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Citation (APA)

Tiberius, C. (2025). Parameter Estimation in Terrestrial Networked Positioning and Timing. In A. Khodabandeh, & S. Verhagen (Eds.), *Navigating the Geodetic Landscape: A Tribute to 45 Years of Excellence: Anniversary Volume on the Occasion of Professor P.J.G. Teunissen's 67th Birthday* (pp. 45-59). Springer. https://doi.org/10.1007/978-3-031-99274-2_3

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Chapter 3

Parameter Estimation in Terrestrial Networked Positioning and Timing



Christian Tiberius

Abstract This chapter presents a terrestrial networked system, which provides position and time to mobile users, just like Global Navigation Satellite Systems (GNSS). An accurate time and frequency reference is distributed, through fiber-optic Gigabit Ethernet, to a network of radio base stations. These stations transmit wideband radio signals, enabling positioning by a mobile receiver with an accuracy of one decimeter, in a multipath-rich built-up area. This terrestrial system is envisioned to complement GNSS in particular in built-up and urban areas, as fiber-optic infrastructure is commonly already in place there.

3.1 Introduction

The Global Positioning System (GPS), and Global Navigation Satellite Systems (GNSS) in general, represent wonderful technology for positioning, navigation and timing, and are a great asset to today's society. Though, typically GNSS perform worst where they are actually needed most. In built-up and urban areas, where people live, work, recreate, and move around, satellite signals are often blocked by objects abundantly present in these environments. Also, signals are frequently reflected by these objects, a phenomenon referred to as multipath. Ideally the receiver is exposed to only direct Line-of-Sight signals. Signal reflections gathered by the receiver, and which have travelled a detour, impact (bias) the measurements of range to the satellites, and thereby degrade the positioning, navigation and timing performance.

We discuss a terrestrial networked radio positioning and timing system, which can complement GNSS in particular in those areas. In SuperGPS, the more popular name for the system, we transpose the strengths of GPS/GNSS, yet at the same time overcome the weaknesses. The focus is again on determining the position and time of a mobile receiver, i.e. on the four dimensional coordinates (x, y, z, t) .

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In this chapter we present an overview of the terrestrial networked positioning and timing system, and the recently developed prototype. We discuss parameter estimation aspects of this terrestrial system, and thereby we happily benefit from the many lessons learned on GNSS positioning, navigation and timing. Then an exposition follows on the phenomenon of multipath, and its effects on the measurements of travel-time of the radio signal. This chapter is concluded with experimental positioning and timing results, obtained with the prototype system deployed at the TU Delft campus.

3.2 System Overview

By means of an artist impression, an overview of the SuperGPS system is shown in Fig. 3.1. An atomic clock is typically the source of the system. In the Netherlands, National Metrology Institute VSL in Delft is home of the Dutch national time scale UTC(VSL). The time and frequency reference are distributed over optical glass fibre (in blue), and synchronized wideband radio transmitters, located for instance in lamp-posts and on building roof-tops around, serve mobile users in the street.

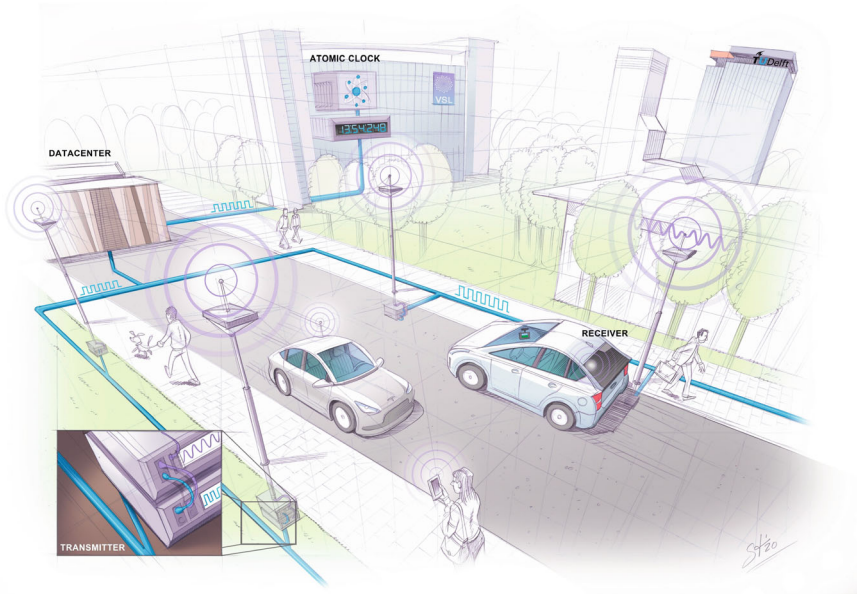


Fig. 3.1 Artist impression of the SuperGPS-system: a network of radio transmitters is synchronized over fibre-optic connections (in blue), to the same atomic clock. Drawing by Stephan Timmers

