

Charge pumping in mesoscopic systems coupled to a superconducting lead

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We derive a general scattering-matrix formula for the pumped current through a mesoscopic region attached to a normal and a superconducting lead. As applications of this result we calculate the current pumped through (i) a pump in a wire, (ii) a quantum dot in the Coulomb blockade regime, and (iii) a ballistic double-barrier junction, all coupled to a superconducting lead. Andreev reflection is shown to enhance the pumped current by up to a factor of 4 in the case of equal coupling to the leads. We find that this enhancement can still be further increased for slightly asymmetric coupling.

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I. INTRODUCTION

It is well known that the quantum transport properties of a mesoscopic system are modified in the presence of a superconducting interface, due to interference between normal and Andreev reflections. Andreev reflection (AR)¹ is the electron-to-hole reflection process which occurs when an electron with energy slightly above the Fermi energy is incident on the boundary between a normal metal and a superconductor: the electron enters the superconductor after forming a Cooper pair, and leaves a hole in the normal metal with energy slightly below the Fermi level which travels back along (nearly) the same path where the electron came from. Because of the phase-coherent character of AR, it is interesting to study its effect on transport in mesoscopic systems, where phase coherence plays an important role. In the last decade, this has led to the discovery of a wealth of quantum interference effects in mesoscopic normal-metal–superconductor (NS) structures,² such as the observation of a large narrow peak in the differential conductance of a disordered NS junction (“reflectionless tunneling”),³ and the discovery of novel Kondo phenomena in quantum dots attached to a normal and a superconducting lead.⁴ In addition, investigations of the conductance in superconductor–carbon-nanotube devices have recently appeared,⁵ which indicate that also in these devices resonant behavior due to AR occurs.

The purpose of this paper is to study the effects of AR on a different type of mesoscopic transport, namely adiabatic quantum pumping. Quantum pumping involves the generation of a dc current in the absence of a bias voltage by periodic modulations of two or more system parameters, such as, e.g., the shape of the system or a magnetic field. The idea was pioneered by Thouless for electrons moving in an infinite one-dimensional periodic potential.⁶ In recent years, adiabatic quantum pumping in quantum dots has attracted a lot of attention.^{7–9} Quantum dots are small metallic or semiconducting islands, confined by gates and connected to electron reservoirs (leads) through quantum point contacts (QPC’s).¹⁰ In addition to investigations of pumping in quantum dots, theoretical ideas have been put forward for charge pumping in carbon nanotubes,¹¹ and for pumping of Cooper pairs.¹² Here we consider a mesoscopic system consisting of an arbitrary normal-metal region, e.g., a quantum dot or

QPC, coupled to a superconductor, as schematically depicted in Fig. 1(a). We start by deriving a general formula for the pumped current through this NS system in terms of its scattering matrix (Sec. II). This is the N -mode generalization of the result of Wang *et al.* for a NS system with single-mode leads.¹³ We then use this result to calculate the current pumped through: a simple peristaltic pump (Sec. III A), a quantum dot in the Coulomb blockade regime (Sec. III B), and a double-barrier junction (Sec. III C), each coupled to a superconducting lead. Comparing these with the pumped current in the corresponding systems attached to normal leads only, shows that AR enhances quantum pumping by up to a factor of >4 in systems with (nearly) symmetric coupling to the leads, while it reduces quantum pumping in the opposite situation of strongly asymmetric coupling.

II. DERIVATION OF NS PUMPING FORMULA

Consider the system in Fig. 1(a). The normal region (which may, e.g., be disordered, or contain a constriction) is coupled to ideal normal leads 1 and 2 containing N modes each. No bias voltage is applied to the system, so all reservoirs are held at the same potential. We assume a constant pair potential $\Delta(\vec{r}) = \Delta_0 e^{i\phi}$ in the superconductor, which is applicable for wide junctions¹⁴ and has previously been used to derive the conductance through a NS junction.¹⁵ We also assume that the NS interface is ideal, i.e., no specular reflection occurs for energies $0 < \epsilon < \Delta_0$, with ϵ the energy measured from the Fermi energy ϵ_F . The scattering matrix S_{NS} of the entire system is given by¹⁶

