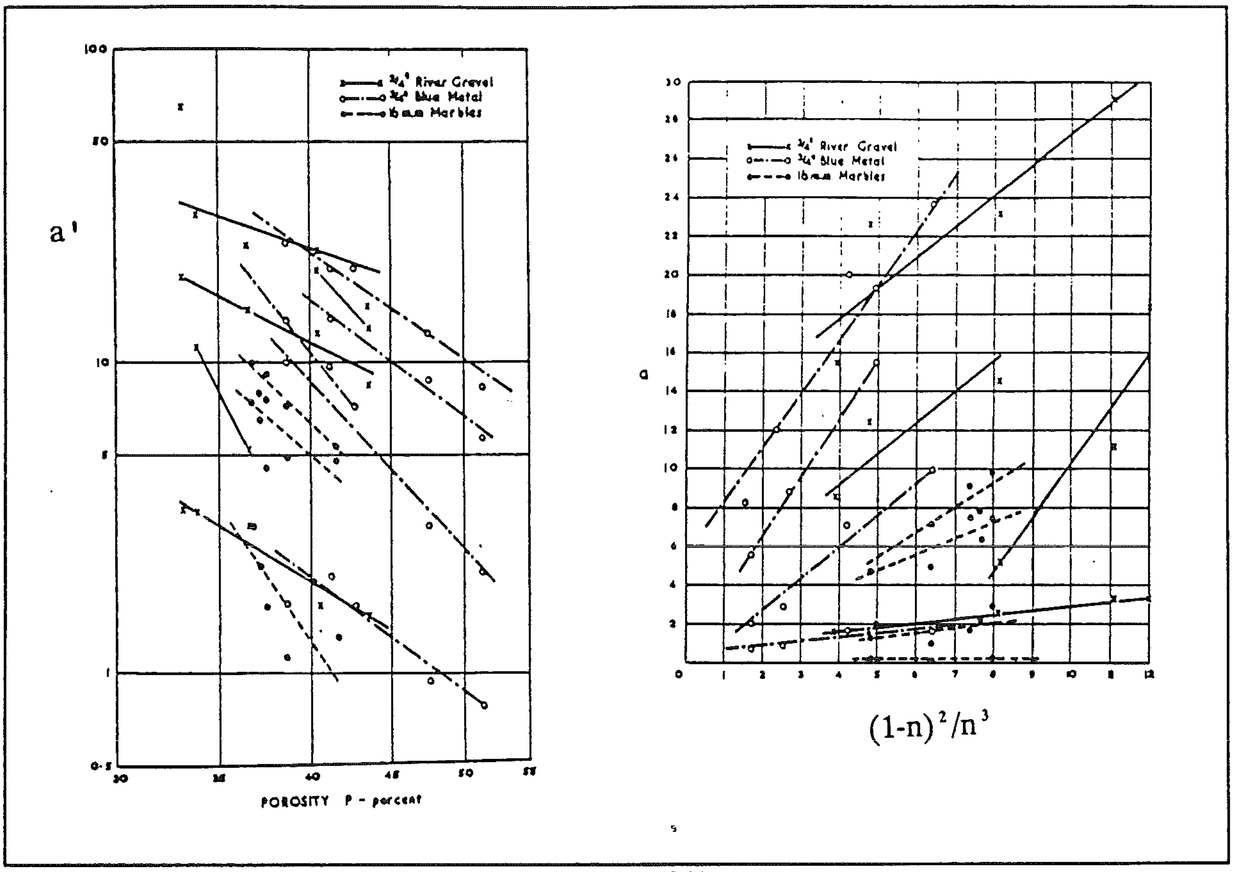


Figure 2.10 Porosity Factors - Data from Dudgeon (1964)

Some researchers have avoided the problem of defining a suitable characteristic dimension and porosity function by using the square root of the intrinsic permeability to represent the length term instead. Ward (1964), for example, began with a dimensional analysis using the square root of the permeability as a characteristic length parameter, and arrived at a general expression for porous media flow

$$\frac{dp}{dl} = \frac{\rho^2 q^3}{\mu} \phi\left(\frac{q\rho\sqrt{k}}{\mu}\right) \qquad [2.53]$$

defining $q\rho\sqrt{k}/\mu = R_k$, a Reynolds' number. He then made use of an observation that plotting the inverse of the Darcy permeability against $\rho q/\mu$ results in a linear relation in the post-Darcy flow regime. This gives the relation

$$\frac{dp/dl}{\mu q} = d + e\frac{\rho q}{\mu} \qquad [2.54]$$

which is, in fact, the familiar series or Forchheimer equation where d and e are the intercept and slope of the line, respectively. By equating equations [2.53] and [2.54] Ward arrived at

$$\frac{dp}{dl} = \frac{\mu}{k}q + \frac{c\rho}{\sqrt{k}}q^2 \qquad [2.55]$$

where c is a dimensionless constant determined to be 0.55 for all porous media and k is defined as 1/d, the intercept in equation [2.54]. Controversy appeared over whether a single curve should exist for all porous media or whether a Moody-type diagram better describes materials of differing permeabilities. In either case permeability tests must be made on each sample to determine its relevant parameters.

Ward incorporated all effects of the properties of the granular material in his permeability term k. His friction factor-Reynolds' number plot (Figure 2.11) is essentially the same as that proposed by Ergun (1952), except that Ergun used a grain diameter as a characteristic length and included porosity effects externally, applied to both f and R_e (Figure 2.9).

