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Combination of global still-water and wave load effects for reliability-based design of floating production, storage and offloading (FPSO) vessels

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Abstract

The purpose of this paper is to establish probabilistic models for still-water loads, based on design data, and the combined still-water and wave load effects for semi-probabilistic and probabilistic design of floating production, storage and offloading vessels (FPSO). A new still-water load model for FPSOs is proposed, based on a Poisson square-wave model, with a modified Weibull distribution for load intensity, which accounts for load control during operation. The long-term variation of wave-induced load effects is modelled by a Poisson square-wave process. A new solution for the combined effect is derived. A procedure for determining characteristic extreme values for individual and combined load effects, and load combination factors, is established. The methodology is used to illustrate load combination factors suitable for typical FPSOs. This approach is also shown to be useful in obtaining realistic load models, in terms of random variables, for use in reliability formulations.

Keywords: FPSO; Load combination; Still-water bending moment; Wave-induced bending moment; Operational control; Load combination factors

1. Introduction

A floating production, storage and offloading unit (FPSO) represents an attractive concept for offshore production of oil or gas. It is the foremost floating production facility, making up almost 60–70% [1] of all floating systems in the world.

FPSO hulls are similar to those of trading tankers, except that they have some extra topside equipment and an arrangement for turret mooring. However, FPSOs operate in a different way than tankers. For instance, cargo is continuously being loaded and unloaded, implying that still-water loads vary constantly. Moreover, the vessel operates with zero speed, with or without weather vaning. Finally, the offshore industry is greatly concerned about safety and applies first principles, as well as reliability-based approaches, in establishing rational design methodology.

The focus of this paper is the assessment of still-water and wave-induced load effects, and their combination, for structural design and safety analysis of FPSOs. Even though they are slightly correlated, the two load effects are commonly estimated separately; therefore, their combined effects need to be determined.

The load combination problem is virtually a superposition of stochastic load processes. The main issue is that the maxima of individual load processes will not occur simultaneously. This implies that the maximum of the combined processes, in general, is smaller than the sum of the maxima of the individual load processes.

Load combination methods are dependent on the load models. There is no adaptable method suitable for all types of load models. For example, the Ferry-Borges method [2] is suitable for Ferry-Borges load processes, the load coincidence method [3] is mainly suitable for Poisson processes, while the point crossing methods [4] is suitable for continuous stochastic processes.

In addition, there are deterministic combination rules, namely, the peak coincidence method, the Turkstra's rule [5] and the SRSS rule [6]. Wen [3] carried out a comprehensive evaluation of different kinds of deterministic combination rules. A detailed evaluation of Turkstra's rule and SRSS rule was made by Næss [7–9]. The general conclusion is that Turkstra's rule is non-conservative and the SRSS rule lacks consistency.

Currently, the design of ocean-going ships is based on adding the characteristic extreme values of still-water and wave loads, which usually leads to over-design. As far as

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the stochastic combination of these loads for ocean-going ships is concerned, Guedes Soares and Moan [10] initially investigated their combination by the upcrossing rate method. Furthermore, Guedes Soares [11] adopted the alternating renewal model to reproduce the time variation of still-water load effects (SWLE), in which the durations of voyages and time in port are taken into account. The classical Ferry-Borges and point crossing methods were adopted to predict combined extreme values and determine load combination factors suitable for ocean-going ships.

As indicated above, the characteristics of still-water and wave loads for FPSOs differ even from those of tankers. Moan and Jiao [12] and Wang and Moan [13] adopted the Poisson square-wave model to reproduce the time variability of the still-water and wave load effects. Based on operational data of still-water bending moments for a particular FPSO, they found that the hogging and sagging still-water bending moments followed exponential and Rayleigh distributions. The wave loading was modelled by a Weibull distribution. They compared different load combination methods and established load combination factors valid for a particular FPSO.

However, exponential and Rayleigh distributions for the still-water bending moments may not be appropriate for FPSOs in general and a more versatile distribution model is desired. In actual operations, still-water bending moments are subject to operational control. This means that, in principle, the maximum allowable value cannot be exceeded. However, some abnormal operations and exceptions may result in the allowable value being exceeded. Therefore, the constructed distribution must be modified to allow for such conditions.

Moreover, unlike the situation for ocean-going ships, FPSOs may experience a continuous change in load conditions owing to the loading–offloading cycle, i.e. being repeatedly in sagging and hogging conditions. In addition, the procedure for loading and offloading the vessel during one cycle has a direct effect on the time variability of still-water bending moments. This means that a short-term model for time variation of still-water load effects over one cycle should be constructed.

Furthermore, high values of topside loads, combined with the presence of the turret, may result in an uneven time fraction in hogging and sagging of a FPSO, which has a direct effect on the combined sagging and hogging extreme bending moments.

In addition, for FPSOs operating in different areas with harsh and benign conditions, extremely diverse wave-induced loads are experienced. This implies that the relative magnitude of still-water and wave-induced loads varies and, hence, so do the load combination factors.

Finally, it is noted that only data for still-water loads are available when the design is carried out. Therefore, combination analysis needs to be based on available data.

The purpose of the present paper is to establish probabilistic models for still-water loads based on the design data, as well as for the combined still-water and wave load effects (e.g. vertical bending moments amidship) for the semi-probabilistic and probabilistic design of FPSOs. In particular, semi-probabilistic methods, based on partial safety factors, are increasingly adopted in modern design codes, while probabilistic methods are applied to calibrate semi-probabilistic ones [14–16]. The important goal is to develop a new solution for the combined still-water and wave load effect by taking into account the different features of still-water loads of a FPSO.

2. Still-water bending moment (SWBM) model

2.1. General

While the still-water load, due to gravity and buoyancy, may contribute to 40–50% of the total global hull-girder load for merchant vessels, the effect may be somewhat less for production vessels in the North Sea. However, for new barge-type models with a large block coefficient, C_B, the still-water load could be much larger. Also, the still-water to wave load in benign waters will be larger than for North Sea conditions.

However, topside weight and the presence of a possible turret result in a distribution of weight that differs from that of tankers. The additional volume at the ends of ship-shaped FPSOs, combined with limited ballast tanks, can create still-water bending moment significantly larger than for traditional tankers.

Some offshore production ships are converted tankers; therefore, one might easily suggest that SWBM statistics for tankers would be applicable to production ships. Moreover, production ships experience a different mode of operation than tankers. For instance, they undergo a continuous cycle of cargo loading and off-loading, while tankers go to sea with a full load or with ballast. Also, the frequency of load condition change is different. Hence, the still-water load effect (SWLE) in production ships differs from those in tankers and other conventional ships. This has been clearly reflected in a statistical analysis of SWLE by Moan and Jiao [12].

The designed FPSO will allow crude oil offloading in a continuous round-the-clock operation at high flow-rate into a shuttle tanker moored in tandem or side-by-side. The crude oil would be directed to storage tanks by aligning valves from the FPSO's central control room. Tanks would be filled in a predetermined sequence to maintain the vessel's hydrostatic stability and to keep stress levels in the hull within allowable limits. However, a shortage of shuttle tankers due to bad weather or a lack of storage capacity may result in lost production several times a year. In general, maximum required loading time might vary significantly between ships since it depends on oil storage capacity and maximum oil production rate of the vessels. Based on available FPSO operating data [1], it can vary between 4 days and 1 month.

Since, the load only needs to comply with the upper limit of bending moments or shear forces, the captain has significant flexibility. Hence, he may not strictly follow the load manual and, hence, produce larger variations in the still-water loads than implied by the manual, and even result in exceeding the allowable value.

Table 1
Still-water bending moment of some existing FPSOs

FPSO	Locations	Sagging (M_{sw}, GN m)		Hogging (M_{sw}, GN m)		$M_{sw,D}/M_{sw,R}$	
		Rule[a]	Design	Rule[a]	Design	Sag	Hog
1	NS	4.74	4.05	5.12	5.07	0.85	0.99
2	NS	5.53	10.8	5.54	1.96	1.95	0.35
3	NS	3.73	1.043	4.07	3.235	0.28	0.80
4	NS	1.15	1.052	1.29	1.309	0.92	1.02
5	WA	4.49	3.277	4.97	4.323	0.73	0.87

NS, North Sea; WA, West Africa.
[a] Rule from Eq. (1).