3.2.1 *Optical Synchronization*

Just like in GNSS, timing is key. Accurate timing across the network of transmitters in SuperGPS is provided through the White Rabbit protocol, Serrano et al. (2009) and Moreira et al. (2009). In short, White Rabbit is an accurate time-frequency reference implemented on digital data communication (Gigabit Ethernet). It has been developed by CERN, the European Organization for Nuclear Research, and deployed in a large network with a wide variety of sensors and actuators.

A White Rabbit network is hierarchical and starts with a so-called grandmaster, connected to an accurate time and frequency reference, such as an atomic clock. Then several layers of so-called switches follow (each switch typically feeds or branches into 18 switches in the next layer), distributing time and frequency further into the tree, and eventually the network ends at so-called nodes, which are the interface to ‘users’; they are for instance connected to sensors and actuators. The interface to electrical components consists of a 1 PPS (pulse-per-second) and a 10 MHz frequency, next to digital time stamps for instance according to UTC(VSL).

3.2.2 *Wideband Radio Ranging and Positioning*

In SuperGPS, just like in GNSS, we rely on one-way ranging, in a set-up with synchronized transmitters, and typically an asynchronous receiver. Using wideband radio signals for ranging brings, next to a higher precision, improved resistance to multipath though increased time resolution. In GNSS, signal bands typically span a few MHz to tens of MHz. In SuperGPS we intend to use hundreds of MHz, or even more.

For the measurement of signal travel-time, the received signal is correlated with a template (known a-priori to the receiver), in a so-called matched filter. The width of the correlation peak for time delay estimation is inversely proportional to the signal bandwidth, see e.g. Kay (1993). A wideband signal offers a good ranging precision (narrow peak) and also the capacity to identify reflections of the signal separately from the desired Line-of-Sight signal. The bottom line is that wideband radio signals allow for precise, unbiased time delay estimation, also in a multipath-rich environment.

For further details on the optical time-frequency distribution and the wideband radio signals we refer to Koelemeij et al. (2022) and Tiberius et al. (2023).

3.2.3 *Prototype System*

A prototype implementation of the system has been deployed on the TU Delft campus, specifically at The Green Village (2023), see Fig. 3.2. Five of the transmitter antennas are visible in red ellipses, as well as the receiver antenna on the roof of the car in front.

Fig. 3.2 Set-up of the prototype SuperGPS system at The Green Village on the TU Delft campus



Optical land-surveying equipment was used to establish an accurate ground-truth trajectory for the car.

Technical details of the prototype system, the equipment used, as well as the challenges, such as sampling a 160 MHz wideband radio-signal at the receiver in a time-critical application, are described in Diouf et al. (2021).

The signal model and signal processing for time delay estimation, in order to determine the signal travel-time, are detailed in Dun et al. (2021).

3.3 P&T Parameter Estimation

Based on measurements of signal travel-time we would like to estimate the receiver position coordinates and the receiver clock-offset. In this section we introduce the model of observation equations, which are non-linear in the receiver coordinates. The intricacies of parameter estimation on the basis of a non-linear model are addressed in the next subsection. In the third subsection an analysis of precision of the parameter estimators is presented, in relation to the geometry of the transmitters and receiver set-up.

3.3.1 Model of Observation Equations

Time delay observable $\underline{\tau}_r^i$, for the radio signal (one-way) travel-time from transmitter i to receiver r results from signal processing in the receiver. Upon multiplication by the speed of light, c , the pseudo-range observation equation at time t reads

$$\underline{p}_r^i(t) = c\underline{\tau}_r^i = d_r^i(t) + c\tau_h + c\delta_r(t) + \underline{e}_r^i \quad (3.1)$$

