

NOISE REDUCTION BY RANGE IMAGE FILTERING

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ABSTRACT: Three-dimensional models are usually a severe simplification of the real world. Laser scans that are used to make these models have a high resolution and potentially contain much more spatial information. This potential can often not be exploited due to the measurement noise present in the data. The question addressed in this paper is how to remove the measurement noise without simplifying the model more than necessary. The proposed method solves this problem by using the redundancy present due to the high spatial resolution. By smoothing the data according to the probability and observation theory, a best linear unbiased prediction of the surface is obtained. This is implemented for range image filtering, and tested on synthetic data. The results show that this is a powerful method for the smoothing of noisy 3D data.

1. INTRODUCTION

1.1. Problem definition

Three-dimensional (3D) models are usually a severe simplification of the real world. In many cases like visualisation or simple simulation, this is appropriate or preferable, since reduction of complexity makes the model easier to store, visualise and process. However, for some simulation purposes the simplification of the model may lead to unrealistic results.

To get realistic shape information, a measurement technique is needed that measures 3D points systematically and automatically with high spatial resolution. For many objects, laser scanners are the best choice for such a measurement technique.

To be able to use laser scanner measurements, the object surface needs to be modelled. The application determines the demands and constraints for the 3D model. A usual approach for processing laser scans is to simplify the resulting 3D point cloud again into planes, spheres, cylinders etcetera by fitting these shapes on the point cloud. In this way, much of the spatially detailed shape information of the real world in the measurements is not used (figure 1). Another common approach is to make a triangulation surface through the measurement points. When the spatial resolution of the scanner is high compared to the measurement precision (standard deviation of the measurement noise), this results in a very bumpy surface (figure 2). A bumpy surface can be a problem for visualisation and simulation purposes when the orientation of the surface or change in the orientation of the surface is of importance, for instance for visualisation of lighting or simulation of fluid flows. For the latter, this is important because the fluid flows are modelled with differential equations, which results in sensitivity for discontinuities in the first and second derivatives. A third approach is to use a

more advanced method to model the noisy point data (e.g. power crust [1]). However, these methods give a still too bumpy result, have no quality description and/or are biased. A last option is manual modelling which is less objective and very time consuming. Currently, the time needed to process laser scan data to a model takes in general 5 to 20 times as much time as capturing the data [2].



Figure 1: Even in the absence of measurement noise, fitting a plane (line) removes dents and bumps in the surface that are clearly visible in the laser scan data (points).



Figure 2: Laser scan data (points) with high spatial resolution compared to the measurement precision gives a bumpy triangulation surface (line).

1.2. Noise reduction by range image filtering

For some applications the severe simplification of the real world in planes and other shapes is inappropriate, but the raw data contains too much measurement noise to be used directly for describing the surface (e.g. in a triangulation) too. To reduce the measurement noise but to keep detailed spatial information, a smoothing method is needed. There are many ways to smooth noisy measurement data. However, the smoothing method should allow error propagation and have meaningful parameters to tune it for the optimal amount of smoothing. In this paper this is achieved by a least squares approach using probability and observation theory. To make this implementable for large data sets, this approach is translated to an image convolution filter that can be applied on the raw data of a laser scanner saved as a range image.

1.3. Paper outline

In section 2 best linear unbiased prediction and range image filtering are introduced. The equations to implement these two methods are given in section 3. To demonstrate the method an experiment is described in section 4. In section 5 the experiment and the method are evaluated. This leads to conclusions and future research listed in section 6.

2. APPROACH

2.1. Best linear unbiased prediction

The least squares approach of smoothing reduces the measurement noise by using the redundancy in the measurements, which is present due to the high spatial resolution. To be able to do this, a relation between nearby points is needed. Therefore, it is assumed that the object surface can be locally described as a continuous function. This function should not be deterministic, but should be made stochastic by adding model noise besides the measurement noise in the least squares solution. This is done in the same way as this is done in Kalman

filtering [3], namely by adding pseudo-observations. To do this, the following aspects should be known: (1) quality of the measurements (covariance matrix of the measurements), (2) the mathematical description of the object surface (e.g. N^{th} order polynomial) and (3) the average deviation of the mathematical description (covariance matrix of the model noise, including spatial correlation). If all assumptions including the values of above aspects hold, the least squares solution gives the best linear unbiased prediction [3] of the surface points. It also gives a quality description for every smoothed point on the surface by applying error propagation. Best linear unbiased prediction (BLUP) is the term used in statistics, which is the same as what is called Kriging in geostatistics. In this paper the name BLUP will be used.