A type of Moody diagram was proposed by Arbhabhirama and Dinoy (1973) after a theoretical development based upon the Kozeny-Carman equation coupled with the Forchheimer type equation given by Ahmed and Sunada (1969) and Ward (1964), ie based upon √k as a characteristic length. Their result was the same as Ward's except that the constant c in equation [2.55] was treated as a variable dependent upon the grain parameters, ie

$$c = g\left[\frac{D}{(\frac{k}{n})^{\frac{1}{2}}}\right] \qquad [2.56]$$

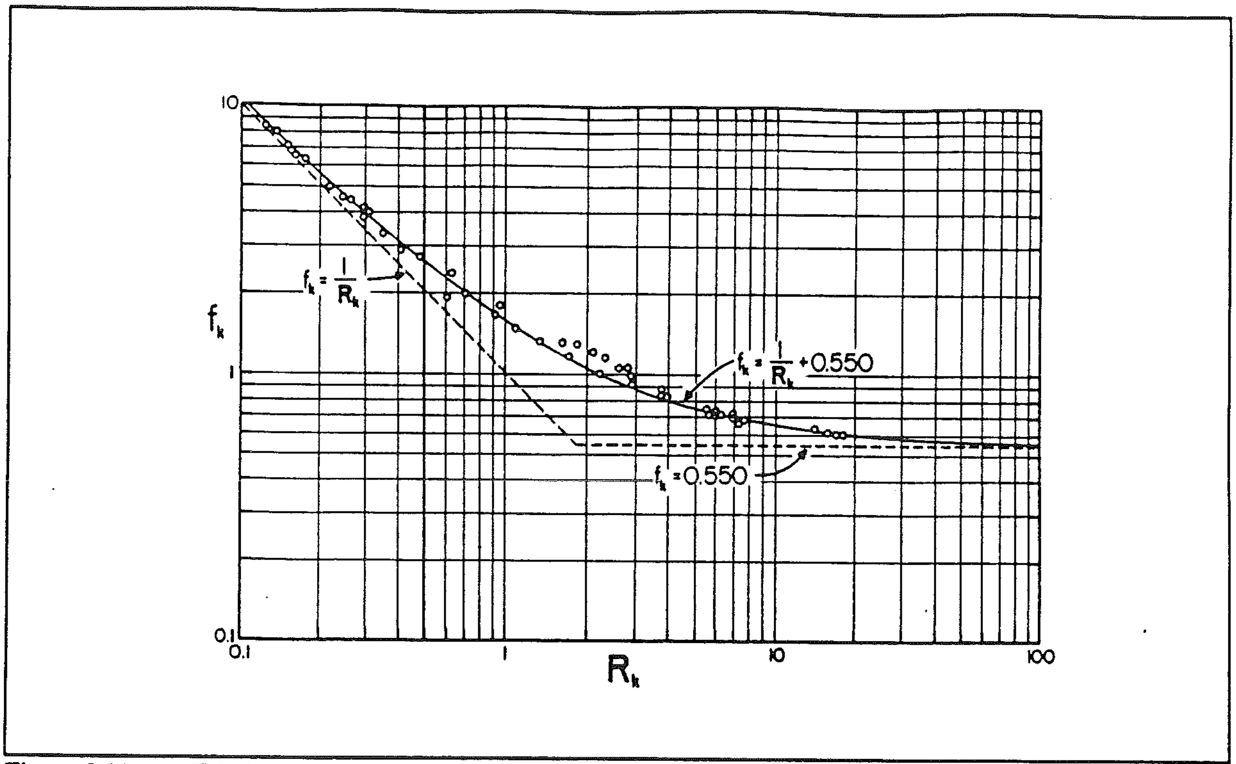


Figure 2.11 Friction Factor - Reynolds' Number plot - Data from Ward (1964)

In the development specific values for other parameters such as tortuosity, etc, were assumed for which no supporting reasons or evidence were given. Their data is plotted on figures 2.12 and 2.13. The main uncertainty of this theory is the application of the Kozeny-Carman equation, which is strictly applicable only to the Darcy regime because all inertia terms of the Navier-Stokes equations are ignored, to flows outside the Darcy regime and coupled with the Forchheimer equation, which is a flow law itself. This, in effect, only transcribes the grain parameters from the Kozenv theory into the Forchheimer equation. Other analyses by Ergun (1952), Burke and Plummer (1950), Ward (1964) and Ahmed and Sunada (1969) support the theory that a single curve can apply to all porous media, thereby refuting equation [2.56] because the "constant" c cannot be dependent upon grain size D.

Ahmed and Sunada (1969) began their development using the full Navier-Stoke's equations for incompressible fluids, adopted to account for turbulent fluctuations in velocity and pressure. The parameters were

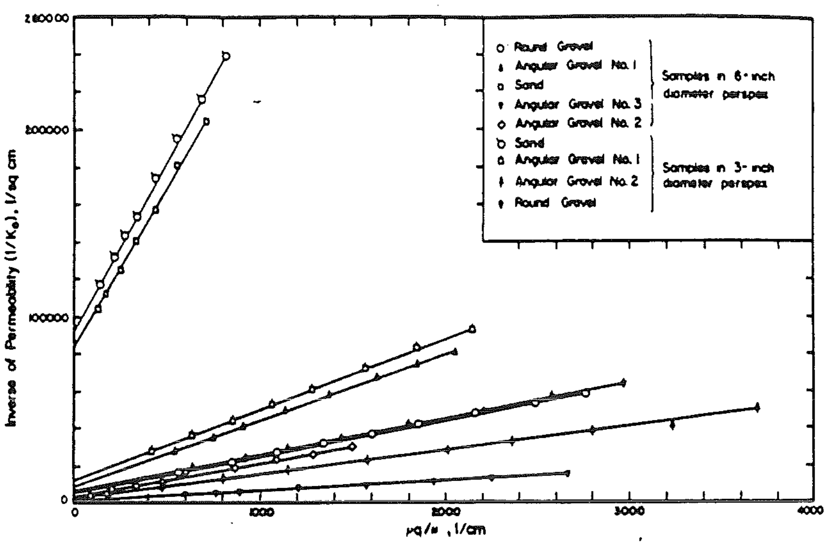


Figure 2.12 Permeability Data from Arbhabhirama and Dinoy (1973)

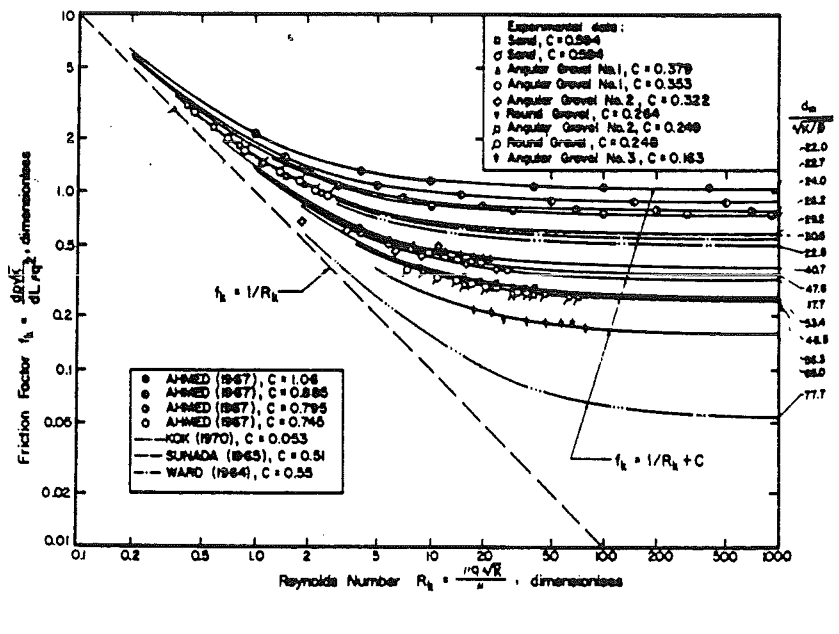


Figure 2.13 Friction Factor - Reynolds' Number plot (Arbhabhirama and Dinoy, 1973)