with d_r^i the geometric distance from transmitter i (typically stationary) to (mobile) receiver r , $d_r^i(t) = \sqrt{(x^i - x_r(t))^2 + (y^i - y_r(t))^2 + (z^i - z_r(t))^2}$, τ_h hardware delays of the transmitter-receiver link, e.g. introduced by the radio front-end (assumed to be identical for all transmitter-receiver links, as the transmitters all consist of identical hardware and configuration; and assumed to be time-invariant as well), $\delta_r(t)$ is the receiver clock offset, as the (mobile) receiver is generally not synchronized to the transmitter infrastructure, and $\underline{e}_r^i$ is the random measurement error (an underscore denotes a random variable). Atmospheric delays are not applicable (also tropospheric delays can be neglected due to the short distances in this system, which range up to hundreds of meters at most). In the sequel, with single epoch snapshot positioning, the hardware delays τ_h are merged with the receiver clock offset $\delta_r(t)$, as these parameters can not be separately estimated.

The standard deviation of the random pseudo-range measurement error $\underline{e}_r^i$ is, in the prototype system, typically at the sub-decimeter level; fundamentally it depends on signal bandwidth and Signal-to-Noise ratio (SNR).

Geometric distance d_r^i is clearly non-linear in the receiver coordinates, and the linearized model of observation equations, with m transmitters, reads

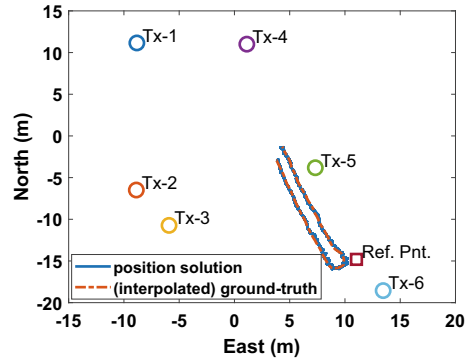
$$E \begin{pmatrix} \Delta \underline{p}_r^1 \\ \Delta \underline{p}_r^2 \\ \vdots \\ \Delta \underline{p}_r^m \end{pmatrix} = \underbrace{\begin{pmatrix} -u_{r,x}^1 & -u_{r,y}^1 & -u_{r,z}^1 & 1 \\ -u_{r,x}^2 & -u_{r,y}^2 & -u_{r,z}^2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -u_{r,x}^m & -u_{r,y}^m & -u_{r,z}^m & 1 \end{pmatrix}}_A \begin{pmatrix} \Delta x_r(t) \\ \Delta y_r(t) \\ \Delta z_r(t) \\ c\delta_r(t) \end{pmatrix} \quad (3.2)$$

with $\underline{p}_r^i$ the pseudo-range observable, $(u_{r,x}^i, u_{r,y}^i, u_{r,z}^i)$ the unit direction vector from receiver r to transmitter i . The vector of unknown parameters contains (increments to) the four dimensional receiver coordinates: position coordinates x_r , y_r , z_r and time (offset) δ_r .

The pseudo-range observables' variance matrix is a diagonal matrix with the measurement variances on the diagonal.

Provided that the observables' variance matrix gets properly propagated, working with so-called Time Difference of Arrival (TDoA) observables is equivalent. Then (for example) differences $\underline{p}_r^i - \underline{p}_r^1$ are used, with $i = 2, \dots, m$, and the receiver clock-offset δ_r cancels in all these combinations; only the receiver coordinates remain as unknown parameters.

Fig. 3.3 Positioning results of a kinematic experiment for a duration of 3.5 min. Taken from Tiberius et al. (2023). Copyright 2023 by the Institute of Navigation (ION)



Positioning results of the prototype system at The Green Village are shown in Fig. 3.3. This concerns a 3.5 min kinematic experiment with a trolley (with 1000 Hz position update-rate). The positions of the six transmitters are indicated by small colored circles. The ground-truth trajectory is shown by the red dashed line, and the resulting TNPS positions are shown by the blue trace. Positions are shown in a local two-dimensional horizontal coordinate system (East, North).

3.3.2 Non-linear Least-Squares

Starting from the observation Eq. (3.1) and collecting the pseudo-range measurements from m transmitters, p_r^1 to p_r^m , in vector y , and the receiver position coordinates and the receiver clock offset in vector x , yields the following non-linear model

$$E(\underline{y}) = F(x) \quad (3.3)$$

and least-squares estimation amounts to, taking the inverse of the observables' variance matrix as a weight matrix, minimizing the following objective function

$$\min_x \|y - F(x)\|_{Q_y}^2 \quad (3.4)$$

Generally no closed form solution is available, and even if it would exist, as $\hat{x} = G(y)$, the estimator is inherently biased, as $E(\hat{x}) \neq G(E(y))$, but instead $E(\hat{x}) = E(G(y))$ (Teunissen 1989). Yan et al. (2008) presents an indication of the effect in a pseudo-range-based decimeter-precision positioning set-up, being a 2 mm effect, in terms of the measurement, over a distance of 5 m. In satellite positioning, with distances beyond 20.000 km, these effects are really negligible, but in local terrestrial positioning systems (and indoor systems in particular) they may not.

