Dynamic Modeling of Fluid Power Transmissions for Wind Turbines

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Abstract

Fluid power transmission for wind turbines is quietly gaining more ground/interest. The principle of the various concepts presented so far is to convert aerodynamic torque of the rotor blades into a pressurized fluid flow by means of a positive displacement pump. At the other end of the fluid power circuit, the pressurized flow is converted back to torque and speed by a hydraulic motor.

The main advantage of a hydrostatic transmission over geared and direct drive systems is the possibility to vary the transmission ratio. Thus it is possible to operate with variable rotor speed (required for maximum energy extraction), whilst using a synchronous generator directly coupled to a grid, thereby eliminating the need for an AC frequency converter and a voltage transformer. Furthermore, hydraulic drives not only have a higher power density than electrical drives, but their use also allows for alternative arrangements of components. This provides the opportunity to significantly reduce the nacelle mass.

Previous publications on fluid power transmissions for wind turbines have mostly been focused on the energy efficiency of the system, based on steady state simulations and measurements. Little has been mentioned about the dynamic behavior, especially regarding the inherent damping characteristics of fluid power transmissions. This paper presents a theoretical model of the fluid power transmission and the analysis of the influence of the main design parameters on the dynamic behavior of the system.

1 Introduction

1.1 Background

The application of fluid power transmission in wind turbines is quietly gaining interest. At the moment, projects to develop this technology are underway in the Scotland (commercial) [1], Norway (commercial) [2], USA, Germany and The Netherlands [3].

In all projects the concept is that the main shaft connects a positive displacement pump to the rotor. The pump converts the torque and speed of the rotor into a pressure flow. At the other end of the fluid power circuit, the pressure flow is again converted into torque and speed by a hydraulic motor. Figure 1 displays a hydraulic diagram of the principle.

The reasoning behind these projects is that this form of transmission presents a number of advantages over geared and direct drive systems, such as:

- Dynamic variable transmission allows for synchronous generator, thus no frequency converter is required.
- Reduced torsional vibrations. Not only the rotor and generator are physically decoupled, but also the damping characteristics of the fluid power transmission will reduce wear and benefit the power quality.

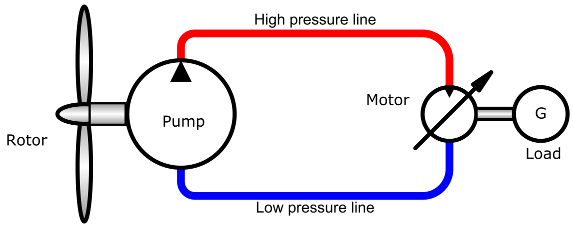


Figure 1: The principle concept of applying fluid power transmission in wind turbines

- High power density resulting in reduced nacelle weight and space.
- Mature technology with high reliability of components

Hydraulics used to be associated with poor energy efficiency and contamination due to leakage. However, as the focus on energy efficiency has grown, industry has responded by producing more efficient drives, higher quality fluids and designing more efficient circuits.

1.2 Problem

Understanding the dynamic behavior of a system is a necessary condition for applying it. The goal of this research is to develop a general dynamic model of a fluid power transmission for wind turbines, in order to gain better insight on the dynamic behavior of fluid power transmissions and explore the influence of the main design parameters.

1.3 Approach

In order to study the influence of the design parameters, a fluid power transmission is modeled for a wind turbine with 1MW rated power capacity. Characteristics of the hydraulic drives are based on commercial available data with rated operational pressures of 350 bar. A synchronous generator (4 poles, 1500 rpm) is used. The approach is to derive a mathematical model from basic physical principles. The theory in this paper is a precedent to the design of the controller.

2 Principles of Fluid Power

2.1 The Hydraulic Drive

The primary function of hydraulic pumps is to convert mechanical energy into hydraulic energy. Pumps are characterized by a volumetric displacement, which describes the amount of volumetric fluid obtained per rotational displacement of the driving shaft. The hydraulic power obtained by a pump is given by the product of the volumetric flow and the pressure difference across the hydraulic drive.

