# Simple parametric relations for solid rocket stage inert mass estimation

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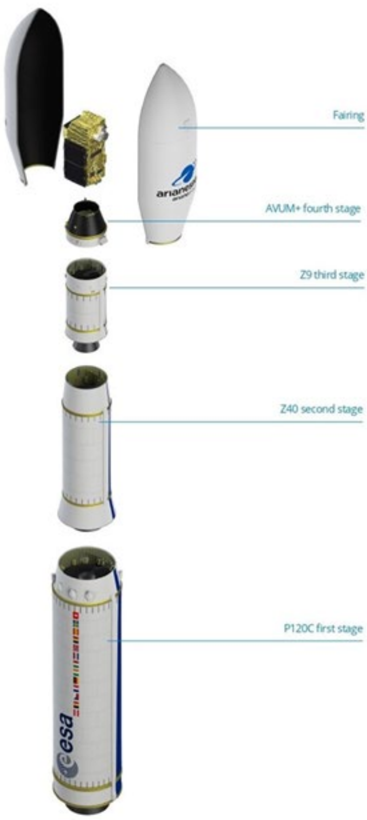
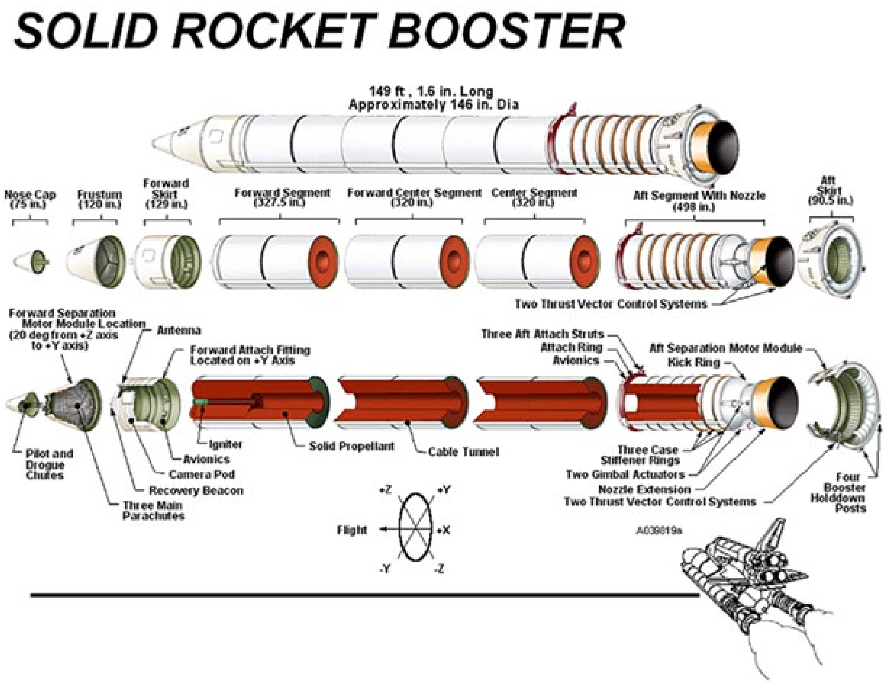
Faculty of
Aerospace engineering




B.T.C. Zandbergen

Figures on front page show Space Shuttle Solid Rocket Booster (courtesy NASA) and Vega C (courtesy Arianespace/ESA)

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**Executive Abstract**

This study quantifies the relationship between propellant mass and inert mass for solid rocket stages with steel and composite casings. Regression models reveal that steel-cased stages exhibit sub-linear scaling due to structural overhead, while composite-cased stages show lower inert mass and near-linear scaling, partly reflecting modern design improvements. Statistical analysis highlights substantial variability arising from stage-specific design choices, safety factors, and subsystem configurations. The results provide practical guidance for preliminary design, including conservative upper-bound estimates for inert mass and insights into the influence of casing material and stage design.

# 1. Introduction

Estimating the inert[1] mass of solid rocket stages relative to their propellant mass is a critical factor in several aspects of rocket design. It facilitates scaling of solid rocket stages for launch vehicles, comparing them to liquid and hybrid rocket stages, and evaluating the impact of different numbers of solid rocket boosters on the overall vehicle mass.

In Zandbergen (2019), two simple[2] empirical relations were proposed to estimate the inert mass ($M_{inert}$) of solid rocket stages as a function of the propellant mass ($M_{propellant}$) and the material used for the casing. These relations are useful, but the study lacks key details, particularly the dataset used for regression analysis and the individual estimation errors. Without this data, it is impossible to verify the accuracy of the relations, evaluate their structural variability, or assess any potential bias in the results.

This work aims to expand upon Zandbergen's empirical relations by incorporating a more detailed analysis. We will not only extend the existing models but also provide supplemental information on the accuracy of the estimations, including an examination of any bias and structural variability in the relationships. Our goal is to improve the reliability of inert mass estimations for solid rocket stages, enhancing their use in design and comparative studies.

# 2. Definition of solid rocket stage

A solid rocket stage consists of the solid rocket motor (SRM) and all additional subsystems required for its operation during flight. These subsystems typically include:

- **Structural elements**, such as the front and aft skirts, interstages, nose cone (for strap-on boosters) and stage attachment structures
- **Avionics and electrical systems**
- **Thrust Vector Control (TVC) hardware**[3] (for stages that are equipped with TVC)
- **Reaction Control Systems (RCS)**
- **Separation systems**
- **Range safety and destruct systems**
- **Recovery and deceleration systems** (for reusable boosters), such as parachutes

The Space Shuttle SRB mass distribution shown in Figure 1 illustrates how different components contribute to the overall mass of the stage. The propellant makes up the largest portion, approximately 86% of the total stage mass. The remaining mass consists of the solid rocket motor (SRM) hardware (inert mass), which holds the propellant and ensures thrust generation (about 11.3%), and the additional subsystems, also part of the inert mass) required for the stage to function as intended (about 3.1%).

[1] Inert mass refers to the mass of a solid rocket stage (also known as the dry mass) excluding the propellant mass.
[2] The term simple in "simple parametric relationships" indicates that the relationship involves a single parameter, limiting the complexity of the expression.
[3] In many Solid Rocket Motors (SRMs), the movable (gimbaled) nozzle is integrated into the motor assembly and is often treated as part of the SRM in hardware documentation. However, the hydraulic or electromechanical power unit and control logic are typically considered part of the vehicle's overall control system. There are, of course, exceptions to this generalization.

While the SRM hardware and subsystems only account for 14% of the total mass, they make up a significantly larger portion of the **inert mass**—approximately 20% (3.1% of 14.4%). Thus, when estimating the inert mass of a solid rocket stage, the additional subsystems should not be overlooked, as they represent a substantial fraction of the stage's inert mass.