$$S_{NS}(\epsilon) = \begin{pmatrix} S^{ee}(\epsilon) & S^{eh}(\epsilon) \\ S^{he}(\epsilon) & S^{hh}(\epsilon) \end{pmatrix}, \quad (1)$$

where $S^{ee} - S^{hh}$ are $N \times N$ scattering matrices given by

$$S^{ee}(\epsilon) = r_{11}(\epsilon) + \alpha^2 t_{12}(\epsilon) r_{22}^*(-\epsilon) M_e t_{21}(\epsilon), \quad (2a)$$

$$S^{eh}(\epsilon) = \alpha e^{i\phi} t_{12}(\epsilon) M_h t_{21}^*(-\epsilon), \quad (2b)$$

$$S^{he}(\epsilon) = \alpha e^{-i\phi} t_{12}^*(-\epsilon) M_e t_{21}(\epsilon), \quad (2c)$$

$$S^{hh}(\epsilon) = r_{11}^*(-\epsilon) + \alpha^2 t_{12}^*(-\epsilon) r_{22}(\epsilon) M_h t_{21}^*(-\epsilon). \quad (2d)$$

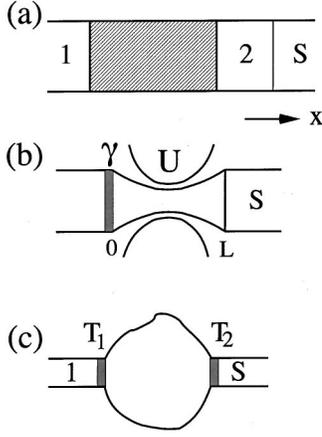


FIG. 1. (a) Normal-metal region (hatched) adjacent to a superconducting lead (S). The normal leads 1 and 2 contain N modes each. (b) NS pump in a 1D wire containing a tunnel barrier γ and an external potential U . (c) Quantum dot coupled via tunneling barriers T_1 and T_2 to a normal (1) and a superconducting (S) lead.

Here $r_{ij}(\epsilon)$ [$t_{ij}(\epsilon)$], $i, j = 1, 2$, denotes the reflection [transmission] amplitude for electrons at energy ϵ , with $0 < \epsilon < \Delta_0$, traveling from lead i [j] to lead i , $\alpha \equiv \exp[-i \arccos(\epsilon/\Delta_0)]$, $M_e \equiv [1 - \alpha^2 r_{22}^*(\epsilon) r_{22}^*(\epsilon)]^{-1}$, and $M_h \equiv [1 - \alpha^2 r_{22}^*(\epsilon) r_{22}(\epsilon)]^{-1}$. The scattering matrix (1) is unitary and satisfies the symmetry relation $S_{NS}(\epsilon, B, \phi)_{ij} = S_{NS}(\epsilon, -B, -\phi)_{ji}$ for time-reversal invariance. Adiabatic quantum pumping in this NS junction is obtained by slow and periodic variations of two external parameters X_1 and X_2 as $X_1(t) = \bar{X}_1 + \delta X_1 \sin(\omega t)$ and $X_2(t) = \bar{X}_2 + \delta X_2 \sin(\omega t + \phi)$. The frequency ω has to be such that $\omega \ll \tau_{\text{dwell}}^{-1}$, with τ_{dwell} the time particles spend in the system, in order for equilibrium to be maintained throughout the entire pumping cycle. The net charge $\delta Q(t)$ emitted into lead 1 due to the modulations δX_1 and δX_2 consists of the amount of negative charge carriers (electrons) minus the amount of positive charge carriers (holes) emitted into lead 1. For infinitesimal variations δX_i , $i = 1, 2$, this charge is given by

$$\delta Q(t) = \frac{e}{2\pi} \sum_{\alpha, \beta \in 1} \left[\text{Im} \left(\frac{\partial S_{\alpha\beta}^{ee}}{\partial X_1} S_{\alpha\beta}^{ee*} - \frac{\partial S_{\alpha\beta}^{he}}{\partial X_1} S_{\alpha\beta}^{he*} \right) \delta X_1(t) + \text{Im} \left(\frac{\partial S_{\alpha\beta}^{ee}}{\partial X_2} S_{\alpha\beta}^{ee*} - \frac{\partial S_{\alpha\beta}^{he}}{\partial X_2} S_{\alpha\beta}^{he*} \right) \delta X_2(t) \right], \quad (3)$$

