

Reliability Based Design of Dredge Sludge Depot for Mechanism Static Liquefaction

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Abstract. Along the coastline and in particular in the deltaic estuaries of the Netherlands, the occurring of numerous flow slides have been recorded. Analysis of these historical instabilities has resulted in empirical guidelines for verification of occurring of flow slides.

The request to tender for the Engineering & Construct project Dredge Sludge project Hollandsch Diep in the Netherlands prompted the development of realistic design tools which enables the prediction of the initiation of static liquefaction in a reliability based framework. The realisation of a disposal area with a volume of approximately 10 million m³ in a restricted circumference required the excavation up to 38m depth and construction of a ring dike, while designing the required slope with sufficient reliability.

Based on international literature, in-house experience and laboratory and field tests a simplified static liquefaction model based on the critical state framework has been developed. For the reliability assessment a robust method has been adopted using stochastic analysis of the dominant parameters, while using a regional database. Several planned and unplanned failures during the realisation of the dredging activities were successfully analysed using the simplified stochastic approach of the modified state parameter model. The article gives insight into the determination and application of the state parameter approach in the reliability based framework and discusses the practical and academic needs for further developments and research.

Keywords. Static liquefaction, reliability based approach, case history

1. Historical Evidence Flow Slides

In the South-western deltaic province Zeeland in the Netherlands a vast amount of flow slides have occurred (see Figure 1) initiated by the morphological processes occurring in this deltaic estuary.

After the 1953 storm surge disaster, the necessity of reinforcement of dikes was evident. The required stability of the foreshore of the dikes initiated the analysis of the vast amount of flow slides by Wilderom (1979). It was believed that these instabilities were initiated by steepening of slopes caused by erosion of the main gully or slope and siltation at the crest of the slope.

The type of instability or combination thereof depends on the nature of the subsoil and its density. The physical processes are dominated by either contractant or dilatant behaviour where both the time aspect and resulting mobilised properties are depending on the hydraulic conductivity. A total of 1129 instabilities have

been reported of which 200 flow slides and instabilities have been analysed in sufficient detail.

These instabilities have occurred in loose young Holocene layers. The slope after occurrence of a flow slide instability has a large scatter with initial direct values between 1:6 – 1:15 and average values for the final slope of 1:15 – 1:20. The average slope of instabilities initiated in older formations is 1:3.5. This type of instability can also occur in areas with foreshores which are susceptible for flow slides. It should be noted that liquefaction flow slides may either initiate or be initiated by retrogressive shear failures, sometimes called breaching.

An overlooked shortcoming of the dataset concerns its poor statistical basis with respect to geometrical properties as a prerequisite for liquefaction flow slides. Silvis (1985) reported a detailed statistical analysis of the dataset and concluded amongst other things that (1) there is no statistical evidence that steepening caused by erosion preceded liquefaction flow slides and

that (2) slopes that did not fail for at least five years after surveying had a similar distribution as slopes that did fail. In other words, the slopes before failure as reported by Wilderom represent slopes occurring in Zeeland in general, rather than being related to the occurrence of liquefaction flow slides.

The Eastern Scheldt, part of the deltaic region, had various retrogressing coastal zones which have been stabilized by coastal protection with $\sim 0.6\text{m}$ of crushed rock on 10m wide mattresses. Another mitigation applied to limit the extent of the erosion process is compaction of the loose sand deposits.

The analysis of instabilities resulted in the following conditions: average steepest slope prior to instability was 1:3; the height of the steepest slope varied between 1 - 9m and is therefore not a clear parameter. The time between the observations and the actual instability varied between 1 day up to over a year, with no clear trend as well. It was furthermore striking to see that most instabilities occurred after tidal-deviations of decimetres to a meter resulting in a higher loading gradient.