In practice the above non-linear model is typically approximated by a linearized one, and the solution is obtained iteratively. Conditions for convergence are given in

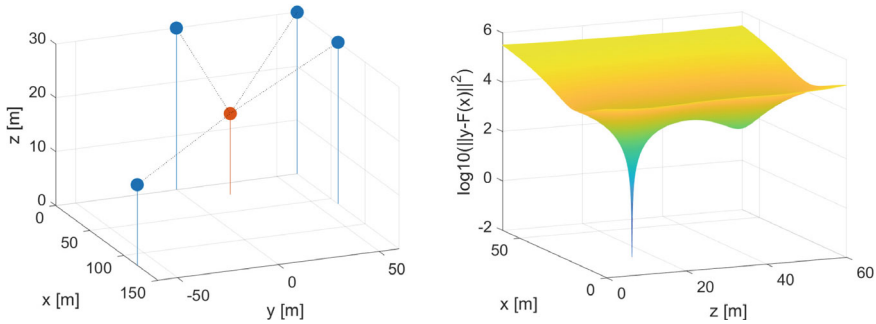


Fig. 3.4 Three dimensional geometry of a synchronized range-based positioning set-up (no receiver clock-offset), at left. There are four transmitters (blue dots) and one receiver (red dot); the redundancy equals 1. The transmitter position coordinates are $(0, 0, 30)$, $(0, 60, 30)$, $(60, 60, 30)$ and $(120, -60, 15)$. The receiver position coordinates are $(20, 20, 15)$. Least-squares objective function in logarithmic scale $\log_{10}(\|y - F(x)\|_{Q_{yy}}^2)$, in the x - z -plane (with $y = 20$), at right

Teunissen (1990b). An example of the above (non-linear) objective function is shown in Fig. 3.4 for a range-based positioning set-up. The global minimum is attained for $x = 20$, $z = 15$, and additionally a local minimum can be seen around $x = 20$, $z = 43$. In this analysis the pseudo-range observables' variance matrix Q_{yy} was set to a scaled identity matrix with a standard deviation of $\sigma = 0.10$ m.

The aforementioned iteration requires an initial guess to start, and clearly, an arbitrary initial guess may lead to a *local* minimum, rather than the desired *global* one. The initial guess should be sufficiently close to the global minimum.

Yan et al. (2013) presents, after a review of iterative descent techniques, so-called direct methods, which, as a result of a simplification of the problem, may be sub-optimal, but do not require iteration. Or, one may use such a direct method to come up with (hopefully) a good initial guess sufficiently close to the global minimum, to start the iteration.

3.3.3 Geometry and Precision

We have restricted positioning with the prototype-system to two dimensions: East and North in the local horizontal plane. The transmitter antennas are all at a 3.5–5.5 m height, and the receiver antenna on the trolley and on the car respectively was at 1.573 m and at 2.006 m height. Hence, there is minimal diversity in height and the elevation angles, at which the receiver ‘sees’ the transmitters, are all fairly small.

The height of the receiver antenna is assumed to be constant and has consequently been substituted in the observation equation to allow for two-dimensional positioning using the prototype system (the experiment was carried out on a local flat, horizontal road). In the following we provide justification for the restriction to two dimensional positioning.

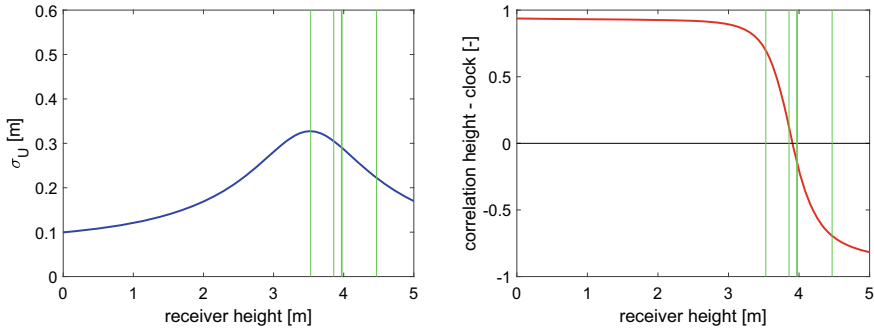


Fig. 3.5 Precision of scenario with six transmitters: standard deviation of estimator for receiver height/Up coordinate $\sigma_{\hat{U}}$ (in blue), at left, and correlation coefficient of height and receiver clock offset (in red), at right, both as a function of the receiver height