To normalize the still-water bending moment, the following formula may be used

$$M_{sws,R} = -0.065C_1L^2B\,(C_B + 0.7) \quad \text{in sagging (kN m)} \tag{1}$$

$$M_{swh,R} = +0.015C_1L^2B(8.17 - C_B) \quad \text{in hogging (kN m)}$$

where L is the ship's length, B its beam, C_B is the block coefficient and C_1 is the wave coefficient given by:

$$C_1 = \begin{cases} 10.75 - \left(\dfrac{300-L}{100}\right)^{3/2} & \text{for } 190 \le L \le 300 \\ 10.75 & \text{for } 300 \le L \le 350 \\ 10.75 - \left(\dfrac{L-350}{150}\right)^{3/2} & \text{for } 350 \le L \le 500 \end{cases} \tag{2}$$

These expressions apply for trading vessels and are not relevant for FPSOs. In general, the global SWLE for ships should be determined by direct calculation as the maximum value under possible extreme load design conditions. While still-water bending moments for original FPSOs were within the regulation requirements for trading tankers, different features of new models have resulted in still-water loads that exceed the rule moment by up to 95% [17] (Table 1). It is obvious that actual design values deviate significantly from Eq. (1).

In the remaining part of this section, the long- and short-term time variability of SWBM is described, followed by a discussion on different parent distribution models of SWBM.

2.2. Long-term variability

As mentioned above, an FPSO undergoes a continuous loading–offloading cycle. In one cycle, it will experience maximum bending moments for both hogging and sagging. The variation in the maximum SWBMs over different cycles describes the long-term variation of SWBM.

Owing to the random arrival of shuttle tankers, different FPSO loading capacities, weather conditions, uncertainty of actual operations and so on, the duration of any one loading–offloading cycle varies. If the correlation between two successive cycles is neglected, a Poisson point process can be used to describe the renewal of different cycles within the lifetime of an FPSO. Furthermore, if the maximum SWBM (sagging or hogging) in one cycle is known, the Poisson square-wave model can be used to describe the long-term variability of SWBM. However, over one cycle, an FPSO will be successively hogging and sagging for different durations. Therefore, two conflicting square-waves are now imbedded in one cycle and their height and width correspond to the maximum hogging and sagging SWBMs and the duration of hogging and sagging, respectively.

As shown in Fig. 1, the proposed model consists of a Poisson point process for the renewal time of successive cycles, with a mean occurrence rate of $\nu_{cy} = 1/E[\tau_{cy}]$, where $E[\tau_{cy}]$ is the mean value of the duration τ_{cy} for any one cycle. At each renewal instant, the SWBM is successively modelled by two square waves. Their heights $M_{swh,ST}$ and $M_{sws,ST}$ correspond to the maximum intensities of hogging and sagging SWBM in one cycle, i.e. random variables with a distribution, which will be discussed later. Their random widths τ_h and τ_s are the durations of hogging and sagging, and are assumed to follow exponential

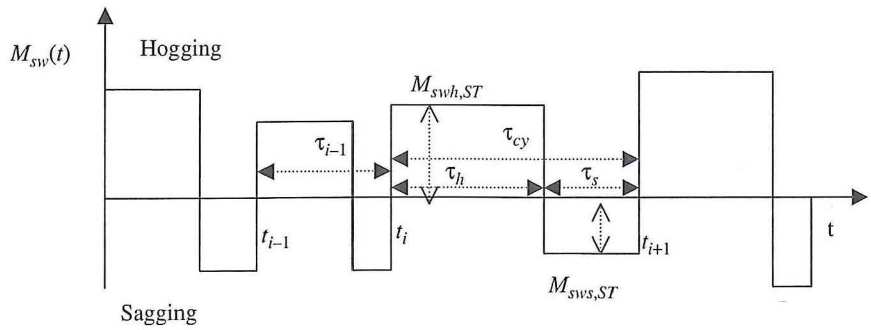

Fig. 1. Modelling long-term SWBM variability.

distributions. Their mean values are equal to $p_h E[\tau_{cy}]$ and $p_s E[\tau_{cy}]$, where p_h and p_s are the probabilities in hogging and sagging for the lifetime of one FPSO, respectively.

By adopting the above-mentioned model, and assuming that intensities $M_{sw,ST}$ ($M_{swh,ST}$ and $M_{sws,ST}$ refer to hogging and sagging) of SWBM in one cycle are independent and identically distributed random variables, and that the square wave height in each cycle is fully substituted by $M_{sw,ST}$, the probabilistic distribution of maximum hogging (or sagging) SWBM $M_{swmax,T}$ over a reference period T can be approximated as follows [3]

$$F_{M_{swmax,T}}(m) = \exp\{-\nu_{cy}T[1 - F_{M_{sw,ST}}(m)]\} \quad (3)$$

Using Eq. (3), all the extreme values of SWBM in any reference period can be predicted; however, in practice, it is the characteristic extreme value in any reference period T that is of interest. This value is usually obtained by calculating the mean number of upcrossing a certain level m for a load process. For the present Poisson model, the mean number of upcrossing some level m can be determined as follows:

$$N_{sw}(m,T) = \nu_{cy}T[1 - F_{M_{sw,ST}}(m)] \quad (4)$$

By considering $N_{sw}(m,T) = 1$, the corresponding characteristic value in the reference period can be determined.

From Eqs. (3) and (4), it is clearly seen that the prediction of all extreme values is based on the distribution function $F_{M_{sw,ST}}(m)$ of the maximum SWBM in one loading–offloading cycle, as addressed below.

2.3. Short-term variability

In one typical loading–offloading cycle, the variation in SWBM is referred to as the short-term variability. This is due to difference in weight distribution, which successively change the buoyancy, resulting in changes in SWBMs.

There are two kinds of weight distribution variations: (1) where loading positions remain unchanged but the cargo or weight varies; and (2) where the loading positions change.

Therefore, we define one load condition, with one fixed combination of loading and offloading positions, as one independent load condition. In one load condition, there is one maximum SWBM, which is defined as the intensity of SWBM corresponding to the load condition.

In fact, any one-load condition is the accumulated results of previous load conditions. Therefore, a strong correlation must exist between them. However, an assumption of independence usually results in conservative estimates.

In one typical loading–offloading cycle, the FPSO tanks will be filled in a predetermined sequence to maintain the vessel's hydrostatic stability and keep stress levels in the hull within allowable limits, i.e. it will experience a fixed sequence of loading and offloading.

However, the initially planned loading sequence might not be strictly followed in actual operation, owing to human 'error' or exceptional situations, such as possible early unloading prior to a storm, or repair of cracks. In addition, since the load only needs to fulfil the upper limit to bending moments or shear forces, the captain has significant flexibility in choosing loading or offloading procedures, among the other features, the loading positions.

In effect, this means that the loading–offloading sequence is subject to uncertainty and the duration of any one loading process, with fixed loading and offloading positions, is also uncertain. Therefore, it is accepted that a stochastic point process be adopted to model the renewal of a series of loading processes with fixed loading and offloading positions, i.e. the renewal of load conditions with time.

Strictly speaking, the renewal of load conditions is an operational control process and it is very difficult to find an appropriate model to describe it. From an engineering point of view, a Poisson model is adopted, due to its simplicity, to imitate the renewal of load conditions.

It is well known that, for a Poisson model, mean duration is sufficient to describe the renewal of load conditions. The mean value should be obtained from statistical analysis of actual operational data. However, in the design stage, the actual operational data is not available, so an estimated mean duration value is desirable. Moreover, data for previous FPSOs are not necessarily representative of future models.

In the design stage, the loading capacity of one FPSO is known based on its oil tanks; therefore, its oil production rate is not difficult to determine. Based on these two parameters, the duration of one loading–offloading oil cycle can be approximately estimated. For example, for the FPSOs, Petrojarl I, A and B, the duration of one cycle is 190/50, 920/200 and 1100/40 days, respectively [1]. Then, based on a predetermined loading sequence from the manual, the number of different combinations of loading–offloading positions or the number of load conditions in one loading–offloading cycle can be identified. Dividing the duration of one cycle by the number of load conditions, the expected load condition duration can be estimated.

The uncertainty of the mean duration of load conditions will affect predicted extreme values. According to the principle of order statistics, the predicted extreme value is larger when the mean duration is smaller, because the number of independent load conditions increases. However, when the operational control of SWBM is taken into account, predicted extreme values are basically independent of the number of load conditions. The numerical analysis in Section 5 highlights this point.

Based on the above considerations, the short-term model, shown in Fig. 2, is assumed for a loading–offloading cycle. In this model, any rectangle stands for one of the above-defined load conditions, its height $M_{sw,i}$ ($M_{sws,i}$ and $M_{swh,i}$ refer to sagging and hogging) stands for the intensity of SWBM under the ith load condition, its width $\Delta\tau_i$ ($\Delta\tau_{i,h}$ and $\Delta\tau_{i,s}$ refer to hogging and sagging durations) stands for the duration of the corresponding load condition, while t_i is the renewal instant of load conditions.

