Modulation Schemes for the Eurofix Datalink

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Preface

This report concludes my research on the modulation schemes for the Eurofix datalink. The Eurofix project is a part of the Beek program 'Integrated Navigation for Traffic and Transportation'. Hopefully, the results of my research will contribute to the realization of a Loran-C data channel for DGPS corrections.

I would like to thank staff and students of the P&N group for their support and companionship. Especially, I would like to thank the students whom I worked with at the Eurofix project for the past eight months, Dennis, Jeroen, Remco, and Tom. The discussions we had were constructive and encouraging creating a good atmosphere to work in. Also, I would like to express gratitude to my mentors, Durk, Edward, and Jos for their help and advise. They always had time to listen and discuss the ideas I had. Their way of supporting and supervising students is an example for other departments and universities.

I hope that my research gave new insights and new points of interest for further research on the Eurofix project.

Gerard Offermans
September 1994

Summary

In the Eurofix system DGPS corrections are transmitted using the Loran-C signal as a datalink by time shifting the Loran-C pulses. The performance of the datalink, however, is bounded by restrictions of both the Loran-C system and the DGPS system. First, the normal Loran-C performance should not degrade noticeably due to the extra data modulation. Second, the DGPS corrections should be regularly updated with a low message error probability. The use of error correcting code reduces the effective data rate but is necessary to achieve a low message error probability

In this report the properties of the Loran-C datalink are discussed and possible modulation schemes for the Loran-C datalink are derived. The design of the modulation schemes is restricted by the balancing requirement imposed by the normal Loran-C performance. The degradation of the normal Loran-C performance is simulated as a function of the signal-to-noise ratio for the different balanced modulation schemes.

The balancing requirement reduces the data channel capacity significantly. Also, the normal Loran-C receiver is not guaranteed a bias-free tracking signal with balanced modulation as the Loran-C performance degradation depends on the alignment of the receiver window with respect to the balancing block. Therefore, simulations have been done to investigate whether unbalanced data transmission yields tolerable Loran-C performance degradation.

A coding scheme based on nearly balanced modulation yields great improvement in the properties of the data channel. The normal Loran-C performance degradation is comparable with the degradation due to balanced modulation.

Abbreviations

CRI	Cross-Rate Interference
CWI	Continuous Wave Interference
DGPS	Differential Global Positioning System
GCD	Greatest Common Devisor
GF	Galois Field
GPS	Global Positioning System
GRI	Group Repetition Interval
IOD	Issue Of Data
I-Q sample	In-phase and Quadrature sample
LF	Low Frequency
Loran	Long range navigation
MEP	Message Error Probability
rms	root-mean-square
RTCM	Radio Technical Commission for Maritime Services
RTCM-SC	RTCM-Special Committee
SA	Selective Availability
SNR	Signal-to-Noise Ratio
UDRE	User Differential Range Error
VHF	Very High Frequency

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1 Introduction

Eurofix is a new radionavigation system that consists of two existing navigation systems: the Global Positioning System and Loran-C. The Loran-C signal can be additionally modulated to act as a datalink over which Differential GPS corrections are broadcast to the GPS users. The modulation, however, is bounded by two requirements. First, the normal Loran-C user should not notice a Loran-C performance degradation due to the extra data modulation. Therefore, restrictions are imposed on the type of modulation that can be used. Second, the GPS user should receive regular updates of the satellite range corrections in order to achieve the required positioning accuracy. This calls for a fast and reliable datalink, which are conflicting requirements. Therefore, a suitable modulation scheme has to be designed which fulfills both requirements.

The Loran-C data channel is disturbed by noise and cross-rate interference affecting the reliability of the datalink. The influences of noise and cross-rate are compensated for by the use of error correcting codes resulting in a slower datalink. Initially, three modulation schemes are derived and compared with respect to data transmission performance. With the aid of computer simulations the Loran-C performance degradation due to the data modulation for hard limited stationary receivers is calculated.

The effective data rate of the Loran-C channel is largely limited by the balanced modulation requirement. Therefore, simulations have been done to investigate the possibilities to relax this requirement. Obviously, better schemes can be designed when the Loran-C balancing restriction on the modulation type is less stringent.

After a brief introduction of the Eurofix system in Chapter 2, Chapter 3 discusses the modulation and demodulation aspects of the Eurofix data channel. After a quick look at the Loran-C channel properties the next Chapter derives three schemes which are based on balanced data modulation and compares them with respect to data transmission and Loran-C performance. Chapter 5 discusses the consequences of unbalanced data modulation on the Loran-C performance degradation. Also, a coding scheme is derived based on nearly balanced data modulation. In the final Chapter of this report conclusions are drawn based on the results of Chapters 4 and 5.

2 Introduction to Eurofix

In this Chapter a short introduction of the navigation system Eurofix is given, as was proposed by Van Willigen [1]. The Eurofix system consists of two existing navigation systems, the Global Positioning System (GPS) and Loran-C. In short, the Loran-C signal will be used as a broadcast data channel for Differential GPS (DGPS) corrections. The first two Sections discuss the DGPS system and its implementation in the Eurofix system, in the following Sections the Loran-C datalink is discussed.

2.1 Differential GPS

The Global Positioning System is a satellite navigation system capable of position determination with high accuracy. The largest errors in the positioning are due to an intentional degradation of the GPS signal (Selective Availability, SA) on the civilian Standard Positioning Service which limits the accuracy to 100 m (95%). However, the Federal Radionavigation Plan [2] states that the accuracy for Harbor and Harbor Approaches should be 8 - 20 meters (95%). In order to meet these requirements the use of Differential GPS is unavoidable.

DGPS operates as follows:
On a well known position on earth a DGPS reference station is placed which monitors all GPS satellites in view. By measuring the Pseudo Range to each satellite and comparing it with the calculated range the station can generate a correction for the erroneous Pseudo Range mainly caused by Selective Availability. The corrections are then broadcast to the user receivers in the vicinity and applied by them to increase the accuracy of their positioning. That is, errors which are the same in reference station and user receiver (correlated errors) can be corrected. The final accuracy of the corrected positioning is decreased by two decorrelating effects of the DGPS corrections:

1. Spatial decorrelation
2. Temporal decorrelation

2.1.1 Spatial decorrelation

Due to the different alignment of the satellites for reference and user receiver the DGPS correction decorrelates. This spatial decorrelation is a function of distance between user and reference station and limits the use of corrections from very distant stations. As a rule of thumb, the rms value of the spatial decorrelation of a correction increases by 0.5 meter with every 100 km user-reference station separation. Although its influence cannot be neglected the spatial decorrelation will not be discussed any further in this report. Furthermore, when the user and reference receiver are very distant from each other the set of visible satellites will be different. Therefore, the user has to be within a 600 - 800 km range of a reference station to be able to correctly use the generated corrections.

2.1.2 Temporal decorrelation

The time necessary to transmit a DGPS correction is bounded by the rate of the datalink. During transmission the correction ages and becomes less valid. Besides, the correction has to be used until the next update is fully received decorrelating even further. As mentioned earlier, the largest error source of the civil GPS positioning is Selective Availability. This SA error consists of two parts: a slowly varying ephemeris error and a faster changing clock error. Especially, the correction for the latter error source decorrelates fast. The additional error due to the temporal decorrelation can be approximated with Formula 2.1 [3].

$$\sigma_{range} = \tfrac{1}{2} a\, t^2 \qquad \text{Formula 2.1}$$

with **a** the rms value of the SA clock error acceleration, approximately 0.004 m/s^2. The quadratic increase of the rms value of the decorrelation error is shown in Figure 2.1.

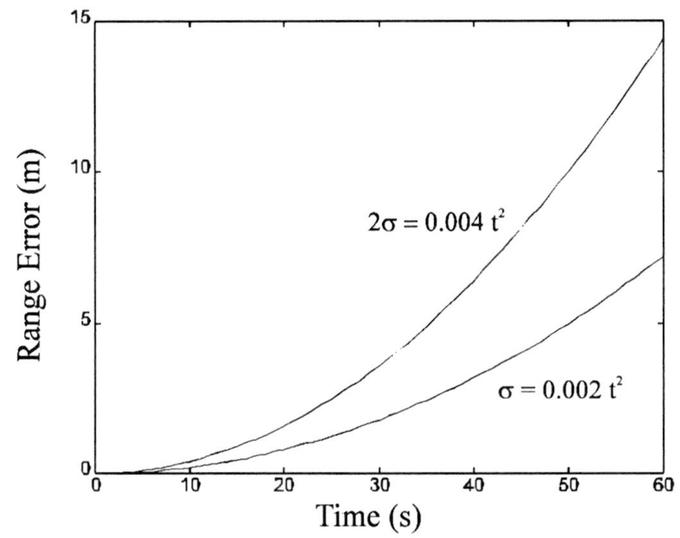

Figure 2.1 σ and 2σ values of the temporal decorrelation as a function of time

If we assume that the final position error distribution is normal, the 95% interval is given by the 2σ value. Apparently, this temporal decorrelation term has great influence on the final accuracy when DGPS corrections are transmitted using a low data rate channel, for instance the Eurofix datalink.

2.2 The asynchronous data format for Eurofix DGPS messages

Beekhuis [3] showed that the RTCM SC-104 type 1 synchronous DGPS message [4] is not suitable for low bit rate data channels. The temporal decorrelation becomes too large due to the long transmission time. Instead, he derived an asynchronous DGPS format with less bits than the RTCM SC-104 type 9 message and thus more suitable for the Eurofix datalink. To assure compatibility with existing standard GPS receivers the smaller asynchronous format should easily be transformed to the synchronous RTCM type 1 message. Therefore, the Eurofix format contains more or less the same information as the RTCM type 1 message. In order to keep the total number of bits as small as possible some parameters were omitted or shortened. The Eurofix asynchronous data format (a correction for one satellite) as proposed by Beekhuis is given in Table 2.1.

Function	Number of bits	Range	Resolution
Message Type	2	4 messages	
Satellite ID	5	32 satellites	
Issue of Data (IOD)	8		
Time Reference (Z-count)	10	1 hour	3.6 seconds
Range correction	12	-655.32...+655.32 m	0.32 m
Range Rate correction	7	-2.048...+2.048 m/s	0.032 m/s
UDRE	1	2 states	
Total number of bits	45		

Table 2.1 Eurofix asynchronous DGPS correction for one satellite [3].

For the meaning of the different functions the reader is referred to [3] and [4]. In comparison, the RTCM type 1 message for 6 satellites contains 360 bits (288 data and 72 parity bits).
Note that the 45 bits Eurofix message does not contain any parity bits. The error correction for the Eurofix messages highly depends on the properties of the Loran-C data channel, and is discussed in Chapter 4.

At the receiving end the asynchronous corrections have to be repacked into a synchronous RTCM type 1 message in order to assure compatibility with standard GPS receivers. Every time a new asynchronous correction is received, the old corrections have to be recalculated to match the new time reference. The new correction and the recalculated ones fit into one synchronous message which is used by the GPS receiver to upgrade its position calculation. The validity of the recalculated corrections, however, is still related to the time reference in which they were generated. So, each repacked RTCM type 1 message contains only one updated satellite, the rest of the corrections are recalculated ones.

2.2.1 User satellite selection in asynchronous DGPS update frame

The final DGPS accuracy depends on the accuracy of the corrected range measurements to each satellite used in the position calculation. Due to the different temporal decorrelation the accuracy of these measurements vary as well. If the DGPS receiver uses four satellites for its positioning the accuracy is also dependent on the place of the corrections in the satellite update frame. Figure 2.2 shows the final accuracy of two DGPS users which each use a different set of four satellites for their position calculation. The DGPS corrections are transmitted sequentially with a fixed order in the satellite update frame.

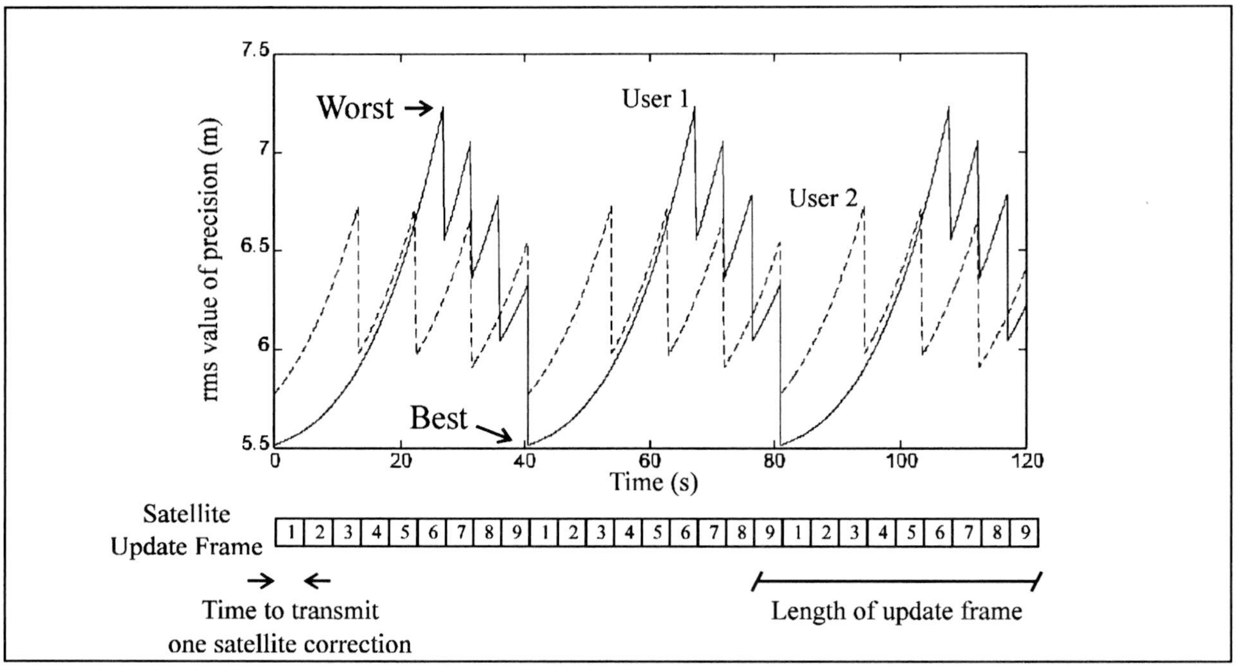

Figure 2.2 Final accuracy dependent on the user satellite selection.

Here, receiver 1 calculates its position using corrected ranges to satellites 6, 7, 8, and 9, receiver 2 uses satellites 3, 5, 7 and 9. Each time a new correction for a used satellite is received, the accuracy of the total positioning is improved dramatically (vertical transitions in the picture). The best performance is achieved when all four satellites are most recently updated, the worst occurs just before the update of the first satellite (see Figure 2.2). Clearly, the mean accuracy for both users is more or less the same but the standard deviation differs.

2.2.2 Integrity and synchronization messages

The Eurofix datalink will mainly be used to transmit DGPS corrections to the users. At the same time, the datalink acts as an Integrity channel [5]. Although both types of messages have to be transmitted as fast as possible and with the lowest possible message error probability (MEP), they set different performance requirements on the datalink. The boundaries for the DGPS messages are set by the minimum update rate (validity of the correction due to temporal decorrelation) and the maximum message error probability. The boundaries for the integrity messages are set by two integrity parameters, the Time to Alarm (10 seconds for DGPS services) and the Probability of Missed Detection [2, 5]. Also, the receiver should be able to easily classify the type of message, in other words, an X-type message should not change into a Y-type due to a simple error pattern. For data frame timing aspects some sort of synchronization should be applied, especially when an Integrity message interrupts a DGPS message.

In short, three basic types of messages can be sent over the datalink which have to be mutually distinctive.

1. DGPS corrections
2. Integrity messages
3. Synchronization messages

All three types of messages can, and probably will, be of different length.

2.2.3 DGPS reference station considerations

In order to meet the 8 - 20 m (95%) accuracy on the Eurofix coverage area the following three parameters should be taken into account in designing the DGPS reference station and the Loran-C datalink.

1. Number of satellites updated by the reference station
2. Update rate for each satellite
3. Message error probability of a satellite correction

Obviously, the three parameters are related to each other. The satellite update rate depends on the number of satellites monitored by the reference station and the transmission time of one correction. In turn, the limit on the transmission time of one correction is set by the message error probability and the rate of the datalink (the lower the MEP, the larger the transmission time). For good performance, a balance has to be found between low message error probability and reasonable satellite update rate. The rate of the datalink has a major impact on both parameters but is also limited by the Loran-C signal structure, as is described in the next Section.

2.3 The Eurofix datalink

The generated DGPS corrections have to be transmitted from reference station to the users. Up to now, corrections have been transmitted using radio transmitters in the LF and VHF radio bands. These transmitters have only limited range and require expensive transmission bandwidth in the crowded radio spectrum. The navigation system Loran-C provides a possible means to transmit corrections over a large area using the existing infrastructure and transmission bandwidth. In this way, the realization of full DGPS coverage on the US continent and Europe is reduced to cleverly modulating the Loran-C signal.

2.3.1 Pulse advance and delay modulation

The DGPS corrections are transmitted to the users by additional modulation of the Loran-C signal. Each Loran-C station transmits a group of eight pulses at 1 μs intervals. The transmission is repeated every Group Repetition Interval (GRI). By time shifting the original Loran-C pulses the modulator creates code bits which are used to transmit the DGPS messages. On the other hand, the introduction of the Eurofix modulation on the Loran-C signal inevitably means a degradation of the normal Loran-C positioning performance. However, under all conditions the normal Loran-C user should be able to navigate within the boundaries specified by the Minimum Performance Standard of the RTCM SC-70 [6]. As a consequence, restrictions have to be imposed on the amount of power and the sort of modulation that can be used for Eurofix data transmission. Current research presumes a 1 μs time advance and delay modulation to transmit data over the Loran-C channel (Figure 2.3).

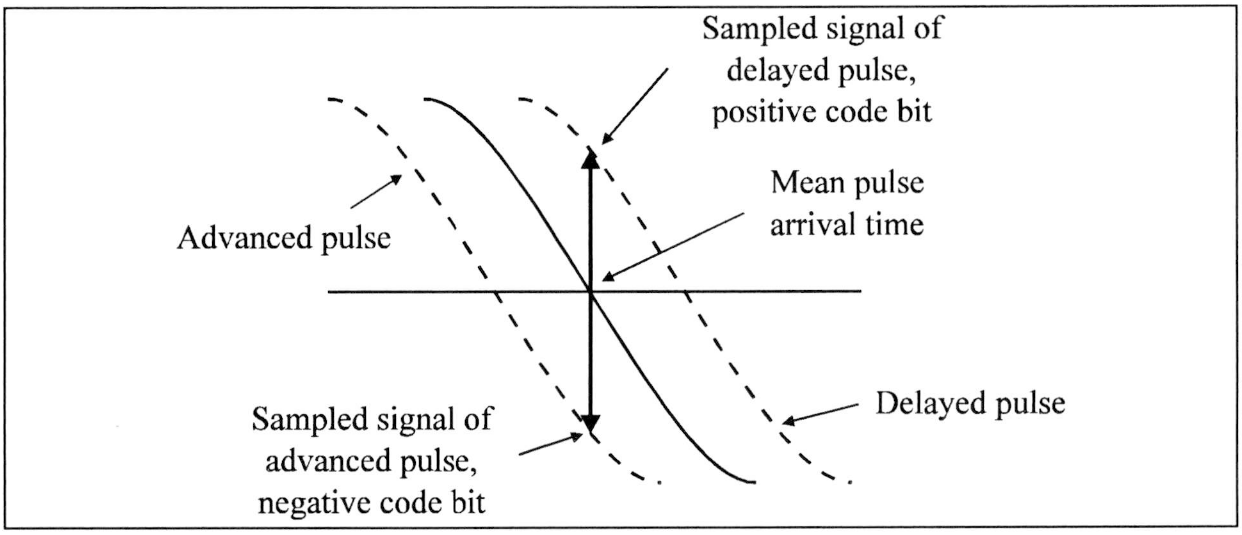

Figure 2.3 Eurofix advance and delay modulation

To assure normal Loran-C performance two restrictions are imposed on the applied modulation:

- First, in order to preserve the normal Loran-C performance the mean value of the time shifts should not change from the original unmodulated one. Therefore the Eurofix modulation has to be balanced.
- Second, the first two pulses of each GRI cannot be modulated as they are used by the Loran-C stations to signal a malfunction of the system (blinking). Further, the tracking at high signal to noise ratios requires unmodulated pulses to avoid dead zones in hard limiter receivers, as will be explained in Section 3.5.3. Hence, only six out of eight pulses are modulated with Eurofix data.

The vector representation of the Loran signal with additional modulation is given in Figure 2.4. The time shifts of 1 μs correspond to a modulation angle $\varphi = 36°$. Due to the modulation the tracking signal is 1.84 dB lower than the tracking signal of an unmodulated pulse, the coding bit signal is 4.62 dB under the maximum signal level. With only 6 out of 8 pulses modulated the average navigation power loss equals $^6/_8 \cdot 1.84 = 1.38$ dB per pulse. The modulation angle could be increased yielding more signal power for data transmission. Whether the higher navigation power loss would be tolerable is not discussed here. In the rest of this report a 1 μs time shift modulation is assumed.

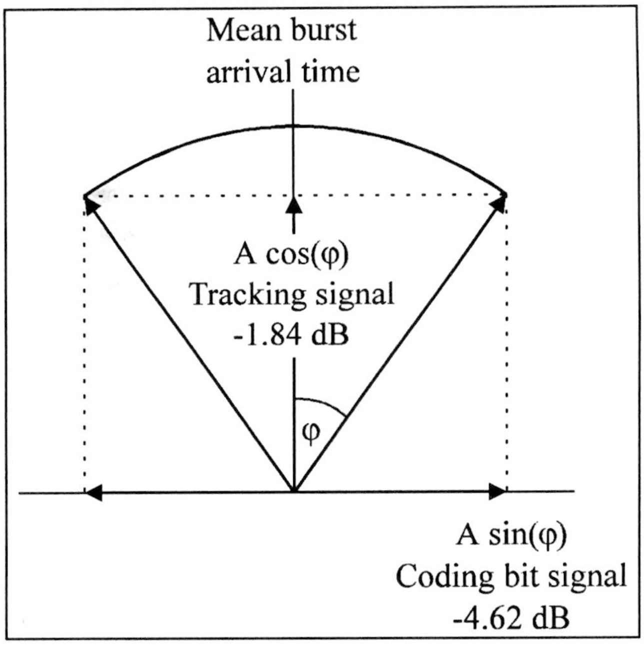

Figure 2.4 Vector representation of the time shifted Loran-C signal.

2.3.2 Data rate limitations of the Loran-C data channel

The data rate of the Eurofix channel depends on the GRI of the used Loran-C chain. This GRI can vary from 40 to 100 ms, mainly depending on the number of secondaries and the distances between stations of the same chain. When the possibilities for data transmission over the Loran-C channel are investigated, the largest possible GRI (e. g. 100 ms) should be used as a worst case example. The data rate of the 100 ms GRI is 60 code bits per second (6 code bits per GRI). Due to the properties and restrictions of the Loran-C channel, a lot of these code bits have to be used for modulation balancing and error correction reducing the effective data bit rate (the number of data bits per second) immensely to approximately 15 bits per second or less.

2.4 Conclusions

The validity of a DGPS correction mainly depends on the age of the correction. Due to temporal decorrelation the correction deteriorates. Therefore, the DGPS message format has to be adjusted for the slow Loran-C datalink. The final user position accuracy does not only depend on the update rate and message error probability of the DGPS messages, but also on the selection of used satellites and their place in the update frame.

The Loran-C signal is additionally modulated by time shifting the original pulses. The normal Loran-C performance imposes two restrictions on the modulation form:

- The modulation has to be balanced
- The first two pulses in each GRI cannot be used for data transmission.

In this way a maximum of 60 code bits per second can be used for data transmission of which many has to be used for balancing and error correction.

3 Modulation and demodulation

This Chapter discusses the modulation and demodulation of the Eurofix data on the Loran-C signal which is disturbed by noise and interference. After an introduction to the Eurofix communication model, the Loran-C channel and the demodulation of the Eurofix data are discussed. Also, the loss of tracking information for the normal Loran-C users is investigated and a modulation requirement is derived.

3.1 Introduction

The Eurofix communication model is given in Figure 3.1.