2.1.1 Generated Flow

The ideal flow Q_{ideal} from the pump is the product of its volumetric displacement V_p per revolution and its rotational speed ω.

$$Q_{ideal} = V_p \cdot \omega \tag{1}$$

The net generated flow from a pump Q_p is expressed as:

$$Q_p = Q_{ideal} - Q_s \tag{2}$$

Here Q_s represents the slip flow due to leakages. Two types of leakage are distinguished at the drive:

1. the internal leakage Q_{li} (also known as cross-port leakage) between the high and low pressure lines.
2. the external leakage Q_{le} from the internal chambers to the case drain.

Due to the small clearances inside a pump, the leakage flows are usually laminar and are described as a function of pressure and/or rotational speed of the drive. A general schematic of the different leakage losses that occur in a hydraulic drive is shown in figure 2.

Complex models have been developed to consider a turbulent leakage flow [4],[5]. However, for the purpose of this study and for simplicity of analysis, a simple description assumed, where the total leakage flow Q_s is directly proportional to the pressure difference across the hydraulic drive p_L in terms of a laminar leakage coefficient C_v [6].

$$Q_s = C_v \cdot p_L \tag{3}$$

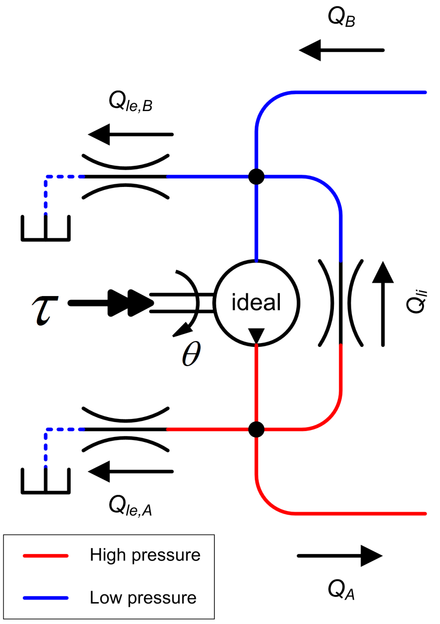


Figure 2: Flow components of a hydraulic pump [7]

2.1.2 Transmitted Torque

The effective torque of the pump τ_p is modeled as:

$$\tau_p = V_p \cdot p_L + \tau_d + \tau_f \tag{4}$$

The damping torque τ_d is a torque loss required to shear the fluid in the small clearances between mechanical elements in motion [8]. It is independent of the load and is assumed to be proportional to the pump speed and viscous damping coefficient B_p:

$$\tau_d = B_p \cdot \omega \tag{5}$$

Friction torque τ_f simulates the effect of dry friction forces on the pump pistons that oppose their motion. The resulting friction torque is proportional to the volumetric displacement and the pressure difference across the hydraulic drive. For a quasi-static analysis a constant Coulomb friction coefficient C_f is defined to describe the friction torque independently of the speed [8]:

$$\tau_f = C_f \cdot V_p \cdot p_L \tag{6}$$

2.2 Hydraulic Hose

2.2.1 Compressibility and Hydraulic Capacitance

The pressurized hose that connects the pump and the hydraulic motor acts as a mechanical spring. It is modeled as a constant volume with an effective bulk modulus associated to the compressibility of such volume. The volumetric flow associated to a pressure variation is given by:

$$Q_c = \frac{V_c}{E_{eff}} \cdot \dot{p_L} \tag{7}$$

The stiffness of a hydraulic circuit is therefore determined by the effective bulk modulus or elasticity E_{eff} of the system and the total volume V_c of the container.The hydraulic capacitance C_H, is defined as the relation between time rate of change of the pressure and the flow speed. It describes the ability of the hydraulic circuit to store energy.

$$C_H = \frac{V_c}{E_{eff}} \tag{8}$$

The effective bulk modulus considers not only the fluid stiffness but also the material compliance and the entrapped air within the hose [9].

$$\frac{1}{E_{eff}} = \frac{1}{E_{fluid}} + \frac{1}{E_{hose}} + \frac{V_{air}}{V_t} \cdot \frac{1}{E_{air}} \tag{9}$$

$$= \frac{1}{E_{fluid}} + \frac{1}{E_{hose}} + \frac{\alpha}{1.4 \cdot p} \tag{10}$$

where, $\alpha = \frac{V_{air}}{V_t}$ is the percentage of entrapped air in the total volume at a reference pressure p