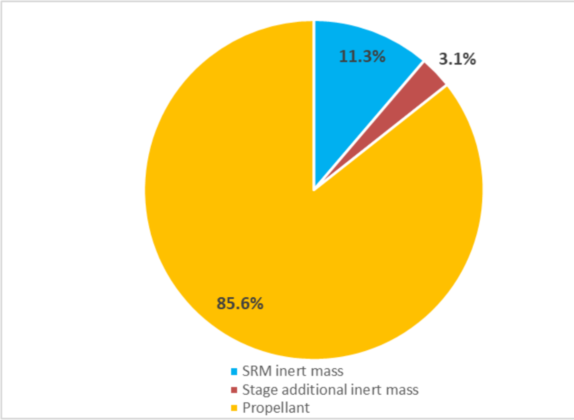


*Figure 1: Mass breakdown of Space Shuttle SRB mass*

For example, the Space Shuttle SRB has a propellant mass of about **503 tons** and an inert stage mass of approximately **84.5 tons**. Of this, around 66 tons is composed of the SRM hardware, including the casing, nozzle, thermal insulation, and ignition system. The remaining 18.5 tons includes subsystems such as the aft skirt (which serves as a thermal shield for the Shuttle External Tank, holds the stage separation systems, and houses the TVC actuators), the front skirt (which holds the forward separation system, parachute system and nose cone), as well as avionics and miscellaneous components.

## 3. Overview of existing relations

The existing empirical relations for estimating solid rocket stage inert mass are based on two specific casing materials: steel and composite-fibre[4]. These relations provide a functional estimate of inert mass, but the actual data used to develop them is not available, limiting the ability to verify or refine the results.

**Steel SRMs (N = 11 data points)**:

$$M_{\text{stage inert}} = 0.1726 \times M_{\text{propellant}} + 1173 \ (in\ kg)$$

- **Propellant mass range**: 9.5 t to 505 ton
- **Additional mass**: 1173 kg accounts for elements that do not scale with propellant mass.
- **Goodness of fit:** $R^2$ = 0.9907, indicating a very strong correlation between predicted and observed data.

[4] The original work also presents a third relation, for which no specific casing material is provided. The data show that this relation is based on a mix of rocket stages with steel casings (N = 10) and composite casings (N = 3). However, since the two relations for steel and composite casings are distinct, equally accurate, and cover a similar range of propellant masses, the third relation does not offer additional value. Given these considerations, it is deemed unnecessary to include this third relation in the main text.

- **RSE (or RMSPE')**[5]: 20.1%, meaning that ~68% of all the observed (or true) values is within a range of ± 20.1% of the predicted values.

**Composite-fibre based SRMS (N = 11)**:

$$M_{\text{stage inert}} = 0.1173 \times M_{\text{propellant}}$$

- Propellant mass range: 3 – 320 ton
- **Lighter mass**: The scaling factor for composite casings is smaller than for steel, reflecting the lighter weight of composite materials.
- **Goodness of fit:** $R^2$ = 0.9833, also showing a strong correlation between predicted and observed data
- **RSE (RMSPE'):** 24.1% (relative to the predicted value)

**Strengths:**

1. **High Accuracy**: Both relations exhibit high $R^2$ values, close to 1, which indicates a strong correlation between the predicted and observed values. This suggests that the relations can reliably predict inert mass within the given material categories.
2. **Material Sensitivity**: The relations account for the material used for the casing, which reflects real-world variations in design. For example, for a propellant mass of 100,000 kg:
    - Steel casing: Estimated inert mass of 18,433 kg.
    - Composite casing: Estimated inert mass of 11,730 kg.

   This shows a factor of difference between the two of approximately 1.55-1.6, highlighting the impact of the material choice on inert mass.

**Limitations:**

1. **Uncertainty in Scaling:** The RMSPE' indicates significant variability in the actual data compared to the predictions. An RMSPE' of 20.1% (for steel-cased solid rocket stages) means that, on average, the observed values are within 20.1% of the predicted inert mass. Similarly, an RMSPE' of 24.1% (for composite SRMs) suggests a 24.1% error relative to the predicted values.
2. **Reproducibility Issues:** The actual datasets used to derive the relations are not available. For the steel and composite relations, the stages used were identified[6], but no mass data was provided. The absence of these data points limits the ability to reproduce or refine the relations.

[5] In Zandbergen's work, the term **RSE** refers to the **root mean squared percentage error** (RMSPE'), which is calculated relative to the **estimated (or predicted) value** rather than the observed value. This means that the percentage error is expressed relative to the predicted inert mass, and not the observed inert mass.

[6] Retrieval of the original data revealed that the dataset included 10 steel-cased rocket stages and 3 composite-cased rocket stages. The identified stages are Space Shuttle SRB, Ariane 5 P230, PSLV PS1, ASLV first stage, H2 SRB, H1 SRB/Castor 2, Titan III SRB, Titan IV SRB, Delta Castor 4A, Ariane 4 PAP, Pegasus Orion 50S, Hercules GEM-60, and Castor 120.

# 4. Accuracy of regression relationships

When assessing the accuracy of a regression model, we seek to answer:

*How well does the model represent the true underlying physical relationship?*

In this work, model accuracy depends on two primary factors:

- **Bias** (systematic error)
- **Structural variability.**

Together, these determine how closely the regression model can approximate reality.

**1. Bias (Systematic Error)**

Bias represents the systematic deviation between the prediction $\hat{y}(x)$ generated by the regression model and the observed or "true" [7] value $y(x)$.

Mathematically, bias at a given input x is defined as:

$$\text{Bias}(x) = \mathbb{E}[\hat{y}(x)] - \text{y}(x)$$

Where $\mathbb{E}[\hat{y}(x)]$ stands for expected value of $\hat{y}(x)$.

In practical terms, this corresponds to a systematic shift in the regression curve. For example, if the regression consistently underestimates inert mass by 5%, it indicates **systematic bias** in the model.

Common sources of Bias include:

- Incorrect functional form (e.g., linear instead of power-law)
- Poor-quality calibration data
- Extrapolating beyond the data range

A key implication is that bias cannot be reduced by increasing data alone. If the model form is incorrect, e.g., using a linear model when a power-law relationship exists, the regression wil not converge to the true relationship, even with infinite data.

**2. Structural Variability**

Even in the absence of bias, real-world systems exhibit inherent variability. This is referred to as structural.

Structural variability arises from genuine physical differences between systems, rather than measurement error. It reflects the fact that no single functional relationship can perfectly capture all design variations.

Sources of Structural Variability:

- Differences in design philosophy
- Material selection
- Manufacturing approaches (e.g. monolithic vs. segmented casing)

[7] The true value is the ideal, perfect, and often unknown measurement of a quantity, while the observed value is the actual numerical result obtained from an experiment or measuring instrument. If measurement error is negligible, the observed value is identical to the true value.

