The stress generated by non-Brownian fibers in turbulent channel flow simulations

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Turbulent fiber suspension channel flow is studied using direct numerical simulation. The effect of the fibers on the fluid mechanics is governed by a stress tensor, involving the distribution of fiber position and orientation. Properties of this function in channel flow are studied by computing the trajectories and orientations of individual particles, referred to as the particle method. It is shown that, due to computer restrictions, the instantaneous stress in channel flow cannot be simulated directly with the particle method. To approximate the stress we compute the second-order moment of the fiber distribution function. This method involves an unknown subgrid term, which is modeled as diffusion. The accuracy of the moment approximation is studied by comparing Reynolds averaged stress to results obtained from the particle method. It is observed that the errors are $\sim 1\%$ for $y^+ > 20$, and $\sim 20\%$ for $y^+ < 20$. The model is improved by applying a wall damping function to the diffusivity. The moment approximation is used to simulate drag-reduced channel flow. A simplified model for fiber stress is introduced as fiber viscosity times rate of strain, where fiber viscosity is defined as the ratio of Reynolds averaged dissipation due to fiber stress and Reynolds averaged dissipation due to Newtonian stress. Fluid velocity statistics predicted by the simple model compare very well to those obtained from the moment approximation. This means that the effect of fibers on turbulent channel flow is equivalent to an additional Reynolds averaged viscosity. © 2007 American Institute of Physics. [DOI: 10.1063/1.2800041]

I. INTRODUCTION

Suspended linear polymers, with a large length to diameter ratio, can induce significant changes in the flow properties of the carrier fluid at volume concentrations as low as 10^{-5} . Most striking is the reduction of the drag coefficient in turbulent pipe flow.¹

A distinction can be made between rigid and flexible polymers. Rigid polymers (fibers) affect the carrier fluid through viscous effects only, whereas flexible polymers induce both viscous as well as elastic effects.² Numerical research points out that elasticity has an adverse effect on drag reduction,^{3,4} which implies that polymer-induced drag reduction is most likely due to viscous effects. Attributing drag reduction to viscous effects is further confirmed by simulations showing that rigid polymers⁵ and flexible polymers⁶ induce very similar changes in the turbulent structures.

A qualitative explanation of drag reduction based on viscosity arguments was provided by Lumley.⁷ It is based on the shear thinning property,² causing polymer viscosity to be approximately zero in the viscous sublayer. Outside this layer turbulence induces nonzero polymer viscosity which in turn dampens the turbulence. This effect induces a thickening of the viscous sublayer and consequently a reduction of the drag. More recent studies confirmed that drag reduction can be predicted by assuming an additional viscosity which is zero in the viscous sublayer and increases with wall-distance. 8,9

The problem of drag reduction is of extreme complexity since it involves the combination of polymer dynamics and turbulence. It is for this reason that a quantitative theory is still lacking, despite numerous experimental, numerical and theoretical works, conducted over the past 50 years. This paper deals with direct numerical simulation (DNS) of fiber suspension channel flow. The aim is to provide a simplified picture of the fiber stress tensor, which accounts for the effect of the fibers on the fluid mechanics. Section II deals with numerical details of the channel flow simulation and the mathematical description of fiber stress. This stress is governed by a constitutive equation, involving the statistical distribution of fiber position and orientation. Properties of this function are derived using a particle method, i.e., by computing the trajectories and orientations of individual particles. From the analysis it follows that a direct computation of fiber stress is unfeasible. We investigate an approximate method to compute fiber stress in Sec. III. The method consists of solving the second-order moments of the distribution function, referred to as moment approximation. The accuracy of this method is determined by making a comparison to the "exact" particle method. In Sec. IV the moment approximation is used to simulate drag-reduced channel flow. A simplified model for fiber stress is introduced as an additional viscosity. This so-called fiber viscosity is defined such that the addi-

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tional dissipation equals the dissipation due to the "exact" fiber stress. The model is verified by a comparison to the simulation using the moment approximation. Conclusions are given in Sec. V.