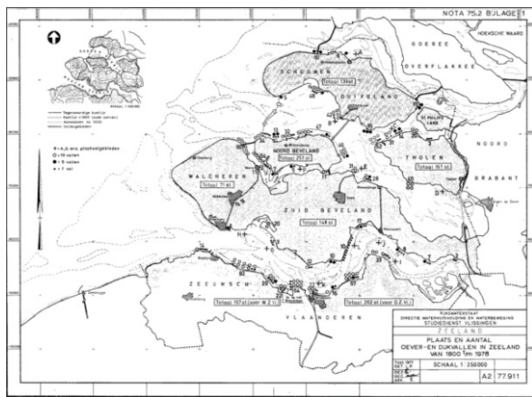


Figure 1. Location and number of flow slides in the province of Zeeland from 1800 – 1978 (Annex I – Wilderom, 1979)

2. Liquefaction Behaviour

2.1. Primary Causes for Instability

As described by Koppejan (1948) “A coastal flow slide in Zeeland means an unexpected downward sliding of a large portion of the foreshore below the dike, sometimes causing disappearance of part of the dike as well.” The

result is described instead of the physical processes causing the instability. Liquefaction is a more appropriate name for the physical liquid behaviour of the sand/ water mixture.

Koppejan (1948) described three primary causes for the occurrence of liquefaction: tidal streams causing a steepening of the slopes, thus moving away from the safe equilibrium situation (I – load gradient); seepage pressures during falling water which, for loose sand with a density smaller than the critical density, diminishes the apparent friction angle of the slope (II - packing) and finally local loss of equilibrium which may develop into mass liquefaction (III - trigger).

2.2. Characterisation of Cohesionless Soils

The importance of proper characterisation of cohesionless soils above valuing experience, which has proven to be the least reliable for design of dams and dikes, has already been advocated by Casagrande (1936).

Drained laboratory tests performed at Harvard University (Casagrande, 1936) showed expansive - and contractive volume change for dense and loose sand respectively while shearing. Continuation of shearing finally results in a constant volume while deforming at constant shearing stress. The concept of the critical voids ratio line (Roscoe et al., 1958) is related to the state of soil, which may be differently defined. One concept proposed by Casagrande (1936) is concerned with changes of volume in drained tests, while Taylor (1948) concerns with changes of effective stress, and consequently strength in undrained tests. The critical state is defined as the soil state (p' , e , q) at which the soil arrives when the shear distortions are continued until the soil flows like a frictional fluid (Schofield & Wroth, 1968).

The factors that control the occurrence of unstable behaviour have been investigated by Lade (1989). Materials exhibiting non-associated flow may become unstable when exposed to certain stress paths inside the Mohr-Coulomb failure surface. However, it is emphasised by Lade (1989) that instability is not synonymous with failure, although it may lead to catastrophic events. In Figure 2 a schematic presentation of the location of the instability line for loose sand is presented in (p' , q) plane, with p' being the

isotropic effective stress and q the deviatoric stress while applying soil mechanics' sign convention (compression positive). The top of the yield surface of the non-associatively flowing loose sand lies within the failure surface, while after large straining and development of softening, the failure surface is reached. Following the work of Roscoe et al. (1958), the softening frictional fluid flow will eventually reach the critical state line (CSL) as failure surface, but the state (p' , e , q) at which the CSL is reached is significantly different from its initial unstable state. The line from the origin in p' , q space to the top of the yield surface is defined as the instability line. The slope of this instability line, M_{IL} is not unique and depends for a certain material on the initial void ratio at the instability point, e .

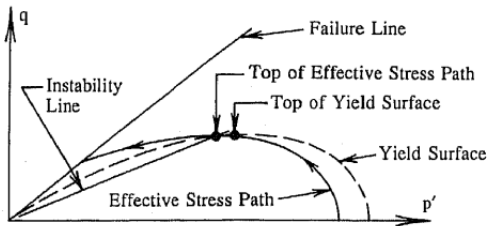


Figure 2. Location of instability line for loose sand (Fig. 7 – Lade, 1992)

Chu et al. (2003) determined the relationship between the slope of the instability line, M_{IL} and modified state parameter $\bar{\psi} = (e - e_{cr})$ with drained and undrained triaxial tests for Singapore sand (see Figure 3), where e is the void ratio at the instability point and e_{cr} is the void ratio at the critical state at the same mean effective stress.

According to Jefferies & Been (2006), the transition in behaviour lies around a modified state parameter $\bar{\psi} = -0.07$, due to the fact that in triaxial compression even samples starting at the critical state show slight softening.