If the model noise is set very small, the BLUP will result in fitting of shapes and thus removing all spatial detail that does not fit in these shapes. If the model noise is set to a very large value however, the spatial relation with nearby points is lost and the BLUP will give for every point the best information available: the measurement of that point with unsmoothed measurement noise. There is a trade-off between these two extremes, more reduction of measurement noise will also reduce the spatial detail. The desired amount of smoothing can thus be defined either by the spatial detail that should be preserved or the desired reduction of the measurement noise. If both are defined, the required combination of scan resolution and scan precision can be determined.

2.2. Range image filtering

Most terrestrial laser scanners are scanning in a regular grid of horizontal and vertical angles, creating a range image rather than a 3D point cloud. The rows and columns of the range image are the vertical and horizontal angles of the terrestrial laser scanner and the pixel values of the image are the measured ranges from the scanner. The range image will represent a spherical coordinate system. The major advantage of the range image is that it can be used to smooth the measurements by image filtering. The equations for BLUP applied on a regular grid become quite simple and can be used to produce an image convolution filter. In this way, large convolution filters can be designed on the basis of information like the measurement noise of the scanner and model noise of the object. Laser scanners that do not produce a range image (like airborne laser scanners) can also be filtered as a range image, but only after resampling the data as a regular grid.

3. THEORY AND IMPLEMENTATION

3.1. Best linear unbiased prediction

The BLUP is a least squares solution of a system of observation equations. The observations are the measured ranges r . To complement the system of observation equations pseudo-observations d are added to describe how the neighbouring ranges are related. An example of the system of observation equations is given in equation 1. In this example the following two assumptions are made. Firstly, for simplicity it applies to a 1D measurement series (e.g. a cross section of a surface) only, as every measurement has only two neighbours. A second assumption is a linear model to describe the surface locally, as can be seen in the values $[\frac{1}{2} \ -1 \ \frac{1}{2}]$ in A_d . In a grid, the average of the neighbours (the two $\frac{1}{2}$'s) should equal the value of the grid point itself. Note that this also means that the values in the vector of pseudo-observations d are all zero.

$$E\left\{\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_n \\ \hline d_2 \\ d_3 \\ \vdots \\ d_{n-1} \end{bmatrix}\right\} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ \hline & \frac{1}{2} & -1 & \frac{1}{2} & \\ & & \frac{1}{2} & -1 & \frac{1}{2} \\ & & & \ddots & \ddots \\ & & & \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_n \end{bmatrix} \quad (1a)$$

$$\text{or short: } E\left\{\begin{bmatrix} r \\ d \end{bmatrix}\right\} = \begin{bmatrix} A_r \\ A_d \end{bmatrix} \cdot r \quad (1b)$$

$$\text{or even shorter: } E\{y\} = Ax \quad (1c)$$

The observations and pseudo-observations are assumed to be normally distributed. An example of probability distribution of the observations and pseudo-observations $D\{y\}$ is given in equation 2, with the following two assumptions. Firstly, a linear decrease of spatial correlation in the model noise is assumed, as the values in Q_d decrease linearly away from the diagonal. Secondly, the correlation distance is 3 times the grid spacing, as the values of Q_d reach zero at this distance from the diagonal.

$$D\left\{\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_n \\ \hline d_2 \\ d_3 \\ \vdots \\ d_{n-2} \\ d_{n-1} \end{bmatrix}\right\} = \begin{bmatrix} \sigma_r^2 & & & & \\ & \sigma_r^2 & & & \\ & & \sigma_r^2 & & \\ & & & \ddots & \\ & & & & \sigma_r^2 \\ \hline & & & & \sigma_r^2 \\ & \sigma_d^2 & \frac{2}{3}\sigma_d^2 & \frac{1}{3}\sigma_d^2 & \\ & \frac{2}{3}\sigma_d^2 & \sigma_d^2 & \frac{2}{3}\sigma_d^2 & \frac{1}{3}\sigma_d^2 \\ & & \ddots & \ddots & \ddots \\ & & \frac{1}{3}\sigma_d^2 & \frac{2}{3}\sigma_d^2 & \sigma_d^2 & \frac{2}{3}\sigma_d^2 \\ & & & \frac{1}{3}\sigma_d^2 & \frac{2}{3}\sigma_d^2 & \sigma_d^2 \end{bmatrix} \quad (2a)$$