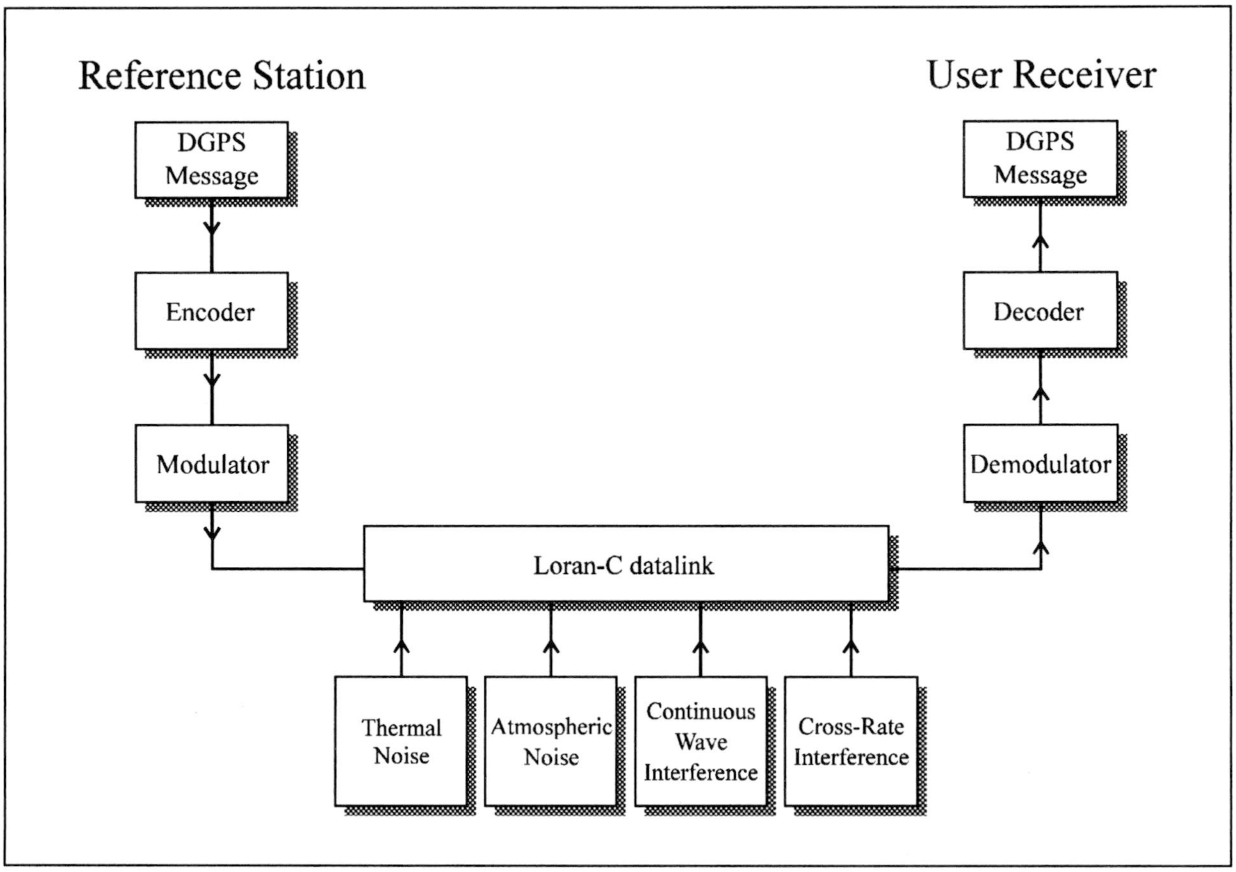

Figure 3.1 The Eurofix communication model.

The purpose of the Eurofix datalink is to transport the DGPS message generated in the reference station to the users. Unfortunately, the Loran-C data channel is disturbed by interference and noise. Therefore, the DGPS message has to be packed into a protecting cover of error correcting code, especially designed for the properties of this particular data channel. The encoded DGPS message is modulated to be transmitted over the Loran-C channel. At the receiving end the message is demodulated yielding an encoded DGPS message which probably contains errors due to the imperfect data channel. The errors are corrected by the decoder and the decoded DGPS message is fed into the GPS receiver.

How the message should be encoded depends highly on the properties of the data channel. Therefore, first an investigation has to be done on the Loran-C channel. The design of the error correcting codes is discussed in Chapter 4.

3.2 Sources of interference on the Loran-C signal

As can be seen in Figure 3.1, the Loran-C channel is disturbed by four kinds of interferences:

- Thermal noise
- Atmospheric noise
- Continuous wave interference
- Cross-Rate interference

Now, each interference source will be briefly discussed, focusing on its effect on the reception of the Loran-C signal.

3.2.1 Thermal noise

Like any other radio transmission system the Loran-C signal is disturbed by thermal noise. The signal-to-noise ratio (SNR) of the Loran-C signal can easily drop below the 0 dB line when the signals are received at far distances from the Loran-C station. The normal Loran-C user compensates for low SNR by integrating the received Loran-C pulses for a certain period of time. With data transmission, however, each pulse carries a piece of information and the signal cannot be upgraded by integration.

3.2.2 Atmospheric noise

Atmospheric noise is a pulse like strong interfering signal generated by electrostatic discharges during lightning. Due to the high transmission power the effects of a discharge can be noticed over thousands of kilometers and can be disastrous for the low power Loran-C signal of distant stations. Due to its unpredictable behavior, atmospheric noise is hard to model. The influences of atmospheric noise are not treated any further in this report.

3.2.3 Continuous wave interference

Interference of radio systems (other than Loran-C) which operate at a frequency near the Loran-C spectrum is called continuous wave interference (CWI). This interference can be very strong, but is always limited to a very small band around a central frequency. The effects of CWI can be successfully suppressed by notch filters in addition to the standard Loran-C bandpass filter. In the remainder of this report the influences of CWI are considered negligibly small.

3.2.4 Cross-rate interference

Probably the largest source of interference of the Loran-C signal is cross-rate. Due to the fact that each Loran-C station transmits the same pulse shape signal at the same frequency, the signals of the desired Loran-C chain will sometimes be disturbed by those of other chains. In normal Loran-C navigation this cross-rate interference will be averaged out by the receiver minimizing its effect on the final positioning. With Eurofix, however, we want to transmit data over the Loran-C channel with each of the last six pulses carrying a piece of information. If a pulse is corrupted due to cross-rate interference there will be a chance that the information is lost. The influences of cross-rate on the data transmission are treated in Chapter 4.

3.3 The composite Loran-C signal; groundwave and skywave

At the antenna of a Loran-C receiver a composite signal is received that consists of a signal transmitted by the desired Loran-C station and a signal of the sources of interference described in the previous Section. In turn, the desired Loran-C signal is a composite signal as well and consists of a direct signal (groundwave) and a reflected signal (skywave) of the Loran-C station.

The skywave of a Loran-C signal is the reflection of the original signal against the ionosphere, see Figure 3.2. Due to the longer transmission path the skywave arrives a time delay τ_d later than the groundwave (generally 35 μs or longer), the time delay τ_d depends on the distance between receiver and transmitter and the height of the reflection layer (60 - 90 km). Due to its lower attenuation, the skywave can be received up to 20 dB louder than the groundwave. Therefore, in normal Loran-C navigation only the first part of the groundwave can be used for positioning.

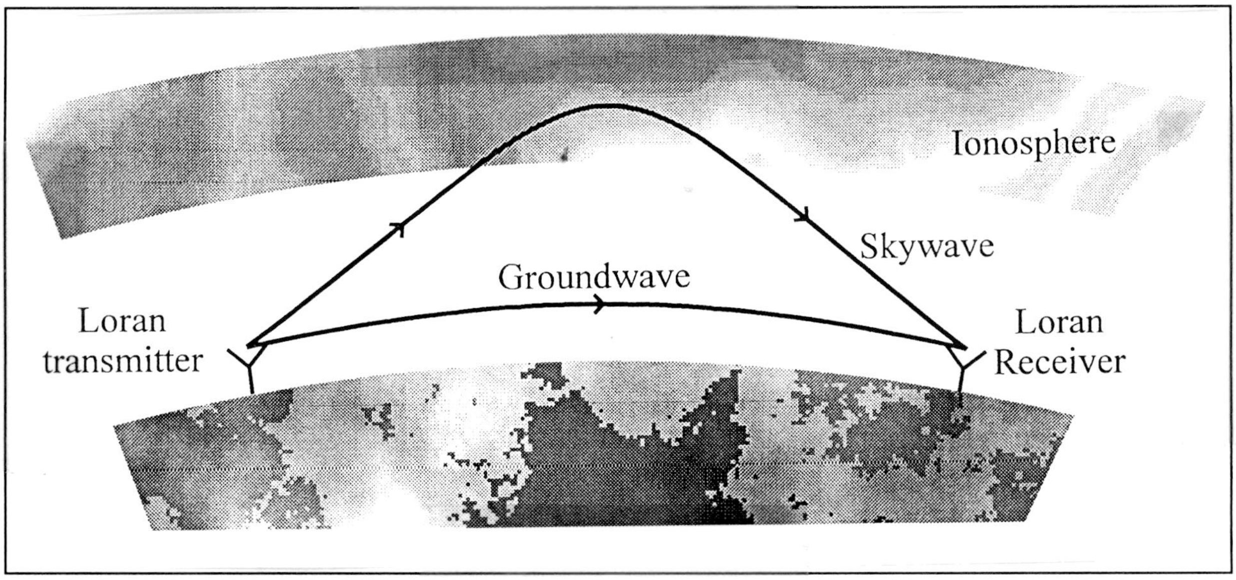

Figure 3.2 Groundwave and skywave of Loran-C signal.

The composite Loran-C signal is a superposition of the groundwave and skywave with a relative phase dependent on the skywave delay τ_d, Formula 3.1.

$$C(t) = A_{gnd} \cdot e(t) \cdot \sin(0.2\pi \cdot t) + A_{sky} \cdot e(t - \tau_d) \cdot \sin(0.2\pi \cdot (t - \tau_d)) \qquad \text{Formula 3.1}$$

where t is time in μs, A_{gnd} and A_{sky} are the amplitudes of the groundwave and skywave respectively, and e(t) is the envelope of the Loran-C signal. The envelope of the composite signal changes as the relative phase changes from 0 to π radians, Figure 3.3.

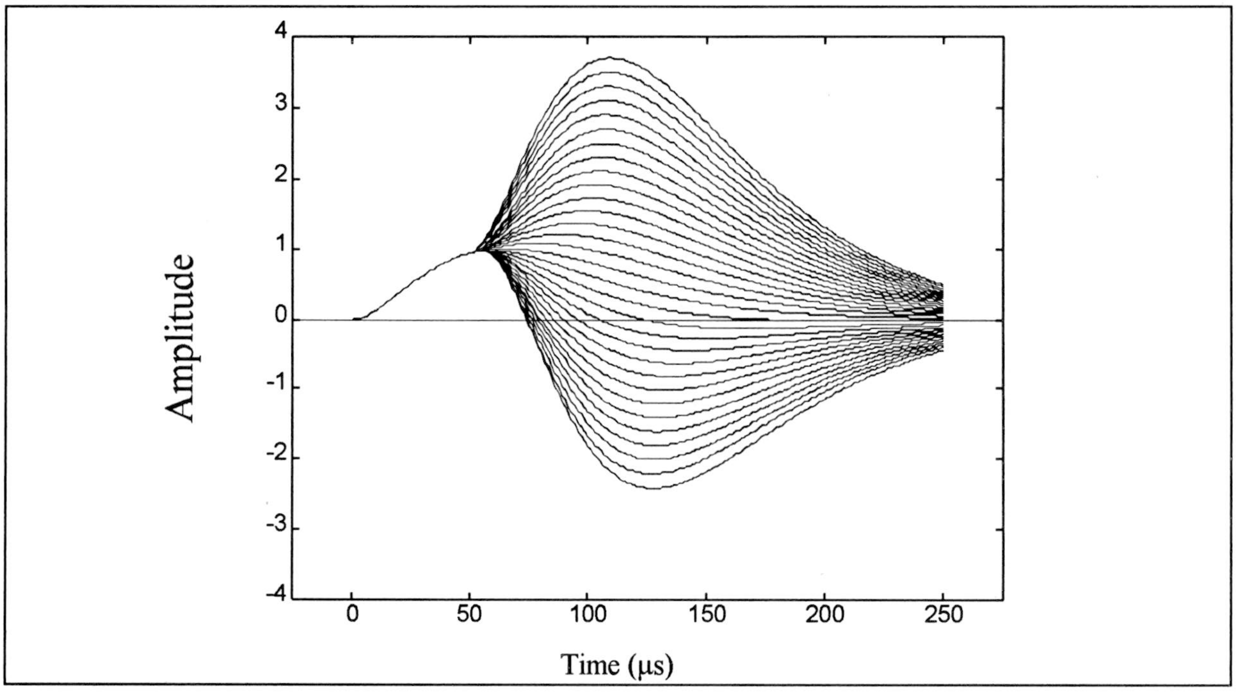

Figure 3.3 Envelope of unfiltered composite Loran-C signal $\tau_d \in [50, 55]$ μs, $A_{gnd} / A_{sky} = 1/3$.

Although the time delay τ_d is unpredictable, it is only slowly varying and thus stable over a longer period of time. By monitoring the received composite signal for a certain period, the energy of the received skywave can also be used for tracking and data demodulation purposes. The next Section discusses two possible Eurofix demodulator types of which one uses the received skywave energy.

3.4 Demodulation of the Eurofix data

This Section describes different techniques to demodulate the Eurofix data. Unlike normal Loran receivers the data recovery part of the Eurofix receiver cannot average a number of pulses to compensate for cross-rate and low signal-to-noise ratios. Every modulated pulse contains a piece of unique information which we like to recover. Therefore, a simple though precise demodulation technique has to be selected. Obviously, knowledge about the precise time of arrival of an unmodulated reference pulse is necessary in order to demodulate correctly. In this Section, however, the timing is assumed to be perfect. Section 3.7 discusses the influence of tracking errors on the accuracy of the demodulation. Various demodulator types have been tested and optimized [7]. Two of them are treated here, the single point demodulator and the cross correlation demodulator [15].

3.4.1 Single point demodulator

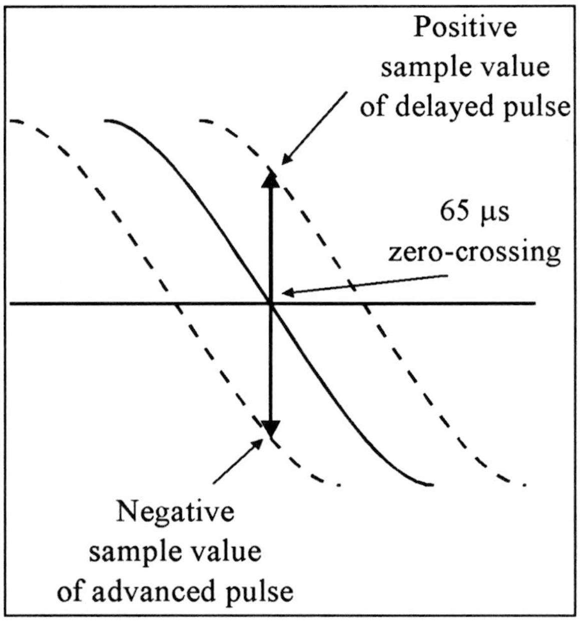

Figure 3.4 Single point demodulation of the Eurofix data.

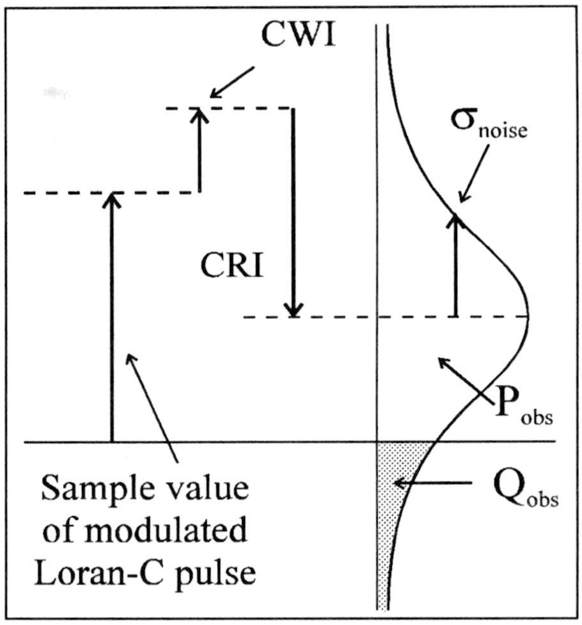

Figure 3.5 Superposition of desired signal, Continuous wave interference, cross-rate interference and noise.

In short, the single point demodulator samples the received Loran-C signal at an expected zerocrossing, for instance the 65μs zerocrossing of the unfiltered signal. The polarity of the sample determines the demodulation output, as can be seen in Figure 3.4.

If the sample has a positive polarity, the modulation is said to be a delayed pulse, if the sample has negative polarity, the modulation is said to be an advanced pulse. The single point receiver can be either of the hard limited type or the linear one. A hard limited demodulator only looks at the polarity of the sample, a linear one also takes the actual sample value into account.

The actual sample value depends on the contribution of all signals received at that particular moment in time. Figure 3.5 shows the probability density function of a received composite signal constructed from the desired Loran-C signal, a continuous wave interference component, a cross-rate component and a contribution of the noise. The area above the horizontal line (P_{obs}) is the probability of a good detection of the modulated data, the area under the horizontal line (Q_{obs}) is the probability of a wrong detection.

Obviously, the sample point (zerocrossing) should be chosen near the top of the pulse envelope where the signal-to-noise ratio is highest. However, the signal may not have been disturbed yet by the skywave. Therefore, the selected zerocrossing depends on the distance between Loran-C station and Eurofix user assuring a skywave free signal.

3.4.2 Cross correlation demodulator

The cross correlation demodulator type has been designed by Megapulse, Inc. and uses all available signal power of ground- and skywaves to determine the modulated time shift of each pulse. The technique is based on the short time stability of the skywave delay τ_d which yields a predictable shape of the received composite Loran-C signal including skywaves. In order to retrieve the modulated time shift, the demodulator takes a set of samples from the actual received pulse and compares it with a stored image of the pulse shape, as can be seen in Figure 3.6.

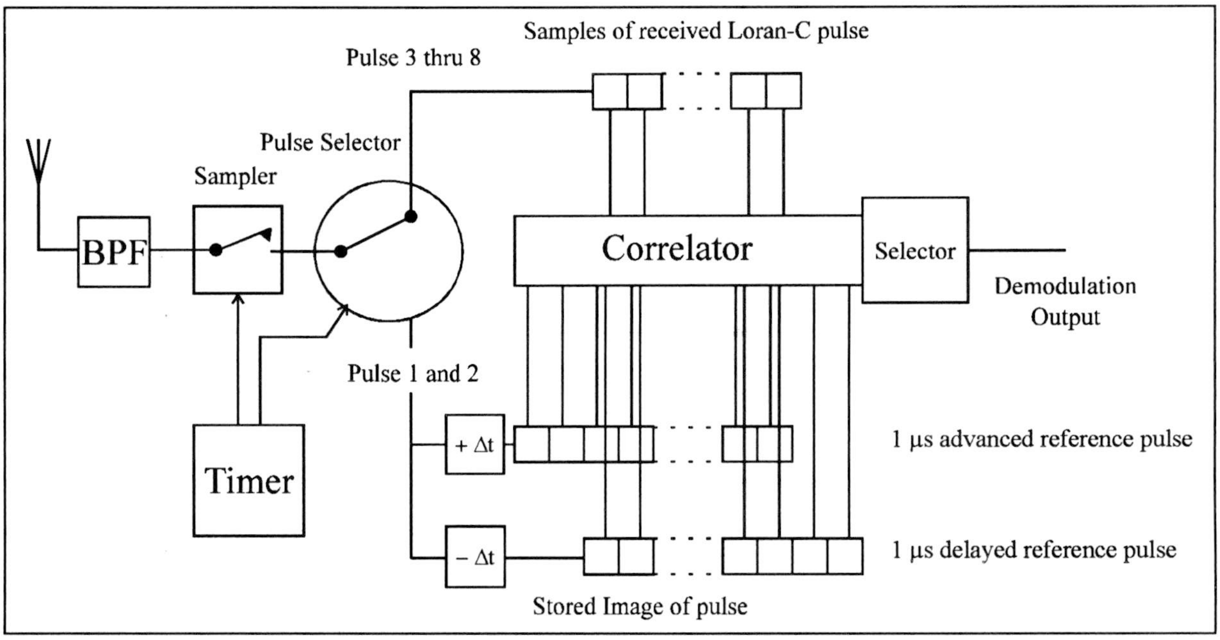

Figure 3.6 Simplified cross correlation demodulator.

After a trigger from the timer the filtered antenna signal is sampled for the duration of the composite signal (approximately 250 μs), the samples of the first two unmodulated pulses are used to upgrade the stored image, the samples of the last six pulses are stored in a register to be demodulated afterwards. After the collection of samples of a modulated pulse is complete, it is once multiplied by a 1 μs advanced version of the stored image and once multiplied by a 1 μs delayed one. The products are then summed to form the + 1μs and - 1 μs correlation values of the received modulated pulse. At the end, the demodulator chooses the time shift with the highest correlation value to be the transmitted time shift. This type of demodulation strongly resembles the Early-Late delay-lock-loop of standard GPS receivers.

This demodulator type can either be hard limited or linear. A hard limited receiver requires a relatively high sample frequency as the information is stored in the zerocrossings of the signal. Megapulse uses a sampling frequency of 2 MHz (one sample every 0.5 μs) which yields a collection of 500 samples per pulse. The sample interval is a compromise between resolution in time and the required amount of memory. The stored image is built up by the sum of 256 sample collections of the first two pulses (taken from 128 consecutive GRIs) to eliminate the influences of noise and interference. In this way 500 registers containing a value between -256 and +256 represent the image of the pulse. A register corresponding to a sample position with high SNR (near the top of a cycle and the top of the envelope) is filled with a high (plus or minus) value, whereas a register corresponding to a sample position with lower SNR (near a zerocrossing of a cycle) is filled with a lower value. In this way the correlation of samples with high SNR contribute more to the final correlation value than samples with lower SNR.

A linear type demodulator can do with a lower sampling frequency. All information about the pulse is contained in I-Q sample pairs, taken at distances equal to at most the correlation time of the bandfiltered noise (e. g. 59 μs with a Loran-C bandfilter of 17 kHz ref. [7]). The In-phase and Quadrature samples themselves should be taken at ¼ of the period of a 100 kHz Loran-C cycle (e. g. 2.5 μs). Using a linear type demodulator which takes an I-Q sample pair every 50 μs would need only 12 registers which contain the amplitudes of the samples (not hard limited). The reference pulse could be built up by the moving average of the samples of the first two pulses in each GRI.

3.4.3 Soft decision demodulation

As was described in the Eurofix communication model, the errors made by the demodulator due to noise and cross-rate have to be corrected by the applied error correcting codes. On the other hand, the demodulator can aid the decoder in correcting errors by supplying extra information about the quality of the received signal at the cost of a more complex demodulator structure. Two possible demodulation strategies and their consequences for the data channel are discussed:

1. Two decision demodulation.
2. Three decision demodulation.

The first type of demodulation is the simplest. Here, the demodulator always takes a decision whether the received pulse was advanced or delayed. Any additional information (e.g. signal-to-noise ratio, amplitude of the sample, obvious cross-rate interference) will be discarded. This demodulator type makes the data channel look like Figure 3.7.

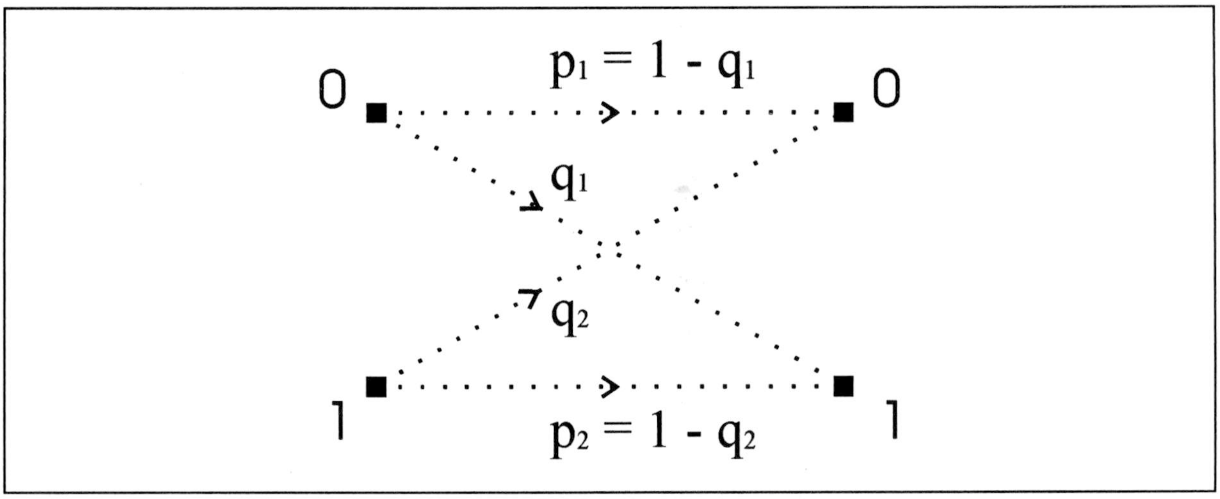

Figure 3.7 The binary data channel.