2.2.2 Pressure Losses and Hydraulic Resistance

Pressure losses are associated to the energy dissipation from viscous effects of a fluid flow. By applying the Darcy-Weisbach equation for pressure losses due to friction, the relation for pressure losses in a pipe for a laminar flow is given by:

$$p_{loss} = \frac{128 \cdot \mu \cdot L}{\pi \cdot D^4} \cdot Q \tag{11}$$

The pressure losses can be expressed as a function of a hydraulic resistance R_H and volumetric flow through the hose.

$$p_{loss} = R_H \cdot Q \tag{12}$$

Hence:

$$R_H = \frac{128 \cdot \mu \cdot L}{\pi \cdot D^4} \tag{13}$$

3 State-Space Representation of Fluid Power Transmission

3.1 Conversion to State Space

The mechanical-hydraulic conversion of the system is analytically described by a second order system. The first equation is obtained from the torque balance at the rotor shaft according to Newton's second law; lumped constants for the rotor and a rigid shaft are assumed such that:

$$J_t \cdot \ddot{\theta} = \tau_{aero} - \tau_{d,rotor} - \tau_p \tag{14}$$

$$= \tau_{aero} - B_r \cdot \dot{\theta} - V_p \cdot (p_L + p_{loss}) - B_p \cdot \dot{\theta} - C_f \cdot V_p \cdot (p_L + p_{loss}) \tag{15}$$

$$= \tau_{aero} - \left[B_r + B_p + V_p^2 \cdot (1 + C_f) \cdot R_H\right] \cdot \dot{\theta} - V_p \cdot (1 + C_f) \cdot p_L \tag{16}$$

The second equation of the hydrostatic transmission, including the pump, hose and hydraulic motor is developed based on the flow continuity equation for the high pressure line. The displacement of the motor V_m is controlled by input parameter e. Leakages of the pump and motor are included so that:

$$C_H \cdot \dot{p_L} = V_p \cdot \dot{\theta}_p - C_{v,p} \cdot (p_L + p_{loss}) - C_{v,m} \cdot p_L - e \cdot V_m \cdot \omega_m \tag{17}$$

$$= V_p \cdot (1 - R_H \cdot C_{v,p}) \cdot \dot{\theta} - (C_{v,p} + C_{v,m}) \cdot p_L - e \cdot V_m \cdot \omega_m \tag{18}$$

The linear dynamic equations of the proposed model are rewritten in state-space form:

$$\dot{\mathbf{x}} = A \cdot \mathbf{x} + B \cdot \mathbf{u} \tag{19}$$

$$\mathbf{y} = C \cdot \mathbf{x} \tag{20}$$

Here, $\mathbf{x}$ is the state vector, $\mathbf{u}$ is the input vector and $\mathbf{y}$ is the output vector. For the hydraulic transmission these vectors are defined as:

$$\mathbf{x} = \begin{bmatrix} \dot{\theta} \\ p_L \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \tau_{aero} \\ e \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \dot{\theta} \\ p_L + p_{loss} \end{bmatrix} \tag{21}$$

The matrices A, B, C and D are defined as:

$$A = \begin{bmatrix} -\frac{B_r+B_p+V_p^2\cdot(1+C_f)\cdot R_H}{J_t} & -\frac{V_p\cdot(1+C_f)}{J_t} \\ \frac{V_p\cdot(1-R_H\cdot C_{v,p})}{C_H} & -\frac{(C_{v,p}+C_{v,m})}{C_H} \end{bmatrix} \tag{22}$$

$$B = \begin{bmatrix} \frac{1}{J_t} & 0 \\ 0 & -\frac{V_m\cdot\omega_m}{C_H} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ R_H\cdot V_p & 1 \end{bmatrix} \tag{23}$$

With a linear mathematical model of this second-order system, the dynamic behavior is characterized through the eigenvalues of the matrix A; hence the natural frequencies and damping ratios are obtained.

