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DOI
10.1016/j.econlet.2024.111606

Publication date
2024

Document Version
Final published version

Published in
Economics Letters

Citation (APA)
Fokkink, R., & de Munnik, H. (2024). Multiplayer boycotts in convex games. *Economics Letters*, *236*, Article 111606. https://doi.org/10.1016/j.econlet.2024.111606

Economics Letters 236 (2024) 111606

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Multiplayer boycotts in convex games

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ARTICLE INFO

JEL classification:
C71

MSC:
91A12

Keywords:
Cooperative games
Impact of boycotts
Shapley value

ABSTRACT

We extend the notion of boycotts between players in cooperative games to boycotts between coalitions. We prove that convex games offer a proper setting for studying these games. Boycotts have a heterogeneous effect. Individual players that are targeted by many-on-one boycotts suffer most, while non-participating players may actually benefit from a boycott.

During the nineteen-eighties Western nations imposed economic sanctions against South-Africa. Their effectiveness has been debated, but the apartheid regime did unravel sooner than anticipated (Manby, 1992). It remains a prime example of the power of boycotts, which are increasingly being used by Western governments (Meyer et al., 2023). Today, economic sanctions are imposed against Iran, North Korea, the Russian Federation, and Venezuela. We extend previous work on boycotts in cooperative games and demonstrate the usefulness of the Shapley value to quantify their impact.

1. Boycott games

A cooperative game with transferable utility is a pair (N, v) where N is the set of *players* and the *characteristic function* v is defined for all *coalitions* $S \subset N$. What is the impact if one coalition decides to boycott another coalition? This question has already been asked and answered by Besner (2022), for single players. We extend his work to coalitions.

Definition 1 (*Besner*)**.** Two players i, j are *disjointly productive* if for all $S \subset N \setminus \{i, j\}$ we have

$$v(S \cup \{i, j\}) - v(S \cup \{j\}) = v(S \cup \{i\}) - v(S).$$

Disjoint coalitions A, B are *disjointly productive* if i, j are disjointly productive for all $i \in A$ and $j \in B$.

We write marginal contribution as

$$dv_i(S) = v(S \cup \{i\}) - v(S)$$

or more generally

$$dv_C(S) = v(S \cup C) - v(S).$$

Lemma 2. *If $A, B \subset N$ are disjointly productive, then for all $A' \subset A$ and $B' \subset B$ and $S \subset N \setminus (A \cup B)$*

$$dv_{B'}(S \cup A') = dv_{B'}(S).$$

Proof. The marginal $dv_{B'}(S \cup A')$ is a sum of marginals of players in B' that are disjointly productive from A'. Therefore, removing A' from the coalition does not change the marginal contribution. □

Definition 3. For any (N, v) and any disjoint pair of coalitions A, B we say that (N, v^{AB}) is the *A, B-boycott game* if

1. $v^{AB}(S) = v(S)$ if $S \cap A = \emptyset$ or $S \cap B = \emptyset$,
2. A and B are disjointly productive in (N, v^{AB}).

If $A = \{i\}$ and $B = \{j\}$ then we write v^{ij} and we have a *one-on-one boycott*. If $A = \{i\}$ and $|B| > 1$ then we write v^{iB} and we have a *many-on-one boycott*.

By Lemma 2 we have $v^{AB}(S \cup A' \cup B') - v^{AB}(S \cup B') = v^{AB}(S \cup A') - v^{AB}(S)$. for S disjoint from $A \cup B$ and $A' \subset A, B' \subset B$. A rearrangement of terms gives

$$v^{AB}(S \cup A' \cup B') = v(S \cup A') + v(S \cup B') - v(S), \tag{1}$$

which defines v^{AB}.

Definition 4. A characteristic function is *supermodular* if

$$v(A \cup B) + v(A \cap B) \geq v(A) + v(B) \tag{2}$$

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https://doi.org/10.1016/j.econlet.2024.111606
Received 15 January 2024; Received in revised form 14 February 2024; Accepted 14 February 2024
Available online 15 February 2024

for all coalitions A, B. A cooperative game with a supermodular characteristic function is called a *convex game*.

Theorem 5. *$v^{AB} \leq v$ for all disjoint A and B if and only if (N, v) is convex.*

Proof. Compare the Eqs. (1) and (2) by taking $A = S \cup A'$ and $B = S \cup B'$. □

The purpose of a boycott is a reduction of the opponent's utility, which is exactly what happens in a convex game. Also, larger boycotts inflict more harm:

Theorem 6. *If $A \subset C$ and $B \subset D$ then $v^{CD} \leq v^{AB}$ for a convex game.*

Proof. For coalitions S and V we write $S_V = S \cap V$. Partition a coalition S into $S_A \cup S_{C\setminus A} \cup S_0 \cup S_B \cup S_{D\setminus B}$, where $S_0 = S \setminus (C \cup D)$. We need to prove that $v^{AB}(S) \geq v^{CD}(S)$, which expands into

$$v(S_A \cup S_{C\setminus A} \cup S_0 \cup S_{D\setminus B}) + v(S_{C\setminus A} \cup S_0 \cup S_{D\setminus B} \cup S_B) - v(S_{C\setminus A} \cup S_0 \cup S_{D\setminus B}) \\ \geq \\ v(S_A \cup S_{C\setminus A} \cup S_0) + v(S_0 \cup S_{D\setminus B} \cup S_B) - v(S_0).$$