The transitions ($1 \rightarrow 0$) and ($0 \rightarrow 1$) are caused by noise, cross-rate and other interferences. The probabilities $\mathbf{q_1}$ and $\mathbf{q_2}$ are not necessarily the same and depend on the actual status of the channel. If for instance in some moment of time a pulse is likely to be hit by cross-rate, the probabilities $\mathbf{q_1}$ and $\mathbf{q_2}$ will change. Further, the cross-rate signal could be aligned in such a way that it forces the demodulation to one value regardless the shift of the desired pulse. These properties make that the channel is not continuous and not symmetric. Note that for a reliable data transmission the mean value of both $\mathbf{q_1}$ and $\mathbf{q_2}$ should be less than 0.5.

The three decision demodulator strategy uses an extra state. If the demodulator detects that the received pulse has been severely hit by cross-rate or other interference, it takes no decision but declares the received pulse not to be demodulated (an erasure occurred). The channel now looks like Figure 3.8.

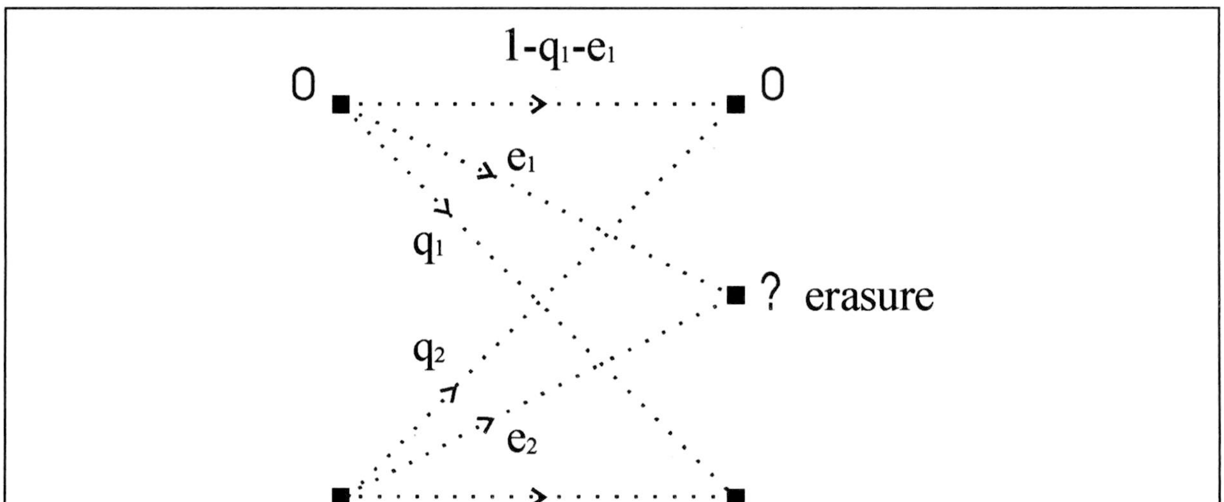

Figure 3.8 The binary erasure data channel.

Again, the probabilities $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{e}_1$, and $\mathbf{e}_2$ depend on the actual status of the channel. Further, the proportion between **q** and **e** will be demodulator dependent. Obviously, the demodulator is not able to distinguish between an error and a good reception (after all, if the demodulator knew it made an error it would correct it right away). The demodulator has to set certain thresholds on the value of the received signal in order to declare a received signal not healthy enough to be demodulated. As an example a linear single point demodulator with fixed thresholds is shown in Figure 3.9.

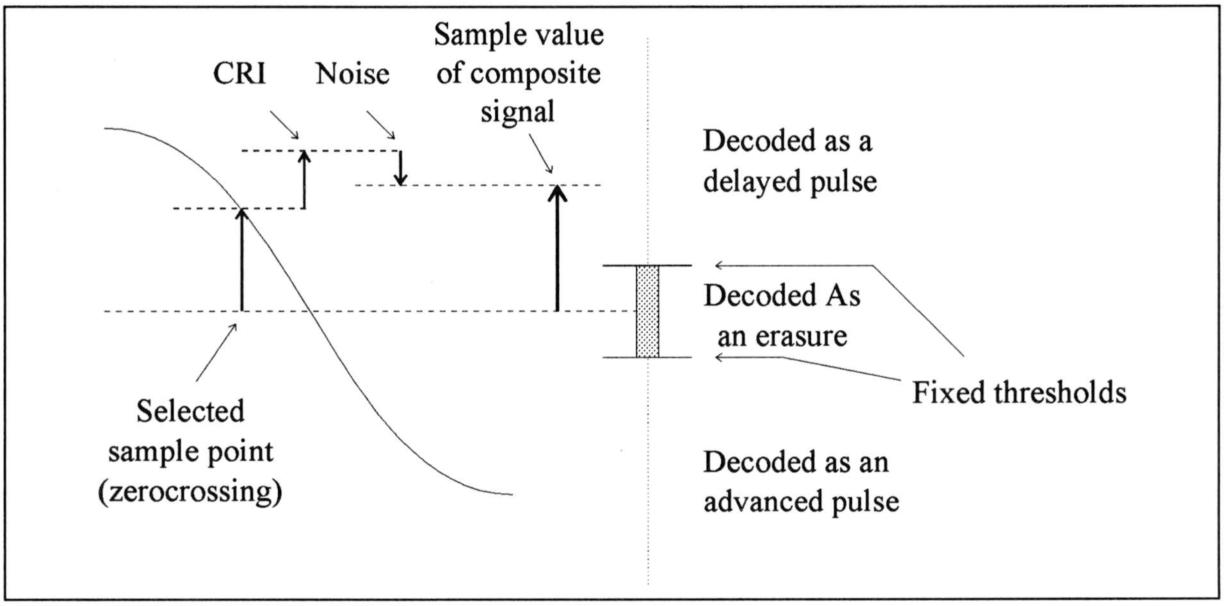

Figure 3.9 Linear single point three decision demodulator with fixed thresholds.

In this example the sample value exceeds the upper threshold so the signal will be demodulated as a delayed pulse.

Apparently, the introduction of erasures means that the demodulator should have extra information about the quality of the signal. Therefore, the demodulator should be either of the linear single point type or the correlation type. Clearly, the hard limited single point receiver is not able to declare a received pulse an erasure because it lacks information about the signal's quality. The decision thresholds can be either fixed or variable. With variable thresholds the proportion of the probabilities **q** and **e** can be kept the same with changing signal-to-noise ratio, as illustrated in Figure 3.10.

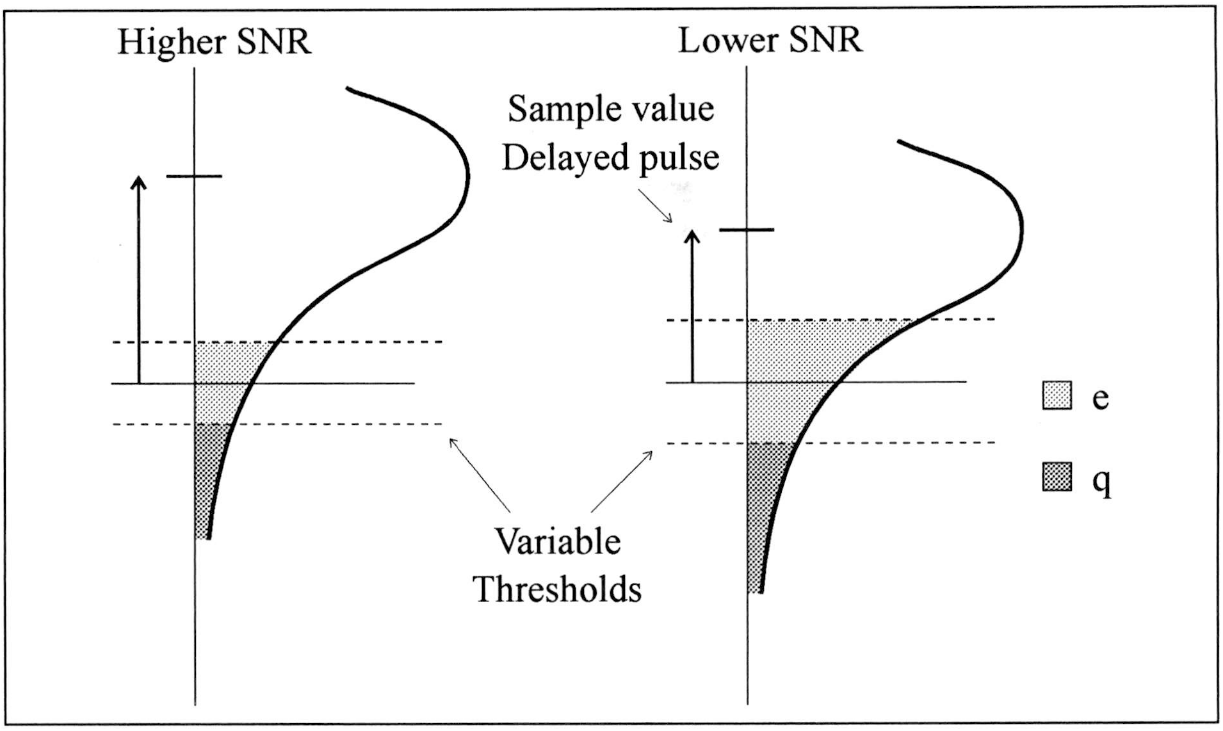

Figure 3.10 Illustration of use of variable thresholds for equal proportion of q and ***e*** *with changing SNR.*

Obviously, by setting demodulation thresholds the error probability decreases, that is, some of the errors are now declared erasures. On the other hand, the probability of good detection decreases as well. Nevertheless, this decrease is compensated for by the increased performance of the decoder. At which values the thresholds must be set or at which **e/q** rate they must be designed is not discussed here.

The choice which demodulator should be used depends on the degree of complexity a receiver manufacturer is willing to apply. A linear cross correlation demodulator performs best, but requires a fast D/A converter. Due to wide spread use of hard limiters and the ability for soft demodulation of the Eurofix data, the hard limited cross correlation demodulator is favored.

3.5 Loran-C navigation degradation

By additionally modulating the Loran-C signal with Eurofix data some navigation power is lost. This loss inevitably means a degradation in the performance of the normal Loran-C positioning. A first restriction in the design of Eurofix, however, stated that this degradation should be 'acceptable'. In this Section the degradation of the Loran-C navigation due to the modulated data is discussed. First a general model of current hard limited Loran-C receivers is given.

3.5.1 A typical hard limited Loran-C receiver

A Loran-C user determines its position by measuring time differences between the reception of master signals and secondary signals [8]. In order to do so, the receiver tracks the zerocrossing of the third cycle in each pulse. The acquisition of the desired time measurement consists of two steps: First, the desired cycle has to be found (cycle identification) and second, the desired zerocrossing has to be tracked. Cycle identification is done on basis of the known shape of the signal's envelope and has to be done very accurately. A cycle slip (10 μs error) corresponds to a positioning error of 3 km!

When the right cycle is identified, the signal is sampled to find the right zerocrossing. Once this zerocrossing is found, the time measurement can be done and the user's position can be calculated. From here on the receiver continues to track the zerocrossing which can change position due to the receivers movement. To eliminate the influences of cross-rate and noise in the determination of the zerocrossing, the signal samples are integrated for a certain period of time, T_{av} . In a hard limited receiver this integration is done using an up-down counter where each positive sample yields an up count and each negative sample a down count. After the integration time, T_{av} , the value of the up-down counter determines the action taken to adjust the approximated location of the zerocrossing (and thus the sample point for the next T_{av} seconds). The tracking process is clarified with an example.

Figure 3.11 shows the probability density function of the signal value at the sample point when the receiver makes a tracking error of about - 400 ns. Due to the tracking error the mean of the signal value is greater than zero which yields the directional information for the tracker. Due to noise (for simplicity the other sources of interference are discarded in this example) the probability of a negative signal value (Q_{obs}) is not equal to zero. This means that there is a real chance for the hard limited signal sample to be negative. The probabilities P_{obs} and Q_{obs} depend on the mean sample value (determined by the tracking error) and the standard deviation of the noise σ_{noise} (depending on signal-to-noise ratio).

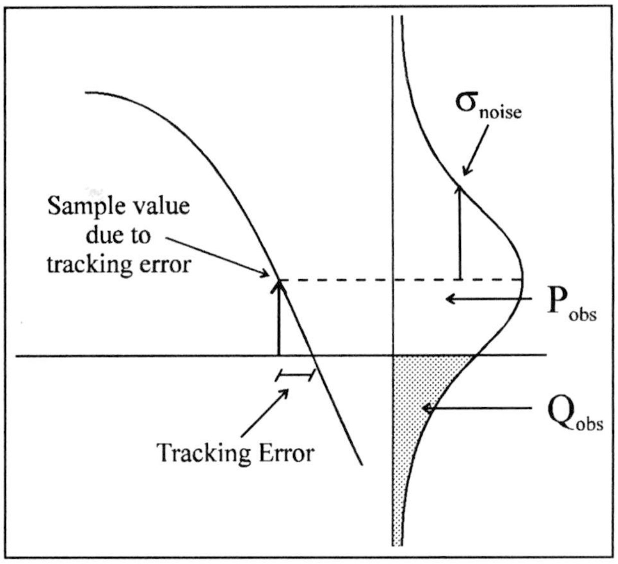

Figure 3.11 Probability density function of the sample value due to a tracking error and noise.

Initially, the up-down counter is filled with the value zero. Then, for a period T_{av} the received Loran-C pulses are sampled on the estimated zerocrossing but due to the tracking error wrong sample point. The up-down counter adds one, every time a positive sample is measured (with probability P_{obs}), and subtracts one, every time a negative sample is measured (with probability Q_{obs}). After T_{av} seconds the sign of the up-down counter determines the direction the tracker has to be corrected to. Figure 3.12 shows a possible pattern the up-down counter can be filled with. The pace the up-down counter is filled with equals the difference between P_{obs} and Q_{obs} . When the signal is sampled at (or very near) the actual zerocrossing both probabilities P_{obs} and Q_{obs} are equal (0.5) and the up-down counter is filled randomly.

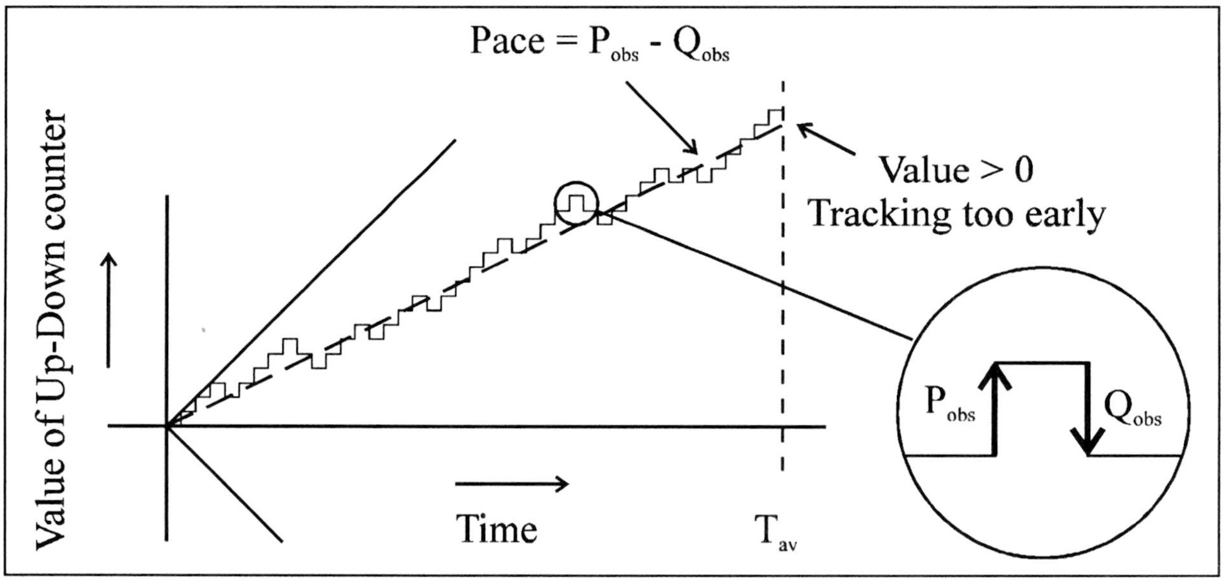

Figure 3.12 Up-down counter in Loran-C receiver.

At the end of the period T_{av} a decision is made based on the random sign of the up-down counter. Theoretically, the tracker of this type of receivers will be idling around the true zerocrossing in this situation.

The dynamic behavior of the Loran-C receiver relates to the averaging time T_{av} and the resolution of the tracking (stepsize of tracking correction). That is, if for instance the receiver has a T_{av} = 7 seconds and the resolution = 50 ns, then it takes at least 56 seconds to correct a 400 ns tracking error. The dynamic behavior has to be fast with Eurofix receivers, as will be shown in Section 3.6. In contrast, a fast normal Loran-C receiver will experience more degradation due to the extra modulation than a slower one. The influence of the modulation on the normal Loran-C tracking is discussed in the following Sections.

3.5.2 Sample value probabilities of modulated Loran-C pulses

The time shifts of the modulated pulses introduce an extra error in the tracking loop of a normal receiver. Figure 3.13 shows the probability density function of the sample value of a time advanced and delayed pulse in comparison with an unmodulated one.

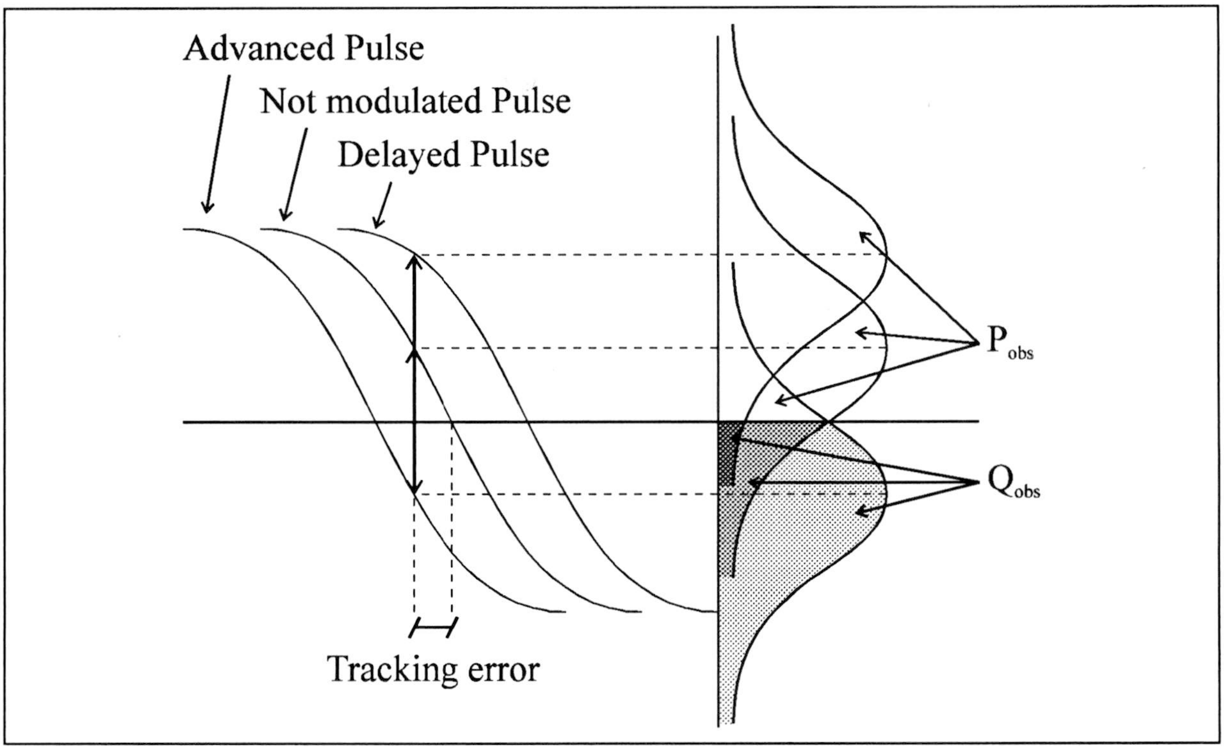

Figure 3.13 Comparison of the probability density functions of the sample values of a delayed, an advanced and an unmodulated pulse.

Clearly, because of the additional time shifts the probabilities P_{obs} and Q_{obs} change and therefore become modulation dependent. This means that the final value of the up-down counter and thus the tracking performance will now depend on the transmitted data. No serious harm is done if only the sum of the probabilities P_{obs} over the interval T_{av} is greater than the sum of all Q_{obs}. One way to achieve this is to force the mean of the modulation to zero by balancing the transmitted time shifts. This means that there has to be an equal number of time advances and time delays in a certain block (a number of GRIs). In this way the average tracking error due to the data modulation is zero. Tracking information (gain of the tracking loop) is preserved as long as the sum of the probabilities P_{obs} of a pair of corresponding modulated pulses (one advanced and one delayed, called a modulated pair from now on) is greater than the sum of the probabilities Q_{obs}.

3.5.3 Dead zone in hard limited receivers

As stated before, the gain of the tracking loop of a modulated pair depends on the sums of P_{obs} and Q_{obs}. As the signal-to-noise ratio increases the Gaussian distribution becomes higher and narrower leveling the sums of P_{obs} and Q_{obs}, as is shown in Figure 3.14.

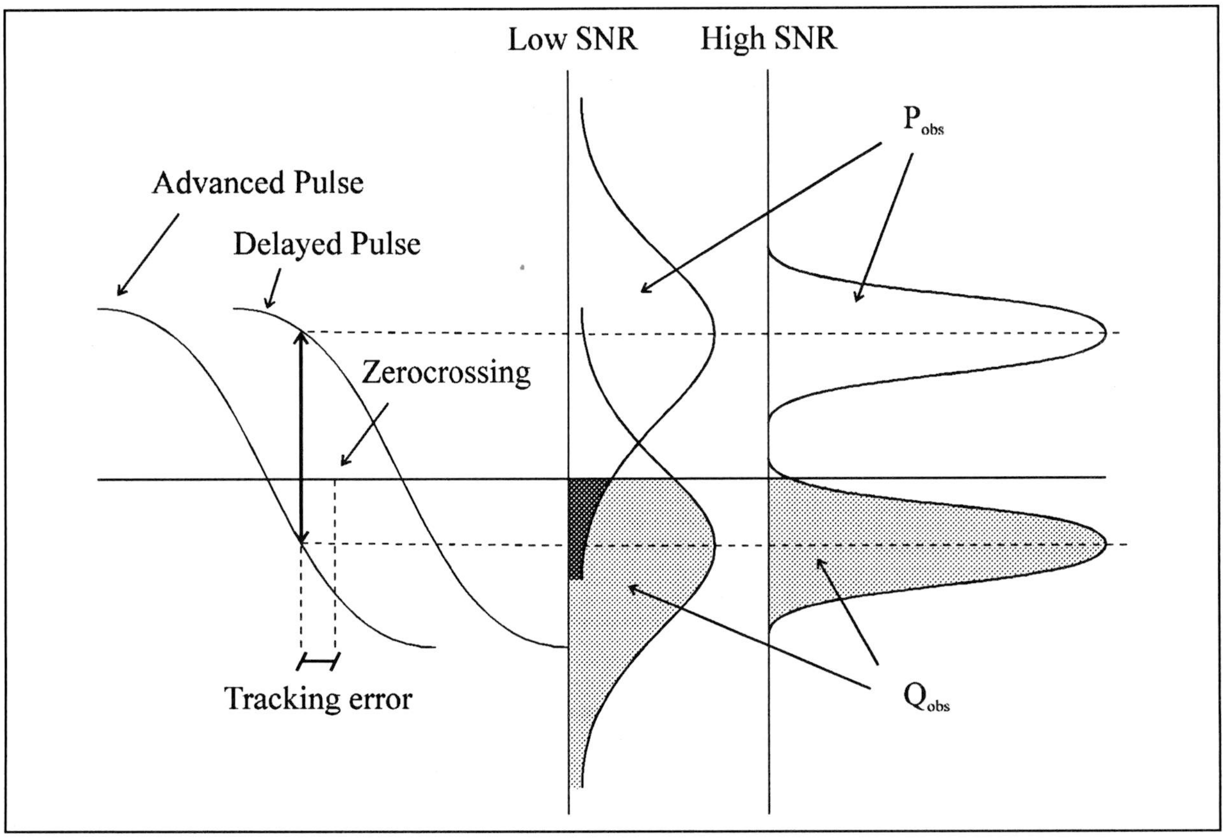

Figure 3.14 Illustration of change in P_{obs} and Q_{obs} due to higher signal-to-noise ratio.