Wanatowski & Chu (2007) analysed the static liquefaction of Singapore sand in plane strain and compared the results with those from triaxial tests. The relationship between M_{IL} and $\bar{\psi}$ was influenced by the intermediate principal stress, thus resulting in a curve for triaxial and one for plane strain conditions as illustrated in Figure 4.

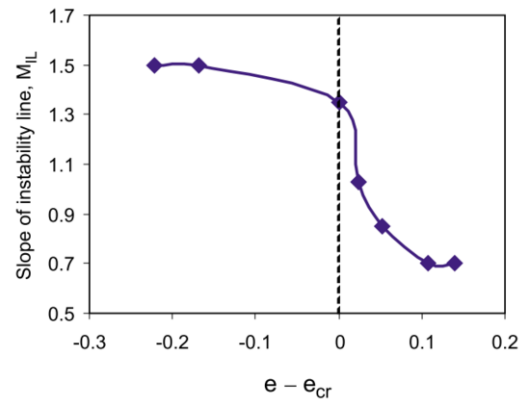


Figure 3. Relationship between the slope of the instability line, M_{IL} and difference between the initial void ratio and void ratio on the failure line, $e - e_{cr}$ (Fig. 5 – Chu et al., 2003)

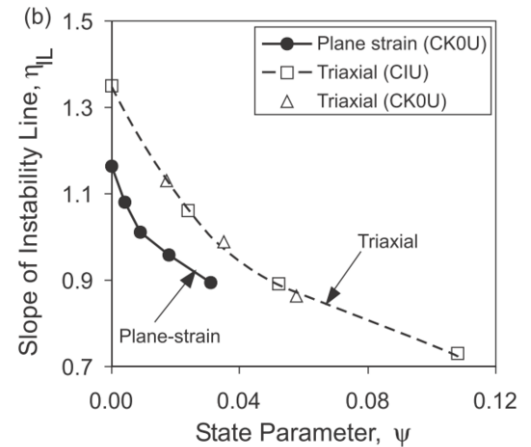


Figure 4. Slope of instability line M_{IL} versus state parameter (Fig. 13b – Wanatowski & Chu, 2007)

When the slope of the instability line is normalized according to Jefferies & Shuttle (2002) with respect to the slope of the critical state line M for triaxial compression and plane strain testing, an unique relationship between M_{IL}/M and $\bar{\psi}$ results. Jefferies and Shuttle (2002) presented the derivations and results for pre-peak strength conditions of this idealized normalization framework for Cambridge Type models. Wanatowski & Chu (2007) showed that, with this concept for Singapore sand with positive modified state parameters (contractive behaviour), the findings on static liquefaction behaviour of sand established in triaxial tests can be extended to plane-strain conditions.

It should be noted that there are specific differences between both tests types which are not captured with this generic approach. In plane strain tests the peak strength is larger, which is in agreement with Rowe (1969), while in triaxial tests the generated excess pore water pressure is larger. Furthermore the intermediate principal stress influences the failure surface. In general it may be concluded that extending undrained triaxial tests to plane strain conditions, following the framework of Jefferies & Shuttle (2002), provides a cautious estimate for predicting the liquefaction potential of soils.

3. Simplified Reliability Based Design

3.1. Modelling

Following the philosophy of Baran & Sweezy (Muir Wood, 2004), modelling is abstracting from non-essentials to obtain an unobstructed view of the important. It is a good model if it provides key to understand reality.

Ideally the stress-strain behaviour from pre-peak to large straining is modelled. Incorporation of the occurrence of static liquefaction in a reliability based design reduces the associated risks compared to for example application of empirical knowledge as summarized by Wilderom (1979). For this reason, emphasis was put on modelling the onset of instability represented by the top of the instability line.

The commercial Finite Element code Plaxis (Brinkgreve et al., 2008) was used as a modelling tool with Hardening soil as the material model for the sand layers. The friction and dilatancy angle in this model (Schanz & Vermeer, 1996) are material constants, while the corresponding critical state friction angle is calculated. The stress-strain behaviour until the top of the instability line is validated, where the maximum stress ratio results in an apparent critical state friction angle. The post-peak behaviour, with softening until the soil flows like a frictional fluid and reaches the critical state line, is not part of the numerical code (Brinkgreve, 1994). However, the generation of excess pore water pressures due to either shear stresses and/ or numerical plasticity due to post peak straining mimics the actual physical behaviour.