3.2 Natural Frequencies

Natural frequencies are determined by the magnitude of each pair of complex conjugated eigenvalues of matrix A. The resulting hydraulic natural frequency ω for the proposed second order system is then given by:

$$\omega_n = \sqrt{\frac{V_p^2}{C_H\cdot J_t}}\cdot K_i \tag{24}$$

With

$$K_i = \sqrt{(1+C_f)\cdot(1+R_H\cdot C_{v,m}) + \frac{(B_r+B_p)\cdot(C_{v,p}+C_{v,m})}{V_p^2}} \tag{25}$$

3.3 Damping Ratios

Damping ratios are obtained from the ratio of the absolute value of the real part and the magnitude of each pair of complex conjugated eigenvalues of matrix A

$$\zeta = \left[\frac{(B_r+B_p)}{2\cdot V_p}\sqrt{\frac{C_H}{J_t}} + \frac{(C_{v,p}+C_{v,m})}{2\cdot V_p}\sqrt{\frac{J_t}{C_H}}\right]\cdot\frac{1}{K_i} \tag{26}$$

Generally the term $\frac{(B_r+B_p)\cdot(C_{v,p}+C_{v,m})}{V_p^2} << 1$ and $R_H\cdot C_{v,m} << 1$, thus giving little influence of the constant K_i over the natural frequency and damping ratio.

3.4 Transfer functions

From the state-space representation, the following transfer function is derived:

$$\mathbf{y} = G(s)\cdot\mathbf{u} \tag{27}$$

Here,

$$G(s) = C\cdot(s\cdot I - A)^{-1}\cdot B \tag{28}$$

Writing out equation 27 yields:

$$\begin{bmatrix} \dot{\theta} \\ p_L + p_{loss} \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}\cdot\begin{bmatrix} \tau_{aero} \\ e \end{bmatrix} \tag{29}$$

The entries of matrix G are:

$$G_{11}(s) = \frac{1}{f(s)} \cdot \left(s + \frac{C_{v,p} + C_{v,m}}{C_H} \right) \cdot \frac{1}{J_t} \tag{30}$$

$$G_{12}(s) = \frac{1}{f(s)} \cdot \frac{V_p \cdot (1 + C_f) \cdot V_m \cdot \omega_m}{C_H \cdot J_t} \tag{31}$$

$$G_{21}(s) = \frac{1}{f(s)} \cdot \left(s + \frac{1 + R_H \cdot C_{v,m}}{R_H \cdot C_H} \right) \cdot \frac{R_H \cdot V_p}{J_t} \tag{32}$$

$$G_{22}(s) = \frac{1}{f(s)} \cdot \left(s + \frac{B_r + B_p}{J_t} \right) \cdot \left(-\frac{V_m \cdot \omega_m}{C_H} \right) \tag{33}$$

The denominator $f(s)$ is given by the characteristic polynomial of matrix A,

$$f(s) = s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2 \tag{34}$$

A block diagram of the proposed linear model is shown in figure 3.

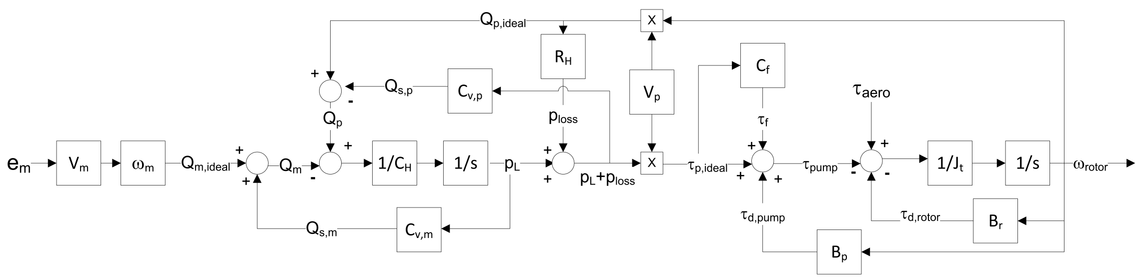


Figure 3: Block diagram of the linear model

4 Case Study: 1MW Transmission

To demonstrate the implementation of the proposed state space model, a wind turbine with fluid power transmission and a rated power of 1MW is considered. In the model the behavior of the wind turbine rotor is determined solely by its mass moment of inertia. The two model input parameters are the aerodynamic torque and the volumetric displacement of the hydraulic motor. The properties of the hydraulic pump and motor are derived from the Hägglunds hydraulic motors [10]. The specific values applied in the model are listed in table 1.