- Safety factors (e.g. manned vs. unmanned missions)
- Design parameter variation (e.g. combustion pressure and nozzle area ratio)
- Configuration differences (e.g. movable nozzle vs. fixed nozzle)

This can be expressed as:

$$y = f(x) + \epsilon_{\text{structural}}$$

Where $\epsilon_{\text{structural}}$ represents the unmodeled or unexplainable variability (real-world differences not captured by the model).

Importantly, this variability is irreducible: It cannot be eliminated by improving measurements or increasing dataset size, it must be characterized probabilistically.

**3. How Bias and Structural Variability Relate to Accuracy**

Having defined bias and structural variability separately, we now consider how they jointly determine overall model accuracy.

The total **model variability** can be decomposed into two components:

$$\text{Total Model Variability}^2 = \text{Bias}^2 + \text{Variance (or Structural Variability)}$$

In the present context, the variance term is interpreted as **structural variability**, i.e., variability arising from real physical differences between systems rather than statistical noise.

**Interpretation:**

| Component | What It Means | Can It Be Reduced? |
|---|---|---|
| Bias | Wrong trend or systematic shift | Yes (better model form) |
| Structural variability | Real physical scatter | No (only modelled probabilistically) |

This decomposition highlights a fundamental trade-off:

- A model may be **unbiased but imprecise**, exhibiting large structural variability
- A model may be **precise but biased**, tightly clustered around an incorrect trend
- The ideal model is both **low-bias and low-variability**, although the latter is ultimately limited by physical reality

**4. In the Context of Inert Mass Estimation**

When modelling inert mass using a relation like:

$$M_{\text{inert}} = a \cdot (M_{\text{prop}})^{b}$$

Within this framework, model accuracy is influenced by three key factors:

- The correctness of the assumed functional form (source of bias),
- The degree of real-world stage-to-stage variability (structural variability),
- How well the dataset spans the relevant design space.

Structural variability arises primarily from variation in **unmodelled design parameters** (e.g., chamber pressure, mixture ratio, structural margins), as well as differences in materials, manufacturing approaches, and configuration choices.

As a result, regression outcomes can fall into four characteristic cases:

- **Unbiased but imprecise** (large structural variability)
- **Precise but biased** (tight scatter around wrong curve)
- **Both biased and variable** (worst case)
- **Unbiased with low structural variability** (ideal)

**5. Engineering Definition of Regression Accuracy**

In engineering practice, the accuracy of a regression model is determined by its ability to predict the true response with minimal systematic bias and properly quantified structural variability.

A model is considered accurate if:

- Mean error ≈ 0 (low bias)
- Prediction intervals capture the observed scatter
- Residuals are random (no trends)

**6. Structural Variability vs. Statistical Noise**

It is important to distinguish between two fundamentally different sources of variability:

- Statistical noise: Random, unwanted variability or "background" fluctuation in data that obscures the true underlying signal
- Structural variability: Real physical differences (cannot be reduced through better measurements)

In the context of rocket design parametric relationships, structural variability usually outweighs statistical noise (random measurement errors or background fluctuations). Measurement errors are generally small compared to variations arising from differing design choices and operating conditions.

Consequently, the observed scatter in regression data should primarily be interpreted as physical variability, not as noise to be eliminated.

**7. Measures of bias and structural variability**

To quantify model performance, this work employs metrics based on percentage errors, reflecting the multiplicative nature of deviations in engineering scaling laws.

This choice is motivated by the observation that errors tend to scale with the magnitude of the predicted value, rather than remaining constant. Such multiplicative behaviour arises from the combined influence of multiple unmodelled design variables.

**A. Percentage Error (PE or % Error)**

Percentage error in this work[8] is defined as:

[8] Sometimes, the percentage error is taken relative to the true value. This can be useful for e.g. the determination of measurement error. In predictive situations, the percentage error taken relative to the predicted value is preferred.

$$PE = \frac{(\hat{y} - y)}{\hat{y}} \times 100$$

Where:

- $\hat{y}$ is the predicted value from the model
- $y$ is the observed value

This formulation expresses error relative to the model prediction, which is consistent with the variability around the regression curve.

**B. Mean Percentage Error (MPE)**

**MPE** (based on PE): quantifies **systematic bias** relative to the predicted value. The formula for MPE is:

$$MPE = \frac{1}{N} \sum_{i=1}^{N} PE_i$$

Where:

- $N$ is the number of data points
- $i$ is an integer index that refers to the i-th specific datapoint

MPE measures bias relative to the regression curve and is primarily used as a consistency check (i.e. whether the regression curve lies centrally within the data). Ideally, MPE **= 0%** (no bias). A positive MPE indicates overestimation, while a negative MPE indicates underestimation.

**C. Standard Deviation (SD) of Percentage Error**

The formula for SD of the percentage error is:

$$SD = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (PE_i - MPE)^2}$$

SD measures the spread of errors relative to the predicted value and is interpreted here as variability.

Under the assumption of approximate normality, SD can be used to define statistical bounds (e.g. ±1σ, ±3σ). These bounds represent the expected spread of real-world designs around the regression curve.

**D. Mean Absolute Percentage Error (MAPE)**

MAPE provides a measure of the average magnitude of error, independent of sign. It is calculated using:

$$MAPE = \frac{1}{N} \sum_{i=1}^{N} | PE_i |$$

MAPE complements MPE by avoiding cancellation of positive and negative errors and as such provides a single measure of overall model accuracy.

**E. Root Mean Squared Percentage Error (RMSPE[9])**

[9] In some earlier publications, this parameter has also been referred to as Standard Error of Estimate (SEE) and Relative Standard Error (RSE). However, these terms were found to create too much confusion with the definition of the Standard Error. Hence, in this work, the term RMSPE was introduced as it better describes what is meant.

$$RMSPE = \sqrt{\frac{1}{N-m}\sum_{i=1}^{N}\left(\frac{\hat{y}_i - y_i}{\hat{y}_i} \times 100\right)^2}$$

Where:

- m gives the number of parameters estimated ($for\ instance, m = 2$ for a linear regression with both slope and intercept, m = 1 when forcing the linear curve through the origin (i.e., no intercept), and $m = 2$ for a power-law type of regression). This parameter is common to prevent overfitting.

RMSPE provides a measure of **fit quality**, accounting for model complexity (via the parameter m).

It has several important interpretations:

- Quantifies scatter around the regression curve
- Penalizes large deviations more strongly than MAPE
- Can be used to define statistical bounds (e.g. ±1σ, ±3σ) under normality assumptions, see an example worked out in appendix A.

In the present context, RMSPE is particularly useful because it characterizes variability relative to the model prediction, aligning with the multiplicative error assumption.

**F. Summary of roles**

| Metric | Primary Role |
|---|---|
| **PE** | Relative scatter description |
| **MPE** | Bias (systematic error) |
| **SD** | Structural variability (σ, bounds) |
| **MAPE** | Overall accuracy |
| **RMSPE** | Fit quality / scatter around model/ structural variability (σ, bounds) |

## 5. Stages with steel casings

The dataset for this analysis was compiled using solid rocket stages with steel casings. These stages were originally identified in Zandbergen (2019) and used as the baseline for our analysis. However, not all mass data from the original stages were available, and several stages were therefore replaced with comparable alternatives. For example, the HI SRB was substituted with the H-II SRB to maintain data consistency.