II. GOVERNING EQUATIONS AND NUMERICAL DETAILS

A. Channel flow

Fiber suspension flow is governed by the incompressible Navier-Stokes equations, supplemented by the divergence of the fiber stress tensor τ (Ref. 2),

$$\rho \frac{D\boldsymbol{u}}{Dt} = \boldsymbol{\nabla} \cdot (-\Pi \boldsymbol{\delta} + 2\mu \boldsymbol{S} + \boldsymbol{\tau}), \quad \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0.$$
(1)

Here \boldsymbol{u} is the fluid velocity vector, \boldsymbol{t} is time, ∇ is the gradient operator in physical space, $D/Dt = \partial/\partial t + \boldsymbol{u} \cdot \nabla$ is the material derivative, $\boldsymbol{\delta}$ is the unit tensor, $\boldsymbol{S} = \frac{1}{2}(\nabla \boldsymbol{u}^T + \nabla \boldsymbol{u})$ is the rate of strain tensor, Π is the pressure, ρ is the solvent mass density, and μ is the solvent dynamic viscosity.

Equation (1) is integrated numerically in the channel geometry. The simulations discussed in Secs. II and III are one-way coupled, meaning that fiber dynamics are influenced by the flow, but not vice versa, i.e., $\tau=0$ in Eq. (1). The flow is driven by means of a constant pressure gradient between two parallel no-slip walls separated a distance H in the y-direction. Periodic boundary conditions are imposed in the homogeneous directions x and z. We use a pseudospectral flow solver. Spatial derivatives are computed with a Fourierbasis for the homogeneous directions and a second-order central finite differences scheme for the wall normal direction. Time integration is achieved with the second-order explicit Adams-Bashforth scheme. Conservation of mass is ensured using a projection method. Poisson's equation is transformed to Fourier space in the homogeneous directions and a tridiagonal solver is used for the resulting tridiagonal matrices. The variables are discretized on a nonequidistant staggered mesh. Pressure and the velocity components in the homogeneous directions are defined in the cell centers. The wall normal velocity component is defined on the cell faces.

The Reynolds number $\text{Re}=\rho U_{\tau}H/\mu=360$ is based on the friction velocity U_{τ} with $U_{\tau}^2 = (1/2)(-d\overline{\Pi}/dx)(H/\rho)$. The overbar denotes Reynolds averaging.¹⁰ The channel dimensions and resolutions in x (streamwise), y (wall-normal), and z (spanwise) are $1.5H \times H \times 0.75H$ and $48 \times 192 \times 48$, which are similar to Ref. 11. We have chosen to use this small domain, since it allows performing simulations, using relatively little computer resources. The small domain influences the numerical solution quantitatively, while qualitatively the solution resembles the solution on large domains.¹¹ In the present study quantitative details are of less importance, since the aim is to compare different fiber stress models. The grid is nonuniform in the y-direction such that y of the *i*th grid-point $0.5\{1 + \arctan[3(i/192)]$ is given by (-0.5)]/arctan(1.5)}. The grid-spacing in ν/U_{τ} -units at the wall and in the channel center are $11 \times 0.88 \times 5.6$ and 11 $\times 2.9 \times 5.6$, where the kinematic viscosity equals $\nu = \mu/\rho$. This resolution resembles the one used in Ref. 12, which is



FIG. 1. Comparison of mean (a) and standard deviation (b) of fluid velocity as a function of wall distance, Kim *et al.* (Ref. 12) (circles), $1.5H \times H \times 0.75H$ channel (dashed lines), $6H \times H \times 3H$ channel (solid lines).

generally regarded as sufficient resolving all important spatial scales. The time step is $\Delta t=3.6 \times 10^{-2} \nu/U_{\tau}^2$. According to Reynolds decomposition \cdots , $(\cdots)'$, and $(\cdots)_{\rm rms}$ denote mean part, fluctuating part, and standard deviation. A variable with superscript + is scaled with μ , ρ , and U_{τ} .

In Fig. 1 fluid velocity statistics are compared to the data of Ref. 12, who also performed DNS of turbulent channel flow at Re=360. The differences are due to different channel dimensions, which is supported by the results of a simulation performed on a larger domain with dimensions $6H \times H \times 3H$. In Fig. 1 it is shown that the corresponding results agree very well with Ref. 12, which verifies our simulation code.