Converting to marginals and rearranging the terms gives

$$dv_{S_A \cup S_{C\setminus A} \cup S_0}(S_{D\setminus B}) + dv_{S_0 \cup S_{D\setminus B} \cup S_B}(S_{C\setminus A}) \\ \geq \\ dv_{S_{C\setminus A} \cup S_0}(S_{D\setminus B}) + dv_{S_0}(S_{C\setminus A}).$$

As v is supermodular, marginals $dv_X(Y)$ increase with X. Therefore, the left-hand side is indeed larger than the right-hand side. □

Convex games provide the right setting for studying boycotts. We now determine their impact on individual players.

2. The impact of a boycott

A *TU-value* φ assigns a vector $\varphi(N, v)$ to a game (N, v). Its coordinates are $\varphi_i(N, v)$, or simply φ_i, for players $i \in N$. We write $\varphi(S) = \sum_{i\in S} \varphi_i$. It is *efficient* if $\varphi(N) = v(N)$. To measure a boycott's impact, we compare its value to the value of the original game.

Definition 7. The *impact* of a boycott is $\varphi(v) - \varphi(v^{AB})$.

In a boycott game, $i \in A$ essentially only contributes to $N \setminus B$. Therefore, it is natural to require that the value of i in (N, v^{AB}) equals its value in $(N \setminus B, v)$. We say that such a value is *boycott respecting.* A value has *balanced impact* if in one-on-one boycotts

$$\varphi_i(v) - \varphi_i(v^{ij}) = \varphi_j(v) - \varphi_j(v^{ij}).$$

Besner (2022) proved that the Shapley value is the unique efficient TU-value that respects boycotts and has balanced impact. We therefore restrict out attention to the Shapley value.

Definition 8. For a coalition $C \subset N$, the *subgame* (N, v_C) is defined by

$$v_C(S) = v(S \cap C).$$

Let $\bar{A}$ denote the complement of A. For disjoint coalitions A, B we have

$$v^{AB} = v_{\bar{A}} + v_{\bar{B}} - v_{\bar{A}\cap\bar{B}}.$$

Since the Shapley value ϕ is additive, the impact of a boycott is

$$\phi(v) - \phi(v_{\bar{A}}) - \phi(v_{\bar{B}}) + \phi(v_{\bar{A}\cap\bar{B}}).$$

A player $i \in A$ is a null-player in the subgames on $\bar{A}$ and $\bar{A} \cap \bar{B}$. For such a player the impact is

$$\phi_i(v) - \phi_i(v_{\bar{B}}).$$

Theorem 9. *In a many-on-one boycott v^{iB} the impact is maximal for i. More explicitly,*

$$\phi_i(v) - \phi_i(v^{iB}) \geq \phi_j(v) - \phi_j(v^{iB})$$

for all $j \in N$.

Proof. The impact on i is $\phi_i(v) - \phi_i(v_{\bar{B}}) \geq \phi_i(v) - \phi_i(v_{N\setminus\{j\}})$ for any $j \in B$ since the game is convex. The impact on $j \in B$ is $\phi_j(v) - \phi_j(v_{N\setminus\{i\}})$ which is equal to $\phi_i(v) - \phi_i(v_{N\setminus\{j\}})$ by balancedness of ϕ. The impact on i is greater than or equal to the impact on $j \in B$.

The impact on a non-participating player $k \in \bar{A} \cap \bar{B}$ is

$$\phi_k(v) - \phi_k(v_{N\setminus\{i\}}) - \phi_k(v_{\bar{B}}) + \phi_k(v_{\bar{B}\setminus\{i\}}).$$

By balancedness this equals

$$\phi_i(v) - \phi_i(v_{N\setminus\{k\}}) - \phi_i(v_{\bar{B}}) + \phi_i(v_{\bar{B}\setminus\{k\}}),$$

which by monotonicity and $\bar{B} \setminus \{k\} \subset N \setminus \{k\}$ is bounded by the impact on i

$$\phi_i(v) - \phi_i(v_{\bar{B}}).$$ □

Example. *Non-participating players may benefit. Consider the three-player cooperative game $v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$ and $v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 6$ and $v(\{1,2,3\}) = 12$. Note that $v(S) = 6(|S| - 1)$ where $|S|$ denotes the number of elements. This is a Myerson communication game (Algaba et al., 2001) on a triangle with Shapley value $(\phi_1, \phi_2, \phi_3) = (4,4,4)$. If 1 boycotts 2 then $v^{12}(\{1,2\}) = 0$ but all other values remain the same. The Shapley value of v^{12} is $(3,3,6)$. Player 3 profits from the boycott. The boycott deletes the edge between 1 and 2 from the triangle so that 3 becomes central.*

Players that participate in a boycott stand to lose. Players that are unaffected stand to gain. The following makes that precise.