As a consequence, at very high SNR the sums of P_{obs} and Q_{obs} of a modulated pair are approximately equal reducing the loop gain to zero for almost any tracking offset. In this so called dead zone, the modulated pair does not contribute to the tracking anymore. If all pulses in the GRI were modulated, the receiver would lose track completely. Besides the normal Loran-C signaling (blinking), this is a second good reason why the first two pulses in each GRI cannot be modulated.

Now, the degradation of the normal Loran-C tracking owing to the modulation can be approximated. Figure 3.15 shows the signal loss due to a balanced modulation as a function of SNR where SNR is calculated with respect to the amplitude of the tracked cycle. The Loran-C receiver is a simulated hard limited receiver with fixed tracking bandwidth of 0.2 Hz (averaging time of 5 seconds) and a small tracking stepsize of 1 ns, the simulated receiver is described in Section 4.6.1.

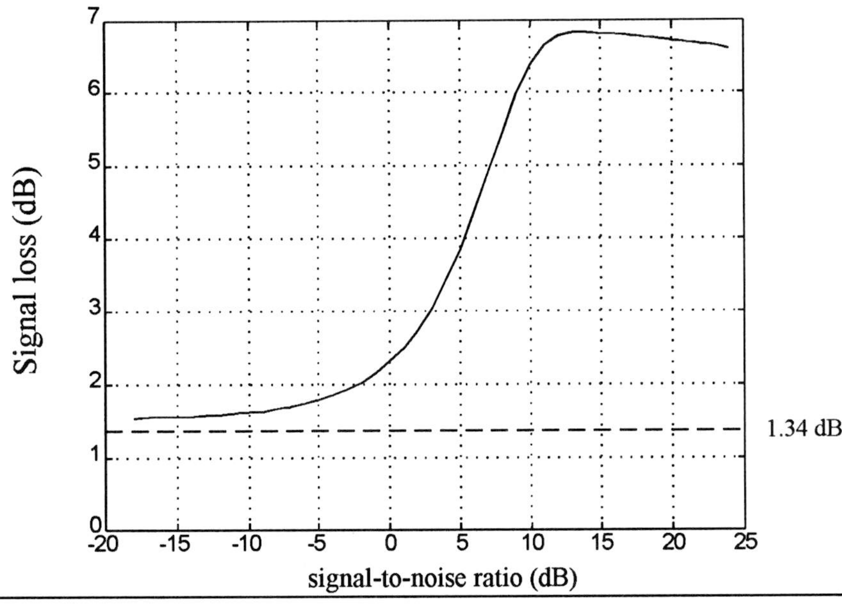

Figure 3.15 Signal loss due to modulation as a function of signal-to-noise ratio.

Theoretically, the SNR loss boundaries can be calculated in the following way: for low SNR, all modulated pulses still contribute to the tracking information, only a navigation signal loss of 1.84 dB per modulated pulse is noted. With only 6 out of 8 pulses modulated this yields a total average loss of 1.38 dB per pulse as was calculated in Section 2.3.1. For high SNR, the balanced modulated pulses cancel completely and therefore do not contribute to the tracking loop gain anymore (dead zone). Only the two unmodulated pulses add to the tracking information yielding a power loss of ¾ which corresponds to a 6.02 dB loss (on power basis). The theoretical calculation assumes a perfect timing. Tracking errors are not taken into account. This explains the slight difference with the simulated signal loss in Figure 3.15.

3.5.4 The sliding window concept

Due to the relatively low signal-to-noise ratios and the effects of cross-rate interference, all Loran-C receivers integrate the received signal before a time measurement is done. The averaging time T_{av} is receiver dependent and can vary from one minute to less than a second. The influence of the data modulation will therefore be receiver dependent.

Each receiver takes samples of a number of consecutive pulses. Therefore, the sample collection of a Loran-C receiver can be seen as a sliding window over all available pulses in time. The size of the window (in samples) relates to the averaging time T_{av} and is equal to T_{av} divided by the Group Repetition Interval of the Loran-C transmitter. As long as the modulation is balanced over the sliding window of the receiver, it does not introduce any severe tracking errors. However, as the window size is receiver dependent a balanced modulation for one receiver could be unbalanced for another.
Also, although the modulation is balanced over a certain window, the receiver could lock onto it on a bad moment in time, yielding an unbalanced modulation within the receiver window. All is explained in Figure 3.16.

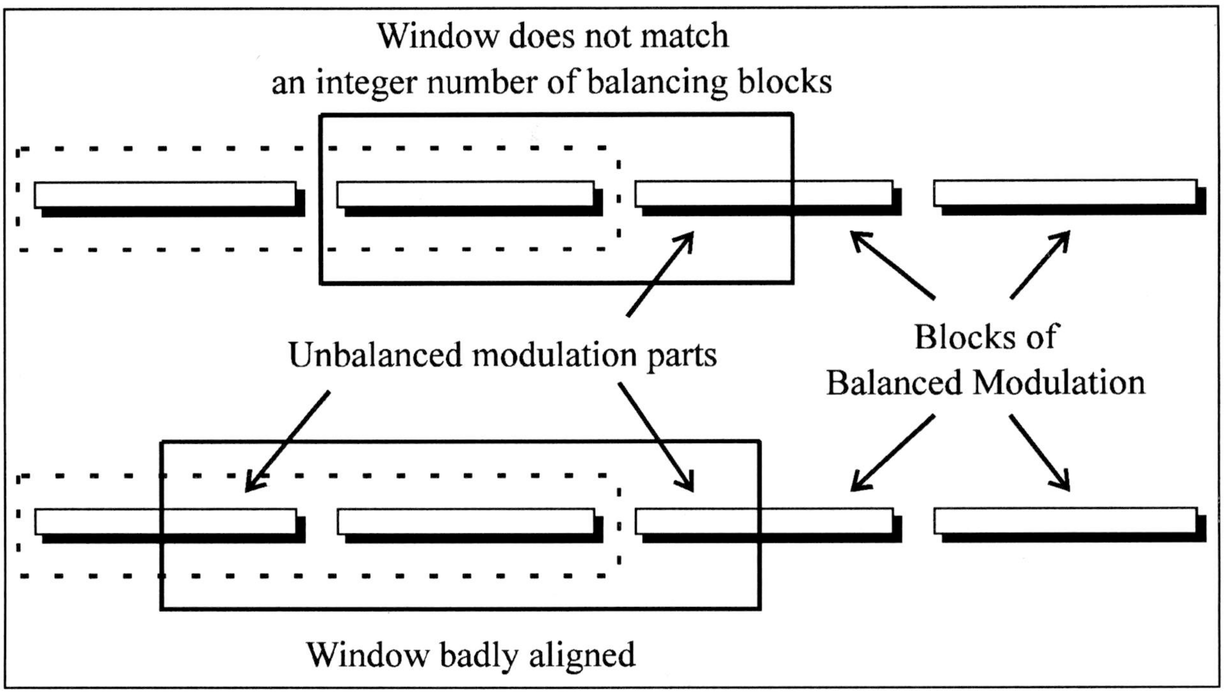

Figure 3.16 Integration window versus balanced modulation blocks.

Clearly, any imbalance in the integration window will cause an extra tracking error. Unfortunately these imbalances are unavoidable. The influence of imbalances depends highly on the size of the integration window and size of the balanced modulation block. If the window contains a large number of balanced blocks, the unbalanced parts are averaged out. Therefore, the balanced modulation block length should be short relative to any possible integration window size to avoid serious tracking errors due to Eurofix data modulation.

3.6 Influence of tracking errors on the data demodulation

For good demodulation of the Eurofix data the incoming pulses should be sampled at precisely the right time. If due to a tracking error the signal is not sampled at the correct sample point, the demodulation becomes inaccurate, as is shown in Figure 3.17.

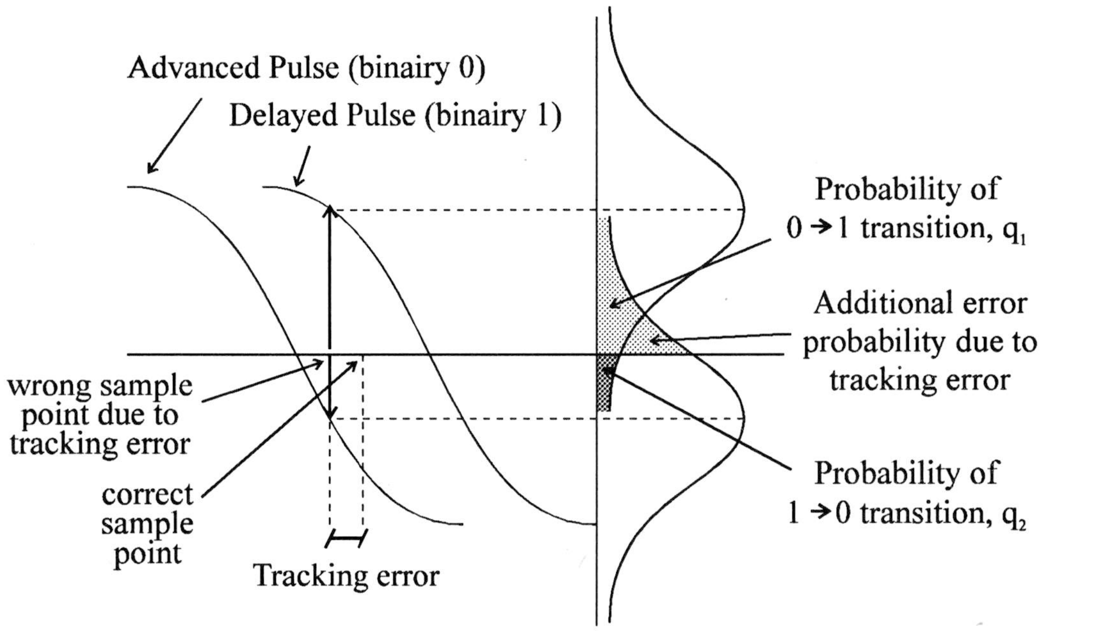

Figure 3.17 Influence of a tracking error on the data demodulation.

Clearly, the probabilities q_1 and q_2 are not equal, making the channel asymmetric and increasing the total demodulation error probability. In order to avoid this increase of demodulation errors the tracking loop of the Eurofix receiver should have a good dynamic behavior (preferably better than current Loran-C receivers). What type of tracking loop should be selected is a subject for further research and is not discussed in this report.

3.7 Conclusions

At the Loran-C antenna a composite signal is received that consists of a signal of the desired Loran-C station and interfering signals. The desired signal consists of a groundwave and a skywave part. The relatively slow varying skywave delay τ_d enables the use of the skywave energy for data demodulation. The effects of cross-rate interference (the largest source of interference) on data transmission are discussed in Chapter 4.

In order to make as few demodulation errors as possible a suitable and preferably simple demodulation algorithm has to be selected. The hard limited cross correlation technique is favored because of its relatively low complexity and its ability to use the skywave energy for data demodulation. Extra care should be taken in the selection of the type of tracking loop for the demodulator.

The demand for low additional Loran-C performance degradation puts two restrictions on the possible modulation forms:

- The modulation has to be balanced in order to avoid tracking errors due to long strings of time advanced or time delayed pulses.
- The block length of the balanced modulation has to be short in comparison with the integration window of the Loran-C receiver.

4 Eurofix error correcting aspects

The Loran-C channel is not an ideal data channel, but is endangered by interferences of all kinds, as mentioned in Section 3.2. Only two of them are discussed here, thermal noise and cross-rate interference. After a brief description of their influences on data transmission, error correcting codes are designed to counteract these influences.

4.1 Thermal noise

The noise in this communication system is assumed to be Gaussian which means that it has a normal distribution with zero mean (no bias) and a standard deviation σ_n equal to the square root of the signal power. Due to noise, random demodulation errors occur. The probability of these random errors depends on the signal-to-noise ratio (the value of σ_n in proportion to the signal strength). One remark about the influences of noise has to be made with respect to the demodulator type: the single point demodulator only takes one sample of signal and noise, yielding a noise error probability directly depending on the SNR. In contrast, the cross correlation demodulator takes more samples of signal and noise, averaging the noise over a longer period which corresponds to an increase of the signal-to-noise ratio. The SNR increase depends on the length of the time interval the samples are taken in compared to the correlation length of the bandfiltered noise.

4.2 Cross-rate interference

Due to the fact that each Loran-C station transmits the same pulse shape signal at the same frequency, the signals of the desired Loran-C chain will sometimes be disturbed by those of other chains. In normal Loran-C navigation this cross-rate interference is averaged out by the receiver minimizing its effect on the final positioning. With Eurofix, however, each modulated pulse carries a piece of information which has to be recovered one way or the other. If a pulse is corrupted by cross-rate interference, there will be a chance that the information is lost. This Section will take a closer look on cross-rate interference and its influences on data transmission.

Cross-rate is a deterministic source of interference and is only influenced by the following four parameters:

- Group Repetition Interval of desired and cross-rating Loran-C stations
- Initial time offset between the two interfering stations
- Groundwave and skywave signal strength of both interfering Loran-C stations
- Skywave delay of both interfering Loran-C stations

Although a receiver could receive cross-rate from many stations of different Loran-C chains, for simplicity only the effect of the loudest cross-rating station will be taken into account here.

Figure 4.1 shows the cross-rate interference for GRIs 6000 and 8000 (60 ms and 80 ms). Each group of eight pulses can be seen as a comb with eight teeth. Cross-rate interference is only noticed if one or more teeth of the cross-rating station coincide with teeth of the Eurofix station.

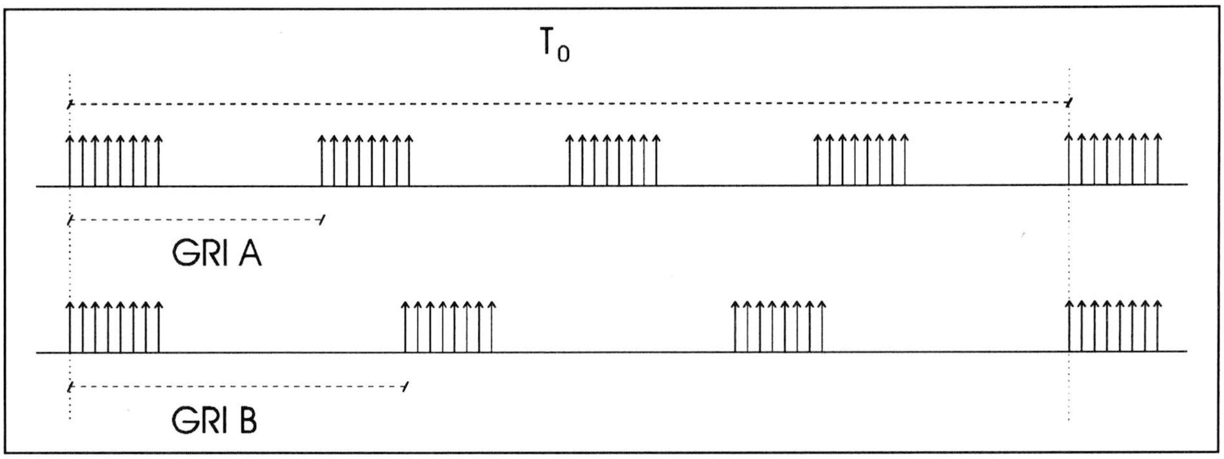

Figure 4.1 Cross-rate interference for GRIs 6000 and 8000

As can be seen from Figure 4.1 the cross-rate pattern is repeated every T_0. This repetition time T_0 (cross over period ref. [9]) can be calculated with the aid of Formula 4.1.

$$T_0 = \frac{GRI_A \times GRI_B}{GCD(GRI_A, GRI_B)} \times 10^{-5} \text{ Seconds} \qquad \text{Formula 4.1}$$

with GCD the Greatest Common Devisor and the GRIs in their 4 digit representation. In our example the cross over period would be 240 ms. Most GRIs of Loran-C chains in Europe and the US are chosen to have large cross over periods with chains in the vicinity (their GCD has to be small).

4.2.1 Sub-periodicity of cross-rate interference

Although two neighboring Loran-C chains may be mutually prime (their GCD = 1) the cross-rate pattern can contain large sub-periodicities. This so-called "short interleaving" [9] can highly influence the performance of the Loran-C datalink. The origin of this sub-periodic behavior lies in the fact that the arrows in Figure 4.1 , the teeth of the comb, are of certain width (approximately 200 μs). If for instance our two cross-rating GRIs are 6001 and 8001 (mutually prime), the cross over period T_0 equals 480 sec. However, after 240 ms (approximately four times GRI_A and three times GRI_B) both arrows (pulses) have only shifted 10 μs relative to each other, which is exactly one 100 kHz cycle. This means that if a group of pulses of GRI_A ($group_A$) is hit by cross-rate of GRI_B ($group_B$), it will be hit again 4 GRIs later with nearly the same alignment.

The alignment of the cross-rating pulse $group_B$ with respect to the Eurofix pulse $group_A$ depends on the time difference Δt between the start of both groups which can be calculated with Formula 4.2.

$$\Delta t = t_0 + \mathbf{n} \times GRI_A - \mathbf{m} \times GRI_B \qquad \text{Formula 4.2}$$

where t_0 is the initial time offset at the user position at some starting point in time, **n** is the number of GRI_A since the starting time and **m** is the nearest integer to keep Δt as small as possible. Δt is the time difference between the start of the n^{th} $group_A$ and the m^{th} $group_B$. If this time difference lies within the interval <-8 ms, 8 ms$>$ the two pulse groups (partly) overlap, if the time difference equals zero both groups start at the same time and all 8 pulses will fully overlap. Further, this time difference determines which pulses in $group_A$ are hit and how severely they are hit.

Example: Suppose the Eurofix station has $GRI_A = 6500$ and the cross-rating station has $GRI_B = 5005$, and initially they were aligned ($t_0 = 0$). After 30 times GRI_A (1.95 seconds) **m** will be 39 (nearest integer) and the time difference $\Delta t = -1.95$ ms. This means that $group_A$ starts 2 pulses earlier than $group_B$, but the coinciding pulses of $group_A$ start 50 μsec later than those of $group_B$. Further, only pulse 3 thru 8 of $group_A$ will be hit by a pulse of $group_B$ with an offset of 50 μsec as shown in Figure 4.2.

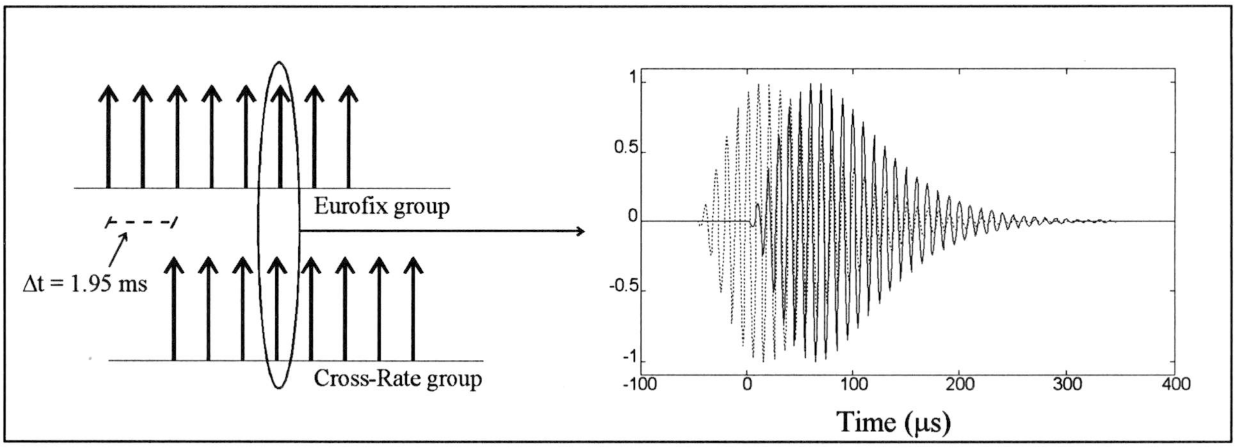

Figure 4.2 Example of cross-rate alignment, left: group alignment, right: pulse alignment (Eurofix pulse solid and cross-rate pulse dotted).

4.2.2 Phase angle between desired and cross-rate signal

The shape of the summed signal of the Eurofix pulse and the cross-rate pulse depends on the relative phase of both signals. Figure 4.3 shows the summed signal for phase differences of 0° and 180°.

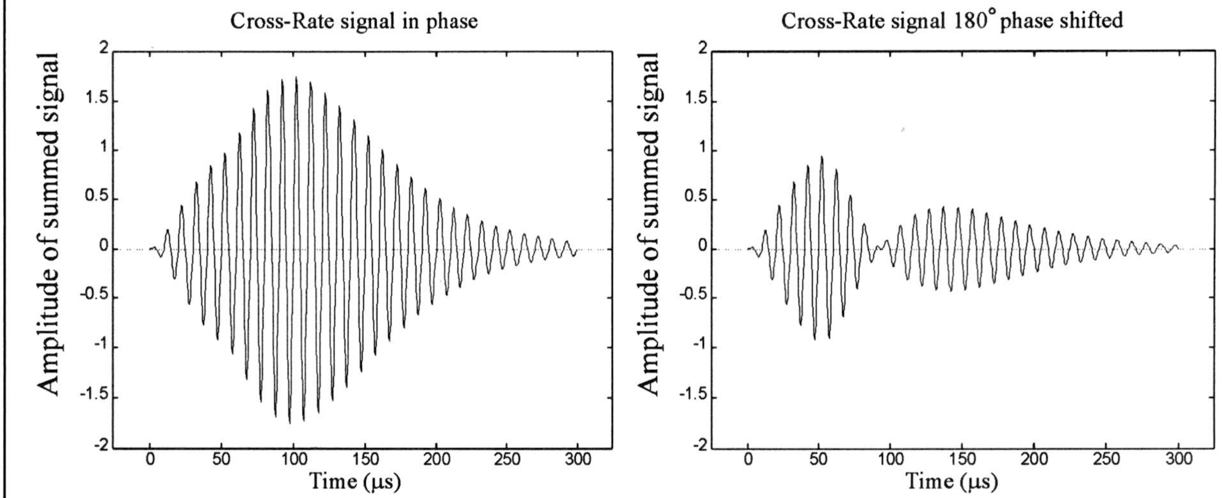

Figure 4.3 Received summed signal containing Eurofix and cross-rate pulse, equal in strength, once in phase (left) and once 180° phase shifted (right).

As can be seen in Figure 4.3 the influence of the cross-rating signal depends on its relative phase with respect to the Eurofix pulse. Although the two interfering GRIs only differ in a multiple of one 100 kHz cycle (last digit in the GRI representation stands for 10 μsec), this relative phase is in general different from zero. First, the Eurofix and cross-rate pulse can have a different Phase Code (equal to either 0° or 180°, see [8]). And second, the user position relative to both interfering transmitters introduces a phase angle. Especially this second reason makes the phase angle between desired and cross-rate signal unpredictable. One should also keep in mind that the user is probably moving, changing the relative phase with every meter. Figure 4.4 explains the second reason, where λ is the wavelength of a 100 kHz cycle (e.g. $\lambda = c/f = 3$ km).