B. Fibers

We assume a suspension of buoyantly free, cylindrical rods of length *L* and diameter *D*, with aspect ratio $r = L/D \gg 1$. The effects of a finite *r* and Brownian motion are ignored. Furthermore it is assumed that the fibers are noninteracting, massless and substantially smaller than the Kolmogorov length-scale.

Under these conditions the fibers translate as material points and rotate as material lines,²

$$\dot{\mathbf{x}} = \mathbf{u}, \quad \dot{\mathbf{p}} = \nabla \mathbf{u}^T \cdot \mathbf{p} \cdot (\boldsymbol{\delta} - \mathbf{p}\mathbf{p}).$$
 (2)

Here x is the particle position vector, p is the particle orientation unit vector and the overdot represents time differentia-

The effects of finite r on rotary motion,¹³

$$\dot{\boldsymbol{p}} = \frac{r^2 - 1}{r^2 + 1} \boldsymbol{\nabla} \boldsymbol{u}^T \cdot \boldsymbol{p} \cdot (\boldsymbol{\delta} - \boldsymbol{p}\boldsymbol{p}) + \frac{2}{r^2 + 1} \boldsymbol{\Omega} \cdot \boldsymbol{p}, \qquad (3)$$

are of order r^{-2} and can therefore be ignored for $r \ge 100$. Here $\Omega = \frac{1}{2} (\nabla u^T - \nabla u)$ is the vorticity tensor.

Brownian motion induces diffusion on fiber position and orientation, where the latter is most pronounced.² Adding diffusion to fiber rotation gives

$$\dot{\boldsymbol{p}} = \boldsymbol{\nabla} \boldsymbol{u}^T \cdot \boldsymbol{p} \cdot (\boldsymbol{\delta} - \boldsymbol{p} \boldsymbol{p}) - d_r \boldsymbol{\nabla}_{\boldsymbol{p}} \ln f, \qquad (4)$$

where $f(\mathbf{p}, \mathbf{x}, t)$ is the probability of finding a fiber with orientation \mathbf{p} at position \mathbf{x} at time t. The rotary diffusivity $d_r = 1/t_B$, where t_B is a diffusion time scale,²

$$t_B \approx \frac{\pi \rho \nu L^3}{[\ln(r) - 0.8] 3k_B T},\tag{5}$$

with $k_R \approx 1.4 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$ the Boltzmann constant and T the temperature. Paschkewitz $et al.^5$ simulated fiber suspension channel flow at Re \approx 700, $c = 7.5 \times 10^{-3}$, r = 100, and $t_B/t_L \approx 25$, with t_L the large eddy turnover time t_L $\approx (H^2/\nu) \text{Re}^{-1}$ (Ref. 10). The drag coefficient was 18.5% smaller than the corresponding value in Newtonian flow. Increasing t_B/t_L from 25 to ∞ induced a further marginal change of 0.6% to the drag coefficient. This means that in the study of drag reduction Brownian motion can be neglected for $t_B/t_I \gtrsim 100$. We can recast this condition into a minimum fiber length. If we assume Re=360, r=100, $T=3 \times 10^2$ K, $\nu = 1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$, $\rho = 10^3 \text{ kg m}^{-3}$, and $H = 5 \times 10^{-2} \text{ m}$, then $L \ge 3 \times 10^{-8}$ m, in order for the neglect of Brownian motion to be justified. This constraint is met by commercially available rigid-rod like polymers. For instance xanthan polysaccharide and schizophyllum polysaccharide used in drag reduction experiments¹⁴ have a length of $L \sim 1 \times 10^{-6}$ m.

The assumption that the particle length *L* is smaller than the Kolmogorov length scale $l_K \approx H \operatorname{Re}^{-3/4}$ (Ref. 10) is valid when $L/H \ll \operatorname{Re}^{-3/4}$. Assuming fiber length to be smaller than one-tenth of the Kolmogorov length scale at Re=360 and $H=5 \times 10^{-2}$ m gives $L \leq 6.0 \times 10^{-5}$ m.