non-dimensionalized and time averaged for steady state flow arriving at

$$\frac{dP}{dx} = \frac{1}{n\forall}\frac{\mu q}{D^2}\iiint \frac{\partial^2 \bar{u}_{ri}}{\partial x_{rj}\partial x_{rj}}(nd\forall) + \frac{1}{n\forall}\frac{\rho q^2}{D}\iiint\left(-\bar{u}_{rj}\frac{\partial \bar{u}_{ri}}{\partial x_{rj}} - u_{rj}\frac{\partial u_{ri}}{\partial x_{rj}}\right)(nd\forall) \qquad [2.57]$$

where P = statistical space averaged pressure
u_r = dimensionless average component of velocity = u/q
u'_r=dimensionless fluctuating velocity component = u'/q

Ahmed and Sunada argued that both integrals in equation [2.57] represent constant values for a particular homogeneous isotropic porous medium. They further assumed that the turbulent velcoities are small compared to the main stream value and were thus neglected. After further development they obtained

$$\frac{dP}{dx} = \frac{\mu}{k}q + \frac{\rho}{\sqrt{ck}}q^2 \qquad [2.58]$$

where $k = cd^2$. Results of different investigators plotted as a single line on their friction factor-Reynolds' number graph (Figure 2.14). The values of k correspond to Darcy's permeability and no attempt was made to relate k to any grain parameters.

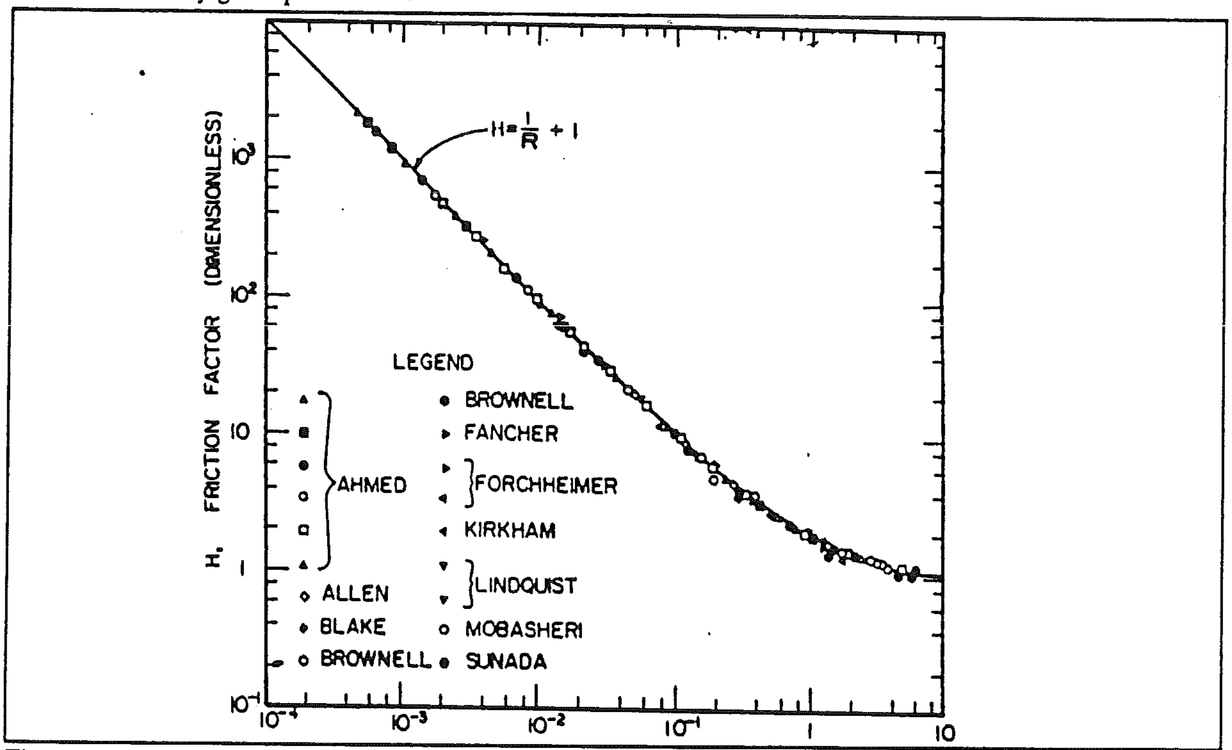


Figure 2.14 Friction Factor - Reynolds' Number Plot by Ahmed & Sunada (1969)

2.3.2 Concerning Porosity Factors

Data from some of Dudgeon's (1964) tests, along with data from Yalin and Franke (1961) and Brownell et al. (1950) have been analyzed to determine the a and b Forchheimer coefficients and compare to Ergun's equations. This data is plotted on Figure 2.15. The same data is presented in Figure 2.16 as a friction factor - Reynolds' number plot, where the friction factor is i/F_r and R_e is based upon mean particle diameter. Contrary to Ergun's (1952) findings, when his porosity factors are incorporated into the f and R_e terms (Figure 2.16b) this data does not collapse onto a single curve. Indeed, the data from Yalin and Franke does shift together but that from the other researchers shifts further apart. It is seen (Figure 2.17 a,b) that the relation between Ergun's porosity functions and the Forchheimer coefficients is not so good as for Egrun's data.

