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Transportation Research Symposium 2025 Conference Abstracts

# Reactive Scheduling of Integrated Quayside Operations Under Uncertainty: A Mixed-Integer Linear Programming Approach

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## Abstract

Planning of container terminal operations is a complex task, which requires the accurate scheduling of operations that are highly interrelated and uncertain. This study aims to investigate the integration of quayside operational planning functions under uncertain parameters by applying a reactive approach. A mixed integer linear programming (MILP) model is formulated to optimally assign and schedule quay cranes to multiple vessels simultaneously. The model will derive a baseline schedule that minimises the cost of waiting and departure delays of vessels. To address the uncertainty, a reactive strategy is formulated to generate a rescheduling plan when two types of disruptions, delays in vessel arrivals and quay crane breakdowns, occur during the operation. The reactive strategy will take the baseline schedule as input and derive a reactive schedule that minimizes the cost of deviations from the baseline schedule. The numerical experiments demonstrate the performance and effectiveness of the proposed approach to solve the integrated formulation under uncertainty.

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*Keywords:* Container terminals; port optimization; quay crane scheduling; uncertainty; reactive strategy.

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## 1. Introduction

Around 80% of the global trade by volume is carried by maritime transportation and handled by seaports around the world, and seaborne container trade accounts for more than 60% of global seaborne trade (Raeesi et al., 2023). Container terminals play a vital role in global containerized trade, serving as the primary gateways that connect

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maritime and land transportation (Geerlings & Van Duin, 2011). According to UNCTAD (2023), a 3% annual growth in containerized trade is predicted for the years 2024 and 2028. With the increased growth in containerized trade and maritime transport, container terminals are busier than ever, and there is a growing need for improved operational efficiency. Thus, the focus on the optimization of container terminal operations has gained more attention from terminal operators as well as from the academic community in recent years (Tengecha & Zhang, 2022; Weerasinghe et al., 2024).

There are a number of planning functions associated with this vessel handling operation (de Silva et al., 2024). This includes the Berth Allocation Problem (BAP), which determines the assigned berths and berthing durations for incoming vessels, the Quay Crane Assignment Problem (QCAP), which determines how many cranes should be assigned to vessels to perform loading and discharging operations, and the Quay Crane Scheduling Problem (QCSP), which determines the sequence of tasks of each crane (Weerasinghe et al., 2024). QCSP is the most complex among all three planning functions, with a higher number of variables and additional constraints, such as maintaining a safety margin between two operating cranes and the physical restriction of moving cranes crossing each other (Rodrigues & Agra, 2024). The decisions made when solving these operational problems are highly interrelated and have a significant impact on one another (Bierwirth & Meisel, 2015). For example, the berthing duration of a vessel depends on the number of QCs assigned and how well the cranes are scheduled to perform the operation (Xiang et al., 2018). Due to the interrelated nature of these operational problems, a growing trend of studying the integrated formulations of problems has been identified in recent years (Rodrigues & Agra, 2022).

The berth planning team of the terminal prepares a baseline schedule, including the berthing schedule, which includes allocated berths and berthing durations of vessels, and crane schedules, which include assigned cranes to each vessel and their detailed schedules based on the information declared by the vessels in advance (Xiang & Liu, 2021). However, vessel handling operations are frequently affected by several uncertain events, including delays in vessels' arrivals and breakdown of QCs, making the implementation of the baseline schedule infeasible (Iris & Lam, 2019). Hence, significant importance is there to addressing those uncertainties in operational problems to minimize the delays in vessel handling operations. However, uncertainty has been rarely incorporated into operational problems in the literature, specifically in the integrated formulations of the problems, due to the added complexity. Generally, two approaches are applied to address the uncertainty: proactive and reactive. Proactive approaches develop the baseline plan by incorporating uncertainty beforehand by allocating time buffers and or reserving QCs, while reactive approaches develop the baseline plan under a deterministic environment and use a reactive strategy to reschedule the baseline plan when disruption occurs due to uncertain events (Rodrigues & Agra, 2022).

Most of the papers studying integrated operational problems have used proactive approaches to address uncertainties (Rodrigues & Agra, 2022; Weerasinghe et al., 2023). However, when severe disruption occurs, the results obtained from these approaches may become infeasible, or they may lead to poor resource utilization due to the use of excessive buffers. On the other hand, reactive approaches can reschedule the baseline schedule in real-time when a disruption occurs, while reducing resource waste (Ji et al., 2022). There is a necessity to study the use of reactive approaches to address the uncertainty in integrated formulations of operational problems.