The neglect of inertia is justified when the particle response time $t_p \approx (\rho_p/\rho)D^2/\nu$ (Ref. 15) is smaller than the Kolmogorov time scale $t_K \approx (H^2/\nu) \text{Re}^{-3/2}$ (Ref. 10), i.e., when $D/H \ll \text{Re}^{-3/4}$. Here ρ_p is the particle mass density, which is assumed to be equal to the fluid mass density ρ . This condition is automatically satisfied when *L* is smaller than the Kolmogorov length scale, since $r=L/D \gg 1$.

When considering interactions, it is convenient to distinguish between different regimes of concentration.² In the dilute regime $cr^2 \leq 1$, the distance *d* between a fiber and its nearest neighbor $d \geq L$ and interactions can be neglected. In the semidilute regime $cr^2 \geq 1 \geq cr$, the spacing between the fibers is $d \leq L$ but $d \geq D$. Although physical contacts are rare, the fibers are affected by hydrodynamic interactions. In the concentrated regime $cr \ge 1$, the spacing is $d \le D$, and fibers are in constant physical contact with each other. Drag reducing suspensions fall within the semidilute regime. Folgar and Tucker measured fiber orientation in laminar shear flow of semidilute suspensions with r=83 and cr^2 ranging between (3-40) (Ref. 16). They interpreted the effect of interactions as a rotary diffusivity d_r proportional to the characteristic shear γ ,

$$d_r = C_I \gamma, \quad \gamma = \sqrt{2S:S}. \tag{6}$$

The proportionality constant C_I defines the relative importance of hydrodynamic interactions on fiber motion. This parameter was found to range between $(3-4) \times 10^{-3}$. These rather low values suggest that neglecting interactions does not introduce errors that are too large.

C. Fiber distribution function

The probability $f(\mathbf{p}, \mathbf{x}, t)$ of finding a fiber with orientation \mathbf{p} at position \mathbf{x} at time t is governed by²

$$\frac{\partial f}{\partial t} + \boldsymbol{\nabla} \cdot (\dot{\boldsymbol{x}}f) + \boldsymbol{\nabla}_{\boldsymbol{p}} \cdot (\dot{\boldsymbol{p}}f) = 0, \qquad \int f d\Omega = 1, \tag{7}$$

where ∇_{p} is the gradient operator on the unit sphere Ω and \dot{x} and \dot{p} are given by Eq. (2). The first equality in Eq. (7) is referred to as the Fokker-Planck equation. It describes the advection of probability in fiber position and orientation space. The second equality in Eq. (7) is needed due to homogeneity of the Fokker-Planck equation. It expresses that the spatial fiber distribution is homogeneous.

A particle method is used to identify properties of f in turbulent Newtonian channel flow. The simulations are oneway coupled. The method is similar to the one used in Ref. 17 and involves computing fiber trajectories in position and orientation space, governed by Eqs. (1) and (2). Fluid velocity and velocity gradient values at the position of the particles are interpolated. The interpolation scheme uses the Fourier-basis in the streamwise direction and a third order polynomial fit in spanwise and wall-normal directions. We use expensive Fourier interpolation rather than polynomial interpolation in the streamwise direction, since the latter induces large wiggles with period Δ_x/u_x in the time signal of particle velocity. Here Δ_x is the grid-spacing in the streamwise direction. The position and orientation of the fibers is advanced in time with the second-order Adams-Bashforth scheme.

First, we approximate the **p**-dependence of f on an individual point, by computing $\mathbf{p}(t)$ of 10^4 fibers which follow the same material point $\mathbf{x}(t)$. The initial conditions for the orientation of the fibers are random. Figure 2(a) shows the development of the eigenvalues a_{ii} of $\langle \mathbf{pp} \rangle$. Here the average $\langle \cdots \rangle$ is taken over particles positioned on the same point. By definition, $a_{11}+a_{22}+a_{33}=1$ and $a_{11} \ge a_{22} \ge a_{33} \ge 0$. It appears that f develops from the isotropic initial distribution $(a_{11} = 1/3)$ via a planar distribution $(a_{33}=0)$ into a unidirectional distribution $(a_{11}=1)$. This demonstrates that after a finite time interval the orientation of a fiber does not depend on its initial conditions. By considering many such traces as shown