where the indices α and β are summed over all N modes in lead 1. This expression is obtained along the same lines as the pumped current in a quantum dot coupled to two normal leads⁷ and based on a formula derived by Büttiker *et al.*,¹⁷ see also Ref. 13. The total charge emitted into lead 1 during one period $\tau \equiv 2\pi/\omega$ is found by integrating Eq. (3) over time,

$$Q(\tau) = \frac{e}{2\pi} \int_0^\tau dt \sum_{\alpha, \beta \in 1} \left[\text{Im} \left(\frac{\partial S_{\alpha\beta}^{ee}}{\partial X_1} S_{\alpha\beta}^{ee*} - \frac{\partial S_{\alpha\beta}^{he}}{\partial X_1} S_{\alpha\beta}^{he*} \right) \frac{dX_1}{dt} + \text{Im} \left(\frac{\partial S_{\alpha\beta}^{ee}}{\partial X_2} S_{\alpha\beta}^{ee*} - \frac{\partial S_{\alpha\beta}^{he}}{\partial X_2} S_{\alpha\beta}^{he*} \right) \frac{dX_2}{dt} \right],$$

and rewriting this as an integral over the area A that is enclosed in parameter space (X_1, X_2) during one period. We then find that the total current $I_{\text{NS}} \equiv \omega/2\pi Q(\tau)$ pumped into lead 1 is given by

$$I_{\text{NS}} = \frac{\omega e}{2\pi^2} \int_A dX_1 dX_2 \sum_{\alpha, \beta \in 1} \Pi_{\alpha\beta}(X_1, X_2) \quad (4a)$$

$$\approx \frac{\omega e}{2\pi} \delta X_1 \delta X_2 \sin \phi \sum_{\alpha, \beta \in 1} \Pi_{\alpha\beta}(X_1, X_2), \quad (4b)$$

with

$$\Pi_{\alpha\beta}(X_1, X_2) \equiv \text{Im} \left[\frac{\partial S_{\alpha\beta}^{ee*}}{\partial X_1} \frac{\partial S_{\alpha\beta}^{ee}}{\partial X_2} - \frac{\partial S_{\alpha\beta}^{he*}}{\partial X_1} \frac{\partial S_{\alpha\beta}^{he}}{\partial X_2} \right]. \quad (5)$$

Equation (4) is valid at zero temperature and to first order in the frequency ω ; For $N=1$ it reduces to the single-mode result of Ref. 13 (apart from a factor of 2 for spin degeneracy in the latter). Equation (4b) applies for bilinear response in the parameters X_1 and X_2 , in which case the integral in Eq. (4a) becomes independent of the pumping contour. I_{NS} is of similar generality as the expression for the pumped current in the presence of two normal-metal leads,⁷

$$I_N = \frac{\omega e}{2\pi^2} \int_A dX_1 dX_2 \sum_{\alpha \in 1} \sum_{\beta \in \{1,2\}} \text{Im} \left(\frac{\partial S_{\alpha\beta}^*}{\partial X_1} \frac{\partial S_{\alpha\beta}}{\partial X_2} \right). \quad (6)$$

Here $S_{\alpha\beta}$ denotes the $2N \times 2N$ scattering matrix of the system. Note that the index β in this case is summed over the modes in both lead 1 and lead 2, since electrons can be incident from either lead. In our NS junction, the charge pumped into the right lead is converted into a supercurrent.

In order to illustrate the result (4), we now proceed to apply it to several NS configurations. Unless otherwise noted, we restrict ourselves to the linear-response regime corresponding to weak pumping. In that regime, only the scattering matrix at the Fermi level $\epsilon=0$ is needed.