To increase the number of data points and improve the statistical reliability of the analysis, we added several other stages to the dataset:

- Stage 1 and 2 of the Japanese J-1 rocket
- Stage 1 of the M-V rocket
- Stage 1 and 2 of the Scout rocket (USA)

For this study, we focused solely on solid rocket stages, excluding solid rocket upper stages. Upper stages often include additional provisions related to payload support and integration with the rest of the launch vehicle, which can distort the scaling relationship between propellant mass and inert mass.

Table 1 presents the final dataset compiled for this study, including typical mass characteristics for solid rocket stages with stainless steel casings.

*Table 1: Typical mass characteristics of solid rocket stages (excl. upper stages) with stainless steel casings*

| ID | Vehicle | Stage | Propellant mass (ton) | Stage inert mass (ton) | Stage total mass (ton) | References |
|---|---|---|---|---|---|---|
| 1 | Space Shuttle | Shuttle SRB | 502.127 | 87.5448 | 590 | [Horton, 1980] |
| 2 | Ariane 5G | Ariane 5 SRB | 238 | 38 | 274.8 | [CAPCOM, "Ariane 5"], [LVC] |
| 3 | Titan IV | UA 1207 | 272.32 | 43.98 | 316.3 | [Backlund, 1971] |
| 4 | Titan III | SRM (CSD) | 195.9 | 34.5 | 230.4 | [Backlund, 1971] |
| 5 | GSLV | PS1 (S139) | 138.2 | 23.13 | 161.3 | [ISRO], [LVC] |
| 6 | M-V, stage 1 | M14 | 71.5 | 12.07 | 83.65 | [Wade. "M14"] |
| 7 | HII | SRB | 59.15 | 11.95 | 71.1 | [Jane's, 1995] |
| 8 | J1 | Stage 1 | 59.15 | 11.75 | 70.9 | [Jane's, 1995] |
| 9 | Scout | S1 (Algol III) | 12.817 | 1.939 | 14.756 | [Tack] |
| 10 | J1 | Stage 2 | 10.4 | 2.3 | 12.7 | [Jane's, 1995] |
| 11 | Delta/Atlas | Castor 4A | 10.2 | 1.543 | 11.743 | [Jane's], [LVC] |
| 12 | Ariane 4 | PAP | 9.45 | 3.08 | 12.53 | [CAPCOM, Ariane 4], [LVC] |
| 13 | PSLV | PSOM | 8.92 | 2.91 | 10.93 | [Wade. "PSLV"] |
| 14 | ASLV | ASLV-1 | 8.9 | 2.9 | 11.8 | [Wade, "ASLV"] |
| 15 | Ariane 3 | PAP | 7.35 | 2.41 | 9.76 | [CAPCOM, Ariane 2-3], [LVC] |
| 16 | Scout | S2 (Castor 2A) | 3.768 | 1.064 | 4.832 | [Tack] |
| 17 | H1 | H1-SRB | 3.729 | 0.695 | 4.424 | [Wade, "H1"] |

It is important to note that the dataset includes both strap-on boosters (parallel-staged elements) and serial-staged elements. Additionally, many of the stages are derived from older launch vehicles, reflecting the fact that modern designs predominantly use composite casings.

One notable discrepancy in the dataset concerns the H1 SRB. Although it is reported to be based on the Castor-2 stage used on the Scout vehicle, the inert mass listed for the H1 SRB differs significantly. This could be due to the inherent difference between a strap-on booster (SRB) and a serial staged element (Castor-2), or it could be an indication of potential errors in the data (like a mix-up of stage and motor data).

Figure 2 illustrates the relationship between **inert mass** and **propellant mass** for the compiled dataset. Regression analysis was performed using Excel, both with and without forcing the regression through the origin. The resulting relationships are summarized in Table 2.

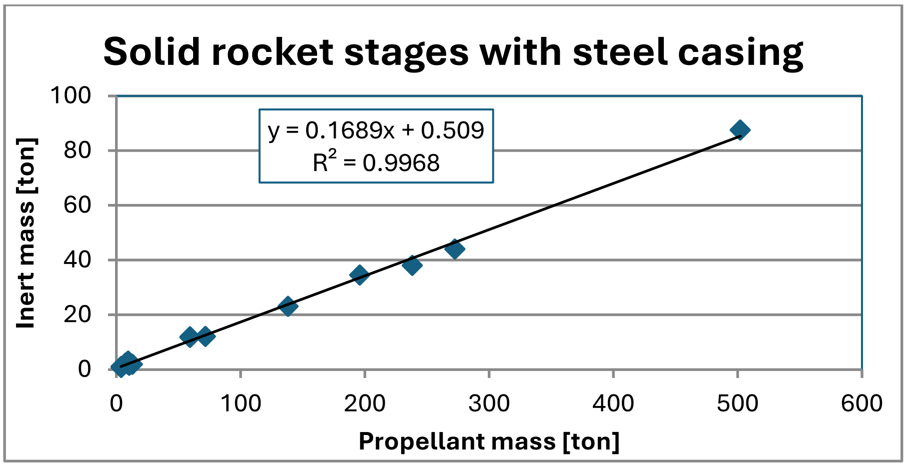


*Figure 2: Inert mass versus propellant mass for a range of solid rocket stages with stainless steel casings.*

*Table 2: Inert mass regression relationships for solid rocket stages with steel casings; All mass values are in tons.*

| ID | Relationship | $R^2$ | MPE (SD) | MAPE | RMSPE | N |
|---|---|---|---|---|---|---|
| S1 | $M_{stage\ inert} = 0.1707\ M_{propellant}$ | 0.9977 | -24% (37%) | 29% | 45% | 17 |
| S2 | $M_{stage\ inert} = 0.1689\ M_{propellant} + 0.5090$ | 0.9968 | -2% (24%) | 17% | 23% | 17 |
| S3 | $M_{stage\ inert} = 0.2851\ M_{propellant}^{0.9030}$ | 0.9925 | -2% (23%) | 16% | 22% | 17 |

Note: The "S" in the ID refers to steel-cased stages. MPE denotes Mean Percentage Error, SD is the Standard Deviation, MAPE is the Mean Absolute Percentage Error, RMSPE is the Root Mean Square Percentage Error relative to the predicted value, and N represents the number of data points.

The dataset spans a wide range of propellant masses, which meants that a high coefficient of determination ($R^2$) does not necessarily imply high predictive accuracy. Large dynamic ranges can yield high $R^2$ values even when the prediction errors remain significant. This is illustrated in the error metrics MPE (SD), MAPE, and RMSPE.