This study aims to fill the research gap by proposing a methodology to integrate the three planning functions under uncertainty using a reactive approach. First, a mixed integer linear programming (MILP) model is developed to optimally assign and schedule QCs to multiple vessels simultaneously. Then, a reactive strategy is incorporated into the model to reschedule the baseline plan when two types of disruptions, delays in vessels' arrivals and QC breakdowns, occur during the operation. Finally, the proposed model is compared with the existing models in the literature, and numerical experiments are conducted to test the validity of the model.

## 2. Methodology

This study assumes the berthing schedules are determined in advance according to the service agreements and aims to propose a mathematical model to derive the crane schedule for multiple vessels simultaneously to further optimize the berthing schedule by reducing the actual berthing duration. A mixed integer linear programming (MILP) technique is used to develop a strategy to solve the integrated problem under uncertainty. The conceptual framework to formulate the integrated model, along with the reactive strategy, is shown in Figure 1.

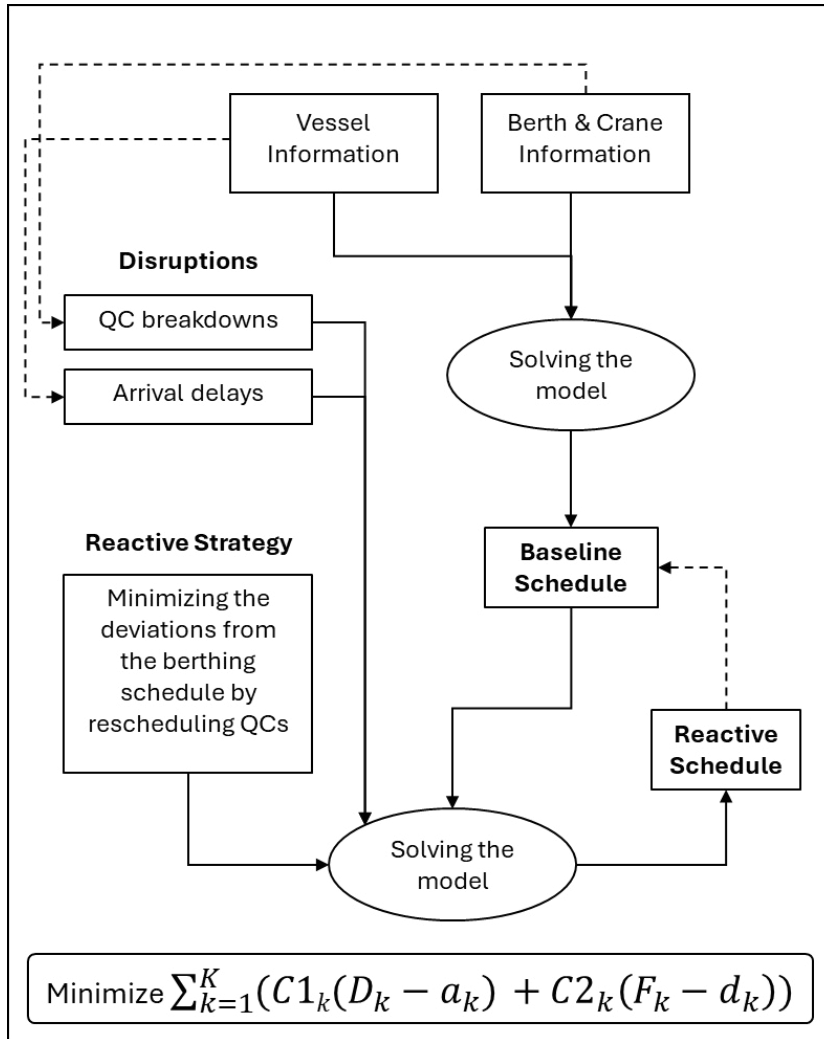


Figure 1: Conceptual framework

First, a MILP model that considers practical constraints such as the non-overlapping constraint and the safety distance between cranes is formulated to derive a baseline schedule that minimizes the cost of waiting and handling time of vessels under a deterministic environment. Then, a reactive strategy is formulated to derive a rescheduling plan when disruptions like deviations of vessel arrival times and crane breakdowns occur at the time of implementation of the baseline schedule. The reactive strategy aims to generate a rescheduling plan that reduces delays from the baseline berthing schedule after the realization of an uncertain event by rescheduling only QCs.