Now, the probabilistic distribution of $M_{sw,i}$ has to be determined. According to the above analysis, $M_{sw,i}$ is the maximum SWBM in the ith load condition. Owing to the

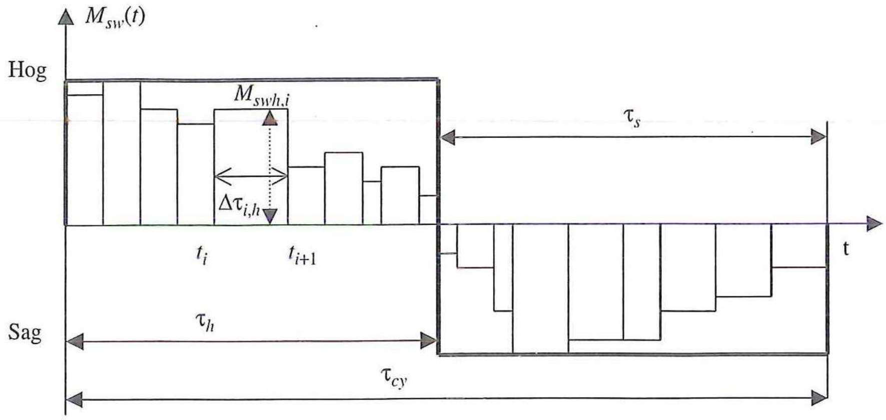

Fig. 2. Modelling short-term SWBM variability.

non-stationarity of the stochastic load process, the probabilistic distributions of all the $M_{sw,i}$ s are different. In particular, the correlation of different $M_{sw,i}$ values is difficult to describe and determine. Therefore, some assumptions have to be made.

In the design stage, the statistical distribution of SWBMs, based on all the design load conditions, can be obtained. In this analysis, the design load conditions are a series of critical load conditions, so the acquired distribution can be assumed as the distribution of the maximum SWBM in one load condition. In addition, the statistical analysis is performed based on all the critical load conditions, so the time variation of SWBMs under different load conditions is ignored. To some extent, it is reasonable to assume $M_{sw,i}$ to be independent and identically distributed random variables M_{sw}.

Based on the above model, the continuous and non-stationary stochastic process in one loading–offloading cycle has been simplified, as an independent and identically distributed stochastic sequence, which follows Poisson law, to update. With such a short-term time variant model, it is very convenient to predict the maximum SWBM in one loading–offloading cycle.

According to the Poisson model, the conditional distribution function of the extreme value of hogging SWBMs $F_{M_{swh,ST}}(m|\tau_h=d_h)$ in the deterministic hogging duration d_h can be determined as follows:

$$F_{M_{swh,ST}}(m|\tau_h=d_h)=F_{M_{swh}}(m)\exp\left\{-\frac{d_h}{E[\Delta\tau_{i,h}]}[1-F_{M_{swh}}(m)]\right\} \tag{5}$$

Here, $F_{M_{swh}}(m)$ is the distribution function of M_{swh} and $E[\Delta\tau_{i,h}]$ is the mean value of the duration $\Delta\tau_{i,h}$ of one hogging load condition. Considering the randomness of τ_h, the unconditional distribution function $F_{M_{swh,ST}}(m)$ is

$$F_{M_{swh,ST}}(m)=\int_0^\infty F_{M_{swh}}(m)\exp\left\{-\frac{t}{E[\Delta\tau_{i,h}]}[1-F_{M_{swh}}(m)]\right\}f_{\tau_h}(t)\mathrm{d}t \tag{6}$$

where $f_{\tau_h}(t)$ is the density function of τ_h. For prediction of extreme values of higher levels, $F_{M_{swh}}(m)$ approaches 1 and can be ignored. In addition, if we assume that τ_h follows the exponential distribution, then, Eq. (6) reduces to

$$F_{M_{swh,ST}}(m)=\frac{1}{1+n_h[1-F_{M_{swh}}(m)]} \tag{7}$$

where $n_h=E[\tau_h]/E[\Delta\tau_{i,h}]$.

With Eq. (7), the hogging or sagging extreme value in one loading cycle can be estimated. Substituting the values into Eqs. (3) and (4), the statistical extreme values of the SWBM for any FPSO in reference time T can be predicted.

2.4. The parent distribution

The parent distribution of SWBM should ideally be constructed from actual operational data. However, at the design stage, only data on assumed load conditions are available; therefore, statistical analysis needs to be performed on the design data.

Based on 453 actual still-water load conditions, Moan and Jiao [12] and Wang and Moan [13] showed that the statistical distribution functions of hogging and sagging SWBMs for FPSO Petrojarl I were well fitted by exponential and Rayleigh distributions, respectively. To cover these and other conditions, a two-parameter Weibull distribution was adopted to model the variation of the SWBM

$$F_{M_{sw,0}}(m)=1-\exp\left[-\left(\frac{m}{a}\right)^b\right] \tag{8}$$

where the scale parameter a and shape parameter b were determined from the mean value and standard deviation, respectively, of the design data of SWBM for the relevant FPSO.

The distribution in Eq. (8) implies an M_{sw} well above the maximum allowable value of SWBMs. By recognizing that on-board control of the SWBM will be exercised, it is reasonable to truncate the Weibull distribution function at the maximum allowable value m_d of SWBM. On the other hand, various abnormal operations and exceptional situations may result in an M_{sw} that exceeds the allowable design value. Guedes Soares [18] introduced a truncated factor to decrease the probability of exceeding the maximum allowable SWBM

$$P[M_{\rm sw} > m_{\rm d}] = T_{\rm R}[1 - F_{M_{\rm sw,0}}(m_{\rm d})] \quad (9)$$

where T_R is the truncated factor, which is a measure of the efficiency of existing on-board controls. Where there is no control, T_R is equal to 1 and the initial Weibull distribution is unchanged. For perfect control, T_R is zero and there is an exact truncated distribution in which the maximum allowable value is never exceeded. When T_R takes a value between 0 and 1, it implies a partially control situation and the initial distribution has to be modified to satisfy the probability axiom.

For values smaller than the allowable value m_d, the modified probability density function $f_{M_{sw}}(x)$ is related to the initial Weibull density function $f_{M_{sw,0}}(x)$ by

$$f_{M_{\rm sw}}(x) = T_{\rm F} f_{M_{\rm sw,0}}(x) \qquad x \leq m_{\rm d} \quad (10)$$

where the correct factor is given by

$$T_{\rm F} = \frac{1 - T_{\rm R}[1 - F_{M_{\rm sw,0}}(m_{\rm d})]}{F_{M_{\rm sw,0}}(m_{\rm d})} \quad (11)$$

Values exceeding the allowable limit are described by the upper tail of another Weibull density function $f_{M_{sw,e}}(m)$. For this new Weibull distribution $F_{M_{sw,e}}(m)$, the probability of exceeding the allowable limit m_d should be as follows:

$$1 - F_{M_{\rm sw,e}}(m_{\rm d}) = T_{\rm R}[1 - F_{M_{\rm sw,0}}(m_{\rm d})] \quad (12)$$

Moreover, at the optimal operational control for SWBM, the probabilistic density functions of the Weibull trail distribution and the truncated distribution are assumed to be equal to each other at m_d that is

$$f_{M_{\rm sw,e}}(m_{\rm d}) = T_{\rm F} f_{M_{\rm sw,0}}(m_{\rm d}) \quad (13)$$

Based on Eqs. (12) and (13), the two parameters of the new Weibull distribution function can be determined.

After the above modifications, the probabilistic density and cumulative distribution functions of SWBM, depending on the extent of undertaken still-water load control, are as follows

$$f_{M_{\rm sw}}(m) = \begin{cases} T_{\rm F} f_{M_{\rm sw,0}}(m) & 0 \leq m \leq m_{\rm d} \\ f_{M_{\rm sw,e}}(m) & m > m_{\rm d} \end{cases} \quad (14)$$

and

$$F_{M_{\rm sw}}(m) = \begin{cases} T_{\rm F} F_{M_{\rm sw,0}}(m) & 0 \leq m \leq m_{\rm d} \\ F_{M_{\rm sw,e}}(m) & m > m_{\rm d} \end{cases} \quad (15)$$

Based on design sagging data for FPSO Petrojarl I, the probabilistic density and cumulative distribution functions of SWBM are displayed in Figs. 3 and 4, respectively.