FIG. 2. (a) Development of the first and third eigenvalues a_{11} and a_{33} of $\langle pp \rangle$ along a fluid path. (b) The two-point correlation Γ of qq(x,t), with q defined in Eq. (8). (c) Absolute value of the Fourier transform of Γ (dots).

in Fig. 2(a), this interval is estimated as $10^3 \nu / U_{\tau}^2$. After this time interval the distribution function has the following form:

$$f(\boldsymbol{p},\boldsymbol{x},t) = \delta(\boldsymbol{p} - \boldsymbol{q}(\boldsymbol{x},t)), \tag{8}$$

with $\delta(\cdots)$ the Dirac-delta function and q(x,t) the direction of a fiber at position x at time t. It is noted that the same behavior occurs (not shown) when the effects of finite aspect ratio are taken into account in the description of fiber rotation, i.e., when rotation is given by Eq. (3).

Next the *x*-dependence of *q* is studied. For this purpose a simulation is carried out of Eqs. (1) and (2) for 10^6 particles with initial random *x* and *p*. After $10^3 \nu/U_{\tau}^2$ time units, fiber orientation is assumed independent of initial conditions. Samples of *q*(*x*,*t*) are taken from which the two-point corre-

lation $\Gamma(|dx|) = \overline{qq(x,t): qq(x+dx,t)} = \overline{\cos^2 \theta}$ is computed. The angle θ is between the orientation vectors of two fibers. The average ... is taken over all particle pairs separated a distance |dx| at several time instances. Fully correlated and uncorrelated orientations correspond to $\Gamma=1$ and $\Gamma=1/3$, respectively. The statistical errors, related to the finite sample size, restrict the range for which Γ is accurately determined to $|dx|^+ \ge 0.05$. It means that the number of particle pairs, which are separated a distance ≤ 0.05 from each other, is inadequate to obtain an accurate prediction for $\Gamma(|dx|)$. The result in Fig. 2(b) shows that $\Gamma(|dx|^{+}=0.05)=0.88$. This indicates that there are significant variations of qq over a spatial separation of 0.05 ν/U_{τ} -units. This suggests that in turbulent flow q(x) is discontinuous, similar to the concentration of a passive scalar at zero diffusivity.¹⁸ This hypothesis is strengthened by the power spectrum of qq, defined as the Fourier-transform of Γ , given in Fig. 2(c). For wavenumber $k > 2\pi/l_{K}$, the spectrum decays as k^{-1} , similar to the energy spectrum of a passive scalar in turbulent flow at small diffusivity.¹⁸ Here l_K is the Kolmogorov length scale, estimated as $2\pi/l_K^+ \approx 2\pi \operatorname{Re}^{-1/4} \approx 1.4$.

D. Fiber stress

The fiber stress τ in Eq. (1) equals the rate of strain projected on the fiber directional vectors by means of a double contraction with the fourth-order moment of the fiber distribution function,²

$$\boldsymbol{\tau} = 2\alpha\mu\boldsymbol{S}:\langle \boldsymbol{pppp} \rangle, \quad \alpha \approx \frac{4cr^2}{3(\ln r - 0.8)}.$$
(9)

Equation (9) involves averaging $\langle \cdots \rangle$ over fibers contained in a volume V, surrounding the point at which the stress is to be determined. The average can be expressed as an integral over Ω and V, weighted with f,

$$\langle \cdots \rangle (\boldsymbol{x}, t) = \frac{1}{V} \int_{V} dV \int_{\Omega} d\Omega f(\boldsymbol{p}, \boldsymbol{y}, t) \cdots .$$
 (10)

By definition the dimensions of V are smaller than the smallest length-scales of ∇u , such that ∇u may be considered constant in V (Ref. 2). The number of particles needed to accurately compute stress in turbulent channel flow is estimated by assuming that ∇u does not change appreciably over 10 ν/U_{τ} -units, being the grid spacing in streamwise direction in well-resolved simulations.¹² Figure 2(b) shows that $1 - \Gamma(|dx|^+ = 10) \approx 0.45$ indicating that fiber orientation within a grid-cell is distributed with standard deviation ~ 1 . The standard deviation of the sum of N orientations is therefore $\sim 1/\sqrt{N}$ (Ref. 10). Thus for a 10% accuracy in stress $\sim 10^2$ particles per grid-cell are necessary. A direct numerical simulation of channel flow at low Re requires $\sim 10^6$ gridcells. To simulate ten large eddy turnover times H/U_{τ} , our computer code would take ~100 years on an AMD Opteron 2 GHz processor.

