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Sparse Array Placement for Bayesian Compressive Sensing Based Direction of Arrival Estimation

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***Abstract*—In this paper, an algorithm to generate a sparse linear antenna array for Direction of Arrival (DoA) estimation that works well in combination with Bayesian Compressive Sensing (BCS) is proposed. The proposed algorithms rely on the provided information inherent to BCS, i.e., the entropy of the recovered estimation vector, to place new sensor antenna elements in an initially empty array, so that the most additional information is gathered about the observed scene. It is shown by means of simulation and radar measurements that BCS methods for DoA estimation using sparse sensor arrays provide promising results in terms of detection probability and estimation accuracy. Furthermore, the proposed algorithms are able to generate sparse sensor arrangements which provide an improved performance when compared against randomly generated sparse arrays.**

***Index Terms*—Bayesian Compressive Sensing, antenna placement, MIMO radar, DoA estimation.**

I. Introduction

DIRECTION of Arrival estimation for multiple targets is a key capability for modern radar systems in applications such as automotive radar. This is typically realised by utilising antenna arrays, with resulting angular resolution depending on the array aperture. In the quest to improve angular resolution, methods such as Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) have been proposed besides conventional DoA estimation methods based on digital beamforming [1]. While effective in ideal conditions, these methods face practical trade-offs, including the requirement of high Signal-to-Noise Ratio (SNR), large number of snapshots, or a priori knowledge about the number of targets to be expected. Furthermore, these methods typically rely on full, uniformly spaced linear array (ULA) geometries.

To address some of these limitations, methods based on the Compressive Sensing (CS) framework [2] have been proposed. These have desirable characteristics, including the possibility of using only a single snapshot [3], improved noise stability [4], and reducing the amount of antennas needed in the sensor array [5]. The central paradigm of CS methodology is to cast the DoA estimation problem into a linear equation. The estimation turns into the task of solving this linear equation, which by nature of the problem is underdetermined. Expecting only a sparse solution set, CS methods can be employed and promise to find this sparse solution.

In this paper, a statistical method [6] to solve the linear equation is adopted using Bayesian inference and the Relevance Vector Machine (RVM) [7], [8], within the Bayesian CS framework [9]. Unlike the application of this framework to DoA estimation with complete ULAs [10]–[13], in this work sparse arrays are considered. Specifically, an approach is proposed that uses the entropy of the recovered estimation vector to guide the placement of new antenna elements to form a sparse array maximising the additional information generated. The proposed approach is validated with simulations and experimental data. While sparse sensor placement has been addressed in the classic CS framework, typically by exploiting the mutual coherence [5], [14]–[18], or by some research in sonar [19], to the best of our knowledge there is limited research on sparse antenna placement strategies using BCS for experimental radar purposes.

The rest of this paper is structured as follows. Section II introduces the signal model and the BCS framework for DoA estimation. Section III presents the proposed approach for array synthesis in MIMO radar architectures. An evaluation of the proposed approach focusing on experimental results is provided in section IV. Finally, section V concludes the paper.

II. Signal Model and BCS Framework

In the considered geometry, K targets are assumed to be present in the far-field of the antenna array, which consists of M elements. Furthermore, it is assumed that the signals are narrowband. The received signal at a specific antenna $m = 1, \dots, M$ is given by the sum of all impinging signals with a time delay τ_{mk} and a noise term n_m. The time delay τ_{mk} is taken with respect to the first element of the antenna array, located at the origin of the reference system [1]:

$$y_m(t) = \sum_{k=1}^{K} s_p(t - \tau_{mk}) + n_m(t) \tag{1}$$

With the far-field assumption, the time delay τ_{mk} can be expressed as $\tau_{mk} = \boldsymbol{\xi}^T \mathbf{r}_m$ [1]. Here, $\mathbf{r}_m$ is the position of the m^{th} antenna in the 3D space and $\boldsymbol{\xi}$ denotes the propagation direction depending on the angles of arrival ϕ and θ.

In this work, linear 1D arrays are considered, so that the expression for the time delays simplifies to:

$$\tau_{mk} = \boldsymbol{\xi}^T \mathbf{r}_m = \xi(m-1)\Delta \sin\theta_k \tag{2}$$

where $\xi = \frac{2\pi}{\lambda}$ is the propagation constant in free space. By expressing the sensor spacing in units of wavelength as $d = \Delta/\lambda$, the incurred time delay at each sensor can be expressed as $\tau_{mk} = 2\pi d(m-1)\sin\theta_k$.