### 3. Modal Formulation

#### Objective Function:

$$\text{Minimize } \sum_{k=1}^K (C1_k(D_k - a_k) + C2_k(F_k - d_k)) \tag{1}$$

The objective function consists of two parts: berthing after the expected berthing time and finishing the handling after the requested departure time. The objective is to minimize the total cost of waiting time and departure delay for all the vessels in the planning horizon.

**Constraints:**

$$\sum_{k=1}^K \sum_{i \in I_v} X_{kijat} \leq 1; \quad \forall q, t \quad (2)$$

$$\sum_{q=1}^Q X_{kijat} \leq 1; \quad \forall k, j \in J_k, t \quad (3)$$

$$\sum_{q=1}^Q \sum_{h=1}^t X_{kjqt} \geq Pkj \cdot V_{kit}; \quad \forall k, j \in J_k, t \quad (4)$$

$$\sum_{a=1}^Q \sum_{t=1}^T X_{kjqt} = Pkj; \quad \forall k, j \in J_k \quad (5)$$

$$V_{kjt} \geq R_{kt} \quad \forall k, j \in J_k, t \quad (6)$$

$$Fk = T - \sum_{t=1}^T R_{kt} + 1 \quad \forall k \quad (7)$$

$$\sum_{t=1}^T R_{kt} \geq 1 \quad \forall k \quad (8)$$

$$lcqt \leq ll_{ki} + M \cdot (1 - X_{kijat}) \quad \forall k, j \in J_k, q, t \quad (9)$$

$$lcqt \geq ll_{ki} - M \cdot (1 - X_{kijat}) \quad \forall k, j \in J_k, q, t \quad (10)$$

$$lcqt \leq G \quad \forall q, t \quad (11)$$

$$lcqt \geq 1 \quad \forall q, t \quad (12)$$

$$lcqt \geq lc_{a+1, t} - m \quad \forall q \neq Q, t \quad (13)$$

$$\sum_{t=1}^{a_k+1} Y_{kt} \leq 0 \quad \forall k \quad (14)$$

$$\sum_{a=1}^Q X_{kijat} \leq Y_{kt} \quad \forall k, j \in J_k, t \quad (15)$$

$$Y_{kt} \leq Y_{k,t+1} + R_{k,t+1} \quad \forall k, t \quad (16)$$

$$Y_{k,t+1} \leq 1 - R_{kt} \quad \forall k, t \quad (17)$$

$$S_{kw} + S_{wk} = 1 \quad \forall k, w, k \neq w \quad (18)$$

$$l_k + jj_k \leq lw + M(1 - S_{kw}) + M(1 - Y_{kt}) + M(1 - Y_{wt}) \quad \forall k, w, k \neq w, t \quad (19)$$

$$B_{kt} \geq Y_{kt} - Y_{k,t-1} \quad \forall k, t \in [2, T] \quad (20)$$

$$\sum_{t=1}^T B_{kt} = 1 \quad \forall k \quad (21)$$

$$\sum_{t=1}^T B_{kt} \cdot t = D_k \quad \forall k \quad (22)$$

$$F_k, D_k, lc_{qt} \geq 0 \quad \forall k, t \quad (23)$$

$$X_{kijat} \geq 0 \quad \forall k, j \in J_k, t \in [t', t' + rt] \quad (24)$$

$$X_{kijat} = X_{kijat}^* \quad \forall k, j \in J_k, q, t \in [1, ut] \quad (25)$$

The foundation of the schedule lies in the QC Assignment decisions. These constraints ensure that a QC is assigned to only one bay at a time, and a bay can be worked on by only one QC at a time. Crucially, they mandate that the sum of all assigned work segments for a bay must precisely meet the bay's total required handling time. This work assignment directly influences the physical location constraints. When a QC is assigned to a specific bay, its location along the quay is fixed to that bay's position. This movement is tightly controlled by the safety distance rule, which dictates a minimum operational gap between adjacent QCs at all times to prevent collision and ensure safety. The cranes must also remain within the total limits of the quay.