The relatively high RMSPE value for relation **S1** is largely caused by underprediction at low propellant masses (below approximately 10 t), where the inert mass is severely underestimated. This is shown in Figure 3, which presents the percentage error distribution for relation S1. The data points are ordered from high to low propellant mass (left to right).

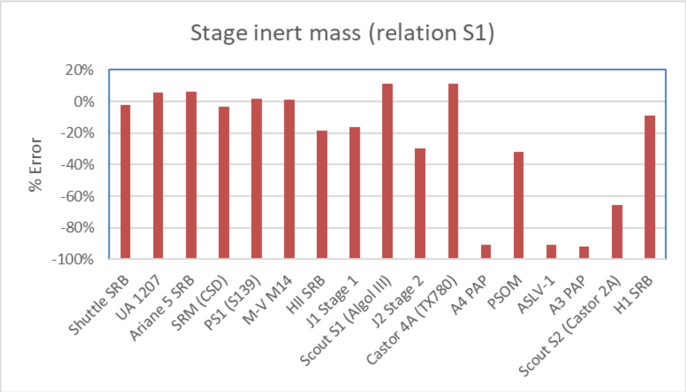


*Figure 3: Overview of percentage error (PE') results for the relation S1 (linear). Negative values indicate an underestimation and positive values an overestimation.*

From this figure, we can see that the error tends to be larger for **smaller stages**. When testing for normality, we found the following distribution of the 17 data points:

- **14 data points** within the ±1 RMSPE range (-45% to +45%)
- **1 data point** within the ±1 to ±2 RMSPE range
- **3 data points** outside the ±2 RMSPE range

This suggests that the error distribution deviates significantly from a normal distribution, with substantial underestimation occurring at lower propellant masses.

When checking some of the 4 data points that show large underestimation more closely, we found that a notable discrepancy exists between the H-1 SRB and the Scout Castor-2A stage, even though the H-1 SRB is reported to be based on the Castor-2 stage. No source could be found that provides both stage-level and motor-level mass data for the H-1 system, which introduces some uncertainty in this comparison. Another interesting observation concerns the Ariane-4 PAP and Castor-4A stages, which are both strap-on boosters. These booster stages have nearly identical propellant masses, yet the stage inert mass of the PAP is approximately twice as high. For the first stage of the J-1 rocket the propellant mass is similar, while the inert mass lies roughly between the values for the PAP and Castor-4A stages. This difference may warrant further investigation.

Relation S2 was introduced to investigate whether adding a constant mass term would improve the regression. The inclusion of a constant term reflects the fact that non-scaling elements such as skirts, separation systems, and avionics contribute a fixed mass that does not depend on propellant mass. This model better represents smaller stages where such fixed components comprise a larger fraction of the total inert mass.

The results from relation S2 show a more normal distribution of errors:

- 11 data points in the ±1 RMSPE range (-23% to +23%)
- 5 data points in the ±1 to ±2 RMSPE range
- 1 data point outside ±2 RMSPE

However, underestimation persists for smaller propellant masses.

To explore whether a power-law relationship could better capture the scaling behaviour between propellant mass and inert mass, a power-law regression model was also tested. This power-law regression also provides a reasonable fit, with similar error metrics to relation S2. The $R^2$ slightly decreases, but the structural variability (SD ≈ 23%, RMSPE ≈ 22%) remains largely unchanged.

The power-law relationship provides a slightly better fit in some cases, but structural variability persists due to design differences (e.g., strap-on boosters vs. serial-staged elements, varying materials and subsystems).

It is noteworthy that the power-law relationship shows a slight improvement, be it that the structural variability remains substantial (SD ≈ 23%, RMSPE ≈ 22%), almost same as for **S2**. In practice, this means that the prediction errors vary considerably in magnitude and direction across the dataset, even though the average error is close to zero.

All in all, the low MPE suggests that the model is approximately unbiased and that the errors are roughly centred around zero. Such behaviour is consistent with a symmetric error distribution, for example a normal distribution.

Next figure shows the percentage error that results for the relation S3. The data points are ordered from high to low propellant mass (left to right).

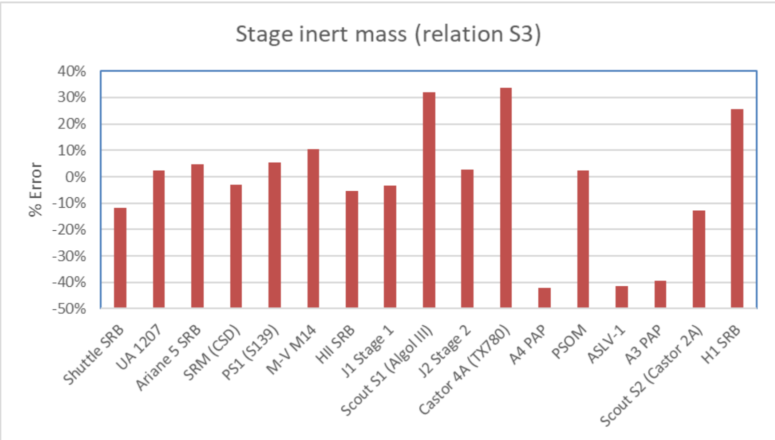


*Figure 4: Percentage error (PE) resulting from applying the relation S3 (power-law). Negative values indicate an underestimation and positive values an overestimation.*

As in the linear case, the largest errors occur for stages with relatively small propellant masses. The normality check also shows reasonable agreement with 11 data points falling within the ±1 RMSPE range (−22% to +22%), 6 between ±1 and ±2 RSMPE, and none outside the ±2 RMSPE range. At the low mass range, the underestimations are still relatively large, which further strengthens our suspicions of possible errors in the data used.

Table 2 also illustrates that SD (Standard Deviation) and RMSPE (Root Mean Square Percentage Error) are related metrics, both serving as measures of variability. However, each provides unique insights into the performance of the regression model. This is evident in the shift from relation S1 to S2 and S3, where both SD and RMSPE are reduced. The reduction in RMSPE is more pronounced, which indicates that RMSPE is more sensitive to underestimations or discrepancies in the regression model, particularly at lower propellant masses, where the model may struggle due to data issues or an incorrect functional form.

**Conclusions**

- The models exhibit varying degrees of structural variability, which is expected given the wide differences in design philosophies, safety margins, and mission requirements between stages.
- The low MPE values suggest that, on average, the models are unbiased, with errors centered around zero.
- Further investigation into low-propellant-mass stages is warranted due to large underestimations at the lower end of the mass spectrum.

Overall, the study highlights the **complexity** of modeling **inert mass** for steel-cased solid rocket stages, and future work will aim to refine these relationships, especially for **smaller stages**.

## 6. Stages with composite casings

Data were compiled for 17 solid rocket stages (SRMs) equipped with composite casings. This dataset includes more recent stages, such as those from the European Vega C rocket, the Japanese Epsilon rocket, and the USA Athena 1 rocket. The dataset is presented in Table 3.