The signal vector received by the antenna array is obtained by stacking each element output into a vector as $\mathbf{y} = [y_1(t), \ldots, y_M(t)]$. The time delays τ_{mk} at the different sensors induce corresponding phase shifts $e^{-j\omega\tau_{mk}}$, leading to the common signal model for received data [1]:

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{a}(\theta_k) s_k + \mathbf{n} \tag{3}$$

where the phase shifts for each element have been stacked into the *steering vector* $\mathbf{a}(\theta_k) \in \mathbb{C}^M$ [1].

The steering vectors for all K targets are combined into a $M \times K$ matrix $\hat{A} = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{M \times K}$, the so-called *steering matrix* [1]. Then, (3) can be compactly expressed as:

$$\mathbf{y} = \hat{A}\mathbf{s} + \mathbf{n} \tag{4}$$

with $\mathbf{s} \in \mathbb{C}^K$ now indicating the amplitudes of the K target signals and $\mathbf{n}$ accounting for the noise contribution.

To linearise the problem of finding θ_k and enable the application of the BCS framework, the angular range over all possible θ is discretised into a grid of G equally-spaced angles. The steering matrix can then be generated as an over-complete dictionary matrix $A \in \mathbb{C}^{M \times G}$ where each column is a steering vector $\mathbf{a}_g = \mathbf{a}(\theta_g)$ with $g = 1, \ldots, G$ corresponding to each possible target direction. Furthermore, since the number of targets K is typically not known, the vector of signal coefficients $\mathbf{s} \in \mathbb{C}^K$ is expanded into a sparse vector $\mathbf{x} \in \mathbb{C}^G$ with unknown support, corresponding to targets presented at those angles. The resulting data model is given as:

$$\mathbf{y} = A\mathbf{x} + \mathbf{n} \tag{5}$$

This over-complete dictionary matrix $A \in \mathbb{C}^{M \times G}$ can now directly be used in the CS or BCS framework. Finally, to complete the signal model for the proposed BCS algorithm, the complex-valued representation given in (5) is expanded into a real valued equation [11]. Using this expansion, the dimensions of the involved quantities are doubled.

The BCS framework recovers the sparse vector $\mathbf{x}$ in (5) (whose support is the set of coefficients in $\mathbf{s}$ from equation (4)), essentially performing the DoA estimation. Within the framework, a prior belief is formulated in terms of a PDF, that the vector of weights $\mathbf{x}$ has a sparse support in a transform basis Ψ, assuming only a few point-like targets in the scene. In this case, the transform basis Ψ is the Fourier transform. Then, the BCS framework computes a posterior PDF about the vector $\mathbf{s}$, also including confidence metrics about the estimates and an estimation for the noise variance.
Recalling the general formulation of a CS data model with the $M \times G$ sensing matrix Φ [20]:

$$\mathbf{y} = JA\mathbf{x} + n = \Phi\mathbf{x} + \mathbf{n} \tag{6}$$

the similarity to (4) can be observed. In this work, the sensing matrix Φ consists of the array steering matrix A and an implied diagonal selection matrix J. The coefficients on the diagonal of J will be found with the proposed algorithms and are either 0 (element not included) or 1 (element included). The additive term n in (6) describes noise from both measurement noise and reconstruction error, which is assumed Gaussian [9]. The Gaussian assumption for the noise will now result in the likelihood model [9]:

$$p(\mathbf{y}|\mathbf{x}, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{K/2}} \exp\left\{-\frac{1}{2\sigma^2}\|\mathbf{y} - \Phi\mathbf{x}\|_2^2\right\} \tag{7}$$