The assignment of work, in turn, is directly linked to the vessel state constraints. If any QC is working on a vessel's bay, the vessel must be in a berthed state. The first time segment where a vessel transitions to the berthed state defines the single mooring time, which establishes the official berthing time used in the waiting cost component of the objective function. Once all required work segments on all bays are completed, the vessel finish variable is set, which defines the finish time used in the departure delay cost component. Immediately after the work is finished, the constraints ensure the vessel is no longer in the berthed state, modeling its departure from the quay.

Finally, the entire schedule must respect the spatial and temporal vessel non-overlapping constraints. These ensure that if two vessels are berthed at the quay simultaneously, their physical spaces along the quay do not overlap. This is managed by considering the relative arrival sequence of the two vessels, their fixed lengths (number of bays), and their starting positions on the quay. In essence, work assignment drives berthing state and location, which must adhere to safety and non-overlapping rules, ultimately determining the berthing and finish times used to calculate and minimize the total operational cost.

### 4. Results and Discussions

The results of solving the model are presented in Figure 1. Figure 2 displays how cranes are rescheduled for a vessel delay. Figure 3 displays how cranes are rescheduled for a crane breakdown.

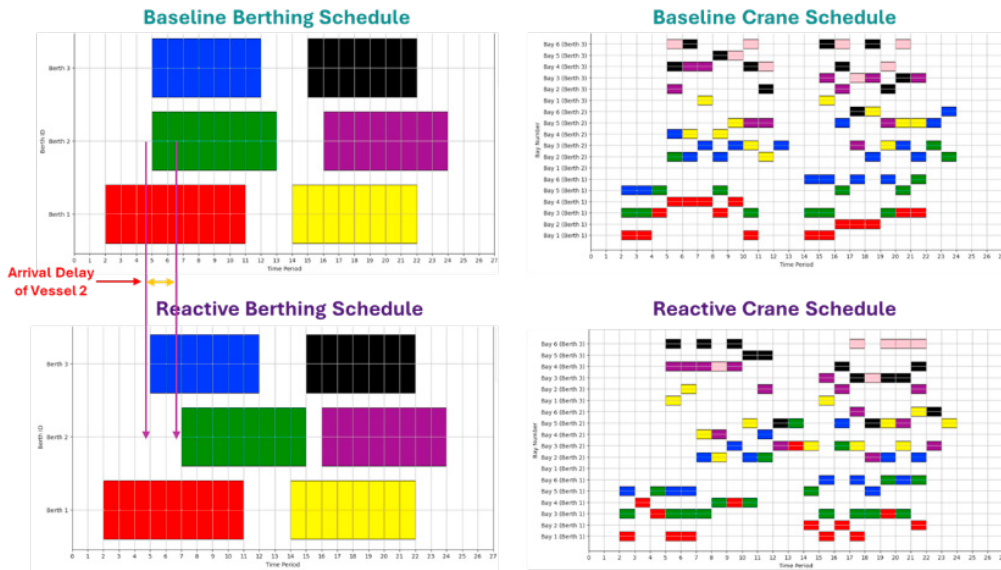


Figure 2: Vessel delay

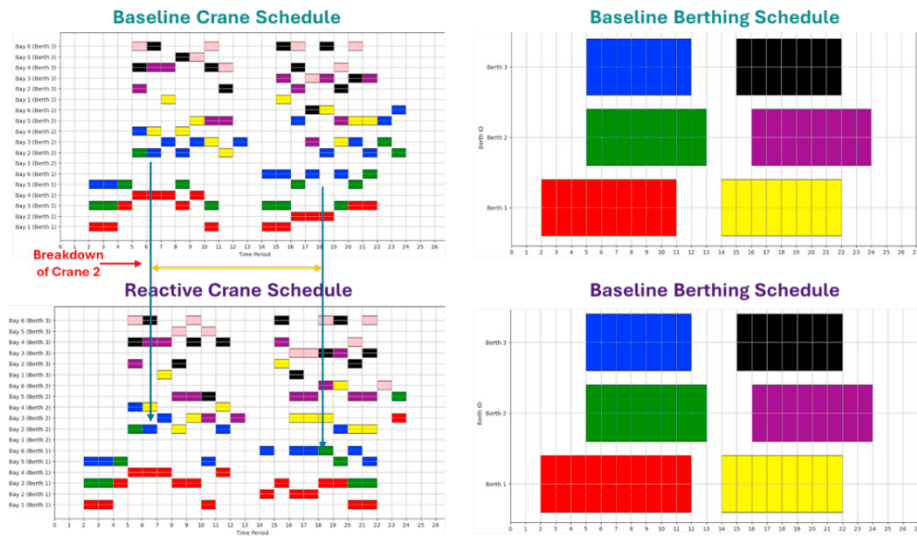


Figure 3: Crane breakdown

The results of the numerical experiments on small-sized problems ensure that the model is developed without inconsistencies and contradictions between variables and constraints. As specified, the safety margin is always maintained between adjacent QCs and the order of the QCs is maintained throughout the operation avoiding

overlaps. Furthermore, the proposed reactive strategy has displayed its efficiency in minimizing the deviations from the baseline berth schedule by only rescheduling QCs for the two types of uncertainties. However, the applicability of the reactive strategy for large-scale instances is yet to be analysed. This requires the combination of solution algorithms with the mathematical model to solve large-scale instances.