III. APPLICATIONS

A. Peristaltic pump in a one-dimensional wire

As a first example, consider a “peristaltic” pump formed by a one-dimensional (1D) wire in contact with a superconducting lead; see Fig. 1(b). The pump is operated by periodically opening and closing a tunnel barrier of height γ and varying an external potential U . Calculating the scattering matrix (1) from the Schrödinger equation with potential $V(x) = \gamma \delta(x) + U \Theta[x(L-x)]$ and substituting into Eq. (4b) yields the current

$$I_{\text{NS}} = \frac{3\sqrt{2}}{32} \frac{\omega e}{\pi k^2} \delta U \sin^2(kL). \quad (7)$$

Comparing this with the analogous expression in the case of two normal leads,⁷

$$I_N = \frac{\omega e L}{8\pi^2 k} \delta U + \frac{\omega e}{16\pi^2 k^2} \delta U [\pi \sin^2(kL) - \sin(2kL)], \quad (8)$$

we see that the presence of AR can both enhance and reduce the pumped current. Maximum enhancement of $I_{NS}/I_N = 3\sqrt{2}/2 \approx 2.1$ occurs for short wires such that $2kL \ll 1$. Note that I_{NS} , unlike I_N , has no classical contribution and is entirely due to quantum interference in the wire.

B. Pumped current peak heights in a nearly-isolated quantum dot

As a second example, we study quantum pumping in a quantum dot which is coupled via two tunneling barriers with transmission probabilities T_1 and T_2 to a normal and a superconducting single-mode lead, see Fig. 1(c). Pumping is achieved by periodic variations of the strength of the two tunneling barriers V_1 and V_2 [with $V_{1,2} \equiv \sqrt{(1-T_{1,2})/T_{1,2}}$ for delta-function barriers] as $V_1(t) = \bar{V}_1 + \delta V_1 \sin(\omega t)$ and $V_2(t) = \bar{V}_2 + \delta V_2 \sin(\omega t + \phi)$. We are interested in the regime of high barriers (where transmission is low, $T_1, T_2 \ll 1$), and weak pumping ($\delta V_m \ll \bar{V}_m, m=1,2$). At low temperatures such that $k_B T \ll \Delta < E_C$ [with Δ the single-particle level spacing and $E_C = e^2/C$ the charging energy of the dot, C being the total capacitance] the quantum dot then remains in the Coulomb blockade regime during the whole pumping cycle and transport through the dot is mediated by resonant transmission through a single level.¹⁸ Substituting the appropriate scattering matrix^{19,20} into Eq. (4b) yields, up to lowest order in T_1 and T_2 and for thermal energies less than the total decay width Γ into the leads $k_B T < \Gamma \ll \Delta$ [with $\Gamma = \Gamma_1 + \Gamma_2 \equiv \hbar \nu (T_1 + T_2)$ and ν the attempt frequency, the inverse of the round-trip travel time between the two barriers],

$$I_{NS} = \frac{\omega e}{4\pi} \delta V_1 \delta V_2 \sin \phi \frac{T_1^{7/2} T_2^{5/2} (\sqrt{T_1} + \sqrt{T_2})}{\left[\frac{1}{4} (T_1^2 + T_2^2) + \left(\frac{\epsilon - \epsilon_{\text{res}}}{\hbar \nu} \right)^2 \right]^3}. \quad (9)$$

Here ϵ_{res} denotes the resonance energy for a completely isolated dot ($T_1 = T_2 = 0$). Note that Eq. (9) is not symmetric with respect to T_1 and T_2 , in contrast with the conductance¹⁵ $G_{NS} = (e^2/h) (T_1^2 T_2^2 / [\frac{1}{4}(T_1^2 + T_2^2) + [(\epsilon - \epsilon_{\text{res}})/\hbar \nu]^2])^2$ through this system. This is due to the fact that I_{NS} depends on $\partial S/\partial X$, whereas G_{NS} depends on the transmission eigenvalues, the eigenvalues of the matrix $t_{12}^\dagger t_{12}$. Compared to the pumped current in a dot coupled to two normal leads,¹⁹

$$I_N = \frac{\omega e}{4\pi} \delta V_1 \delta V_2 \sin \phi \frac{(T_1 T_2)^{3/2} (\sqrt{T_1} + \sqrt{T_2}) (T_1 + T_2)}{\left[\frac{1}{4} (T_1 + T_2)^2 + \left(\frac{\epsilon - \epsilon_{\text{res}}}{\hbar \nu} \right)^2 \right]^2}, \quad (10)$$

we find that

$$\frac{I_{NS}}{I_N} = \frac{T_1^2 T_2 \left[\frac{1}{4} (T_1 + T_2)^2 + \left(\frac{\epsilon - \epsilon_{\text{res}}}{\hbar \nu} \right)^2 \right]^2}{(T_1 + T_2) \left[\frac{1}{4} (T_1^2 + T_2^2) + \left(\frac{\epsilon - \epsilon_{\text{res}}}{\hbar \nu} \right)^2 \right]^3}. \quad (11)$$