Figure 3.5 presents an analysis of the precision, according to model (3.2) (with $\sigma_p = 0.03$ m), with six transmitters and the receiver at start of kinematic track, see Fig. 3.3. The precision of the East and North coordinates (not shown here) is well within 10 cm, and only at a few centimeter level in the middle of the area of interest (typically 2 cm). The standard deviation for the height/Up coordinate is much poorer, see Fig. 3.5, where the vertical green lines mark the transmitter-heights.

The bottom-line is that in theory three dimensional positioning is possible, but due to the limited set up of the prototype, the vertical coordinate has a poor precision; its standard deviation ranges up to several decimeters.

In addition we consider the scenario when the receiver is at the origin (0, 0) of Fig. 3.3 at zero height. Then, with fives transmitters (Tx-6 muted), the correlation between receiver height and receiver clock offset is 0.999, see Fig. 3.6, hence, extremely large! And for a zero receiver height, the standard deviation for the height/Up coordinate estimator is 0.73 m. In the design matrix A of model (3.2), see Table 3.1, the elements of the first two columns vary, spanning the full range $[-1, +1]$, see also Fig. 3.3, transmitters are in all directions, but the elements of the third column are all around -0.35 .

For this five transmitter scenario and the receiver in the origin, all unit-direction vectors, see Table 3.1, nearly lie on a cone with the centerline (approximately) to the local zenith.

Table 3.1 Design-matrix of positioning geometry with five transmitters

0.59	-0.75	-0.30	1.00
0.76	0.56	-0.34	1.00
0.44	0.80	-0.41	1.00
-0.10	-0.94	-0.33	1.00
-0.82	0.43	-0.39	1.00

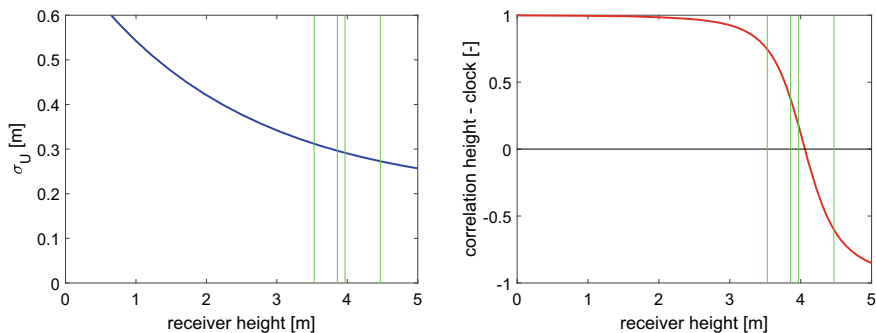


Fig. 3.6 Precision of scenario with five transmitters: standard deviation of estimator for receiver height/Up coordinate $\sigma_{\hat{U}}$ (in blue), at left, and correlation coefficient of height and receiver clock offset (in red), at right, both as a function of the receiver height

All unit direction vectors from the receiver to the transmitters in the linearized model have the same angle with a vector a , that is $u_r^{iT} a = \text{constant}$ for all $i = 1, \dots, m$, and by the presence of the unknown receiver clock offset (corresponding to a column of all ones in the design-matrix), this results in a singularity (and a rank defect of the design-matrix), Teunissen (1990a). In the above example, vector a is (approximately) pointing to the local zenith $a \approx (0, 0, 1)$, and hence maximum correlation occurs between the estimators for the local height and the receiver clock offset. Estimation of the height coordinate (and the receiver clock offset as well) can be done only with very poor precision. In the limiting case the receiver height is un-determined. All elements of the third column, pertaining to the Up- or height-coordinate of the receiver, are approximately equal. The cone geometry is shown in Fig. 3.7.

