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Sensor placement in stochastic models for maximal transient and steady-state estimation performance

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1 Introduction

Precision mechatronics is facing increasingly complex positioning challenges due to thermomechanical deformations. A key example is the semiconductor industry requiring subnanometer positioning accuracy for chip manufacturing. This precision is hindered, among other, by the thermal expansion of the wafer stage in lithography machines. This deformation cannot be measured directly and thus needs to be estimated. The estimate can be obtained by positioning optimaly temperature sensors. This is known as the sensor placement problem.

We compare two sensor placement approaches that maximizes either the transient or the steady-state estimation performance. We highlight the fact that a finite-horizon formulation can give better estimation performance compared to the commonly-used steady-state approach.

2 Problem formulation for stochastic models

The discrete-time LTI stochastic system is given by,

$$\Sigma_G := \begin{bmatrix} x_{k+1} \\ y_k \\ z_k \end{bmatrix} = \begin{bmatrix} A & B_u & B_w & 0 \\ C & 0 & 0 & I \\ L & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ u_k \\ w_k \\ v_k \end{bmatrix}, \qquad (1)$$

with k the discrete time, x the state, y the measurements, u the known input and z the performance output to be estimated. The sensor layout is characterized by C. The disturbance w and the measurement noise v are zero-mean white noises and the initial state x_0 is a Gaussian random variable:

$$w_k \sim \mathcal{N}(0,Q), \quad v_k \sim \mathcal{N}(0,R), \quad x_0 \sim \mathcal{N}(\hat{x}_{0|0}, P_{0|0}) \qquad (2)$$

with known symmetric covariance matrices $Q \succeq 0$, $R \succ 0$, $P_{0|0} \succeq 0$ and initial estimate $\hat{x}_{0|0}$. The initial state and noises are assumed to be mutually independent. The observer design characterizes the output estimation error e_k. Two sensor placement problems are written as minimizations:

- The finite-horizon problem

$$\min_{C \in \Omega_C} \frac{1}{k_f} \sum_{k=1}^{k_f} \mathbb{E}\left[\|e_k\|^2\right], \qquad (3)$$

  where the cost function measures the average estimation error on the interval $[1, k_f]$, i.e., transient output estimation accuracy.
- The infinite-horizon co-design problem

$$\min_{C \in \Omega_C} \lim_{k \to \infty} \mathbb{E}\left[\|e_k\|^2\right], \qquad (4)$$

  where the cost function measures the infinite-time or steady-state output estimation accuracy.

Under our assumptions, the optimal observer is the Kalman filter as stated in [1].

3 Application

The two estimation approaches are evaluated and compared for a simple 3D thermomechanical system. Results show the dependence of the optimal sensor location with the considered horizon for estimation accuracy (see Fig 1).

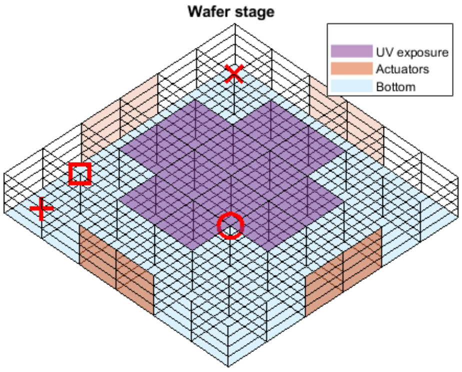

Figure 1: Optimal sensor locations for several horizons k_f (see Table 1).

Horizon k_f	2	20	500	10000	∞
Optimal sensor location	+	×	□	○	○

Table 1: Optimal sensor location for the finite- and infinite-horizon problems with symbols of Figure 1.

References

[1] P. R. Kumar and P. P. Varaiya, "Stochastic Systems: Estimation, Identification, and Adaptive Control", Prentice Hall, 1986.