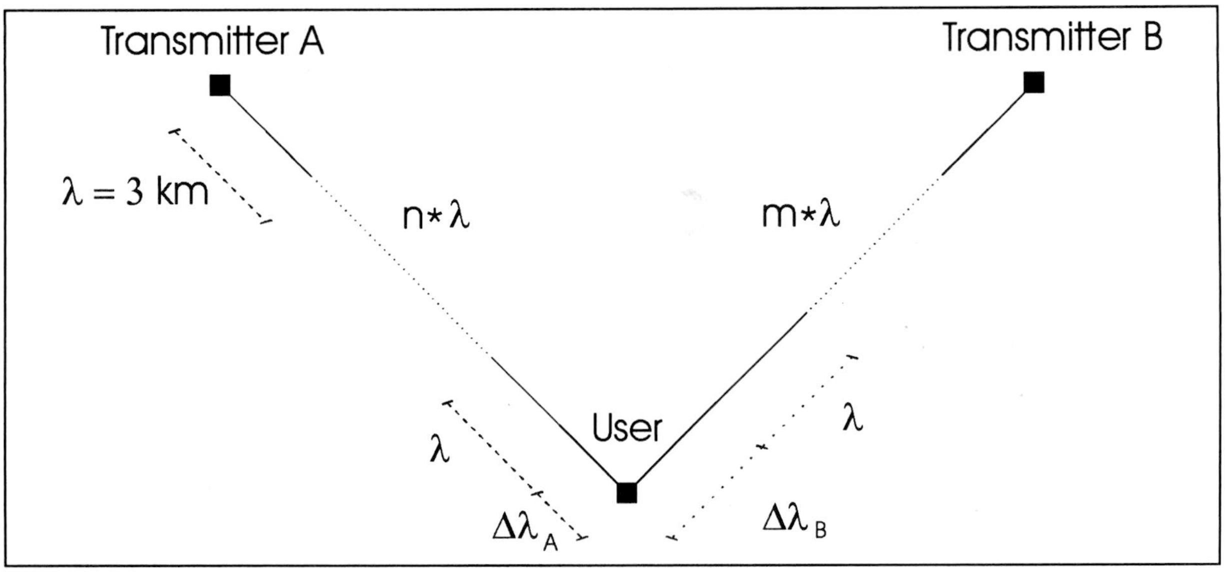

Figure 4.4 Additional phase angle due to user position relative to Loran-C transmitters.

The phase angle between the Eurofix and cross-rate pulse depends on the difference of the distances between user and Loran-C stations (measured in 100 kHz wavelengths). If for instance the distance between user and transmitter A equals 396 km (exactly 132 times λ) and between user and transmitter B 451 km (150 times λ and $\Delta\lambda = 1/3$) than the phase angle is 120° (only if both transmitters are coherent).

In stationary receivers cross-rate from one Loran-C station can only occur with two opposite phase angles determined by the user's location and only switching due to the Phase Coding of both received signals. This phase angle is slowly changing as the skywave delay τ_d of the cross-rate signal changes. In mobile receivers the phase angle can change due to the movement of the mobile. This change, however, is also relatively slow, because a phase change of 180° due to movement corresponds to a displacement of at least 750 m (when the user is moving from one transmitter to the other on the baseline between them). On the whole, the phase angle between desired signal and cross-rating signal is user position dependent and can reach any value between 0° and 360°. On a short period of time, however, the value of the phase angle of a cross-rate signal is limited to two opposite values (actually 3 pairs of opposite values can occur if we assume that the cross-rate station is also modulated with Eurofix data)

4.2.3 The influence of cross-rate on data demodulation

A cross-rate signal does not necessarily have to destroy the Eurofix information. Depending on the phase angle the signal could also aid the demodulation. The alignment of the cross-rate signal remains the same within a received group of 8 pulses (except for the Phase Codes). Therefore, if a received pulse group is hit by cross-rate, the probability of more than one erroneous demodulation in the group increases. The occurrence of multiple errors in one pulse group is called a burst error.

The influence of cross-rate is also demodulator dependent. The influence on the single point demodulator depends only on the value of the cross-rate sample on the specified sample point. If for instance the cross-rate sample equals zero, because the sample point coincides with a zerocrossing of the cross-rate it has no influence at all. The influence of cross-rate on the cross correlation demodulator is more complex. It depends on alignment, strength and shape (groundwave and skywave) of the cross-rate signal, and on the properties of the cross correlation demodulator (e. g. length of time frame, hard limited or linear).

The influence of cross-rate interference also depends highly on the amount of power of the cross-rating signal with respect to the Eurofix signal. Both signals are composite signals containing a groundwave and a skywave part. Generally, the cross-rate station is far away and therefore the skywave of the cross-rating signal is the largest threat for reliable data transmission.

4.2.4 Typical cross-rate patterns

As was shown, a long list of parameters determines the influence of cross-rate interference on the desired signal. Clearly, the most important parameter is the time difference Δt, given in Formula 4.2. This time difference decides whether a cross-rating pulse group is within reach of the pulse group of the Eurofix station. Below, four examples of typical cross-rate patterns are given where it is assumed that cross-rate is effective if the pulse is within a 150 μs reach. The pictures show the number of pulses of a desired pulse group that is hit by cross-rate of stations of another chain. The GRI number corresponds to the value of **n** of Formula 4.2.

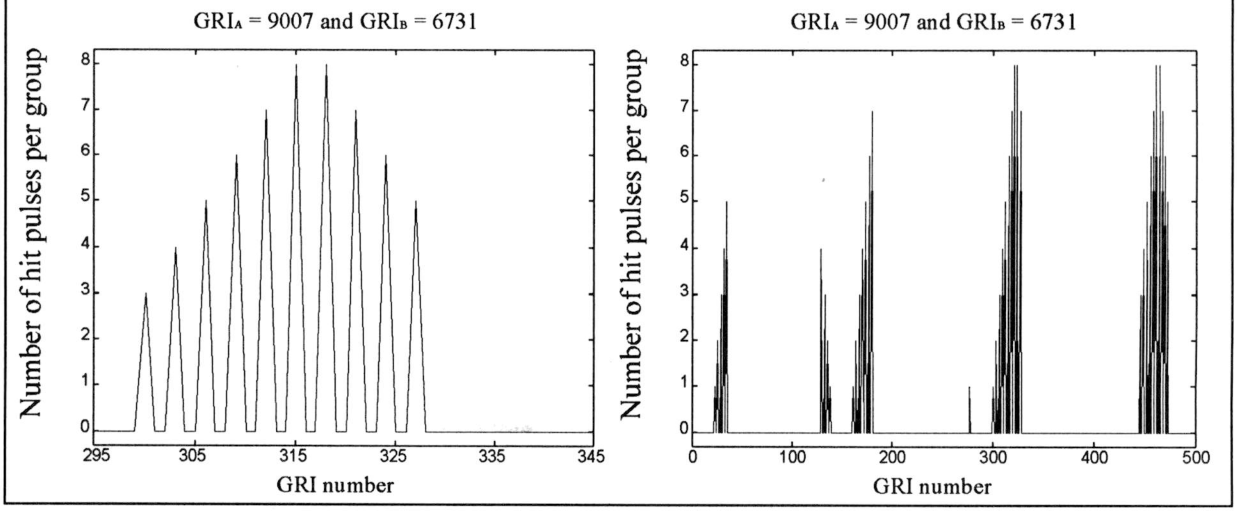

Figure 4.5 a&b Cross-rate pattern of two new European Loran-C chains, Eurofix chain Lessay 9007 and cross-rate chain Ejde 6731.

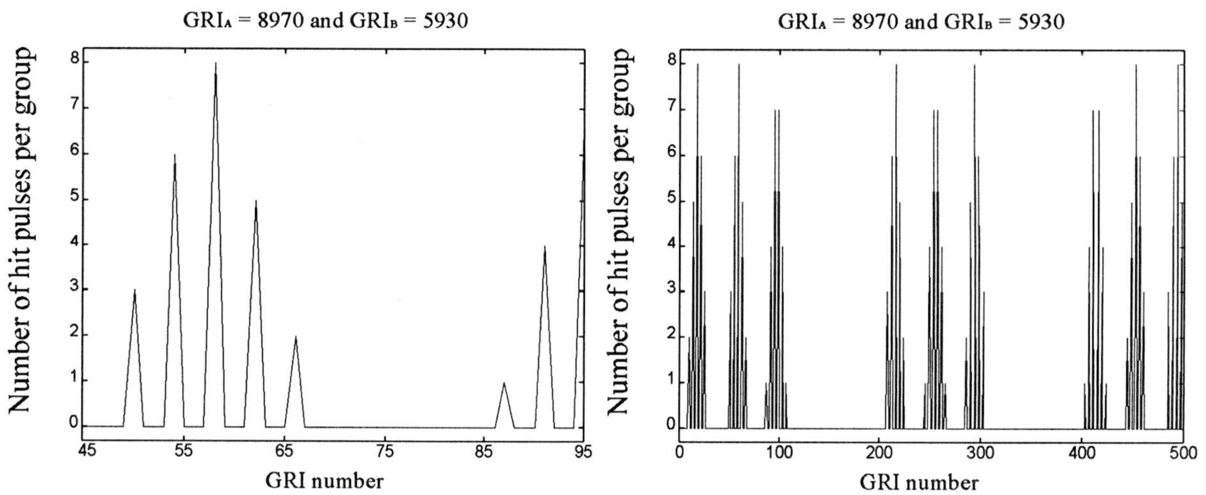

Figure 4.6 a&b Cross-rate pattern of two American Loran-C chains, Eurofix chain Great Lakes chain 8970 and cross-rate chain Canadian East Coast chain 5930.

Clearly, both pictures show a strong sub-periodic effect. In the European example the sub-periodic interval equals 3 and in the American example 4. Also, both patterns show distinct groups of cross-rate (b parts of the Figures). In such a group the peaks are separated at the sub-periodic distances.

Figure 4.7 shows a different cross-rate pattern. Here, cross-rate is more regularly spread yielding relatively large sub-periodic intervals of 13, 19 and 31. As mentioned before, the sub-periodic intervals are predetermined and depend only on the GRIs of the interfering chains. Obviously, both clustered cross-rate patterns with small sub-periodic intervals and more spread cross-rate patterns with larger sub-periodic intervals occur. The cross-rate patterns and sub-periodic intervals of the other European and American chains are given in Appendix A.

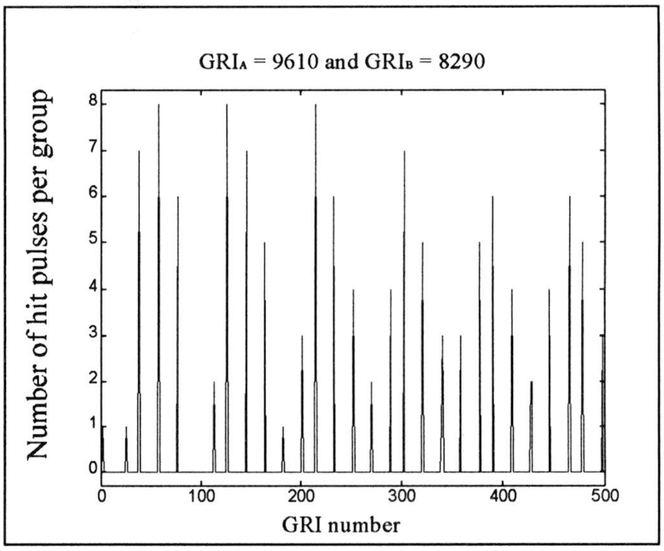

Figure 4.7 Cross-rate pattern of two American Loran-C chains, Eurofix chain South Central U.S. chain 9610 and cross-rate chain North Central U.S. chain 8290.

4.3 Compensation for the cross-rate influences

Although the influence of cross-rate depends on strength and relative phase of the signal, the patterns of the previous Section are taken to be the worst case on which the error correcting codes will be designed. The error patterns due to cross-rate show the following two properties:

- First, the errors are likely to come in bursts of maximum length 6.
- Second, the errors occur within regular intervals determined by the sub-periodic properties of the cross-rate signal.

On top of the errors due to cross-rate from one transmitter, errors occur due to noise and cross-rate from other transmitters (either in the same chain with the same pattern only shifted in time, or in another chain with another pattern). This increase of error sources makes the design of appropriate codes a very complex process. In first approximation, only the influences of one cross-rate signal and noise are taken into account.

In the remainder of this Section, two types of coding schemes are derived and compared in terms of effective data rate, and error correcting capabilities. Those who are not familiar with the coding theory are referred to Appendix B, introduction to error correcting codes.

In coding theory, different types of error correcting codes and techniques can be found that are suitable to correct burst errors (e.g. interleaving, product codes, Reed-Solomon codes). In this Section coding schemes are designed using a product code to counteract the influences of cross-rate because of its low complexity and ease to decode. First, the parameters and properties of a product code are derived.

4.3.1 Product codes

A product code is built up by two single codes, C_h and C_v ref. [10]. The direct product of these codes $C_h \otimes C_v$ is a matrix with code words of C_h on its rows and code words of C_v on its columns. If both single codes are systematic (the first **k** bits are the information bits, the last **r** = **n** - **k** bits are check bits) the matrix looks like Figure 4.8.

$$\left[\begin{array}{cccc:cccc}
x_{1,1} & x_{1,2} & \cdots & x_{1,k_h} & y_{1,1} & \cdots & & y_{1,r_h} \\
x_{2,1} & x_{2,2} & \cdots & x_{2,k_h} & y_{2,1} & \cdots & & y_{2,r_h} \\
\vdots & \vdots & & \vdots & \vdots & & & \vdots \\
x_{k_v,1} & x_{k_v,2} & \cdots & x_{k_v,k_h} & y_{k_v,1} & \cdots & & y_{k_v,r_h} \\
\hdashline
z_{1,1} & z_{1,2} & \cdots & z_{1,k_h} & s_{1,1} & \cdots & & s_{1,r_h} \\
\vdots & \vdots & & \vdots & \vdots & & & \vdots \\
z_{r_v,1} & z_{r_v,2} & \cdots & z_{r_v,k_h} & s_{r_v,1} & \cdots & & s_{r_v,r_h}
\end{array}\right]$$

Figure 4.8 Matrix representation of a product code.

Here, x are information bits, y are check bits of the horizontal code, z are check bits of the vertical code, and s are the check bits of either z or y.

The properties of the product code are as follows:

length of the code:	$\mathbf{n} = \mathbf{n_h} \cdot \mathbf{n_v}$	
dimension of the code:	$\mathbf{k} = \mathbf{k_h} \cdot \mathbf{k_v}$	(number of information bits)
Hamming distance:	$\mathbf{d} = \mathbf{d_h} \cdot \mathbf{d_v}$	
Code rate:	$\mathbf{R} = \mathbf{R_h} \cdot \mathbf{R_v} = \mathbf{k} / \mathbf{n}$	

The strength of this type of code lies in the decoding strategy. The code words are transmitted row per row, and at the receiving end stored in the matrix again. First, the receiver decodes the columns of the matrix, correcting errors that occurred during transmission (the number of errors that can be corrected depends on the Hamming distance $\mathbf{d_v}$ of the vertical code). After vertical decoding a horizontal vector remains, which is decoded to correct for errors that slipped through the vertical decoder (again, the number of errors that can be corrected depends on the Hamming distance $\mathbf{d_h}$). In this way, the product code can correct a burst error of a length which is equal to the length $\mathbf{n_h}$ of the horizontal code.

On basis of this product code, appropriate modulation and coding schemes are designed for Eurofix data transmission. One of the single codes (either C_h or C_v) has to be a balanced one, as the Loran-C performance requires balanced modulation. In the next two Sections two coding schemes are derived, one based on vertical balancing, the column balanced scheme, and the other based on horizontal balancing, the word balanced scheme.

4.4 Column balanced modulation

The first strategy used to design a modulation scheme is the column balanced strategy developed by Megapulse, Inc. This Section describes the modulation scheme and derives its properties, e.g. number of data bits per GRI, error correcting capabilities, and the different modulation patterns for different types of messages.

Figure 4.9 describes the modulation scheme which will be used to transmit the normal DGPS corrections. Here a '0' stands for 'no time shift' (e.g. the first two pulses in each GRI), a '+' sign stands for a time delay, and a '-' sign for a time advance. These shifts are called pulse bits from now on.

	GRI A	GRI B
GRI 1 & 2	0 0 + + + + + +	0 0 + + + + + +
GRI 3 & 4	0 0 - - - - - -	0 0 - - - - - -
GRI 5 & 6	0 0 + + + + + +	0 0 + + + + + +
GRI 7 & 8	0 0 - - - - - -	0 0 - - - - - -

Code bit
vertically extracted

Figure 4.9 Column balanced modulation pattern for normal DGPS corrections.

Message type classification

As can be seen, a code bit is represented by four pulse bits (two advances and two delays in order to be balanced). The two advanced and two delayed pulses can be ordered in six possible ways which are used to represent logical ones and zeros in three different classes, as is shown in Table 4.1.

Message Type ------------- GRI Number	Normal DGPS Corrections		Synchronization		Integrity Message	
1 & 2	+	−	+	−	+	−
3 & 4	−	+	+	−	−	+
5 & 6	+	−	−	+	−	+
7 & 8	−	+	−	+	+	−
Code bit value	1	0	1	0	1	0

Table 4.1 Vertical modulation patterns used to transmit code bits in three different classes.

The classification of the different message types can be done very accurately. Each message class uses its own modulation type for the duration of the entire message. In other words, the class-identification is repeated 6 times per 4 GRIs. Therefore, a misclassification of the message due to demodulation errors is unlikely.

Error correcting capabilities

By representing each code bit as four pulse bits in different GRIs, the scheme provides protection against cross-rate interference. The code C_v of the product code can be seen as a four times repetition code with Hamming distance $\mathbf{d}_v = 4$. This means that the vertical code can correct one error and detect two. Nevertheless, if the sub-periodic interval of the cross-rate interference is 2, 3 or 4 GRIs, the receiver may not be able to decode a code bit correctly anymore. This is illustrated with an example in Figure 4.10. If for instance the first and fifth GRI are hit by cross-rate (sub-periodic interval equal to four), the user could receive an unexpected shift combination.

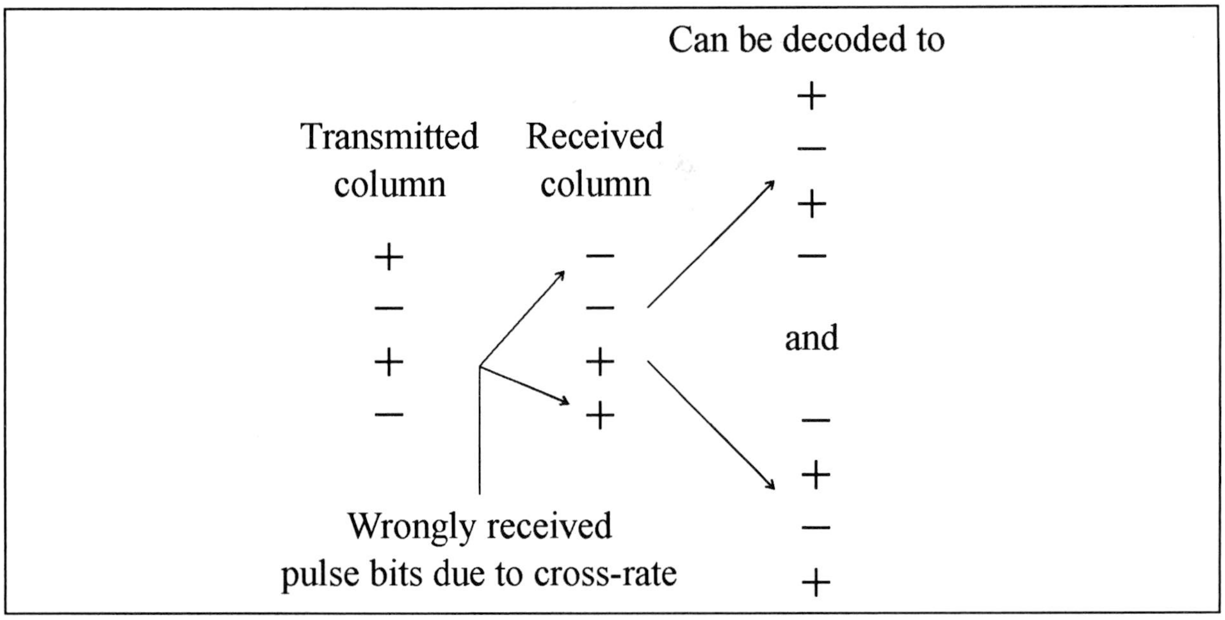

Figure 4.10 Effect of sub-periodic interval of cross-rate.

If the receiver tries to decode the received pulse bits, it cannot correct the two errors. As a result, the decoder outputs an erasure on this particular position in the message. This erasure has to be corrected by the horizontal code C_h which is applied over the entire DGPS message (about 45 data bits). However, if too many erasures occur, the horizontal code cannot correct them and as a result the DGPS message is lost.

Figure 4.9 showed that the modulation is balanced over eight GRIs. It can also be seen that the scheme is vulnerable for cross-rate with sub-periods of 2, 3 and 4 GRIs which means that reliable data transmission is endangered. To assure reliable data transmission for these cross-rate situations the code should be extended by interleaving over a larger number of GRIs. But then, the code is balanced over a larger block of GRIs possibly introducing a larger Loran-C positioning error.

Finally, with this modulation scheme 12 code bits are transmitted in 8 GRIs which means an effective data rate of 1.5 bits per GRI (37.5 - 15 bits per second). The code rate of the four times repetition code is equal to $R_v = 0.25$. The additional horizontal code C_h necessary to correct for occasional erasures will only reduce both data rate and code rate.

Loran-C performance degradation

Although the Loran-C performance degradation due to the Eurofix modulation depends highly on the tracking bandwidth of the Loran-C receivers, some general properties of this modulation scheme can be derived. The maximum imbalance occurs when the receiver window partly overlaps two balanced blocks, one at the beginning of the window and the other at the end as demonstrated in Figure 4.11 for normal DGPS messages.

The small rectangles each depict a modulated GRI, the shaded ones are balanced and the grey ones are not balanced because their corresponding balancing GRIs fall out of the receiver window. It can be seen that the maximum imbalance equals 4 GRIs. The influence of this imbalance depends on the receiver window size and becomes smaller if the window size increases. In Section 4.6 the Loran-C performance degradation is simulated using a hard limiter type receiver. There, the influence of the Eurofix data using a column balanced modulation scheme is shown as a function of signal-to-noise ratio and receiver window size.

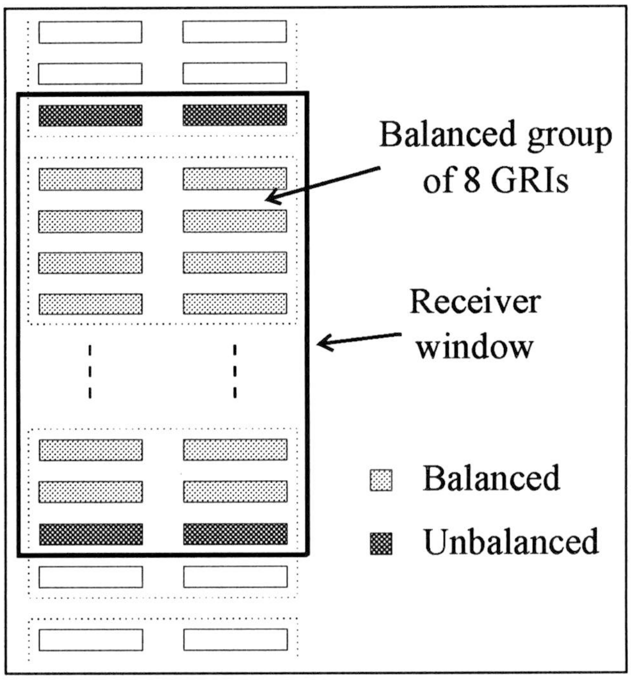

Figure 4.11 Maximum imbalance of the column balanced scheme for normal DGPS messages.

4.5 Word balanced modulation

The next strategy that can be used to design a modulation scheme is word balancing. In contrast with the previous modulation scheme, the modulation is balanced over a number of *consecutive* shifts. The number of GRIs over which the modulation is balanced, is smaller than with the column balanced scheme. Consequently, the Loran-C performance degrades slightly less. A disadvantage of the scheme is that the modulation cannot be systematic anymore. As a consequence, the reception of an erroneous word means the loss of all data bits in that word. The next Sections describe two different schemes with increasing complexity.

4.5.1 Balanced over 8 shifts

By balancing the modulation over a certain amount of shifts, two considerations have to be taken into account. First, balancing over a large number of shifts makes the code more effective, in other words, it takes relatively less parity bits **r** to balance **k** data bits, the code rate of the balanced code increases. On the other hand, by making the block larger the chance of cross-rate damaging the block increases. Because the code cannot be systematic anymore, this means the destruction of all bits in the block. Second, an appropriate block length has to be selected so that the total number of possible messages is slightly larger than some power of two. In this way only a few unused words are discarded and the code is maximally used.