A full posterior PDF is sought for $\mathbf{x}$ and σ^2, while the matrix Φ and the CS measurement vector $\mathbf{y}$ are known. To enforce sparsity onto the weights $\mathbf{x}$, a sparsity promoting prior probability is imposed.The adopted method from [8], [9] utilises a hierarchical prior, which has similar sparsity promoting characteristics of a Laplace prior but results in closed form expressions. For this, a zero-mean Gaussian density is defined as the prior probability on each element in $\mathbf{x}$:

$$p(\mathbf{x}|\alpha) = \prod_{i=1}^{N} \mathcal{N}(x_i|0, \alpha_i^{-1}) \tag{8}$$

where $\alpha = [\alpha_1, \ldots, \alpha_N]$ is the precision of each Gaussian density, and $\sigma_i^2 = \alpha_i^{-1}$ the *variance* of each distribution for x_i. Now *hyperpriors* are imposed upon the *hyperparameters* α and the noise variance σ^2, where its precision is denoted by $\beta = 1/\sigma^2$ [8]. Since the hyperparameters resemble *scale* parameters, the hyperpriors are chosen to be Gamma distributions with only positive values [8]:

$$p(\alpha) = \prod_{i=0}^{N} \Gamma(\alpha_i|a, b) \tag{9}$$

$$p(\beta) = \Gamma(\beta|c, d) \tag{10}$$

In [8], the hyperparameters a, b, c and d are all set to zero to obtain uniform or "improper" hyperpriors, implying that results will not depend on the unit of measurement and simplifying subsequent derivations. Moreover, this enables the posterior probability to accumulate at very large values for some hyperparameters α. The posterior of the corresponding weights will then peak around zero, "turning-off" those weight inputs and their associated basis functions (columns in Φ). This elimination process is basically creating sparsity, and the remaining weights are termed "relevant" vectors [8].

The posterior over the weights $p(\mathbf{x}|\mathbf{y}, \alpha, \sigma^2)$ can be evaluated in closed form as a multivariate Gaussian distribution [8], [9], using Bayes' equation:

$$p(\mathbf{x}|\mathbf{y}, \alpha, \sigma^2) = \frac{p(\mathbf{y}|\mathbf{x}, \sigma^2)p(\mathbf{x}|\alpha)}{p(\mathbf{y}|\alpha, \sigma^2)} \tag{11}$$

$$= \frac{1}{(2\pi)^{(N+1)/2}|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}\|\mathbf{x} - \mu\|_{\Sigma^{-1}}^2\right\} \tag{12}$$

with mean and covariance given as:

$$\Sigma = (\sigma^{-2}\Phi^T\Phi + \tilde{A})^{-1} \tag{13}$$

$$\mu = \sigma^{-2}\Sigma\Phi^T\mathbf{y} \tag{14}$$

and with $\tilde{A} = \text{diag}(\alpha_0, \alpha_1, \ldots, \alpha_N)$. Note here the dependence on the unknown hyperparameters α as well as on β.

By marginalising over the unknown weights $\mathbf{x}$, a closed form solution is obtained as [8]:

$$p(\mathbf{y}|\alpha, \sigma^2) = \int p(\mathbf{y}|\mathbf{x}, \sigma^2) p(\mathbf{x}|\alpha) d\mathbf{x} \tag{15}$$

$$= \frac{1}{(2\pi)^{N/2}|C|^{1/2}} \exp\left\{-\frac{1}{2}\mathbf{y}^T C^{-1}\mathbf{y}\right\} \tag{16}$$

with $C = \sigma^2\mathbf{I} + \Phi A^{-1}\Phi^T$. This expression is known as the marginal likelihood or "evidence function" for the hyperparameters [7], [8], [21]. The process of its maximisation, or more commonly its logarithm is:

$$\mathcal{L}(\alpha) = \log p(\mathbf{y}|\alpha, \sigma^2) \tag{17}$$

$$= -\frac{1}{2}\left[Nlog(2\pi) + log(|C|) + \mathbf{y}^T C^{-1}\mathbf{y}\right] \tag{18}$$

This is termed as *type-II maximum likelihood* or "evidence procedure" [8]. Considering some known initial values for α and β, expression (13) and (14) can be evaluated. Following this, α and β can be re-estimated using (17), in an iterative procedure such as the Expectation Maximization algorithm. An efficient, "constructive" algorithm for the RVM inversion has been derived in [22] and is implemented in this work to utilize BCS to design sparse arrays for radar-based DoA.

III. Proposed Sparse Array Generation Method

The measurement or sensing matrix Φ in (6) has M rows as $\Phi = [\mathbf{r}_1, \ldots, \mathbf{r}_M]^T \in \mathbb{C}^{M\times G}$, with G denoting the number of discretized steering directions. In designing an array for DoA estimation, the question arises on how new measurements could be added to a set of initial measurements (i.e., new antenna elements as new rows in Φ), such that they improve the DoA estimation using the BCS framework. This problem has been theoretically addressed in [23], and inspires the formulation of the approach specifically proposed in this work for designing sparse arrays for radar-based DoA estimation [6]. The crucial information used is the covariance matrix in (13) made available by the BCS framework, but not by other classical CS techniques.