$$\text{or short: } D\left\{\begin{bmatrix} r \\ d \end{bmatrix}\right\} = \begin{bmatrix} Q_r \\ Q_d \end{bmatrix} \quad (2b)$$

$$\text{or even shorter: } D\{y\} = Q_y \quad (2c)$$

The least squares solution is given by:

$$\hat{x} = (A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} y \quad (3)$$

$$D\{\hat{x}\} = Q_{\hat{x}} = (A^T Q_y^{-1} A)^{-1} \quad (4)$$

3.2. Range image filtering

The filtered value of a range measurement is thus a weighted sum of the observed range measurements (equation 3). The weight for each observed measurement depends only on the distance to the measurement to be filtered (if the precision of all ranges is assumed to be the same). Therefore, it is not necessary to smooth all ranges by solving them all together in a system of observation equations of a least squares prediction. This would be complicated anyway, because the matrices that need to be inverted would be very large. Instead, the weights can be determined by solving the system for a small number of observations and then use these weights to smooth all ranges by image convolution.

The weights are defined by the matrix F :

$$(A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} = F \quad (5)$$

This matrix F can be split in two parts, so equation 3 can be written as:

$$\hat{r} = [F_r \quad F_d] \cdot \begin{bmatrix} r \\ d \end{bmatrix} \quad (6)$$

Because the vector d consists of zero values only, this reduces to:

$$\hat{r} = F_r \cdot r \quad (7)$$

Except for some influence of the effects of the edges of the smoothed area, the rows of F_r are identical around the diagonal and fading to zero away from the diagonal. This is consistent with intuitive expectation that only ranges in the neighbourhood of a specific range have an influence on its smoothing. The filter for the convolution can thus be obtained by taking the values around the diagonal.

The equations 1-7 could be adapted to a 2D situation. However, when the model is homogeneous (e.g. no differences in values for the measurement noise) the resulting 2D filter should be the same in all directions. Therefore, the 1D result (figure 5) can also be used to derive the 2D filter (figure 6) by “spinning” it around.

4. EXPERIMENT

4.1. Test object

The method is tested on synthetic data, so the amount of measurement noise can be varied and the smoothed result of the BLUP can be compared to the reality without measurement noise to validate it.

The test data is a roof of 5 by 10 metres with two sloped parts and two flat parts. To be able to show the flexibility of the BLUP there are some features present. One sloped roof side is sagged the other one is corrugated. One flat roof part has two dome shaped features, the other part has random dents and bumps (figure 3).

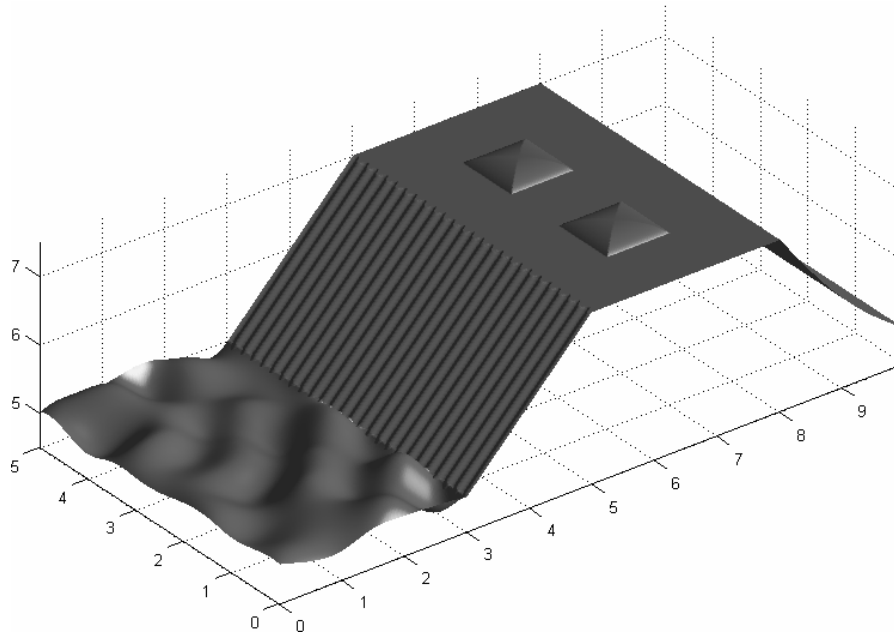


Figure 3: Test object, a roof with some features to demonstrate the flexibility of BLUP.