Due to the inadequate number of particles, instantaneous stress cannot be computed from our particle simulation. On the other hand, Reynolds averaged stresses can be computed and are plotted in Fig. 3(a), using $\alpha = 1$. To compute these averages it is assumed that $\overline{\langle \cdots \rangle} = \cdots$. It is noted that these



FIG. 3. Mean fiber stress using $\alpha = 1$ (a) and mean fiber orientation (b) in Newtonian flow, computed with the particle method.

stresses are not coupled to the fluid equations of motion. An interesting feature is that at the wall all stress components are zero, i.e., fibers oriented in the direction of zero strain. This is related to the fact that at the wall $S = S_{xy}(e_xe_y + e_ye_x) + S_{yz}(e_ye_z + e_ze_y)$ and the observation that at the wall $p_y = 0$, as shown in Fig. 3(b). Here e_i are the Cartesian unit vectors.

III. MOMENT APPROXIMATION

A. Moment evolution equation

As pointed out in the previous section, instantaneous fiber stress cannot be computed using a particle method. A statistical method based on Eq. (7) can be used. Due to its discontinuous nature, f cannot be solved directly and approximations are needed. In this work the so-called moment approximation is investigated.

The transport equation for the second moment of f is derived by multiplying Eq. (7) by pp and applying the averaging operator [Eq. (10)] to the result,

$$\frac{1}{V} \int_{V} dV \int_{\Omega} d\Omega \left(pp \left[\frac{\partial f}{\partial t} + \nabla \cdot (uf) + \nabla_{p} \cdot [\nabla u^{T} \cdot p \cdot (\delta - pp)f] \right] \right) = 0.$$
(11)

$$\frac{1}{V} \int_{V} dV \left[\frac{\partial qq}{\partial t} + \nabla \cdot (uqq) - \nabla u^{T} \cdot qq - qq \cdot \nabla u + 2\nabla u : qqqqq \right] = 0.$$
(12)

Applying Eq. (10) and assuming ∇u to be constant in V yields

$$\frac{D\langle pp \rangle}{Dt} - \nabla u^T \cdot \langle pp \rangle - \langle pp \rangle \cdot \nabla u + 2\nabla u : \langle pppp \rangle = s, \quad (13)$$

with

$$s = -\frac{1}{V} \int_{V} dV \nabla \cdot \{ [u(y) - u(x)] qq \}$$
$$= -\frac{1}{V} \int_{V} dV \nabla \cdot [\nabla u(x) \cdot (y - x) qq].$$
(14)

Here y is a position vector varying over V, x is the position of the center of V, $\nabla u(x)$ is the velocity gradient at x and $\langle pp \rangle$ and $\langle pppp \rangle$ are the second and fourth-order moments of f. Equation (13) cannot be solved directly. Two unknowns have to be modeled: the fourth-order moment and the subgrid-term s.

The fourth moment appears in the equation of change for the second moment due to the nonlinear dependence of fiber rotation on fiber orientation [Eq. (2)]. To obtain a closed set of equations a model must be adopted to express $\langle ppp \rangle$ in terms of $\langle pp \rangle$. Accurate models have been developed by parameterizing the distribution function¹⁹ and by fitting "exact" solutions of the Fokker-Planck equation.^{20,21} In this work, we use the closure developed by Wetzel,²² who extended the method introduced by Cintra and Tucker.²⁰ The closure expresses the principal values of $\langle pppp \rangle$ as functions of the principal values of $\langle pp p \rangle$ by means of a fit to numerical solutions to Eq. (7) for simple flows. The fit coefficients are constrained to produce correct $\langle pppp \rangle$ for the three limiting cases of isotropic, biaxial, and uniaxial distribution functions.