*Table 3: Typical mass characteristics of solid rocket stages with composite casings*

| ID | Vehicle | Stage | Propellant mass [t] | Dry mass [t] | Total mass [t] | References |
|---|---|---|---|---|---|---|
| 1 | Vega | P80 | 87.71 | 8.533 | 96.243 | [Arianespace] |
| 2 | Vega | Zefiro 23 | 23.814 | 2.486 | 26.3 | [Arianespace] |
| 3 | Vega | Zefiro 9 | 10.567 | 1.433 | 12.00 | [Arianespace] |
| 4 | Vega C | Stage 1 (P120C) | 141.634 | 13.393 | 155.027 | [Fiat Avio] |
| 5 | Vega C | Stage 2 (Z40) | 36.239 | 4.238 | 40.477 | [Fiat Avio] |
| 6 | Vega C | Stage 3 (Z9) | 10.567 | 1.433 | 12.00 | [Fiat Avio] |
| 7 | Athena 1 | Stage 1 | 48.809 | 4.36 | 53.169 | [Wade, "Athena"], [LVC] |
| 8 | Athena 1 | Stage 2 | 9.779 | 0.865 | 10.644 | [Wade, "Athena"], [LVC] |
| 9 | Pegasus | Stage 1 | 12.152 | 2.154 | 14.288 | [Pegasus], [Stargazer] |
| 10 | Pegasus | Stage 2 | 3.025 | 0.375 | 3.4 | [Pegasus] |
| 11 | Epsilon | Stage 1 (SRB-A3) | 66.3 | 8.7 | 75.0 | [Epsilon] |
| 12 | Epsilon | Stage 2 (M35) | 15.0 | 2.0 | 17.0 | [Epsilon] |
| 13 | HIIA | SRB-A | 65.04 | 10.5 | 75.54 | [JAXA], [LVC] |
| 14 | Delta 7925 | GEM-40 | 11.765 | 1.467 | 13.232 | [Jane's], [LVC] |
| 15 | M-V | M34 | 10.1 | 0.9 | 11.00 | [Jane's] |
| 16 | PSLV | Stage 3 (PS3) | 7.26 | 0.718 | 7.978 | [Jane's], [LVC] |
| 17 | Titan IVB | SRMU (ATK/Aliant) | 315.44 | 36.565 | 352.005 | [Boury] |

Not all of the collected data represent complete stage inert masses. For instance, the mass of Pegasus Stage 1, as reported in the Pegasus User's Guide, appears to refer only to the motor mass. In Stargazer (2006), the wing mass is reported as approximately 286 kg, but separate data for the aft skirt assembly, aerodynamic control fins, wing structure, or front skirt assembly are unavailable.

Similar uncertainties exist for the Athena 1 stages. The Lower Interstage connects the Castor 120 first stage to the second stage, while the Upper Interstage links the Orbus 21D second stage to the Orbital Adjustment Module (OAM). As such, it's uncertain whether all reported inert masses reflect complete stages or just the SRMs.

An additional observation concerns the Titan IVB SRMU, the only stage in the database with a segmented casing. Segmented casings typically require heavy joint structures to connect individual segments, which is expected to increase the inert mass relative to monolithic motor casings.

The data for **composite-cased SRMs** were plotted in Figure 5, with a **linear regression curve** (relation **C1**) superimposed.

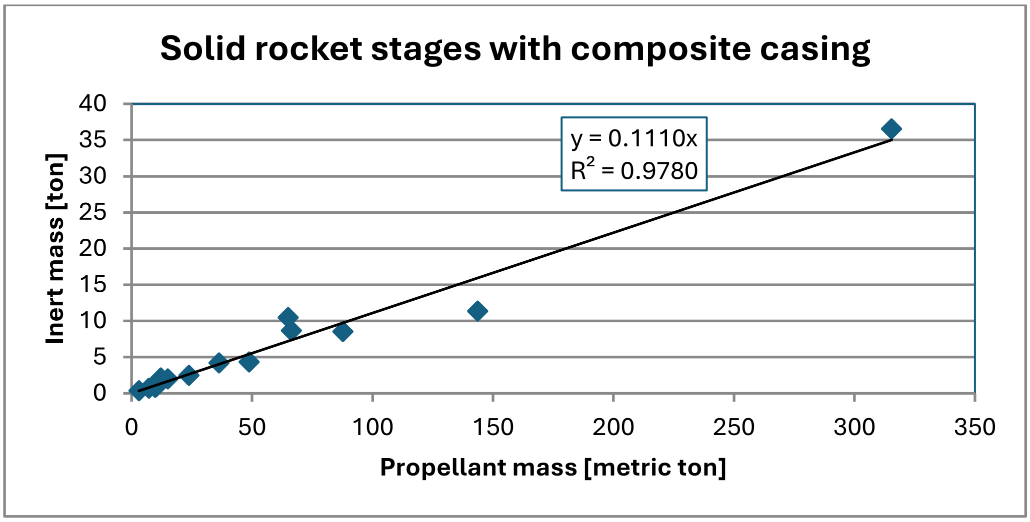


*Figure 5: Inert mass versus propellant mass for a range of solid rocket stages with composite casings.*

*Table 4: Inert mass regression relationships for solid rocket stages with composite casings; All mass values are expressed in tons.*

| ID | Relationship | $R^2$ | MPE (SD) | MAPE | RMSPE | N |
|---|---|---|---|---|---|---|
| C1 | $M_{stage\ inert} = 0.1110\ M_{propellant}$ | 0.9780 | -6% (24%) | 20% | 25% | 17 |
| C2 | $M_{stage\ inert} = 0.1275\ M_{propellant}^{0.9678}$ | 0.9661 | -2% (23%) | 18% | 24% | 17 |

Table 4 summarizes the regression relationships for composite-cased stages. The ID letter "C" denotes composite casing. Relation C1 represents a linear fit forced through the origin, while relation C2 is a power-law regression.

In Figure 6, the percentage errors resulting from applying relation C1 are shown. The data points are ordered from high to low propellant mass (left to right).

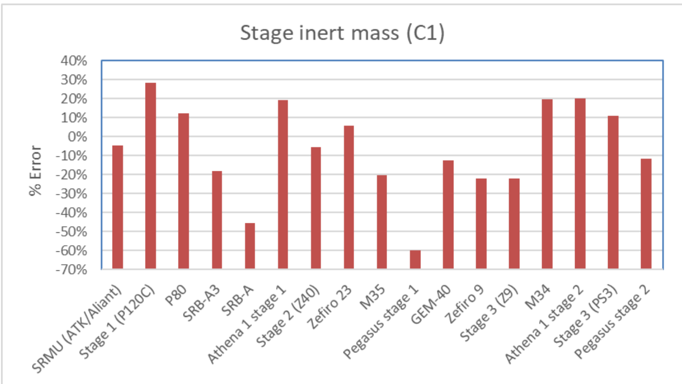


*Figure 6: Percentage error (PE') resulting from applying the relation C1. Negative values indicate an underestimation and positive values an overestimation.*

For relation C1, the mean percentage error (MPE) is -6%, indicating a slight overestimation. The calculated standard deviation (SD) is approximately 24%, while the RMSPE is 25%. When assessing the normality of the error distribution, we find the following:

- 12 data points fall within the ±1 RMSPE range (−25% to +25%),
- 4 points fall between ±1 and ±2 RMSPE,
- 1 point falls outside the ±2 RMSPE range.