The modelling-phases in the simplified modified state parameter (SMSP) approach are: * Determination of CPTU-based estimation of state parameter (Been et al., 1986); * Explicit approximate correction for undrained response for anisotropic soil conditions (after Imam et al., 2005); * Implementation of IL concept in Plaxis (HS) in discrete layers; * Stress generation using incremental weight generation; * Staged construction/ excavation with updated mesh and staged adaptation of soil parameters; * Failure analysis by both incremental strength reduction and weight increase; * Approximate simulation of excess pore pressure generation in undrained shear by applying Skempton's β parameter.

3.2. Stochastic Analysis

The consequences of static liquefaction exceed those of conventional instabilities. The uncertainties associated with this particular mechanism related to all components of the geotechnical design (Orr & Farrell, 1999) require a balanced approach to formulate a reliability boundary.

The variability of all components affecting the geotechnical design influence the required factor of safety for a pre-set reliability. When the factor of safety is defined in terms of characteristic values it is pointed out by Fenton & Griffiths (2008) that the mean factor of safety does not adequately reflect the actual design safety. Instead of calibration of load and resistance factors, the method of a stochastic analysis is adopted (Fenton & Griffiths, 2008).

The significance of this method is for instance illustrated by Hicks & Onisiphorou (2005), who investigated the failure of the Nerlerk berm by linking random field theory for modelling spatial variability with finite elements for computing geostructural response. The results demonstrate that it may be possible for a predominantly dilative fill to liquefy, due to the presence of semi-continuous loose zones arising from deposition-induced anisotropy.

During the design & construct phase of the project only academic liquefaction and stochastic models were available. The simplified stochastic, 1st order second moment, approach by Duncan (2000, 2001) has been adopted in combination with the FEM Plaxis to assess the main

stochastic variables influencing static liquefaction. The following stochastic variables are assumed uncorrelated in the analyses: density of the loose Holocene and denser Pleistocene sand-layers; tidal fluctuation; slope geometry; relation modified state parameter versus slope of instability line; slope critical state line; sensitivity FEM prediction. The standard deviation of each parameter is determined with the 6σ rule.

A total number of $2n + 1$ analyses are required for the aforementioned modelling phases, subsequently allowing for the determination of the difference in safety factor opposed to the mean values. The safety factor is assumed to behave according to a lognormal distribution. The required factor of safety corresponding to a 5% confidence level for the occurrence of static liquefaction depends on the construction method for a given design slope but is 1.3 on average acting on the slope of the instability line. The analysis performed is a global liquefaction analysis incorporating the site specific variability (Fenton & Vanmarcke, 1998).

4. Dredge Sludge Depot Hollandsch Diep

4.1. General

The dredged sludge depot Hollandsch Diep is an E&C contract and involves the dredging of a disposal pit with a capacity of around 10 million m^3 in the river Hollandsch Diep. The river Hollandsch Diep is typically 4-5m deep and the depot after construction has an approximate depth, width and length of 38m, 500m and 1200m respectively.

The reference design of the Public Works Department assumed slopes of 1(v):4(h) feasible, based on the empirical experience by Wilderom (1973) and awarded a bonus for making the slopes even steeper, thus even increasing the volume of the depot given a fixed outer boundary.

4.2. Soil Characterization

The subsoil conditions in the project area are very heterogeneous and typically consist of very loose fine Holocene silty sand layers from river bottom till ~18m – NAP. Under these Holocene

layers a plastic Pleistocene clay layer (Formation of Kedichem) with varying thickness is overlying the deeper silty Pleistocene sand layers. The medium grainsize d_{50} of both types of sand layers varies between 70-200 μm with a uniformity coefficient d_{60}/d_{10} of 2-4. Local experience suggested that the area is highly susceptible to static liquefaction.