The subgrid term *s* represents the effects of the unresolved variations of qq on $\langle pp \rangle$. The length-scales of these variations are smaller than the linear dimensions of *V*. We use diffusion to model this term,

$$\boldsymbol{s} = D\nabla^2 \langle \boldsymbol{p} \boldsymbol{p} \rangle, \tag{15}$$

where *D* is referred to as the artificial diffusivity. This is the conventional approach used in numerical simulations of polymer moment equations^{5,6} since it ensures stable and smooth numerical solutions.²³

B. Performance

The accuracy of the moment approximation [Eqs. (1), (9), (13), and (15)] is investigated by comparing Reynolds averaged stress to the "exact" values obtained from the particle simulation.

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FIG. 4. Mean stress components using $\alpha = 1$ in Newtonian flow, computed with the particle method (solid lines), moment approximation using subgrid model Eq. (15) (dashed lines), and moment approximation using subgrid model Eq. (16) (dashed-dotted lines).

The accuracy is determined by two factors: the fourthorder moment closure and the subgrid model. In a previous work, we studied the accuracy of the fourth-order moment closure separately.²⁴ Very accurate results in turbulent channel flow were obtained using the closure scheme developed by Cintra and Tucker,²⁰ which is believed to be less accurate than the scheme used here.²² Therefore we assume that errors introduced by the moment approximation are mainly due to the inadequacy of the subgrid model.

The numerical methods to solve Eqs. (13) and (15) are similar to the methods used for Eq. (1) as described in Sec. II A. In addition the moment and stress components are defined in the cell-centers and zero wall-normal derivatives at the walls are used as boundary conditions for these variables. Simulations are carried out for different values of the artificial diffusivity *D*. This parametric study revealed that the error in mean stress decreases with decreasing *D*. However, for $D \leq \nu$, the solution $\langle pp \rangle$ exhibits spurious wiggles. In Fig. 4 the stress for $D=\nu$ is compared to the "exact" stress. The moment approximation predicts accurate stress for $y^+>20$, while it overestimates stress for $y^+<20$. The moment approximation does not predict zero stress at the wall, in contrast to the "exact" result. In Fig. 4 it is demonstrated that by damping the diffusivity in the near-wall region $(y^+<30)$,

$$\boldsymbol{s} = D\boldsymbol{\nabla} \cdot f_{w} \boldsymbol{\nabla} \langle \boldsymbol{p} \boldsymbol{p} \rangle, \quad f_{w} = \begin{cases} \sin^{2}(\pi y^{*}/60) & \text{if } y^{*} < 30\\ 1 & \text{if } y^{*} > 30 \end{cases}$$
(16)

the moment approximation is improved to predict zero stress at the wall, as well as a qualitatively correct behavior in the near wall region.

IV. DRAG-REDUCED FLOW

A. Comparison to Newtonian flow

The moment approximation [Eqs. (1), (9), (13), and (16)] is used to simulate non-Newtonian fiber suspension channel flow. The simulations are two-way coupled, i.e., the fluid affects the fibers and vice versa. The parameters are $\alpha = 20$, $D = \nu$, and Re=360. Further simulation details are given in Sec. II A. In Figs. 5(a) and 5(b) fluid velocity statistics in fiber suspension flow and Newtonian flow are compared. The larger mean velocity in the fiber suspension implies that the fibers reduce the drag coefficient $C_D = (U_\tau/U_b)^2$. The bulk velocity is defined as $U_b = (1/H) \int_0^H \bar{u} dy$. The outward shift of the intercept of the linear profile ($y^+ < 10$) and the logarithmic profile ($40 < y^+ < 100$) indicates a thickening of the vis-



FIG. 5. Mean (a) and standard deviation (b) of the fluid velocity. Comparison between drag-reduced flow computed with full constitutive equation (solid lines), drag-reduced flow computed with simplified constitutive equation (circles), and Newtonian flow (dashed lines).

cous and buffer layers. The parallel upward shift of the mean velocity profile in the logarithmic layer $(40 < y^+ < 100)$ indicates that the flow is in the "small drag reduction" (SDR) regime, whereas in the "large drag reduction" regime the slope increases.²⁵ Typical for SDR-flow, the turbulent velocity intensity $u_{\rm rms}$ is increased in x and reduced in y and z. These results are consistent with findings of previous numerical research.⁵