4.2. Requirements

Based on: (1) the characteristics of the measurement system that will be used, (2) the object and (3) the requirements of the resulting 3D model, the best choices for the measurements and the parameters of the BLUP can be determined. Let's assume that the used scanning device is one of the constraints and its measurement precision is 0.10 m. Suppose it is required to reduce the standard deviation of the measurement noise to 0.025 m and that all spatial details of size 0.5 m and larger should be modelled.

The first choice to be made is the model to locally describe the object surface. A locally plane surface, like in equation 1a, seems a safe choice here.

Next, the correlation distance of model noise should be determined, which depends on the model (locally a plane surface) and on how well the object complies with this assumption. The correlation distance is the distance in which the model noise is likely to be the same (the length of a bend in case of a plane surface assumption). This should be estimated by looking at the object. A small correlation distance value is a good choice in this case, because the object surface has some sharp bents. A value of 5 times the grid spacing is used. Within this range, the model noise is assumed to decrease linearly, like in equation 2a.

That leaves only the standard deviation of the model noise as parameter to tune the smoothing. Because the covariance matrix of the smoothed measurements is given by equation 4, experimenting with the model noise shows that a standard deviation of 0.00075 m is needed to obtain a standard deviation of 0.025 m. At this value the weights of the filter reduce to values around zero at a distance of circa 17 times the resolution (figure 5). To make sure features of 0.5 metres are still preserved, this should correspond with about 0.25 m. The required resolution is therefore at least 0.0147 cm. This means smoothing over circa $\pi \cdot 17^2$ measurements, however measurements close to the filtered measurement will have most influence.

4.3. Measurements

In this case of synthetic data the object can be easily sampled with the required resolution. And the assumed distribution of the measurement noise can be exactly applied to the data. The roof is sampled with 0.01 m resolution to resemble laser scan data. Next, normally distributed measurement noise is added with a standard deviation of 0.10 m (figure 4).

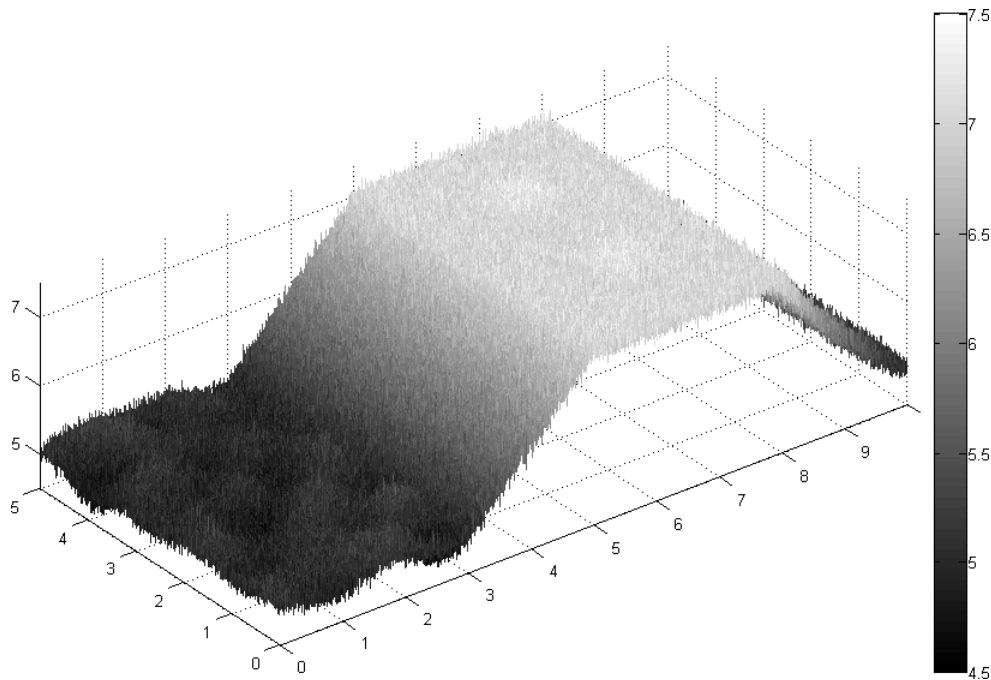


Figure 4: Synthetic measurement data of the test object with a standard deviation of 0.10 m.

4.4. Range image filtering

Based on the requirements the image filter is determined (figure 6) by deriving it from the 1D case (figure 5), as described in section 3.