It is obvious that, with closer operational control of SWBM, values exceeding the allowable value are closer to m_d. In addition, there is little deviation among the mean values and standard deviations of SWBM for different truncation factors.

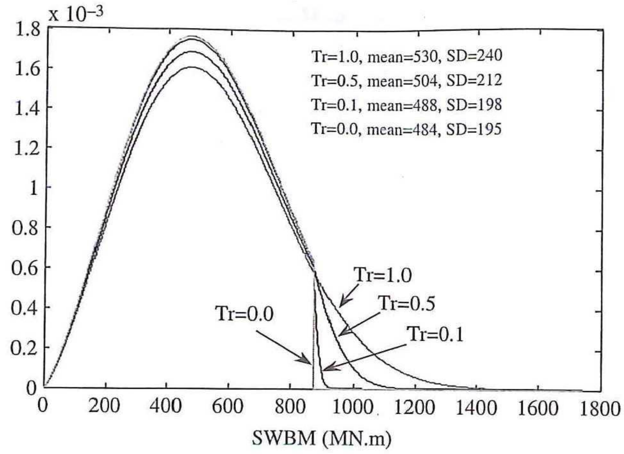

Fig. 3. SWBM density function for FPSO Petrojarl I.

3. Vertical wave-induced bending moment (VWBM) model

In a long-term framework, wave elevation is a non-stationary process that is modelled by taking wave elevation as a sequence of discrete short periods of stationary Gaussian waves, which are characterised by parameters, such as significant wave height and average period.

Short-term VWBM corresponds to a steady (random) sea state, which is considered stationary, with a duration of several hours. Long-term statistics are derived by using the total probability theorem for all short-term sea states over the relevant long-term scatter diagram.

Long-term VWBM is then modelled as a Poisson square-wave process (Fig. 5). The peak of each individual VWBM, M_w, is consequently approximated by the following two-parameter Weibull distribution

$$F_{M_{\rm w}}(m) = 1 - \exp\left[-\left(\frac{m}{g}\right)^q\right] \quad (16)$$

where g and q are the scale and shape parameters, respectively. In the case of non-linear load effects, both parameters in Eq. (16) may be different for hogging and sagging VWBM. Here,

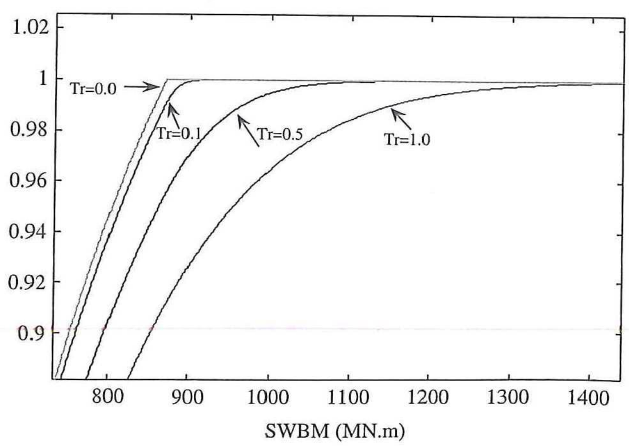

Fig. 4. Upper part of the SWBM distribution function for FPSO Petrojarl I.

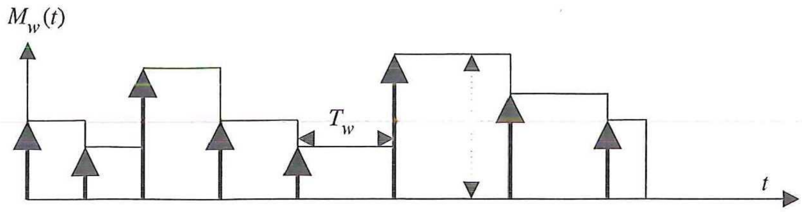

Fig. 5. Long-term VWBM variability (sagging or hogging).

the non-linear difference between hogging and sagging VWBM is taken into account by assuming the traditional regulation values of VWBM to be the characteristic extreme value over 20 years. The corresponding uncertainty is evaluated from direct calculation by the DnV standard code, Nauticus [19].

Even if actual values used for FPSOs are determined by direct analysis, it is convenient to use IACS rule values [20] for hull girder moments as a reference value

$$M_{w,Rs}(-) = -0.11C_1L^2B(C_b + 0.7) \quad \text{(kN m)} \tag{17}$$

sagging moment

$$M_{w,Rh}(+) = 0.19C_1L^2BC_b \quad \text{(kN m)} \tag{18}$$

hogging moment

where L and B are in metres, C_b should not be smaller than 0.6, and C_1 depends on the vessel length L

$$C_1 = \begin{cases} 10.75-(3-0.01L)^{1.5} & 90<L<350 \\ 10.75 & 300<L<350 \\ 10.75-(0.0067L-2.33)^{1.5} & 350<L \end{cases} \tag{19}$$

In addition, independence between individual peaks is assumed due to the long-term statistical average characteristic of the statistical distribution of M_w.

With the Poisson model, the distribution of extreme values for wave-induced bending moments in reference period T can be determined as follows:

$$F_{M_{w\ \max,T}}(m) = F_{M_w}(m)\exp\{-\nu_wT[1-F_{M_w}(m)]\} \approx \exp\{-\nu_wT[1-F_{M_w}(m)]\} \tag{20}$$

The mean number of upcrossing a level of m in reference period T is

$$N_w(m,T) = \nu_wT[1-F_{M_w}(m)] \tag{21}$$

where ν_w is the mean occurrence rate of peak values for VWBMs, which can be determined by long-term statistical analysis of VWBMs. Based on Eqs. (16), (20) and (21), all the characteristic extreme values of the VWBMs can be predicted.

4. Combined extreme values of SWBM and VWBM

The load combination method is based on load models and the correlation between loads. It seems that the assumption of independence between still-water and wave-induced bending moments at least for trading vessels may be adequate [11,13]. No information about correlation is available for FPSOs as yet. Hence, independence between the SWBM and VWBM is assumed.

The combination of hull girder bending moments needs to be achieved separately for hogging and sagging, as shown in Fig. 6. For example, the main contribution to hogging moments occurs in period τ_h. However, there is a possibility that the hogging moments occur in period τ_s if the hogging contribution from wave load is large. Nevertheless, the probability is very small, so, in the context of engineering application, the second contribution is ignored.

In the deterministic hogging duration $\tau_h = d_h$, according to Poisson model in Section 3 (Eq. (20)), the conditional distribution $F_{M_{w,\tau h}}(m|\tau_h = d_h)$ of the maximum VWBM $M_{w,\tau h}$ can be determined as follows

$$F_{M_{w,\tau h}}(m|\tau_h = d_h) = F_{M_{w,h}}(m)\exp\{-\nu_wd_h[1-F_{M_{w,h}}(m)]\} \tag{22}$$

where $F_{M_{w,h}}(m)$ is the distribution function of the long-term hogging peak values $M_{w,h}$ for wave-induced vertical bending moments.

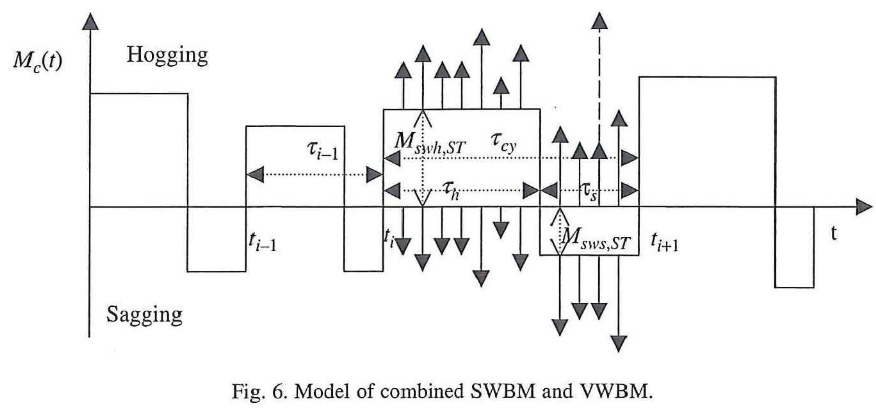

Fig. 6. Model of combined SWBM and VWBM.