This distribution suggests that the error distribution is approximately normal.

A power-law regression model was also tested for the composite-cased stages. Although the coefficient of determination ($R^2$) decreased slightly (from 0.9780 to 0.9661), the resulting fit showed a small decrease in mean percentage error (MPE) and standard deviation (SD). Overall, the power-law model (C2) offers a slight advantage over the simple linear regression (C1) in terms of model accuracy.

# 7. Comparison

In this section, we first compare the results obtained in this study with previously published results. We then examine the influence of casing material on stage inert mass. Finally, we discuss the implications of these findings for design margins and upper-bound estimation using statistical metrics.

## 7.1 Comparison with historical results

Table 5 summarizes the regression relationships derived in this study alongside earlier results reported by Zandbergen (2019). Overall, the results are broadly consistent, as many of the same stages were used in both datasets. However, some stages were replaced or added when data for the original stages could not be retrieved.

Despite the general agreement, several small but systematic differences are evident between the historical and updated regression relationships. For steel-cased stages, the slope of the regression decreases slightly from 0.1726 in the historical relation to approximately 0.1689–0.1707 in the updated linear fits (S1–S2). This indicates a modest reduction in predicted inert mass for larger stages in the updated dataset. For composite-cased stages, the regression coefficient deviates more from the historical value, with the proportional relation C1 yielding a coefficient of 0.1110 compared to 0.1173 in the previous data. These differences are relatively small and are likely due to minor changes in the dataset rather than any fundamental shift in scaling behavior. Nonetheless, they highlight the sensitivity of parametric relationships

*Table 5: Summary of inert mass estimation relationships; All mass values expressed in tons.*

| ID | Regression relationship | Propellant mass range | $R^2$ |
|---|---|---|---|
| **Steel (historical)** | $M_{stage\ inert} = 0.1726\ M_{propellant} + 1.173$ | 9.5-505 ton | 0.9907 |
| **Composite (historical)** | $M_{stage\ inert} = 0.1173\ M_{propellant}$ | 3-320 ton | 0.9833 |
| **S1** | $M_{stage\ inert} = 0.1707\ M_{propellant}$ | 3.5-505 ton | 0.9977 |
| **S2** | $M_{stage\ inert} = 0.1689\ M_{propellant} + 0.5090$ | 3.5-505 ton | 0.9968 |
| **S3** | $M_{stage\ inert} = 0.2851\ M_{propellant}^{0.9030}$ | 3.5-505 ton | 0.9925 |
| **C1** | $M_{stage\ inert} = 0.1110\ M_{propellant}$ | 3.0-320 ton | 0.9780 |
| **C2** | $M_{stage\ inert} = 0.1275\ M_{propellant}^{0.9678}$ | 3.0-320 ton | 0.9661 |

## 7.2 Key Observations from the Comparison

1. **Casing Material and Historical Trends:**

- The reduction in stage inert mass for composite casings is clear, with composite-cased stages showing approximately 35% lower inert mass compared to steel-cased stages (0.1110 vs. 0.1707). However, it is important to note that some of this difference is not solely due to the material itself. Steel-cased stages in the dataset are mostly dominated by booster designs, whereas composite-cased stages tend to include more serial-staged elements. Additionally, modern rockets benefit from lighter nose sections, interstages, electromechanical TVC systems (rather than hydraulics), and reduced navigation and avionics mass. Therefore, even contemporary steel-cased stages could achieve similar mass reductions to composite stages, though inert mass would still be higher. Furthermore, monolithic composite casings inherently avoid the mass associated with segment joints found in segmented SRBs.

2. **Scaling Behavior:**

    - The power-law exponent for steel-cased stages (~0.90) is slightly lower than for composite-cased stages (~0.97), indicating sub-linear scaling for steel stages. This reflects the greater structural overhead associated with steel casings, which limits scaling efficiency at higher propellant masses. Composite casings, on the other hand, exhibit near-linear scaling, consistent with their lower structural overhead and higher mass efficiency.

3. **Variability in Stage Mass:**

    - As summarized in Table 6, the relatively large spread in the data is evident from the SD and RMSPE. Variability arises from a variety of factors, including the type of stage (parallel or serial staged), design philosophy, safety factors, material properties, combustion pressure, thrust profile, burn time, and other mission-specific parameters. Even stages with similar propellant masses can differ substantially in inert mass due to these factors.

4. **Implications for Design Margins:**

    - Relation S1, which shows the highest RMSPE, suggests that for smaller stages (propellant mass < 10 t), a higher design margin may be necessary to mitigate risk. For stages with propellant mass above 10 t, the effective RMSPE is closer to 18%, indicating that the same margin may not be needed for larger stages. Appendix A provides a detailed method for estimating an upper bound on stage inert mass using MPE and RMSPE.

5. **SLS Steel-Cased SRBs:**

    - The U.S. Space Launch System (SLS) booster still employs steel-cased SRBs, likely due to cost considerations and the extensive operational experience with steel, especially for reusable hardware. Currently, no operational, monolithic, fibre-based, reusable SRBs of a size comparable to the SLS boosters exist.

6. **Overall Conclusion:**

    - Despite some variability, the regression relationships derived in this study are broadly consistent with historical results while providing a more comprehensive dataset and error quantification. The combination of material-dependent scaling and structural variability underscores the importance of considering both systematic trends and stage-specific design features when estimating inert mass.