The balanced code used for Eurofix is a special case of the equal weight codes A(n, d, w), where n is the length of the code (the total number of bits), w is the number of ones in each code word (weight of the code word), and d is the Hamming distance (always even). The properties of this code are described in Appendix B. In this particular case w = ½ n, and **d** = 2. The total number of possible words A(n, 2, ½ n) can be calculated with Formula 4.3.

$$A\left(n, 2, \tfrac{1}{2}n\right) = \binom{n}{\tfrac{1}{2}n} \qquad \text{Formula 4.3}$$

Note that n is always an even number. Using this Formula the maximum number of code words for any n can be derived. Table 4.2 gives the properties of this code for n from 2 to 12.

n	A(n, 2, ½ n)	$^2\log(A)$	Code Rate	bit rate (bits per GRI)
2	$2 = 2^1 + 0$	1	0.5	3
4	$6 = 2^2 + 2$	2.58	0.65	3
6	$20 = 2^4 + 4$	4.32	0.72	4
8	$70 = 2^6 + 6$	6.12	0.77	4.5
10	$252 = 2^7 + 124$	7.97	0.80	4.2
12	$924 = 2^9 + 412$	9.85	0.82	4.5

Table 4.2 Maximum number of code words of A(n, 2, ½ n).

Note that the numbers behind the decimal point in the third column are not real bits, they are formed by the superfluous words only to be used for special messages such as synchronization, integrity messages, etc. As can be seen, best performance is given by choosing n = 8. Then, the total number of code words is 70 of which 64 are used to transmit 6 code bits. The remaining 6 words are used for special messages. Balancing over 8 shifts means that the transmitted words cross the GRI boundaries. Therefore, three 8 shift words are handled as a block to fit into a four-GRI frame. To compensate for wrongly detected shifts due to cross-rate the block of three words is repeated three times. The modulation scheme now described is given in Figure 4.12.

```
     1 GRI
/ - - - - - - - - - - - - /

|                        |                        |                        |
| 0 0 + + + + - -   0 0 - - | + + + +   0 0 - - - - | + +   0 0 + + - - - - |
| 0 0 + + + + - -   0 0 - - | + + + +   0 0 - - - - | + +   0 0 + + - - - - |
| 0 0 + + + + - -   0 0 - - | + + + +   0 0 - - - - | + +   0 0 + + - - - - |
|                        |                        |                        |

/ - - - - - - - - - - - - - - - - - - - /
  1 word of 6 code bits
```

Figure 4.12 Modulation scheme using 8 balanced shifts to transmit 6 bits.

Message type classification

Classification of the message type (normal DGPS corrections or error message) is possible by transmitting one of the remaining 6 words (remember, only 64 of the 70 words were used to transmit data). This classification, however, is less robust than the one used in the column balanced concept. Here, the classification word only differs in 2 positions from words used for data transmission. Due to a simple error pattern, the classification word can change into a data word. Especially, if the classification frame of an integrity message is corrupted the receiver cannot distinguish between normal DGPS corrections and this integrity message. Therefore, to meet the Probability of Missed

Detection integrity requirement [5] the classification frame and the integrity message should be repeated. Classification for the different messages could be done by repeating the classification frame 6 or 9 times instead of the normal 3 times repetition (vertical code). The larger number of repetitions increases the reliability of the message classification but at the same time slows down the datalink.

Error correcting capabilities

The product code of this modulation scheme is built up by a balanced code horizontally and a three times repetition code vertically as is shown in Figure 4.13.

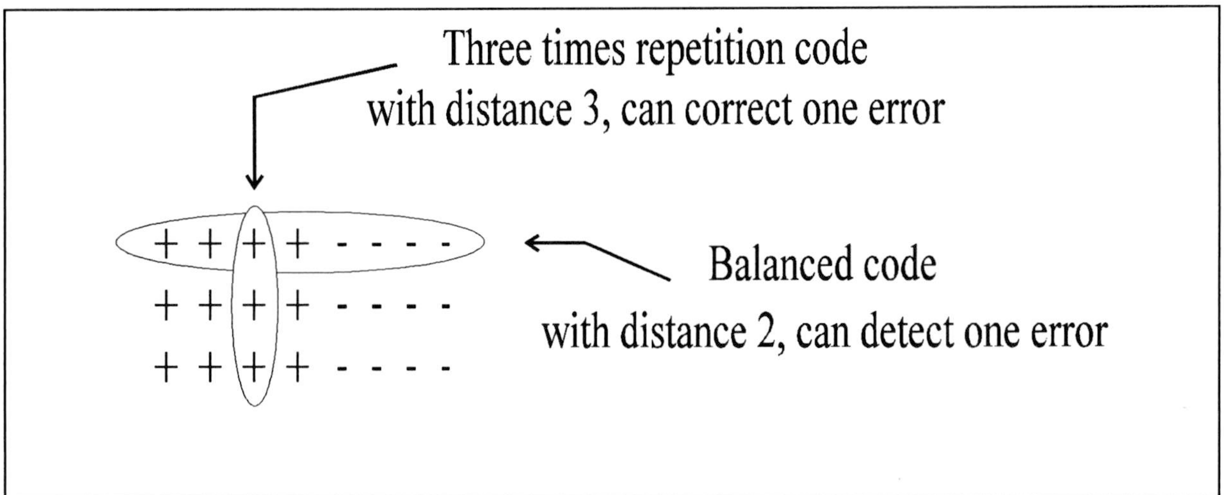

Figure 4.13 Error correction of the word balanced modulation scheme.

The equal weight code horizontally has a Hamming distance of $\mathbf{d}_h = 2$ and the repetition code vertically has a distance $\mathbf{d}_v = 3$. As mentioned earlier, a product code has at least a Hamming distance $\mathbf{d} = \mathbf{d}_h \cdot \mathbf{d}_v$ (the product of the Hamming distances of each single code). The strength of the code, however, lies in the decoding technique. First, the receiver decodes the received frame vertically, one wrongly detected pulse bit can be corrected, and then checks the decoded message horizontally. So, if a GRI is hit by cross-rate and the corresponding word is corrupted, the vertical majority vote can correct the error. However, if the column contains two errors, the decoder takes the wrong decision and decodes the column to the wrong value. The horizontal check can only detect this error, the frame cannot be decoded and 6 code bits are lost. Extra error correction should compensate for the loss, but the extra parity bits slow down the effective data bit rate. If no further error correction is applied a message containing one or more undecodable frames is discarded which degrades the final DGPS accuracy.

The encoding and decoding of the data bits can be done using a look-up table or by applying some efficient coding algorithm [11]. Here, the use of a look-up table is more effective as the number of table entries remains small. Appendix C lists the look-up table for this code where the code words are chosen as systematically as possible.

As can be seen from Figure 4.12, the modulation scheme is vulnerable for sub-periodic intervals of 4 and 8 GRIs. The scheme can be extended using smaller or larger interleave periods for the data words. But then, the corresponding three shifts of each word will not necessarily be on the same pulse position in the GRI. Also, the three corresponding words are differently spread over the GRIs making it vulnerable for more sub-periodic intervals of the cross-rate pattern. In Figure 4.14 the consequences of code extension from three to four 8 shift words are shown. The four 8 shift words do not fit into an integer number of GRIs anymore. The last two pulse bits of the fourth word have to be transmitted at the beginning of the next row and as a consequence the entire row is shifted two places. Apparently, the pulse bits of three corresponding words (for vertical decoding) are located on different pulse positions. Insight in the properties of the modulation scheme with respect to sub-periodic cross-rate patterns has become less clearly.

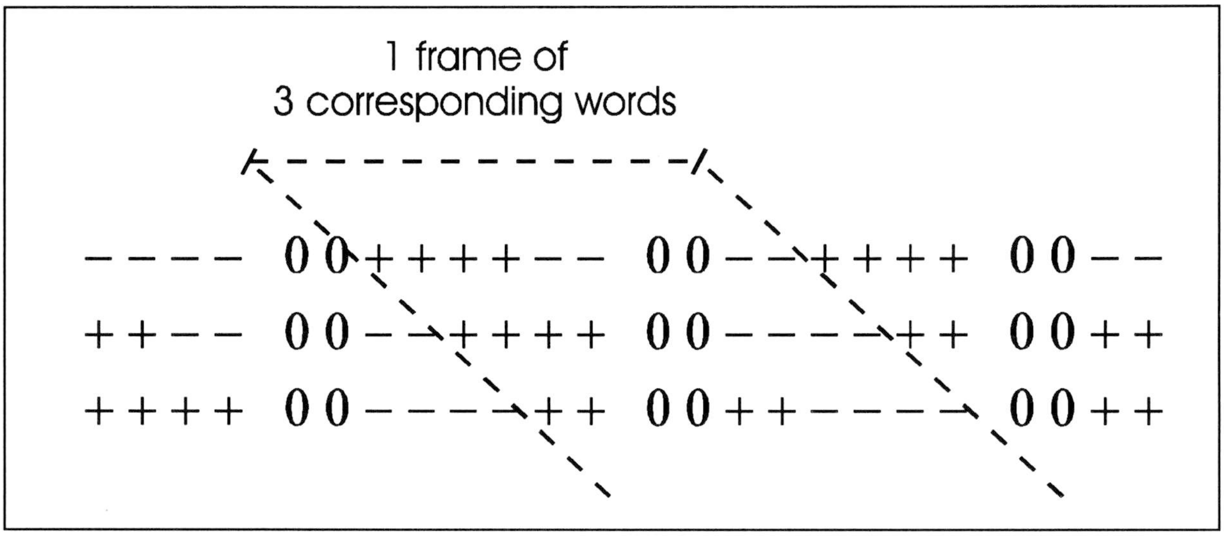

Figure 4.14. Consequences of an interleave period of four instead of three words.

In conclusion, the scheme uses 12 GRIs to transmit 18 bits which yields a data rate of 1.5 bits per GRI. If a frame is declared undecodable the whole message is rejected affecting the final DGPS accuracy. Lastly, the scheme can be extended to counteract the influences of certain sub-periodic cross-rate intervals at the cost of more complexity and a weaker insight on the behavior of the scheme.

Loran-C performance degradation

Again, the Loran-C performance degradation depends on the receiver window size. Maximum imbalance occurs if the receiver window partly overlaps two balanced words. It can be shown that the maximum imbalance is only 8 shifts long. Section 4.6 compares the performance of this scheme with the performance of the previous one in respect of Loran-C degradation. Probably, this scheme performs slightly better due to the smaller balancing block length.

4.5.2 Balanced over more shifts adding extra error correction

The previous modulation scheme has two major drawbacks:

- First, if in one column two errors occur due to cross-rate and noise the frame cannot be decoded and as a consequence the whole message is lost. At high signal-to-noise ratios this will not happen very often, but as the SNR decreases the number of discarded messages will increase and reaches an intolerable level.
- Second, the previous scheme is vulnerable for sub-periodic cross-rate intervals of 4 and 8 GRIs. Code extension is possible but yields a complex scheme. The scheme described in this Section has better error correcting properties at the cost of a lower effective data rate.

To provide extra error correction the Hamming distance of the total scheme has to be increased. The previous scheme had a distance of $\mathbf{d} = \mathbf{d_h} \cdot \mathbf{d_v} = 6$ and was used to first decode vertically and then *check* horizontally (note that the horizontal code was only error detecting and not correcting). The increase in error correcting capabilities of the scheme will be sought in the horizontal code. Appendix B states that an equal weight code (and thus the balanced code as well) always has an even Hamming distance. Thus, a balanced code has to be found with Hamming distance $\mathbf{d_h} = 4$.
Table 4.3 gives the maximum number of code words of the equal weight code with distance 4, A(n, 4, ½ n) [12].

n	A (n, 4, ½ n)	$^2\log(A)$	Code Rate	bit rate (bits per GRI)
2	--	--	--	--
4	$2 = 2^1 + 0$	1	0.25	1.5
6	$4 = 2^2 + 0$	2	0.33	2
8	$14 = 2^3 + 6$	3.81	0.48	2.25
10	$36 = 2^5 + 4$	5.16	0.52	3
12	$132 = 2^7 + 4$	7.04	0.59	3.5

Table 4.3 Maximum number of code words of A(n, 4, ½ n) [12].

Again, the numbers behind the decimal point in the third column are not real bits, but are formed by the superfluous words (used to transmit special messages). Note that the Code Rate and the data speed decreased enormously. As can be seen from Table 4.3, only two good choices for n remain, n = 10 and n = 12. For these choices the number of superfluous words is relatively small and at the same time the code rate is still acceptable. Unfortunately, the scheme with n = 10 only provides extra error correction but does not solve the problem of variable interleave periods. The scheme with n = 12 solves both problems and is discussed below.

Figure 4.15 shows the modulation scheme.

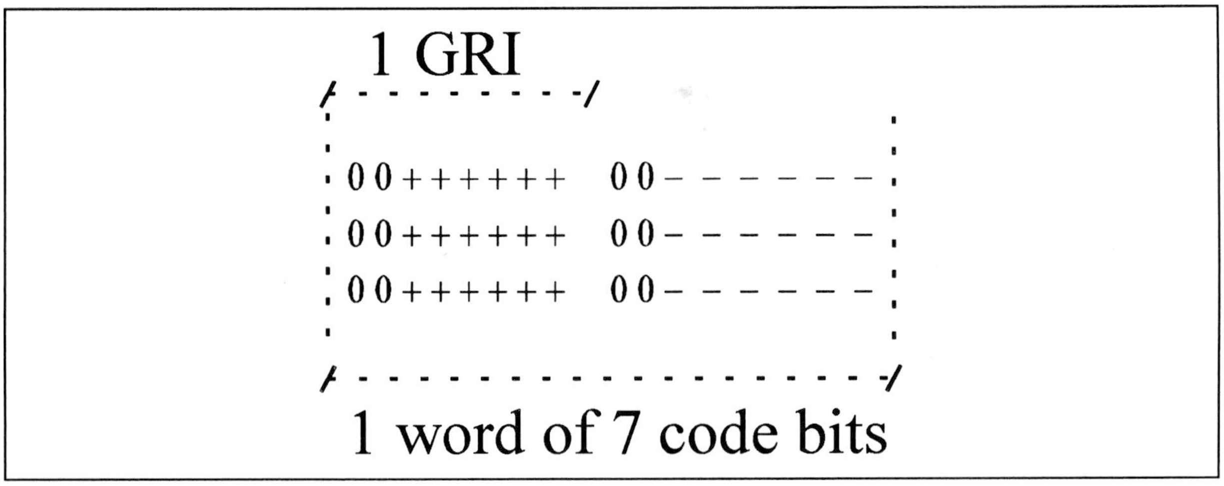

Figure 4.15 Modulation scheme based on the balanced code A(12, 4, 6).

Error correcting capabilities

Again, this scheme describes a product code, only now it is built up by a balanced code with distance $\mathbf{d_h} = 4$ horizontally, and the three times repetition code, $\mathbf{d_v} = 3$, vertically. Clearly, this code will be able to correct more errors than the previous one. First, the receiver decodes vertically and then horizontally. But now, the horizontal decoder is able to correct one error made by the vertical decoder due to cross-rate and noise, as is explained with an example.

<u>Example:</u>

Suppose the Eurofix transmitter sent the frame:
(For ease of survey, the + and – signs are replaced by 1 and 0).

$$\begin{pmatrix} 1&1&1&1&1&1&0&0&0&0&0&0 \\ 1&1&1&1&1&1&0&0&0&0&0&0 \\ 1&1&1&1&1&1&0&0&0&0&0&0 \end{pmatrix}$$

Due to cross-rate and noise the following frame was received:
Clearly, a burst error of length 6 occurred in the second row and random errors occurred in the first, second ,and third row.

$$\begin{pmatrix} 1&1&1&1&1&1&0&0&0&1&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&1 \\ 1&1&1&0&0&1&0&1&0&0&0&0 \end{pmatrix}$$

Vertical decoding yields the vector :

$$\begin{pmatrix} 1&1&1&1&0&1&0&0&0&0&0&0 \end{pmatrix}$$

Because of the extra error correction ($\mathbf{d_h} = 4$) the vertical decoding error on the fifth place can be corrected:

$$\begin{pmatrix} 1&1&1&1&1&1&0&0&0&0&0&0 \end{pmatrix}$$

Figure 4.15 shows that this scheme is vulnerable for sub-periodic cross-rate intervals of 2 and 4. But as the 12 shift word exactly fits into a two-GRI frame, any possible interleave period can be chosen to compensate for sub-periodic cross-rate intervals without losing balance. Figure 4.16 shows a possible extension of the scheme using an interleave period of three words.

```
1 GRI
/--------/
00++++++  00------  00+++---  00-+-+-+  00++--++  00-+---+
00++++++  00------  00+++---  00-+-+-+  00++--++  00-+---+
00++++++  00------  00+++---  00-+-+-+  00++--++  00-+---+
/------------------/------------------/------------------/
    first word         second word         third word
  of 7 code bits     of 7 code bits      of 7 code bits
```

Figure 4.16 Possible extension of the scheme using a larger interleave period.

The extension of the code has one drawback. As the block length of the interleaved extended code increases, the Time to Alarm requirement for integrity messages is endangered. Before an integrity message can take hold of the data channel, the started block of DGPS correction data has to be finished first. In worst case (from the integrity point of view), the words are interleaved over the total message length. In that case, it takes at most 4.2 seconds (the transmission time of a 49 bit message over a Loran-C channel with GRI = 100 ms) before the channel is available for integrity messages. Whether this delay is acceptable depends on the DGPS integrity requirements.

To conclude, this scheme transmits 7 code bits in 6 GRIs, which yields an effective data rate of 1.17 bits per GRI. Again, the remaining 4 words can be used to signal special messages.

Loran-C performance degradation

The Loran-C degradation will be merely the same as the previous scheme. Only now, the maximum imbalance of the scheme is 2 GRIs, 1.5 times larger than the 8 shift balanced scheme and 2 times smaller than the column balanced scheme. All three schemes are compared in the next Section.

4.6 Comparing the schemes with respect to Loran-C performance

In this Section the three schemes are compared using a simulated hard limited Loran-C receiver. The Loran-C performance degradation is calculated as a function of signal-to-noise ratio, receiver tracking bandwidth and receiver tracking stepsize. For simplicity reasons, the influences of cross-rate interference, continuous wave interference, and atmospheric noise are discarded. First, the simulated Loran-C receiver is discussed.

4.6.1 The simulated Loran-C receiver

The simulated Loran-C receiver is a stationary hard limited receiver tracking a zerocrossing of the Loran-C pulse. The receiver uses an up-down counter with fixed averaging time T_{av} in the tracking loop to adjust the timing of the sample point. After integration of the samples taken at the estimated zerocrossing for a period T_{av}, the sign of the up-down counter determines the timing adjustment, as is described in Section 3.5.1. Depending on the sign of the up-down counter, the sample point is advanced or delayed with a fixed tracking stepsize in order to improve the estimate of the zerocrossing. The value of the up-down counter and thus the corresponding probabilities P_{obs} and Q_{obs} depend on the averaging time T_{av}, the tracking error , the signal-to-noise ratio, and the applied modulation scheme.

The tracking of a Loran-C signal can be simulated by a Markov process of order 1. This is easiest explained with an example.

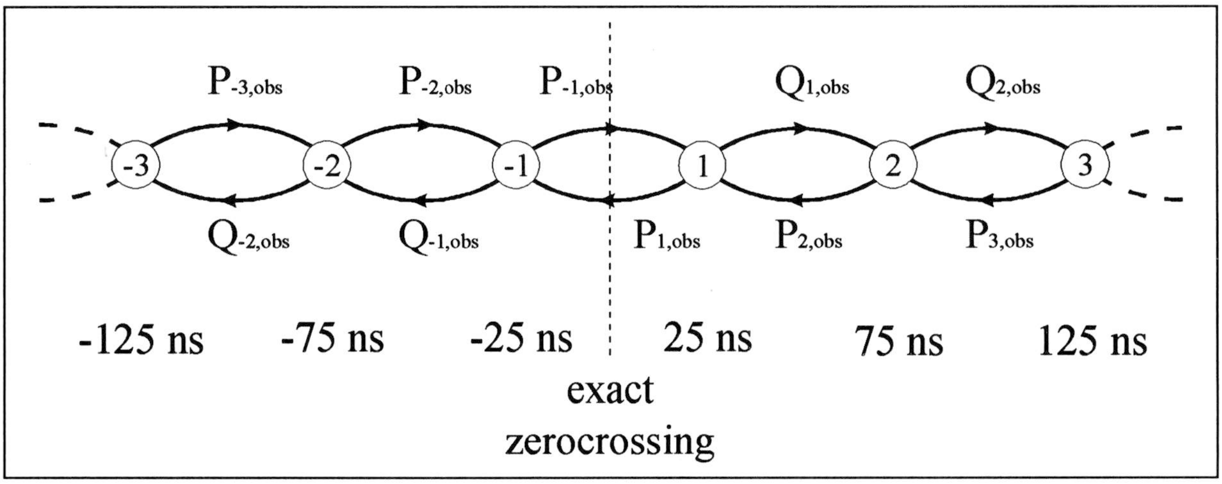

Figure 4.17 Loran-C tracking modeled as a Markov process.

Figure 4.17 describes the tracking of a Loran-C receiver with a fixed tracking stepsize of 50 ns. Without loss of generality, the states of the Markov process are chosen symmetrically around the actual zerocrossing. The transition probabilities $P_{i,obs}$ and $Q_{i,obs}$ relate to the probability density function of the value of the up-down counter which in turn depends on the value of the tracking error of state i. The transition probabilities can be calculated as a function of the tracking error.

The probability of each state in the Markov chain can now be calculated by Formula 4.4 ref. [13].

$$P(S_i) = \sum_j P(S_j) \cdot P(S_i / S_j)$$

with

$$\sum_j P(S_j) = 1 \qquad \text{Formula 4.4}$$

Formula 4.4 gives a set of linear equations which can be solved by linear algebra. The set of state probabilities describes the discrete distribution of the tracking error of the receiver.

By calculating the tracking error distribution as a function of the signal-to-noise ratio for the different modulation schemes, a tool has been made to compare the various schemes.

4.6.2 Comparison of the schemes

Figure 4.18 shows the rms value of the tracking error of a simulated hard limiter receiver as a function of the signal-to-noise ratio, for normal Loran-C transmission (no modulation), and for Loran-C transmission with extra data modulation using the three modulation schemes. The receiver has a fixed averaging time of 2 seconds, and a fixed tracking stepsize of 50 ns. The signal-to-noise ratio is defined with respect to the rms value of a sine wave with the same amplitude as the tracked cycle.

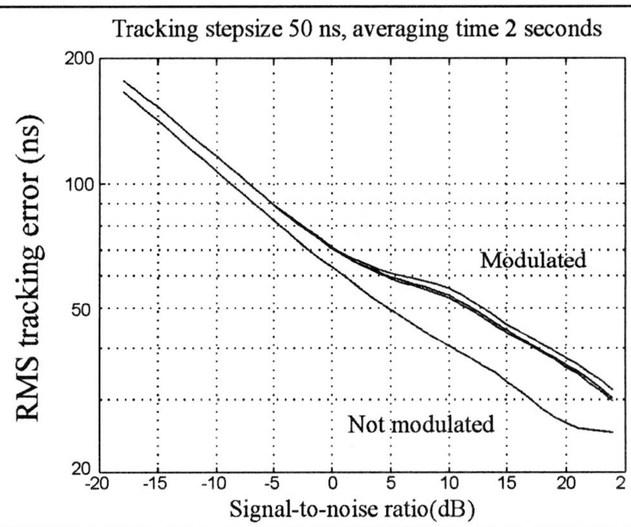

Figure 4.18 RMS value of the tracking error of a simulated Loran-C receiver as a function of the signal-to-noise ratio.

As can be seen, the three lines corresponding to the modulated Loran-C transmission show a larger tracking error than the line of the normal Loran-C transmission. The picture shows two points of interest. First, at approximately +5 dB SNR the slope of the modulated transmission changes. This points to the beginning of the dead zone of the hard limited receivers (see Section 3.5.3).
Second, at approximately +20 dB SNR the slope of the normal transmission changes. Apparently, the tracking cannot be improved any further than 25 ns. This is easily explained, as the tracker is just changing from state '-25 ns' to state '+25 ns' in the Markov chain at high SNR.

Figures 4.19 a-d show the Loran-C performance degradation of the three modulation schemes relative to the performance of the receiver when no modulation is applied for different receiver parameters.