To ensure the maximum achievable array aperture and thus resolution, the algorithm starts with the most outer antennas included. From there, running the inversion algorithm once, estimates of the weight's mean values μ_x and their covariances Σ_x are obtained. The estimated DoA can be obtained by the index of the non-zero coefficients (or mean values). The decisive information lies in the covariance Σ_x, from which the computation of the *differential entropy* of the reconstructed signal is proposed to determine the overall reconstruction uncertainty [6], [23]:

$$\begin{aligned} h(\mathbf{x}) &= \frac{1}{2}\log|\Sigma_x| + c \\ &= -\frac{1}{2}\log|\tilde{A} + \alpha_0\Phi^T\Phi| + c \end{aligned} \tag{19}$$

where $\tilde{A} = diag(\alpha_0, \alpha_1, \ldots, \alpha_N)$.

The goal is now to select a new measurement, in this case a new antenna element position, which yields the maximum

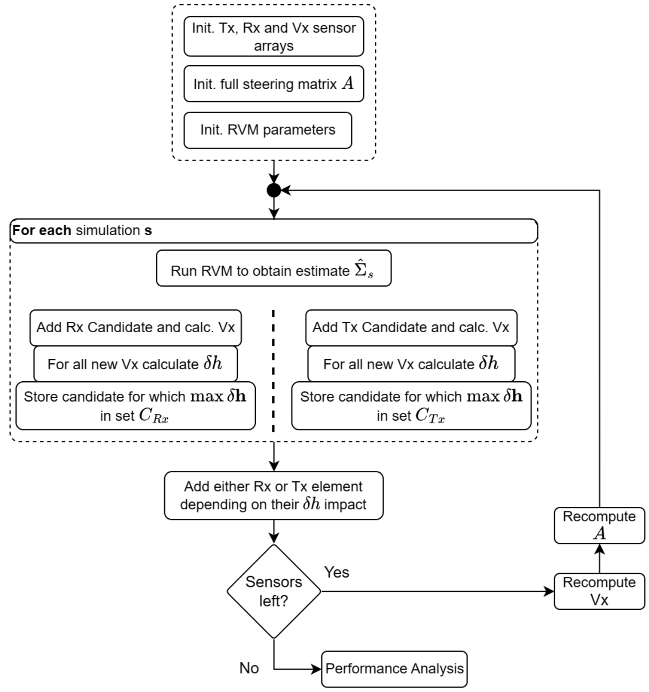

Fig. 1. Block diagram of the proposed BCS entropy based sparse array design algorithm for the MIMO case.

reduction in entropy or average uncertainty of the variable. Inspired by [23], an update equation is proposed which allows testing a new candidate row for how much it reduces the entropy if it is included into the sensing matrix [6]:

$$h_{new} = h_{old} - \frac{1}{2}\log\left(1 + \alpha_0\mathbf{r}_{new}\hat{\Sigma}_x\mathbf{r}_{new}^H\right) \tag{20}$$

Here, α_0 denotes the noise variance, $\hat{\Sigma}_x$ is the estimate of the covariance matrix obtained in the previous run of the BCS algorithm, and $\mathbf{r}_{new}$ denotes a new candidate row of the sensing matrix Φ, which in the DoA case is equivalent to a row of the steering matrix. Therefore, adding a new candidate row amounts to including a new candidate antenna position to the sparse array. The important term is the second one in (20). In order to minimise the entropy as much as possible with each new antenna, this term has to be maximised for each new candidate. This term is defined as inspired by [19]:

$$\delta h(\mathbf{r}_{new}) = \log\left(1 + \alpha_0\mathbf{r}_{new}\hat{\Sigma}_x\mathbf{r}_{new}^H\right) \tag{21}$$

Unlike linear arrays, the MIMO radar array architecture is more challenging, since the resulting virtual receiver array that is used in the DoA estimation is the result of two separate arrays, the physical transmitter array and the receiver array. Running the algorithm for the virtual array, thereby treating it as a ULA, will lead to the problem of factorising this array into a physical transmitter and receiver array afterwards. This is not trivial, and it might be the case that a feasible factorisation cannot be found (especially when overlapping virtual elements are considered). To address this, an iterative approach is proposed and new antennas are added to either the transmitter or receiver arrays. This decision is again made based on the BCS entropy, but evaluated on the resulting virtual array [6]. This ensures that no virtual arrays are obtained that cannot be realised in terms of physical transmitter and receiver arrays. A block diagram of the procedure is shown in Fig. 1.