*Table 6: Statistical error metrics for regression relationships; percentage values rounded to nearest integer for clarity*

| ID | $R^2$ | MPE (SD) | MAPE | RMSPE | N |
|---|---|---|---|---|---|
| **Steel (historical)** | 0.9907 | - | | 20% | 11 |
| **Composite (historical)** | 0.9833 | - | | 24% | 11 |
| **S1** | 0.9977 | -24% (37%) | 29% | 45% | 17 |
| **S2** | 0.9968 | -2% (24%) | 17% | 23% | 17 |
| **S3** | 0.9925 | -2% (23%) | 16% | 22% | 17 |
| **C1** | 0.9780 | -6% (24%) | 20% | 25% | 17 |
| **C2** | 0.9661 | -2% (23%) | 18% | 24% | 17 |

## 7.3 Synthesis of Results

The analysis of steel- and composite-cased solid rocket stages reveals the following key findings:

- **Mass Reduction:** Composite casings are associated with approximately 35% lower inert mass coefficients in the regression relationships. However, this difference may also reflect architectural and subsystem design differences between the steel and composite datasets.
- **Scaling Behavior:** Composite stages exhibit near-linear scaling with propellant mass, while steel stages scale sub-linearly due to higher structural overhead.
- **Bias:** All relations, except S1, show low bias. For relation C1, MPE is small, but the MPE is still relatively high. Relation C2 shows a further improvement in MPE, indicating a reduction in the upper bound relative to the prediction.
- **Structural Variability:** Significant variability in inert mass exists, arising from design choices, safety factors, subsystem configurations, and SRB segmentation. The nearly identical values for RMSPE and SD suggest that the variability is primarily driven by unmodeled parameters.
- **Design Guidance:** Appendix A demonstrates how to use MPE and RMSPE to estimate an upper bound for stage inert mass, providing a probabilistic basis for margin calculations.

Overall, the results offer quantitative guidance for estimating stage inert mass and provide valuable insight into how casing material, stage design, and historical development practices influence scaling behavior.

# 8. Conclusions and recommendations

In this work, parametric relationships for estimating stage inert mass have been derived based on historical data for steel- and composite-cased solid rocket stages. The regression relationships obtained here were compared with earlier reported results and were found to be broadly consistent. This comparison confirms the validity of the dataset and the methods, while providing a more comprehensive quantification of errors.

For steel-cased stages, the power-law regression model provided a better fit than a purely linear model, particularly capturing sub-linear scaling behaviour. For composite-cased stages, the improvement of a power-law fit over the linear model was marginal. The scaling exponent for both materials was found to be below 1, indicating that inert mass increases sub-linearly with propellant mass. This reflects the fact that certain structural elements and subsystems do not scale proportionally with propellant mass; for instance, increasing propellant mass by lengthening a motor case does not necessarily require proportional increases in the front skirt, nose section, or navigation equipment.

A substantial reduction in inert mass is observed for monolithic composite-cased stages, approximately 35% lower than steel-cased stages. However, part of this reduction is attributable not only to the material itself but also to historical and design factors: modern rockets benefit from lighter nose sections, interstages, electromechanical thrust vector control systems, and reduced navigation and avionics mass. Segmentation in steel-cased boosters further adds mass, whereas composite stages are typically monolithic. Thus, even steel-cased stages built today could achieve similar efficiencies if modern design practices are applied.

The statistical analysis demonstrates that even for negligible bias, significant structural variability across stages, as reflected in SD, and RMSPE'. This variability arises from differences in, for instance, design philosophy, safety factors, subsystem configuration, and mission-specific requirements. Even stages with similar propellant masses may have markedly different inert masses. Such variability should not be considered inherent or unavoidable, but rather as an opportunity for design optimization.

Historical data for some stages remain incomplete or uncertain, particularly regarding whether reported masses correspond to entire stages or just the motor. This limitation highlights the need for more detailed data collection and possibly more granular modelling of individual mass components for future studies.

**Practical Design Guidance:**

The regression relationships derived in this study can serve as a rapid tool for preliminary stage design. To account for structural variability and ensure safe margins, upper bounds should be estimated using the statistical methods described in **Appendix A**. For small or low-mass stages, where relative errors are largest, designers should apply higher margins or consider dedicated regressions. Material choice, stage segmentation, and subsystem design significantly affect inert mass; incorporating modern design practices—including monolithic casings, optimized TVC systems, and reduced avionics mass—can yield significant efficiency gains, even for steel-cased stages. By combining regression-based estimates with conservative upper-bound calculations, engineers can balance mass efficiency, safety, and performance early in the design process.

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# Appendix A: Upper bound estimation of stage inert mass

When designing launch vehicles, it is often necessary to establish an **upper bound** for stage inert mass to ensure adequate design margins. This appendix outlines how statistical error metrics— RMSPE' (Root Mean Square Percentage Error relative to the predicted value), and MPE' (Mean Percentage Error relative to the predicted value)—can be used to estimate an upper bound for inert mass, assuming a multiplicative error model and a normal error distribution.

**A1 Methodology**

Assume that the estimated inert mass for a stage is $\mathrm{M_{est}} = 1000$ kg. The goal is to calculate an upper bound $\mathrm{M_{UB}}$ with roughly **84% confidence**, meaning that the probability of the actual inert mass exceeding $\mathrm{M_{UB}}$ is less than 16%. This corresponds to +1 standard deviation in a normal distribution[1].

The metrics are derived from the statistical analysis in **Table 6**. For example, for C2 (composite, power-law), we have the following metrics based on 17[2] data points.

- RMSPE' = 23%
- MPE' = +2% (nearly unbiased[3])

**A2 Upper bound calculations**

1) Using RMSPE only (assuming MPE' is zero or close to zero)

Since

- RMSPE' = 23%

We compute:

- Upper multiplier $= 1 + \mathrm{RMSPE'} = 1 + 0.23 = 1.23$

So:

- $\mathrm{M_{UB}} = 1000 \times 1.23 = 1230\ kg$

2) With MPE'.

Since:

- MPE' = +2% (nearly unbiased)
- RMSPE' = 23%

We compute:

$$\text{Upper} = -2\% + 23\% = 21\%$$

Upper Multiplier:

$$\text{Upper multiplier} = 1 + \text{Upper} = 1 + 0.21 = 1.21$$

[1] The assumption of normality in the distribution of errors can be assessed by checking how well the data points fit within the standard deviation ranges. Specifically, for a normal distribution, approximately 68% of data points should lie within 1 standard deviation (RMSPE') of the mean, and about 95% within 2 RSMPE'. If MPE' is small, this can be verified using the observed data distribution, as outlined in Section 4 of the main text.

[2] With N = 17, the sample size is relatively small for reliably checking normality. For more robust verification, it is generally better to have a sample size of at least **N ≥ 30**, as larger samples provide more reliable tests for normality and reduce the impact of random variability.

[3] For a normal distribution, it is important that bias is close to zero. If bias is not close enough, consider another form for the regression curve.

So:

- $\mathrm{M_{UB}} = 1000 \times 1.21 = 1210\ kg$

**A3 Key Insights**

- **Uncertainty metrics**: For upper bound estimation use uncertainty metrics relative to the predicted value (not the observed value).
- **Checking the normality of the residuals** is a key step before relying on error-based metrics. If the residuals are non-normal, traditional methods as used here may not provide reliable estimates of uncertainty or upper bounds. In such cases, alternative methods should be considered to handle the non-normality and to obtain more accurate estimates.
- The results for the upper bound are based on several assumptions, primarily that the errors follow a normal distribution. Given the small sample size (N = 17 for steel and N = 17 for composites), these estimates are approximate and should be interpreted with caution. In practice, normality might not always hold, especially with larger or different datasets. **Future analyses with more data would help confirm or refine these conclusions**.