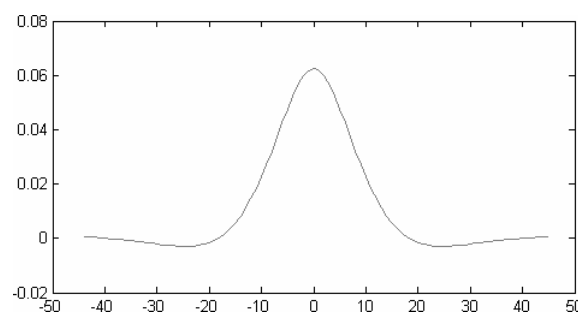


Figure 5: 1D filter used to obtain the filter of figure 6.

Image convolution of the range image (figure 4) with this filter (figure 6) provides the best linear unbiased prediction of the smoothed ranges (figure 7).

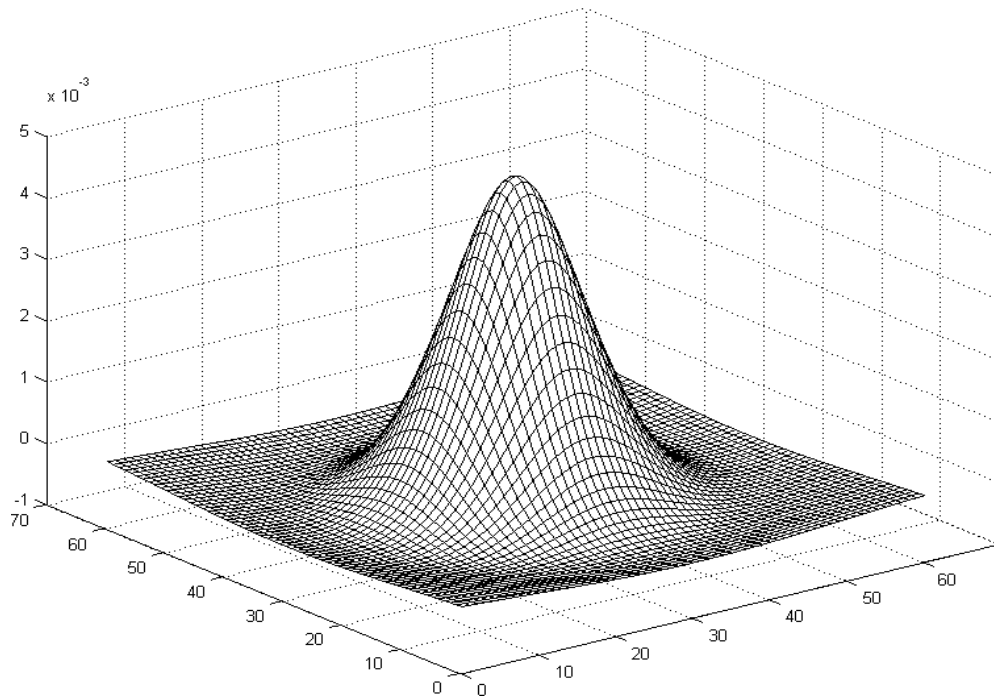


Figure 6: Image filter applied on the test data to obtain the smoothed data.

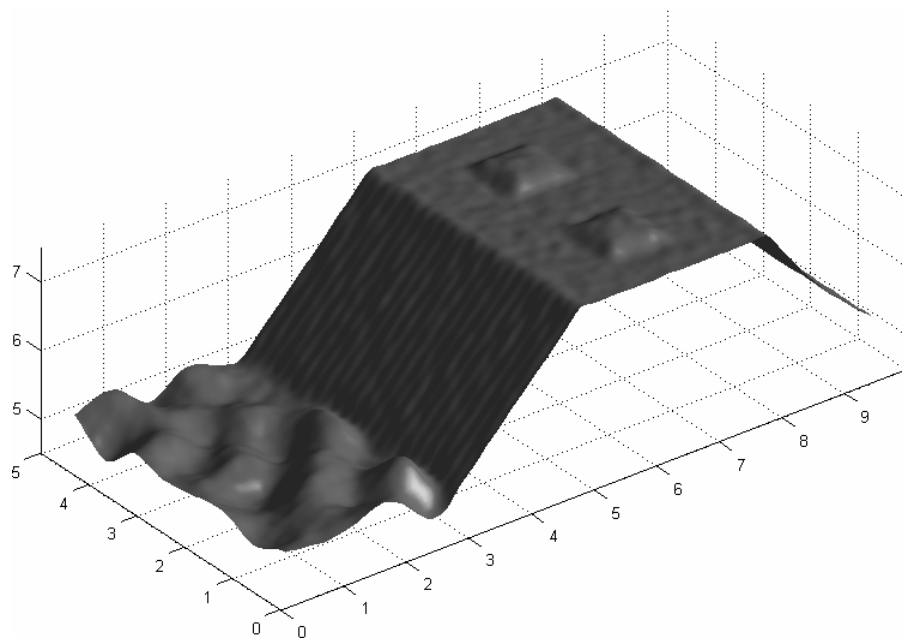


Figure 7: BLUP of the noisy measurement data of figure 4.

