Correcting movement errors in frequency-sweeping interferometry

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Received February 24, 2005; accepted April 19, 2005

Absolute distance measurements can be performed with an interferometric method that uses only a single tunable laser. This method has one major drawback, because a small target movement of the order of one wavelength during a measurement will be interpreted as a movement of one synthetic wavelength. This effect is usually mitigated by adding a second (nonscanning) laser. We show that absolute distance measurements can be performed with only one laser if the movements encountered are smooth, on the time scale of one measurement. In this case the movement errors can be compensated with a simple algorithm that combines several subsequent measurements. First experimental results show good agreement with theory.

OCIS codes: 120.3180, 120.5050, 140.3600, 280.3400.

Several different techniques can be used to measure absolute distances, among which are time-of-flight measurements, high-frequency modulation schemes, and interferometric methods. Of these, only the last two have the potential to achieve resolutions below a millimeter over several hundred meters. Most of these methods are not truly absolute distance measurements, but systems that are sensitive to changes on the scale of a certain synthetic wavelength. The total distance can be calculated only with *a priori* (low-resolution) knowledge of the distance or by cascading a number of systems with decreasing synthetic wavelengths.[1,2] Truly absolute, single-stage distance measurements can be made by sweeping a laser over a known wavelength range and measuring the phase difference interferometrically during the sweep, usually called frequency-sweeping interferometry.[3,4] This method suffers from a basic drawback that has large consequences if the target moves during a measurement. A movement of the target over one optical wavelength is interpreted as the movement over one synthetic wavelength. This problem is usually solved by adding a second laser, which reduces the sensitivity to movements of the order of the synthetic wavelength itself.[5,6]

We explore a solution without a second laser, by measuring in the presence of movements and correcting for the movement errors in the data analysis. Our intended application is absolute distance metrology between satellites. The Darwin Space Interferometer, for example, would require knowledge of the absolute distance between two satellites with an accuracy of better than 100 μm over a distance of 250 m.[7]

Consider an optical interferometer with a fixed optical path length difference L that is equipped to measure phase as a function of time. For fixed or slowly changing optical frequency ν, the phase ϕ is proportional to both the length and the frequency. Because the phase is usually measured modulo 2π, the absolute phase is unknown. By unwrapping the phase over time it is, however, possible to measure phase differences. If the light source is a tunable laser that is swept from optical frequency ν_1 to ν_2, the total phase difference will be

$$\phi_2 - \phi_1 = 2\pi\frac{\nu_2 L}{c} - 2\pi\frac{\nu_1 L}{c} = 2\pi\frac{\Delta\nu}{c}L = 2\pi\frac{L}{\Lambda}, \quad (1)$$

with c the speed of light, $\Delta\nu$ the frequency difference of the sweep, and Λ the so-called synthetic wavelength defined by

$$\Lambda = c/\Delta\nu. \quad (2)$$

Reversing Eq. (1), the length L should be calculated as

$$L = \frac{\phi_2 - \phi_1}{2\pi}\Lambda = \frac{\Delta\phi}{2\pi}\Lambda. \quad (3)$$

The length is thus directly proportional to the total phase difference $\Delta\phi$, which can be determined by measuring the phase before and after the sweep and counting the number of fringes during the sweep. Error analysis of Eq. (3) yields the error in the length measurement δL:

$$\delta L = \left[\left(\Lambda\frac{\delta\Delta\phi}{2\pi}\right)^2 + \left(L\frac{\delta\Lambda}{\Lambda}\right)^2\right]^{1/2} \approx \left[2\left(\Lambda\frac{\delta\phi}{2\pi}\right)^2 + 2\left(L\frac{\nu}{\Delta\nu}\frac{\delta\nu}{\nu}\right)^2\right]^{1/2}. \quad (4)$$

The first term is constant and depends on the error in the phase difference $\delta\Delta\phi$, which is determined by two phase measurements with an error $\delta\phi$ each. The second term scales with the length L and depends on the error in the synthetic wavelength $\delta\Lambda$. As indicated by Eq. (2), the synthetic wavelength is determined by a difference between two large frequencies with error $\delta\nu$ each. This leads to the amplification by a large factor $\nu/\Delta\nu$.