Application of porosity factors, or functions, serve only to translate the individual curves on the axes by an amount Dx and Dy. It may be noted that Yalin and Franke's data (in Figure 2.16 a) is distributed so that the highest curve (B) has the highest porosity and the lowest curves (A and C) have the lowest porosities. It is therefore possible to find a "porosity factor" that will shift these curves closer together by considering the amount $\delta x, \delta y$ each must move. As an example, a "porosity factor" of n^5 applied to the friction factor will cause a vertical shift of the curves that bring them closer together (Figure 2.18). By applying similar correction factors to the Reynolds' number the curves will shift horizontally and a better fit can be obtained. Dudgeon (1964) noted the same result for various values of the dimension D, which gets applied to both axes, as D is embedded in both the Reynolds' number and Froude number. Applying a further correction to the D term will therefore cause an additional shift $\delta x, \delta y$ to the curves. This is the basis for the "equivalent diameter" approach to defining universal curves for all porous media (Yalin, 1971).

Therefore, for data from ordered media, such as that from Yalin and Franke and Ergun, the amount of shift can be fairly well estimated. But, for more disordered media such a shift can not be so well defined. This

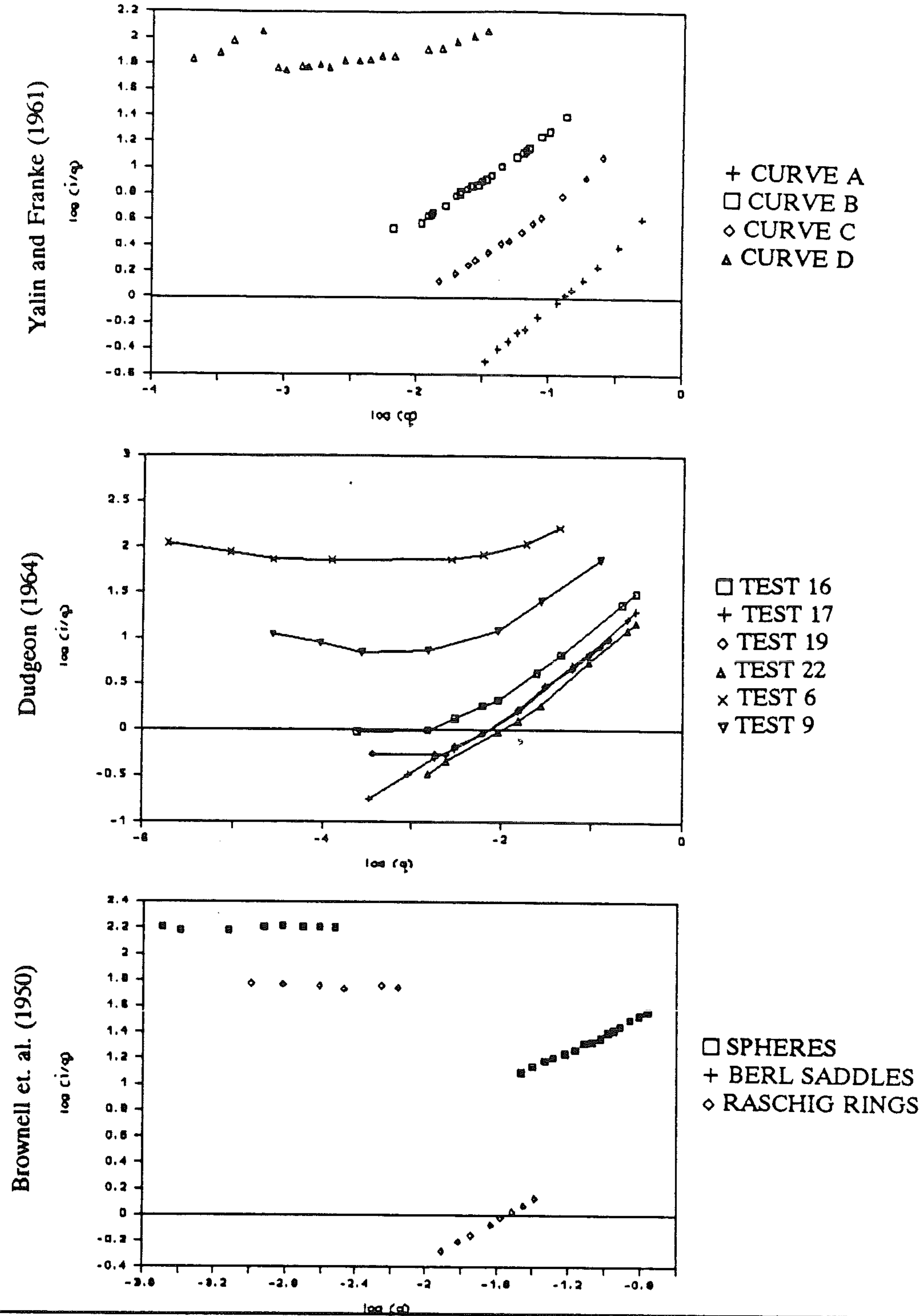


Figure 2.15 i/q versus q (log-log) - Data from Different Researchers

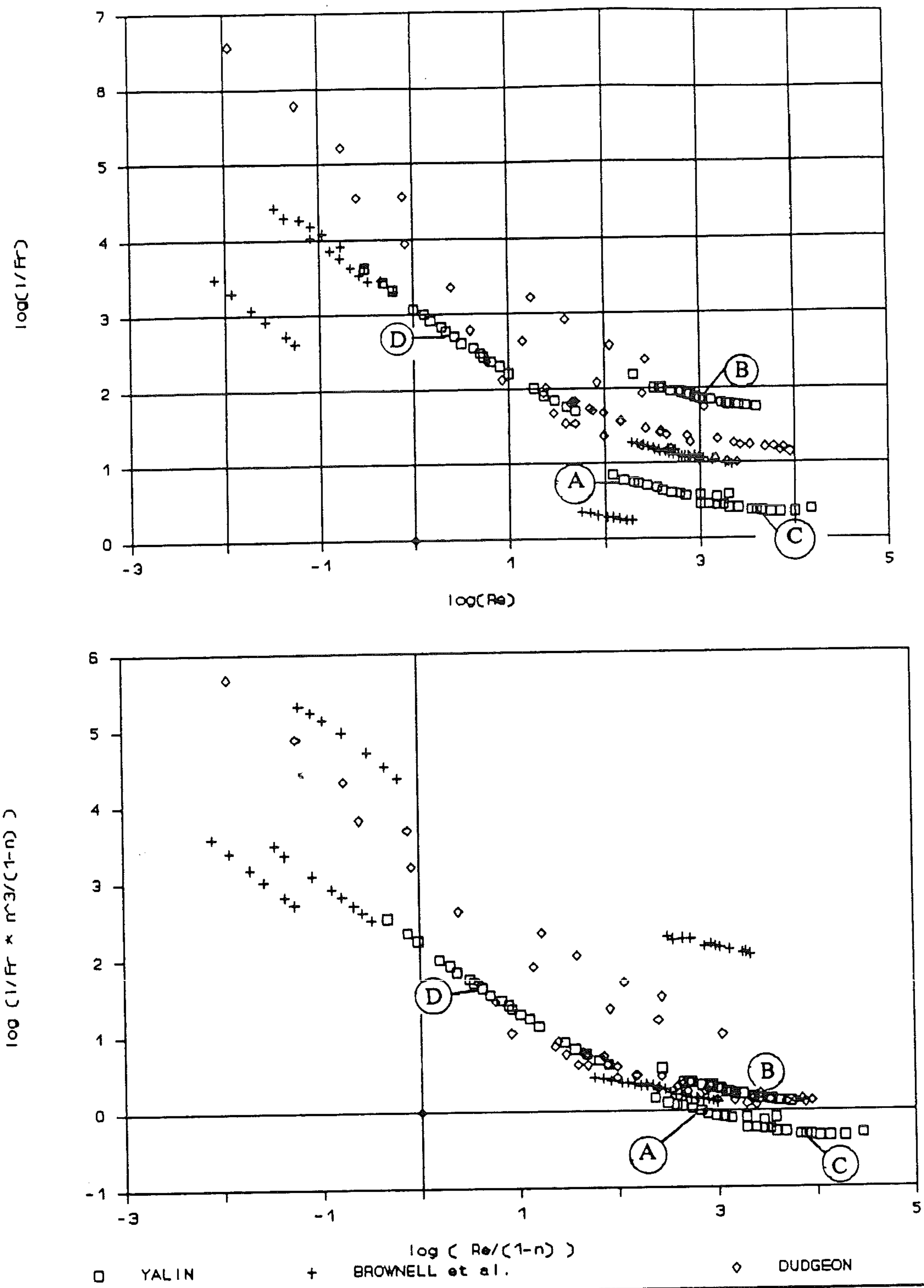
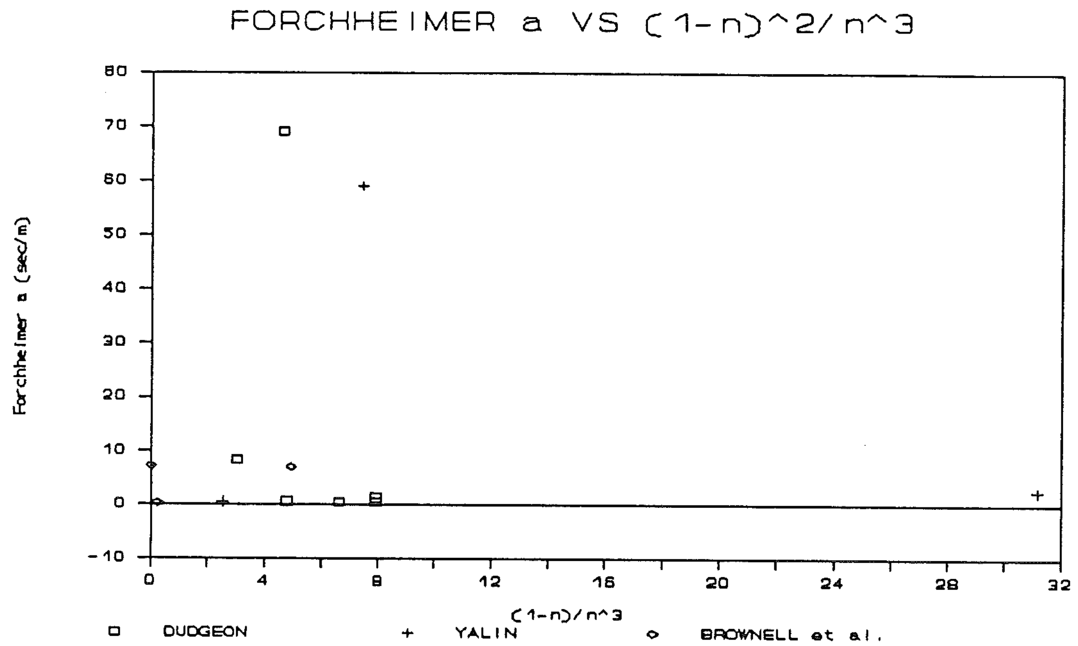


Figure 2.16 Effect of Porosity Factors on f - Re Plots



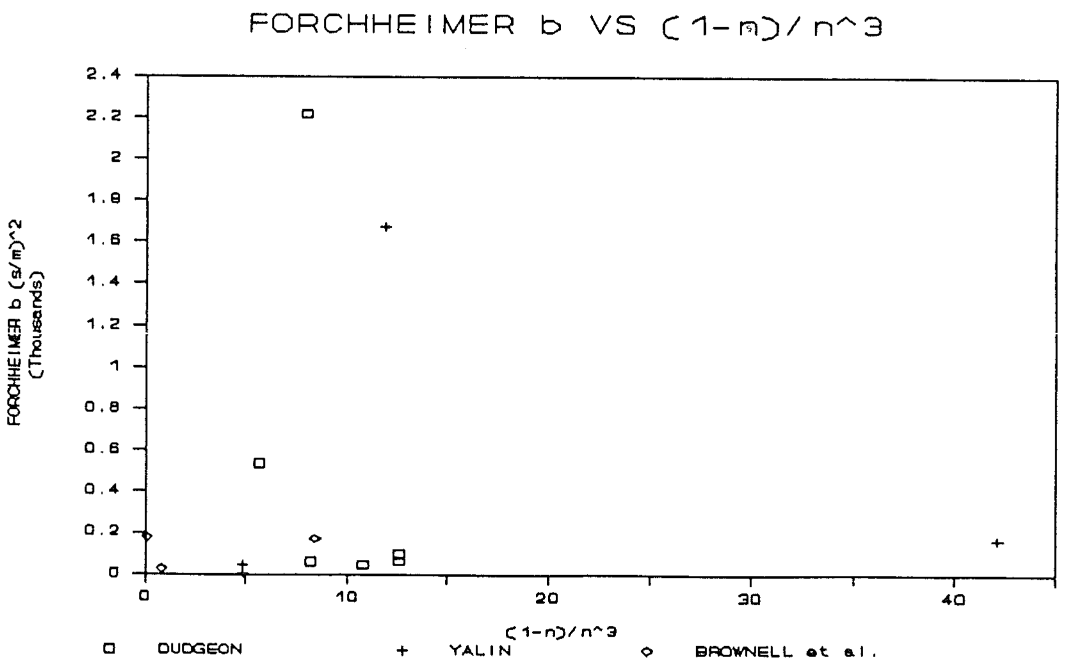


Figure 2.17 Forchheimer Coefficients Versus Porosity Factors from Ergun (1950)