Based on the assumption of independence between the SWBM and VWBM, the unconditional distribution $F_{M_{\rm swwh}}(m)$ of the combined maximum hogging bending moment $M_{\rm swwh}$ for hogging duration $\tau_{\rm h}$ can be determined as follows

$$
\begin{aligned}
F_{\rm swwh}(m) &= P[M_{\rm swwh} < m] \\
&= P[M_{\rm swwh} < m | \tau_{\rm h} = t] P[\tau_{\rm h} = t] \\
&= P[M_{\rm swh,ST} + M_{{\rm w},\tau_{\rm h}} < m | \tau_{\rm h} = t] P[\tau_{\rm h} = t] \\
&= P[M_{\rm swh,ST} < m - y | M_{{\rm w},\tau{\rm h}} = y \cap \tau_{\rm h} = t] P[M_{{\rm w},\tau{\rm h}} = y | \tau_{\rm h} = t] P[\tau_{\rm h} = t] \\
&= \int_0^\infty {\rm d}y \int_0^\infty F_{\rm swh,ST}(m - y | M_{{\rm w},\tau{\rm h}} = y \cap \tau_{\rm h} = t) f_{M_{{\rm w},\tau{\rm h}}}(y | \tau_{\rm h} = t) f_{\tau_{\rm h}}(t) {\rm d}t \\
&= \int_0^\infty \frac{\nu_{\tau{\rm h}} F_{M_{\rm swh}}(m-y) f_{M_{\rm w,h}}(y)}{\nu_{\Delta\tau{\rm h}}[1 - F_{M_{\rm swh}}(m-y)] + \nu_{\rm w}[1 - F_{M_{\rm w,h}}(y)] + \nu_{\tau{\rm h}}} + \frac{\nu_{\tau{\rm h}} \nu_{\rm w} F_{M_{\rm swh}}(m-y) f_{M_{\rm w,h}}(y)}{\{\nu_{\Delta\tau{\rm h}}[1 - F_{M_{\rm swh}}(m-y)] + \nu_{\rm w}[1 - F_{M_{\rm w,h}}(y)] + \nu_{\tau{\rm h}}\}^2} {\rm d}y \\
&= \int_0^m \frac{\nu_{\tau{\rm h}} F_{M_{\rm swh}}(y) f_{M_{\rm w,h}}(m-y)}{\nu_{\Delta\tau{\rm h}}[1 - F_{M_{\rm swh}}(y)] + \nu_{\rm w}[1 - F_{M_{\rm w,h}}(m-y)] + \nu_{\tau{\rm h}}} + \frac{\nu_{\tau{\rm h}} \nu_{\rm w} F_{M_{\rm swh}}(y) f_{M_{\rm w,h}}(m-y)}{\{\nu_{\Delta\tau{\rm h}}[1 - F_{M_{\rm swh}}(y)] + \nu_{\rm w}[1 - F_{M_{\rm w,h}}(m-y)] + \nu_{\tau{\rm h}}\}^2} {\rm d}y
\end{aligned}
\tag{23}
$$

where the density function $f_{M_{{\rm w},\tau{\rm h}}}(y|\tau_{\rm h} = t)$ of the maximum VWBM in the deterministic hogging duration $\tau_{\rm h} = d_{\rm h}$ can be determined from Eq. (22), and $\tau_{\rm h}$ is still assumed to follow an exponential distribution as before: $\nu_{\Delta\tau{\rm h}} = 1/E[\Delta\tau_{i,{\rm h}}]$ and $\nu_{\tau h} = 1/E[\tau_h]$. $M_{\rm swwh}$, $M_{\rm swh,ST}$ and $M_{{\rm w},\tau h}$ are, respectively, combined, still-water and wave-induced maximum hogging bending moments for hogging duration $\tau_{\rm h}$. $F_{\rm swh,ST}$ ( ) is the distribution function of $M_{\rm swh,ST}$ and $F_{M_{\rm swh}}$( ) is the parent distribution function of the hogging SWBM $M_{\rm swh}$. Correspondingly, $f_{M_{\rm w,h}}$( ) is the probabilistic density function of the long-term hogging peak values $M_{\rm w,h}$ of VWBM.

Eq. (23) is equivalent to the parent distribution of the combined bending moment, in which the information on uncertainty and time variability of SWBM and VWBM is incorporated satisfactorily. Numerical analysis in Section 5 shows that the Eq. (23) is a very robust solution.

For FPSO Petrojarl I, based on design sagging data, the equivalent combined parent distribution functions, with different operational control for SWBM, are shown in Fig. 7. The rational variation of the combined parent distribution functions with the truncation factors can be clearly seen.

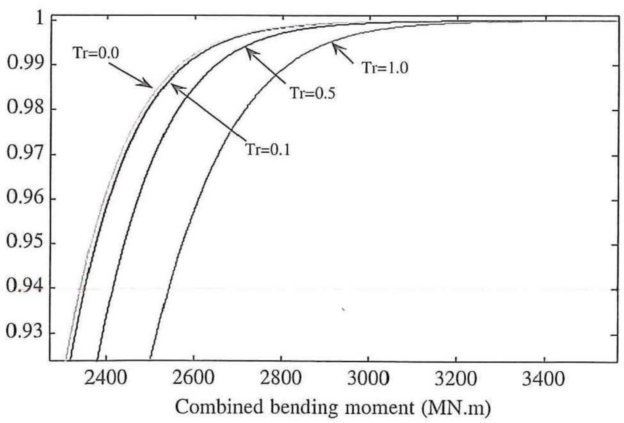

Fig. 7. Upper part of equivalent combined parent distribution function.

If the maximum combined hogging bending moment in one loading–offloading cycle is taken as $M_{\rm swwh}$, the distribution function $F_{M_{{\rm cmax},T}}(m)$ of the combined extreme bending moment $M_{\rm cmax,\,T}$ in reference period T can be obtained analogous to Eq. (3) as follows

$$F_{M_{{\rm c\ max},T}}(m) = \exp\{-\nu_{\rm cy} T[1 - F_{M_{\rm swwh}}(m)]\} \tag{24}$$

Also, the mean number of upcrossing some level m of the combined load process can be determined as follows:

$$N_{\rm c}(m, T) = \nu_{\rm cy} T[1 - F_{M_{\rm swwh}}(m)] \tag{25}$$

Based on Eqs. (23)–(25), all types of the characteristic extreme values for the combined load process can be predicted.

5. Numerical analysis

In the following numerical analysis, three different and one general FPSOs are considered. The necessary initial data are summarized in Table 2, which are based on Refs. [1,12,17].

Here, L, B and $C_{\rm B}$ are length, breadth and block coefficient of the FPSOs under consideration. $\mu_{\rm sw}$ and $\sigma_{\rm sw}$ are the mean value and standard deviation of SWBM, which have been normalized with respect to the rule reference value. H and S stand for respectively, hogging and sagging conditions; $E[\tau_{\rm cy}]$ is the mean duration of one typical loading–offloading cycle. $p_{\rm s}$ and $p_{\rm h}$ are the probabilities in sagging and hogging conditions of one FPSO. $m_{\rm d}$ is the allowable value, which is also normalized with respect to the rule reference value. For FPSO Petrojarl I, the rule reference sagging moment, according to the DnV Rule [12], is 1100 MN m. For comparison, the hogging SWBM is also normalized by it, although the rule hogging moment exists. Similarly, for FPSO A and B, the bending moments are normalized by the rule hogging moment

Table 2
SWBM parameters for three typical FPSOs

FPSO	L (m)	B (m)	C_B	μ_{sw}	σ_{sw}	$E[\tau_{cy}]$ (days)	$p_s(p_h)$	m_d
Petrojarl	194.2	32	0.82	48.2(S)	21.8(S)	4	0.75(0.25)	79.2(S)
Design NS				36.6(H)	19.2(H)			72.6(H)
Petrojarl	194.2	32	0.82	29.7(S)	16.8(S)	4	0.75(0.25)	79.2(S)
Actual NS				12.9(H)	11.8(H)			72.6(H)
FPSO A	278	45	0.85	12.7(S)	9.5(S)	4	1/6(5/6)	60 (S)
Design NS				29.7(H)	22.4(H)			75.5(H)
FPSO B	280	54	0.83	31.1(S)	18.6(S)	28	3/8 (5/8)	73 (S)
Design WA				40.1(H)	22.5(H)			87 (H)

NS, North Sea; WA, West Africa.

of Eq. (1), because the hogging condition is dominated. The reference data for the three FPSOs are all based on design load conditions.

As can be seen for FPSO Petrojarl I (Table 2), based on design load conditions, the SWBM data have larger mean values and standard deviations, but smaller coefficients of variation, compared to actual operational load conditions.

Environmental parameters for two typical operating locations for FPSOs are given in Table 3. Data in the second row of Table 3 is based on Ref. [12]; the remaining data is based on DnV standard code Nauticus [19]. k is the ratio of the characteristic value over 20 years and the rule reference value of VWBM from Eqs. (17)–(19); q is the Weibull shape parameter and T_w is the mean long-term peak values of VWBM.