Prior to award of contract, a total of 102 CPT's and 32 boreholes was performed by the Client, while sieve analyses, minimum and maximum dry density testing and conventional triaxial testing were undertaken in the laboratory. After award of contract, advanced drained and undrained triaxial tests were carried out to determine the required soil parameters for the SMSP model. The resulting data points of the modified state parameter versus the slope of the instability line are presented in Figure 5, together with the trendline according to Chu et al. (2003). However, it can be seen that the trendline has to be relocated in order to fit the obtained data set. The transition between drained and undrained material behaviour lies around $\bar{\psi} \sim -0.10$, which boundary is in agreement with Jefferies & Been (2006) and others.

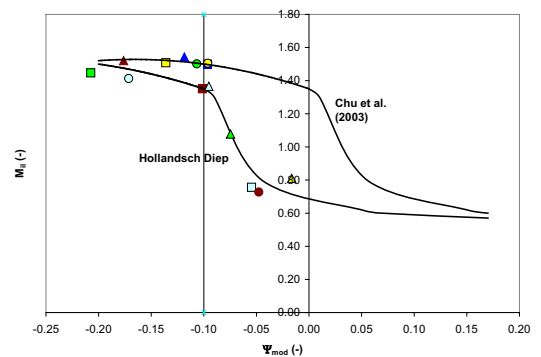


Figure 5. Laboratory data of Modified State parameter versus slope of instability line $M_{IL} (=q/p')$

The CPT(U) data and some density cone data were converted into relative density and state parameter profiles by using correlations from Lunne et al. (1997) and Shuttle et al. (1998). The amount of uncertainty involved in the direct or indirect state parameter determination was significant ($\bar{\psi} \pm 0.05$) where the indirect, relative density based, method resulted in a lower boundary ($\bar{\psi} = -0.10$), which was adopted for project application.

4.3. Model Verification, Resulting Geometry

The predictions of the SMSP model were verified by various intentionally induced failures in the middle of the dredge depot. Steep front slopes were cut with a Cutter Suction Dredger (CSD) until failure occurred within several days after construction. Pre- and post-surveys were compared and analysed with the model predictions. The results were used to determine the effect of several uncertainties on instability resulting in a quantification of the safety margin with implicit definition of equipment uncertainty for the following construction/ excavation stages.

As a result of the analysis with the SMSP model, the design slopes of the consortium Sassenplaat typically had 1(v):4(h) – 1(v):5(h) slopes in the Holocene sand layers and 1(v):5(h) in the Pleistocene sand layers. To allow for inaccuracies inherent in a dredging process with different types of equipment, the Pleistocene sand slopes are occasionally even excavated at 1(v):7(h).

4.4. Unplanned Instabilities

During the excavation of the slopes of the dredge depot several small and larger failures occurred, involving volumes ranging from $\sim 6000 - 350.000 \text{ m}^3$. On all accounts the heterogeneous, very loose, clayey, very silty sand layers above or below the Kedichem clay layer were involved in the instability. The most likely triggering events were generally construction related: temporary steeper slopes, the falling of anchors and spuds and on other occasions the building up of excess pore water pressures due to the construction activities in the slope.

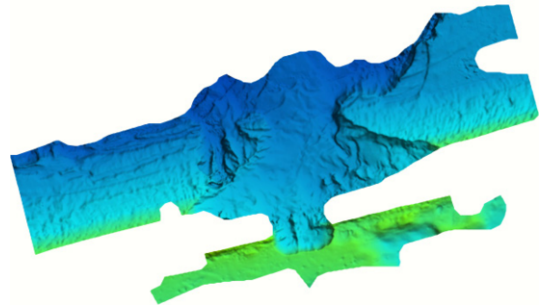


Figure 6. a) Dike failure at water line , b) Underwater survey of dike failure of 26th August 2007 ($\sim 350.000 \text{ m}^3$) (courtesy: Combination Sassenplaat)

In Figures 6a and 6b, a picture of the dike breakthrough and the bathymetric survey after the largest instability are presented respectively. One of the instabilities involved both the full slope and constructed dike section (see De Jager et al., 2008). The extensiveness of the instability was most likely the consequence of an unfortunate combination of the following factors: the temporary existence of multiple, locally steep slopes as a result of the dredging operations, the use of water jets to increase the dredging production, non-occurrence of Kedichem clay layer, and the presence of a very loose clayey silty sand pocket/ trough in the Pleistocene layer.