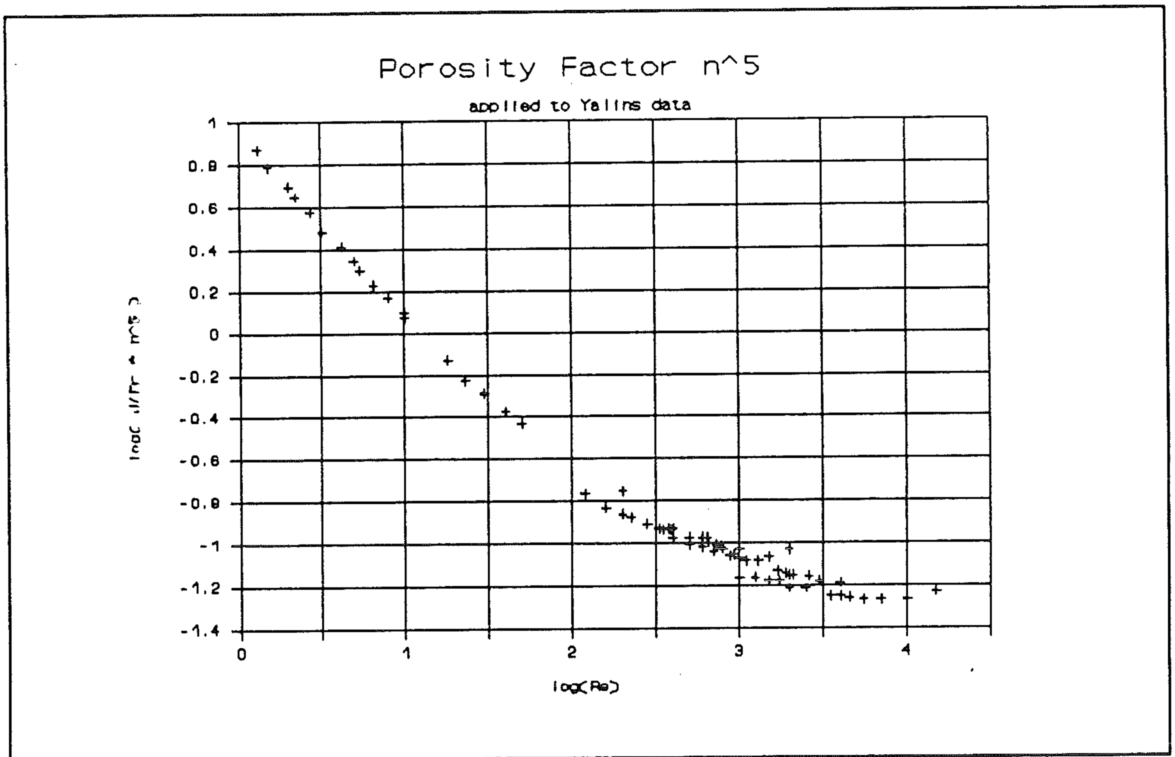


Figure 2.18 Porosity Factor n^5 Applied to data of Yalin & Franke (1961)

is evident from inspection of the data from Brownell et al. (1950): In Figure 2.15 (c) the curves for spheres and berl saddles are almost identical (which is interesting enough considering the wide differences in diameters and porosities!), whereas in Figure 2.16 (a) (after applying Ergun's correction factors) these curves have shifted further apart.

2.3.3 Friction Factor - Reynolds' Number Plots in Unsteady Flow

Several figures in the preceding sections presented (stationary flow) data from different researchers as a friction factor, which is a function of the Reynolds' number. The rationale for this was discussed in section 2.2.2 (Equation [2.20]). From equations [2.20] and [2.43] it has been shown that the form of the function relating the friction factor and Reynolds' number (plotted on log - log axes) in the laminar (Darcy) regime by a straight line with slope -1 and in the fully turbulent regime by a horizontal line. Now the effects of non-stationary flow will be considered.

In addition to the characteristic parameters given in section 2.2.2, non-stationary flow requires additional parameters to be considered. For the specific case of oscillatory flow, these parameters include the period and amplitude, T and A respectively, of oscillation and time, t. However, since q, T, A and t are interdependent quantities, only three of the four may be included in the list of characteristic parameters. Choosing q, T and t, the relation described by equation [2.17] now becomes, for any dimensionless quantity, Y_a,

$$Y_a = \overline{\Phi}_a\left(\frac{qD}{\nu}, n, \frac{t}{T}\right) \qquad [2.59]$$

where $\overline{\phi}_a$ represents some function of the variables within the brackets and is different from that in equation [2.17]. It is thus to be expected that the form of the friction factor - Reynolds' number function will be different from that of the stationary flow case.

In order to investigate the effects of temporal inertia, consider the laminar oscillatory flow situation described in section 2.2.3 (Equations [2.32] and [2.33]). In this situation the velocity and gradient were described by sine functions with a phase difference of $\theta = \tan^{-1}(C'\omega/a)$ between them. C' is the acceleration coefficient for laminar flow (no added mass term) and a is the laminar Forchheimer coefficient. This relation is represented by the plus signs on the friction factor - Reynolds' number plot in Figure 2.19. The points on the graph were computed using equations [2.32] and [2.33] with values for the different parameters listed on the Figure. Absolute values of velocity and gradient data were taken in order to compute the logarithms.

The squares on the graph display the steady flow law defined by equation [2.43] (Forchheimers' equation). Differences in the resistance functions are obvious. The triangles on the graph represent the oscillatory flow law including the turbulent term $(bq \cdot |q|)$, in equations [2.44] and [2.62], and the same acceleration coefficient (C′) is used as for the laminar flow curve (ie. no added mass effects are included). From these plots the effect of the inertia coefficient can be seen. It is the form of these oscillatory functions which is of prime interest.