5.1. Predicted extreme values of SWBM

For FPSO Petrojarl I, operational data of SWBM are available. Therefore, based on the data for Petrojarl I in Table 2, the extreme sagging values of SWBM are initially predicted to evaluate the developed model for SWBM (Table 4).

For the four different truncation models, all the predicted extreme values increase, as expected, with the design period. For extreme values with the same return period, e.g. 20 years, they decrease, as expected, with the decrease in truncation factors. The sensitivity of the predicted extreme values to the number of load conditions also decreases with the decrease in truncation factors.

For the initial parent distribution for SWBM, predicted extreme values, based on design data, are larger than those based on actual operational data. For fully and partially truncated models, regardless of the design or operational data applied, predicted extreme values are very close. This means that, for the initial parent distribution, predicted extreme values are dominated by the uncertainty of SWBM, while for the fully truncated models, they are dominated by the operational control of SWBM.

The results obtained by Wang and Moan [13] (Table 4, in italic) are smaller than those of the developed model, but are very close to the extreme values based on operational data by the developed model. The reason is that the model of SWBM by Wang and Moan [13] is also based on operational data. The slight difference between them is due to the different SWBM model and the different method of predicting extreme values.

5.2. Combined extreme values and combination factors

The purpose of load combination analysis is to determine the factors which can be applied to the secondary loadings, compared to the primary loadings, in the context of the level 1 or 2 method of structural reliability. If $M_{sw,T}$ and $M_{sw,T}$ are, respectively, defined as the characteristic values of SWBM and VWBM in reference time T, the combined characteristic extreme value $M_{c,T}$ can be expressed in the following formats

$$M_{c,T} = M_{w,T} + \psi_{sw} M_{sw,T} \quad \text{or} \quad M_{c,T} := M_{sw,T} + \psi_w M_{w,T} \tag{26}$$

where ψ_w and ψ_{sw} are factors between 0 and 1. In Eq. (26), there are two kinds of expressions. The expression adopted is dependent on which individual load process is dominant. Obviously, load combination factors will depend on the return periods of characteristic extreme values for combined and individual loads. Generally, load combination factors decrease with increasing design lifetime, because the probability of simultaneous occurrence of individual extreme load values decreases with time.

When load combination factors, based on the characteristic extreme values in reference period T, have been determined, the extreme value of the combined load process in reference period T can be approximated as

$$M_{c\ max,T} \approx M_{w\ max,T} + \psi_{sw} M_{sw\ max,T} \tag{27}$$

where $M_{c\ max,T}$, $M_{wmax,T}$ and $M_{swmax,T}$ are the extreme values of the combined, wave-induced and still-water load effects in reference period T, respectively, and are random variables. Eq. (27) is a very convenient formula, used in reliability analysis. It implies that the initial time variant problem, in

Table 3
Long-term VWBM parameters for three typical locations

Location	$k = M_{w,20}/M_{w,rule}$ [a]	q	T_w
North Sea (harsh) Petrojarl I	1.1233	1.0	6.29
North Sea (harsh)	1.1	1.0	10
West Africa (benign)	0.3	0.8	8

[a] $M_{w,rule}$ given by Eq. (17) or (18).

Table 4
Extreme SWBM values for FPSO Petrojarl I

	$M_{sw,T}$(MN m)\T (years)							
	Design data				Operational data			
	1	20	50	100	1	20	50	100
Initial parent distribution (Tr=*1.0)*								
$E[\Delta\tau_i]=1$ h	1508	1710	1765	1805	1204	1413	1472	1515
$E[\Delta\tau_i]=1$ day	1246	1496	1562	1609	942 (*896*)[a]	1191 (*1100*)[a]	1258 (*1155*)[a]	1307 (*1195*)[a]
$E[\Delta\tau_i]=4$ days (1 cycle)	1104	1388	1461	1512	807	1082	1155	1208
Truncated distribution (Tr=*0.5)*								
$E[\Delta\tau_i]=1$ h	1143	1234	1258	1276	1005	1101	1127	1146
$E[\Delta\tau_i]=1$ day	1016	1137	1167	1189	874	999	1031	1053
$E[\Delta\tau_i]=4$ days (1 cycle)	944	1086	1120	1144	782	946	982	1007
Partial truncated distribution (Tr=*0.25)*								
$E[\Delta\tau_i]=1$ h	993	1040	1052	1061	925	974	986	996
$E[\Delta\tau_i]=1$ day	926	990	1006	1017	852	922	938	950
$E[\Delta\tau_i]=4$ days (1 cycle)	886	963	981	994	772	894	913	926
Partial truncated distribution (Tr=*0.1)*								
$E[\Delta\tau_i]=1$ h	914	935	940	944	886	907	912	916
$E[\Delta\tau_i]=1$ day	883	913	920	925	840	885	892	897
$E[\Delta\tau_i]=4$ days (1 cycle)	863	900	909	914	766	872	881	886
Partial truncated distribution (Tr=*0.0)*								
$E[\Delta\tau_i]=1$ h	871	872	872	872	870	871	871	871
$E[\Delta\tau_i]=1$ day	866	871	871	871	833	869	871	871
$E[\Delta\tau_i]=4$ days (1 cycle)	850	871	871	871	762	863	868	870

[a] Data from Wang and Moan (1996) [13].

which loads are stochastic processes, can be reduced to a time invariant process, in which loads are random variables that are easily dealt with. Moreover, the explicit formula, in term of both load effects $M_{w\max,T}$ and$M_{sw\max,T}$, allows model uncertainty to be readily incorporated in the reliability formula.

In reliability analysis of marine structures, annual extreme values are of interest. Eq. (27) implies the following mean values and standard deviations for annual extreme values of the combined load effects

$$\mu_{c,\text{ annual}} = \mu_{w,\text{ annual}} + \psi_s \mu_{sw,\text{ annual}} \tag{28}$$

and

$$\sigma^2_{c,\text{ annual}} = \sigma^2_{w,\text{ annual}} + \psi_s^2 \sigma^2_{sw,\text{ annual}} \tag{29}$$

where $\mu_{c,annual}$, $\mu_{w,annual}$ and $\mu_{sw,annual}$ are mean annual extreme values of combined, wave-induced and still-water load effects respectively; $\sigma_{c,annual}$, $\sigma_{w,annual}$ and $\sigma_{sw,annual}$ are the corresponding standard deviations. In Section 5.3, it will be shown that Eq. (27) gives an accurate representation of the extremes, i.e. Eqs. (28) and (29) give an accurate estimate of the true $\mu_{c,annual}$ and $\sigma_{c,annual}$ of the combined load processes.

5.3. Numerical analysis for combined extreme values

5.3.1. Case study 1: Petrojarl I

Based on the methods outlined in Sections 2–4 and data in Tables 2 and 3, the predicted characteristic extreme values and load combination factors were obtained for FPSO Petrojarl I, and are shown in Tables 5 and 6.

All predicted combined extreme values increase, as expected, with the service period. Similar to the extreme values for SWBM, the combined extreme values, based on the different truncation models, decrease with the decrease in truncation factors. However, load combination factors generally increase with a decrease in truncation factor.

The load combination factors of VWBM are larger than those for SWBM; the reason being that wave-induced load is generally larger than still-water load in the harsh conditions. However, load combination factors for SWBM are of primary interest in this case.

Load combination factors, based on the initial and partially truncated models, decrease, as expected, with increasing design period, which agrees well with the general conclusion of extreme analysis. However, the load combination factors of SWBM, based on the fully truncated model, are nearly constant with varying service period, and are larger than those based on initial and partially truncated models. This fact can be explained as follows.