## 5. Conclusion

Optimization of vessel handling operations in container terminals under uncertainties by incorporating practical constraints is highlighted as a challenging problem that involves the integration of various planning functions and accurate representation of uncertainties and practical constraints. Our study mainly focuses on this aspect, and a methodology is proposed to integrate the operational problems under uncertain environments by applying a reactive approach. At first, a MILP model is formulated to assign and schedule QCs to multiple vessels simultaneously to minimize the cost of waiting time and departure delay of vessels. A baseline schedule can be generated from the model using deterministic parameters, and to mitigate the disruptions from uncertainties, a reactive strategy is incorporated into the model. The reactive model aims to minimize the deviations from the baseline berthing schedule by rescheduling QCs in real-time. Future studies may focus on employing factors such as unscheduled vessel arrivals and deviations in container handling times to the model, as well as the development of a solution algorithm to derive solutions for large-scale problems in reasonable computational times.

## Appendix A. Model formulation: indices, parameters and decision variables

### A.1. Indices

k	-	Index of vessels, $k = 1, \dots, K$
q	-	Index of QCs from left to right, $1 = 1, \dots, Q$
t	-	Index of time segments, $t = 1, \dots, T$
j	-	Index of bays, $j = 1, \dots, jj_k$

### A.2. Parameters

K	-	Total number of vessels
Q	-	Total number of QCs
T	-	Total number of time segments
$J_k$	-	Set of bays in vessel k
$jj_k$	-	Total number of bays in vessel k
$U_k$	-	Allocated berth for vessel k
$P_{kj}$	-	The number of time segments needed to complete the work on bay j of vessel k
$l_k$	-	Position of the first bay of vessel k along the quay
$ll_{kj}$	-	Position of bay j of vessel k along the quay
G	-	Total length of the quay
$a_k$	-	Expected time of arrival of vessel k
$d_k$	-	Requested departure time of vessel k
$C1_k$	-	Cost of berthing after arrival time per unit time for vessel k
$C2_k$	-	Cost of completing the handling after the requested departure time per unit time for vessel k
M	-	A large number

### A.3. Decision Variables:

$X_{kjqt}$	Binary, equal to 1, crane $q$ is assigned to bay $j$ of vessel $k$ at time $t$ , and 0 otherwise
$V_{kjt}$	Binary, equal to 1, if the handling in the bay $j$ of vessel $k$ is completed at time $t$ , and 0 otherwise
$R_{kt}$	Binary, equal to 1, if all the work on vessel $k$ is finished at time $t$ , and 0 otherwise
$Y_{kt}$	Binary, equal to 1, if vessel $k$ is berthed in its berth at time $t$ , and 0 otherwise
$S_{kw}$	Binary, equal to 1, if vessel $k$ is arriving at the port before vessel $w$ , and 0 otherwise
$B_{kt}$	Binary, equal to 1, if the mooring time of the vessel $k$ is at time $t$ , and 0 otherwise
$F_k$	Integer, Indicate the finishing time of work on the vessel $k$
$D_k$	Integer, Indicate the berthing time of the vessel $k$
$lc_{qt}$	Integer, Indicate the location of crane $q$ at time $t$

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