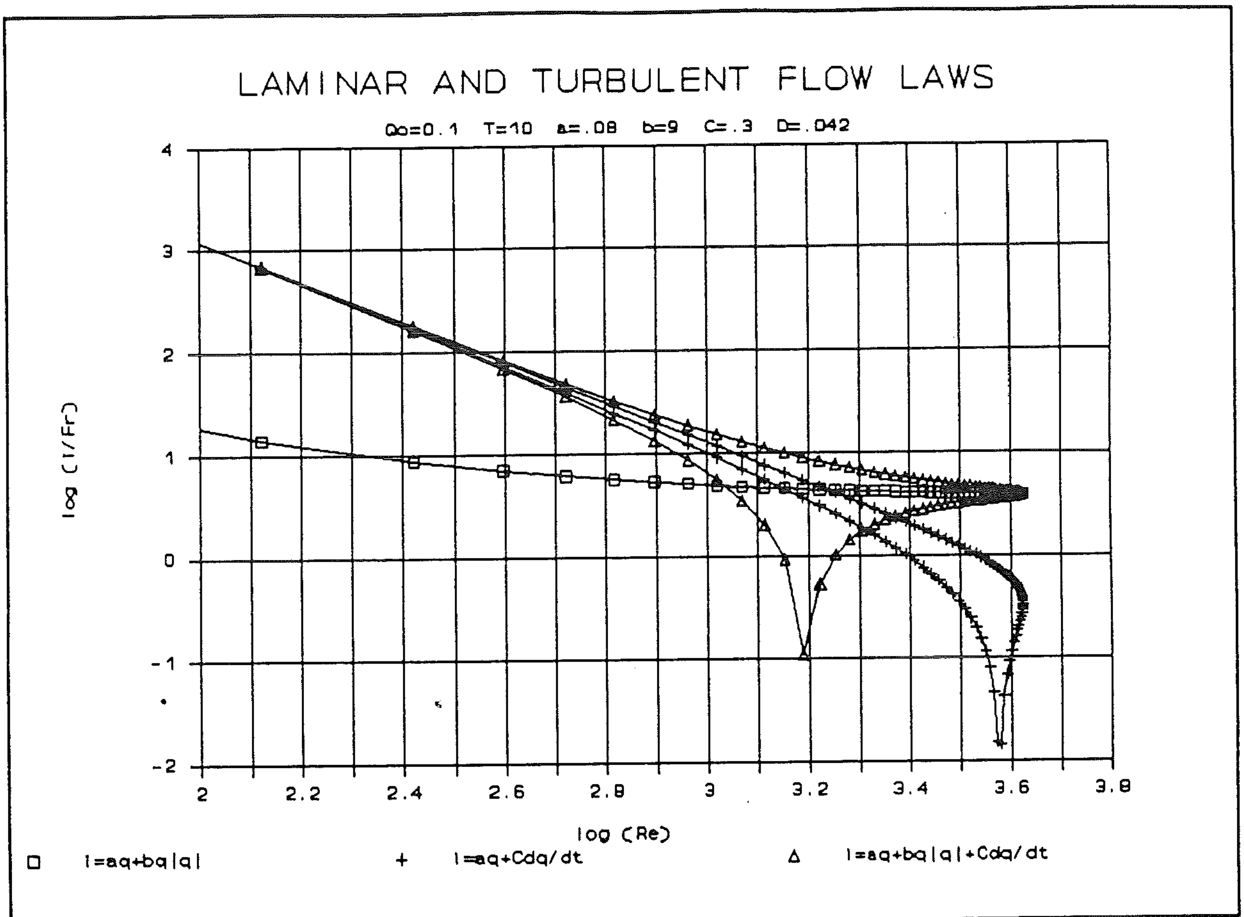


Figure 2.19 f - R_e Graph: Oscillatory and Steady Flow Functions

The first point to notice is the fact that the oscillatory functions tend to be linear with slopes of -2 for lower flowrates. This contrasts with the steady flow condition which is characterized by a slope of -1 in the laminar regime and slope of zero in fully developed turbulent flow. The interpretation of the -2 slope of the oscillatory functions (laminar and turbulent) is that, for most flowrates, the inertial term dominates over the others and the gradient, therefore, is dependent more on the acceleration than it is on the velocity. This can be seen from

$$\frac{i}{F_r} = \frac{agD}{q} \pm bgD + \frac{CgD}{q^2}\frac{dq}{dt} = \phi(\frac{qD}{\nu}, n, \frac{t}{T}) \qquad [2.60]$$

where, for small q values, the stationary laminar and turbulent terms will be negligible compared to the inertia term. Therefore i/F_r will be proportional to $1/q^2$ and so must be proportional to R_e^{-2}, leaving

$$\frac{i}{F_r} = \left(\frac{\nu}{D}\right)^2 \frac{1}{q^2} \cdot \phi'\left(n, \frac{t}{T}\right) \qquad [2.61]$$

Then, clearly, the gradient, i, is not dependent upon the velocity, q but only on the terms remaining within the functional relation ϕ'. Similar to the friction factor - Reynolds' number plots for stationary flow, the oscillatory flow plots will be (for a given set of parameters a, b, C, D, T, q_o) a family of curves with the same form as that in Figure 2.19 but varying in position with different values of n.

A second point of interest in Figure 2.19 is the "dip" which appears near the high end of the abscissa range. The entire lower portion of the oscillatory curves occur when the velocity approaches zero from a maximum or minimum. The "dip" in Figure 2.19 occurs during the time span between the maximum and zero-crossing of the velocity record; the actual minimum value occurs at the zero-crossing of the gradient record. The location of the minimum (and thus the form of the "dip") is therefore dependent upon the magnitude of the phase shift between the velocity and hydraulic gradient. This is the main difference between the oscillatory "laminar" and "turbulent" functions shown in Figure 2.19. When the turbulent resistance term is added to the "laminar" oscillatory equation it produces an additional phase shift, thus changing the shape and location of the "dip". This feature is not a physical one (as is the sudden decrease in the Drag Coefficient - Reynolds' number plots for flow past a sphere, seen where the boundary layer separates from the sphere) but results from the mathematical relationships of phase-varying sine functions.