When the truncated model is adopted, extreme SWBM values are practically constant; the increase in combined extreme values with design period is primarily caused by the increase in extreme VWBM values. Hence, the differences between the combined and VWBM extreme values remain nearly the same and the load combination factors of SWBM

Table 5
Extreme values and combination factors for Petrojarl I 1 (1) $E[\Delta\tau_i]=1$ day

	T (years)							
	Sagging				Hogging			
	1	20	50	100	1	20	50	100
The initial distribution (Tr = *1.0*)								
$M_{sw,T}$ (MN m)	1246	1496	1562	1609	967	1250	1325	1378
$M_{w,T}$ (MN m)	1834	2190	2299	2381	709	2041	2142	2219
$M_{c,T}$ (MN m)	2800 (2364)[a]	3190 (2720)[a]	3300 (2829)[a]	3380 (2911)[a]	2310 (2112)[a]	2680 (2444)[a]	2790 (2545)[a]	2870 (2622)[a]
ψ_{sw}	*0.78*	*0.67*	*0.64*	*0.62*	0.62	0.51	0.49	0.47
ψ_w	0.85	0.77	0.76	0.74	0.79	0.70	0.68	0.67
Partial truncated distribution (Tr = *0.5*)								
$M_{sw,T}$ (MN m)	1016	1137	1167	1189	843	983	1017	1042
$M_{w,T}$ (MN m)	1834	2190	2299	2381	1709	2041	2142	2219
$M_{c,T}$ (MN m)	2640	3000	3110	3190	2210	2550	2650	2730
ψ_{sw}	*0.79*	*0.71*	*0.69*	*0.68*	0.59	0.52	0.50	0.49
ψ_w	0.89	0.85	0.85	0.84	0.80	0.77	0.76	0.76
Partial truncated distribution (Tr = *0.1*)								
$M_{sw,T}$ (MN m)	883	913	920	925	782	822	830	835
$M_{w,T}$ (MN m)	1834	2190	2299	2381	1709	2041	2142	2219
$M_{c,T}$ (MN m)	2570	2930	3040	3120	2170	2500	2610	2680
ψ_{sw}	*0.83*	*0.81*	*0.81*	*0.80*	0.59	0.56	0.56	0.55
ψ_w	0.92	0.92	0.92	0.92	0.81	0.82	0.83	0.83
Truncated distribution (Tr = *0.0*)								
$M_{sw,T}$ (MN m)	866	871	872	872	772	798	799	799
$M_{w,T}$ (MN m)	1834	2190	2299	2381	1709	2041	2142	2219
$M_{c,T}$ (MN m)	2560 (2318)[a]	2920 (2674)[a]	3030 (2783)[a]	3110 (2865)[a]	2160 (2128)[a]	2500 (2460)[a]	2600 (2561)[a]	2680 (2638)[a]
ψ_{sw}	*0.84*	*0.84*	*0.84*	*0.84*	0.58	0.58	0.57	0.58
ψ_w	0.92	0.94	0.94	0.94	0.81	0.83	0.84	0.85

Numbers in italics are the relevant numbers.
[a] By Turkstra's rule.

remain practically constant. Secondly, with increasing design period, extreme SWBM values have reached the maximum allowable level and the maxima of SWBM and VWBM will occur simultaneously with a higher probability. Hence, larger combination factors are obtained.

By comparing Tables 5 and 6, it can be seen that ψ_{sw} slightly increases when $E[\Delta\tau_i]$ is made smaller, i.e. the number of load conditions increases.

For Petrojarl I, which operates in the North Sea, the combined load is dominated by VWBM, which should be taken as the primary load effect. Correspondingly, SWBM should be taken as the secondary load effect. Therefore, the combination factors for SWBM are of interest. Furthermore, because Petrojarl I mainly operate in sagging conditions, the combination factors for sagging SWBM are relevant. These factors are indicated by italics in Tables 5 and 6. Table 5 also shows the combined extreme values obtained by Turkstra's rule. It is apparent that the extreme values are underestimated by Turkstra's rule.

In reliability analysis, annual extreme values and their uncertainty are also pertinent. Table 7 shows the means and standard deviations of annual extreme values for combined and individual loadings in italics. As expected, they decrease with decreasing truncation factors. In addition, based on Eqs. (28) and (29) and the acquired annual load combination factors of SWBM (shown in italics in Tables 5 and 6), the estimated means and standard deviations of annual extreme values are given in Table 7. It is evident that the approximate results are very close to those based on exact combination analysis. This implies that the predicted load combination factors can be applied in combining the annual maximum for SWBM and VWBM in the failure function for reliability analysis.

5.3.2. Case study 2: FPSO A

Table 8 shows load combination factors for FPSO A operating in the North Sea. It is interesting to note that, when the operational control effect is neglected, the predicted hogging SWBM and VWBM extreme values are close. Hence, the corresponding combination factors are nearly equal. When the operational control effect is taken into account, the extreme values of hogging SWBM obviously decrease with decreasing truncation factors, and the corresponding load combination factors are different. Unlike FPSO Petrojarl I, which mostly operates in sagging conditions, FPSO A operates in hogging conditions. Hence, the extreme values of sagging SWBM are very small and much smaller than the corresponding m_d, and the truncation does not affect the extreme values of sagging SWBM. For this reason, the combination factors of sagging SWBM are very small and do not vary with different operational control of SWBM. Since wave loads are dominant, the relevant combination factors are those for hogging SWBM, which are shown in Table 8 (in

Table 6
Extreme values and combination factors for Petrojarl I 1 (2) $E[\Delta\tau_i]=3$ h

	T (years)							
	Sagging				Hogging			
	1	20	50	100	1	20	50	100
The initial distribution (Tr=*1.0*)								
$M_{sw,T}$ (MN m)	1425	1640	1698	1741	1170	1413	148	1528
$M_{w,T}$ (MN m)	1834	2190	2299	2381	1709	2041	2142	2219
$M_{c,T}$ (MN m)	3000	3370	3480	3560	2500	2850	2960	3040
ψ_{sw}	*0.82*	*0.72*	*0.70*	*0.68*	0.68	0.57	0.55	0.54
ψ_w	0.86	0.79	0.78	0.76	0.78	0.70	0.69	0.68
Partial truncated distribution (Tr=*0.5*)								
$M_{sw,T}$ (MN m)	1103	1203	1229	1247	945	1057	1087	1108
$M_{w,T}$ (MN m)	1834	2190	2299	2381	1709	2041	2142	2219
$M_{c,T}$ (MN m)	2760	3120	3230	3310	2340	2680	2780	2860
ψ_{sw}	*0.84*	*0.77*	*0.76*	*0.74*	0.67	0.60	0.59	0.58
ψ_w	0.90	0.88	0.87	0.87	0.82	0.80	0.79	0.79
Partial truncated distribution (Tr=*0.1*)								
$M_{sw,T}$ (MN m)	905	928	934	938	813	839	845	850
$M_{w,T}$ (MN m)	1834	2190	2299	2381	1709	2041	2142	2219
$M_{c,T}$ (MN m)	2660	3010	3120	3200	2280	2610	2710	2790
ψ_{sw}	*0.91*	*0.88*	*0.88*	*0.87*	0.70	0.68	0.67	0.67
ψ_w	0.96	0.95	0.95	0.95	0.86	0.87	0.87	0.87
Truncated distribution (Tr=*0.0*)								
$M_{sw,T}$ (MN m)	871	872	872	872	795	799	799	799
$M_{w,T}$ (MN m)	1834	2190	2299	2381	1709	2041	2142	2219
$M_{c,T}$ (MN m)	2640	3000	3110	3190	2270	2600	2700	2780
ψ_{sw}	*0.93*	*0.93*	*0.93*	*0.93*	0.71	0.70	0.70	0.70
ψ_w	0.96	0.97	0.97	0.97	0.86	0.88	0.89	0.89

Numbers in italics are the relevant numbers.

italics) and exhibit the same variation in truncation factors as FPSO Perojarl I.

5.3.3. Case study 3: FPSO B

Tables 9 and 10 show analogous results for FPSO B operating in West Africa. For FPSOs in benign waters, still-water load is dominant and the combination factor ψ_w of VWBM is the most significant. It can be seen that ψ_w is large, although the extreme values of VWBM are small. The reason being that wave-induced load is a rapid time-variant process and its maxima meet the maxima of SWBM with a greater probability, resulting in higher combination factors. Therefore, combination factors are not only dependent on the relative magnitude of individual loads but also on their time variation. Because FPSO B mainly operates in hogging conditions, the most important combination factors are those for hogging VWBM, as shown in Tables 9 and 10 (in italics). The difference between Tables 9 and 10 is due to the different shape parameters for Weibull distribution of the long-term VWBM peak values. Evidently, combination factors are not sensitive to larger shape parameters.