Backcalculation of the initiation of instability resulted in a safety factor ranging between 0.60 and 1.10 resulting in a high probability of failure. Unfortunately, it's not possible to predict the evolving processes after the initiation of static liquefaction with the SMSP model, as it is not possible to carry out coupled calculations.

4.5. Lessons Learned

The state parameter determination and the statistical characterization of spatial variability for the dredge excavation were verified and no anomalies were discovered for the layers which were characterized as sand, silty sand layers following the Soil Behaviour Type (SBT) (Robertson, 1990).

The state parameter determination proposed by Robertson (2010), which is in line with the methodology proposed by Jefferies & Been (2006), has been applied for the Hollandsch Diep project to verify if the characteristic CPTU's plot susceptible to liquefaction (see Figure 7).

The thin red dashed line in Figure 7 is the liquefaction susceptibility boundary for $\bar{\psi} = -0.07$ including a grain characteristic correction factor (K_c) according to Robertson (2010). It can be observed that the majority of all relevant CPTU datapoints taken prior to the instability (apart from the Kedichem clay layer) plot as not susceptible for static liquefaction.

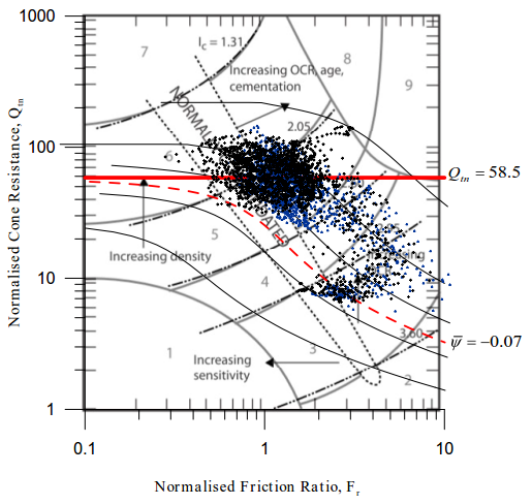


Figure 7. SBT chart based on normalized CPT parameters (modified from Robertson 1990) with CPTU data instability Hollandsch Diep and liquefaction boundaries

For static liquefaction, which typically has a constant load and may develop itself ranging from hours to days, application of this undrained behaviour correction factor seems inappropriate. When no correction factor is applied a constant Q_m boundary of 58.5 results, represented by the thick red line in Figure 7. The CPTU SBT chart

with uncorrected boundary plots the Hollandsch Diep data now as susceptible for static liquefaction. The same verification has been performed for several Dutch project locations where tens controlled and uncontrolled instabilities occurred. In all cases the corrected liquefaction boundary as proposed by Robertson (2010) proved not a cautious indicator.

However, Figure 7 only considers the susceptibility of the soil as measured by the CPT and cannot account for the anisotropy of the soil, the pre-failure geometry, the value of the trigger and the imposed stress path by the used construction method.

5. Conclusions and Recommendation

The Simplified Modified State Parameter (SMSP) framework, in combination with a stochastic analysis, developed for the design of the dredge excavation sludge depot proved a pragmatic model which provides insight in the main phenomena.

The occurrence of instabilities underlines the importance of proper soil characterization while taking into account both regional - and local variability and anisotropy. Furthermore the thorough work method and tool dependent validation of the model predictions during/ prior to project execution with controlled instabilities has since then been applied on multiple projects and provides invaluable added information.

Following the experience obtained for the Hollandsch Diep project it is proposed by the authors to use a constant normalized cone resistance Q_m boundary of 73, as a cautious indicator for liquefaction susceptibility. This corresponds to an uncorrected modified state parameter $\bar{\psi} = -0.10$.

It is worthwhile to investigate whether application of different model concepts, using user defined models: NorSand (Wanatowski et al., 2013) or UBC-sand (Tsegaye, 2010), reduces the variation in reliability in the stochastic analysis.

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