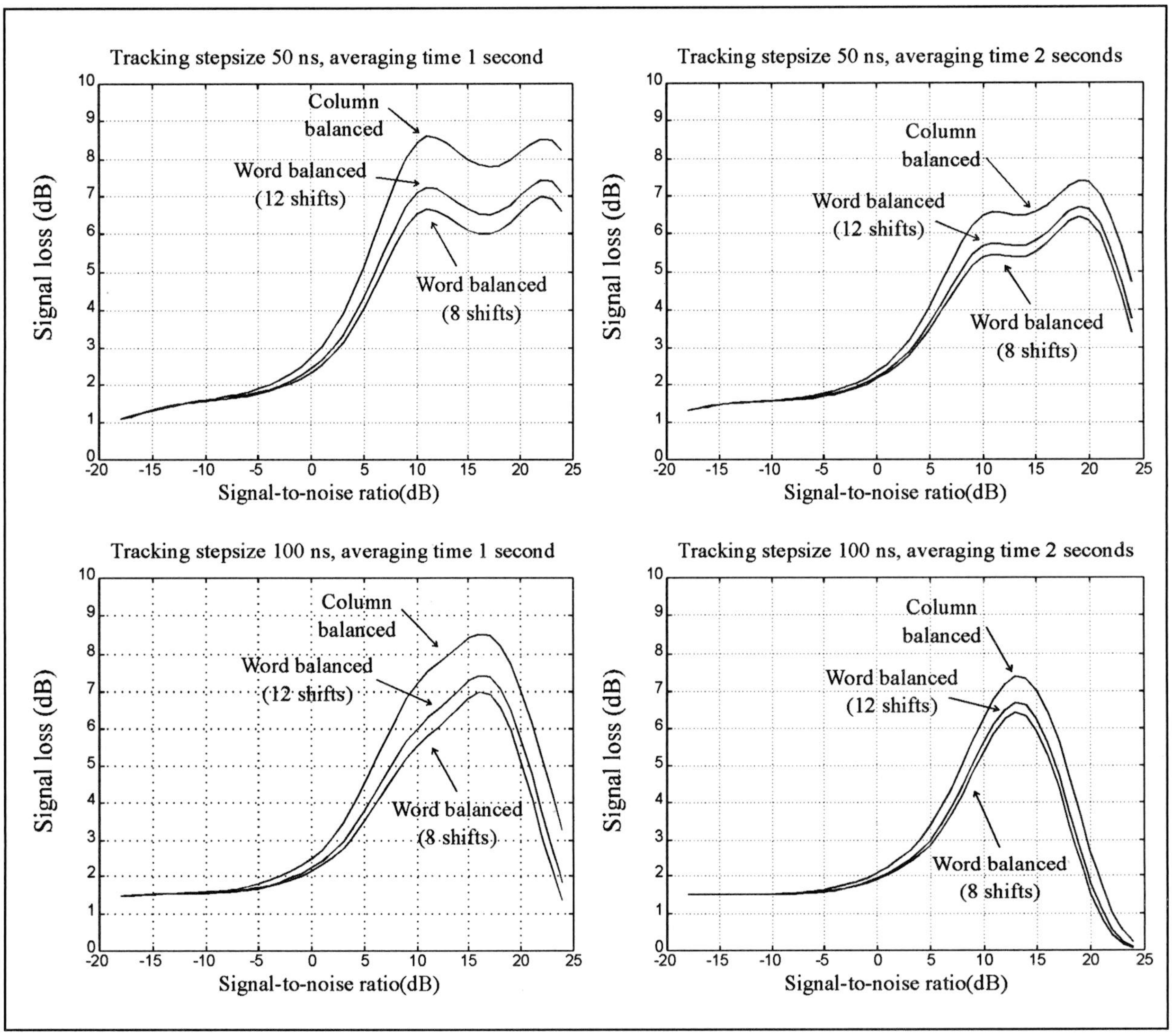

Figure 4.19 a-d Loran-C degradation as a function of the signal-to-noise ratio for three different modulation schemes and four different receivers.

Although the curves of the four receiver types are rather different, some properties of the Loran-C degradation due to data modulation can be derived:

- at low SNR all modulation schemes only yield a small signal power loss (approximately 1.5 dB), the modulated pulses still contribute to the tracking performance.
- as SNR increases, the modulated pulses lose their influence on the tracking performance (dead zone). The tracking signal is reduced with approximately 6 to 7 dB.
- at high SNR the performance of the receiver is limited by the tracking stepsize. At some point, the performance of the receiver with modulated Loran-C signals can still be improved, whereas the fixed tracking stepsize leaves no room for improvement for the unmodulated Loran-C transmission.

As was to be expected, the column balanced modulation introduces a slightly higher performance degradation than the word balanced ones. The differences between the column and word balanced schemes decrease as the averaging time increases.

4.7 Conclusions

Cross-rate interference can have large influence on data transmission. Especially, cross-rate patterns with small sub-periodic intervals endanger reliable data transmission. The influences of cross-rate have to be compensated for by error correcting codes.

The column balanced modulation scheme provides vertical error correction to compensate for cross-rate. The additional horizontal code can be designed over the entire DGPS message (large block) which in general gives better performance than codes over shorter blocks. Unfortunately, the scheme is hard to extend to compensate for sub-periodic intervals of 2, 3, and 4.

The 8 shift word balanced scheme on its own will probably not be robust enough for the hostile Loran-C data channel. Additional error correction has to be applied to improve the performance. However, due to the unsystematic 8 shift balanced code that will be a complex matter.

The 12 shift word balanced scheme probably performs best at the cost of a lower effective data rate. The scheme's ability to easily extend the interleave period makes protection for any sub-periodic cross-rate interval possible.

The Loran-C performance degradation is comparable for all three schemes. The word balanced schemes perform slightly better due to a shorter balancing block than the column balanced scheme. At low SNR the loss in tracking signal is small (1.5 dB), at higher SNR the loss increases (to approximately 7-8 dB) due to the dead zone of hard limited Loran-C receivers.

All three schemes have a relatively low effective data rate, mainly caused by the balancing requirement. Better codes could be designed and higher effective data rates could be reached if only the modulation would not necessarily have to be balanced. It is reasonable to think that the information source outputs ones and zeros in equal proportions. Perhaps the majority of the messages is already balanced anyway. The next Chapter investigates the possibilities of data transmission without this annoying balancing requirement.

5 Unbalanced data modulation

In the previous Chapter three coding schemes were designed based on balanced modulation of the Loran-C signal. However, the effective data bit rates of the three schemes are relatively low which leaves practically no room for extra error correction if the message error probability seems to be too high. The three schemes operate at the system's boundaries which means that decreasing the effective data rate any further could have large influences on the final DGPS accuracy. If the balancing requirement could be abandoned, more efficient codes can be designed with higher effective data rates.

This Chapter investigates the possibilities of unbalanced data transmission. First, the consequences for the normal Loran-C performance are discussed, and then, a code is designed using the maximum transmission bandwidth.

5.1 Loran-C performance degradation due to unbalanced data transmission

Inevitably, unbalanced data transmission introduces a larger tracking error than balanced data transmission. In this Section the performance degradation of unbalanced modulated Loran-C transmission is derived and compared to balanced modulation and no modulation. Figures 5.1 a&b show the theoretically expected performance degradation of unmodulated data transmission calculated with the simulated Loran-C receiver with averaging time 5 seconds and tracking stepsize 1 ns.

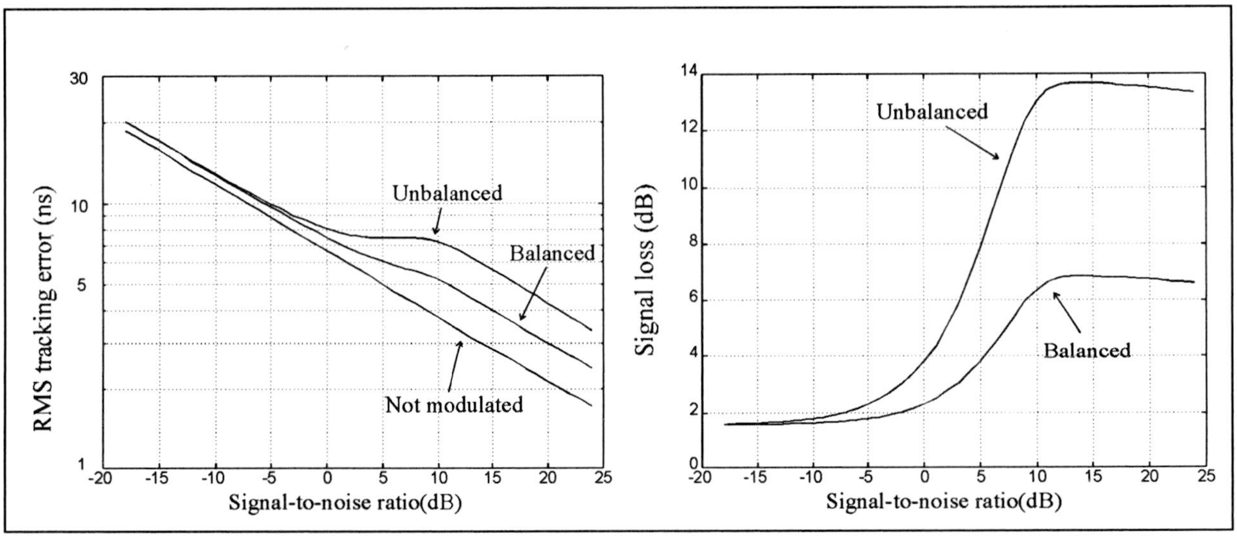

Figure 5.1a: RMS tracking error of Loran-C receiver with unmodulated, balanced modulated and unbalanced modulated transmission. b: Signal loss of balanced and unbalanced modulation relative to not modulated transmission.

No importance has to be attached on the actual values of the rms tracking errors in Figure 5.1a, merely the path of the three curves and their mutual relation is important. Clearly, the unbalanced modulation yields a larger decrease of Loran-C performance than the balanced modulation. Especially, the horizontal part of the unbalanced modulation curve in Figure 5.1a is of great interest. Apparently, for signal-to-noise ratios between +3 and +9 dB the tracking is not improved with increasing SNR. For this SNR interval the data modulation is dominant over the two unmodulated pulses in the tracking loop. At larger SNR values, the two unmodulated pulses take over and tracking performance is increased.

Fortunately, the difference in performance between unbalanced and balanced modulation becomes smaller for the simulated Loran-C receiver. In Figures 5.2 a-d the performance of the unbalanced modulation is compared with the column balanced modulation. Note that the column balanced modulation resulted in the highest Loran-C performance degradation of the three balanced modulation schemes.

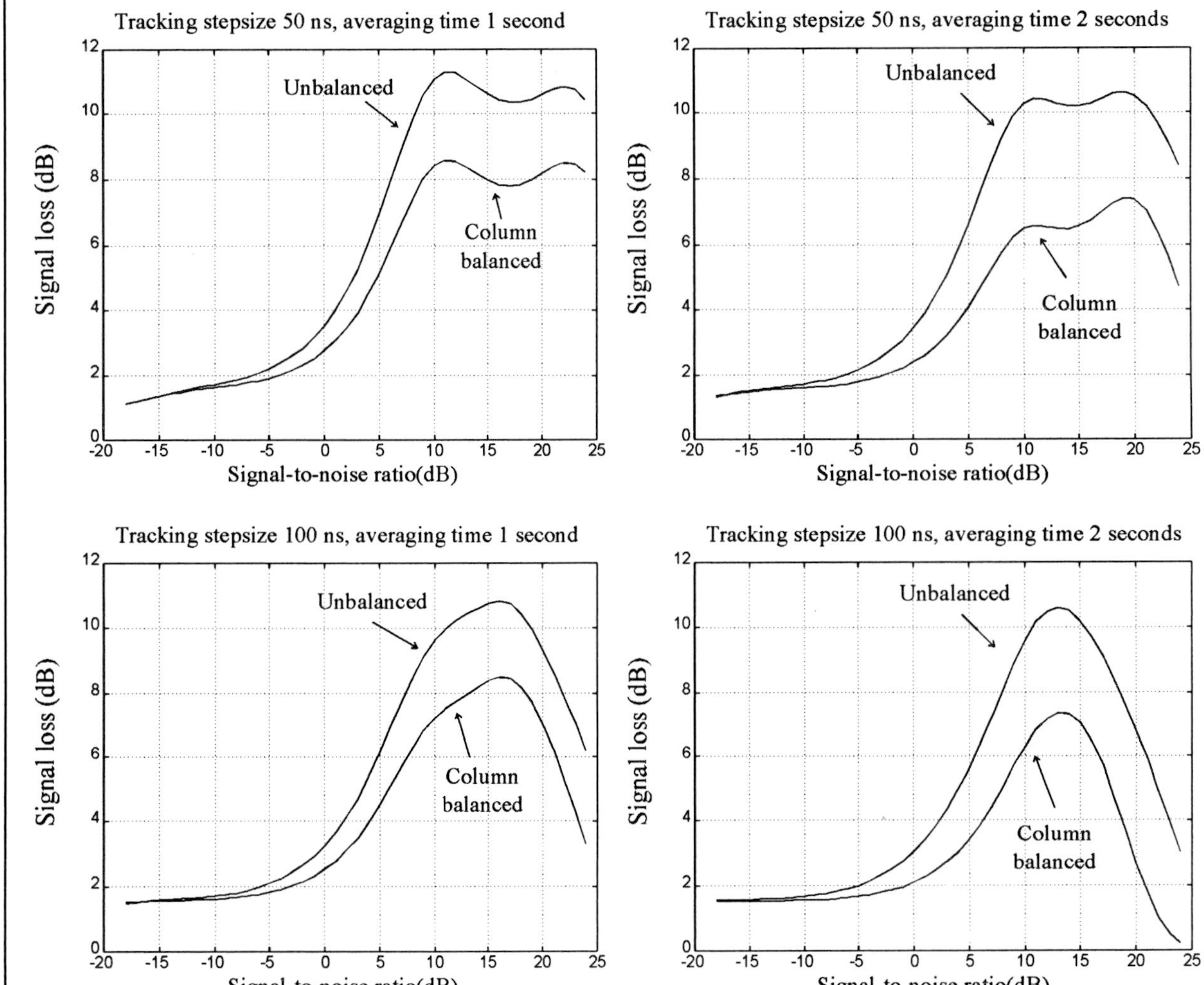

Figure 5.2 a-d Loran-C degradation as a function of the signal-to-noise ratio for unbalanced and column balanced data transmission for four different receivers.

The performance degradation of unbalanced modulation shows approximately the same path shape. The only difference is that the unbalanced path has a higher peak value. However, Figure 5.3 shows that at SNR values between +3 and +9 dB the slope of the tracking error of unbalanced modulation becomes horizontal.

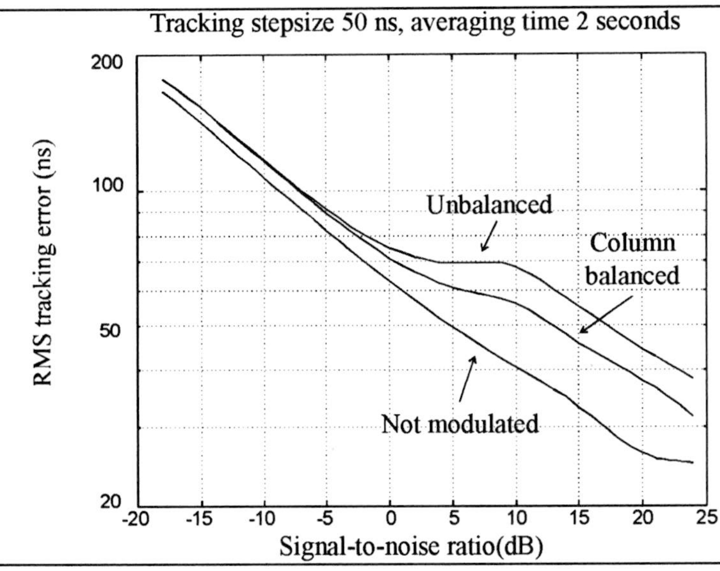

Figure 5.3 RMS value of the tracking error of a Loran-C receiver as a function of SNR

The knowledge that with increasing SNR the navigation performance remains the same is undesirable, to put it mildly. On the other hand, the performance will probably still be better than required to meet the Loran-C specifications. Whether the datalink properties can be improved at the cost of more Loran-C degradation is a political decision.

5.2 Coding scheme design with full data channel capacity

A code which has full data channel capacity at its disposal probably has better properties. Good error correcting capabilities as well as a sufficient effective data rate can easier be achieved without the balancing restriction. In this Section a code is designed which is a concatenation of two other codes. First, the properties of a concatenated code are discussed.

5.2.1 Concatenated code design

The principle of a concatenated code is explained in Figure 5.4, ref. [10].

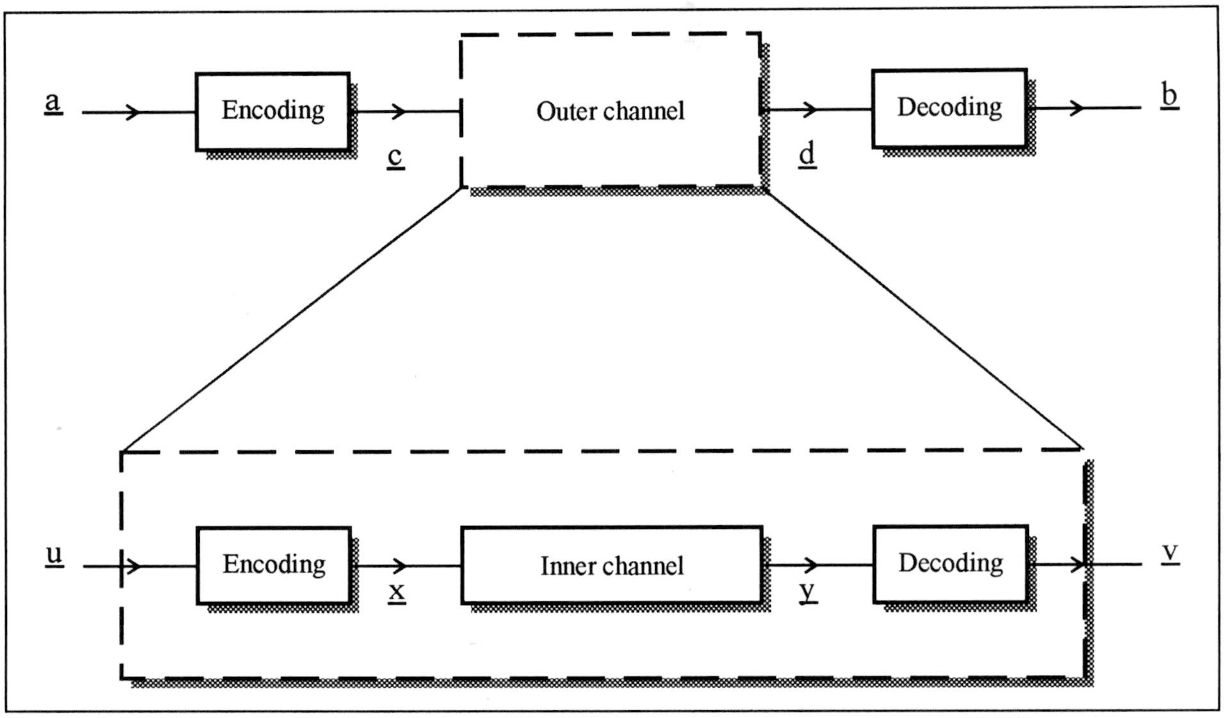

Figure 5.4 Concatenated code.

A concatenated code consists of an inner code and an outer code. The inner code is a binary code with length **n** and dimension **k** (number of data bits), the outer code is a non-binary code over GF($2^{\mathbf{k}}$) with length **N** and dimension **K** (number of data symbols, a Galois Field GF(2^k) is a collection of 2^k elements, Appendix B). The total encoding and decoding will be done in three steps:

- First, the incoming message is divided into **K** parts of length **k** which are elements of GF($2^{\mathbf{k}}$) and can be seen as symbols a_i .
- Second, the vector $\underline{a}$ containing **K** symbols a_i is the input vector for the outer code which adds **N** - **K** check symbols, forming the encoded vector $\underline{c}$.
- Third, each of the **N** symbols c_i is treated as a binary vector $\underline{u}$ of length **k** and fed to the inner code. The inner code adds **n** - **k** check bits to every group of **k** bits forming the vector $\underline{x}$.

Then, the **N** vectors $\underline{x}$ are transmitted over the inner channel to the receiver. At the receiving end the process is reversed.

A concatenated code is able to effectively correct for burst errors and random errors [10].

With all coding tools at hand and all knowledge about the Loran-C data channel available, a suitable unbalanced coding scheme can be designed. But first, properties of the Loran-C data channel have to be examined again.

The Loran-C channel is mainly disturbed by two sources, noise and cross-rate interference. Noise can cause single errors randomly placed with a probability depending on the signal-to-noise ratio. Cross-rate interference can cause burst errors with a maximum length of 6 bits (6 modulated pulses per GRI). The code has to be able to correct for both error sources.

The length of the inner code is set by the properties of the burst errors due to cross-rate interference. As the burst error falls always within one GRI, it is best corrected for if it coincides with only one symbol of the outer code. This means that the length of the inner code is set at 6. The dimension **k** of the inner code can now be chosen 3 or 5.
The (6,3) shortened Hamming code has distance 3 and is capable of correcting one error. The price that has to be paid for a single error correcting code is heavy. Already, 3 out of 6 bits are used for error correction.
The (6,5) parity code has distance 2 and thus is not able to correct errors. However, the code can detect any odd number of errors (e.g. 1, 3, and 5). If it has detected errors the entire symbol is declared an erasure and has to be corrected by the outer code.
The unbalanced code will be designed using the (6,5) parity code as inner code.

The outer code is chosen to be a Reed-Solomon code over GF(2^5). A t-error correcting Reed-Solomon code over GF(2^k) has the following properties:

maximum length	$\mathbf{N} = 2^k - 1$
maximum dimension	$\mathbf{K} = 2^k - 1 - 2t$
Hamming distance	$\mathbf{d} \geq 2t + 1$

The dimension **K** of the outer code is set by the total amount of bits in a message, the 45 bits for the asynchronous DGPS correction (Section 2.2) set **K** equal to 9 ($\mathbf{k} \cdot \mathbf{K} = 5 \cdot 9 = 45$). The number of check symbols depends on the degree of error correction that is needed for reliable data transmission. As can be seen above, two check symbols have to be added for every extra error to be corrected. The maximum performance with respect to error correction is achieved by choosing $\mathbf{N} = 31$ and $\mathbf{K} = 9$. In this way 22 check symbols protect 9 information symbols, so that a message containing 11 erroneous symbols can still be corrected. With the aid of the inner code generating erasures this performance can be increased even further. The effective data bit rate of this code is 1.45 bits per GRI, comparable with the balanced schemes.

The encoding of the concatenated code will be explained with the example of Figure 5.5. In this example a 2-error correcting Reed-Solomon code is used which means that 4 extra parity symbols protect the 9 data symbols.

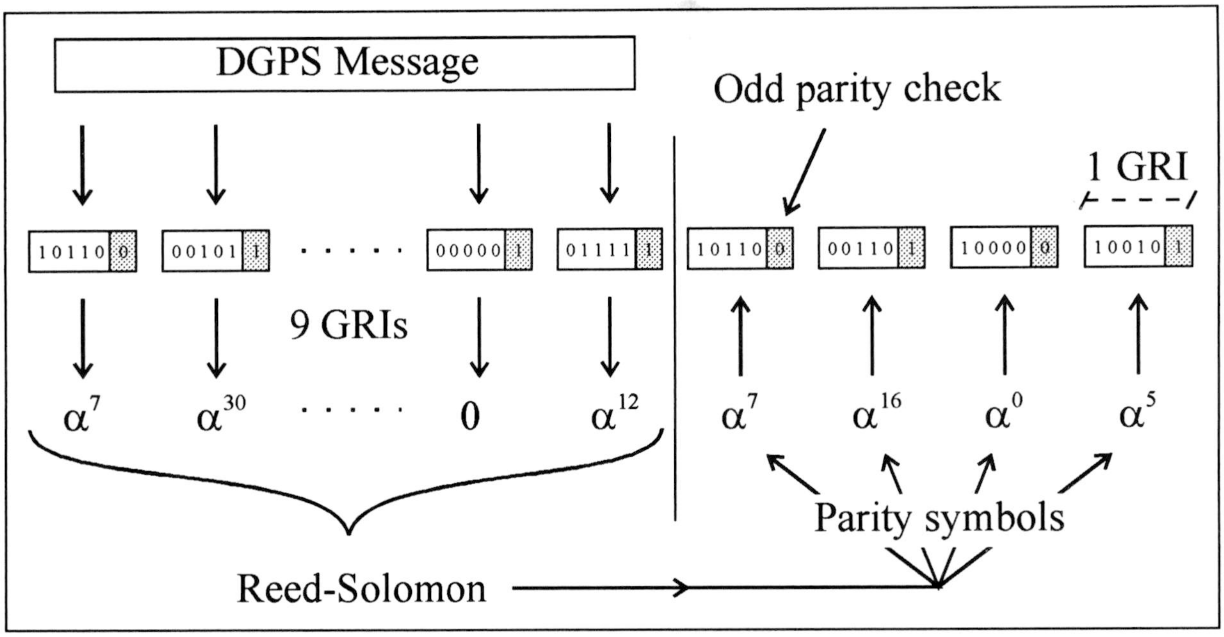

Figure 5.5 Example of the encoding of a DGPS message.