5. EVALUATION

5.1. Experiment

The differences with the BLUP and the data without measurement noise are generally below 0.03 m. The largest deviations occur in the corrugated part of the roof, because these corrugations are completely filtered out (figure 8). This is a direct result of the choice to

preserve only features larger than 0.5 m. However, the BLUP corresponds to the mean shape of the corrugated part of the roof. Other areas with large differences are locations where the model does not hold, like the locations of sharp bents. For the rest all features are present, even if they were almost drowned in the measurement noise for the human eye. This clearly shows the power of a well designed smoothing method.

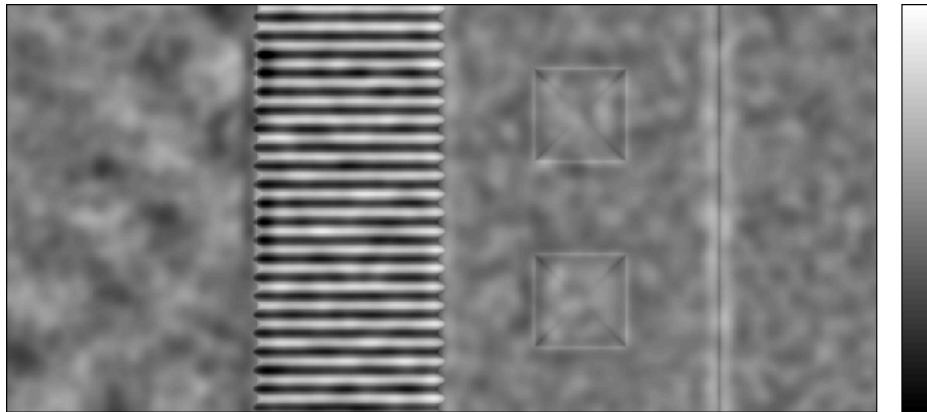


Figure 8: Differences between the BLUP (figure 7) and noise-free data (figure 4); Colour scale from -0.075 m (black) to +0.075 m (white).

5.2. Method

It is shown that the method of BLUP is a powerful and easily implementable method to smooth noisy 3D data. However, there are some remaining issues.

Most objects will have some sharp bends or edges that are not smooth. These features can be described as points or lines having discontinuities in surface orientation, or having a too strong curvature. To prevent these features from being smoothed, these points and lines need to be detected or entered by an operator first. Next, the method should be adapted to preserve them.

In the experiment measurements in a regular grid of Cartesian coordinates were used. However, laser scanners give either non-Cartesian coordinates in a grid or Cartesian coordinates without a grid structure.

To efficiently describe the complete surface of the smoothed object some kind of data reduction should be used, as many data points are redundant. A piecewise function could be used to describe the surface in between these points.

6. CONCLUSIONS

6.1. Conclusions

For some applications a compromise between triangulation and shape fitting is needed, because a triangulation contains too much measurement noise whereas shape fitting simplifies reality too much.

This paper presents how best linear unbiased prediction (BLUP) can be used to design convolution filters to obtain such a compromise. In geostatistics BLUP is often called Kriging.

BLUP is a way to design optimised smoothing filters. Features in the point cloud that are almost drowned in the measurement noise for the human eye are still reconstructible. Therefore it can be concluded that BLUP is better than manual surface fitting.

6.2. Future research

At break lines (a change in surface slope) and edges (a jump in surface height) the assumptions of BLUP are incorrect. Due to the model noise, the BLUP adapts as quickly as possible, but if the location of the break lines and edges would be known or be estimated from the data, the model could be adapted to accommodate these discontinuities and give a better prediction. How these break lines and edges can be found or entered and how the method should be adapted will be investigated.

Implementation of BLUP is easy in case of Cartesian coordinate data in a grid structure. However, this poses some limitations on its use, as airborne laser scan data are not structured in a grid and terrestrial laser scan data consist of spherical coordinates. How to adapt the method to deal with this will be part of future research too.

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