If the distance to be measured is not stationary during one measurement, an additional error arises. What is needed is the instantaneous difference of the phases measured at the frequencies ν_1 and ν_2, which is measured directly in conventional two-wavelength interferometry. In our case these phases are measured at different moments in time, thereby confus-

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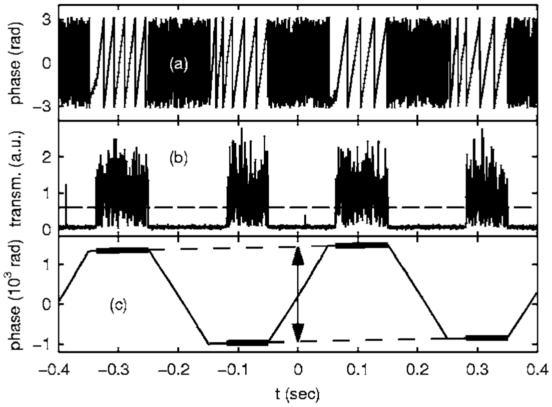

Fig. 1. Raw signals: (a) interferometric phase, (b) transmission of the Fabry–Perot cavity. The dashed line is the threshold value. (c) Unwrapped phase of (a). The dashed lines and the arrow describe the four-point algorithm.

ing the effects of path length and optical frequency change. To analyze the sensitivity to movement, the length is expressed as a function of time t:

$$L(t) = L_0 + L_1 t + L_2 t^2/2 + L_3 t^3/6 + \cdots . \tag{5}$$

This is a standard Taylor expansion, where the subscripts denote the order of the derivative. The phases measured before and after the frequency sweep are now

$$\phi_1 = 2\pi \frac{\nu_1 L(-\tau/2)}{c}, \quad \phi_2 = 2\pi \frac{\nu_2 L(\tau/2)}{c}, \tag{6}$$

where τ is the time between the phase measurements. Substituting Eqs. (5) and (6) into Eq. (3) yields the calculated length L_{calc}:

$$L_{\text{calc}} = \frac{\phi_2 - \phi_1}{2\pi}\Lambda = L_0 + \frac{\nu}{\Delta\nu}L_1\tau + \frac{\tau^2}{8}L_2 + \cdots , \tag{7}$$

where ν is the average of ν_1 and ν_2. The first term gives the correct distance at time $t=0$. The second term, however, is proportional to the speed L_1 and contains the same large factor $\nu/\Delta\nu$ that was previously encountered in relation (4). Note that this term is also dependent on $\Delta\nu$ and thus on the direction of the frequency sweep. Since in practice the laser is repeatedly swept up and down, the calculated distances will oscillate around the true value. An improvement is to take the average of two such consecutive measurements. This requires three phase measurements at $t=-\tau$, 0, τ at optical frequencies ν_2, ν_1 and ν_2, respectively:

$$\phi_1 = 2\pi \frac{\nu_2 L(-\tau)}{c}, \quad \phi_2 = 2\pi \frac{\nu_1 L(0)}{c},$$

$$\phi_3 = 2\pi \frac{\nu_2 L(\tau)}{c}. \tag{8}$$

The length must now be calculated as

$$L_{\text{calc}} = \frac{\phi_1 - 2\phi_2 + \phi_3}{4\pi}\Lambda = L_0 + \frac{\nu\tau^2}{2\Delta\nu}L_2 + \frac{\tau^2}{4}L_2 + \cdots . \tag{9}$$

The first large error term is now no longer dependent on the speed L_1, but on the acceleration L_2. There is, however, a certain asymmetry of using two measurements at the higher frequency and only one at the lower frequency. By once more combining two consecutive (three-point) measurements, we arrive at a four-point algorithm. The measured phases are now

$$\phi_1 = 2\pi \frac{\nu_2 L(-3\tau/2)}{c}, \quad \phi_2 = 2\pi \frac{\nu_1 L(-\tau/2)}{c},$$

$$\phi_3 = 2\pi \frac{\nu_2 L(\tau/2)}{c}, \quad \phi_4 = 2\pi \frac{\nu_1 L(3\tau/2)}{c}, \tag{10}$$

and the length should be calculated as

$$L_{\text{calc}} = \frac{\phi_1 - 3\phi_2 + 3\phi_3 - \phi_4}{8\pi}\Lambda = L_0 + \frac{3\tau^2}{8}L_2 - \frac{\nu\tau^3}{4\Delta\nu}L_3 + \cdots . \tag{11}$$

Again, there is no dependency on L_1. There still is a dependency on the acceleration L_2, but this is a small term comparable with the actual movement during a sweep. The first large error term only appears in third order.

Relation (4) shows that $\Delta\nu$ should be chosen as large as possible to reduce the measurement errors. The type of laser limits this to a value of the order of 100 GHz. There is usually also a limit on the tuning rate $\Delta\nu/\tau$, caused by either the tuning mechanism or the detection electronics. Inspection of Eq. (11) shows that, with the value of $\tau/\Delta\nu$ fixed, the first large error term scales with τ^2. For reducing the movement error, τ, and thus $\Delta\nu$, should be made smaller. These two conflicting requirements on the size of $\Delta\nu$ can be used as an opportunity to balance the sizes of the errors in relations (4) and (11).