For symmetric coupling ($T_1 = T_2$) close to resonance, ($\epsilon - \epsilon_{\text{res}} \ll \hbar \nu T_{1,2} \equiv \Gamma_{1,2}$, AR enhances the pumped current by a factor of 4.²¹ In case of strongly asymmetric coupling close to resonance, on the other hand, AR reduces the pumped amplitude: $I_{NS}/I_N \sim (T_1/T_2)^2 \ll 1$ for $T_1 \ll T_2$, and $I_{NS}/I_N \sim T_2/T_1 \ll 1$ for $T_2 \ll T_1$. The enhancement by a factor of 4 for symmetric barriers consists of two contributions of a factor of 2: one factor of 2 is due to the contribution of both electrons and holes to the current, which is also responsible for the doubling of conductance $G_{NS}/G_N = 2$ in this NS structure.¹⁵ A second factor of 2 comes from the asymmetry of the NS dot with respect to injection of charge carriers into the leads, since electrons can only leave the system through the left, normal lead. This leads to an extra doubling of the pumped current compared to the normal case where electrons are injected into both the left and the right leads. This extra factor of 2 does not occur in the presence of an applied bias, as for conductance, since the bias causes charge carriers to flow from one side to the other in both the normal and the NS system. Note that due to the asymmetry of Eq. (11) with respect to T_1 and T_2 the maximum attainable enhancement is even larger than 4: for a slightly asymmetric junction (with $T_1/T_2 \sim 1.26$) one obtains $I_{NS}/I_N \sim 4.23$. In this case quantum interference between electrons and holes in the NS system is maximal. If the barrier asymmetry is further increased, I_{NS}/I_N decreases and eventually becomes less than 1 for strongly asymmetric coupling, when pumping is dominated by one barrier only.

At temperatures higher than the decay width, $\Gamma \ll k_B T \ll \Delta$, the pumped current exhibits Coulomb oscillations as a function of an applied gate voltage.¹⁹ The peak heights of these oscillations can be obtained by thermally averaging Eq. (9) as $I_{NS, \text{peak}} \equiv -\int d\epsilon I_{NS} f'(\epsilon, T) \approx (1/4k_B T) \int_{\epsilon_{\text{res}} - (1/2)\hbar \nu \sqrt{T_1 + T_2}}^{\epsilon_{\text{res}}} d\epsilon I_{NS}$, where $f(\epsilon, T) \equiv [1 + \exp(\epsilon/k_B T)]^{-1}$ denotes the Fermi function. We obtain

$$I_{NS, \text{peak}} = \frac{\omega e (8 + 3\pi) \hbar \nu}{16\pi k_B T} \delta V_1 \delta V_2 \sin \phi \frac{T_1^{7/2} T_2^{5/2} (T_1^{1/2} + T_2^{1/2})}{(T_1^2 + T_2^2)^{5/2}}. \quad (12)$$

This thermal average does not explicitly include the effect of the charging energy E_C on the pumped current. A full linear response theory for Coulomb blockade conductance oscillations including charging energy was developed in Ref. 18. There it was shown that for temperatures $k_B T \ll \Delta$ only one level N_{min} participates in the transport [N_{min} is defined as the level which minimizes the energy $E_N + U(N) - U(N-1) - \epsilon_F$, with E_N the energy of the N th level of the dot, and $U(N)$ the electrostatic energy of a dot containing N electrons], and the oscillation peaks are well described by the thermal average. One can show that for the same reason the pumped current peaks in this temperature range are well de-

scribed by the thermal average (12), with the understanding that $T_{1,2}$ in Eq. (12) refer to the level N_{\min} . From Eq. (12) and the analogous normal-state result,¹⁹

$$I_{N,\text{peak}} = \frac{\omega e(2+\pi)\hbar\nu}{16\pi k_B T} \delta V_1 \delta V_2 \sin\phi \frac{(T_1 T_2)^{3/2}(T_1^{1/2} + T_2^{1/2})}{(T_1 + T_2)^2}, \quad (13)$$

we obtain

$$\frac{I_{NS,\text{peak}}}{I_{N,\text{peak}}} = \left(\frac{8+3\pi}{2+\pi} \right) \frac{T_1^2 T_2 (T_1 + T_2)^2}{(T_1^2 + T_2^2)^{5/2}}. \quad (14)$$