Fig. 3.7 Geometry of all receiver (r)—transmitter (i) unit direction vectors on a cone, with center-line by vector a , with $i = 1, \dots, 5$ (Teunissen 1990a)

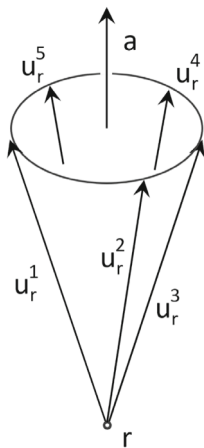
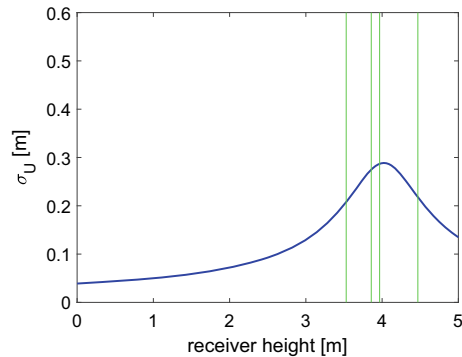


Fig. 3.8 Precision of scenario with five transmitters without receiver clock offset: standard deviation of estimator for receiver height/Up coordinate σ_U (in blue), as a function of the receiver height



When we leave out the receiver clock offset and estimate only the three receiver position coordinates, the precision of the receiver height is shown in Fig. 3.8. For a zero receiver height we clearly reach a sub-decimeter precision, but at a receiver height of about 4 m, the standard deviation attains a maximum of 0.29 m: then all transmitters, and the receiver as well, are (approximately) in a (horizontal) plane, meaning that the horizontal coordinates can be precisely estimated (as the receiver ‘sees’ transmitters in all directions), but the vertical component is still hardly or poorly estimable; the range or distance measurements hardly contain any information about the vertical component of the receiver position.

With a local positioning system, as typically there are practical restrictions on the installation of transmitters, one should be well aware of the various particular geometries discussed in this section.

3.4 Multipath

This section presents a short exposition on the phenomenon of multipath, and its effects on time-delay measurements such as the pseudo-ranges. A visual demonstration of multipath is given by means of image processing: each row of pixels is considered to be transmitted signal and subsequently received with a delay, and, additionally a couple of extra copies (of the same row-signal, but delayed and attenuated) are received.

The results of the various multipath scenarios are shown in Fig. 3.9. The image at top left shows the original photo, which has been transmitted and was received without attenuation and a delay of 10 pixels, but has been shifted back by 10 pixels to the left, to correct for this delay of the Line-of-Sight-signal, hence we just see the original image. The Line-of-Sight-delay-correction of 10 pixels has also been applied to the other images.

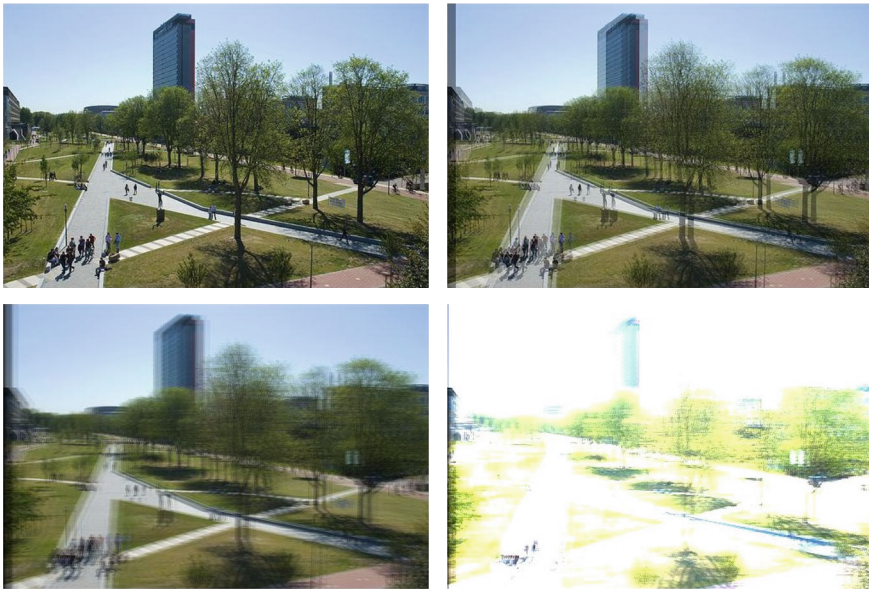


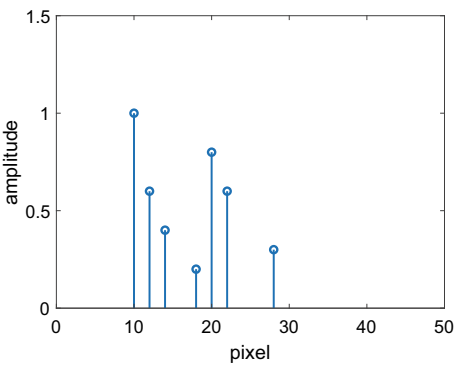
Fig. 3.9 Multipath: Line-of-Sight signal delayed, and corrected for (top left), Line-of-Sight signal and one reflection (top right), Line-of-Sight signal with 6 copies (bottom left), and Line-of-Sight signal with 6 copies but not corrected for additional brightness by reflections (bottom right)