B. Lumley's scenario

The aim of the present work is to provide a simplified, yet accurate, description of the effect of fibers on fluid mechanics. The reduced model is based on arguments provided by Lumley, who explains drag reduction by an additional viscosity, i.e., by modeling fiber stress as⁷

$$\boldsymbol{\tau} = 2\boldsymbol{\mu}_{\boldsymbol{f}} \boldsymbol{S}. \tag{17}$$

Lumley argues that the fiber viscosity μ_f is induced by turbulence. As an effect turbulence is dampened, which results in a thickening of the viscous sublayer and consequently a reduction of the drag. To explore this idea we begin by defining μ_f based on the integral energy balance, which in nondimensional form reads



FIG. 6. (a) Normalized fiber viscosity $\mu_f/(\alpha\mu)$ as a function of wall distance in drag-reduced flow (solid line) and Newtonian flow (dashed line). (b) Newtonian dissipation $\vec{\epsilon}^+$ (lines) and normalized fiber dissipation $\vec{\epsilon}^+_f/\alpha$ (circles) in Newtonian flow (solid line and filled circles) and drag-reduced flow (dashed line and open circles).

$$2U_b^{\dagger} = \int_0^{\text{Re}} (\overline{\epsilon}^{\dagger} + \overline{\epsilon_f}^{\dagger}) dy^{\dagger}.$$
 (18)

Here $\epsilon_f = S: \tau = 2\alpha\mu S: \langle pppp \rangle: S$ is dissipation of kinetic energy due to fiber stress and $\overline{\epsilon} = 2\mu \overline{S:S}$ is dissipation due to Newtonian stress.

Since the drag coefficient $C_D = 1/U_b^{+2}$ is directly related to the fiber dissipation $\overline{\epsilon_f}$, we define μ_f such that the dissipation predicted by Eq. (17), $\overline{\epsilon_f} = 2\mu_f \overline{S:S}$ equals the "real" dissipation,

$$\mu_f = \mu \frac{\epsilon_f}{\overline{\epsilon}} = \mu \alpha \frac{S:\langle pppp \rangle: S}{\overline{S:S}}.$$
(19)

Figure 6(a) shows $\mu_f/(\mu\alpha)$ as a function of wall distance in Newtonian and drag-reduced flow. The normalized fiber viscosity $\mu_f/(\alpha\mu)$ is smaller in drag-reduced flow as compared to Newtonian flow. Figure 6(b) shows that this is linked to a decrease of $\overline{\epsilon}_f/\alpha$, while $\overline{\epsilon}$ is remarkably similar in both flows.

A simulation is carried out of Eqs. (1) and (17) with $\mu_f(y)$ taken from the drag-reduced channel flow simulation, given in Fig. 6(a). Resulting non-Newtonian flow statistics are compared to the simulation of the full constitutive equations in Figs. 5(a) and 5(b). The striking agreement implies that fiber-induced drag reduction can be regarded as an effect

due to an additional Reynolds averaged viscosity. This means that characteristic directions of the fiber stress tensor, are not of key importance for drag reduction. Also its fluctuations in time and space are not relevant.

V. CONCLUSIONS

It is demonstrated that a direct computation of the stress generated by non-Brownian fibers in turbulent channel flow is computationally unfeasible. We have approximated the stress by computing the second-order order moment of the fiber distribution function. The method involves a subgrid term, which is modeled as diffusion. It is shown that the accuracy of the method is improved by applying a walldamping to the diffusivity.

Non-Newtonian fiber suspension channel flow is simulated and compared to Newtonian flow. The fibers induce a reduction of the drag coefficient. A simplified constitutive model as a viscous stress is studied. The proposed fiber viscosity μ_f is defined such that the resulting Reynolds averaged dissipation equals the dissipation of the "exact" fiber stress. The simplified model induces changes in the flow which are nearly identical to those predicted by the full constitutive equations. It is therefore concluded that the characteristic directions of the fiber stress tensor, as well as its fluctuations in time and space can be ignored in the study of drag reduction. Instead the effect of the fibers can be considered as a Reynolds averaged isotropic viscosity, which reduces the complexity of the problem considerably. This result validates important assumptions made in theoretical efforts towards a theory of drag reduction.^{8,26}

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