It should be noted that in the MIMO architecture, adding a new antenna element to either of the two physical arrays might lead to more than one new element in the virtual array. Unlike in the ULA case, this in a sense reduces the degrees of freedom to place new antennas in order to maintain the physical feasibility of the resulting sparse array. Moreover, due to the expansion of the complex steering matrix to real values, each of the new virtual sensors leads to two new rows in the expanded matrix, similar as in the ULA case. In total, this can cause the addition of a multitude of rows to the steering matrix during just one iteration of the algorithm for the MIMO case, which should be kept in mind.

IV. Evaluation of the Proposed Approach

In this section, the proposed approach for sparse array design for DoA is evaluated. Several simulated case studies have been performed and reported in greater detail in [6], whereas here for conciseness only the experimental results are reported, as to the best of our knowledge experimental validation of BCS techniques applied to MIMO radar DoA are lacking in the open literature. Moreover, a simpler version of the proposed algorithm is explained in [6], where a physical ULA is used instead. This simplified version can be used for other applications, such as communication antenna arrays with ULA receptors. A tuning parameter for the BCS method has been adjusted to suppress the surrounding energy, which is described in [6] Appendix C. This parameter can be used to influence the false alarm rate when assessing the detector performance. In Fig. 3 and Fig. 4, it can be seen that the produced false alarms mostly fall below or to a similar energy level as the FFT method.

In terms of performance metrics, the outputs of the BCS DoA estimation method are treated as a joint detector & estimator, and the performances of each sparse array are assessed by evaluating ROC curves. In this context, the *Jaccard Index* can also be evaluated as another metric to narrow down the feasible sparse array stages as well as the suitable detector thresholds [19].

Summarising, the complete assessment pipeline includes:

1) Generate a sequence of sparse arrays that are filled iteratively by the proposed antenna placement algorithm.
2) Utilise each generated array with the BCS method to obtain DoA estimates.
3) Apply a range of possible detection thresholds to each estimation output, providing a matrix of array versus threshold data.
4) A two-dimensional map of array sparsity versus threshold can be plotted, with the Jaccard index indicating the intensity values of each pixel [6].

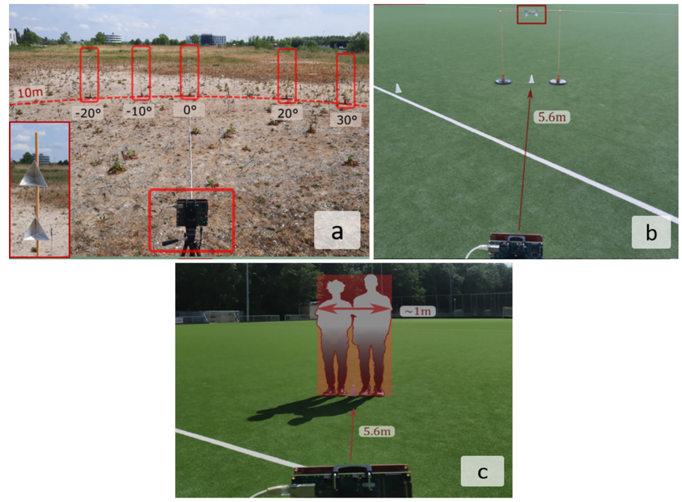

Fig. 2. Measured scenarios: separated corner reflectors (a); corner reflectors close to each other (b); two persons forming an extended target (c).

5) The cell with maximum Jaccard index will indicate the sparse array topology and detection threshold scoring the best performance.
6) Having now a fixed sparse array, plot the ROC curve for that parameter pair by varying a detection threshold, and calculate the Root Mean Square Error of the DoA estimation.