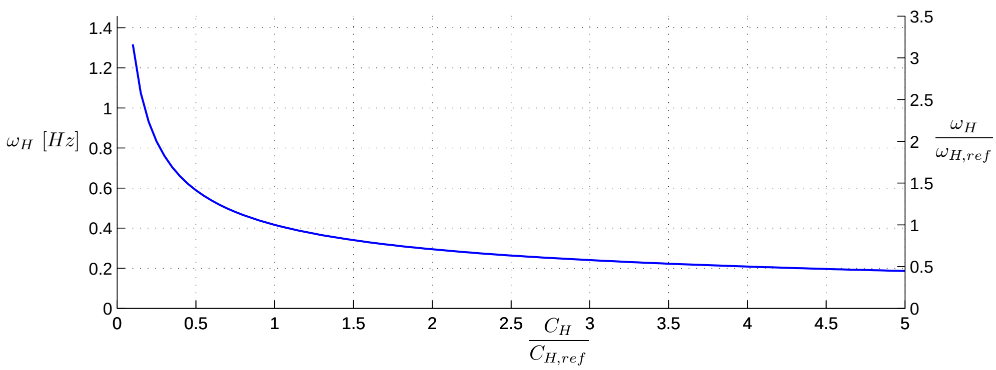


Figure 4: Natural frequency variation

Table 1: Reference values for 1 MW hydrostatic transmission wind turbine

Parameter	**Symbol**	**Unit**	**Value**
Rotor mass moment of inertia	J_t	[kg· m^2]	5.86e5
Pump volumetric displacement	$\tilde{V}_p$	[L/rev]	52.8
Hydraulic motor volumetric displacement	$\tilde{V}_m$	[L/rev]	1.2
Nominal pressure of hydraulic drives	p_{nom}	[Pa]	350e5
Nominal volumetric efficiency of hydraulic drives	η_{vol}	[-]	0.95
Pump nominal speed	n_p	[rpm]	35
Hydraulic motor nominal speed	n_m	[rpm]	1500
Pump laminar volumetric leakage coefficient	$C_{v,p}$	[m^3/s/Pa]	4.4e-11
Hydraulic motor laminar volumetric leakage coefficient	$C_{v,m}$	[m^3/s/Pa]	4.51e-11
Pump dry friction coefficient	C_f	[-]	0.02
Pump viscous damping coefficient	B_p	[Nm· s]	500
Rotor shaft viscous damping coefficient	B_r	[Nm· s]	3500
Hose internal diameter	D	[m]	0.05
High pressure hose length	L	[m]	10
Percentage of entrapped air	α	[-]	0.01
Dynamic viscosity of fluid	μ	[Pa· s]	36.68e-3
Bulk modulus of fluid	E_{fluid}	[Pa]	1.4e9

The influence of the hydraulic capacitance on the natural frequency of the hydrostatic transmission is shown in figure 4. The hydraulic capacitance reflects the consequences of the amount of oil as well as any presence of dissolved air in the system. For this particular case study, a low natural frequency is observed due mainly to the 10m flexible hose. This frequency might be excited by the operational frequencies of the rotor, nevertheless the associated high damping ratio will compensate any dynamic amplification at these conditions.

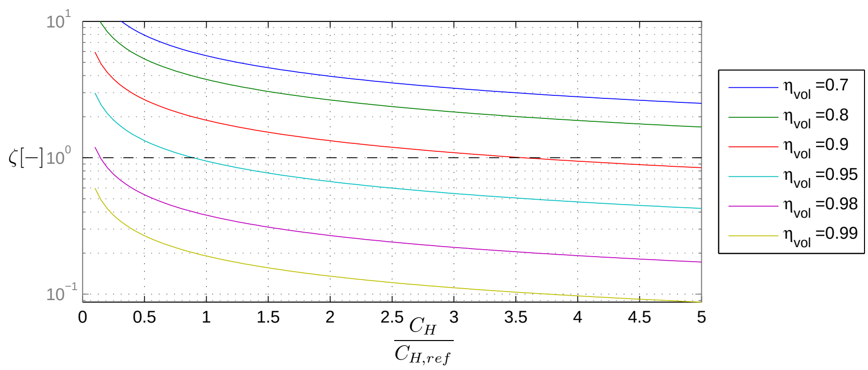
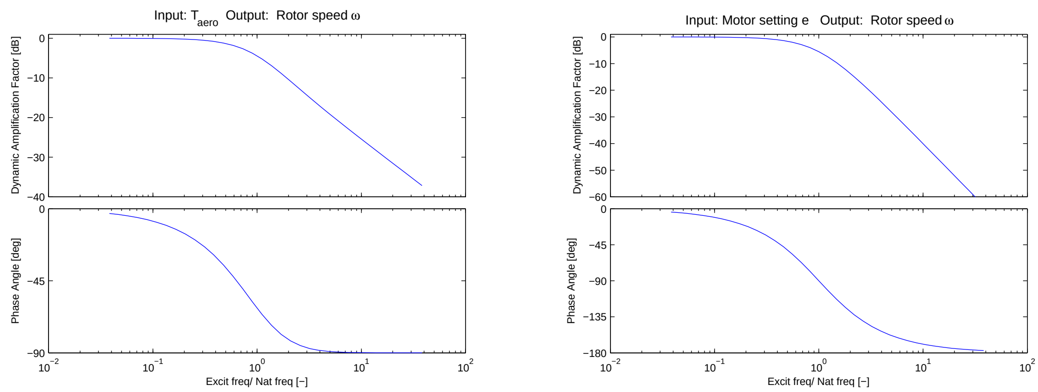