The image at top right shows the Line-of-Sight signal, plus one copy with a relative delay of 10 pixels, and a (relative) amplitude of 0.8. One can clearly see a ghost-image.

The lower left image shows the result of a multipath-channel, with the direct Line of Sight-signal and up to six copies, shown in the impulse response graph of Fig. 3.10 (the impulse response is modelled here by delta Dirac pulses, hence copies of the signal are only delayed and attenuated, not distorted). This image has become quite blurred, and the locations of objects are hard to pin-point accurately. In a similar way, a radio-receiver gets ‘confused’ when having to determine the time-of-arrival or the travel-time of an incoming radio-signal in a multipath-environment. Note that a radio-channel with just six copies is still a fairly moderate multipath-channel. In reality one may experience way more multipath in built environment (in the order of several tens of signal copies/reflections).

To complete this visual demonstration of multipath we show the lower right image in Fig. 3.9. As each reflection *adds* energy to the received signal, the pixels should get brighter the more copies there are. This shows clearly here. With six copies, the image at bottom right is a lot brighter than the image top left with just the delayed Line-of-Sight signal. The other three images actually have been corrected for this ‘brightness’-effect in order to allow for a fair mutual comparison (all three Red-Green-Blue values equal to 0 yields a black pixel, and all three values equal to 255 yields a white pixel). As a side-note we comment that multipath in built environment

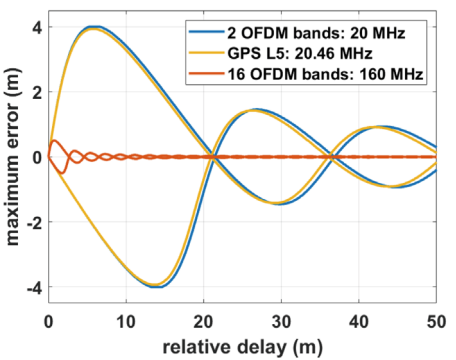
Fig. 3.10 Multipath channel impulse response, with the delay in pixels on the horizontal axis (the Line-of-Sight signal arrives with a delay of 10 pixels), and the amplitude, relative to the Line-of-Sight, along the vertical axis



is a disadvantage to ranging and positioning, but is a blessing to telecommunication, as multipath increases the received energy per transmitted bit and thereby improves performance (for instance in terms of Bit-Error-Rate (BER)).

Resistance of ranging to multipath is provided by signal bandwidth. Figure 3.11 shows so-called multipath error envelopes, as a function of the delay of the reflection relative to the Line-of-Sight signal, and assuming in this analysis a relative amplitude of the reflection of 0.6. The maximum multipath error is given along the vertical axis, see Tiberius et al. (2023) for further details. The yellow and blue lines, both representing a 20 MHz wide signal, demonstrate a maximum error of about 4 m. The red line, representing 160 MHz of signal bandwidth, as used in the SuperGPS prototype system, has a maximum error of about 0.5 m, hence, one order of magnitude smaller.

Fig. 3.11 Multipath error envelopes. In blue and yellow signals with 20 MHz of bandwidth, in red a signal with 160 MHz of bandwidth



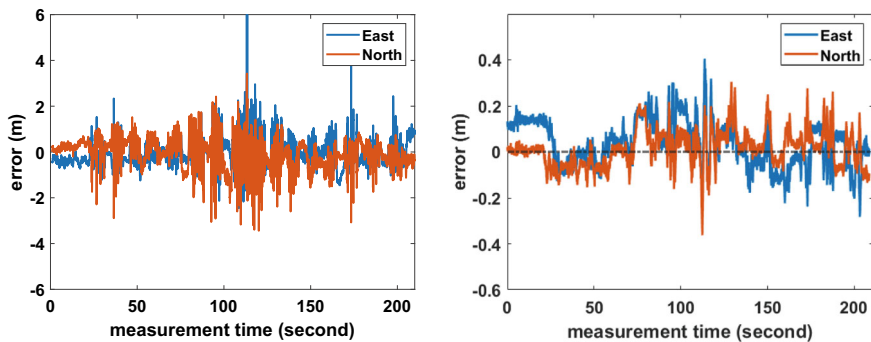


Fig. 3.12 Position errors, in terms of local horizontal East and North coordinates, as a function of time (estimated positions minus ground-truth). At left using 20 MHz of signal bandwidth for ranging, and at right 160 MHz. Taken from Tiberius et al. (2023). Copyright 2023 by the Institute of Navigation (ION)