The generated sparse arrays have been tested with experimental data collected with the TI AWR2243 Cascade board, which has a total of 86 unique virtual antenna positions for DoA azimuth estimation. The data is always collected with all the 86 virtual antennas, but in post-processing only the antenna elements selected by the proposed BCS sparse array design approach will be used for DoA processing. The radar parameters used for all measured scenes include: 77 GHz start frequency, 5 MHz/μs slope, 80 μs ramp end-time, 400 MHz bandwidth, 256 ADC range samples & 128 chirps in 4 frames. Three types of scenes have been measured as shown in Fig. 2: 5 corner reflectors with good separation between them (Fig. 2a); 2 corner reflectors very close together (Fig. 2b); 2 persons acting as an extended target of opportunity (Fig. 2c). While several target arrangements have been measured in each type of scene, only a selection of results are reported here for conciseness, with more details provided in [6].

A. Five corner reflectors

The measured scene is shown in Fig. 2a. Five corner reflectors have been placed at a fixed distance of 5 or 10 m, with varying angles to the line of sight of the radar. The maximum unambiguous range has been set to about 108 m to avoid too many target returns from outside the range of interest. The range resolution has been set to a value of 37.5 cm, so that in post-processing the range bin where the targets are placed can be easily identified. Using all the 86 unique virtual antennas, the theoretically resulting angular resolution is for a zero-degree azimuth angle equal to $\Delta\theta_0$ =1.33°.

Sparse arrays designed with the proposed BCS algorithm have been evaluated with an increasing number of antenna elements, i.e. assuming to progressively activate more elements out of the possible 86 non-overlapping ones in the chosen radar board. After generating sparse arrays with the proposed algorithm, acceptable detection performance is reached when up to 37 out of 86 possible virtual antennas are included, as shown in Fig. 3 compared to a conventional FFT-based spectrum using the full 86 virtual sensors. Although this work is focused on angular resolution, a detailed analysis of estimation error is shown in [6], where it can be seen how the reduction in the number of antennas does not have a large impact on it.

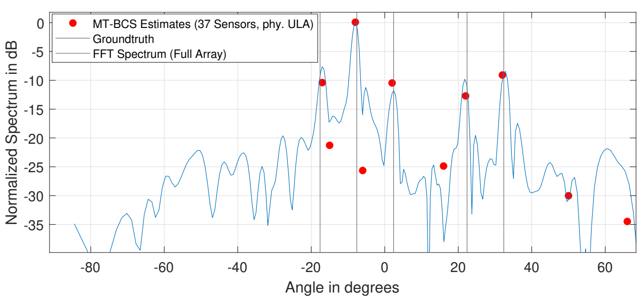

Fig. 3. Example of BCS-based DoA estimation (red dots) for 5 corner reflectors in the scene.

B. Two corner reflectors

Here, two corner reflectors are placed at 5.6 m from the radar, initially very close to each other (see Fig. 2b). Starting from a distance of 1.4 cm from corner to corner (13 cm from centre to centre), they have been shifted apart in steps of 3 cm (corresponding to 0.3° at 5.6 m distance), until their separation has increased by 12 cm (corresponding to 1.22°).

The evaluation is performed by averaging the results of 100 iterations of the BCS method, using one chirp each time and with a generated sparse array with 15 elements. Fig. 4 compares DoA estimations using the proposed BCS method with 0.5° angular discretization and an FFT beam-former. At the closest separation of 1.33° between the two targets, the FFT (blue line) is unable to resolve the targets, while the BCS (red dots) can, with approximately 15 dB separation in magnitude with respect to the remaining false detections. Note that the BCS estimation was performed using only one single snapshot. The shown results have been calculated assuming an underlying physical ULA, but the same can be observed when a physical MIMO array is assumed and a similar number of virtual sensors are included in the array. When the two targets are separated by an additional $\approx 0.33°$, so that also the FFT method is able to separate them, the BCS method can work with even fewer sensors, i.e., only 5 antenna elements compared to the 15 elements needed for the previous case. The results for this case are shown in Fig. 5.

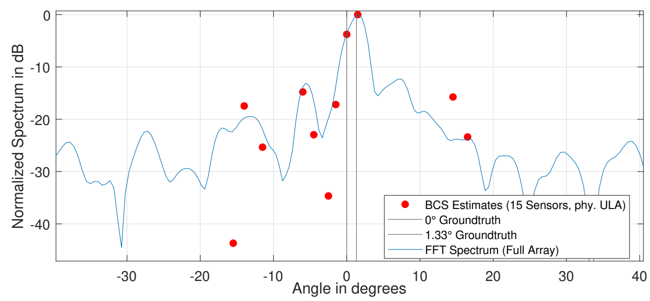

Fig. 4. Comparison FFT vs proposed BCS approach for DoA estimation for two corner reflectors separated by 1.33°.