First, the generated DGPS message of 45 bits is cut into 9 parts of 5 bits each, forming 9 symbols of GF(32). Second, the 9 data symbols are fed into the Reed-Solomon encoder yielding the 4 parity symbols. Third, each of the 13 symbols is now encoded by the inner odd parity code which means that the parity bit is filled with 1 if the 5 bit symbol has even parity, and is filled with 0 if the symbol has odd parity. Now, 13 GRIs can be modulated with the encoded DGPS message.

The effective data rate is dramatically improved in comparison with the balanced schemes. Now, an entire DGPS message is transmitted in 13 GRIs which yields an effective data rate of nearly 3.5 bits per GRI. With this scheme 2 erroneous symbols or 4 symbol erasures can be corrected. Erasures occur when in the received GRI 1, 3 or 5 pulse shifts are inverted. If the error correcting capabilities are not sufficient enough, the scheme can easily be extended by simply adding more parity symbols (two for every extra error).

5.2.2 Nearly balanced coding scheme

The inner code of the concatenated scheme is a parity code. Here, the odd parity is chosen which yields a balanced GRI for more than half the number of code words. Table 5.1 shows the imbalance distribution of the 6 bit words. After encoding, the weight of the 6 bit symbols becomes odd (1, 3, or 5). The imbalances for the 6 bit word can be calculated as follows:
In each 6 bit word an equal number of ones and zeros cancel each others influence (advance vs. delay). The remaining ones (or zeros) are not compensated for and yield an imbalance. For instance, a word of weight 1 (1 delay and 5 advances) corresponds to an imbalance of - 4 (4 uncompensated advances).

	Before encoding					
Weight of 5 bit symbol	0	1	2	3	4	5
Frequency	1	5	10	10	5	1
	After encoding					
Weight of 6 bit word	1	1	3	3	5	5
Imbalance	- 4		0		+ 4	
Frequency	6		20		6	

Table 5.1 distribution of the weights and imbalances before and after a (6,5) odd parity code.

As can be seen in Table 5.1, the majority of the 6 bit words is balanced. Obviously, this yields great improvement in the Loran-C performance compared to unbalanced data transmission.

5.2.3 Loran-C performance degradation of the nearly balanced code

In Figure 5.6 the rms tracking error of the nearly balanced coding scheme is compared with the unbalanced and unmodulated transmission.

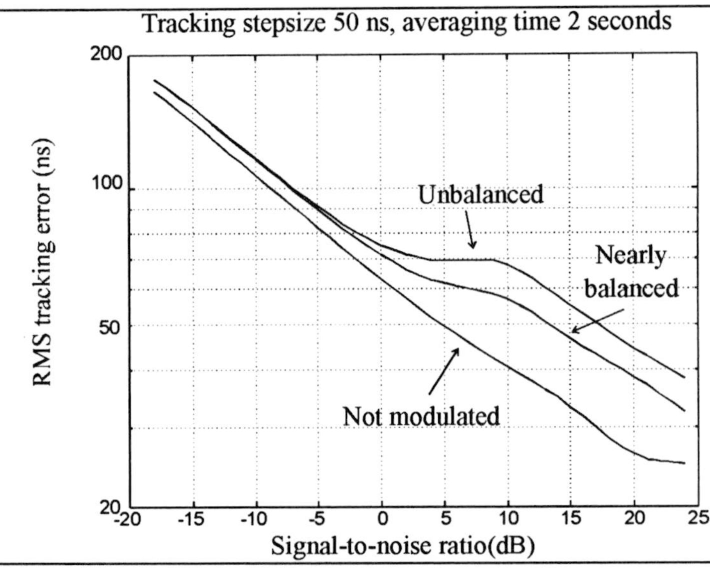

Figure 5.6 RMS tracking error of the nearly balanced coding scheme compared with unbalanced and unmodulated transmission.

Apparently, the performance is significantly improved in comparison with unbalanced data transmission. Also, the horizontal slope has disappeared. Reason enough to compare the nearly balanced coding scheme with the balanced schemes. Figures 5.7a-d compare the nearly balanced coding scheme with the unbalanced and column balanced schemes.

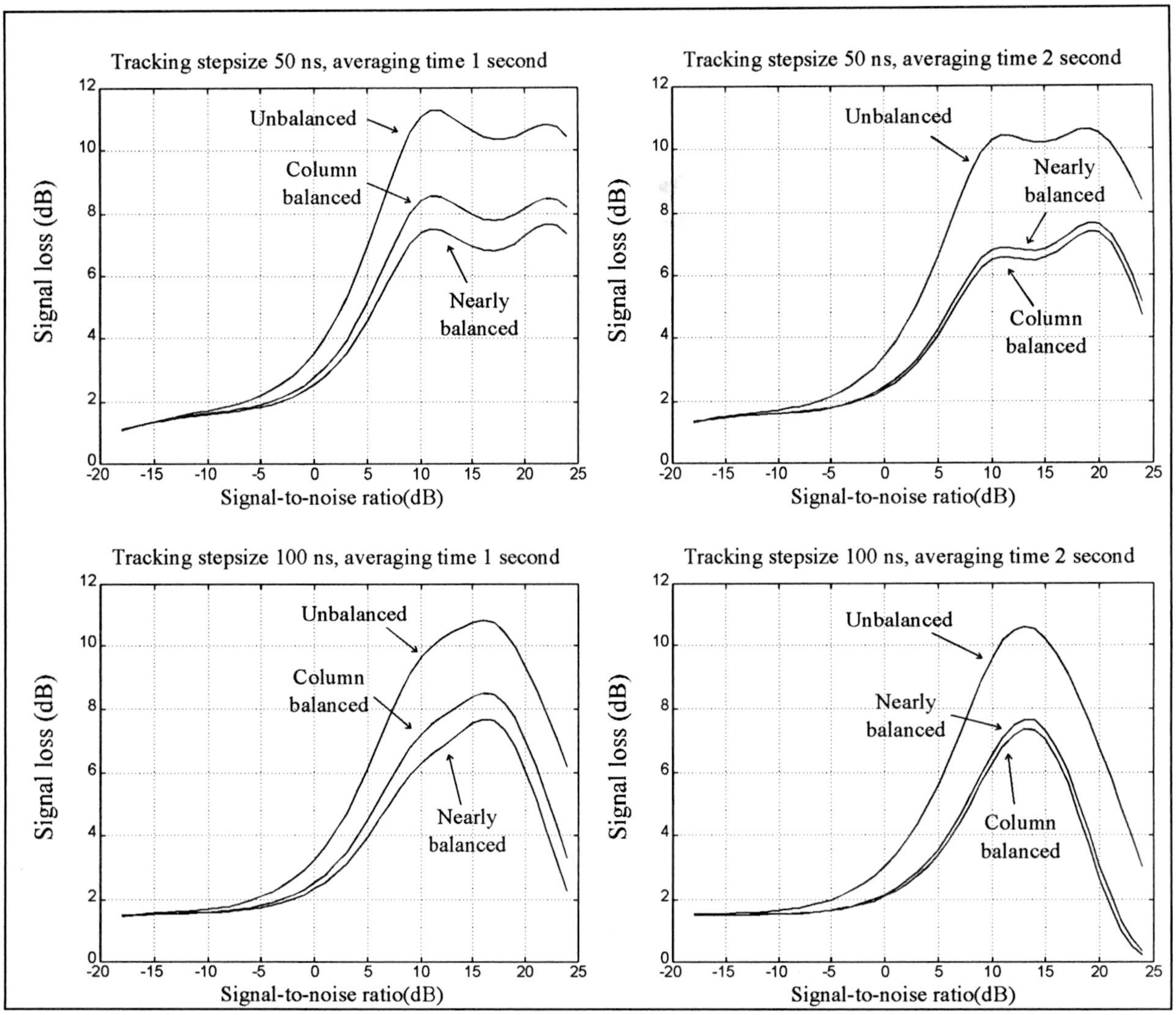

Figure 5.7a-d Loran-C degradation as a function of the signal-to-noise ratio for the nearly balanced coding scheme compared to the unbalanced and column balanced data transmission.

As can be seen, the nearly balanced coding scheme is competitive with the column balanced scheme. For small receiver integration times the scheme even performs better. The explanation for that has to be sought in the imbalance due to imperfect alignment of the tracking window and the balancing block for the column balanced scheme.

5.3 Conclusions

Although unbalanced data transmission probably yields an intolerable Loran-C performance degradation, it opened a whole new area of possible coding schemes. Simulations showed that as long as the majority of the transmitted words is balanced and the mean value of all possible code words equals zero (in terms of modulation) the Loran-C performance degradation is comparable with balanced modulation. It should be noted, however, that the simulations were only performed for signal-to-noise variations in a hard limited stationary receiver. The results of these simulations should be verified by real life experiments.

As far as Eurofix data transmission is concerned, the relaxed balancing requirement has great effect on the properties of the Eurofix datalink. Now, better codes can be designed which are more efficient in terms of error correcting capabilities and effective data rates. The coding scheme derived in this Chapter is only one example of probably a large variety of possible coding schemes.

6 Conclusions and recommendations

The design of a modulation scheme for the Eurofix datalink is not a trivial matter. The scheme has to meet both the Loran-C performance requirements as well as the DGPS update rate requirements. From this report the following conclusions can be drawn:

6.1 Conclusions

With respect to DGPS accuracy:

- Due to the different temporal decorrelation of the asynchronous DGPS satellite corrections the accuracy of the corrected Pseudo Range for each satellite is different. Therefore, the final user DGPS accuracy depends also on the selected set of satellites and how they are placed in the satellite update frame.

With respect to the Loran-C data channel:

- The sub-periodic properties of the cross-rate interference patterns have large influences on the data transmission. The interleave period of the modulation scheme should not coincide with the sub-periodic interval of the main cross-rating Loran-C chain in the neighborhood.

With respect to balanced modulated Eurofix data transmission:

- At low signal-to-noise ratios the loss of navigation signal due to modulation is approximately 1.5 dB, whereas at higher signal-to-noise ratios this loss is increased due to the dead zone in hard limited receivers. Then, the loss varies from 6 to 8 dB depending on the applied modulation scheme and receiver configuration.
- The Loran-C performance degradation also depends on the alignment of the receiver window with respect to the block the modulation is balanced over. Especially, receivers with small integration windows will notice a degradation due to local imbalances of the modulation scheme. This effect is decreased as the modulation is balanced over a smaller block length.
- The three balanced modulation schemes have low effective data rates of 1.5 bits per GRI or less, mainly caused by the balancing requirement.

With respect to unbalanced modulated Eurofix transmission:

- At low signal-to-noise ratios the Loran-C degradation of the unbalanced modulation is comparable with the balanced modulation (1.5 dB signal loss). However, at higher signal-to-noise ratios the unbalanced modulation yields a significant degradation of the Loran-C performance (approximately 14 dB signal loss). On the other hand the minimum standard performance will probably still be met.
- A concatenated coding scheme consisting of an odd parity code and Reed-Solomon code yields a great improvement in the effective data rate (1.5 to 3.5 bits per GRI, depending on the degree of error correction).
- The odd parity code yields a nearly balanced coding scheme which has Loran-C performance comparable with balanced codes.
- Although the unbalanced data transmission on itself probably yields an intolerable Loran-C performance degradation, it opened the door to a wide new area of possible nearly balanced coding schemes.

6.2 Recommendations and further research

The conclusions drawn in this Chapter are all based on theoretical analyses and computer simulations. Real life testing of the modulation schemes should verify whether the results are correct.

Also, the Loran-C performance degradation is only simulated for changing signal-to-noise ratios in a stationary receiver. Influences of cross-rate interference, continuous wave interference, atmospheric noise, and tracking errors due to movement of the receiver were discarded. Research at the behavior of the Loran-C performance with these error sources turned on could yield different results.

The influence of data modulation on fast acquisition linear receivers should also be investigated. One should keep in mind that the linear receiver can easily demodulate the Eurofix data, subtract it from the received signal, and use the upgraded pulse to improve tracking performance.

Finally, the door is opened to a whole new set of nearly balanced coding schemes. The scheme derived in Chapter 5 is only one example. Perhaps, coding schemes better adjusted to the typical Loran-C environment are lying there waiting to be picked up.

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Appendix A Sub-periodic cross-rate properties of European and American Loran-C chains

The sub-periodic intervals of the new European and existing American Loran-C chains are shown in Tables A.1 and A.2. The names and GRIs of the Eurofix stations are given in the first column, and in the first row the GRIs of the interfering chains are given. Information about the location of the stations in each chain can be found in [9] and [14].

		7001	9007	7499	6731
Bø	7001	X	18	1	23 25
Ejde	9007	14 35	X	10 15	**3**
Sylt	7499	1	12 18	X	8 9
Lessay	6731	24 26	**4**	9 10	X

Table A.1 Sub-periodic cross-rate intervals of the new European Loran-C chains.

		5990	9940	8290	9610	8970	7980	9960	5930
Canadian West Coast Chain	5990	X	15 20	7	24				
U.S. West Coast Chain	9940	9 12	X	**10 15**	30	18 19			
North Central U.S. Chain	8290	5	**12 18**	X	15 22	1 27		**12 18**	
South Central U.S. Chain	9610	15	31	13 19	X	**14**	15	2 27	
Great Lakes Chain	8970		20 21	1 36	**15**	X	16 24	20 30	**4**
Southeast U.S. Chain	7980				18	18 27	X	25	**3**
Northeast U.S. Chain	9960			**10 15**	2 26	18 27	20	X	9
Canadian East Coast Chain	5930					**6**	**4**	15	X

Table A.2 Sub-periodic cross-rate intervals of the American Loran-C chains.

The sub-periodic intervals are calculated with the aid of Formula 4.1 of Section 4.2. In this calculation the cross-rate pulses are effective if they hit within a 150 μs interval of the desired pulses. The boldfaced numbers in both Tables correspond to pairs of GRIs whose cross-rate pattern is badly aligned and a large number (> 4) of hit pulses in consecutively hit GRIs occupy the same pulse positions. Especially, the GRI pairs with a small boldfaced sub-periodic interval endanger reliable data transmission. Some cells in Table A.2 are intentionally left blank. The corresponding cross-rate chain is considered to have far less influence than other neighboring chains.

Appendix B Introduction to error correcting codes

In this Appendix the definitions and properties of some parameters of the error correcting theory are discussed. For more background information the reader is referred to [10] and [11].

The error correcting codes used in this report are all binary *block codes.* A block code C of length **n** can be seen as a set of $2^{\mathbf{k}}$ vectors which is a coset of all possible binary vectors of length **n**. A code is called linear if the sum of any two code words is a code word itself. Linear codes can be made systematic if the first **k** bits of the code word correspond to the information vector $\underline{u}$. The length **n** of the code is then built up by **k** information bits (dimension of the code) and **r** check bits as is shown in Formula A.1.

$$\underline{x} = \left(u_1\, u_2 \cdots u_k \,\middle|\, s_1 \cdots s_r\right) \qquad \text{Formula A.1}$$

The performance of a code can be described with two parameters, the Hamming distance and the code rate.

Hamming distance

The Hamming distance $\mathbf{d}(\underline{a},\underline{b})$ between two vectors $\underline{a}$ and $\underline{b}$ is defined by the number of places the two vectors differ in. For example, the Hamming distance of the following two vectors $(1\ 0\ 0\ 1\ 1\ 1\ 0)$ and $(0\ 1\ 0\ 1\ 1\ 0\ 1)$ equals 4.

The Hamming distance of a code C is defined as the minimum Hamming distance between any two vectors (code words) of code C.

The Hamming distance is a measure for the error correcting capabilities of a code. A code that has a Hamming distance $\mathbf{d} \geq 2\mathbf{t} + 1$ is capable of correcting any **t** errors or detecting any **2t** errors. Also, the code can exchange the correction of an error with the correction of two erasures. An erasure is a symbol which the demodulator outputs if it cannot make a decision whether the received bit was a 1 or a 0. An erasure can be seen as an error of which the position is known and only the true value has to be found.

In short, a code can correct any combination of **t** errors and **e** erasures as long as $2\mathbf{t} + \mathbf{e} + 1 \leq \mathbf{d}$, the Hamming distance of the code.

Code rate

The code rate of a code is a measure for the amount of redundancy that has to be added to the information bits to achieve the desired error correcting performance. The code rate of a binary code is given by Formula A.2.

$$R = \frac{{}^{2}\log(M)}{n} \qquad \text{Formula A.2}$$

where M (cardinality) is the total number of code words of the code C. In general M equals $2^{\mathbf{k}}$, the code rate is then described by

$$R = \frac{k}{n} \qquad \text{Formula A.3}$$

Note that the code rate $R \leq 1$.

A good code preferably has a high Hamming distance and a high code rate.

Equal weight codes A(n, d, w)

The equal weight code is a class of non-linear and unsystematic codes. This means that the **k** information bits cannot appear as the first part of a code word. All vectors of the code have the same weight (the number of ones in a code word). The Hamming distance of an equal weight code is always even, because any two code words differ in an even number of places. A balanced code is a subclass of the equal weight codes. Then, **w** equals ½ **n**.

The maximum number of code words of the A(n, 2, w) code can be calculated with the binomial expression of Formula A.4.

$$A(n,2,w) = \binom{n}{w} = \frac{n!}{k!\,(n-k)!} \qquad \text{Formula A.4}$$

The set of code words of an equal weight code with larger distance (always even) is a subset of the code words of A(n, 2, w).

Galois fields

For the construction of good error correcting codes Galois fields are often used. A Galois field GF(q) is a finite field with q elements on which the operations sum and multiplication are defined. q has to be a prime number or a power of a prime number (normally a power of 2). The elements of a Galois field can be used as symbols (instead of bits) over which efficient codes can be designed. The symbols of GF(2^m) can be represented as vectors of m bits (m-tuple representation) and used to create efficient binary codes (e.g. the binary Reed-Solomon code of Section 5.2.2). For a thorough explanation of the Galois fields and corresponding codes the reader is referred to [10] and [11].

Appendix C
Look-up table for the A(8, 2, 4) balanced code

This table contains the conversion of an original 6 bit word into the balanced A(8, 2, 4) code (4 ones and 4 zeros). The conversion is done as systematically as possible, a code is called systematic, if the information bits (word of 6) appear at the beginning of the code word (the 8 shift word). The Table lists 50 of 64 6-bit messages systematically. The remaining 14 information vectors contain more than 4 ones or 4 zeros, and thus cannot be encoded systematically. An alternative has to be found among the remaining 20 legal code words.

Entry		Information vector		Code word	Comment
0		0 0 0 0 0 0		0 0 0 1 1 1 1 0	Not Systematic
1		0 0 0 0 0 1		0 0 1 0 1 1 1 0	Not Systematic
2		0 0 0 0 1 0		0 0 1 1 0 1 1 0	Not Systematic
3		0 0 0 0 1 1		0 0 0 0 1 1 1 1	
4		0 0 0 1 0 0		0 0 1 1 1 0 1 0	Not Systematic
5		0 0 0 1 0 1		0 0 0 1 0 1 1 1	
6		0 0 0 1 1 0		0 0 0 1 1 0 1 1	
7		0 0 0 1 1 1		0 0 0 1 1 1 0 1	
8		0 0 1 0 0 0		0 1 1 0 1 0 1 0	Not Systematic
9		0 0 1 0 0 1		0 0 1 0 0 1 1 1	
10		0 0 1 0 1 0		0 0 1 0 1 0 1 1	
11		0 0 1 0 1 1		0 0 1 0 1 1 0 1	
12		0 0 1 1 0 0		0 0 1 1 0 0 1 1	
13		0 0 1 1 0 1		0 0 1 1 0 1 0 1	
14		0 0 1 1 1 0		0 0 1 1 1 0 0 1	
15		0 0 1 1 1 1		0 0 1 1 1 1 0 0	
16		0 1 0 0 0 0		0 1 0 0 1 1 1 0	Not Systematic
17		0 1 0 0 0 1		0 1 0 0 0 1 1 1	
18		0 1 0 0 1 0		0 1 0 0 1 0 1 1	
19		0 1 0 0 1 1		0 1 0 0 1 1 0 1	
20		0 1 0 1 0 0		0 1 0 1 0 0 1 1	
21		0 1 0 1 0 1		0 1 0 1 0 1 0 1	
22		0 1 0 1 1 0		0 1 0 1 1 0 0 1	
23		0 1 0 1 1 1		0 1 0 1 1 1 0 0	
24		0 1 1 0 0 0		0 1 1 0 0 0 1 1	
25		0 1 1 0 0 1		0 1 1 0 0 1 0 1	
26		0 1 1 0 1 0		0 1 1 0 1 0 0 1	
27		0 1 1 0 1 1		0 1 1 0 1 1 0 0	
28		0 1 1 1 0 0		0 1 1 1 0 0 0 1	
29		0 1 1 1 0 1		0 1 1 1 0 1 0 0	
30		0 1 1 1 1 0		0 1 1 1 1 0 0 0	
31		0 1 1 1 1 1		0 1 1 1 0 0 1 0	Not Systematic

Table A.3. Look-up table for the A(8, 2, 4) balanced code.

Entry		Information vector		Code word	Comment
32		1 0 0 0 0 0		1 0 0 1 0 1 1 0	Not Systematic
33		1 0 0 0 0 1		1 0 0 0 0 1 1 1	
34		1 0 0 0 1 0		1 0 0 0 1 0 1 1	
35		1 0 0 0 1 1		1 0 0 0 1 1 0 1	
36		1 0 0 1 0 0		1 0 0 1 0 0 1 1	
37		1 0 0 1 0 1		1 0 0 1 0 1 0 1	
38		1 0 0 1 1 0		1 0 0 1 1 0 0 1	
39		1 0 0 1 1 1		1 0 0 1 1 1 0 0	
40		1 0 1 0 0 0		1 0 1 0 0 0 1 1	
41		1 0 1 0 0 1		1 0 1 0 0 1 0 1	
42		1 0 1 0 1 0		1 0 1 0 1 0 0 1	
43		1 0 1 0 1 1		1 0 1 0 1 1 0 0	
44		1 0 1 1 0 0		1 0 1 1 0 0 0 1	
45		1 0 1 1 0 1		1 0 1 1 0 1 0 0	
46		1 0 1 1 1 0		1 0 1 1 1 0 0 0	
47		1 0 1 1 1 1		1 0 1 1 0 0 1 0	Not Systematic
48		1 1 0 0 0 0		1 1 0 0 0 0 1 1	
49		1 1 0 0 0 1		1 1 0 0 0 1 0 1	
50		1 1 0 0 1 0		1 1 0 0 1 0 0 1	
51		1 1 0 0 1 1		1 1 0 0 1 1 0 0	
52		1 1 0 1 0 0		1 1 0 1 0 0 0 1	
53		1 1 0 1 0 1		1 1 0 1 0 1 0 0	
54		1 1 0 1 1 0		1 1 0 1 1 0 0 0	
55		1 1 0 1 1 1		1 0 0 1 1 0 1 0	Not Systematic
56		1 1 1 0 0 0		1 1 1 0 0 0 0 1	
57		1 1 1 0 0 1		1 1 1 0 0 1 0 0	
58		1 1 1 0 1 0		1 1 1 0 1 0 0 0	
59		1 1 1 0 1 1		1 1 0 0 0 1 1 0	Not Systematic
60		1 1 1 1 0 0		1 1 1 1 0 0 0 0	
61		1 1 1 1 0 1		1 1 0 0 1 0 1 0	Not Systematic
62		1 1 1 1 1 0		1 1 0 1 0 0 1 0	Not Systematic
63		1 1 1 1 1 1		1 1 1 0 0 0 1 0	Not Systematic
R1				1 0 1 0 1 0 1 0	Synchronisation
R2				0 1 1 0 0 1 1 0	Integrity Message
R3				0 1 0 1 1 0 1 0	
R4				0 1 0 1 0 1 1 0	
R5				1 0 0 0 1 1 1 0	
R6				1 0 1 0 0 1 1 0	

Table A.3 continued. Look-up table for the A(8, 2, 4) balanced code.

The difference between systematic and unsystematic encoding is explained with an example.

Example: The information vector corresponding to table entry 3 contains 4 zeros and 2 ones. By adding another 2 ones at the end of the message the desired systematic balanced code word is found. However, table entry 0 contains 6 zeros and balancing would require 6 ones. The code word has only room for two check bits and therefore this vector cannot be encoded systematically. The spare code word 00011110 is chosen because it best resembles the original message.