5.3.4. Case study 4: sensitivity analysis for a generic FPSO

The effect of different relative SWBM and VWBM magnitudes on load combination factors is evaluated as shown in Table 11. Here, their relative magnitude is defined

Table 7
Mean and standard deviation of annual extreme values for Petrojarl I

	Initial Tr=1.0		Tr=0.5		Tr=0.1		Tr=0.0	
	μ_{annul}	σ_{annual}	μ_{annul}	σ_{annual}	μ_{annul}	σ_{annual}	μ_{annul}	σ_{annual}
$E[\Delta\tau_i]=1$ day								
SWBM (MN m)	1293	111	1038	54	888	14	866	6
VWBM (MN m)	1902	152	1902	152	1902	152	1902	152
Combined (MN m)	*2871*	*168*	*2706*	*156*	*2636*	*155*	*2625*	*155*
Eqs. (28) and (29)	*2911*	*175*	*2722*	*158*	*2639*	*152*	*2629*	*152*
$E[\Delta\tau_i]=3$ h								
SWBM (MN m)	1466	94	1122	44	909	10	870	1
VWBM (MN m)	1902	152	1902	152	1902	152	1902	152
Combined (MN m)	*3064*	*162*	*2828*	*155*	*2719*	*154*	*2704*	*154*
Eqs. (28) and (29)	*3104*	*170*	*2845*	*156*	*2729*	*152*	*2711*	*152*

Numbers in italics are the relevant numbers.

Table 8
Combination factors for FPSO A in harsh conditions $E[\Delta\tau_i]=1$ day

	T (years)											
	Initial			Tr=0.5			Tr=0.1			Tr=0.0		
	1	20	100	1	20	100	1	20	100	1	20	100
Sagging conditions												
ψ_{sw}	0.20	0.16	0.14	0.20	0.15	0.14	0.19	0.15	0.14	0.19	0.14	0.14
ψ_w	0.78	0.71	0.68	0.78	0.72	0.71	0.78	0.72	0.73	0.78	0.72	0.73
Hogging conditions												
ψ_{sw}	*0.79*	*0.70*	*0.66*	*0.77*	*0.66*	*0.61*	*0.79*	*0.76*	*0.74*	*0.79*	*0.78*	*0.79*
ψ_w	0.82	0.69	0.64	0.85	0.77	0.74	0.88	0.88	0.88	0.88	0.90	0.91

Numbers in italics are the relevant numbers.

Table 9
Combination factors for FPSO B in benign conditions (1) $h=0.8$, $E[\Delta\tau_i]=1$ day

	T (years)											
	Initial			Tr=0.5			Tr=0.1			Tr=0.0		
	1	20	100	1	20	100	1	20	100	1	20	100
Sagging conditions												
ψ_{sw}	0.93	0.87	0.84	0.92	0.85	0.81	0.91	0.87	0.85	0.90	0.88	0.88
ψ_w	0.84	0.68	0.61	0.83	0.70	0.64	0.82	0.79	0.78	0.81	0.81	0.82
Hogging conditions												
ψ_{sw}	0.96	0.91	0.89	0.95	0.89	0.86	0.95	0.92	0.90	0.94	0.93	0.93
ψ_w	*0.87*	*0.70*	*0.63*	*0.86*	*0.72*	*0.67*	*0.86*	*0.83*	*0.82*	*0.86*	*0.87*	*0.88*

Numbers in italics are the relevant numbers.

Table 10
Combination factors for FPSO B in benign conditions (2) $h=1.332$, $E[\Delta\tau_i]=1$ day

	T (years)											
	Initial			Tr=0.5			Tr=0.1			Tr=0.0		
	1	20	100	1	20	100	1	20	100	1	20	100
Sagging conditions												
ψ_{sw}	0.95	0.91	0.90	0.94	0.89	0.87	0.93	0.91	0.89	0.93	0.91	0.91
ψ_w	0.90	0.79	0.74	0.89	0.79	0.75	0.88	0.85	0.83	0.87	0.86	0.87
Hogging conditions												
ψ_{sw}	0.97	0.94	0.93	0.97	0.93	0.91	0.96	0.94	0.92	0.96	0.95	0.95
ψ_w	*0.92*	*0.80*	*0.75*	*0.91*	*0.81*	*0.76*	*0.91*	*0.87*	*0.85*	*0.91*	*0.90*	*0.91*

Numbers in italics are the relevant numbers.

Table 11
Sensitivity analysis for a general FPSO

T	1 year			20 years			100 years		
Initial distribution (Tr=1.0)									
c	0.5	2.5	5.0	0.5	2.5	5.0	0.5	2.5	5.0
ψ_{sw}	0.94	*0.82*	*0.74*	0.91	*0.71*	*0.64*	0.91	*0.67*	*0.60*
ψ_w	*0.76*	0.84	0.89	*0.64*	0.76	0.85	*0.61*	0.73	0.83
Tr=0.5									
ψ_{sw}	0.93	*0.83*	*0.77*	0.89	*0.76*	*0.71*	0.88	*0.73*	*0.68*
ψ_w	*0.79*	0.89	0.93	*0.68*	0.86	0.92	*0.65*	0.85	0.91
Tr=0.1									
ψ_{sw}	0.94	*0.90*	*0.85*	0.93	0.88	*0.83*	0.92	*0.87*	*0.83*
ψ_w	*0.86*	0.95	0.96	*0.84*	0.95	0.96	*0.83*	0.95	0.97
Tr=0.0									
ψ_{sw}	0.97	*0.93*	*0.87*	0.97	*0.92*	*0.88*	0.98	*0.93*	*0.88*
ψ_w	*0.93*	0.97	0.97	*0.94*	0.97	0.98	*0.96*	0.97	0.98

The numbers in italics are the relevant numbers.

by ratio c of extreme VWBM values over 20 years and the allowable SWBM value. The allowable SWBM value is assumed to be 100 units, while the mean and standard deviation of SWBM are 60 and 30 units, respectively. The mean duration of a load condition and the loading–offloading cycle are 3 h and 4 days, respectively. The probability of sagging or hogging is assumed to be 0.75. The shape parameter and long-term peak VWBM value are 1.0 and 10 s, respectively.

In general, with variations in relative magnitudes for individual loads, load combination factors for SWBM and VWBM vary to different degrees, which are dependent on the operational control of SWBM. For the initial SWBM model, the load combination factors show clear variations with relative magnitudes of SWBM and VWBM, while those for the fully truncated model do not display evident variations, and those for the partially truncated models are between the previous two models.

6. Conclusions

Based on a Poisson square-wave model, a new still-water load model for FPSOs is proposed. Using SWBM design data, SWBM intensity is modelled by a Weibull distribution, which is further modified to account for operational control of SWBM. Long-term variations in wave-induced load effects are also modelled by a Poisson square-wave process. A new approach for combined SWBM and VWBM is derived. A procedure for determining characteristic extreme values for combined, still-water and wave-induced bending moments is established. Valid load combination factors, suitable for typical FPSOs, are provided, in which a time-variant formula for reliability analysis can be reduced to a time-invariant process. Numerical analyses are performed to assess the sensitivity of the results to different parameters.

It is shown that the extreme values of still-water and combined loads can be greatly overestimated if operational control is not accounted for. On the other hand, the control is unlikely to be perfect. Hence, the partially truncated model is recommended to account for control of SWBM. A truncation factor should be determined from actual operational data; however, available operational data for FPSOs are insufficient to reliably determine truncation. Therefore, it is recommended to, conservatively, base it on the operational experience of trading vessels, such as tankers [11], and use a truncation factor of 0.5.

When operational control is ignored, the number of independent still-water load conditions has a significant effect on predicted extreme values. When operational control is taken into account, extreme SWBM values are mainly dominated by the maximum allowable value and insensitive to the number of independent load conditions.

An important goal of this research was to explore valid load combination factors for semi-probability design of FPSOs. In general, combination factors depend on the parent distribution, time variation and relative magnitude of individual loads. For still-water load with a slow time-variation, the SWBM combination factor is mainly dominated by its relative magnitude to wave-induced load. For wave-induced load with a rapid time-variation, the VWBM combination factor is determined, not only by its relative magnitude to still-water load, but also its time variation. The fast time-variation of VWBM will result in an increase in the corresponding combination factor, despite its smaller relative magnitude in some conditions. For instance, in benign waters, the wave-induced load is much smaller than the still-water load, but with larger combination factors.

In the harsh condition of the North Sea, in which wave-induced load is dominant, and for FPSOs mainly operating in sagging conditions, the relevant combination factors of sagging SWBM with a truncation factor of 0.5 are about 0.80, 0.75 and 0.70 for return periods of 1, 20 and 100 years, respectively, while for FPSOs operating in hogging conditions, the respective relevant hogging combination factors are about 0.8, 0.65 and 0.60.

In the benign conditions of West Africa, in which still-water load is dominant, based on a particular FPSO working mainly in hogging conditions, the relevant combination factors of hogging VWBM are about 0.85, 0.70 and 0.65 for return periods of 1, 20 and 100 years, respectively.

It is also shown that maximum combined still-water and wave loading can be well represented by the maxima of still-water and wave loading and the obtained load combination factors. In this way, structural reliability analysis is simplified.

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