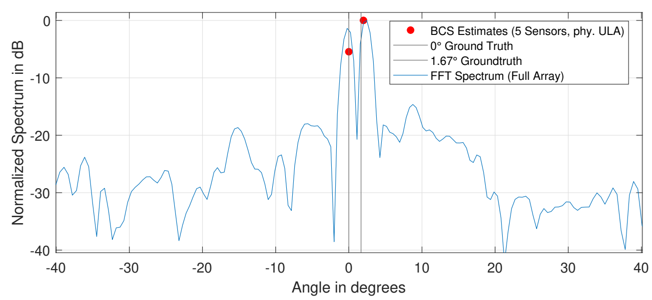

Fig. 5. Comparison FFT vs proposed BCS approach for DoA estimation for 2 corner reflectors separated by 1.66°.

C. Extended target

A final test has been performed to evaluate the proposed BCS method with an extended target, namely two persons standing shoulder to shoulder with a span of about 1m equivalent to about 10°. This scenario is shown in Fig. 2c). Fig. 6 shows the results of an FFT beam-former (blue) compared with DoA values (red dots & green crosses) estimated by the proposed BCS method using a sparse antenna array. The BCS approach returns high-valued detections in the region where the extended target is located and also the highest peaks of the FFT are present. These peaks are most likely related to the scattering centers of the extended target. For the physical ULA based array and this data capture, around 30 sensors seem to be sufficient such that the BCS algorithm obtains non-zero coefficients covering the angular span of the extended target. Similarly, the MIMO based generated sparse array with 30 virtual elements is able to recover those coefficients, which is displayed by the green cross marks in Fig. 6. In both cases, fewer than 50% of the sensors from the original, full (virtual) ULA of 86 sensors are needed.

Summarizing the results of this section, good performances with experimental data can be obtained with sparse arrays generated with the proposed BCS method. With experimental data, more antenna elements are needed compared to when only simulated data is used [6], but the proposed approach is still able to design arrays with fewer elements that still maintain performances to separate targets located in proximity.

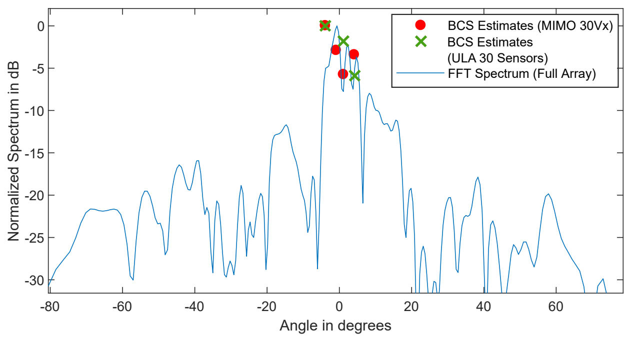

Fig. 6. Experimental result for extended target DoA estimation using 30 virtual antennas in both the physical ULA assumption and the MIMO assumption.

V. Conclusions

In this paper, an approach is proposed to generate sparse antenna arrays for MIMO radar DoA using the uncertainty measures provided by the Bayesian Compressive Sensing framework. The approach works by selecting a suitable number of antenna elements based on an entropy minimisation operation. Experimental results with corner reflectors and an extended target show that the proposed approach results in faster uncertainty reduction compared to random selection. This decreased uncertainty leads to a noticeable performance improvement in the detection of targets, while maintaining accuracy in their DoA estimation. From a practical perspective, these results suggest the feasibility to utilise sparse arrays combined BCS for DoA estimation, offering the possibility to reduce hardware complexity, cost, and energy consumption by using fewer physical antenna elements. Compared to established super-resolution algorithms such as MUSIC, BCS based approaches can work with only one single snapshot, as well as in sparse array architectures. However, potential drawbacks in terms of the computational implementation overhead [24] and possible SNR reduction need to be further explored in future work. The proposed method has been tested against random antenna selection and should be further tested against other sparse antenna array architectures.

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