3.5 Positioning Results

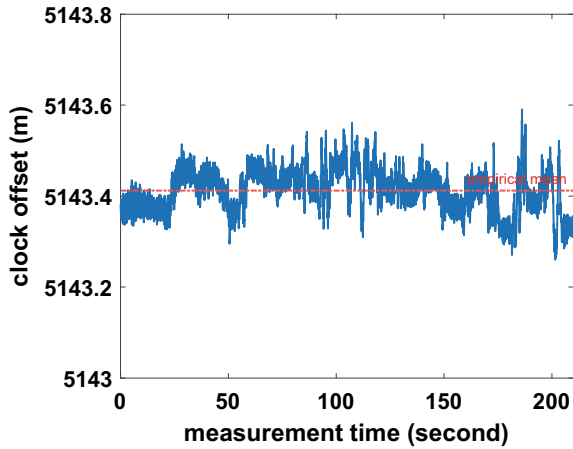
Positioning results are presented in Fig. 3.12. Shown are the position error in local East (blue) and North (red) of the (mobile) receiver, over a 3.5 min trajectory, at left with 20 MHz of radio signal bandwidth (RMSE are 82 and 68 cm resp. for East and North), and at right with 160 MHz (10 and 8 cm resp. for East and North (RMSE)). The vertical scale is different by one order of magnitude. The effect of signal bandwidth is prominent.

3.6 Timing Results

Applications of timing lie in power utilities (think of frequency monitoring in power grid, detection of faults, and measuring and managing energy balance), in finance (think of stock exchanges and trading platforms; regulatory requirements on time stamping transactions), data centers (e.g. server synchronization), media (synchronization of equipment for production and broadcasting), and telecommunication (e.g. synchronization of for instance 5G mobile network base stations, for hand-over and radio frequency optimization).

Timing results of the SuperGPS prototype are shown in Fig. 3.13. In this run the receiver, while slowly moving, was actually—through optical glass-fibre—synchronized to the transmitter infrastructure, so ideally we would find the receiver clock δ_r close to zero. However, we are actually estimating $c\tau_h + c\delta_r(t)$ (including transmitter and receiver hardware delays), cf. (3.1), as one can tell from the large value of about 5143 m. With this graph we can assess the precision of wireless time

Fig. 3.13 Estimated receiver clock offset, expressed in meter, for a synchronized receiver. Taken from Tiberius et al. (2023). Copyright 2023 by the Institute of Navigation (ION)



transfer. The empirical standard deviation $\sigma_{\hat{\delta}_r}$ is 0.04 m, or at the 0.1 ns level for $\sigma_{\hat{\delta}_r}$, but we cannot conclude about the absence of biases, and this also underlines that wireless time transfer is only possible with meticulous equipment calibration.

3.7 Concluding Remarks

In this chapter we presented a terrestrial hybrid optical-wireless system for positioning and timing, particularly suited for use in built-up and urban areas. A prototype system was developed and deployed in an outdoor scenario at the TU Delft campus. This system experimentally demonstrated a one decimeter position accuracy and sub-nanosecond timing precision.

Acknowledgements This research-project was supported by the Dutch Research Council (NWO) under grant 13970—SuperGPS, and was carried out by a team consisting of Gerard Janssen, Jeroen Koelemeij, Erik Dierikx, Cherif Diouf, Han Dun, and the author. We would like to thank Lolke Boonstra and Terence Theijn from TU-Delft ICT-FM and Rob Smets of SURF, the collaborative organization for ICT in Dutch Education and Research for their expertise and support of the optical infrastructure. We also thank Loek Colussi and Frank van Osselen of Agentschap Telecom (now Rijksinspectie Digitale Infrastructuur—RDI) and René Tamboer and Tim Jonathan of The Green Village for their support in realizing the prototype.

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