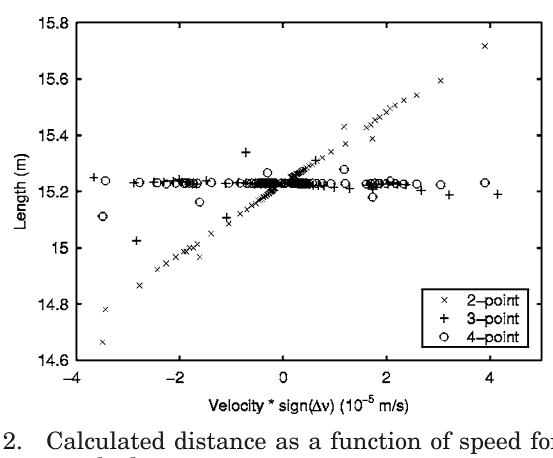

Fig. 2. Calculated distance as a function of speed for the different methods.

The method described above was used to calculate the length in a setup for measuring absolute distances that we are developing.[8] The laser source is a Littman-type external cavity laser diode with a wavelength of 633 nm. The optical frequency can be tuned continuously over several tens of GHz with the aid of a piezo actuator. The laser also has a current modulation input, which is used for high-frequency feedback. To simulate long distances, we built a homodyne Mach–Zehnder interferometer with a 10 m polarization-maintaining fiber in one of the arms.[6] The phase is measured in quadrature by using polarization multiplexing, and we compensate for polarization mixing in the data analysis.

In the theory explained above, we assumed that the laser was repeatedly swept up and down and that a single phase measurement is performed at each end point. In practice, however, we sweep the laser with the piezo in a trapezoidal pattern. The frequency is swept up or down over 7.5 GHz during 0.1 s. Subsequently, we lock the laser for 0.1 s to a very high-finesse Fabry–Perot cavity by using the Pound–Drever method.[9] Between sweeps, the wavelength stays fixed for a short period, during which multiple phase readings can be made. The frequency difference $\Delta\nu$ is now defined at several free spectral ranges of the cavity, so we can reference the unknown distance to the known length of the cavity.

Figure 1 shows a short section of the raw signals of a 20 s measurement run. During this period the fiber was briefly heated with a strong lamp to cause a short expansion and subsequent contraction of the fiber. In Fig. 1(a) the phase is plotted, which shows periods with slow fringes due to the movement only and periods with fast fringes (not discernible) due to the movement and the wavelength sweep. Figure 1(b) shows the transmission of the Fabry–Perot cavity, which shows when the laser locks to the cavity. Only those points where the transmission of the Fabry–Perot cavity is above a certain threshold are included in the calculation, to guarantee that the wavelength is well defined. In Fig. 1(c) the unwrapped phase is shown. The graph closely resembles the waveform applied to the piezo, but it is tilted because of the path length change during the sweep. To calculate the length with the two- and three-point algorithms, we use the average of all the phase readings during one locked period. The four-point algorithm is slightly modified, since we now can fit two straight lines through the upper and lower edges of the graph [see Fig. 1(c)]. The phase difference is then calculated by evaluating the two lines at $t=0$. The slope of the fitted lines can also be used to calculate the speed of the path length change.

Plotting calculated length versus speed allows the sensitivity for target movement of the various methods to be studied; see Fig. 2. As can be seen from Eqs. (7), (9), and (11), the error depends on the direction of the frequency sweep. The speed is therefore first multiplied by the sign of the sweep. The two-point method shows a very strong dependency on speed, as expected. The slope agrees with theory to within experimental error. The slopes of the three- and four-point method show dependencies that are 25 and 240 times lower, respectively, although both should have shown no dependency at all. This might be the result of the particular length trajectory (exponential decay due to heating–cooling of the fiber), which has a strong correlation among the various derivatives. The standard deviation of roughly 100 consecutive length measurements is 1.6 mm and 130 μm for the three- and four-point method, respectively. The difference between these two methods is much larger than could be explained by averaging over more points alone. A few points show large errors due to the discontinuities in the trajectory of the movement. These were excluded from the evaluation. As in Fig. 2, the calculated lengths for the three- and four-point methods can be plotted against the acceleration L_2. As expected, this shows a dependency on the acceleration for the three-point method and a much reduced sensitivity for the four-point method, but the effect is less apparent because of other noise sources.

In summary, we have shown that we can do length measurements with a one-laser frequency-sweeping scheme with a much reduced sensitivity to target movement. To achieve this we combine four consecutive phase measurements instead of the normal two. First measurements show a repeatability of 130 μm at 15 m. It must be noted that our method does not reduce the sensitivity to fast disturbances such as turbulence or vibration experienced in an industrial environment. For our application, which only experiences smooth movements, this scheme could allow for using only one laser. It might also be applicable to systems that experience large (but relatively slow) thermal drifts, but otherwise operate in a benign environment.

This project was funded by TNO-TPD, a division of the Dutch Organization for Applied Scientific Research. B. Swinkels's e-mail is b.l.swinkels@tnw.tudelft.nl.

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