Figure 5: Damping ratio variation

From figure 5, it is seen that the volumetric efficiencies of the hydraulic drives have a major influence on the amount of damping. The volumetric efficiency is directly related to the leakages of the hydraulic drives. These leakages contribute to most of the load damping of the system. However, special attention is needed in order to avoid a sluggish response from overdamped systems.

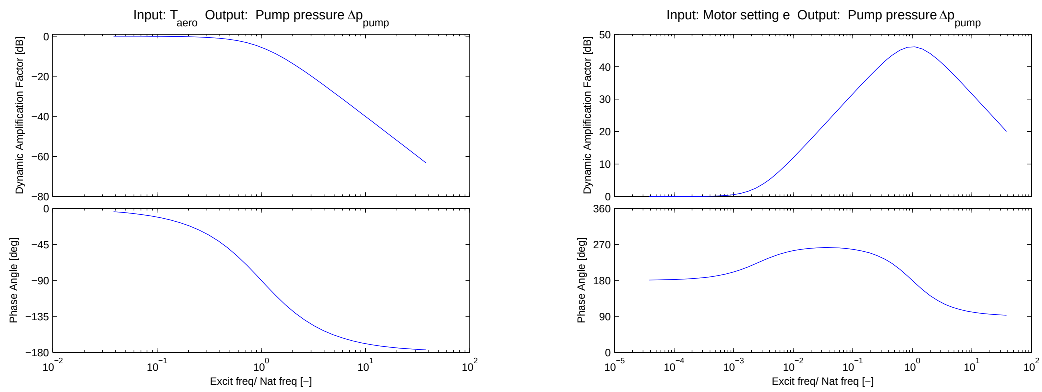
The Bode plots in figures 6 and 7 show the system response in terms of magnitude and phase difference of the steady-state output, with respect to the harmonic excitation at different frequencies. The dynamic amplification factor is defined as the magnitude of the steady-state output at every input frequency (dynamic gain), divided by the magnitude of the steady-state output at static conditions (static gain).

It is of particular interest to demonstrate the inherent damped behavior of the rotor speed with respect to a harmonic excitation of the aerodynamic torque and motor setting as observed in figures 6(a) and 6(b). Figure 7(b) shows the high influence of the motor setting on the pressure variations specially at relatively low frequencies. This aspect is significant from the control point of view, since the motor setting is the controlled parameter of the system.



(a) (b)

Figure 6: Frequency response of the rotor speed to: 6(a) aerodynamic torque, 6(b) hydraulic motor setting



(a) (b)

Figure 7: Frequency response of the pressure difference across the pump to: 7(a) aerodynamic torque, 7(b) hydraulic motor setting

5 Conclusion

The inherent damping characteristics of the hydraulic drives are mostly associated to the leakage losses and the effective hydraulic capacitance. Although low natural frequencies of the hydraulic transmission could be present in the operational frequencies of the rotor, the proposed model shows a very well damped response of the rotor speed. Pressure variations due to harmonic inputs of the motor setting are significant at low frequencies and should be avoided by means of a well-designed controller. Special attention should be given for overdamped systems in order to avoid a sluggish response.

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