Also here, AR enhances the pumped current in case of symmetric tunnel barriers, while a reduction occurs for asymmetric barriers. Maximum enhancement of $I_{NS,\text{peak}}/I_{N,\text{peak}} \sim 2.55$ is reached for $T_1/T_2 \sim 1.292$. This factor is less than 4, because the average over energy ϵ from which the peak heights (12) are obtained also involves contributions of I_{NS} [Eq. (9)] further away from resonance, for which I_{NS}/I_N is much less than 4 [consider, e.g., Eq. (11) for $\epsilon - \epsilon_{\text{res}} = \frac{1}{2}\hbar\nu\sqrt{T_1^2 + T_2^2}$]. This results in lower maximal enhancement of the pumped current peaks (14) at higher temperatures $k_B T \gg \Gamma$.

Another interesting result is obtained in this system by relaxing the assumption of weak pumping and considering quantum pumping by varying the two tunneling barriers in such a way that the loop which describes the pumping cycle in parameter space encircles the entire resonance line. For normal-metal contacts this problem has recently been studied²⁰ and led to the prediction that at zero temperature the charge transferred during one pumping cycle is quantized, $Q = e$ (for spinless electrons). The transferred charge in our NS system is obtained by substituting the scattering matrix of a 1D double-barrier junction given in Ref. 19 into Eq. (4a) and integrating over the resonance line $V_1^{-1} + V_2^{-1} = |(\epsilon - \epsilon_{\text{res}})/\hbar\nu| \ll 1$, with $V_i^{-1} \equiv \sqrt{T_i/(1-T_i)}$ for $i=1,2$. We then obtain

$$Q = \frac{3\sqrt{2}e}{2} \int_{-1}^1 dz \frac{(1+z)^2(1-z^2)^3}{[1+6z^2+z^4]^{5/2}} = e, \quad (15)$$

so AR neither enhances nor reduces quantum pumping. This occurs because charge is effectively transferred by a shuttle mechanism (first through one barrier and then through the next), which is unaffected by Andreev interference effects and fixed by the pumping loop. Since only pairs of electrons can enter the superconductor, but the strong Coulomb interaction (charging energy) forbids simultaneous pumping of two electrons with opposite spin, this pumping process is *not allowed* in a nearly closed NS quantum dot. As pointed out in Ref. 22 it can, however, occur in a double-barrier junction in which electron-electron interactions may be neglected. Both in case of two normal and in case of one normal and one superconducting single-mode lead a quantized amount of charge of $2e$ is then transferred during each pumping cycle.²²

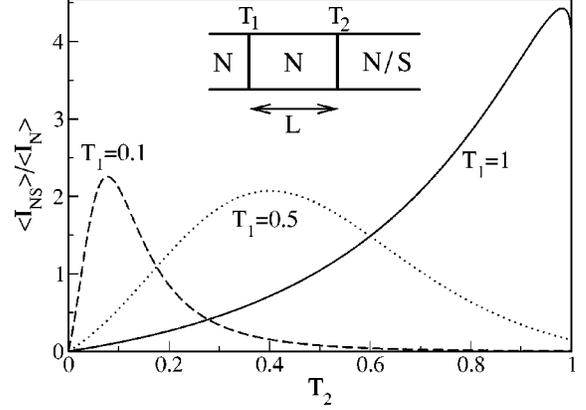


FIG. 2. Ratio of the average pumped currents $\langle I_{NS} \rangle$ and $\langle I_N \rangle$ in ballistic $NININ$ and $NINIS$ junctions as a function of T_2 for $T_1 = 1, 0.5$, and 0.1 . The inset shows the double-barrier junction considered.