2.3.4 Effects of Temporal Inertia

The full equation governing unsteady flow in the post-linear regime was given in section 2.3.1 (Equation [2.44]). This section deals with the acceleration coefficient, C, given in that equation. The acceleration coefficient must include the same force that was required to accelerate the water mass for the case of laminar flow (section 2.2.3). An additional inertial coefficient, C_I, is required for flow outside of the laminar regime. This inertial force is required to produce the convectional acceleration of the fluid around the solid particles. This principle is commonly used in the analysis of wave forces on piles (Dean and Dalrymple, 1984). The extra inertial force is

$$F_I = C_I \rho \forall_s \frac{dv_p}{dt} \qquad [2.62]$$

where $\forall_s$ is the volume of solid particles. The inertia coefficient, C_I has a value of 2.0 for potential flow around a circular cylinder and is composed of two terms, ie.

$$C_I = 1 + k_m \qquad [2.63]$$

where k_m is the added mass coefficient and is a function of the shape of the object. The unity term in equation [2.63] arises directly from the pressure gradient across the obstruction, similar to the buoyant force on an object in a hydrostatic fluid. The added mass term is caused by disturbances to the flow field resulting from the object's shape. It is the force necessary to produce an additional pressure gradient which accelerates neighbouring fluid past the particles. In other words, the total kinetic energy of the system (including the moving object and the fluid set in motion around the object) is regarded as the kinetic energy of an object alone having the same size as the original object but with an increased mass . A similar phenomenon occurs when a fluid is accelerated past a stationary object, ie. the fluid appears to experience an increase of mass.

The force balance for the system including laminar and turbulent losses, fluid inertia and added mass can be written (assuming the driving force, F, is the applied pressure gradient dp/dx)

$$F = \frac{dp}{dx} dx \cdot A_T = \frac{\rho g}{K} \forall_T q + \rho \forall_T b' q|q| + \rho \forall_F \frac{dv_F}{dt} + C_I \forall_S \frac{dv_F}{dt} \qquad [2.64]$$

where A_T = Total area perpendicular to flow direction
$\forall_F$ = Volume of fluid = $n\forall_T$
$\forall_T$ = Total volume (solid + fluid) = $dx \cdot A_T$
$\forall_S$ = Volume of solids = $\forall_T (1-n)$
$b' = b/\rho$

Expressing [2.64] in terms of hydraulic gradient, i, and with $q = nv_p$ then

$$i = aq + bq|q| + \frac{1}{g}\left[1 + \frac{C_I(1-n)}{n}\right]\frac{dq}{dt} \qquad [2.65]$$

where the acceleration coefficient, C, includes the added mass and fluid inertia, ie

$$C = \frac{1 + C_I\left(\frac{1-n}{n}\right)}{g} \qquad [2.66]$$

C has dimensions of (sec^2 / m). The added mass term, k_m, can be solved from [2.63] and [2.66] as

$$k_m = [Cg - 1]\left(\frac{n}{1-n}\right) - 1 \qquad [2.67]$$

and should have a value greater than 1 (Dean and Dalrymple, 1984). From this expression it is seen that for a porosity of n=0.3, C must be greater than 0.58 for $k_m \geq 1$ and, for a porosity of n = 0.5, C must be greater than 0.30.

Hannoura and McCorquodale (1978a, 1985) and Abdel-Gawad and McCorquodale (1985) tried to solve equation [2.65] for the added mass term. They first conducted steady flow tests on granular materials to determine values of their a and b coefficients. Then unsteady tests were conducted, where the pressure gradient and water velocities were measured simultaneously. The steady flow values of a and b were assumed to apply to the unsteady flow case and then equation [2.65] could be solved directly for the acceleration coefficient C. Unfortunately, no conclusive data was obtained. The primary reason for this may have been due to the methods used to accelerate the water, which are described in the references listed above. Tests were conducted with uni-directional flow and with very short time durations (less than 1 second). Results from are presented in Figures 2.20 and 2.21. Figure 2.20 is a plot of i/q versus q (similar to Figure 2.7) showing both the steady and unsteady data. As expected, the steady flow data follows a linear trend but the unsteady data seems to show no pattern at all. Figure 2.21 shows results for the inertia coefficient, C_1. Values for C_1 vary widely and many are even negative which, theoretically, should not occur.

If equations [2.44] and [2.65] are correct, the unsteady data should be expected to fall uniformly around the steady flow line on Figure 2.20. One of the main goals of the present study is to check this presumption using an oscillatory flow field.

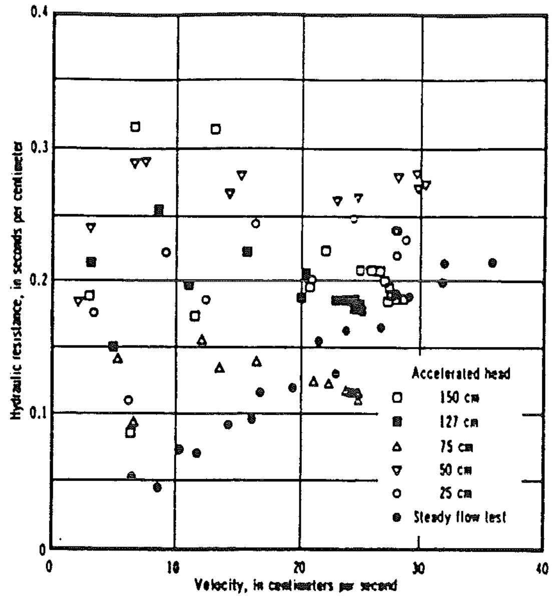


Figure 2.20 i/q vs q - Data from Hannoura and McCorquodale (1978a)

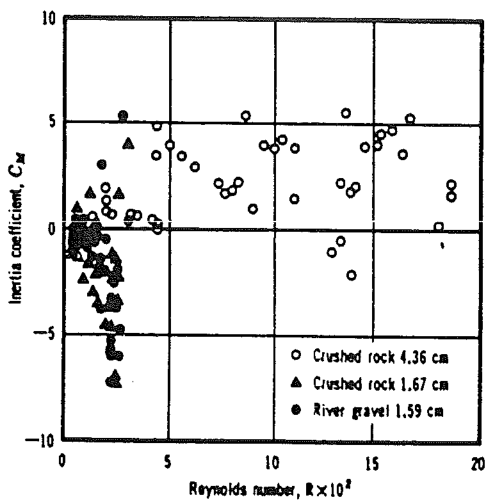


Figure 2.21 Inertia Coefficients from Hannoura and McCorquodale (1978a)

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