Definition 10. A player i is *invariant* under a boycott if $v(S) = v^{AB}(S)$ for all coalitions that contain i.

Theorem 11. *If (N, v) is a convex game and $A, B \subset N$ are disjoint, then the impact of a boycott between A and B is non-negative for all $i \in A \cup B$ and non-positive for invariant players.*

Proof. If $i \in A$ participates in a boycott, then the impact $\phi_i(v) - \phi_i(v_{\bar{B}})$ is non-negative by monotonicity. If k is invariant, then

$$dv_k^{AB}(S) = v^{AB}(S \cup \{k\}) - v^{AB}(S) = v(S \cup \{k\}) - v^{AB}(S) \geq dv_k(S).$$

Since its marginal contribution is non-decreasing under the boycott, so is its expected marginal contribution, which is equal to the Shapley value. The impact is non-positive. □

3. The impact of boycotts on trade blocks

To gain insight in the impact of boycotts on trade networks we consider some sample games. Computing the Shapley value of a network game requires non-trivial algorithms, which is why we consider simple networks known as *block graphs* (Algaba et al., 2001).

A single homogeneous trade block. Consider the characteristic function $v(S) = |S| - 1$. Its Shapley value is $\phi_i(v) = 1 - \frac{1}{n}$ for all i, where $n = |N|$. If A of size a boycotts B of size b then $\phi_i(v^{AB}) = 1 - \frac{1}{n-b}$ for $i \in A$ and $\phi_j(v^{AB}) = 1 - \frac{1}{n-a}$ for $j \in B$. Non-participating players are invariant and benefit slightly. The boycott has minimal impact, unless a coalition is substantial. Trade blocks are sheltered against internal trade wars.

A single heterogeneous trade block. Now there is a special $x \in N$. We have $v(S) = |S| - 1$ if $x \notin S$ and $v(S) = 3(|S| - 1)$ if $x \in S$. The Shapley value is $\phi_i(v) = 2 - \frac{1}{n}$ for $i \neq x$ and $\phi_x(v) = n - \frac{1}{n}$. In a many-on-one boycott of A versus x, we have $\phi_i(v^{Ax}) = 1 - \frac{1}{n-1}$ for $i \in A$

Table 1
Russian exports of mineral fuels 2021–23 — billions of US dollars.
Source: Bruegel Russian Foreign Trade Tracker, 2023.

Importer	2021		2022		2023
	Spring	Fall	Spring	Fall	Spring
EU27	49.8	70.6	89.5	61.7	18.5
UK	3.5	3.6	3.2	0.1	0.0
China	21.6	31.0	37.9	45.6	45.9
India	1.6	2.6	9.6	23.7	27.9
Turkey	2.5	3.0	18.8	23.1	15.1
World	97.5	134.5	178.4	165.0	116.0

and $\phi_x(v^{Ax}) = n - a - \frac{1}{n-a}$. The impact on non-participating players is negligible. This situation is similar to a consumer boycott, in which it is the question if the number of participating consumers (whose value halves) is enough to make the producer x change its policy (Delacote, 2009).

Boycotts between trade blocks. Trade networks tend to fall apart into large internal markets with few outside connections (Wilhite, 2001). Let $N = I \cup J \cup K$ with equal sized blocks $|I| = |J| = |K| = n$. The blocks have interconnecting key players $i \in I, j \in J, k \in K$. We have $v(S) = |S|$ if there are less than two key players within S. Key players connect the trade and double the value. For instance, $v(S) = |S| + |S \cap I| + |S \cap K|$ if $i, k \in S$ but $j \notin S$. If all three key players are in, then $v(S) = 3|S|$. The Shapley value is $\phi_x(v) = \frac{5}{3}$ for ordinary players and $\phi_y(v) = \frac{4}{3}n + \frac{5}{3}$ for key players. Obviously, key players are valuable.

If I boycotts J then trade between these blocks disintegrates. The value $v(N) = 9n$ reduces to $v^{ij}(N) = 7n$. Trade block K is unaffected. Trade blocks are sheltered against trade wars between other blocks. The value of non-key players in $I \cup J$ reduces to $\frac{4}{3}$, and for key players it halves $\frac{2}{3}n + \frac{4}{3}$. Key players will therefore be hesitant to join. Suppose that key player i drops out of the boycott. Now the ordinary players in J retain Shapley value $\frac{5}{3}$ while players in $I \setminus \{i\}$ remain at $\frac{4}{3}$. The maximum impact is on i and j with value $n + 2$. This boycott hurts I more than J.

Actual trade networks are much more versatile. After the Western nations imposed sanctions on the Russian Federation in 2022, trade from Europe to Russia diverted through members of the Eurasian Economic Union while India became a major consumer of Russian oil (Schott, 2023). In February 2023, the EU and the UK stopped imports of crude oil from Russia altogether. In anticipation Russia redirected its trade to the non-participating countries China, India and Turkey, compensating for the impact in 2023, see Table 1. The extension of our work to dynamic trade networks is a challenging computational task (Skibski et al., 2019).

Data availability

No data was used for the research described in the article.

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