C. Ballistic double-barrier junction

Finally, we compare the pumped current I_N in a NI_1NI_2N junction vs I_{NS} in a NI_1NI_2S junction, where $I_{1,2}$ denote tunnel barriers with transmission probability per mode $T_{1,2}$; see the inset of Fig. 2. In linear response, assuming N -mode leads and ballistic transport between the barriers, the pumped currents are given by

$$I_N = I_C T_1^{3/2} T_2^{3/2} \sum_{n=1}^N \frac{A_N + B_N \cos \phi_n + C_N \sin \phi_n}{(D_N + E_N \cos \phi_n)^2}, \quad (16)$$

and

$$I_{NS} = 4I_C T_1^3 T_2^{5/2} \sum_{n=1}^N \frac{A_{NS} + B_{NS} \cos \phi_n + C_{NS} \sin \phi_n}{(D_{NS} + E_{NS} \cos \phi_n)^3}. \quad (17)$$

Here $I_C \equiv (\omega e/2\pi) \delta V_1 \delta V_2 \sin\phi$,²³ and A_N, \dots, E_{NS} are given by $A_N \equiv \sqrt{R_1 T_2} + \sqrt{R_2 T_1}$, $B_N \equiv -\sqrt{R_1 R_2}(\sqrt{R_1 T_2} + \sqrt{R_2 T_1})$, $C_N \equiv -1 + R_1 R_2 - \sqrt{R_1 R_2 T_1 T_2}$, $D_N \equiv 1 + R_1 R_2$, $E_N \equiv -2\sqrt{R_1 R_2}$, $A_{NS} \equiv 2\sqrt{R_2}$, $B_{NS} \equiv \sqrt{R_2 T_1 T_2} - \sqrt{R_1}(2 - T_2)$, $C_{NS} \equiv -(\sqrt{T_1(2 - T_2)} + \sqrt{R_1 R_2 T_2})$, $D_{NS} \equiv (1 + R_1)(1 + R_2)$, and $E_{NS} \equiv -4\sqrt{R_1 R_2}$. For $L \gg \lambda_F$, with λ_F the Fermi wavelength and $NT_i \gg 1$ the current is not dominated by a single resonance and the phases ϕ_n are uniformly distributed from 0 to 2π . Replacing the sums in Eqs. (16) and (17) by integrals over ϕ_n then yields the average pumped currents

$$\langle I_N \rangle = I_C \frac{T_1^{3/2} T_2^{3/2}}{\pi(1 - R_1 R_2)^2} [2(-1 + R_1 R_2 + \sqrt{R_1 R_2 T_1 T_2}) - \pi(\sqrt{R_1 T_2} + \sqrt{R_2 T_1})], \quad (18)$$

and

$$\langle I_{NS} \rangle = 4I_C \frac{T_1^3 T_2^{5/2}}{\pi(D_{NS}^2 - E_{NS}^2)^{5/2}} [2C_{NS} D_{NS} (D_{NS}^2 - E_{NS}^2)^{1/2} - A_{NS} (E_{NS}^2 + 2D_{NS}^2)/2 + 3B_{NS} D_{NS} E_{NS}/2], \quad (19)$$

whose ratio is plotted in Fig. 2. As for a nearly isolated quantum dot, $\langle I_{NS} \rangle / \langle I_N \rangle$ is largest in case of nearly symmetric coupling. Only for $T_1 = 1$ is the maximum enhancement by a factor of >4 obtained, since for $T_1 < 1$ fewer electrons reach the NS interface due to normal reflections at the barriers, which in the absence of resonances reduces the effect of Andreev reflection.

IV. CONCLUSION

In conclusion, we have studied adiabatic quantum pumping in mesoscopic NS systems. Compared to the conductance in these systems we predict two striking differences: (i) For a nearly isolated quantum dot with symmetric ($T_1 = T_2$) tunneling barriers, and a transparent ($T_1 = T_2 = 1$) double-

barrier NS junction, AR enhances the pumped current by a factor of 4, which is twice the maximum enhancement of the conductance in these systems. (ii) In the case of quantum pumping this enhancement is not an absolute maximum, whereas in the case of conductance it is. These differences are due to, respectively, the absence of an external bias and the asymmetric dependence on the tunnel barriers in case of quantum pumping. We hope that these fascinating effects of Andreev reflection on quantum pumping will find experimental confirmation, e.g., in present-day available nearly closed quantum dots.²⁴

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